

Introduction to Vectors

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Contents

1	Intro to vectors	1
1.1	Linear combinations	1
1.2	2-Dimensional Vectors	1
1.2.1	Special linear combinations	2

1 Intro to vectors

$$cv + dw = c \begin{bmatrix} 1 \\ 1 \end{bmatrix} + d \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} c + 2d \\ c + 3d \end{bmatrix} \quad (1.0)$$

1.1 Linear combinations

- A linear combination of $c = 1$ & $d = 1$ for the equation 1 will produce the following output:

$$v + w = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \quad (1.1)$$

- \therefore a linear combination is the addition of vectors and multiplying their scalars

1.2 2-Dimensional Vectors

$$\text{Column Vector} \quad v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (1.2.0)$$

- A column vector has 2 separate numbers v_1 and v_2 which produces a 2-dimensional vector v

- Although v_1 and v_2 cannot be added, we can add other vectors together

$$\text{Vector Addition} \quad v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \text{ and } w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \text{ add to } v + w = \begin{bmatrix} v_1 + w_1 \\ v_2 + w_2 \end{bmatrix} \quad (1.2.1)$$

- Subtraction follows the same idea, such that $v - w$ is $v_1 - w_1$ and $v_2 - w_2$
- Scalar multiplication can be represented as:

$$\text{Scalar Multiplication} \quad 2v = \begin{bmatrix} 2v_1 \\ 2v_2 \end{bmatrix} = v + v \text{ and } -v = \begin{bmatrix} -v_1 \\ -v_2 \end{bmatrix} \quad (1.2.2)$$

- In the above equation, 2 is the scalar, c , of the vector v
- The sum of $-v$ and v is the zero vector, 0 and has components 0 and 0

Thus, the sum of cv and wd is the linear combination, $cv + dw$.

1.2.1 Special linear combinations

There are 4 special linear combinations, sum, difference, zero and a scalar multiple cv :

- $1v + 1w = \text{sum of vectors}$
- $1v - 1w = \text{difference in vectors}$
- $0v - 0w = \text{zero vector}$
- $cv - 0w = \text{vector } cv \text{ in the direction of } v$

