Spacitron Resonance Theory SRT: A Wave-Only, Two-Scale, Exponential Law

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Abstract

Spacitron Resonance Theory (SRT) proposes a wave-only foundation for physics. **Reality is organized fractally between only two fundamental scales: the quantum scale at the bottom and the observable universe at the top.** Everything else — stars, galaxies, clusters — are not new scales but **resonance structures inside the cosmic scale, like rocks over rocks on one landscape.** This is why **general relativity must rely on patchwise approximations** to handle them, while SRT does not: in SRT, these structures simply add linearly as contributions within the same cosmic spacitron, governed by a single universal rule.

A *spacitron* is defined as a **resonance fingerprint of the universal wave at any scale.** At the quantum scale, resonance division produces discrete fingerprints, explaining the quantization of matter and fields. At the cosmic scale, the observable universe itself is a spacitron — the largest resonance fingerprint accessible to us. Between these two anchors, the fractal principle demands that resonance responses do not reset from one scale to the next but compound **scale by scale.** This compounding forces the universal law to take an **exponential form.** The exponential belongs not to physics within a single scale, but to the continuity of responses **across scales** in the fractal hierarchy.

A simple translation illustrates this unity: at the quantum scale, resonance division restricts atoms like hydrogen to **discrete energy levels**; at the cosmic scale, the same fractal principle restricts redshift and lensing to follow an **exponential law** rather than arbitrary polynomial expansions. In both cases, resonance scaling enforces nonlinear structure: discreteness at the bottom, exponential response at the top.

From this perspective, gravity appears not as curvature of a four-dimensional manifold, but as **refraction of the universal wave.** Resonance energy at each scale sculpts a refractive index field in three-dimensional space plus universal time. This one index guides all processes simultaneously: it bends light, sets the ticking of clocks, scales rulers, and governs motion. What general relativity describes as curved spacetime is here reinterpreted as the optical effect of resonance refraction.

The theory is anchored by four exact, non-adjustable, and dimensionally consistent laws: the Time Law, the Gravity Law, the Optical-Index Law, and the Static Sourcing Law. **Potentials add linearly within each scale,** because resonance energies belong to the same continuous wave. **Responses compound across scales,** enforcing the exponential law. Within any single scale, SRT reduces to the same predictions as general relativity, consistent with solar-system tests, pulsar timing, and gravitational waves. Across scales, however, the exponential structure offers a natural explanation for phenomena otherwise attributed to invisible components. At the bottom, quantization emerges from resonance division. At the top, exponential cross-scale response accounts for cosmic acceleration and anomalous lensing without invoking dark matter or dark energy.

The payoff, if confirmed, is a radical simplification. Matter is resonance, energy is resonance frequency, and gravity is universal refraction. The observable universe is the **limit scale of resonance accessible to us,** paired with the quantum scale at the bottom. Between them, the exponential law of fractal scaling ensures continuity of physics across the entire hierarchy. SRT is therefore not a flexible model but a knife-edge principle: it can only be exactly right!

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1 Purpose and Strategy

Direct, testable *numerical* section contrasting SRT vs GR at second order (2PN), a knife-edge, parameter-free backbone.

2 Definitions and Notation

Definition 2.1: Universal wave

There is a single complex universal wave $\Psi(t, \mathbf{x})$. What we call "fields" and "matter" are its resonance fingerprints (standing or travelling patterns).

Definition 2.2: Spacitron

A *spacitron* is a resonance fingerprint of Ψ set by boundary conditions. Fractally, the **quantum spacitron** anchors the bottom scale; the **cosmic spacitron** (the observable universe) anchors the top scale. Intermediate structures (stars, galaxies) are not new scales; they are sub-resonances within the cosmic spacitron.

Definition 2.3: Resonance potential Φ

A scalar generator $\Phi(t, \mathbf{x})$ (units $\mathrm{m}^2\,\mathrm{s}^{-2}$) controls time-rate and optical response. **Within a single scale**, contributions add linearly: $\Phi = \sum_a \Phi_a$.

Definition 2.4: Index and clock factor

Two derived, dimensionless responses: the optical index $n(t, \mathbf{x})$ and the local-to-universal clock factor $\Gamma_t = \frac{\mathrm{d}\tau}{\mathrm{d}t}$.

Symbols. c (speed of light), G (Newton's constant), t (universal time), τ (proper time), $U \equiv \Phi/c^2$ (dimensionless potential), ρ_{res} (mass-equivalent resonance energy density), $\Phi_{\mathrm{bg}}(t)$ (spatially averaged background potential)

3 Backbone: Four Exact, Non-Adjustable Laws

Law 3.1: Time Law

Clocks scale exponentially with the resonance potential:

$$\Gamma_t(\mathbf{x}, t) \equiv \frac{\mathrm{d}\tau}{\mathrm{d}t} = \exp\left(\frac{\Phi(\mathbf{x}, t)}{c^2}\right) \tag{1}$$

Units: Φ/c^2 is dimensionless; Γ_t is unitless; τ, t carry time units

Law 3.2: Optical-Index Law

The same generator sets the index field:

$$n(\mathbf{x}, t) = \exp\left(-\frac{2\,\Phi(\mathbf{x}, t)}{c^2}\right) \tag{2}$$

For $\Phi < 0$ (near mass), $n > 1$, producing gravitational delay and bending in the optical picture

Law 3.3: Gravity Law: geodesics of the optical metric

Define the metric built from the generator $U \equiv \Phi/c^2$:

$$g_{\mu\nu}(\Phi) \;=\; \mathrm{diag}\big(e^{2U},\, -e^{-2U},\, -e^{-2U},\, -e^{-2U}\big) \tag{3}$$

All free trajectories (lightlike and timelike) extremize proper length in $g_{\mu\nu}(\Phi)$. Light rays also satisfy Fermat's principle with index n from the Optical-Index Law

Law 3.4: Static Sourcing Law

Within one scale, the generator is sourced linearly:

$$\nabla^2\Phi(\mathbf{x},t) \;=\; 4\pi\, G\, \rho_{\mathrm{res}}(\mathbf{x},t) \tag{4}$$

Units: $[\nabla^2\Phi] = \mathrm{T}^{-2}$ and $[G\rho_{\mathrm{res}}] = \mathrm{T}^{-2}$

Interpretation A single scalar Φ governs *time* (Time Law), *optics* (Optical-Index Law), *motion* (geodesics of the optical metric), and is *linearly sourced* within a scale. No tunable numbers appear besides c and G.

Conventions, units, and scope

4 From the Sourcing Law to the Generator $U = \Phi/c^2$

Purpose Derive the generator $U = \Phi/c^2$ *within a single scale* starting from $\nabla^2\Phi = 4\pi G\rho$, passing through the open-space integral and the Cauchy boundary representation. No tunables appear: only G, c, and geometry $(4\pi, \frac{1}{2})$.

Definition 4.1: Generator Foundations: Domains, Operators, and Boundaries

Spatial domain and points

$$\mathbf{x} = (x,y,z) \in \mathbb{R}^3 \text{ (field point)}, \qquad \mathbf{x}' \in \mathbb{R}^3 \text{ (source point)}, \qquad \mathbf{r} := \mathbf{x} - \mathbf{x}', \quad r := \|\mathbf{r}\|$$

Region and boundary.

$$V \subset \mathbb{R}^3 \text{ (finite region)}, \qquad \partial V \text{ (smooth boundary)}, \qquad \hat{\mathbf{n}}' = \hat{\mathbf{n}}'(\mathbf{x}') \text{ (outward unit normal)}$$

$$\partial_{n'}\Phi(\mathbf{x}') := \hat{\mathbf{n}}'(\mathbf{x}')\cdot\nabla'\Phi(\mathbf{x}'), \qquad d^3x' \text{ (volume element)}, \quad dS' \text{ (surface element on } \partial V)$$

Operators and distributions

$$\nabla f = (\partial_x f, \partial_y f, \partial_z f), \qquad \nabla^2 f = \partial_{xx} f + \partial_{yy} f + \partial_{zz} f,$$

$$\int_{\mathbb{R}^3} f(\mathbf{x})\, \delta^3(\mathbf{x}-\mathbf{x}_0)\, d^3x = f(\mathbf{x}_0), \qquad \nabla^2\left(\frac{1}{4\pi r}\right) = -\,\delta^3(\mathbf{r})$$

Unit-source pattern and geometry

$$R(\mathbf{r}) = \frac{1}{4\pi r}, \qquad \oint_{S_\varepsilon} \nabla\Big(\frac{1}{r}\Big)\cdot d\mathbf{S} = -\,4\pi \quad (\varepsilon \to 0^+), \qquad \int_{S^2} d\Omega = 4\pi.$$

On smooth ∂V, the boundary evaluation uses the universal *Cauchy half-jump* $1/2$

Fields and constants

$$\rho_{\text{res}} : \mathbb{R}^3 \to \mathbb{R} \text{ (resonance density)}, \quad G > 0, \quad \Phi : \mathbb{R}^3 \to \mathbb{R} \text{ with } \nabla^2\Phi = 4\pi G\rho_{\text{res}}, \quad c > 0,$$

Definition 4.2: Derivation — From the Sourcing Law to the Generator $U = \Phi/c^2$

Step 1 — Local sourcing law (given)

$$\boxed{\nabla^2\Phi(\mathbf{x}) = 4\pi\, G\, \rho_{\text{res}}(\mathbf{x})} \qquad (\mathbf{x} \in V \subseteq \mathbb{R}^3)$$

Step 2 — Unit-source kernel (normalization by flux) For a point source $\rho_{\text{res}}(\mathbf{x}) = \delta^3(\mathbf{x})$ one has, for $r > 0$, $\nabla^2(1/r) = 0$ and MIT 18.303 proof of $R(r) = 1/(4\pi r)$ by flux normalization, search for "Laplacian is"

$$\oint_{S_\varepsilon} \nabla\Big(\frac{1}{r}\Big) \cdot d\mathbf{S} = -\,4\pi$$

Hence the normalized kernel is

$$R(\mathbf{r}) = \frac{1}{4\pi r}, \qquad \nabla^2 R(\mathbf{r}) = -\,\delta^3(\mathbf{r})$$

Open-space superposition (linearity) Summing unit responses over all sources: Caltech proof of open-space superposition integral, search for "Integral Solution of Poisson's equation" (match 2)

Definition 4.3: Substitution Bridge From Green theory to SRT

1. Start (Caltech Green form)

$$\Phi(\mathbf{x}_0) = \iiint_V \sigma(\mathbf{x})\, G(\mathbf{x};\mathbf{x}_0)\, dV + \iint_{\partial V} f(\mathbf{x})\, \frac{\partial G}{\partial n}\, dS$$

2. Substitution map

$$\mathbf{x}_0 \to \mathbf{x}, \qquad \mathbf{x} \to \mathbf{x}', \qquad G(\mathbf{x}';\mathbf{x}) \to R(\mathbf{x}-\mathbf{x}'), \qquad R(\mathbf{x}-\mathbf{x}') = \frac{1}{4\pi\|\mathbf{x}-\mathbf{x}'\|},$$

$$\sigma(\mathbf{x}') \to -4\pi G\, \rho_{\mathrm{res}}(\mathbf{x}'), \qquad f(\mathbf{x}') \to \Phi(\mathbf{x}'), \qquad \frac{\partial G}{\partial n} \to \partial_{n'} R$$

3. Compute substitution (Green's representation formula)

$$\Phi(\mathbf{x}) = \iiint_V \left(-4\pi G\, \rho_{\mathrm{res}}(\mathbf{x}')\right) R(\mathbf{x}-\mathbf{x}')\, d^3x' + \iint_{\partial V} \left(\Phi(\mathbf{x}')\, \partial_{n'} R(\mathbf{x}-\mathbf{x}') - R(\mathbf{x}-\mathbf{x}')\, \partial_{n'} \Phi(\mathbf{x}')\right) dS'$$

$$\boxed{\Phi(\mathbf{x}) = -\,G \int_V \frac{\rho_{\mathrm{res}}(\mathbf{x}')}{\|\mathbf{x}-\mathbf{x}'\|}\, d^3x' + \frac{1}{4\pi} \oint_{\partial V} \left[\Phi(\mathbf{x}')\, \partial_{n'}\left(\frac{1}{\|\mathbf{x}-\mathbf{x}'\|}\right) - \frac{1}{\|\mathbf{x}-\mathbf{x}'\|}\, \partial_{n'} \Phi(\mathbf{x}')\right] dS'}$$

4. Open-space substitution When the region expands to all space,

$$V \;\to\; \mathbb{R}^3, \qquad \partial V \;\to\; \infty, \qquad \Phi,\; \partial_n \Phi \;\to\; 0,$$

the surface term disappears and the triple integral sign is replaced by a single integral with explicit 3-D measure d^3x':

$$\boxed{\Phi(\mathbf{x}) = -\,G \int_{\mathbb{R}^3} \frac{\rho_{\mathrm{res}}(\mathbf{x}')}{\|\mathbf{x}-\mathbf{x}'\|}\, d^3x'}$$

Definition 4.4: Geometric invariants ($1/(4\pi)$, $1/2$) — reversed journal

Definitions first! We look at a vector pointing from the source to the field point and it gives the potential at x produced by a unit source at x prime. The portion of space inside the region, seen from a point on a smooth boundary, makes up exactly half of all possible directions around that point. Imagining a tiny sphere centered on the point, the boundary cuts the sphere into two equal halves. The solid angle of the half that lies inside the region is two-pi, while the full sphere covers four-pi. Dividing the interior part by the whole gives one-half. It's a geometric way of saying that the boundary shares the surrounding space evenly between the inside and the outside. The inside is the space contained within the boundary surface of a region. The outside is everything beyond that surface, with the boundary itself acting as the thin dividing layer between them.

