Antenna and Path Loss Calculations

Question 1

(a) Find the far-field distance for an antenna with a maximum dimension of 1 m and an operating frequency of 900 MHz.

(b) If the transmitter produces 50 Watts of power, express the transmit power in units of:

  (i) dBm

  (ii) dBW

(c) If 50 Watts is applied to a unity gain antenna with a 900 MHz carrier frequency, find the received power in dBm at a free space distance of 100 m from the antenna. What is $P_r(10\,\text{Km})$? Assume unity gain for the receiver antenna.

Solution

(a) Find the far-field distance for an antenna with a maximum dimension of 1 m and an operating frequency of 900 MHz.

The far-field distance (d_{far}) is given by:

$$d_{\text{far}} = \frac{2D^2}{\lambda}$$

Where:

- $D = 1\,\text{m}$ (maximum antenna dimension)
- $\lambda = \frac{c}{f} = \frac{3\times10^8\ \text{m/s}}{900\times10^6\ \text{Hz}} \approx 0.333\,\text{m}$ (wavelength at 900 MHz)

Thus, the far-field distance is:

$$d_{\text{far}} = \frac{2\times(1)^2}{0.333} \approx 6\,\text{m}$$

Therefore, the far-field distance is approximately **6 meters**.

(b) If the transmitter produces 50 Watts of power, express the transmit power in units of:

  (i) dBm

  The formula to convert Watts to dBm is:

$$P_{\text{dBm}} = 10\log_{10}(P_{\text{W}}\times 1000)$$

  For $P_{\text{W}} = 50\,\text{Watts}$:

$$P_{\text{dBm}} = 10\log_{10}(50\times 1000) \approx 47\,\text{dBm}$$

(ii) dBW

The formula to convert Watts to dBW is:

$$P_{\text{dBW}} = 10\log_{10}(P_{\text{W}})$$

For $P_{\text{W}} = 50$ Watts:

$$P_{\text{dBW}} = 10\log_{10}(50) \approx 17\,\text{dBW}$$

(c) If 50 Watts is applied to a unity gain antenna with a 900 MHz carrier frequency, find the received power in dBm at a free space distance of 100 m from the antenna. What is $P_r(10\,\text{Km})$? Assume unity gain for the receiver antenna.

The received power in free space is given by the Friis transmission equation:

$$P_r(d) = P_t G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2$$

For a 900 MHz carrier frequency:

- **At $d = 100\,\text{m}$:**

$$P_r(100) = 50 \times 1 \times 1 \left(\frac{0.333}{4\pi \times 100}\right)^2$$

$$P_r(100) \approx 3.515 \times 10^{-6}\,\text{Watts}$$

Now convert the received power to dBm:

$$P_r(100)\,(\text{dBm}) = 10\log_{10}(3.515 \times 10^{-6} \times 1000) \approx -24.54\,\text{dBm}$$

- **At $d = 10\,\text{km}$:**

$$P_r(10,000) = 50 \times 1 \times 1 \left(\frac{0.333}{4\pi \times 10,000}\right)^2$$

$$P_r(10,000) \approx 3.515 \times 10^{-10}\,\text{Watts}$$

Now convert the received power to dBm:

$$P_r(10,000)\,(\text{dBm}) = 10\log_{10}(3.515 \times 10^{-10} \times 1000) \approx -64.54\,\text{dBm}$$

Summary:

- The far-field distance is approximately **6 meters**.
- The transmit power is approximately **47 dBm** and **17dBW**.
- The received power at 100 m is approximately **-24.54 dBm**.
- The received power at 10 km is approximately **-64.54 dBm**.

Question 2

(a) Suppose the standard deviation of power due to shadowing is equal to 8 dB. What is the probability that the path loss will exceed the mean path loss by at least 5 dB?

(b) What is the probability that the path loss will exceed the mean path loss by at least 10 dB?

Solution

In log-normal shadowing, the path loss variations (in dB) follow a normal distribution with:

- Mean (μ) = 0 dB (relative to mean path loss)
- Standard deviation (σ) = 8 dB

For exceeding mean path loss by at least 5 dB:
We need $P(X \geq 5)$, where X is normally distributed.
Using the standard normal distribution, we need:

$$Z = \frac{X - \mu}{\sigma} = \frac{5 - 0}{8} = 0.625$$

$$P(X \geq 5) = P(Z \geq 0.625)$$

$$P(Z \geq 0.625) = 1 - P(Z \leq 0.625)$$

From the standard normal table:

$$P(Z \leq 0.625) \approx 0.734$$

Therefore:

$$P(X \geq 5) = 1 - 0.734 = 0.266 \text{ or } 26.6\%$$

For exceeding mean path loss by at least 10 dB:

$$Z = \frac{10 - 0}{8} = 1.25$$

$$P(X \geq 10) = P(Z \geq 1.25)$$

$$P(Z \geq 1.25) = 1 - P(Z \leq 1.25)$$

From the standard normal table:

$$P(Z \leq 1.25) \approx 0.894$$

Therefore:

$$P(X \geq 10) = 1 - 0.894 = 0.106 \text{ or } 10.6\%$$

Thus, the probabilities are as follows:

- The probability of exceeding mean path loss by at least 5 dB is 26.6%.
- The probability of exceeding mean path loss by at least 10 dB is 10.6%.