

Introduction to Vectors

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1 Intro to vectors

$$cv + dw = c \begin{bmatrix} 1 \\ 1 \end{bmatrix} + d \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} c + 2d \\ c + 3d \end{bmatrix} \quad (1.0)$$

1.1 Linear combinations

- A linear combination of $c = 1$ & $d = 1$ for the equation 1 will produce the following output:

$$v + w = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \quad (1.1)$$

- \therefore a linear combination is the addition of vectors and multiplying their scalars

1.2 2-Dimensional Vectors

$$\text{Column Vector} \quad v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (1.2.0)$$

- A column vector (2-dimensional) has 2 separate numbers v_1 and v_2 which produces a 2-dimensional vector v
- Although v_1 and v_2 cannot be added, we can add other vectors together

$$\text{Vector Addition} \quad v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \text{ and } w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \text{ add to } v + w = \begin{bmatrix} v_1 + w_1 \\ v_2 + w_2 \end{bmatrix} \quad (1.2.1)$$

- Subtraction follows the same idea, such that $v - w$ is $v_1 - w_1$ and $v_2 - w_2$
- Scalar multiplication can be represented as:

$$\text{Scalar Multiplication} \quad 2v = \begin{bmatrix} 2v_1 \\ 2v_2 \end{bmatrix} = v + v \text{ and } -v = \begin{bmatrix} -v_1 \\ -v_2 \end{bmatrix} \quad (1.2.2)$$

- In the above equation, 2 is the scalar, c , of the vector v
- The sum of $-v$ and v is the zero vector, 0 and has components 0 and 0

Thus, the sum of cv and wd is the linear combination, $cv + dw$.

1.3 Special linear combinations

There are 4 special linear combinations, sum, difference, zero and a scalar multiple cv :

- $1v + 1w = \text{sum of vectors}$
- $1v - 1w = \text{difference in vectors}$
- $0v - 0w = \text{zero vector}$
- $cv - 0w = \text{vector } cv \text{ in the direction of } v$

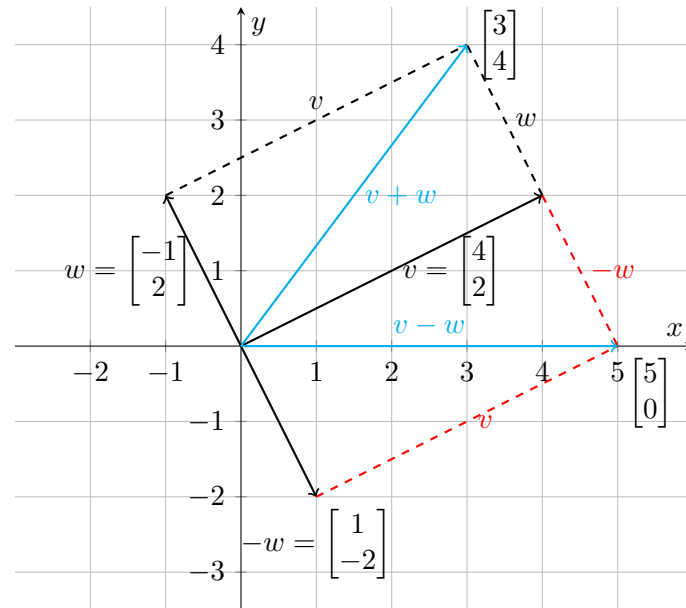


Figure 1: Vector addition and difference

Vector addition and difference is visually represented above, and it highlights its relationship.

- When calculating $v + w$, we place the start of the w vector at the end of v to arrive at the vector $(3, 4)$
- We may also place the start of v at the end of w and arrive at the same vector of $(3, 4)$
 - \therefore linear combination $w + v$ is the same as $v + w$
- However, $v - w$ is calculated such that it is $v + (-w)$, and the vector w is inverted
 - We then treat it as addition, adding the vector $-w$ from the head of vector v
 - Or adding vector v to the head of $-w$
- To summarise, vector addition/difference places the tail of the consequent vector to the head of the preceding vector

1.4 3 Dimensional Vectors

$$v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \text{ where } \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ corresponds to points } (x, y, z) \quad (1.2.3)$$

3-dimensional vectors follow the same principles as 2-dimensional vectors in the way linear combinations are calculated.

1.4.1 Row Vector

A row vector is the transpose of the column vector and vice versa:

$$v_{col} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \text{ is the transpose of } v_{row} v_1 = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}$$
$$\therefore v_{col} = v_{row}^T \text{ and } v_{col}^T = v_{row}$$

1.5 All combinations in linear equations

Key points:

- All combinations of a single vector cu fill a line through $(0, 0, 0)$
- All combinations of 2 vectors $cu + dv$ fill a plane through $(0, 0, 0)$
- All combinations of 3 vectors $cu + dv + ew$ fill the *three-dimensional space*

2 Intro to vectors Problem Set

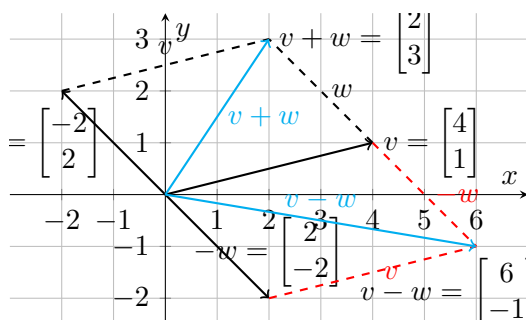
2.1 Problem Set 1.1

1 Geometrical Description.

(a) A line in R^3 (since vector $(3, 6, 9)$ is a scalar multiple of $(1, 2, 3)$) (b) A plane in R^3 (c) All in R^3

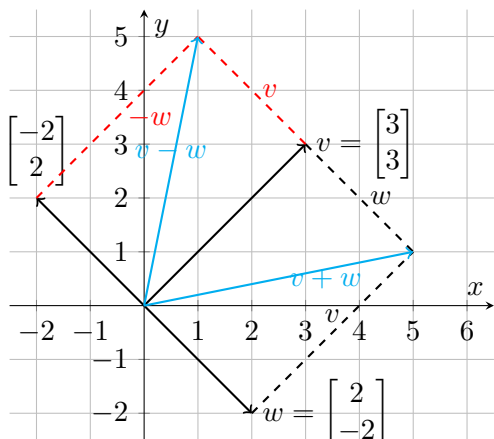
Planes and filling R^3 occur when the vectors are linearly independent.

2 Draw $v = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ and $w = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ and $v + w$ and $v - w$.



3 If $v + w = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ and $v - w = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$, compute and draw vectors v and w .

Working:



$$v + w - (v - w) = w + w$$

$$= 2w$$

$$2w = \begin{bmatrix} 5 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ -4 \end{bmatrix}$$

$$w = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$v + w - w = \begin{bmatrix} 5 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$v = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$