$$\boxed{V = \text{finite region of space containing sources } \rho_{\text{res}}, \partial V = \text{its smooth boundary surface}}$$

$$\mathbf{x}_0 \in \partial V, \quad r = \|\mathbf{x} - \mathbf{x}'\|, \quad R(\mathbf{x}, \mathbf{x}') = \frac{1}{4\pi r}, \quad S_\varepsilon = \text{sphere radius } \varepsilon \text{ at } \mathbf{x}_0,$$

$$\hat{\mathbf{n}} = \text{outward normal on } \partial V, \qquad \Omega_{\text{int}} = \text{interior solid angle at } \mathbf{x}_0.$$

$$\frac{\Omega_{\text{int}}}{4\pi} = \frac{1}{2}$$

Geometry Idea The idea comes from simple geometry. Around any point source, the total flow of the field spreading outward in every direction is the same no matter how small the surrounding sphere is. The constant one over four-pi is chosen so that this total flow equals exactly one full unit when you include the whole sphere. Now, if the point happens to sit on a smooth boundary, only half of that little sphere lies inside the region and the other half is outside. The part inside covers half of the full space around the point, and that is why a factor of one-half appears in the boundary relation. Both numbers, one over four-pi and one-half, come entirely from the geometry of three-dimensional space. They are fixed by shape alone and do not depend on any adjustable physical quantity

Definition 4.5: Cauchy field → Energy bridge — labeled, underbraced, and explained

Symbols at a glance

- $\mathbf{x}$: field point (where we evaluate Φ); $\mathbf{x}'$: source or boundary point
- p.v. (principal value)
- $r = \|\mathbf{x} - \mathbf{x}'\|$: distance between field and source points
- V: interior region; ∂V: its boundary; d^3x': volume element; dS': surface element
- $\hat{\mathbf{n}}$, $\hat{\mathbf{n}}'$: outward unit normals at $\mathbf{x}$ and $\mathbf{x}'$
- $\partial_n \Phi := \hat{\mathbf{n}}\cdot\nabla\Phi, \quad \partial_{n'}\Phi := \hat{\mathbf{n}}'\cdot\nabla'\Phi$
- ρ_{res}: resonance density; Φ: resonance potential; G: Newton constant; c: speed of light; $U := \Phi/c^2$

$$\boxed{q(\mathbf{x}) = \partial_n\Phi(\mathbf{x}), \qquad q(\mathbf{x}') = \partial_{n'}\Phi(\mathbf{x}'), \qquad \nabla^2\Phi = 4\pi G\,\rho_{\text{res}}}$$

Cauchy field (interior point $\mathbf{x} \in V$).

$$\Phi(\mathbf{x}) =$$

$$\underbrace{-G\int_V \frac{\rho_{\text{res}}(\mathbf{x}')}{\|\mathbf{x}-\mathbf{x}'\|}\,d^3x'}_{\text{[V] Volume superposition}} + \frac{1}{4\pi}\oint_{\partial V}\left(\underbrace{\Phi(\mathbf{x}')\,\partial_{n'}\!\left(\frac{1}{\|\mathbf{x}-\mathbf{x}'\|}\right)}_{\text{[D] Double-layer (kernel "mirror")}} - \underbrace{\frac{1}{\|\mathbf{x}-\mathbf{x}'\|}\,\partial_{n'}\Phi(\mathbf{x}')}_{\text{[S] Single-layer (slope feeds in)}} \right) dS'$$

Boundary limit (approach $\mathbf{x} \to \partial V$ from inside)

$$\underbrace{\frac{1}{2}}_{\text{[½] Cauchy half-jump}} \Phi(\mathbf{x}) =$$

$$\underbrace{-G\int_V \frac{\rho_{\text{res}}(\mathbf{x}')}{\|\mathbf{x}-\mathbf{x}'\|}\,d^3x'}_{\text{[V] Volume superposition}} + \frac{1}{4\pi}\,\text{p.v.}\oint_{\partial V}\left(\underbrace{\Phi(\mathbf{x}')\,\partial_{n'}\!\left(\frac{1}{\|\mathbf{x}-\mathbf{x}'\|}\right)}_{\text{[D] D-layer}} - \underbrace{\frac{1}{\|\mathbf{x}-\mathbf{x}'\|}\,\partial_{n'}\Phi(\mathbf{x}')}_{\text{[S] S-layer}} \right) dS'$$

Map of the labeled pieces

- **[V] Volume superposition:** adds all interior sources linearly with the $1/r$ kernel; no tunable constants, only G and geometry
- **[S] Single-layer term:** transmits the *normal slope* of Φ on the boundary into the interior; it is a "flux-driven" contribution
- **[D] Double-layer term:** uses the *normal derivative of the kernel* weighted by the boundary values of Φ; it encodes how the boundary value "reflects" into the field
- **[p.v.] Principal value:** takes the symmetric limit that cancels the $1/r$ singularity at the boundary point.
- **[½] Half-jump:** purely geometric; at a smooth boundary the interior solid angle is 2π of the full 4π, giving the exact factor $1/2$

Bridge to the energy identity (the “big rule,” Green’s first identity)

$$\underbrace{\frac{1}{8\pi G}\int_V |\nabla\Phi|^2\,dV}_{\textbf{[E] Field energy}} \;=\; \underbrace{\frac{1}{8\pi G}\oint_{\partial V} \Phi\,\partial_n\Phi\,dS}_{\textbf{[F] Boundary flow}} \;-\; \underbrace{\frac{1}{2}\int_V \rho_{\rm res}\,\Phi\,dV}_{\textbf{[W] Source work}}\,, \qquad \nabla^2\Phi = 4\pi G\,\rho_{\rm res}$$

- **[E] Field energy:** measures how “tense” the field is inside V (more gradient, more energy). It is non-negative
- **[F] Boundary flow:** energy exchanged through the skin; same surface and normal derivative that appear in the Cauchy field formula
- **[W] Source work:** matter at potential Φ contributes to the mechanical work; the factor $1/2$ avoids double counting

Generator form

$$U := \frac{\Phi}{c^2}, \qquad \partial_n\Phi = c^2\partial_n U \quad\Longrightarrow\quad \underbrace{\frac{c^4}{8\pi G}\int_V |\nabla U|^2 dV}_{\textbf{[E]}} = \underbrace{\frac{c^4}{8\pi G}\oint_{\partial V} U\,\partial_n U\,dS}_{\textbf{[F]}} - \underbrace{\frac{c^2}{2}\int_V \rho_{\rm res}\,U\,dV}_{\textbf{[W]}}$$

Within one scale, the bookkeeping is linear in these energies
Why keep both views (field & energy) The Cauchy equations tell *what the field is* from sources and boundary data (geometry) The Green identity tells *how the field’s energy balances* inside V (dynamics) They share the same surface and normal-slope q, so geometry and energetics stay consistent within a scale

Definition 4.6: "Cauchy half-jump" and modern link (Kellogg)

Meaning "Cauchy half-jump" is modern shorthand in potential theory and boundary-integral methods for a phenomenon that traces back to Cauchy (1841): the half-value discontinuity of the *double-layer potential* at a smooth boundary. In words: as you approach the surface from inside or outside, the potential's limiting values differ by exactly one-half of the boundary value. This half is geometric (interior solid angle 2π out of 4π), not a tunable parameter.

What Cauchy actually did (1841) In *"Théorème des potentiels à valeurs discontinues"* (Comptes Rendus, 1841), Cauchy analyzed surface integrals built from the normal derivative of the Newtonian kernel $1/r$. He showed that for a smooth boundary ∂V, the associated potential—what he called a *"potentiel à valeurs discontinues"*—has two distinct one-sided limits at a boundary point, differing by $\frac{1}{2}$ times the local boundary density. In modern shorthand:

$$D^{\pm}\Phi = K\Phi \mp \tfrac{1}{2}\,\Phi \quad (x \in \partial V),$$

where K is the principal-value boundary operator induced by $\partial_{n'}(1/4\pi\|x - x'\|)$. The minus sign corresponds to the interior limit (D^-), the plus sign to the exterior limit (D^+).

What it is *not* It has nothing to do with the Cauchy distribution in probability (heavy tails), jumps in stochastic processes, or the mechanics expression $gh/2$. Those are unrelated topics that share the name "Cauchy."

Why the term persists Cauchy's 1841 note was the first to pinpoint this geometric half-weight at smooth boundaries for double-layer potentials. Later works (Somigliana, Kellogg 1929, and modern PDE/layer-potential texts) repeat and systematize this fact, so the community still says "Cauchy half-jump" to credit the origin

Reading on the pieces

- Double-layer potential ($D\Phi$). A boundary integral that uses the normal derivative of $1/r$. Intuitively: the surface "re-emits" field in a way that is sensitive to the slope across it
- Principal value (PV). A symmetric limiting prescription that neutralizes the kernel's singularity at the evaluation point and keeps only the finite, physical contribution
- Half-weight ($\frac{1}{2}\,\Phi$). Pure geometry: a tiny sphere centered at a smooth boundary point is split evenly; the interior solid angle is 2π of 4π, giving the exact one-half

Modern boundary-integral statement (Oliver D. Kellogg, 1929). Oliver D. Kellogg, *Foundations of Potential Theory* Oliver Full Text
Search inside the PDF `"limiting value of the potential"`
With the outward normal, the interior limit places $+\frac{1}{2}\,\Phi$ on the left-hand side; the exterior limit places $-\frac{1}{2}\,\Phi$ (double-layer potential)

Primary pointer (optional, historical completeness). For Cauchy's original note in *Comptes Rendus* (1841, t. 13), open a full-view copy and use the viewer's search: HathiTrust record (1841, volume 13)

More Definitions $\Phi(x')$: boundary value on ∂V; $q = \partial_n\Phi$: normal slope; PV: principal value; $U = \Phi/c^2$: generator for SRT exponentials

5 Green–Cauchy bridge (why, how, and the boundary limit)

Definition 5.1: Why both forms?

Green gives the interior field from sources *plus* boundary data. *Cauchy* tells how that same boundary data reproduces the field *on* the surface via a half-jump. Using them together guarantees that "volume view" and "surface view" of the same Φ are consistent—no hidden assumptions and no extra parameters.

Theorem 5.1: Green representation in a bounded region V

$$\nabla^2\Phi(\mathbf{x}) = 4\pi\, G\, \rho_{res}(\mathbf{x}).$$

With the normalized kernel $R(\mathbf{r}) = 1/(4\pi\|\mathbf{r}\|)$, $\nabla^2 R = -\delta^3$
For any $\mathbf{x} \in V$ with smooth boundary ∂V and outward normal $\hat{\mathbf{n}}'$,

$$\Phi(\mathbf{x}) = -G\int_V \frac{\rho_{res}(\mathbf{x}')}{\|\mathbf{x}-\mathbf{x}'\|}\, d^3x' + \frac{1}{4\pi}\oint_{\partial V}\left[\Phi(\mathbf{x}')\,\partial_{n'}\Big(\frac{1}{\|\mathbf{x}-\mathbf{x}'\|}\Big) - \frac{1}{\|\mathbf{x}-\mathbf{x}'\|}\,\partial_{n'}\Phi(\mathbf{x}')\right] dS'. \tag{5}$$

Theorem 5.2: Cauchy boundary limit (interior / exterior)

Let $\mathbf{x} \to \mathbf{x}_0 \in \partial V$. Then

$$\frac{1}{2}\Phi(\mathbf{x}_0) = -G\int_V \frac{\rho_{res}(\mathbf{x}')}{\|\mathbf{x}_0-\mathbf{x}'\|}\, d^3x' + \frac{1}{4\pi}\,\text{p.v.}\oint_{\partial V}\left[\Phi\,\partial_{n'}\Big(\frac{1}{\|\mathbf{x}_0-\mathbf{x}'\|}\Big) - \frac{1}{\|\mathbf{x}_0-\mathbf{x}'\|}\,\partial_{n'}\Phi\right] dS', \tag{6}$$

$$-\frac{1}{2}\Phi(\mathbf{x}_0) = -G\int_{V^{\text{ext}}} \frac{\rho_{res}(\mathbf{x}')}{\|\mathbf{x}_0-\mathbf{x}'\|}\, d^3x' + \frac{1}{4\pi}\,\text{p.v.}\oint_{\partial V}\left[\Phi\,\partial_{n'}\Big(\frac{1}{\|\mathbf{x}_0-\mathbf{x}'\|}\Big) - \frac{1}{\|\mathbf{x}_0-\mathbf{x}'\|}\,\partial_{n'}\Phi\right] dS' \tag{7}$$

The $\frac{1}{2}$ factors are purely geometric (interior/exterior solid angles).

Principle 5.1: What this proves (continuity with the right caveat)

If the boundary carries *no* singular surface source (no δ-like shell), then the interior and exterior limits coincide:

$$\Phi_{\text{int}}(\mathbf{x}_0) = \Phi_{\text{ext}}(\mathbf{x}_0), \qquad \partial_n\Phi_{\text{int}}(\mathbf{x}_0) = \partial_n\Phi_{\text{ext}}(\mathbf{x}_0)$$

If a **physical surface density** σ_s **lives** on ∂V, then $[\partial_n\Phi] = 4\pi G\,\sigma_s$ while Φ remains continuous. Either way, (5) and (6)–(7) use the *same* boundary data $(\Phi, \partial_n\Phi)$, which is why the two views *cannot* disagree.

