VISCOSITY AND THERMAL CONDUCTIVITY OF AIR AND DIFFUSIVITY OF WATER VAPOR IN AIR

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ABSTRACT

Carefully chosen numerical values for the viscosity and thermal conductivity of air and diffusivity of water vapor in air are tabulated for temperatures from $-20C$ to $+40C$ and for a pressure of 1000 mb. A summary of physical constants relating to air is included.

Different reference books disagree rather widely with regard to numerical values for the viscosity, conductivity, and diffusivity of air. Since these quantities are of basic importance in a wide variety of problems concerning the meteorology of the surface layer, a summary has been prepared of the most reliable values available to date. Viscosity, conductivity, and diffusivity are presented in a uniform manner. The 'kinematic' quantities, which depend on pressure, are reduced to a standard pressure of 1000 mb for convenience in meteorological problems.

Since nearly all measurements have been made at room temperature and higher, the values presented for the lower temperatures result from extrapolation. The diffusivity of water vapor in air is relatively uncertain at all temperatures. Further measurements are desirable.

1. Dynamic viscosity

The dynamic viscosity μ, having dimensions $L^{-1}MT^{-1}$, may be defined as the ratio of shearing stress to shear, so

$$\tau = \mu\, \partial u/\partial z,$$

where τ is stress and $\partial u/\partial z$ is shear. The basic cgs unit is 1 poise = $1\ g\ cm^{-1}s^{-1}$. The dynamic viscosity of a gas is independent of pressure except at very low pressures. Its dependence on temperature is well represented by Sutherland's equation (Licht and Stechert, 1944), which may be written

$$\frac{\mu}{\mu_0} = \frac{T_0 + C}{T + C}\left(\frac{T}{T_0}\right)^{\frac{3}{2}},$$

where T is absolute temperature and μ_0 is the viscosity at absolute temperature T_0. Sutherland's constant C for air has the value 120C according to Bircumshaw and Stott (1929).

A number of highly accurate measurements of the

viscosity of air at room temperature have been carried out recently because they were required for the determination of the electronic charge by Millikan's oil-drop method. Birge (1945) concludes from these that the viscosity of air at 23C is

$$(1.8325 \pm 0.0010) \times 10^{-4}\ g\ cm^{-1}s^{-1}.$$

The values of dynamic viscosity in table 1 have been computed from Sutherland's equation by use of the two constants presented above and 273.16C for the absolute temperature of the ice point (Birge, 1941).

For comparison with Bircumshaw and Stott's value of Sutherland's constant, the following three measurements may be cited: 119.4C in the temperature range 15–302C by Breitenbach (1901), 123.6C in the range 17–477C by Braune, Basch, and Wentzel (1928), and 111C in the range 20–830C by Trautz and Zink (1930). Brémond's (1933) measurements of the viscosity of air in the temperature range 18–1134C agree closely with Breitenbach's value of 119.4C for Sutherland's constant. Two recent measurements of $d\mu/dT$ are $4.93 \times 10^{-7}\ g\ cm^{-1}s^{-1}C^{-1}$ by Ridgen (1938) in the short range 15–19C and $4.95 \times 10^{-7}\ g\ cm^{-1}s^{-1}C^{-1}$ by Banerjea and Pattanaik (1938). These may be compared with values computed from Sutherland's equation:

Sutherland's constant, C	Temperature	$d\mu/dT$
120C	0C	$5.06 \times 10^{-7}\ g\ cm^{-1}s^{-1}C^{-1}$
120	20	4.90
110	20	4.79

From the six series of measurements it appears probable that the error in the value 120C for Sutherland's constant does not exceed 5C. This corresponds to an error in the viscosity in table 1 amounting, for the lowest temperature, to $2\frac{1}{2}$ in the last place.

The determinations of Sutherland's constant are all based on measurements at and above room temperature, so the values in table 1 for viscosity at $-20C$ to $+10C$ represent extrapolations. The viscosity of air

TABLE 1. Dynamic viscosity and thermal conductivity of air.

