A Modified Bayesian Filtering Framework for ECG Beat Segmentation

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***Abstract*—In this paper, we have presented a modified EKF structure based on the previously introduced signal decomposition based ECG Dynamic Model (EDM) for ECG beat segmentation. The new EKF can simultaneously estimate each of the ECG components including P, Q, R, S and T waveforms as well as the ECG signal. In this framework, instantaneous Gaussian functions of the P, Q, R, S and T components are considered as hidden state variables that are distinctly estimated from sample to sample. The result have shown that each of the CWs have been accurately estimated from multiple ECG beats. The proposed EDM can also be useful for synthetic ECG generation and ECG denoising applications.**

***Keywords*—ECG Dynamic Model, ECG delineation, Extended Kalman filtering, ECG Beat Segmentation**

I. Introduction

ECG Dynamic Model (EDM) was first introduced by McSharry *et al.*[1] using a set of nonlinear state space equations in Cartesian coordinate system. This model has been used for generating rather realistic synthetic electrocardiogram signals in many normal and abnormal cases [2]. McSharry model was modified by Sameni *et al.* [2] by reducing the number of state variables using polar coordinate system. Thereafter, application of the modified ECG dynamical model has been studied in conjunction with Bayesian filtering framework such as Extended Kaman Filter (EKF), for modeling fetal ECG [3], ECG denoising [2], removing cardiac contaminants [4], generating multi-channel ECG as well as simulating of abnormal rhythms[5], [6]. However, applications of the so-called modified EDM in EKF filter for ECG denoising are restricted to the whole ECG beat and their corresponding phases [7]. While extracting the physiological components such as P wave, QRS complex and T wave is of great importance in some clinical applications. To overcome to this limitation, Sayadi *et al.* introduced a modified EKF structure namely as EKF17, which exploits α, b and θ parameters of Gaussian kernels, corresponding to P, Q, R, S and T waves, as hidden state variables [8], [7], [9]. Although this model has the capability to reach intra beat components and it is shown to work well in ECG beat segmentation, but 17 state variables may impose high complexity to the model.

In this paper, we utilize the signal decomposition based EDM, previously introduced in [10], with the Extended Kalman Filter and investigate it's application in extraction of P, Q, R, S and T components from the noisy input ECG observation. The proposed method has the benefit that simultaneously estimates ECG components while reducing the number of state variables. The results show good performance of the proposed method for estimating the ECG components.

The rest of the paper is organized as follows: in section II, a brief review of some of the common ECG dynamic models is presented. In section III, the previously developed signal decomposition based EDM and it's application in the EKF structure is described. In section IV, the results of the proposed method for P, Q, R, S and T estimation are presented. Finally, a brief summary and some general remarks are presented in the section V.

II. Background

A. ECG Dynamic Model

The well-known McSharry *et al.* ECG dynamical model [1] consists of the set of nonlinear state space equations in Cartesian coordinate system as follows:

$$\begin{cases} \dot{x} = \rho x - \omega y \\ \dot{y} = \omega x + \rho y \\ \dot{z} = -\sum_{i\in\{\text{P,Q,R,S,T}\}} \alpha_i \Delta\theta_i \exp[-\dfrac{\Delta\theta_i^2}{2b_i^2}] - (z - z_0) \end{cases} \tag{1}$$

Where x, y and z are the state variables, $\rho = 1 - \sqrt{x^2 + y^2}$, $\Delta\theta_i = (\theta - \theta_i) mod 2\pi$, $\theta = \arctan(y, x)$ and ω is the angular velocity corresponding to the Heart Rate Variability (HRV). Accordingly, the state variable z models ECG as a summation of the Gaussian kernels. Where α_i, b_i and θ_i adapt amplitude, width and center of the Gaussian kernels to the different morphological ECG signals and z_0 models the base line wander [11].