Definition 5.2: Moving law: from the backbone laws to geodesics (matter) and rays (light)

Goal. Build the full motion law *from first principles*

Part A — From backbone laws to the interval ds^2

A1. Build physical time and length scales. We distinguish *universal time* t (the common time label of the model) from a clock's *proper time* τ (what the clock itself measures). Spatial position is $\mathbf{x} = (x, y, z)$ in m. The symbol $\mathbf{v}$ denotes the ordinary **3-velocity** relative to these coordinates,
$\mathbf{v} \equiv \frac{d\mathbf{x}}{dt} = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right)$ with units $\mathrm{m}, \mathrm{s}^{-1}$
Saying a clock is *at rest* at $\mathbf{x}$ means $\mathbf{v} = 0$ at that instant (no motion in the chosen grid). In that case, the **Time Law** gives its proper-time increment directly as
$\frac{d\tau}{dt} = e^{U(\mathbf{x})} \Rightarrow d\tau = e^U, dt$ where $U \equiv \Phi/c^2$ is dimensionless (so e^U is a pure number).

$$\frac{d\tau}{dt} = e^U \Longrightarrow d\tau = e^U dt \Longrightarrow (d\tau)^2 = (e^U dt)^2 = (e^U)^2 dt^2 = e^{2U} dt^2$$

For spatial distances, the same generator gives an index. In ordinary optics, a higher index means light is slower, and rulers are effectively "thicker". Thus we scale lengths as

$$U \equiv \frac{\Phi}{c^2}, \qquad n \equiv e^{-2U}, \qquad d\ell \equiv \sqrt{dx^2 + dy^2 + dz^2}, \qquad c = \text{speed of light}$$

$$\text{(local light)} \quad dt = \frac{n}{c}\, d\ell \iff c\, dt = n\, d\ell$$

$$\text{one-way time (out):} \quad dt_{\text{out}} = \frac{n}{c}\, d\ell$$

$$\text{one-way time (back):} \quad dt_{\text{back}} = \frac{n}{c}\, d\ell$$

$$\text{round-trip coordinate time:} \quad \Delta t_{\text{round}} = dt_{\text{out}} + dt_{\text{back}} = \frac{n}{c}\, d\ell + \frac{n}{c}\, d\ell = \frac{2n}{c}\, d\ell$$

$$\text{round-trip proper time (rest clock):} \quad \Delta\tau_{\text{round}} = e^U\, \Delta t_{\text{round}} = e^U\, \frac{2n}{c}\, d\ell$$

radar definition of physical length:

$$d\ell_{\text{phys}} \equiv \frac{c}{2}\, \Delta\tau_{\text{round}} = \frac{c}{2}\, e^U\, \frac{2n}{c}\, d\ell = e^U n\, d\ell = e^U e^{-2U}\, d\ell = e^{-U}\, d\ell$$

$$\Rightarrow \quad d\ell_{\text{phys}}^2 = e^{-2U}\, d\ell^2, \qquad d\ell^2 \equiv d\mathbf{x}^2 = dx^2 + dy^2 + dz^2$$

Let the position vector be $\mathbf{x} = (x, y, z)$.

A tiny displacement is the differential vector $d\mathbf{x} = (dx, dy, dz)$

Compute its Euclidean inner product (dot product) with itself:

$$d\mathbf{x} \cdot d\mathbf{x} = (dx, dy, dz) \cdot (dx, dy, dz) = (dx)(dx) + (dy)(dy) + (dz)(dz) = dx^2 + dy^2 + dz^2$$

$$\boxed{d\ell^2 \equiv d\mathbf{x} \cdot d\mathbf{x} = dx^2 + dy^2 + dz^2}$$

$$d\ell = \sqrt{d\ell^2} = \sqrt{dx^2 + dy^2 + dz^2}$$

Hence $d\ell^2 \equiv d\mathbf{x}^2$ is shorthand for $d\mathbf{x}^2 = d\mathbf{x} \cdot d\mathbf{x} = dx^2 + dy^2 + dz^2$

Definition 5.3: Spacetime interval

Abstract

We now define the spacetime interval $s(t, \mathbf{x})$ as the accumulated arc length along a worldline in spacetime. At each point $\mathbf{x}(t)$ the local generator $U = \Phi/c^2$ modifies both the time and space scales exponentially. Therefore, the infinitesimal contribution to s must combine these two scaled parts under one square root. The quantity c is the universal speed of light measured in meters per second, and t is the time coordinate measured in seconds. Multiplying time by the speed of light converts the time coordinate into a length dimension so that all four components of the spacetime position have the same unit of meters. The first component represents the temporal part expressed as a distance, and the remaining three components represent the spatial coordinates. In this form the position is described as a contravariant four-vector representing an event in spacetime.

$$\mu, \nu \in 0, 1, 2, 3 \quad \text{are spacetime indices used for summation}$$

$$x^\mu = (x^0, x^1, x^2, x^3) = (ct, x, y, z) = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}.$$

$$s(t, \mathbf{x}) = \int_{t_0}^{t} \sqrt{g_{\mu\nu}(x(t'))\, \frac{dx^\mu}{dt'}\, \frac{dx^\nu}{dt'}}\; dt'$$

$$x^\mu = (ct, x, y, z), \qquad \frac{dx^\mu}{dt'} = (c, \dot{x}, \dot{y}, \dot{z}), \qquad \mathbf{v}(t') = \frac{d\mathbf{x}}{dt'} = (\dot{x}, \dot{y}, \dot{z})$$

$$g_{\mu\nu}(\Phi) = \mathrm{diag}\big(e^{2U}, -e^{-2U}, -e^{-2U}, -e^{-2U}\big), \qquad U \equiv \frac{\Phi}{c^2}$$

$$g_{\mu\nu} \frac{dx^\mu}{dt'} \frac{dx^\nu}{dt'} = e^{2U(\mathbf{x}(t'))} c^2 - e^{-2U(\mathbf{x}(t'))} \left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right)$$

$$\dot{x}^2 + \dot{y}^2 + \dot{z}^2 = \mathbf{v}(t') \cdot \mathbf{v}(t') \equiv v^2(t')$$

$$s(t, \mathbf{x}) = \int_{t_0}^{t} \sqrt{e^{2U(\mathbf{x}(t'))} c^2 - e^{-2U(\mathbf{x}(t'))} v^2(t')}\; dt'$$

$$ds = \sqrt{e^{2U(\mathbf{x})}c^2 - e^{-2U(\mathbf{x})}v^2}\ dt$$

$$ds^2 = \left(e^{2U}c^2 - e^{-2U}v^2\right) dt^2$$

$$d\mathbf{x}^2 = \left(\frac{d\mathbf{x}}{dt}\cdot\frac{d\mathbf{x}}{dt}\right) dt^2 = v^2\, dt^2$$

$$ds^2 = e^{2U}c^2 dt^2 - e^{-2U}d\mathbf{x}^2$$

$$d\ell^2 \equiv d\mathbf{x}^2 = dx^2 + dy^2 + dz^2$$

$$ds^2 = e^{2U}c^2 dt^2 - e^{-2U}d\ell^2$$

$$\frac{d\tau}{dt} = \frac{1}{c}\, ds/dt = \frac{1}{c}\sqrt{e^{2U}c^2 - e^{-2U}v^2} = e^U\sqrt{1 - e^{-4U}\frac{v^2}{c^2}}$$

$$\text{(timelike)}\quad ds = c\, d\tau, \qquad s = \int ds = c\int d\tau = c\,(\tau - \tau_0)$$

$$\text{(null)}\quad ds^2 = 0 \implies e^{2U}c^2dt^2 = e^{-2U}d\ell^2 \implies c\, dt = e^{-2U}d\ell \implies |d\mathbf{x}/dt| = c\, e^{2U}$$

$$n \equiv e^{-2U} \quad\implies\quad c\, dt = n\, d\ell, \qquad |d\mathbf{x}/dt| = \frac{c}{n}$$

$$\text{(reparam.)}\quad s(\lambda) = \int_{\lambda_0}^{\lambda}\sqrt{g_{\mu\nu}\, x'^{\mu}x'^{\nu}}\ d\lambda', \quad x'^{\mu} = \frac{dx^{\mu}}{d\lambda'},\ \ \text{monotone } \lambda \ \Rightarrow\ s \text{ invariant}$$

$$\text{(four-velocity)}\quad u^{\mu} = \frac{dx^{\mu}}{d\tau}, \qquad g_{\mu\nu}u^{\mu}u^{\nu} = c^2$$

$$e^{2U}c^2\left(\frac{dt}{d\tau}\right)^2 - e^{-2U}\left(\frac{d\mathbf{x}}{d\tau}\cdot\frac{d\mathbf{x}}{d\tau}\right) = c^2$$

$$\frac{d\mathbf{x}}{d\tau} = \frac{d\mathbf{x}}{dt}\frac{dt}{d\tau} = \mathbf{v}\,\frac{dt}{d\tau} \implies \left(\frac{dt}{d\tau}\right)^2\left(e^{2U}c^2 - e^{-2U}v^2\right) = c^2$$

$$\frac{d\tau}{dt} = \frac{1}{c}\sqrt{e^{2U}c^2 - e^{-2U}v^2}$$

$$e^{\pm 2U} = 1 \pm 2U + 2U^2 + \mathcal{O}(U^3) \quad\implies\quad ds^2 \approx \left[(1+2U+2U^2)c^2 - (1-2U+2U^2)v^2\right]dt^2$$

$$\text{(weak field, low speed)}\quad \frac{d\tau}{dt} \approx 1 + U - \frac{v^2}{2c^2} + \mathcal{O}\left(U^2,\, U\frac{v^2}{c^2},\, \frac{v^4}{c^4}\right)$$

$$\text{(speed bound)}\quad ds^2 \geq 0 \implies v^2 \leq c^2e^{4U},\ \ \text{with equality (null) at } v = c\, e^{2U}$$

$$\boxed{s(t,\mathbf{x}) = \int_{t_0}^{t}\sqrt{e^{2U(\mathbf{x}(t'))}c^2 - e^{-2U(\mathbf{x}(t'))}\left(\frac{d\mathbf{x}}{dt'}\cdot\frac{d\mathbf{x}}{dt'}\right)}\, dt'}$$

6 Why Exponential? Scale-Continuity

Theorem 6.1: Scale-Continuity forces exponentials

Assume: (i) **potentials add within a scale**, $\Phi = \sum_a \Phi_a$; (ii) any **response** $\mathcal{R}$ (clock rate, wavelength, ruler length) **multiplies across nested scales**; (iii) $\mathcal{R}$ is continuous, nonzero, and normalized $\mathcal{R}(0) = 1$. Then necessarily

$$\mathcal{R}(\Phi) = \exp\left(\kappa \, \frac{\Phi}{c^2}\right),$$

for some constant κ. Laboratory limits fix $\kappa = 1$ for clocks (**Time** Law) and $\kappa = -2$ for gravity (the optical **index** Law)

Sketch Additivity and multiplicativity give **Cauchy's equation** $\mathcal{R}(\Phi_1 + \Phi_2) = \mathcal{R}(\Phi_1)\mathcal{R}(\Phi_2)$. Continuity implies $\ln \mathcal{R}(\Phi) = k\,\Phi$. Dimensional consistency forces division by c^2 The constants are fixed by local limits.

Explicit cross-scale composition If nested environments $s = 1, \ldots, N$ contribute $\Phi^{(s)}$ along the same path,

$$\frac{d\tau}{dt} = \prod_s \exp\left(\frac{\Phi^{(s)}}{c^2}\right) = \exp\left(\frac{1}{c^2} \sum_s \Phi^{(s)}\right), \qquad n = \exp\left(-\frac{2}{c^2} \sum_s \Phi^{(s)}\right)$$

Potentials add within a scale; responses multiply across scales

7 Standard Tests

Experiment 7.1: Single-Scale Reduction — First-order (weak gravity) tests

Setup Work within one resonance scale and assume gravity is weak. Define, the dimensionless potential, the pure number:

$$U \equiv \frac{\Phi}{c^2}, \qquad |U| \ll 1$$

so the line element is

$$ds^2 = e^{2U} c^2 dt^2 - e^{-2U} d\ell^2, \qquad g_{\mu\nu} = \mathrm{diag}\big(e^{2U}, -e^{-2U}, -e^{-2U}, -e^{-2U}\big)$$

Plain words: Φ is the gravitational potential (how "deep" the gravity well is); $U = \Phi/c^2$ is its unitless version. Clocks tick by $e^U \approx 1 + U$; rulers scale by $e^{-U} \approx 1 - U$; light travels as if in an index $n = e^{-2U} \approx 1 - 2U$

Weak-field metric (what changes at first order) Keep only the first term in U:

$$g_{00} \simeq 1 + 2U, \tag{8}$$
$$g_{ij} \simeq -(1 - 2U)\,\delta_{ij} \tag{9}$$

Plain words: Time runs slightly slower where Φ is more negative, and lengths are slightly stretched in the same places. This is the same first-order metric used for Einstein's classic weak-gravity tests (often summarized as "$\beta = \gamma = 1$" in PPN language)

Free fall (drop test) For slow motion ($v \ll c$),

$$\mathbf{a} = -\nabla\Phi + \mathcal{O}\Big(U\,\frac{v^2}{c^2}\Big)$$

Plain words: The acceleration $\mathbf{a}$ is just the spatial slope of Φ; all bodies fall the same way (∇ means "take the spatial slope")

Gravitational redshift (clocks at two heights) From $d\tau/dt = e^U$

$$\frac{\Delta\nu}{\nu} = +\Delta U \;=\; +\frac{\Delta\Phi}{c^2}$$

Plain words: A clock deeper in the gravity well ticks slower; the frequency shift is set only by the potential difference $\Delta\Phi$

Definition 7.1: Setup & symbol key (this section)

$$n \;=\; e^{-2U}, \qquad U \equiv \frac{\Phi}{c^2}, \qquad \Phi(r) = -\frac{GM}{r},$$

G (Newton's constant)M (central mass)r (radial distance)

c (speed of light)n (effective optical index)

Small-potential expansion ($|U| \ll 1$): $e^{-2U} \approx 1 - 2U + 2U^2 + \cdots$, hence

$$n(r) \approx 1 + \frac{2GM}{rc^2} \quad (\text{since } U = \Phi/c^2 = -GM/(rc^2) < 0).$$

We will use straight-line (unbent) paths for first-order integrals and restore symmetry by doubling the half-space contributions.

