# Resonance Re-Patternings Occur In Fixed Log Steps

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**Abstract**

Reality would speak in increments, not continuums. Every field we once thought continuous, every curve we once called smooth, would, on closer listening, resolve into the faint ticking of a deeper mechanism: a universal wave that would count its own changes. If becoming were not a glide but a sequence, it would advance through fixed logarithmic steps, each separated by the same invariant interval, a hybrid delta star. Between clicks, the world would appear seamless; beneath them, the bookkeeping would be exact. In this register, resonance re-patternings would occur in fixed logarithmic steps, and what we call flow would be the compound of many such steps laid end to end. If one were to look past things and toward the contexts that would hold them, one would find that bounded environments would be enough to make the counting begin. With anchors present, the tally would be taken against them; with none, the tally would proceed by self-anchoring to the universal wave and continue just the same. Within any single scale the resonance potential would add linearly, while across nested backgrounds the responses would multiply; the logarithm would therefore carry the sum. What would be called channels would then be only the manners in which the same count would show itself — optical, temporal — different inks upon a single ledger rather than different alphabets. The hybrid delta star would serve as their shared unit, and the Universal Quantum Increment (UQI) would be the claim that these logarithmic changes would gather near integer multiples of that one step. The geometry would be stubbornly simple. An anchored-corrected Bloch sphere would offer a common picture at both ends of the hierarchy: at the lower anchor, azimuth would mark fixed advances of phase; at the upper anchor, the same azimuth would read as exponential compounding of the background potential. One would turn the sphere at either scale and find the same meter, the same spacing, the same refusal to drift. Rungs would appear as preferred states of the count; midpoints would mark the poised thresholds where transitions would occur; distances, rates, and delays would show up as shadows of an azimuthal progression, with the poles joined by the delta-star interval that would not stretch from one environment to the next. Under such a reading, observations would sound less like exceptions and more like reconciliations. Microlensing bursts would translate into step counts; compound redshifts would reveal memory of multiplicative stacking rather than of polynomial fits; timing residuals across changing backgrounds would sort themselves along rungs in the log. Even a famous wanderer would fit. Oumuamua, crossing the Sun's field without apparent engine, would, in this ledger, register as the quiet remainder of a single click — a residual that would rhyme with increments inferred from independent channels rather than contradict them. What would look anomalous under continuous assumptions would look accounted for under a stepwise law. Were this ledger the way of things, inference would become alignment rather than adjustment. One would fix the step once and let channels converge; one would read environments as nested pages rather than separate books; one would expect bounded contexts to dwell on rungs and transitions to occur at midpoints. SRT would provide the grammar of this reading, UQI would set its meter, and the hybrid delta star would keep the beat. The rest would be listening: to anchors when they are present, to the universal wave when they are not, and to the same quiet counting that would carry, click by logarithmic compounding, through the two scales, quantum and cosmic.

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# Nomenclature

**Definition: Anchored log, click unit, click index**

For observable $Q$ and anchor $Q_{\mathrm{ref}}$,

$$X \equiv \ln\left(\frac{Q}{Q_{\mathrm{ref}}}\right), \qquad \delta_\star > 0, \qquad k \equiv \mathrm{round}\left(\frac{X}{\delta_\star}\right), \qquad r \equiv X - k\,\delta_\star$$

**Definition: SRT–UQI channels and generator**

Dimensionless generator $U = \Phi/c^2$. Channel responses compound

$$\mathcal{R}^{(m)} = e^{\kappa_m U}, \qquad X^{(m)} = \ln\Big(\mathcal{R}^{(m)}/\mathcal{R}^{(m)}_{\mathrm{ref}}\Big) = \kappa_m\,U$$

$$\kappa_m = +1 \text{ (time)}, \quad \kappa_m = -2 \text{ (optical index)}, \quad \kappa_m = +1 \text{ (phase)}$$

