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Relations of Lift and Drag Coefficients of Flow around Flat Plate

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Abstract. In this paper, when Reynolds number is within the range of 10000 to 1000000, the horizontal component of the total pressure of flow around flat plate at high angle of attack was regarded as lift of high angle of attack, and the vertical component was regarded as drag of high angle of attack. The horizontal component of total pressure at small angle of attack was regarded as shape drag, and the total drag coefficient at small angle of attack was considered to the sum of the shape drag and frictional drag at zero angle of attack. For the two states of large and small angle of attack, the application scopes of the formulas of lift and drag coefficients were given. Final, the relations of lift and drag coefficients were obtained by eliminating all angles of attack. Research results show that lift - drag curve of small angles of attack is parabola, and the lift - drag curve of high angles of attack is circle.

Introduction

Flat plate can be considered the simplest airfoil which camber and thickness are close to 0. Flat plate airfoil although rarely appear in the actual work, but it is more convenient for analytical calculation due to its simple structure. The lift and drag formulas of flow around a flat plate can be used to study the performance trends of the wind turbine. Lift and drag coefficient are the main properties of the airfoil, this article will explore the relationship of lift coefficient and drag coefficient.

Drag coefficient of flow around a flat plate at small angle of attack

A number of experimental data show that when the angle of attack is π/2 and Reynolds number is within the range of 10^4 to 10^6, the pressure drag coefficient of the two-dimensional flat plate perpendicular to the flow direction is 1.98 [1,2], 2.01 [3] or 2.06 [4]. By those data and mathematical analysis, we obtained the approximate formulas of total pressure coefficient and its horizontal component coefficient C_{ND} and the vertical component coefficient C_{NL} of flow around a flat plate at high angles of attack [5], in which the horizontal component coefficient C_{ND} can be called shape drag because it is closely related with the angle of attack α.

$$C_{ND} = 2\sin^2 \alpha \tag{1}$$

Equation (1) is applied to the flow around high angles of attack, and not suitable for flow around small angles of attack because the friction drag of the surface of the flat plate was not considered. When the angle of attack is 0, the shape drag is closed to 0, but the total drag (shape drag + friction drag) is not 0. So we need to consider the friction drag. In fact, in a small angle of attack state friction drag is always there, which can be considered constant. Let $2C_f$ represents friction drag coefficient at zero angle of attack, such the total drag coefficient C_D within the scope of small angles of attack is:

$$C_D = 2C_f + 2\sin^2 \alpha \tag{2}$$

Friction drag coefficient $2C_f$ can be calculated by the boundary layer theory in laminar or turbulent state. Absolute value of $2C_f$ is very small, usually 0.003 to 0.05, so it is difficult to directly observe from image of drag coefficient curve of flow at high angles of attack. Although the value of

$2C_f$ is small, it should not be overlooked because its value is nearly to the shape drag of small angle of attack.

Overview of lift and drag coefficient of flow around a flat plate at any angle of attack

The lift coefficient formula of flow around a flat plate at small angle of attack is [6]

$$C_L = 2\pi \sin\alpha \tag{3}$$

When Reynolds number is within the range of 10^4 to 10^6, the vertical component coefficient C_{NL} of the total pressure of flow around a flat plate at a high angle of attack was given by reference [1], which can be called lift coefficient at high angle of attack. The formula is

$$C_{NL} = \sin 2\alpha \tag{4}$$

The non-dimensional estimate equations of lift and drag coefficients of flow around a flat plate at any angle of attack were given in table 1.

Table 1 The equations of lift and drag coefficients of flow around a flat plate at any angle of attack

Equations	Scope of application
$C_L = 2\pi\sin\alpha$	estimate to lift coefficient at small angle of attack
$C_{NL} = \sin 2\alpha$	estimate to lift coefficient at high angle of attack
$C_{L\max} = 2\pi\sin\alpha, C_{L\min} = \sin 2\alpha$	estimate to lift coefficient at stall area
$C_D = 2C_f + 2\sin^2\alpha$	estimate to drag coefficient at small angle of attack
$C_{ND} = 2\sin^2\alpha$	estimate to drag coefficient at high angle of attack
$C_{D\max} = 2C_f + 2\sin^2\alpha, C_{D\min} = 2\sin^2\alpha$	estimate to drag coefficient at stall area

Since friction drag coefficient $2C_f$ is constant and can be measured easily at zero angle of attack, so all the lift and drag coefficients have their certain expressions, which is very conducive for differentiation, integration operations.

The relation of lift coefficient and drag coefficient

Lift and drag coefficients are the function of angle of attack α, and the angle of attack can be considered as reference variables, so the function relation of lift and drag coefficients can be obtained by elimination of the angle of attack.