Experiment 7.2: First Order

Light bending (ray skimming a mass) Start from the optical-index law $n = e^{-2U}$ with $U = \Phi/c^2$ and $\Phi = -GM/r$. Weak field $\Rightarrow n \simeq 1 - 2U \simeq 1 + \dfrac{2GM}{rc^2}$: the mass acts like a very slight index increase toward $r\downarrow$. As you move closer to the mass (smaller r), the effective refractive index n(r) increases a little, which slows the light and gently bends it inward.

Plain words: Plain words: In the weak field, n plays the role of an effective optical index for empty space near a mass. Its value is only slightly greater than one, meaning that light moves a tiny bit slower the closer it passes to the mass. The variation of n with distance, not its absolute size, is what matters: as r decreases the index rises, so the ray is gently bent toward the region of higher n, exactly as in an ordinary lens made of glass. Here α is the total deflection angle of the light ray — how much its direction changes after passing the mass. The variable z marks distance along the reference line the ray would follow if space were perfectly flat; the mass sits near $z = 0$, and the ray comes in from $z = -\infty$ and leaves toward $z = +\infty$. Each slice dz contributes a tiny sideways bend, and adding all those small deflections from one side to the other gives the total angle α. A thin gradient bends the ray by the accumulated transverse slope:

$$\alpha \;\approx\; \int_{-\infty}^{+\infty} \partial_\perp \ln n \, dz \;\simeq\; \int_{-\infty}^{+\infty} \partial_\perp n \, dz \quad (\text{since } n \simeq 1).$$

Geometry on the reference line: $r^2 = b^2 + z^2$ and $\partial_\perp n = (\partial n/\partial r)\,(b/r)$ with

$$\frac{\partial n}{\partial r} = \frac{\partial}{\partial r}\left(1 + \frac{2GM}{rc^2}\right) = -\,\frac{2GM}{c^2}\,\frac{1}{r^2}.$$

Plain words: The geometry of the reference line is introduced so that every point along the unbent path can be described by a simple right triangle: the constant impact parameter b gives the perpendicular distance from the mass, while z runs along the line of travel. This makes the radial distance r follow $r^2 = b^2 + z^2$ everywhere on the path. Using this relation lets us express the gradient of the index purely in terms of z and integrate straight through the encounter without tracking the actual curved trajectory, which is accurate enough for the weak-field first-order case.

Plug in and integrate (the integrand is even in z, so we double $0\to\infty$):

$$\alpha = \frac{2GM\,b}{c^2} \int_{-\infty}^{+\infty} \frac{dz}{(b^2+z^2)^{3/2}} = \frac{2GM\,b}{c^2} \cdot 2 \cdot \frac{1}{b^2} = \boxed{\frac{4\,GM}{b\,c^2}}.$$

Plain words: A gentle index hill toward the mass nudges the ray; summing all tiny nudges along the pass gives the standard $4GM/(bc^2)$.
Numerical check (Sun's limb):

$$\alpha = \frac{4GM_\odot}{R_\odot c^2} \approx 8.49 \times 10^{-6} \text{ rad} \;\approx\; 1.74''.$$

Experiment 7.3: Shapiro Time Delay

Extra Travel Time Past A Mass Plain words: This relation compares how long a light signal takes to cross a region with a potential Φ against how long it would take in perfectly flat space. The small excess time Δt comes from the fact that the effective index n is slightly greater than one, so locally the light travels a little slower. Writing $dt = (n/c)\, ds$ and integrating along the path adds up all those tiny slow-downs into one total delay; replacing $n - 1$ by $-2U = -2\Phi/c^2$ converts the optical picture back into the gravitational potential form, showing directly how the potential depth determines the extra time of flight. Locally, light takes $dt = (n/c)\, d\ell$. Relative to flat space:

$$\Delta t = \frac{1}{c}\int (n-1)\, d\ell \;\simeq\; \frac{1}{c}\int (-2U)\, d\ell = \boxed{-\frac{2}{c^3}\int \Phi\, d\ell}$$

Plain words: Substituting the explicit potential $\Phi = -GM/r$ links the time delay directly to the geometry of the path. Along the straight reference line the element of distance is simply $ds \simeq dz$, and each point of the ray sits at a radial distance $r = \sqrt{b^2 + z^2}$ from the mass. Expressing everything in z makes the integral easy to evaluate: it adds up the small slow-downs on the inbound side ($z < 0$) and the outbound side ($z > 0$) of the pass. Because the situation is symmetric, the two halves contribute equally, producing the neat inverse-sine form shown below.
With $\Phi = -GM/r$ and the straight reference line ($ds \simeq dz$, $r^2 = b^2 + z^2$),

$$\Delta t = \frac{2GM}{c^3}\int_{-\infty}^{+\infty} \frac{dz}{\sqrt{b^2 + z^2}} = \frac{2GM}{c^3}\left[\sinh^{-1}\!\left(\frac{z}{b}\right)\right]_{-Z_1}^{+Z_2}.$$

Plain words: When the source and receiver are both very far from the mass, the angles involved are tiny and the ends of the path lie in the nearly flat region of space. In this limit the inverse–sine term simplifies to a logarithm, which depends only on the product of the two endpoint distances and the square of the closest approach b. The result shows that the extra delay grows slowly, only as the logarithm of distance, so even over astronomical scales the effect remains small but measurable.
For distant endpoints ($r_{1,2} \gg b$, so $\sinh^{-1} x \approx \ln(2x)$),

$$\Delta t \;\approx\; \frac{2GM}{c^3}\ln\!\left(\frac{4\, r_1 r_2}{b^2}\right).$$

Plain words: Near the mass, the "optical medium" is a touch denser, so the clocking of light slows and the signal arrives late by a small, logarithmic amount.
Numerical check (Earth–Sun–Earth, $r_1 = r_2 = 1$ au, $b \approx R_\odot$):

$$\frac{2GM_\odot}{c^3} \approx 9.85\,\mu\text{s}, \quad \ln\!\left(\frac{4r_1 r_2}{b^2}\right) \approx 12.13 \;\Rightarrow\; \Delta t \approx 119\,\mu\text{s (one-way)} \;\approx 238\,\mu\text{s (round trip)}.$$

Experiment 7.4: Perihelion advance

Slow Rotation Of The Ellipse Plain words: This is the standard Newtonian equation for a bound orbit, written in terms of the inverse radius $u = 1/r$. The term on the right, GM/h^2, sets the basic size of the orbit for a given angular momentum h, while the left-hand side describes how r changes as the body moves around the central mass. In this form each closed ellipse of classical mechanics appears as a simple cosine in $u(\varphi)$, making it easy to see how small relativistic corrections later disturb that perfect symmetry.
Write the Newtonian orbit in Binet form for $u = 1/r$ with specific angular momentum $h = r^2\dot{\varphi}$:

$$\frac{d^2u}{d\varphi^2} + u = \frac{GM}{h^2}.$$

Plain words: The exponential form of the potential slightly strengthens the inward pull when the orbiting body is closest to the mass. In the equation this appears as the extra term proportional to u^2, which grows rapidly as r becomes small. It is a tiny correction—suppressed by the factor $1/c^2$—but over many revolutions it causes the ellipse to drift forward. This $3GMu^2/c^2$ term is the mathematical trace of that slow, cumulative precession. The first post-Newtonian correction from the exponential law adds a small term:

$$\frac{d^2u}{d\varphi^2} + u = \frac{GM}{h^2} + 3\,\frac{GM}{c^2}\,u^2.$$

Plain words: To find how the small correction changes the orbit, we assume the path is almost the same as a Keplerian ellipse—just rotating very slowly with each revolution. The parameter ϵ measures that tiny mismatch: if $\epsilon = 0$ the ellipse closes perfectly, and if ϵ is non-zero the perihelion advances a little each loop. Solving to first order gives $\epsilon = 3GM/[a(1-e^2)c^2]$, so the shift per orbit is $2\pi\epsilon$. The final expression shows that the effect grows with the mass of the central body and with how tightly bound the orbit is, matching the famous $43''$-per-century advance of Mercury. Use a nearly Keplerian ansatz $u(\varphi) = u_0\,[1 + e\cos((1-\epsilon)\varphi)]$ with $u_0 = GM/h^2$ and $\epsilon \ll 1$. Keeping only first order in the small parameters yields

$$\epsilon = \frac{3\,GM}{a(1-e^2)\,c^2}\,, \qquad \Delta\varpi = 2\pi\,\epsilon = \boxed{\frac{6\pi\,GM}{a(1-e^2)\,c^2}}.$$

Here we used the Kepler relation $h^2 = GM\,a(1-e^2)$.
Plain words: The ellipse misses closing by a hair each loop; those hairs add up to a slow, steady drift of the perihelion.
Numerical check (Mercury): $a = 5.7909 \times 10^{10}\,\mathrm{m}$, $e = 0.2056$, $r_g = GM_\odot/c^2 \approx 1476.6\,\mathrm{m}$,

$$\Delta\varpi = \frac{6\pi r_g}{a(1-e^2)} \approx 5.02 \times 10^{-7}\ \text{rad/orbit} \ \approx 0.1035''/\text{orbit}.$$

About 415.2 orbits/century $\Rightarrow$ $0.1035'' \times 415.2 \approx 43.0''/\text{century}$.

8 Action Principle and Causal Continuation (Parameter-Free)

Experiment 8.1: Causal continuation — action → wave law → static law (1-scale, non-expert view)

Goal. Keep the static superposition law $\nabla^2\Phi = 4\pi G\rho_{\rm res}$ *exact* within one scale, but allow small time-changes in Φ that travel no faster than c. We build the unique linear, Lorentz-compatible continuation and show it *reduces back* to the static law.
Plain words: Every symbol in the action plays a precise role:

- The integral $\int d^4x$ means we are summing over all points in space and time.
- The prefactor $\frac{1}{8\pi G}$ sets the field's stiffness so that in the static limit we recover the classical Poisson law $\nabla^2\Phi = 4\pi G\rho_{\rm res}$. The factor of 4π here is not cosmetic—it matches the geometric flux convention used in Newtonian gravity.
- Inside the square brackets, $(\nabla\Phi)^2$ is the spatial energy density of the field: it tells how much "tension" or curvature the potential carries from point to point.
- The second term, $-(1/c^2)(\partial\Phi/\partial t)^2$, adds the time part of that energy. The minus sign ensures Lorentz symmetry: the combination space–minus–time is what makes wave propagation at speed c possible.
- Together, these two terms make Φ obey a wave equation rather than a purely static one. Small ripples in Φ now spread through space with the limiting speed c, but when those ripples stop ($\partial_t\Phi = 0$) the equation falls back to the familiar static superposition law.
- $S_{\rm matter}[g_{\mu\nu}(\Phi), \psi]$ collects everything belonging to ordinary matter. The matter fields ψ do not act on Φ directly; they only respond to the metric $g_{\mu\nu}(\Phi)$ that Φ defines. This preserves the equivalence principle: every form of matter "feels" the same geometry.
- The metric $g_{\mu\nu}(\Phi)$ is the bookkeeper that turns the scalar field into spacetime geometry. When Φ is small it reproduces the weak-field metric used earlier, and its exponential structure ensures that energy and light both see the same curved background.

Altogether, this compact expression says that the potential Φ is not just an instantaneous bookkeeping device—it is a genuine dynamical field, storing energy through its gradients and communicating changes at the universal speed c. When nothing moves, it quietly reduces to the standard static law.

Step 1 — Postulate a linear field action (no tunables)

$$\boxed{S[\Phi,\psi] = \int d^4x\, \frac{1}{8\pi G}\left[(\nabla\Phi)^2 - \frac{1}{c^2}\left(\frac{\partial\Phi}{\partial t}\right)^2\right] + S_{\rm matter}[g_{\mu\nu}(\Phi),\psi]}$$

Plain words: The first bracket is a standard *wave energy* for Φ (space-gradients and time-derivatives on equal footing with speed c). Matter fields ψ do not push Φ directly; they push it only through the metric $g_{\mu\nu}(\Phi)$. Why $1/(8\pi G)$? This normalization is fixed so that the static limit gives $\nabla^2\Phi = 4\pi G\rho_{\rm res}$ *with the correct* 4π and so that the field energy is positive.

Step 2 — Vary the action (how matter sources Φ) Plain words:

- The variation of the action shows how matter becomes a source for the field Φ
- The first group, $\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}$, comes from the field's own energy terms in the action.
  - $\nabla^2\Phi$ measures how the potential curves through space.
  - $\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}$ adds the time-curvature part, scaled by the speed of light.
  - Together they form a **wave operator**, meaning that changes in Φ spread at speed c instead of acting instantaneously.
- The second group, $\frac{1}{2}\sqrt{-g}\,T^{\mu\nu}\frac{\partial g_{\mu\nu}}{\partial\Phi}$, represents the influence of matter.
  - $T^{\mu\nu}$ is the stress–energy tensor: it contains the energy density, momentum, and pressure of matter.
  - $\sqrt{-g}$ is the geometric volume factor that appears in all curved-spacetime integrals.
  - $\frac{\partial g_{\mu\nu}}{\partial\Phi}$ tells how much the metric shifts when Φ changes; this is the channel through which matter affects the field.
- The equation therefore balances two sides: the curvature and time-variation of Φ on the left, and the matter distribution that shapes it on the right.
- When the field is static ($\partial_t\Phi = 0$), the time term vanishes and the relation collapses neatly back to the Poisson form $\nabla^2\Phi = 4\pi G\rho_{\text{res}}$.

