Introduction to Vectors

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Contents

1 Intro to vectors

$$cv + dw = c\begin{bmatrix}1\\1\end{bmatrix} + d\begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}c+2d\\c+3d\end{bmatrix} \tag{1.0}$$

1.1 Linear combinations

- A linear combination of $c = 1$ & $d = 1$ for the equation 1 will produce the following output:

$$v + w = \begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}3\\4\end{bmatrix} \tag{1.1}$$

- $\therefore$ a linear combination is the addition of vectors and multiplying their scalars

1.2 2-Dimensional Vectors

$$\textbf{Column Vector} \qquad v = \begin{bmatrix}v_1\\v_2\end{bmatrix} \tag{1.2.0}$$

- A column vector (2-dimensional) has 2 seperate numbers v_1 and v_2 which produces a 2-dimensional vector v
- Although v_1 and v_2 cannot be added, we can add other vectors together

$$\textbf{Vector Addition} \qquad v = \begin{bmatrix}v_1\\v_2\end{bmatrix} \text{ and } w = \begin{bmatrix}w_1\\w_2\end{bmatrix} \text{ add to } v + w = \begin{bmatrix}v_1+w_1\\v_2+w_2\end{bmatrix} \tag{1.2.1}$$

- Subtraction follows the same idea, such that $v - w$ is $v_1 - w_1$ and $v_2 - w_2$
- Scalar multiplication can be represented as:

$$\textbf{Scalar Multiplication} \qquad 2v = \begin{bmatrix}2v_1\\2v_2\end{bmatrix} = v + v \text{ and } -v = \begin{bmatrix}-v_1\\-v_2\end{bmatrix} \tag{1.2.2}$$

- In the above equation, 2 is the scalar, c , of the vector v
- The sum of $-v$ and v is the zero vector, 0 and has components 0 and 0

Thus, the sum of cv and wd is the <u>linear combination</u>, $cv + dw$.

1.3 Special linear combinations

There are 4 special linear combinations, sum, difference, zero and a scalar multiple cv:

- $1v + 1w =$ sum of vectors
- $1v - 1w =$ difference in vectors
- $0v - 0w =$ *zero vector*
- $cv - 0w =$ vector cv in the direction of v

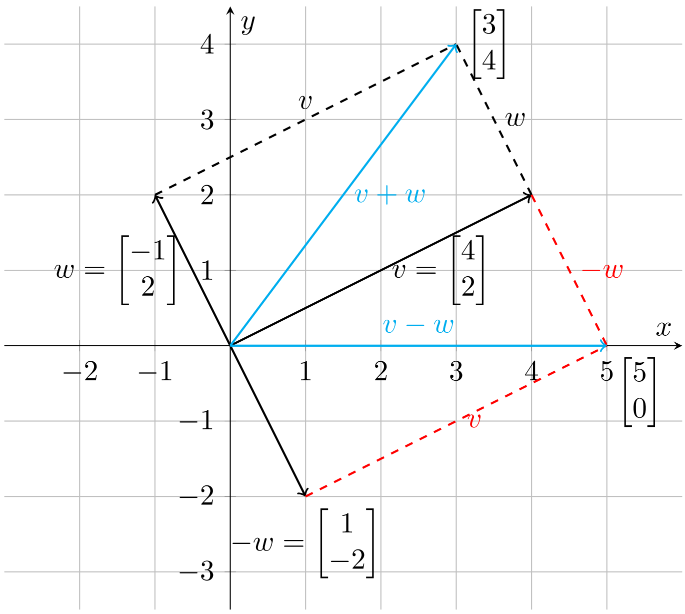

Figure 1: Vector addition and difference

Vector addition and difference is visually represented above, and it highlights its relationship.

- When calculating $v + w$, we place the start of the w vector at the end of v to arrive at the vector $(3, 4)$
- We may also place the start of v at the end of w and arrive at the same vector of $(3, 4)$
  - $\therefore$ linear combination $w + v$ is the same as $v + w$
- However, $v - w$ is calculated such that it is $v + (-w)$, and the vector w is inverted
  - We then treat it as addition, adding the vector $-w$ from the head of vector v
  - Or adding vector v to the head of $-w$
- To summarise, vector addition/difference places the tail of the consequent vector to the head of the preceding vector

1.4 3 Dimensional Vectors

$$v = \begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix} \text{ where } \begin{bmatrix}x\\y\\z\end{bmatrix} \text{ corresponds to points } (x, y, z) \tag{1.2.3}$$

3-dimensional vectors follow the same priniciples as 2-dimensional vectors in the way linear combinations are calculated.

1.4.1 Row Vector

A row vector is the transpose of the column vector and vice versa:

$$v_{col} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \text{ is the transpose of } v_{row} v_1 = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}$$

$\therefore v_{col} = v_{row}^T$ and $v_{col}^T = v_{row}$

1.5 All combinations in linear equations

Key points:

- All combinations of a single vector cu fill a line through $(0, 0, 0)$
- All combinations of 2 vectors $cu + dv$ fill a plane through $(0, 0, 0)$
- All combinations of 3 vectors $cu + dv + ew$ fill the ***three-dimensional space***