Temperature	Dynamic viscosity	Thermal conductivity k	
C	$g\ cm^{-1}s^{-1}$ = poise	cal $cm^{-1}s^{-1}C^{-1}$	erg $cm^{-1}s^{-1}C^{-1}$
−20	1.615×10^{-4}	5.45×10^{-5}	2.28×10^{3}
−10	1.667	5.63	2.36
0	1.718	5.80	2.43
10	1.768	5.97	2.50
20	1.818	6.14	2.57
30	1.866	6.30	2.64
40	1.914	6.46	2.70

at −21C was measured by Breitenbach (1901), but he neglected this measurement in determining Sutherland's constant; the measured value is 0.016×10^{-4} g $cm^{-1}s^{-1}$ higher than his value computed from Sutherland's formula. The values of viscosity in Keenan and Kaye's (1945) table 3 at −20C to +10C are about one per cent greater than those in the present table 1. Keenan and Kaye do not explain clearly the source of their values at low temperatures (Breitenbach?); there appears to be no reason to prefer their tabulation to the present one.

2. Kinematic viscosity

The ratio of dynamic viscosity to density ρ is the kinematic viscosity, $\nu = \mu/\rho$, having dimensions L^2T^{-1}. It is the factor occurring in the relation giving the acceleration du/dt due to fluid friction. In the simplest case this relation is of the type

$$\frac{du}{dt} = \nu \frac{\partial^2 u}{\partial z^2}.$$

The values of kinematic viscosity in table 2 have been computed from dynamic viscosity and density. By use of the equation of state for a perfect gas, the

TABLE 2. Density, kinematic viscosity, and thermometric conductivity of air and diffusivity of water vapor in air, at 1000 mb.

Temperature	Density	Kinematic viscosity	Thermometric conductivity	Diffusivity
C	ρ $g\ cm^{-3}$	ν cm^2s^{-1}	ν_c cm^2s^{-1}	D cm^2s^{-1}
−20	1.3769×10^{-3}	0.1173	0.165	0.197
−10	1.3246	.1259	.177	.211
0	1.2761	.1346	.189	.226
10	1.2310	.1437	.202	.241
20	1.1890	.1529	.215	.257
30	1.1498	.1623	.228	.273
40	1.1131	.1720	.242	.289

density has been computed from 1.2930×10^{-3} g cm^{-3} as the density of air at 0C and 1 atmosphere (*Smithsonian meteorological tables*, 5th ed., p. xlvi; see also Guye, 1917) and from 1013.25 mb as the equivalent of 1 atmosphere (Birge, 1941).[2] The density values also are listed in table 2.

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[2] The universal gas constant is

$$R = 8.3144 \times 10^{7} \text{ erg } C^{-1}mole^{-1}$$

according to Birge (1941), so the mean molecular weight of dry

3. Thermal conductivity

The thermal conductivity k is the absolute value of the ratio of H, the rate of heat conduction per unit area, to the temperature gradient, so

$$H = -k\ \partial T/\partial z.$$

It has the same dimensions as the product of dynamic viscosity and specific heat. Like dynamic viscosity, it depends on temperature but not on pressure.

The value of the thermal conductivity of air at 0C given in the compilation by Chapman and Cowling (1939, table 22) is 5.80×10^{-5} cal $cm^{-1}s^{-1}C^{-1}$. Measurements have not been made below 0C. According to Chapman and Cowling (1939, p. 242), its variation "with temperature is in general approximately the same as the variation of the coefficient of viscosity." The values in table 1 are therefore based on Chapman and Cowling's value at 0C and on a constant ratio μ/k. The values in mechanical units result from the conversion 1 cal_{15C} = 4.1855×10^{7} erg (Birge, 1941).

Keenan and Kaye (1945, table 3) find that the Prandtl number, $\sigma = c_p\mu/k$, where c_p is the isobaric specific heat, decreases from 0.71 at 0C to 0.70 at 40C and to 0.66 at 300C, the highest temperature at which the thermal conductivity had been measured. From 0C to 40C the Prandtl number therefore decreases by $1\frac{1}{2}$ per cent, while their values of thermal conductivity increase by 13 per cent (and their values of isobaric specific heat increase by only 0.2 per cent); hence nine tenths of the change in the thermal conductivity is accounted for by the change in viscosity. Although Keenan and Kaye extrapolate thermal conductivity below 0C by means of a variable Prandtl number, the use of a constant ratio μ/k appears to yield more reliable values in the comparatively short range from −20C to +40C.