Using $\delta S_{\text{matter}} = \frac{1}{2}\int\sqrt{-g}\,T^{\mu\nu}\,\delta g_{\mu\nu}\,d^4x$ we get

$$-\frac{1}{4\pi G}\left(\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}\right) + \frac{1}{2}\sqrt{-g}\,T^{\mu\nu}\,\frac{\partial g_{\mu\nu}}{\partial\Phi} = 0,$$

Plain words:

- When we vary the field part of the action,

$$S_\Phi = \frac{1}{8\pi G}\int\left[(\nabla\Phi)^2 - \frac{1}{c^2}\left(\frac{\partial\Phi}{\partial t}\right)^2\right]d^4x,$$

  each spatial derivative produces a factor of two from $\partial(\nabla\Phi)^2/\partial(\nabla\Phi) = 2\nabla\Phi$.
- That factor of two cancels half of the prefactor $1/8\pi G$, leaving $1/4\pi G$ in front of the Laplacian term:

$$\frac{1}{8\pi G}\times 2 \longrightarrow \frac{1}{4\pi G}.$$

  This is where the familiar 4π in the denominator of the field equation originates.
- The combination $\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}$ then comes directly from integrating the divergence of $\nabla\Phi$ and $\partial_t\Phi$ by parts, converting the squared derivatives in the action into the second derivatives that define the wave operator.
- The variation of the matter action $\delta S_{\text{matter}} = \frac{1}{2}\int\sqrt{-g}\,T^{\mu\nu}\,\delta g_{\mu\nu}\,d^4x$ contributes the coupling term $\frac{1}{2}\sqrt{-g}\,T^{\mu\nu}\frac{\partial g_{\mu\nu}}{\partial\Phi}$, which tells how the stress–energy of matter feeds back into the field.

- Putting both pieces together gives

$$-\frac{1}{4\pi G}\left(\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}\right) + \frac{1}{2}\sqrt{-g}\,T^{\mu\nu}\frac{\partial g_{\mu\nu}}{\partial\Phi} = 0.$$

  The $1/4\pi G$ factor is therefore not arbitrary—it is the natural outcome of the two-from-the-derivative and the $1/8\pi G$ in the action, chosen so that the static limit reproduces the classical Poisson equation $\nabla^2\Phi = 4\pi G\rho_{\text{res}}$.

hence the linear wave equation

$$\boxed{\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} \;=\; 2\pi G\,\sqrt{-g}\,T^{\mu\nu}\,\frac{\partial g_{\mu\nu}}{\partial\Phi}}$$

with $U = \Phi/c^2$ and the SRT single-scale metric

$$g_{\mu\nu}(\Phi) = \operatorname{diag}\bigl(e^{2U}, -e^{-2U}, -e^{-2U}, -e^{-2U}\bigr)$$

A direct derivative gives

$$\boxed{\frac{\partial g_{\mu\nu}}{\partial\Phi} = \frac{2}{c^2}\operatorname{diag}\bigl(e^{2U},\, e^{-2U},\, e^{-2U},\, e^{-2U}\bigr)}$$

Plain words: Matter curves the *rulers and clocks* via $g_{\mu\nu}(\Phi)$; the sensitivity $\partial g_{\mu\nu}/\partial\Phi$ tells the Φ-wave how strongly matter acts as a source

Experiment 8.2: Static, weak, non-relativistic limit

Recover The Superposition Law Plain words:

- Setting $\partial_t \Phi = 0$ removes all wave terms, leaving only the spatial curvature $\nabla^2\Phi$—the purely static field pattern.

- In this slow-motion limit, the geometry is almost flat ($\sqrt{-g} \simeq 1$) and the only important component of the stress–energy tensor is the energy density $T^{00} \simeq \rho_{\rm res} c^2$.

- Substituting the slow-motion approximations into the source term makes the origin of the 4π factor transparent. In the matter part of the equation, only the component T^{00} is significant because matter is essentially at rest. This component represents the energy density of the rest mass: $T^{00} \simeq \rho_{\rm res} c^2$. The metric component that couples to it, $g_{00} = e^{2\Phi/c^2}$, changes with Φ according to $\dfrac{\partial g_{00}}{\partial \Phi} = \dfrac{2}{c^2} e^{2\Phi/c^2} \simeq \dfrac{2}{c^2}$ for weak fields. When we insert these into the matter term $\frac{1}{2}\sqrt{-g}\, T^{00} \frac{\partial g_{00}}{\partial \Phi}$, the square-root factor is unity in the flat limit, and we obtain

$$\tfrac{1}{2} T^{00} \tfrac{\partial g_{00}}{\partial \Phi} \;\simeq\; \tfrac{1}{2}\left(\rho_{\rm res} c^2\right)\left(\tfrac{2}{c^2}\right) \;=\; \rho_{\rm res}.$$

  The first $1/2$ in front of the term cancels the $2/c^2$ from the derivative of the metric, leaving exactly the rest-mass density $\rho_{\rm res}$ as the effective source. Because the field part of the equation already carried the normalization $1/4\pi G$, this substitution closes the loop: the constants combine into $\nabla^2\Phi = 4\pi G \rho_{\rm res}$, reinstating the classical Poisson form with the correct numerical coefficient and sign, and without any adjustable parameters.

- The result is therefore $\nabla^2\Phi = 4\pi G \rho_{\rm res}$, exactly the Newtonian superposition law, with no leftover constants or arbitrary scaling.

- This closure shows that the extended wave form of the theory smoothly contracts to the standard static field when time changes vanish—one law covering both regimes without adjustment.

For slowly changing configurations ($\partial_t \Phi = 0$), small U, and matter at rest,

$$\sqrt{-g} \approx 1, \qquad T^{00} \approx \rho_{\rm res}\, c^2, \qquad T^{ij} \ll T^{00}$$

Keeping the leading term on the right:

$$\nabla^2 \Phi \;\simeq\; 2\pi G \left(\rho_{\rm res}\, c^2\right) \frac{2}{c^2} \;=\; 4\pi G\, \rho_{\rm res}.$$

Plain words: In the static limit the wave law *collapses back* to the exact linear superposition law, with the same $4\pi G$ you started from. No extra factors, no fits.

Experiment 8.3: Causality and positivity checks

Plain words:

- The expression for ε_Φ comes directly from the field part of the action: each spatial gradient $(\nabla\Phi)^2$ represents stored "elastic" energy in how the potential curves through space, while the time-derivative term $(\partial_t\Phi)^2/c^2$ adds the energy of motion when the field changes in time.
- The factor $1/8\pi G$ fixes the scale so that when Φ is static the energy density reduces to the familiar Newtonian form $\frac{(\nabla\Phi)^2}{8\pi G}$ used in gravitational self-energy calculations.
- The sum of the two terms is always positive, meaning that waves or ripples in Φ carry real, positive energy and can travel as physical signals limited by the speed c—never instantaneously.
- By choice, we do not include this Φ-energy itself as a source term on the right-hand side of the field equation. Within a single observational scale, that omission keeps the theory perfectly linear: two independent potentials simply add without cross-terms or self-coupling.
- This "single-scale linearity" makes the bookkeeping clean—matter determines the field, but the field's own stored energy does not feed back into the same equation. Across widely separated scales that feedback could be restored, but inside one domain it is deliberately suppressed so the superposition principle remains exact.

Field energy density from the action

$$\boxed{\varepsilon_\Phi = \frac{1}{8\pi G}\left[(\nabla\Phi)^2 + \frac{1}{c^2}\left(\frac{\partial\Phi}{\partial t}\right)^2\right] \geq 0}$$

Plain words: Disturbances in Φ carry positive energy and move at speed c; nothing propagates instantaneously.

Modeling choice (single-scale linearity). We *do not* add Φ's own stress–energy as a source on the right. Plain words: Within one scale, this keeps Φ exactly linear so that potentials add (clean superposition). Matter sources; the Φ field itself does not.

Experiment 8.4: Optional variant

Curved propagation Plain words:

- The operator $\Box_g$ is the curved-space generalization of the ordinary wave operator. It replaces simple partial derivatives by ones that include the geometric factors of the metric $g_{\mu\nu}$ and its determinant g.
- The prefactor $\frac{1}{\sqrt{-g}}$ and the enclosed $\sqrt{-g}\, g^{\mu\nu}$ together ensure that the divergence is taken correctly in a curved volume element; this guarantees that energy and momentum remain conserved even when the background geometry is not flat.
- In regions where the field is weak and the metric is nearly Minkowskian, $\Box_g \Phi$ reduces smoothly to the flat operator $\nabla^2\Phi - \frac{1}{c^2}\partial_t^2\Phi$, so the static limit and all first-order results stay intact.
- Allowing the metric to enter the operator makes the theory slightly nonlinear: the geometry created by Φ can now bend the very field that defines it. This is called "back-reaction."
- The curved operator is therefore optional but useful whenever that feedback is important—such as in strong fields, near compact objects, or in cosmological calculations—while in laboratory or solar-system conditions the simpler linear form remains an excellent approximation.

If desired, promote the flat wave operator to the metric one:

$$\Box_g \Phi = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \left(\sqrt{-g}\, g^{\mu\nu} \frac{\partial \Phi}{\partial x^\nu} \right)$$

Plain words: This keeps the same static limit but makes superposition only *approximately* linear in weak fields (useful when back-reaction of geometry on Φ is important).

9 Quantum-Scale Resonance Division (Bottom Anchor)

Principle 9.1: Resonance division

Quantization is *boundary-induced resonance division* of the universal wave. Standing-wave closure $\oint \mathbf{k} \cdot d\boldsymbol{\ell} = 2\pi n$ with $p = \hbar k$ and the SRT time gauge (Time Law) yields discrete spectra without adding a separate quantization axiom.

Example (hydrogen, WKB sketch) In a central well, phase closure reproduces $E_n \propto -1/n^2$ at leading order; within a *single* scale, SRT's metric factors rescale time but preserve the known spectrum, while enabling cross-scale compounding if paths sample multiple scales.

10 Cosmic-Scale Exponential Response (Top Anchor)

Experiment 10.1: Cosmic redshift & distance from the background generator (single scale)

Setup (what is the "background" here?) Let $\Phi_{\mathrm{bg}}(t)$ be the *spatially averaged* potential of the cosmic spacitron (no local lumps) Define the unitless generator $U_{\mathrm{bg}}(t) \equiv \Phi_{\mathrm{bg}}(t)/c^2$ From the Time Law,

$$\frac{d\tau}{dt} = e^{U_{\mathrm{bg}}(t)} = e^{\Phi_{\mathrm{bg}}(t)/c^2}$$

Plain words: the *background* sets how fast ideal clocks tick everywhere at the same cosmic time; local structures are treated separately.
From this Time Law, the ratio of proper times for emitter (deep) and observer (high) is:

$$\frac{(d\tau/dt)_0}{(d\tau/dt)_{\mathrm{e}}} = e^{(\Phi_0 - \Phi_{\mathrm{e}})/c^2} > 1$$

Redshift as two clean factors (background × structures) Consider a photon emitted at time t_{e} and received at t_0 SRT's time and optical laws give

$$\boxed{1 + z = \underbrace{\exp\Big(-\frac{\Phi_{\mathrm{bg}}(t_0) - \Phi_{\mathrm{bg}}(t_{\mathrm{e}})}{c^2}\Big)}_{\text{background}} \times \underbrace{\exp\Big(-\frac{\Phi_{\mathrm{struc}}(\mathrm{obs}) - \Phi_{\mathrm{struc}}(\mathrm{em})}{c^2}\Big)}_{\text{endpoints (static lumps)}} \times \underbrace{\exp\Big(-\frac{1}{c^2}\int_{\mathrm{path}} \partial_t \Phi_{\mathrm{struc}}\, dt\Big)}_{\text{time-varying lumps (ISW-like)}}} \tag{C1}$$

Plain words: redshift splits into (i) a *background* factor (how the average clock speed changed between emission and observation), and (ii) a *line-of-sight* factor from crossing lumps (galaxies, clusters), derived from the optical index $n = e^{-2\Phi/c^2}$ If the path runs through "average" space, the second factor is 1

$$\exp\Big(-\frac{\Phi_{\mathrm{bg}}(t_0) - \Phi_{\mathrm{bg}}(t_{\mathrm{e}})}{c^2}\Big) = \frac{a(t_0)}{a(t_{\mathrm{e}})}, \qquad a(t) \equiv \exp\Big(-\frac{\Phi_{\mathrm{bg}}(t)}{c^2}\Big)$$

With the usual normalization of the scale factor a(t) derived directly from the background potential $a(t_0) = 1$,

$$\boxed{1 + z = \frac{1}{a(t_{\mathrm{e}})} \exp\Big[-\frac{\Phi_{\mathrm{struc}}(\mathrm{obs}) - \Phi_{\mathrm{struc}}(\mathrm{em})}{c^2} - \frac{1}{c^2}\int_{\mathrm{path}} \partial_t \Phi_{\mathrm{struc}}\, dt\Big]} \tag{C2}$$

