CAT 1: Transient Analysis of RL, RC, and RLC Circuits

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Determine the dimensions of the core, the number of turns, the cross-section area of conductors in primary and secondary windings of a 100 kVA, 2200/480 V, 1-phase, core type transformer, to operate at a frequency of 50Hz, by assuming the following data. Approximate Volt per turn = 7.5 Volt. Maximum flux density = 1.2 Wb/m^2. Ratio of effective cross-sectional area of core to square of diameter of circumscribing circle is 0.6. Ratio of height to width of window is 2. Window space factor = 0.28. Current density = 2.5 A/mm^2.

Given Data

- 100 kVA, 50 Hz
- 2200/480 V
- $E_t = 7.5$ V
- $K_w = 0.28$
- $A_e/d^2 = 0.6$
- $B_m = 1.2$ Wb/m^2
- $H/W = 2$
- Single-phase
- Current density, $\delta = 2.5$ A/mm^2

Step 1: Calculate the number of turns in primary and secondary windings

Number of primary turns (N_1)

$$N_1 = \frac{V_1}{E_t} = \frac{2200}{7.5} = 293.33 \approx 294 \text{ turns}$$

Number of secondary turns (N_2)

$$N_2 = \frac{V_2}{E_t} = \frac{480}{7.5} = 64 \text{ turns}$$

Step 2: Calculate the maximum value of flux (Φ_m)

From the equation:

$$E_t = 4.44 \times f \times \Phi_m$$

$$\Phi_m = \frac{E_t}{4.44 \times f} = \frac{7.5}{4.44 \times 50} = 0.0338 \text{ Wb}$$

Step 3: Calculate the cross-sectional area of the core (A_e)

Since

$$\Phi_m = B_m \times A_e$$

$$A_e = \frac{\Phi_m}{B_m} = \frac{0.0338}{1.2} = 0.0282 \text{ m}^2$$

Step 4: Calculate the diameter of the circumscribing circle (d)

Given that

$$\frac{A_e}{d^2} = 0.6$$

$$d^2 = \frac{A_e}{0.6} = \frac{0.0282}{0.6} = 0.047 \text{ m}^2$$

$$d = \sqrt{0.047} = 0.2168 \text{ m} = 216.8 \text{ mm}$$

Step 5: Calculate the primary and secondary currents

Primary current (I_1)

$$I_1 = \frac{\text{kVA} \times 1000}{V_1} = \frac{100 \times 1000}{2200} = 45.45 \text{ A}$$

Secondary current (I_2)

$$I_2 = \frac{\text{kVA} \times 1000}{V_2} = \frac{100 \times 1000}{480} = 208.33 \text{ A}$$

Step 6: Calculate the cross-sectional area of primary and secondary conductors

Primary conductor area (a_1)

$$a_1 = \frac{I_1}{\delta} = \frac{45.45}{2.5} = 18.18 \text{ mm}^2$$

Secondary conductor area (a_2)

$$a_2 = \frac{I_2}{\delta} = \frac{208.33}{2.5} = 83.33 \text{ mm}^2$$

Step 7: Calculate the window dimensions

Total copper area in the window (A_{cu})

$$A_{cu} = (N_1 \times a_1 + N_2 \times a_2)$$

$$A_{cu} = (294 \times 18.18 + 64 \times 83.33) = 5344.9 + 5333.1 = 10678 \text{ mm}^2$$

Window area (A_w)

$$A_w = \frac{A_{cu}}{K} = \frac{10678}{0.28} = 38135.7 \text{ mm}^2 = 0.0381 \text{ m}^2$$

Window dimensions

Given $H/W = 2$, where H is the height and W is the width of the window:

$$A_w = H \times W = 2W \times W = 2W^2$$

$$2W^2 = 0.0381$$

$$W^2 = 0.01905$$

$$W = 0.138 \text{ m} = 138 \text{ mm}$$

Height of window (H)

$$H = 2W = 2 \times 138 = 276 \text{ mm}$$

Summary of Results

Core dimensions

- Cross-sectional area (A_e) = 0.0282 m^2
- Diameter of circumscribing circle (d) = 216.8 mm

Number of turns

- Primary winding (N_1) = 294 turns
- Secondary winding (N_2) = 64 turns

Cross-sectional area of conductors

- Primary winding (a_1) = 18.18 mm^2
- Secondary winding (a_2) = 83.33 mm^2

Window dimensions

- Width (W) = 138 mm
- Height (H) = 276 mm

Example 2

Calculate the dimension of the core, the number of turns, and the cross-sectional area of conductors in the primary and secondary windings of a 100 kVA, 2300/400V, 50Hz, 1-phase, shell-type transformer. The given data is as follows:

Given Data

- Apparent power: 100 kVA
- Voltage: 2300/400 V
- Frequency: 50 Hz
- Transformer type: Single-phase, shell-type
- Ratio of magnetic and electric loading: 480×10^{-8}
- Maximum flux density: $B_m = 1.1$ Wb/m^2
- Current density: $\delta = 2.2$ A/mm^2
- Window space factor: $K_w = 0.3$
- Stacking factor: $S_f = 0.9$
- Depth of core / Width of central limb = 2.6
- Height of window / Width of window = 2.5

Step 1: Core Area Calculation

The core area A_c is calculated using the formula:

$$A_c = \frac{S}{4.44 \times f \times B_m \times N} \tag{1}$$

Given:

$$\begin{aligned} S &= 100,000 \text{ VA} \\ f &= 50 \text{ Hz} \\ B_m &= 1.1 \text{ Wb/m}^2 \\ N &= 480 \times 10^3 \end{aligned}$$

$$A_c = \frac{100,000}{4.44 \times 50 \times 1.1 \times 480,000} = \frac{100,000}{117,216,000} \approx 0.0423 \text{ m}^2 \tag{2}$$

Step 2: Core Cross-Section Dimensions

Using the stacking factor (0.9):

$$
\begin{aligned}
\text{Core Width} &= \frac{A_c}{d} = \frac{0.0423}{2.6} \approx 0.1533 \text{ m} \\
\text{Core Depth} &= d = 0.39859 \text{ m}
\end{aligned}
$$

Step 3: Window Area Calculation

$$A_w = H_w \times W_w = 2.5 \times 0.1083 = 0.2708 \text{ m}^2 \tag{3}$$

Step 4: Number of Turns (Primary and Secondary)

$$T_p = \frac{V_1}{4.44 \times f \times B_m \times A_c} = \frac{2300}{4.44 \times 50 \times 1.1 \times 0.0423} = \frac{2300}{103.1583} \approx 223 \text{ turns} \tag{4}$$

Similarly, for the secondary:

$$T_s = \frac{400}{4.44 \times 50 \times 1.1 \times 0.0423} = \frac{400}{103.1583} \approx 39 \text{ turns} \tag{5}$$

Step 5: Cross-sectional Area of Conductors

Using current density:

$$A_g = \frac{I}{k} \tag{6}$$

For primary current:

$$
\begin{aligned}
I_p &= \frac{100,000}{2300} \approx 43.48 \text{ A} \\
A_g &= \frac{43.48}{2.2} \approx 19.76 \text{ mm}^2
\end{aligned}
$$

For secondary current:

$$
\begin{aligned}
I_s &= \frac{100,000}{400} = 250 \text{ A} \\
A_s &= \frac{250}{2.2} \approx 113.64 \text{ mm}^2
\end{aligned}
$$