The McSharry model has been simplified by Sameni *et al.* [2] using the polar coordinate system as follows:

$$\begin{cases} \theta_{k+1} = (\theta_k + \omega\sigma)\ mod\ 2\pi \\ z_{k+1} = z_k - \sum_{i\in\{\text{P,Q,R,S,T}\}} \delta \frac{\alpha_i \omega}{b_i^2} \Delta\theta_i \exp[-\dfrac{\Delta\theta_i^2}{2b_i^2}] + \eta \end{cases} \tag{2}$$

where θ is the *cardiac phase* defined as an explicit state variable that indicates the angular location of the P, Q, R, S and T components and models intra beat variations. z is the amplitude of the ECG signal, δ is the sampling period and η is a noise term corresponding to the inaccuracy of the model. The summation is taken over finite number of Gaussian signals used for modeling P, Q, R, S and T waves. The parameters α_i, b_i and θ_i are amplitude, width and center of the Gaussian kernels respectively.

Another modification of the EDM is proposed by Sayadi *et al.* [11] known as Wave-based ECG Dynamic Model (WEDM) in which each of the Characteristic Waveforms (CW), including P, QRS complex and T wave, are modeled distinctly to produce an ECG beat. The proposed methods of [2] and [11] in conjunction with Bayesian filtering framework have been used in denoising applications.

B. Kalman Filter

The well-known *Kalman Filter* (KF) and it's nonlinear extended version *Extended Kalman Filter* (EKF) are methods for estimating the hidden states of a system, having the dynamic model of the system and set of observations. Consider a system with hidden state vector $\mathbf{x}_k$ and observation vector $\mathbf{y}_k$ at time instant k has the following dynamic model:

$$\begin{cases} \mathbf{x}_{k+1} = f(\mathbf{x}_k, \mathbf{w}_k, k) \\ \mathbf{y}_k = g(\mathbf{x}_k, \mathbf{v}_k, k) \end{cases} \tag{3}$$

where $f(.)$ is the evolution state function, $g(.)$ represents the relationship between the hidden state and observations, $\mathbf{w}_k$ and $\mathbf{v}_k$ are process noise and measurement noise with the corresponding covariance matrices $\mathbf{Q}_k = E\{\mathbf{w}_k\mathbf{w}_k^T\}$ and $\mathbf{R}_k = E\{\mathbf{v}_k\mathbf{v}_k^T\}$ respectively.

To estimate the state vector $\mathbf{x}_k$ using EKF structure, a linear approximation of the (3) near a desired reference point $(\hat{\mathbf{x}}_k, \hat{\mathbf{w}}_k, \hat{\mathbf{v}}_k)$ should be calculated as follows [8], [2], [12]:

$$\begin{cases} \mathbf{x}_{k+1} = f_k(\mathbf{x}_k, \mathbf{w}_k, k) \\ \approx f_k(\mathbf{x}_k, \mathbf{w}_k, k) + A_k(\mathbf{x}_k - \hat{\mathbf{x}}_k) + F_k(\mathbf{w}_k - \hat{\mathbf{w}}_k) \\ \mathbf{y}_k = g(\mathbf{x}_k, \mathbf{v}_k, k) \\ \approx g(\hat{\mathbf{x}}_k, \hat{\mathbf{v}}_k, k) + C_k(\mathbf{x}_k - \hat{\mathbf{x}}_k) + G_k(\mathbf{v}_k - \hat{\mathbf{v}}_k) \end{cases} \tag{4}$$

where

$$\begin{cases} A_k = \dfrac{\partial f(\mathbf{x}_k, \hat{\mathbf{w}}_k, k)}{\partial \mathbf{x}_k}_{|\mathbf{x}=\hat{\mathbf{x}}_k} \\ F_k = \dfrac{\partial f(\hat{\mathbf{x}}_k, \mathbf{w}, k)}{\partial \mathbf{w}}_{|\mathbf{w}=\hat{w}_k} \\ C_k = \dfrac{\partial g(\mathbf{x}_k, \hat{\mathbf{v}}_k, k)}{\partial \mathbf{x}_k}_{|\mathbf{x}=\hat{\mathbf{x}}_k} \\ G_k = \dfrac{\partial g(\hat{\mathbf{x}}_k, \mathbf{v}, k)}{\partial \mathbf{v}}_{|\mathbf{v}=\hat{v}_k} \end{cases} \tag{5}$$

After linearization, state vector $\mathbf{x}$ can be estimated using the following time propagation and measurement propagation equations:

Time Propagation:

$$\begin{cases} \hat{\mathbf{x}}_{k+1}^- = f_k(\hat{\mathbf{x}}_k^+, \mathbf{w}_k, k)_{|\mathbf{w}=\mathbf{0}} \\ \mathbf{P}_{k+1}^- = \mathbf{A}_k\mathbf{P}_k^+\mathbf{A}_k^T + \mathbf{F}_k\mathbf{Q}_k\mathbf{F}_k^T \end{cases} \tag{6}$$