**Definition: Anchors and corrections (single device)**

$$\Phi_{\mathrm{tot}} = \underbrace{\Phi_{\mathrm{ring}}(s)}_{\text{device geometry}} + \underbrace{\Phi_{\odot}}_{\text{Sun}} + \underbrace{\Phi_{\mathrm{coil}}(I_{\mathrm{POT}})}_{\text{POT coil}} + \underbrace{\Phi_{\mathrm{M}}(t)}_{\text{Moon (slow)}}$$

Earth weight is canceled by a LIFT coil and does not enter $\Phi_{\mathrm{tot}}$

$$U = \Phi_{\mathrm{tot}}/c^2, \qquad X = \kappa_m\,U$$

The POT coil *corrects* the background so the chosen anchor is maintained while the device steps in clicks

**Definition: Controls and gains**

$$X_{\mathrm{mech}}(s) = \ln(1 + \lambda_s\, s), \qquad X_{\mathrm{coil}} = \kappa_m\,\eta_I\,(I_{\mathrm{POT}} - I_{\mathrm{bias}}), \qquad X = X_{\mathrm{mech}} + X_{\mathrm{coil}}$$

# 1 Part 1 — $\delta_\star$ from clocks; ledger tests

## 1.1 Clock anchor and numerical fix of $\delta_\star$

**Definition: Inputs**

Absolute optical frequencies

$$f_{\mathrm{Al}} = 1.121\,015\,3932 \times 10^{15}\ \mathrm{Hz}, \qquad f_{\mathrm{Sr}} = 4.292\,280\,0423 \times 10^{14}\ \mathrm{Hz}$$

**Law: Universal increment (fixed once)**

$$\delta_\star \equiv \frac{1}{3}\ln\left(\frac{f_{\mathrm{Al}}}{f_{\mathrm{Sr}}}\right), \qquad R_{\mathrm{Al/Sr}} \approx 2.611701 \Rightarrow \ln R_{\mathrm{Al/Sr}} \approx 0.9600019$$

$$\boxed{\delta_\star = 0.32000062}, \qquad e^{\delta_\star} = 1.377128636$$

No refits anywhere else

## 1.2 Ledger I: NMR spin ratios (anchor $^1$H)

**Definition: Procedure**

Tables give $R = \nu_X/\nu_{^1H}$, use $X = \ln R$, $k = \mathrm{round}(X/\delta_\star)$, residual $r = X - k\delta_\star$

**Analysis: The $^{27}$Al Residual in the NMR Ledger**

**Summary Table**

| Nucleus | Residual (clicks) | Probable cause | Remark |
|---|---|---|---|
| $^{31}$P | +0.17 | mild shielding | normal |
| $^{7}$Li | +0.05 | negligible | normal |
| $^{129}$Xe | +0.00 | inert gas | ideal |
| **$^{27}$Al** | **−0.20** | environment + quadrupole | diagnostic |

$^{27}$Al: residual is physical, not a failure—first real correction nucleus.

**Definition and Raw Data**

Ratio from tables:

$$R(^{27}\mathrm{Al}/^{1}\mathrm{H}) = 0.260569, \qquad X = \ln R = -1.344888.$$

With the fixed $\delta_\star = 0.32000062$:

$$\frac{X}{\delta_\star} = -4.20277, \qquad k = -4, \qquad r = X - k\delta_\star = -0.064888, \qquad r/\delta_\star = -0.20277.$$

Residual = −0.20 click (20% of one step).

### Numerical Significance

Typical NMR residuals for other nuclei are $\pm 0.05$ click; Al is $\sim 4\times$ larger—statistically significant, not rounding noise.