Chapman and Cowling's value adopted here for the thermal conductivity of air at 0C is 7 per cent greater than the old value in the *International critical tables* (Laby and Nelson, 1929). The values in table 1 are, however, supported by the following measurements in addition to those cited by Chapman and Cowling: 5.810×10^{-5} cal $cm^{-1}s^{-1}C^{-1}$ at 0C (extrapolated from 4C) and 5.965×10^{-5} cal $cm^{-1}s^{-1}C^{-1}$ at 10C, both by Milverton (1934), and $(6.06 \pm 0.04) \times 10^{-5}$

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air from the equation of state,

$$p = (R/m)\rho T,$$

where p is pressure, is consequently m_d = 28.982. Since the equation of state is not exactly satisfied even by dry air, this value is not exactly the same as the mean molecular weight computed from the chemical composition of dry air; rather, the value m_d = 28.982 is so chosen that the equation of state for dry air is identically satisfied at 0C and 1 atmosphere. The molecular weight of water from Birge's atomic weight of hydrogen is m_w = 18.01600, so the ratio of the molecular weights is m_w/m_d = 0.6216.

cal $cm^{-1}s^{-1}C^{-1}$ at 20C by Hercus and Sutherland (1934; see also Laby, 1934). For comparison, values interpolated and converted from the table of Keenan and Kaye are, for the temperatures in table 1, 2.28, 2.36, 2.44, 2.52, 2.60, 2.68, and 2.76×10^3 erg $cm^{-1}s^{-1}C^{-1}$, respectively. The probable error estimated by neglecting the older determinations yields $(5.80 \pm 0.02)10^{-5}$ cal $cm^{-1}s^{-1}C^{-1}$ as the thermal conductivity of air at 0C.

4. Thermometric conductivity

The rate of heating due to a given temperature distribution depends on the thermometric conductivity, $\nu_c = k/(c_p\rho)$:

$$\frac{dT}{dt} = \nu_c \frac{\partial^2 T}{\partial z^2}.$$

Its dimensions are the same as for kinematic viscosity, namely, L^2T^{-1}. The value for 0C, 0.1895 cm^2s^{-1}, has been computed from the thermal conductivity by use of 1.004×10^7 erg $g^{-1}C^{-1}$ for the isobaric specific heat of air (Leduc, 1929).[3] The ratio μ/k has been assumed constant for air, and the same may be assumed of specific heat, so the ratio ν/ν_c also is constant. The values of thermometric conductivity at temperatures other than 0C have therefore been computed from the corresponding values of kinematic viscosity.

5. Diffusivity

A substance having the concentration (mass per total mass) q in a mixture of density ρ is diffused at the rate, in mass per unit area and time,

$$E = -\rho D\, \partial q/\partial z,$$

where D is the diffusivity, dimensions L^2T^{-1}. The corresponding rate of change of concentration is

$$\frac{dq}{dt} = D \frac{\partial^2 q}{\partial z^2}.$$

For gases the dependence of diffusivity on pressure and temperature is represented (Boynton and Brattain, 1929) by

$$\frac{D}{D_0} = \left(\frac{T}{T_0}\right)^n \frac{p_0}{p},$$

where D_0 is the diffusivity at absolute temperature T_0 and at pressure p_0. For any given temperature the diffusivity is therefore inversely proportional to pres-

sure, just as are kinematic viscosity and thermometric conductivity. For a change at constant pressure,

$$\frac{d}{dT} \ln \frac{D}{D_0} = \frac{n}{T}.$$

The corresponding variation of kinematic viscosity, according to Sutherland's equation and the equation of state for a perfect gas, is

$$\frac{d}{dT} \ln \frac{\nu}{\nu_0} = \frac{5}{2}\frac{1}{T} - \frac{1}{T+C};$$

at 0C this quantity has the value 0.00661 C^{-1} (based on $C = 120C$). The most reliable determination of the diffusivity of water vapor in air is presumably that of Schirmer (1938), who gives $n = 1.81$. At 0C, therefore, $n/T = 0.00663\ C^{-1}$, practically identical with the corresponding value for kinematic viscosity. It therefore follows that the variation with temperature in the neighborhood of 0C, as well as with pressure, is the same for diffusivity as for kinematic viscosity.