Measurement Propagation:

$$\begin{cases} \hat{\mathbf{x}}_k^+ = \mathbf{x}_k^- + \mathbf{K}_k\left[\mathbf{y}_k - g(\hat{\mathbf{x}}_k^-, \mathbf{v}_k, k)_{|\mathbf{v}=\mathbf{0}}\right] \\ \mathbf{K}_k = \mathbf{P}_k^-\mathbf{C}_k^T\left(\mathbf{C}_k\mathbf{P}_k\mathbf{C}_k^T + \mathbf{G}_k\mathbf{R}_k\mathbf{G}_k^T\right)^{-1} \\ \mathbf{P}_k^+ = \mathbf{P}_k^- - \mathbf{K}_k\mathbf{C}_k\mathbf{P}_k^- \end{cases} \tag{7}$$

Where $\hat{\mathbf{x}}_k^- = \hat{\mathbf{E}}\{\mathbf{x}|\mathbf{y}_{k-1}, \cdots \mathbf{y}_1\}$ is an *a prior* estimate of $\mathbf{x}$ at time instant k having previous observations $\mathbf{y}_1$ to $\mathbf{y}_{k-1}$ and $\hat{\mathbf{x}}_k^+ = \hat{\mathbf{E}}\{\mathbf{x}|\mathbf{y}_k, \cdots \mathbf{y}_1\}$ is a *posterior* estimate that is obtained by correction of $\hat{\mathbf{x}}_k^+$ after observing $\mathbf{y}_k$. The matrices $\mathbf{P}^-$ and $\mathbf{P}^+$ are also an *a prior* and *posterior* state covariance matrices before and after observing $\mathbf{y}_k$.

III. Method

In a previous work [10], a general framework was proposed for morphological modeling of cardiac signals. In this work we aim to use the proposed models for ECG beat segmentation. As suggested in [10], dynamic representation of the signal can obtained by taking derivative from the basis functions of the signal model, instead of the signal itself.

$$\begin{cases} \dfrac{d}{dt}\theta(t) = \omega(t) \\ \dfrac{d}{dt}\phi_i(t) = -\omega(t)\left(\dfrac{\theta(t) - \theta_i}{b_i^2}\right)\phi_i(t) \\ z(t) = \displaystyle\sum_{i\in\{\mathrm{P,Q,R,S,T}\}} \alpha_i\phi_i(t) \end{cases} \tag{8}$$

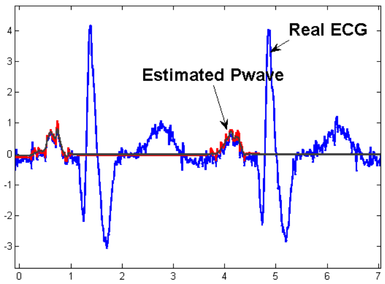
A vectorial and discrete form of the new EDM, with the assumption of the small sampling period δ, can be expressed as:

$$\begin{cases} \theta_{k+1} = (\theta_k + \omega\delta)mod(2\pi) \\ \Phi_{k+1} = \mathbf{H}_k\Phi_k + \mathbf{u} \\ z_k = \boldsymbol{\alpha}^T\Phi_k \end{cases} \tag{9}$$

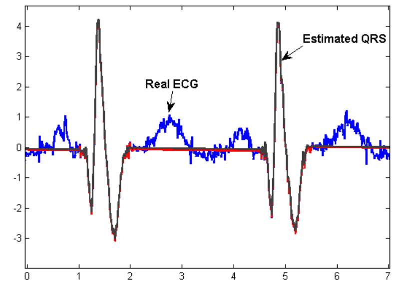
Where z_k is the amplitude of the ECG signal and θ_k is the cardiac phase at time instant k, $\Phi_k = [\phi_P[k], \phi_Q[k], \phi_R[k], \phi_S[k], \phi_T[k]]^T$ is a vector containing basis functions choosing as state variables and $\boldsymbol{\alpha}$ are a vector of their corresponding coefficients. $\mathbf{H}_k$ is a diagonal matrix with entries of $(1 - \omega\delta\dfrac{\theta_k - \theta_i}{b_i^2})$, $i \in \{P, Q, R, S, T\}$. The vector $\mathbf{u}$ is the noise term corresponding to the inaccuracy of the model. As it seems, the proposed EDM consists of six state variables containing Gaussian basis functions of P, Q, R, S and T components, and also θ at each index k. Therefore, this model can simultaneously be used for medeling the ECG signal as well as its Characteristics Waveforms such that each of the ECG components can be obtained using signal expansion of their corresponding basis function and coefficient:

$$\begin{cases} P(t) = \alpha_P\phi_P(t) \\ Q(t) = \alpha_Q\phi_Q(t) \\ R(t) = \alpha_R\phi_R(t) \\ S(t) = \alpha_S\phi_S(t) \\ T(t) = \alpha_T\phi_T(t) \end{cases} \tag{10}$$

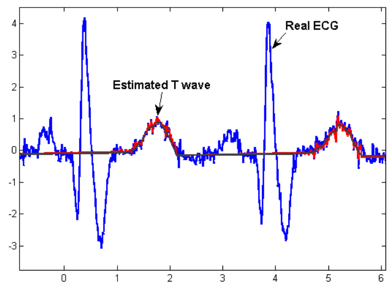
It is important to note that each of the basis functions is updated over time such that it is distinct from sample to sample. As a result, it models the shape of each ECG component with more than one Gaussian functions. Therefore it can accurately extract ECG components even in high noisy



(a) Estimated P wave



(b) Estimated QRS complex



(c) Estimated T wave

Fig. 1. Estimated P, QRS complex and T waves for a typical ECG signal

scenarios. This property leads to high accuracy of the model and estimated parameters.

The new EDM can be utilized in Bayesian framework for various applications such as synthetic ECG generation, denoising and P, Q, R, S and T waves detection using the following process and observation equations:

process equation:

$$\begin{cases} \theta_{k+1} = (\theta_k + \omega_k \delta) mod(2\pi) \\ \phi_{P_{k+1}} = (1 - \omega\delta \dfrac{\theta - \theta_{P_k}}{b_P^2}) \phi_{P_k} + u_P \\ \phi_{Q_{k+1}} = (1 - \omega\delta \dfrac{\theta - \theta_{Q_k}}{b_Q^2}) \phi_{Q_k} + u_Q \\ \phi_{R_{k+1}} = (1 - \omega\delta \dfrac{\theta - \theta_{R_k}}{b_R^2}) \phi_{R_k} + u_R \\ \phi_{S_{k+1}} = (1 - \omega\delta \dfrac{\theta - \theta_{S_k}}{b_S^2}) \phi_{S_k} + u_S \\ \phi_{T_{k+1}} = (1 - \omega\delta \dfrac{\theta - \theta_{T_k}}{b_T^2}) \phi_{T_k} + u_T \end{cases} \quad (11)$$

where θ and ϕ_i are state variables, $\omega, \theta_i, b_i, u_i$ are i.i.d Gaussian random variables considered to be process noise.

observation equation:

$$\begin{cases} \psi_k = \theta_k + v_{1_k} \\ s_k = \displaystyle\sum_{i \in \{\text{P,Q,R,S,T}\}} \alpha_i \phi_i(t) + v_{2_k} \end{cases} \quad (12)$$

Where s_k is the noisy observation and ψ_k is the noisy cardiac phase of ECG signal at time instant k. v_1 and v_2 are zero mean random variables considered to be observation noise. As a result, the state variables vector, $\mathbf{x}_k$, the observation vector, $\mathbf{y}_k$, the process noise vector, $\mathbf{w}_k$, and the observation noise vector, $\mathbf{v}_k$, are defined as follows:

$$\begin{aligned} \mathbf{x}_k &= [\theta_k, \phi_{P_k}, \phi_{Q_k}, \phi_{R_k}, \phi_{S_k}, \phi_{T_k}]^T \\ \mathbf{y}_k &= [\psi_k, s_k] \\ \mathbf{w}_k &= [b_i, \theta_i, u_i, \omega, \delta] \\ \mathbf{v}_k &= [v_{1_k}, v_{2_k}] \end{aligned} \quad (13)$$

with the corresponding process noise and measurement noise covariance matrices $\mathbf{Q}_k = E\{\mathbf{w}_k \mathbf{w}_k^T\}$ and $\mathbf{R}_k = E\{\mathbf{v}_k \mathbf{v}_k^T\}$ respectively.