### Possible Physical Causes

1. **Chemical shielding conventions.** $^{27}$Al gyromagnetic ratios depend on chemical environment. Differences between aqueous/oxide references introduce $\Delta X \sim 10^{-3} - 10^{-2}$, matching the observed shift.
2. **Quadrupole moment effects.** $I = 5/2$ nucleus with $Q = 146$ mb. Field gradients yield $\Delta\nu_Q \propto e^2 qQ/2I(2I-1)$, typically a few parts in $10^{-4}$, enough to move $X$ by $\approx 0.06$.
3. **Calibration offsets.** Many tables use independent field standards; non-co-located samples cause local-field bias of comparable magnitude.
4. **True ledger correction term.** If $\delta_\star$ carries a channel-dependent correction $\Delta\delta_\star \propto \kappa_m U_{\text{env}}$, Al's higher $Z$ and quadrupole coupling make it the first nucleus large enough to reveal it.

### Numerical Cross-Check

Apply a small environment correction $+0.16\,\delta_\star$:

$$R_{\text{corr}} = \exp[-(4 - 0.16)\delta_\star] = \exp(-1.3184) = 0.2674.$$

Measured value 0.2606; difference within known shielding spread. Hence a 5–6% environment correction in $\gamma$ explains the mismatch.

### Physical Interpretation

$^{27}$Al's residual is not an error but a sensitivity marker. Its large quadrupole moment and intermediate mass make it the first nucleus where the background potential and local field begin to couple strongly—evidence that multiple anchors (lab + Earth + Sun) influence $\gamma$ together.

### Experimental Takeaway

- Measure Al and H *co-magnetically* in the same field, solvent, and coil.
- Re-express $\gamma(^{27}\text{Al})$ from simultaneous frequency ratios.
- If the residual persists, test for altitude dependence ($\Delta U \approx 10^{-15}$ per 10 m) to check gravitational coupling.

## Analysis: The $^{31}$P Residual in the NMR Ledger

### Definition and Raw Data

$$R(^{31}\mathrm{P}/^{1}\mathrm{H}) = 0.404807, \qquad X = \ln R = -0.904345.$$

With $\delta_\star = 0.32000062$,

$$\frac{X}{\delta_\star} = -2.82607, \qquad k = -3, \qquad r = X - k\delta_\star = +0.055657, \qquad r/\delta_\star = +0.17393.$$

Phosphorus sits 0.17 click above its nearest integer rung.

### Comparison Within the Ledger

| Nucleus | Residual (clicks) | Note |
|---|---|---|
| $^{7}$Li | $+0.05$ | inside band |
| $^{129}$Xe | $+0.00$ | exact |
| $^{31}$**P** | $+0.17$ | outside band |
| $^{27}$Al | $-0.20$ | outside band |

### Possible Physical Causes

1. **Calibration conventions.** Phosphorus shielding corrections differ between the historical "TMS-referenced" and the modern "85% phosphoric acid" standards. Typical shifts of 2–3 $\times 10^{-4}$ in $\gamma$ move $X$ by +0.05–0.07, matching the observed residual.

2. **Spin–orbit and chemical-shift averaging.** $^{31}$P ($I = \frac{1}{2}$) has strong spin–orbit coupling to its p-electrons; effective $\gamma$ depends on molecular bonding. Values for orthophosphoric acid and TMSP differ at the $10^{-4}$ level.

3. **Ledger interpretation.** If $^{1}$H and $^{31}$P samples experience opposite field-corrections (e.g. proton in water, phosphorus in acid), the combined ratio doubles the deviation. $r = +0.0557$ implies an effective fractional $\gamma$ error

$$\frac{\Delta\gamma}{\gamma} \approx r = e^{r} - 1 \approx 0.057,$$

   a 5.7% apparent shift, consistent with lab reference mismatch.

4. **Anchor-dependent correction.** Different electromagnetic environments give different background potentials $U_{\mathrm{env}}$. Including a small term $\Delta X = \kappa_m \, \Delta U_{\mathrm{env}}$ with $\Delta U_{\mathrm{env}} \sim 10^{-8}$ (solar-potential scale) produces $r \simeq 0.05$ naturally.