Plain words: $a(t)$ is not an extra assumption—it *emerges* from the background generator. $H = \dot{a}/a$ is just the time-rate of that background change. $a(t)$ is a single function that tells how the universe itself changes with time. It describes how space becomes optically "thinner" and how distances between objects appear to grow as the background potential of the universe evolves. When $a(t)$ increases, it means light waves stretch, clocks tick faster, and the universe looks larger and more diffuse; when $a(t)$ was smaller in the past, everything was denser, time flowed more slowly, and light was more compressed. In SRT, this scale factor is not an arbitrary parameter—it directly reflects the changing energy of the background resonance field that underlies all matter and space. We expressed the redshift terms without factorization at first because the effects that produce it arise in different ways and need to be identified before combining them. The background term comes from the average cosmic potential, the static term comes from fixed differences between the emitter and observer, and the path term comes from time changes in the potential as the light travels. Writing them separately makes it clear that each acts independently and multiplies its influence on the photon's frequency. Once that independence is established, the factorization follows naturally from the rule that responses at each stage multiply — in other words, small exponential changes at different points combine into one overall exponential when added in the exponent. That is why we could later merge them into a single compact form representing the same total effect. When the factorized expression is expanded, a difference involving the first term appears in both sub-terms because the total change in potential is measured relative to the same reference point — the observer. In the factorized form, the exponential for each contribution shares this common endpoint, so when you expand it back into separate pieces, the background and structural parts each carry their own comparison between emission and observation. That duplication of the upper reference level is not an error but a reflection of how every contribution to redshift is defined with respect to the same final clock: both the cosmic background and the local structure terms describe how conditions at emission differ from those at observation, so the observer's potential naturally reappears in both

Definition 10.1: Algebraic bridge (C1 → C2)

$$1+z = \exp\Big(-\tfrac{\Phi_{\rm bg}(t_0)-\Phi_{\rm bg}(t_{\rm e})}{c^2}\Big)\,\exp\Big(-\tfrac{\Phi_{\rm struc}(\rm obs)-\Phi_{\rm struc}(\rm em)}{c^2}\Big)\,\exp\Big(-\tfrac{1}{c^2}\int_{\rm path}\partial_t\Phi_{\rm struc}\,dt\Big)$$

$$=: e^A\,e^B\,e^C = e^A\,e^{B+C}$$

$$A := -\tfrac{\Phi_{\rm bg}(t_0)-\Phi_{\rm bg}(t_{\rm e})}{c^2},\quad B := -\tfrac{\Phi_{\rm struc}(\rm obs)-\Phi_{\rm struc}(\rm em)}{c^2},\quad C := -\tfrac{1}{c^2}\int_{\rm path}\partial_t\Phi_{\rm struc}\,dt$$

$$a(t) := \exp\Big(-\tfrac{\Phi_{\rm bg}(t)}{c^2}\Big),\qquad a(t_0)=1 \implies e^A = \frac{a(t_0)}{a(t_{\rm e})} = \exp\Big[-\frac{\Phi_{\rm bg}(t_0)-\Phi_{\rm bg}(t_{\rm e})}{c^2}\Big]$$

$$\boxed{1+z = \frac{1}{a(t_{\rm e})}\,\exp\Big[-\frac{\Phi_{\rm struc}(\rm obs)-\Phi_{\rm struc}(\rm em)}{c^2} - \frac{1}{c^2}\int_{\rm path}\partial_t\Phi_{\rm struc}\,dt\Big]}$$

Law 10.1: Meaning of "high-z" (large redshift regime)

Definition of redshift

$$1+z = \frac{\lambda_{\rm obs}}{\lambda_{\rm em}} = \frac{\nu_{\rm em}}{\nu_{\rm obs}},$$

where λ is wavelength and ν frequency. $z>0$ means the light is stretched (redshifted); $z<0$ means compressed (blueshifted).

Meaning of high-z. "High-z" means *large redshift*—light emitted when the Universe (or the cosmic spacitron background) was much younger and smaller. Typical guideposts:

$$\begin{aligned}
z &\lesssim 0.1 &&: \text{ nearby / local Universe,}\\
z &\sim 1 &&: \text{ intermediate epoch (half cosmic age),}\\
z &\gtrsim 5 &&: \text{ high-}z\text{ (first billion years),}\\
z &\gtrsim 10 &&: \text{ very high-}z\text{ (reionization era).}
\end{aligned}$$

High-z therefore means photons emitted when the background potential $\Phi_{\rm bg}(t)$ was very different from its present value. The exponential factor grows with that difference, making

small variations in $\dot{\Phi}_{\rm bg}$ or $\ddot{\Phi}_{\rm bg}$ observable as curvature in the distance–redshift relation.
Plain words. "High-z" simply means looking far back in time. At high redshift the light is highly stretched, the Universe younger and denser, and in SRT the cosmic background potential—and thus the resonance scale factor $a(t) = e^{-\Phi_{\rm bg}/c^2}$—was markedly different from today.

Experiment 10.2: Distance from null propagation in the optical metric

Plain words:

- Fermat's law gives the local travel time of light as $dt = (n/c)\,d\ell$, where n is the effective refractive index of space and $d\ell$ is the small physical distance element measured along the ray.
- In the SRT background, $n = e^{-2U}$ with $U = \Phi/c^2$. When only the smooth cosmic potential $\Phi_{\rm bg}(t)$ is present (no local structure), this index depends on time but not on position:
$$n_{\rm bg}(t) = e^{-2U_{\rm bg}(t)} = \exp\Big[-2\Phi_{\rm bg}(t)/c^2\Big] \equiv [a(t)]^2.$$
- The function $a(t)$ is the **scale factor**: it measures the relative stretching of all spatial distances with cosmic time. Its present value is $a_0 = a(t_0)$, and by convention $a_0 = 1$ today. When a was smaller in the past, light had to cross a "thicker" optical background ($n_{\rm bg}$ larger).
- The relation $dt = (a^2/c)\,d\ell$ then means that the optical path element changes as the Universe expands: as $a(t)$ grows, the factor $1/a^2$ in $d\ell = (c/a^2)\,dt$ acts as a time-varying optical thinning of space. Light effectively moves through an index that decreases with time.
- Integrating this relation for a radial null path from emission time $t(z)$ to the present t_0 defines the **comoving distance**
$$D(z) = \int_{t(z)}^{t_0} \frac{c}{a^2(t)}\,dt = \int_{a(z)}^{a_0} \frac{c\,da}{a^3 H(a)},$$
where $H(a) = \dot{a}/a$ is the **Hubble function** (the fractional rate of expansion) and z is the **redshift**, defined by $1 + z = a_0/a$.
- The first integral expresses distance directly in time units; the second rewrites it in terms of the evolving scale factor and its expansion rate. Both have the same meaning: they give how far a photon travels through the changing optical background between emission and observation.
- In this interpretation, cosmic expansion and gravitational potential are two sides of the same exponential law. The growth of $a(t)$ represents a gradual relaxation of the background potential $\Phi_{\rm bg}$, which makes the Universe appear optically thinner to passing light.

For light, Fermat's law gives $dt = \frac{n}{c}\,d\ell$ with $n = e^{-2U}$ In the background only ($\Phi_{\rm struc} = 0$),
$$n_{\rm bg}(t) = e^{-2U_{\rm bg}(t)} = \exp\Big(-\frac{2\Phi_{\rm bg}(t)}{c^2}\Big) = \big[a(t)\big]^2.$$
Hence
$$dt = \frac{a^2(t)}{c}\,d\ell \implies d\ell = \frac{c}{a^2(t)}\,dt$$
Integrating a radial null path from emission to observation defines the comoving distance, **D is the optical comoving distance**
$$\boxed{D(z) = \int_{t(z)}^{t_0} \frac{c}{a^2(t)}\,dt = \int_{a(z)}^{a_0} \frac{c\,da}{a^3\,H(a)}\,, \qquad (a_0 \equiv a(t_0))} \tag{C3}$$

Definitions:

$$a(t) = \exp\left[-\frac{\Phi_{\rm bg}(t)}{c^2}\right], \qquad H(t) = \dot{a}/a = -\,\dot{\Phi}_{\rm bg}(t)/c^2, \qquad \frac{d\Phi_{\rm bg}}{da} = -\frac{c^2}{a}$$

Plain words: the background acts like a time-varying "optical thinning" ($1/a^2$) of space for light travel. Rewriting with a and H gives the standard-looking distance integral, but here it comes straight from $\Phi_{\rm bg}$

Note on terminology: "optical comoving distance." In standard cosmology, the comoving distance is defined in the Friedmann–Robertson–Walker (FRW) spacetime, whose line element is $ds^2_{\rm FRW} = c^2 d\tau^2 - a^2(\tau) d\ell^2$. There, $a(\tau)$ is the FRW **scale factor**, and comoving observers measure proper time $d\tau$ along worldlines at fixed spatial coordinates. The usual FRW comoving distance is then $\chi = \int c\, d\tau / a(\tau)$.
In Spacitron Resonance Theory, our coordinate time t is the *universal time* defined by the Time Law, $d\tau/dt = e^{\Phi_{\rm bg}/c^2} = a^{-1}(t)$ with $a(t) \equiv e^{-\Phi_{\rm bg}(t)/c^2}$. Light propagation in the exponential optical metric obeys $c\, dt = a^2(t)\, d\ell$; therefore, integrating $d\ell = c\, dt/a^2$ defines

$$D(z) = \int_{t(z)}^{t_0} \frac{c}{a^2(t)}\, dt = \int_{a(z)}^{a_0} \frac{c\, da}{a^3 H(a)},$$

which we call the **optical comoving distance**. It differs from the FRW χ by one extra factor of a because the SRT optical metric uses a^2 as the effective refractive index of space.
We use this form because in SRT all cosmic distances and redshifts are derived directly from the single background generator $\Phi_{\rm bg}(t)$, with $a(t) = e^{-\Phi_{\rm bg}/c^2}$ and $H(t) = \dot{a}/a = -\dot{\Phi}_{\rm bg}/c^2$. This choice preserves the same observational structure as FRW cosmology but grounds the scaling in the exponential Time Law rather than in an assumed expanding manifold.

What structures do magnification and shear The full index is

$$n(\mathbf{x}, t) = e^{-2\Phi(\mathbf{x},t)/c^2} = e^{-2\Phi_{\rm bg}(t)/c^2}\, e^{-2\Phi_{\rm struc}(\mathbf{x},t)/c^2} = a^2(t)\, \exp\Big(-\tfrac{2}{c^2}\Phi_{\rm struc}\Big).$$

Plain words:

- The operator $\nabla_\perp$ means the gradient taken *perpendicular* to the line of sight—across the sky rather than along it. Wherever the refractive index n varies sideways, light rays are deflected toward the regions of higher n, producing apparent bending and magnification.
- The relation $\nabla_\perp \ln n = -\frac{2}{c^2}\nabla_\perp \Phi_{\rm struc}$ connects this optical gradient directly to the local potential $\Phi_{\rm struc}$ of matter "lumps" such as galaxies or clusters. Steeper potentials make steeper index gradients and therefore stronger lensing.
- The quantity ψ is the **lensing potential**: it gathers the line-of-sight contribution of the structure potential through the integral $\psi \propto \int (\Phi_{\rm struc}/c^2)\, ds$, where s measures distance along the light path. The second derivative $\nabla_\perp\nabla_\perp\psi$ forms the distortion matrix that encodes both focusing (magnification) and stretching (shear) of the image.
- The scalar μ is the **magnification**, given by $\mu \simeq 1/\det(I - \nabla_\perp\nabla_\perp\psi)$. When this determinant approaches zero, the focusing becomes extreme—multiple images or arcs appear.

- In this picture, the smooth background potential $\Phi_{\rm bg}$ fixes the overall optical "level"—the baseline index of space—while local overdensities $\Phi_{\rm struc}$ provide the fine gradients that act as gravitational lenses. If several structures lie along one line of sight, their exponential responses multiply, so their lensing effects compound naturally through the same $e^{-2\Phi/c^2}$ law.

Transverse gradients bend and magnify:

$$\nabla_\perp \ln n = -\frac{2}{c^2}\nabla_\perp \Phi_{\rm struc}, \quad \mu \sim \frac{1}{\det\left(I - \nabla_\perp \nabla_\perp \psi\right)}, \quad \psi \propto \int \frac{\Phi_{\rm struc}}{c^2}\, ds$$

Plain words: the background sets the overall "optical level," while *lumps* supply the gradients that focus (magnify) and shear images. Multiple structures along one line of sight compound their effects multiplicatively in the exponent.

E. Units and quick checks

$$[U_{\rm bg}] = 1, \quad [H] = {\rm s}^{-1}, \quad [D] = {\rm m}, \quad [1+z] = 1$$

If structures are absent, $1+z = \frac{a(t_0)}{a(t_{\rm e})} = \frac{1}{a(t_{\rm e})}$ and $D(z) = \int c\, dt/a^2(t)$ Small potential changes give $\ln(1+z) \approx -\frac{\Phi_{\rm bg}(t_0) - \Phi_{\rm bg}(t_{\rm e})}{c^2}$
as expected from the Time Law

Takeaway. Cosmic redshift and distances follow directly from one object—the background generator $\Phi_{\rm bg}(t)$. $a(t) = e^{-\Phi_{\rm bg}/c^2}$ and $H = -\dot{\Phi}_{\rm bg}/c^2$ are not extra hypotheses; they are *relabels* of $\Phi_{\rm bg}$ Local structures only enter through the path exponential, consistent with the same optical law $n = e^{-2\Phi/c^2}$

11 Worked Examples and Dimensional Checks

Experiment 11.1: Single scale): point mass, uniform sphere, units, gauge

Plain words:

- The potential Φ adds linearly from all matter sources: each mass contributes its own $-GM/r$, and the total field is just their sum. The dimensionless form $U = \Phi/c^2$ then drives every physical channel—time, optics, and dynamics—through simple exponentials.
- For a single point mass M at the origin, the source term is $\rho_{\rm res}(\mathbf{x}) = M\,\delta^3(\mathbf{x})$. Solving the static field equation gives the familiar $1/r$ potential $\Phi(r) = -GM/r$ with $r = |\mathbf{x}|$.
- The factor $\Gamma_t(r) = e^{\Phi/c^2}$ describes how a local clock ticks relative to one far from the mass. Because Φ is negative, $\Gamma_t < 1$: clocks deeper in the potential well run more slowly.
- The effective refractive index for light is $n(r) = e^{-2\Phi/c^2}$. Since $\Phi < 0$, this makes $n > 1$, meaning that light propagates slightly slower near the mass and its path bends toward regions of stronger gravity.
- The first-post-Newtonian bending angle $\alpha_{(1\mathrm{PN})} = \dfrac{4GM}{b\,c^2}$ shows how this optical effect reproduces the known deflection of a grazing ray, where b is the impact parameter (closest approach).
- The radial derivative $a_r = -\partial_r\Phi = -GM/r^2$ gives the familiar inward acceleration; it provides the Newtonian limit of the same potential and anchors the theory to everyday gravitational intuition.
- In summary: Φ adds up linearly from all sources, while its exponential responses—Γ_t for time and n for light—translate that single scalar field into all observable gravitational effects: slower clocks, bent rays, and the inverse-square pull.
- The notation $\mathcal{O}\big((GM/(b\,c^2))^2\big)$ marks the next-order corrections: as $r\to\infty$ or the field becomes weaker, all higher-order terms vanish rapidly, leaving the leading $4GM/(b\,c^2)$ as the asymptotic value.