IV. Application to ECG Beat Segmentation

The proposed method has been utilized for estimating P, Q, R, S and T components of the ECG signal. It has been implemented on $Matlab^{\Re}$ using a selected ECG with sampling rate of 1000 Hz from the MIT-BIH PTB Diagnostic ECG Database [13].

Before implementation of the algorithm, the EKF should be initialized with meaningful values for parameters, state variables and covariance matrices. For estimating the parameters α_i, b_i, and θ_i of the Gaussian kernels, the ensemble average of the ECG were extracted as a template of one ECG beat. Next, the parameters are estimated by applying a nonlinear least squares error algorithm to fit the ECG template, using open-source packages of OSET [14]. The other parameters and covariance matrices are initialized following the methods developed in [2], [12]. It is worth nothing that the value of the entries of the observation noise covariance matrix, $\mathbf{R}_k$, indicates the degree of reliability of each observation. Accordingly, setting the value of $\mathbf{R}_k$ to the suitable value according to our prior knowledge about the quality of the signal has high influence to achieve to the desired accuracy. For

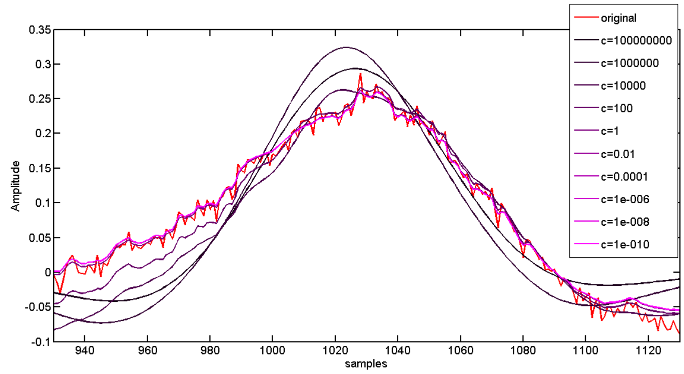


Fig. 2. Estimated T waves with respect to different values of observation covariance matrix $\mathbf{R}$ (the weight c_R)

example, if the data are too noisy or if there is no observation available the value of $\mathbf{R}_k$ should be selected high which means the Kalman filter mostly tried to track dynamics rather than relying on the observations. In similar manner, for precise observations the value of $\mathbf{R}_k$ should be selected low, so the algorithm would mostly relied on the measurements.

The results of the estimated ECG components are shown in (Fig.1). As it seems the proposed method can accurately estimates each of the P, QRS and T waveforms. Moreover, the accuracy in our estimation is significant in comparison to what obtained in [7] because of the time variant property of the basis function in our method.

V. Discussion and Conclusion

In this paper, we have proposed an efficient state space model based on Bayesian filtering for ECG beat segmentation. In the proposed method, the Gaussian basis functions of the ECG components, P, Q, R, S and T, are utilized as hidden state variables. These hidden-state variables are estimated as a time series through the EKF structure. Henceforth, the ECG components are estimated simultaneously using signal expansion over the estimated basis functions. The results demonstrate that applying the signal decomposition based EDM in EKF structure for ECG beat segmentation has the capability of tracking the ECG components. This model can also be useful for ECG denoising and synthetic ECG generation by choosing different filter parameters.

In summary, the signal decomposition based EKF structure has some advantages in comparison to the recent works in this context. As compared with the EKF structure in [2], that has used only two state variables θ and z, it uses six state variables with the advantage of reaching intra beat components or ECG beat segmentation. As compared with the EKF structure in [9], used for ECG beat segmentation, our method exploits from the benefit of more flexibility and less complexity in the estimation of the ECG components. In fact, the time series property of the basis functions resulted on the ECG components that is obtained by means of time variant Gaussian functions, so many basis functions rather than just N Gaussian for each beat. As a result the proposed method leads to much more better accuracy in estimation of the ECG components than methods that use only one time invariant Gaussian function for modeling each of the CWs. Moreover, exploiting of signal decomposition in EDM reduces seventeen state variables of [7], [9] to just six in our method that leads to complexity reduction.

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