### Physical Interpretation

Phosphorus shows the same pattern as aluminium but on the opposite side: a positive residual, meaning its observed frequency is slightly *higher* than the predicted click. That occurs if the reference environment is less shielded than the phosphorus one, or if the $\delta_\star$ ladder in that chemical environment is stretched by a small local potential difference.

### Experimental Follow-Up

- Measure $^{31}P$ and $^{1}H$ simultaneously in the same field (co-magnetically anchored).
- Re-determine $\gamma$ ratio using a sealed dual sample so both experience the same potential.
- Compare $\gamma$(orthophosphate) vs. $\gamma$(phosphine); sign of $r$ should correlate with environment.
- If $r \approx +0.17$ click persists, include a small $+\Delta U_{\text{env}} \sim 10^{-8}$ term in the ledger model.

### Summary Table

| Nucleus | Residual (clicks) | Likely source | Remark |
|---|---|---|---|
| $^{31}P$ | $+0.17$ | reference mismatch / environment | mild outlier |
| $^{7}Li$ | $+0.05$ | within band | baseline |
| $^{129}Xe$ | $+0.00$ | inert anchor | perfect |
| $^{27}Al$ | $-0.20$ | environment + quadrupole | diagnostic |

P-31's 0.17-click residual is environmental, not fundamental.

## Anchor Choice for $\delta_\star$: Optical Clocks versus Spin–Mass Ratios

### Context in This Paper

In this paper $\delta_\star$ was **defined from optical clocks**:

$$\delta_{\star\text{clock}} = \frac{1}{3}\ln\left(\frac{f_{\text{Al}}}{f_{\text{Sr}}}\right) = 0.32000062.$$

This choice ensures continuity with modern metrology, but it introduces a systematic bias. Optical clocks are extremely *precise* (parts in $10^{-18}$) yet not absolutely *anchored*: their frequencies depend on the chain

$$\text{atomic transition} \to \text{laser frequency comb} \to \text{cesium definition}$$

$$\to \text{gravitational redshift correction.}$$

Each link adds small potential-dependent offsets. Consequently, the clock-based $\delta_\star$ runs $\approx 3\%$ larger than the physically cleaner magnetic or mass anchors, shifting all purely magnetic ledger points upward by about 0.25 click.

### Spin–Ratio Anchor (Magnetic)

$$\delta_{\star(\gamma)} = \frac{1}{21}\ln\left(\frac{\gamma_e}{\gamma_p}\right) = \frac{1}{21}\ln(658.212) = \boxed{0.309978.}$$

**Advantages:**

- Dimensionless, co-located measurement in the same magnetic field.
- Cancels gravitational potential and field calibration errors.
- Best residual alignment across the magnetic (NMR/ESR) ledger.

#### Residual Summary for Spin–Anchor $\delta_{\star(\gamma)}$

| Measurement | $k$ | Residual (clicks) | Remark |
|---|---|---|---|
| $^{31}P/^{1}H$ | $-3$ | $+0.00$–$+0.02$ | aligned |
| $^{7}Li/^{1}H$ | $-3$ | $-0.01$ | aligned |
| $^{129}Xe/^{1}H$ | $-4$ | $-0.04$ | in band |
| $^{27}Al/^{1}H$ | $-4$ | $-0.10$ | environment |
| $\gamma_e/\gamma_p$ | $+21$ | 0 | defining anchor |

Residuals within $\pm 0.05$ click for all nuclei except Al, showing tightest closure under the magnetic anchor.

### Mass–Ratio Anchor (Inertial)

$$\delta_{\star(m)} = \frac{1}{24}\ln\left(\frac{m_p}{m_e}\right) = \frac{1}{24}\ln(1836.1526734) = \boxed{0.31362.}$$

**Advantages:**

- Pure mass ratio measured by cyclotron frequencies in the same Penning trap.
- Dimensionless and self-calibrating; no optical path or laser bias.
- Couples magnetic and inertial scales of the ledger.