A. Point mass Source: $\rho_{\rm res}(\mathbf{x}) = M\,\delta^3(\mathbf{x})$ Solution (outside the point):

$$\boxed{\Phi(r) = -\frac{GM}{r}}, \qquad r = |\mathbf{x}|$$

Clocks and index:

$$\Gamma_t(r) = e^{\Phi/c^2} = e^{-GM/(rc^2)}, \qquad n(r) = e^{-2\Phi/c^2} = e^{2GM/(rc^2)}$$

Plain words: Near mass, clocks slow ($\Gamma_t < 1$) and light sees $n > 1$, so it bends and is delayed.

$$\boxed{\alpha_{(1\mathrm{PN})} = \frac{4\,GM}{b\,c^2} + \mathcal{O}\Big((GM/(bc^2))^2\Big)}$$

B. Uniform solid sphere ("cosmic proxy") Plain words:

- The model assumes a sphere of constant mass density ρ filling a radius R and empty space beyond it ($r > R$). The variable r measures the distance from the center of the sphere, and G is the gravitational constant.

- Inside the sphere ($r \le R$) the potential

$$\Phi(r) = -2\pi G\rho\left(R^2 - \frac{r^2}{3}\right)$$

  increases quadratically with r. The gradient of this potential,

$$\partial_r\Phi(r) = \frac{2}{3}\pi G\rho\,(2r),$$

  gives the local gravitational field strength. It grows linearly with radius: $a_r = -\partial_r\Phi = -\frac{4}{3}\pi G\rho\, r$. This means the inward pull is proportional to distance from the center, just like a spring's restoring force.

- At the surface ($r = R$) the enclosed mass is $M = \frac{4}{3}\pi\rho R^3$ and the external potential joins smoothly to the exterior solution $\Phi(r) = -GM/r$. Both Φ and $\partial_r\Phi$ are continuous there, so there is no physical discontinuity in acceleration or in time rate across the boundary.

- The function $\Gamma_t(r) = \exp\bigl[\Phi(r)/c^2\bigr]$ describes how fast a local clock runs compared with one far away. Because Φ is negative, $\Gamma_t(r) < 1$: clocks deeper inside tick more slowly. The complementary optical index

$$n(r) = \exp\Bigl[-2\Phi(r)/c^2\Bigr] = \exp\left[\frac{4\pi G\rho}{c^2}\left(R^2 - \frac{r^2}{3}\right)\right]$$

  governs the speed of light in the potential. Since $n > 1$ near the center, light travels slightly slower there.

- The parameters have simple meanings:
  - ρ — uniform mass density of the sphere [kg m^{-3}].
  - R — radius of the sphere [m].
  - r — radial position within or outside the sphere [m].
  - $M = \frac{4}{3}\pi\rho R^3$ — total mass of the sphere [kg].
  - $\Phi(r)$ — gravitational potential [m^2 s^{-2}].
  - $\Gamma_t(r)$ — local time-dilation factor (dimensionless).
  - $n(r)$ — effective refractive index for light (dimensionless).
  - a_r — radial gravitational acceleration [m s^{-2}].

- The potential's smoothness at $r = 0$ ensures the field is finite and symmetric, matching the expectation that the center of a homogeneous sphere experiences no net force.

- Physically, the interior acts as a mild gravitational lens with uniform curvature: time runs slightly slower and light slows slightly toward the center. In the cosmological analogy, this sphere provides a finite, isotropic "background potential" that represents an average cosmic environment. Its exponential factors $\Gamma_t(r)$ and $n(r)$ can be inserted directly into the redshift and distance laws to model how the cosmic background potential governs frequency shift and light propagation in large-scale space.

Source: constant density ρ for $r \le R$, zero for $r > R$ Inside ($r \le R$) the potential is finite and quadratic:

$$\boxed{\Phi(r) = -\,2\pi G\rho\left(R^2 - \frac{r^2}{3}\right)}, \qquad \partial_r\Phi(r) = \frac{4}{3}\pi G\rho\, r$$

Outside ($r \ge R$): $\Phi(r) = -GM/r$ with $M = \frac{4}{3}\pi\rho R^3$ Continuity at $r = R$: the two forms match ($\Phi \to -GM/R$) Clocks and index inside:

$$\Gamma_t(r) = \exp\Big(-\frac{2\pi G\rho}{c^2}\Big[R^2 - \frac{r^2}{3}\Big]\Big), \quad n(r) = \exp\Big(+\frac{4\pi G\rho}{c^2}\Big[R^2 - \frac{r^2}{3}\Big]\Big)$$

Plain words: The field is *linear* inside ($a_r \propto r$), the potential is *smooth* at the center, and the surface matches the point-mass exterior These background factors can be inserted directly in the redshift law, check box above.

C. Dimensional integrity (sanity checks) Plain words:

- The exponentials used throughout are always applied to dimensionless quantities. Dividing the potential Φ (which has physical units of $\mathrm{m}^2/\mathrm{s}^2$) by c^2 removes the units, leaving a pure number that can safely appear inside $\exp(\cdot)$.
- The dimensional checks confirm consistency: $[\Phi] = \mathrm{m}^2\mathrm{s}^{-2}$, $[G] = \mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}$, and $[\rho_{\mathrm{res}}] = \mathrm{kg\,m}^{-3}$. Combining G and ρ_{res} as $4\pi G\rho_{\mathrm{res}}$ yields s^{-2}, the same as $\nabla^2\Phi$, so the static field law is dimensionally balanced.
- This balance means the field equation needs no extra constants or scaling factors; the numerical coefficients such as 4π come purely from geometry (the surface area of a unit sphere).
- The action therefore depends only on the two universal constants c and G, which set the conversion between geometry, time, and energy. There are no free tuning parameters—once c and G are fixed by measurement, the entire set of exponential laws follows without adjustment.

Every exponential is of a unitless argument:

$$\left[\frac{\Phi}{c^2}\right] = 1, \quad [\Phi] = \mathrm{m}^2\,\mathrm{s}^{-2}, \quad [G] = \mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}, \quad [\rho_{\mathrm{res}}] = \mathrm{kg\,m}^{-3}$$

Static law units: $[\nabla^2\Phi] = \mathrm{s}^{-2}$ and $[4\pi G\rho_{\mathrm{res}}] = \mathrm{s}^{-2}$ match. The action (8.1) uses only c and G; there are *no* adjustable numbers.

D. Gauge of the zero level (what adding a constant does) Plain words:

- Adding a constant C to the potential Φ shifts its absolute zero but does not alter any physical gradients. Because forces depend on $-\nabla\Phi$, a uniform offset has no measurable effect—only differences or spatial changes in Φ matter.
- The transformations $\Gamma_t \to e^{C/c^2}\Gamma_t$ and $n \to e^{-2C/c^2}n$ simply rescale all local clock rates and optical indices by constant factors. These factors cancel in any ratio or derivative, so no experiment confined to one scale can detect the offset.
- Within a single domain—laboratory, planetary, or galactic—the observable quantities are $\Delta\Phi$ and $\nabla\Phi$: how much the potential changes or how steeply it slopes. The absolute level of Φ remains a matter of convention.

- Across multiple domains or cosmological epochs, the situation changes: the exponential factors along a path accumulate the differences in Φ between environments. This compounding of Φ shifts underlies the redshift and time-dilation relations derived earlier—the measurable effect is the *change* of potential between emission and observation, not its absolute value.

Let $\Phi \to \Phi + C$. Then

$$\Gamma_t \to e^{C/c^2}\Gamma_t, \qquad n \to e^{-2C/c^2} n$$

Plain words: This is a global rescale of clocks and index; forces ($-\nabla\Phi$) and all *differences* in potential are unchanged. Within one scale, only $\Delta\Phi$ and $\nabla\Phi$ are observable; across scales, compounding tracks the *change* of Φ along paths (as in the redshift factorization)

12 Light Bending

Light Bending (Numerical Evidence, 1PN–2PN)

Alphabet of symbols

- **Meaning of the azimuthal coordinate φ.** The symbol φ (Greek letter phi) marks the *azimuthal angle* within the orbital plane. It measures how far the light ray has rotated around the central mass, counted from the direction of closest approach ($\varphi = 0$ at $r = r_0$). Its differential $d\varphi$ enters the solid-angle element $d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$
- $A(r)$: time-part of the metric (multiplies $c^2 dt^2$)
- $B(r)$: space-part of the metric (multiplies $dr^2 + r^2 d\Omega^2$)
- $d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$ (solid-angle element).
- $m := GM/c^2$ (mass written as a length; for the Sun $m \approx 1.4766 \times 10^3$ m)
- $\mu := m/r$ (dimensionless potential size at distance r)
- r_0: closest distance of the light ray to the mass (closest approach)
- b: impact parameter (how far the straight line would miss the center)
- $\varepsilon := m/b$ (our one and only smallness parameter).
- E, L: conserved "energy-like" and "angular-momentum-like" constants that label the ray; they only appear via $b = (Lc)/E$.
- **Bundled coefficients** (just shortcuts that keep formulas short): $A_* := 2(b_1 - a_1)$, $C_* := 2b_2 - 2a_2 + 4a_1^2 - 4a_1 b_1$

Why ε is the right knob (where it comes from) Along the path, the potential size is $|\Phi|/c^2 \sim m/r$ For a glancing ray, the relevant scale is $r \sim b$ Hence $|\Phi|/c^2 \sim \varepsilon = m/b \ll 1$. $\Rightarrow$ A tidy series in ε: 1PN $= O(\varepsilon)$, 2PN $= O(\varepsilon^2)$.
How the *exponential metric* appears and already contains 2PN Let $U := \Phi/c^2 = -m/r$. Empirically (redshift) clocks at radius r slow by e^U, so $\sqrt{A} = e^U \Rightarrow A = e^{2U}$ Isotropy of space with local light speed c then implies $B = e^{-2U}$ Expanding to second order:

$$A = 1 + 2U + 2U^2 + O(U^3), \qquad B = 1 - 2U + 2U^2 + O(U^3$$

Comparing with $A = 1 + 2a_1\mu + 2a_2\mu^2$ and $B = 1 + 2b_1\mu + 2b_2\mu^2$ (with $\mu = U$):

$$a_1 = -1,\ a_2 = +1, \quad b_1 = +1,\ b_2 = +1 \ \Rightarrow\ A_* = 4,\ C_* = 8 \quad \text{(these already encode 2PN)}$$

- **Why the ray stays in one plane.** Because the metric is spherically symmetric, every possible trajectory can be rotated into a single plane through the center We choose the *equatorial plane* $\theta = \pi/2$ This makes the solid-angle element $d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$ reduce to $d\Omega^2 = d\varphi^2$ along that plane This step removes no physics—it only simplifies the geometry.

- **Where the square-root denominator comes from** Starting from the optical metric $ds^2 = A\, c^2 dt^2 - B(dr^2 + r^2 d\varphi^2)$ for light ($ds^2 = 0$), the conserved quantities are

$$E = Ac^2\dot{t}, \qquad L = B\, r^2\dot{\varphi}$$

Eliminating $\dot{t}$ and $\dot{\varphi}$ and defining the measurable impact parameter $b = (Lc)/E$ gives the radial equation

$$\dot{r}^2 = \frac{E^2}{c^2 AB}\left(1 - \frac{B\,r^2}{A\,b^2}\right)$$

The square-root in Step 1, $\sqrt{\frac{B\,r^2}{Ab^2} - 1}$, is the inverse of the bracketed term—it measures how curvature alters the simple flat-space relation between r and φ

- **Why $b^2 = \dfrac{B(r_0)\,{r_0}^2}{A(r_0)}$.** At the point of closest approach r_0, $\dot{r} = 0$ (the radial motion stops momentarily). Setting the bracket above to zero gives this exact relation— it is simply the *turning-point condition* that fixes the geometric scale of the trajectory

- **Origin of $S(u)$.** Changing variables to $u = r_0/r$ compresses the infinite range $r \in [r_0, \infty)$ into $u \in (0, 1]$ and keeps every term dimensionless Substituting $r = r_0/u$ into the path equation gathers all geometry and curvature into one positive function:

$$S(u) = \frac{1 - u^2}{u^2}\Big[1 + \mu_0\,g_1(u) + {\mu_0}^2 g_2(u)\Big], \qquad \mu_0 := \frac{m}{r_0}$$

The first factor $(1 - u^2)/u^2$ is the purely geometric "flat-space" contribution; the bracket collects the small corrections caused by curvature

- **Definitions of the helper functions $g_1(u)$ and $g_2(u)$** Expanding the ratio of metric functions as $\frac{B}{A} = 1 + A_* \mu + C_* \mu^2 + O(\mu^3)$, and comparing its value at r_0/u to its value at r_0 gives

$$\frac{(B/A)(r_0/u)}{(B/A)(r_0)} = 1 + \mu_0\,g_1(u) + {\mu_0}^2\,g_2(u) + O({\mu_0}^3),$$

where

$$g_1(u) = -\frac{A_*}{1+u}, \qquad g_2(u) = -C_* + \frac{{A_*}^2}{1+u}$$

In words: g_1 represents the first-order change of the metric ratio as the light moves outward; g_2 adds the smaller, second-order correction These two dimensionless functions store all curvature information for later integration.

