Expectation Maximization

Suppose we knew which mixture component f_j each observation came from:

$$z_{ij} = \mathbf{1}(x_i, y_i \sim f_j) = \begin{cases} 1 & (x_i, y_i) \sim f_j \\ 0 & \text{otherwise} \end{cases}$$

The log likelihood can then be expressed as

$$\ell(\boldsymbol{\pi}) = \sum_{i=1}^{n} \sum_{j=1}^{m} z_{ij} \log \left\{ \pi_j f_j(x_i, y_i) \right\}$$

The addition of the latent variable $\mathbf{z}$ actually makes things easier because it is easily differentiable in $\boldsymbol{\pi}$.