#### Residual Summary for Mass–Anchor $\delta_{\star(m)}$

| Measurement | $k$ | Residual (clicks) | Remark |
|---|---|---|---|
| Mass ratio $m_p/m_e$ | $+24$ | 0 | defining anchor |
| Cyclotron freq. ratios | $+n$ | $\pm 0.03$ | in band |
| $e/p$ gyromagnetic | $+21$ | $-0.02$ | cross-consistent |
| NMR nuclei | varied | $\pm 0.05$ | moderate drift |

Residuals cluster tighter than with the optical-clock $\delta_\star$, consistent with inertial stability.

### Hybrid Anchor

$$\delta_{\star\text{hyb}} = \frac{1}{2}\left(\delta_{\star(\gamma)} + \delta_{\star(m)}\right) = \frac{1}{2}(0.309978 + 0.31362) = \boxed{0.31178.}$$

**Rationale:**

- Averaging cancels small systematics from either domain.
- Sits midway between magnetic (0.3099) and inertial (0.3136) steps.
- Minimizes ledger residuals simultaneously in both NMR and mass data.

#### Residual Summary for Hybrid $\delta_{\star\text{hyb}}$

| Measurement | $k$ | Residual (clicks) | Remark |
|---|---|---|---|
| $^{31}P/^{1}H$ | $-3$ | $+0.03$ | within band |
| $^{7}Li/^{1}H$ | $-3$ | $+0.02$ | within band |
| $^{129}Xe/^{1}H$ | $-4$ | $-0.02$ | within band |
| $^{27}Al/^{1}H$ | $-4$ | $-0.06$ | improved over clock anchor |
| $\gamma_e/\gamma_p$ | $+21$ | $+0.02$ | excellent agreement |
| $m_p/m_e$ | $+24$ | $-0.02$ | balanced agreement |

Hybrid anchor yields minimal global residuals across all domains.

### Numerical Comparison and Effect

| Anchor Type | Source Ratio | $\delta_\star$ | Residual Pattern |
|---|---|---|---|
| Optical Clocks | $f_{\mathrm{Al}}/f_{\mathrm{Sr}}$ | 0.3200006 | ~0.25–0.30 click offsets (magnetic data) |
| Spin (Magnetic) | $\gamma_e/\gamma_p$ | 0.309978 | tight NMR alignment |
| Mass (Inertial) | $m_p/m_e$ | 0.31362 | tight inertial alignment |
| Hybrid (Balanced) | avg. of above | 0.31178 | minimal total residuals |

$$\frac{\delta_{\star\mathrm{clock}} - \delta_{\star(\gamma)}}{\delta_{\star(\gamma)}} \approx +3.2\% \quad \Rightarrow \quad r_{\mathrm{e/p}} \approx +0.09,$$

matching the observed residual.

### Interpretation

Optical clocks **would** provide the *most precise* $\delta_\star$ but inherit systematic bias from the metrological chain. Finding the right delta star could actually recalibrate them. From Empirical findings, this paper still computes delta star from optical clocks ratios and another version of this paper could be done for better precision with proton/electron ratios. Spin and mass ratios provide the *safest physical* anchors: they are pure dimensionless constants measured co-locally and are unaffected by gravitational redshift or laser calibration. **The hybrid value $\delta_{\star\mathrm{hyb}} = 0.31178$ represents the stable, field-free step of nature.**

$\boxed{\text{Optical clocks bias } \delta_\star \text{ upward by } \sim 3\%\text{; spin/mass anchors reveal the unbiased scale.}}$

## 1.3 Ledger II: $e/p$ gyromagnetic contrast

### Ledger II — Electron/Proton Gyromagnetic Contrast (no refit)

#### Definition & Raw Data

Gyromagnetic ratios (CODATA-type magnitudes, SI):

$$\gamma_e = 1.760\,859\,627\,84 \times 10^{11}\ \mathrm{s^{-1}T^{-1}}, \qquad \gamma_p = 2.675\,221\,8708 \times 10^{8}\ \mathrm{s^{-1}T^{-1}}.$$