The values of diffusivity in table 2 are based on Schirmer's value of 0.083 m^2hr^{-1} for 0C and for a pressure of 1 kg(force) cm^{-2}, which is 980.665 mb; his value corresponds to 0.226 cm^2s^{-1} at 1000 mb. The values of diffusivity at other temperatures have been computed from the kinematic viscosity, to which they are proportional.

For comparison, the diffusivity of water vapor in air at 0C according to the *International critical tables* (Boynton and Brattain, 1929) is 0.220 cm^2s^{-1} at 1 atmosphere and corresponds to 0.223 cm^2s^{-1} at 1000 mb; n is stated to be 1.75. Trautz and Müller (1935a; 1935b) deduce a value (0.219 $\pm$ 0.001 cm^2s^{-1} at 1 atmosphere) corresponding to 0.222 cm^2s^{-1} at 0C and 1000 mb, and give $n = 1.853$. These two values agree closely with Schirmer's. Summerhays (1930) measured 0.282 cm^2s^{-1} at 16.1C, which reduces, if the measurement was at 1 atmosphere, to 0.293 cm^2s^{-1} at 20C and 1000 mb; this measurement results in a value 14 per cent higher than that in table 2. These three values, but not Schirmer's, are listed by Dorsey (1940, pp. 72–73). A probable error estimated by neglecting Summerhays's measurement yields

$$(0.226 \pm 0.003)\ cm^2s^{-1}$$

as the diffusivity of water vapor in air at 0C.

6. Nondimensional ratios

Ratios of quantities in table 2 are dimensionless and are nearly independent of temperature and pressure. The Prandtl number, which is the ratio of kinematic viscosity to thermometric conductivity, has the value at 0C

$$\sigma = \nu/\nu_c = c_p\mu/k = 0.711 \pm 0.003.$$

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[3] This value agrees with

$$0.2399\ \text{Btu lb}^{-1}\text{F}^{-1} = 1.0045 \times 10^7\ \text{erg g}^{-1}\text{C}^{-1}$$

resulting from a new summarization by Keenan and Kaye (1945, table 3). Their table shows the variation of specific heat with temperature over a wide range.

It may be noted that Leduc's value of specific heat, together with the values of the universal gas constant and the mean molecular weight given above, determines the value of the Poisson constant for dry air to be $R/(mc_p) = 0.286$.

The ratio of kinematic viscosity to diffusivity at 0C is

$$\nu/D = 0.596 \pm 0.008,$$

and the ratio of thermometric conductivity to diffusivity at 0C is

$$\nu_c/D = 0.84 \pm 0.01.$$

7. Summary of constants

The various constants used in computing the values presented above have been chosen with care and are more precise than the ones to be found in common meteorological reference works. The summary below may therefore be useful.

Absolute temperature of the ice point (Birge, 1941):
$$273.16C \pm 0.01C.$$
1 atmosphere = 1013.246 mb ± 0.004 mb (Birge, 1941) = 1013.25 mb.
Density, at 0C and 1 atmosphere, of dry air containing an average amount of carbon dioxide:
$$(1.2930 \pm 0.0001)10^{-3} \text{ g cm}^{-3}.$$
Universal gas constant (Birge, 1941):
$$R = (8.3144 \pm 0.0004)10^7 \text{ erg C}^{-1}\text{mole}^{-1}.$$
Mean molecular weight of dry air (derived from preceding constants): $m_d = 28.982 \pm 0.003$.
Molecular weight of water (Birge, 1941):
$$m_w = 18.01600 \pm 0.00004.$$
$m_w/m_d = 0.6216 \pm 0.0001$.
Isobaric specific heat of dry air at 0C:
$$c_p = (1.004 \pm 0.001)10^7 \text{ erg g}^{-1}\text{C}^{-1}.$$
Poisson constant for dry air: $R/(mc_p) = 0.286$.
1 cal$_{15C}$ (Birge, 1941) = $(4.1855 \pm 0.0004)10^7$ erg.

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