The relation of lift coefficient and drag coefficient at small angles of attack

Eliminate the angle of attack from equation (2) and (3),

$$C_D = 2C_f + 2\sin^2\alpha = 2C_f + 2\left(\frac{C_L}{2\pi}\right)^2 \tag{5}$$

The formula may also be expressed in the following form

$$C_L^2 = 2\pi^2\left(C_D - 2C_f\right) \tag{6}$$

This equation shows that the lift - drag curve of small angle of attack is a parabola, which vertex at $(2C_f, 0)$, and the focus is on $(\pi^2/2, 0)$. Curve shape is shown in Fig. 1.

Lift - drag curve is usually referred to as the Eiffel polar curve [7].The lift - drag ratio has maximum value at the tangent intersection point of a straight line through the origin and the Eiffel polar curve, and the corresponding angle of attack may prove to be the best angle of attack [4].

The relation of lift coefficient and drag coefficient at high angles of attack

Eliminate the angle of attack from equation (1) and (4),

$$C_{NL}{}^2 = \sin^2 2\alpha = 4\sin^2\alpha\left(1-\sin^2\alpha\right) = 2C_{ND}\left(1-C_{ND}/2\right) = C_{ND}\left(2-C_{ND}\right) \tag{7}$$

The formula may also be expressed in the following form

$$C_{NL}{}^2 + \left(C_{ND}-1\right)^2 = 1 \tag{8}$$

This equation shows that the lift - drag curve of high angles of attack is a circle, which center is at (1, 0), and its radius is 1.

The curve shape of lift - drag coefficient at high angles of attack was shown in Fig. 1 too.

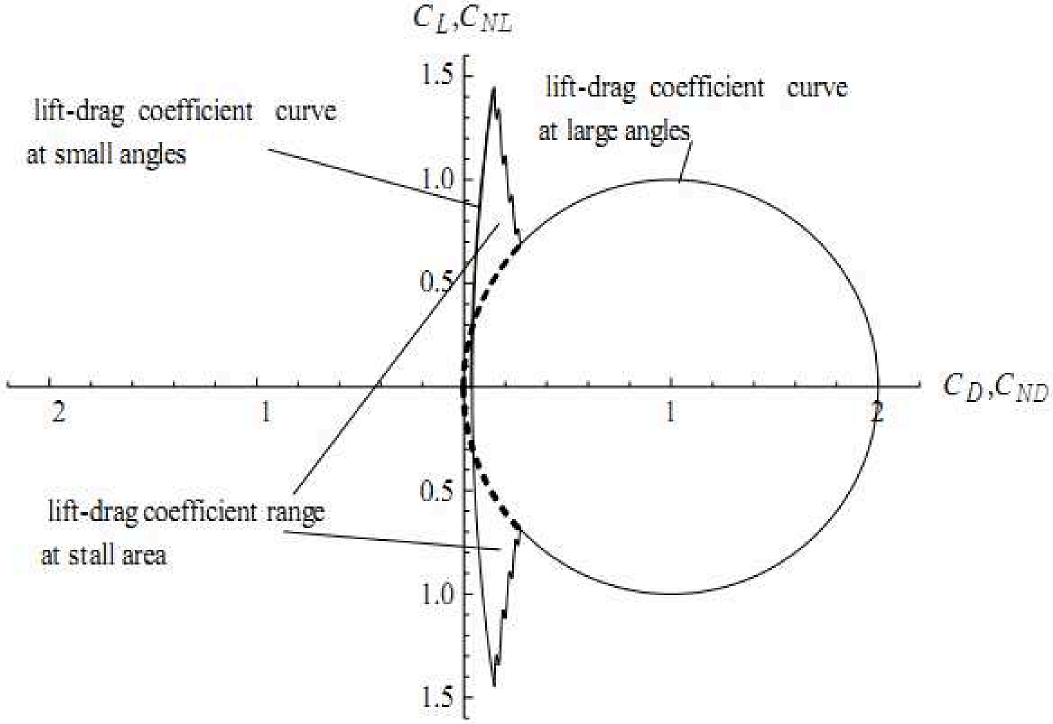

Fig. 1 The relations of lift - drag coefficients at small and high angles of attack

As can be seen from Fig. 1, at the states of high angles of attack the maximum drag coefficient is equal to 2, and the maximum lift coefficient is equal to 1; the lift coefficients at high angles of attack are the minimum lift coefficients of stall area, and the lift coefficients at small angles of attack are the maximum lift coefficients of stall area.

Conclusions

(1) The drag coefficient at small angle of attack can be expressed as the sum of shape drag and the friction drag at zero angle of attack, which facilitates drag coefficient formula expression..

(2) In the conditions at small angles of attack, lift - drag curve is a parabola; in the conditions at high angles of attack, the lift - drag curve is a circle.

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