Solutions to CLRS

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Chapter 24

Exercise 24.1-5

The desired value for each v in V is similar to the $\delta(v)$ in the original Bellman-Ford algorithm. It has the similar property such as optimal substructure, upperbound property, convergence property and path-relaxation property. So it can be solved using Bellman-Ford algorithm with the d function changed.

Exercise 24.4-5

In fact, the added dummy source s is not really needed. We just need to modify the Bellman-Ford algorithm to initialize all $d[v]$ to 0 and it will work well.

Why? The Bellman-Ford algorithm has 2 proedures: relax edges and check negative-weight cycles. We will prove that the algorithm has the same effects without the added edges (s, v) as with the added edges in the 2 procedures. When doing relaxation, each added edge (s, v) is used only once to relax each $d[v](v \in V)$ to 0 then it will not be used to relax any edge because after having relaxed $d[v]$ to 0 the relax condition $d[v] < d[s] + w(s, v)$ (simplified to $d[v] < 0$) will not be satisfied again. So we initialize all $d[v]$ to 0 and the edges (s, v) will be useless. When checking negative-weight cycles, the conditon $d[v] > d[s] + w(s, v)$ is not satisfied whether or not negative weight cycles exist. So the added edges (s, v) is not needed to check the existence of negative-weight cycles.