Introduction to Vectors

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1 Intro to vectors

$$cv + dw = c \begin{bmatrix} 1 \\ 1 \end{bmatrix} + d \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} c + 2d \\ c + 3d \end{bmatrix} \tag{1.0}$$

1.1 Linear combinations

- A linear combination of $c = 1$ & $d = 1$ for the equation 1 will produce the following output:

$$v + w = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \tag{1.1}$$

- $\therefore$ a linear combination is the addition of vectors and multiplying their scalars

1.2 2-Dimensional Vectors

$$\textbf{Column Vector} \qquad v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{1.2.0}$$

- A column vector (2-dimensional) has 2 seperate numbers v_1 and v_2 which produces a 2-dimensional vector v
- Although v_1 and v_2 cannot be added, we can add other vectors together

$$\textbf{Vector Addition} \qquad v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \text{ and } w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \text{ add to } v + w = \begin{bmatrix} v_1 + w_1 \\ v_2 + w_2 \end{bmatrix} \tag{1.2.1}$$

- Subtraction follows the same idea, such that $v - w$ is $v_1 - w_1$ and $v_2 - w_2$
- Scalar multiplication can be represented as:

$$\textbf{Scalar Multiplication} \qquad 2v = \begin{bmatrix} 2v_1 \\ 2v_2 \end{bmatrix} = v + v \text{ and } -v = \begin{bmatrix} -v_1 \\ -v_2 \end{bmatrix} \tag{1.2.2}$$

- In the above equation, 2 is the scalar, c , of the vector v
- The sum of $-v$ and v is the zero vector, 0 and has components 0 and 0

Thus, the sum of cv and wd is the <u>linear combination</u>, $cv + dw$.

1.3 Special linear combinations

There are 4 special linear combinations, sum, difference, zero and a scalar multiple cv:

- $1v + 1w =$ sum of vectors
- $1v - 1w =$ difference in vectors
- $0v - 0w =$ *zero vector*
- $cv - 0w =$ vector cv in the direction of v

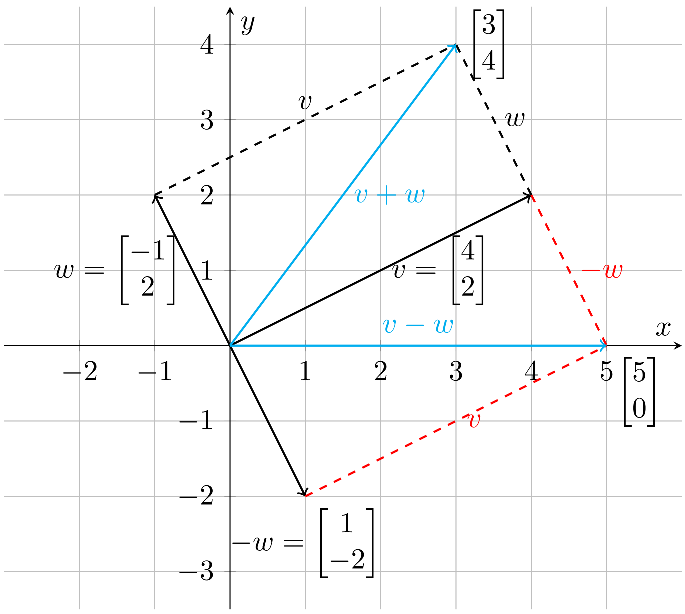


Figure 1: Vector addition and difference

Vector addition and difference is visually represented above, and it highlights its relationship.

- When calculating $v + w$, we place the start of the w vector at the end of v to arrive at the vector $(3, 4)$
- We may also place the start of v at the end of w and arrive at the same vector of $(3, 4)$
  - $\therefore$ linear combination $w + v$ is the same as $v + w$
- However, $v - w$ is calculated such that it is $v + (-w)$, and the vector w is inverted
  - We then treat it as addition, adding the vector $-w$ from the head of vector v
  - Or adding vector v to the head of $-w$
- To summarise, vector addition/difference places the tail of the consequent vector to the head of the preceding vector

1.4 3 Dimensional Vectors

$$v = \begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix} \text{ where } \begin{bmatrix}x\\y\\z\end{bmatrix} \text{ corresponds to points } (x, y, z) \tag{1.2.3}$$

3-dimensional vectors follow the same priniciples as 2-dimensional vectors in the way linear combinations are calculated.

1.4.1 Row Vector

A row vector is the transpose of the column vector and vice versa:

$$v_{col} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \text{ is the transpose of } v_{row} v_1 = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}$$

$\therefore v_{col} = v_{row}^T$ and $v_{col}^T = v_{row}$

1.5 All combinations in linear equations

Key points:

- All combinations of a single vector cu fill a line through $(0, 0, 0)$
- All combinations of 2 vectors $cu + dv$ fill a plane through $(0, 0, 0)$
- All combinations of 3 vectors $cu + dv + ew$ fill the ***three-dimensional space***

1.6 Lengths and dot product

2 Intro to vectors Problem Set

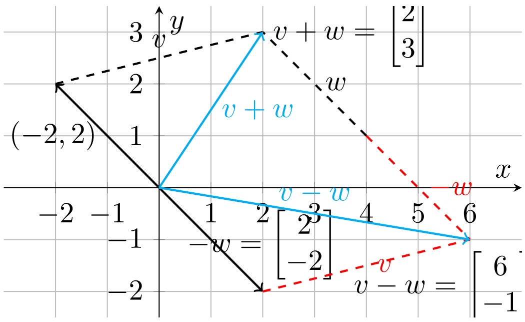
2.1 Problem Set 1.1

1 Geometrical Description.

(a) A line in R^3 (since vector $(3,6,9)$ is a scalar multiple of $(1,2,3)$) (b) A plane in R^3 (c) All in R^3

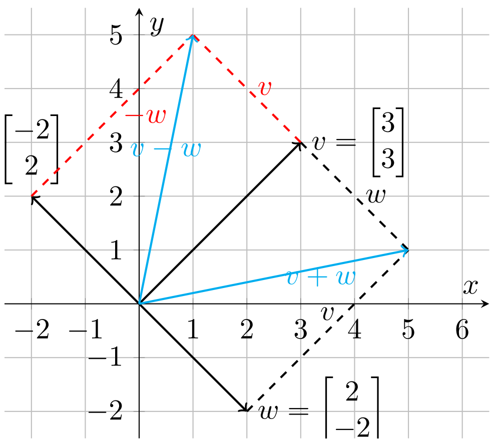
Planes and filling R^3 occur when the vectors are linearly independant.

2 Draw $v = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ and $w = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ and $v + w$ and $v - w$.



3 If $v + w = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ and $v - w = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$, compute and draw vectors v and w.

Working:



$$
\begin{aligned}
v + w - (v - w) &= w + w \\
&= 2w \\
2w &= \begin{bmatrix} 5 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 5 \end{bmatrix} \\
&= \begin{bmatrix} 4 \\ -4 \end{bmatrix} \\
w &= \begin{bmatrix} 2 \\ -2 \end{bmatrix} \\
v + w - w &= \begin{bmatrix} 5 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 \\ -2 \end{bmatrix} \\
v &= \begin{bmatrix} 3 \\ 3 \end{bmatrix}
\end{aligned}
$$