

## Find the expected value of $\ell(\boldsymbol{\pi})$ using the current expected value of the latent variable

The expected value of  $\ell(\boldsymbol{\pi})$ , with respect to the conditional distribution of  $\mathbf{z}$ , given observed data and  $\hat{\boldsymbol{\pi}}^{(t-1)}$  is

$$\mathbb{E}_{\boldsymbol{\pi}} \left[ \ell(\boldsymbol{\pi}) | \mathbf{x}, \mathbf{y}, \hat{\boldsymbol{\pi}}^{(t-1)} \right] = \sum_{i=1}^n \sum_{j=1}^m \mathbb{E}_{\boldsymbol{\pi}} [z_{ij} | x_i, y_i] \{ \log f_j(x_i, y_i) + \log \pi_j \}$$

Since  $z_{ij}$  is an indicator, its expected value is simply the probability that data point  $i$  comes from model  $j$

$$\begin{aligned} \mathbb{E}_{\boldsymbol{\pi}} [z_{ij} | x_i, y_i] &= \Pr_{\boldsymbol{\pi}}(z_{ij} | x_i, y_i) \\ &= \frac{p(x_i, y_i | z_{ij} = 1) p(z_{ij} = 1)}{p(x_i, y_i)} \\ &= \frac{\pi_j f_j(x_i, y_i)}{\sum_{j=1}^m \pi_j f_j(x_i, y_i)} \end{aligned}$$