- **Definition of the half-deflection integral I** Once the variable change is made, the total bending angle comes from twice the integral of $d\varphi/dr$ between r_0 and infinity:

$$I = \int_0^1 \frac{du}{\sqrt{S(u)}}, \qquad \alpha_{\text{total}} = 2I - \pi$$

I is called the *half-deflection integral* because it represents one side of the symmetric path.

- **How we handle the square root in Step 4** Since curvature corrections are tiny ($\mu_0 \ll 1$), we expand the bracket of $S(u)$ using the binomial identity $(1 + x)^{-1/2} = 1 - \frac{x}{2} + \frac{3x^2}{8} + O(x^3)$ Substituting $x = \mu_0\,g_1 + {\mu_0}^2 g_2$ gives a clean power series in μ_0, allowing each order (1PN, 2PN, . . .) to be integrated separately. Integrating term by term over $u \in (0, 1]$ yields the expression for I used in Step 4:

$$I = \frac{\pi}{2} + \frac{A_*}{2}\mu_0 + {\mu_0}^2\left(\frac{\pi}{4}C_* - \frac{1}{4}{A_*}^2\right)$$

Step 1 — One-plane ray and the basic integral By symmetry set $\theta = \pi/2$. The bending is obtained from

$$\frac{d\varphi}{dr} = \frac{1}{r\sqrt{\dfrac{B\,r^2}{A\,b^2} - 1}}, \qquad b^2 = \frac{B(r_0)\,{r_0}^2}{A(r_0)}$$

Step 2 — Form the ratio B/A and organize by powers

$$\frac{B}{A} = 1 + A_*\,\mu + C_*\,\mu^2 + O(\mu^3)$$

Step 3 — Make the integral Let $u := r_0/r \in (0,1]$ and $\mu_0 := m/r_0$ Then the half-deflection integral I becomes

$$I = \int_0^1 \frac{du}{\sqrt{S(u)}}, \qquad S(u) = \frac{1-u^2}{u^2}\Big[1 + \mu_0\, g_1(u) + {\mu_0}^2\, g_2(u)\Big],$$

with simple helpers

$$g_1(u) = -\frac{A_*}{1+u}, \qquad g_2(u) = -C_* + \frac{{A_*}^2}{1+u}$$

Step 4 — Expand the square-root and integrate (elementary) Using $1/\sqrt{1+x} \approx 1 - \frac{x}{2} + \frac{3x^2}{8}$ and integrating $u \in (0,1]$:

$$I = \frac{\pi}{2} + \frac{A_*}{2}\,\mu_0 + {\mu_0}^2\left(\frac{\pi}{4}C_* - \frac{1}{4}{A_*}^2\right)$$

Total bending angle: $\alpha = 2I - \pi$

Step 5 — Trade μ_0 for ε (the observable knob) From $b^2 = B(r_0){r_0}^2/A(r_0)$ one gets

$$\mu_0 = \varepsilon\Big[1 + \tfrac{1}{2}A_*\varepsilon\Big] + O(\varepsilon^3)$$

Substituting cancels the ${A_*}^2$ pieces and yields the compact 2-term law:

$$\boxed{\alpha = A_*\,\varepsilon + \frac{\pi}{2}\,C_*\,\varepsilon^2 + O(\varepsilon^3)}$$

Reading this: 1PN $\propto \varepsilon$ (dominant), 2PN $\propto \varepsilon^2$ (tiny correction)

Step 6 — Numbers (Sun-grazing test) and theory labels

- **Common inputs:** $m = 1.4766 \times 10^3$ m, $b = R_\odot = 6.96 \times 10^8$ m, $\varepsilon = 2.12 \times 10^{-6}$
- **Exponential (SRT):** $a_1 = -1, a_2 = +1, b_1 = +1, b_2 = +1 \Rightarrow A_* = 4,\ C_* = 8$
- **GR (Schwarzschild, isotropic expansion):** $a_1 = -1, a_2 = +1, b_1 = +1, b_2 = 3/4 \Rightarrow A_* = 4,\ C_* = 15/2$

$$\textbf{1PN (both):}\ \alpha_{(1)} = 4\,\varepsilon = 1.75''$$

$$\textbf{2PN (SRT):}\ \alpha_{(2)} = 4\pi\,\varepsilon^2 = 11.66\ \mu\text{as},$$

$$\textbf{2PN (GR):}\ \alpha_{(2)} = \tfrac{15\pi}{4}\varepsilon^2 = 10.93\ \mu\text{as}$$

Wrap-up

- **What the bundles do** A_* and C_* are just "pre-combined" coefficients; they condense long algebra into two numbers that directly give the 1PN and 2PN strengths.
- **Scaling you can feel** Double $b \Rightarrow$ 1PN halves; 2PN quarters That is why 2PN is much harder to see
- **Take-home number** SRT vs GR differ by ~ 0.73 μas at the solar limb (far below arcsecond)

13 Shapiro Time Delay

Shapiro Time Delay Numerical Evidence, 1PN–2PN

Alphabet of symbols

- $A(r)$, $B(r)$: as in Section 12 (time-part and space-part).
- $m := GM/c^2$, $\mu := m/r$
- b: impact parameter along the (nearly straight) reference line.
- $\varepsilon := m/b$ (smallness parameter).
- **Bundled coefficients:** $A_* := 2(b_1 - a_1)$, $C_* := 2b_2 - 2a_2 + 4a_1^2 - 4a_1 b_1$
- **Optical index :** $n(r) = \sqrt{B/A} = 1 + N_1 \frac{m}{r} + N_2 \frac{m^2}{r^2} + \cdots$ Here $N_1 = \frac{A_*}{2}$, $N_2 = \frac{C_*}{2} - \frac{A_*{}^2}{8}$

How the exponential metric feeds 2PN here too With $U = -m/r$, $A = e^{2U}$, $B = e^{-2U}$ (as argued in Section 12) Expanding and matching gives $a_1 = -1, a_2 = +1, b_1 = +1, b_2 = +1 \Rightarrow A_* = 4, C_* = 8 \Rightarrow N_1 = 2, N_2 = 2$ (already a 2PN statement)
Why ε still organizes PN. Along the straight reference line $r(z) = \sqrt{b^2 + z^2}$ we have $m/r \sim \varepsilon$. Hence $n - 1 \sim N_1 \varepsilon + N_2 \varepsilon^2 + \cdots$, so 1PN $\sim (m/c)$ times a logarithm, and 2PN $\sim (m^2/(bc)) \propto \varepsilon(m/c)$

- **Definition of the longitudinal coordinate z.** In the near-straight reference geometry used for the Shapiro delay, the *light ray* is approximated by a line parallel to the optical axis. The coordinate z measures distance *along* this line, increasing from the emitter at $-Z_1$ to the receiver at $+Z_2$. The shortest separation from the massive body to this line is the *impact parameter* b. Each point of the ray therefore lies at a true radial distance
$$r(z) = \sqrt{b^2 + z^2}.$$
This substitution converts the curved-space geometry into a one-dimensional integral along a nearly straight path.

- **Why trigonometric functions appear in the integrals.** The variable change $z = b\tan\psi$ makes the algebra of the distance relation $r^2 = b^2 + z^2$ especially simple: $r = b\sec\psi$ and $dz = b\sec^2\psi\, d\psi$. This substitution transforms the geometric denominators $1/r$ and $1/r^2$ into powers of $\cos\psi$, producing integrals such as $\int d\psi/\cos\psi$ and $\int d\psi/\cos^2\psi = \tan\psi$, which are the trigonometric forms evaluated in Step 2. They are purely geometric—no physics has changed—just an easier way to compute the same distances.

- **Where the delay Δt comes from.** In flat space the travel time of light across a distance element is $dt_{\text{flat}} = d\ell/c$. In the curved optical medium of the gravitational field, light experiences an effective index $n(r) = \sqrt{B/A} > 1$, so the local time element is slowed to $dt = (n/c)\, d\ell$. The *extra time* accumulated relative to the flat case is therefore
$$\Delta t = \frac{1}{c}\int (n(r) - 1)\, d\ell,$$
which, along the straight reference path where $d\ell \simeq dz$, becomes the working integral used in Step 1. This Δt is the Shapiro delay—the small excess time required for the signal to cross the slower optical region near the mass.

Step 1 — Excess light-travel time as a simple path integral

$$\Delta t = \frac{1}{c}\int \big(n(r)-1\big)\,dz = \frac{m}{c}\,N_1\int\frac{dz}{r} + \frac{m^2}{c}\,N_2\int\frac{dz}{r^2} + O\big((m/r)^3\big),$$

with $r(z)=\sqrt{b^2+z^2}$ and endpoints at large radii r_1, r_2

Step 2 — Two elementary integrals (non-expert friendly forms)

$$\int\frac{dz}{r} = \sinh^{-1}\frac{Z_2}{b} + \sinh^{-1}\frac{Z_1}{b} \xrightarrow{r_{1,2}\gg b} \ln\frac{4r_1r_2}{b^2},$$

$$\int\frac{dz}{r^2} = \frac{1}{b}\Big[\arctan\frac{z}{b}\Big]_{-Z_1}^{Z_2} \xrightarrow{r_{1,2}\gg b} \frac{\pi}{b}$$

Step 3 — General 1PN–2PN law using the bundles

$$\boxed{\Delta t = \frac{A_*}{2}\frac{m}{c}\ln\frac{4r_1r_2}{b^2} + \Big(\frac{C_*}{2} - \frac{A_*{}^2}{8}\Big)\frac{\pi\,m^2}{b\,c} + O\big(\tfrac{m^3}{b^2c}\big)}$$

Reading this: the first term is 1PN (logarithmic, geometry-wide); the second is 2PN (falls off like $1/b$).

Step 4 — Numbers for Sun-conjunction (one-way) Use $m = 1.4766\times10^3$ m, $b = R_\odot = 6.96\times10^8$ m, $r_1 = r_2 = 1\,\mathrm{au}$. Then

$$\ln\frac{4r_1r_2}{b^2}\approx 12.13,\qquad \frac{2m}{c} = 9.85\ \mu\mathrm{s},\qquad \frac{m^2}{b\,c} = 10.45\ \mathrm{ps}$$

SRT (exponential): $A_* = 4,\ C_* = 8 \Rightarrow N_1 = 2,\ N_2 = 2.$

$$\Delta t_{(1)} = \frac{2m}{c}\ln\frac{4r_1r_2}{b^2} = 119.5\ \mu\mathrm{s},\qquad \Delta t^{\mathrm{SRT}}_{(2)} = 2\pi\frac{m^2}{b\,c} = 65.7\ \mathrm{ps}.$$

GR (Schwarzschild, isotropic expansion): $C_* = 15/2 \Rightarrow N_2 = 7/4$

$$\Delta t^{\mathrm{GR}}_{(2)} = \frac{7\pi}{4}\frac{m^2}{b\,c} = 57.45\ \mathrm{ps}$$

Difference at the limb (one-way): 8.21 ps. Round-trip: 16.4 ps

Plain-words wrap-up

- **What the bundles mean** A_* sets the 1PN strength; C_* fine-tunes the 2PN strength. They are just convenient stand-ins for combinations of (a_i, b_i).
- **Scaling intuition.** The leading (1PN) delay grows only logarithmically with geometry; the 2PN delay shrinks like $1/b$, so grazing the limb maximizes it.
- **Take-home number.** A limb-grazing radio link shows an ≈ 8 ps difference (one-way) between SRT-exponential and GR at 2PN—small but within picosecond timing reach.

14 Conclusion

The theory is built around a single scalar field that governs both the rate of clocks and the propagation of light through exact exponential relations. Motion follows geodesics within an exponential optical geometry, while sources add linearly inside each physical scale. When the principle of scale continuity is applied, the exponential form is no longer an assumption but a necessity—additivity within a scale and multiplicativity across scales can coexist only through an exponential law.

From a parameter-free action, SRT yields the standard static sourcing law and a Lorentz-compatible wave equation with positive field energy and no arbitrary constants. Within a single scale, the predictions coincide exactly with General Relativity, reproducing all verified post-Newtonian results. Across scales, however, the exponential structure compounds naturally, reshaping how redshift, time delay, and lensing accumulate, and leading to distinct, quantitative differences.

The numerical analysis identifies two clear signatures at the solar limb that provide a direct and decisive test of the theory. For light bending, SRT predicts a second-order deflection larger than General Relativity by about a micro-arcsecond. For the Shapiro time delay, SRT predicts an excess of roughly some nanoseconds. Both effects arise without any adjustable parameters and are fully determined by the exponential structure of the field.

These differences are small, may be they are not yet (public data online can't confirm) measurable with modern and near-future technology: micro-arcsecond astrometry, interferometry, and picosecond-precision laser ranging now lie at this frontier. Spacitron Resonance Theory therefore stands in a uniquely decisive position—completely parameter-free and fully falsifiable. Either the exact exponential relations it predicts will be confirmed by precision observation, or they will be ruled out. In both outcomes, the test is absolute!

References

[1] Standard references on PPN bookkeeping and classic tests

[2] Optical-metric and Fermat approaches to gravitation; textbooks and reviews

[3] Comparative analyses of exponential/isotropic metrics at 2PN order