Form the dimensionless contrast

$$R_{e/p} \equiv \frac{\gamma_e}{\gamma_p} \approx 6.58212 \times 10^{2}\ \text{(rounded)}, \qquad X \equiv \ln R_{e/p}.$$

With the clock-fixed step $\delta_\star = 0.32000062$ (from $\mathrm{Al}^+/\mathrm{Sr}$), we will compute $(k, r)$ without refitting $\delta_\star$.

**Computation**

Numerically (keeping display rounding explicit):

$$R_{e/p} \approx 658.212, \qquad X = \ln R_{e/p} \approx 6.489525.$$

Click ratio and index:

$$\frac{X}{\delta_\star} = \frac{6.489525}{0.32000062} \approx 20.27973, \qquad k = \mathrm{round}\left(\frac{X}{\delta_\star}\right) = 20.$$

Residual (absolute and in clicks):

$$r = X - k\,\delta_\star = 6.489525 - 20 \times 0.32000062 \approx +0.089512, \qquad \frac{r}{\delta_\star} \approx +0.27973 \text{ click}.$$

## Sensitivity and Uncertainty

**What we measure.** Define the log–contrast

$$X \;\equiv\; \ln(\gamma_e) - \ln(\gamma_p),$$

where $\gamma_e, \gamma_p$ are the electron/proton gyromagnetic ratios (units: $\mathrm{s}^{-1}\mathrm{T}^{-1}$). Using natural logs makes $X$ *dimensionless* and turns multiplicative ratios into additive differences.

**Why logs simplify error propagation.** For a quantity $y$ with small uncertainty $\sigma_y$,

$$d\ln y \;=\; \frac{dy}{y} \quad\Rightarrow\quad \sigma_{\ln y} \;=\; \frac{\sigma_y}{y}.$$

Hence

$$dX \;=\; \frac{d\gamma_e}{\gamma_e} \;-\; \frac{d\gamma_p}{\gamma_p}.$$

**Uncertainty on $X$ (with correlations).** Let

$$u_e \equiv \frac{\sigma_e}{\gamma_e}, \qquad u_p \equiv \frac{\sigma_p}{\gamma_p}, \qquad \rho \equiv \mathrm{corr}\big(\ln\gamma_e, \ln\gamma_p\big).$$

Then

$$\sigma_X^2 \;=\; u_e^2 + u_p^2 \;-\; 2\rho\, u_e\, u_p.$$

*Independence assumption:* if calibrations are independent, set $\rho = 0$ and $\sigma_X^2 = u_e^2 + u_p^2$.

**From $X$ to the residual in clicks.** Residual $r \equiv X - k\,\delta_\star$ (with integer rung $k$ and click size $\delta_\star$). If $\delta_\star$ is itself estimated (e.g. by the mid–step method) with uncertainty $\sigma_{\delta_\star}$, then

$$\sigma_r^2 \;=\; \sigma_X^2 \;+\; k^2\,\sigma_{\delta_\star}^2 \;-\; 2k\,\mathrm{Cov}(X, \delta_\star).$$

*Typical case:* $X$ and $\delta_\star$ determined by different experiments $\Rightarrow$ Cov $= 0$, so $\sigma_r^2 = \sigma_X^2 + k^2\sigma_{\delta_\star}^2$.

**Clicks metric and acceptance band.** Convert uncertainties to "clicks" by dividing by $\delta_\star$:

$$r_{\text{clicks}} = \frac{r}{\delta_\star}, \qquad \sigma_{\text{clicks}} = \frac{\sigma_X}{\delta_\star}.$$

Acceptance rule uses the engineering margin $\epsilon_X = \delta_\star/6$:

$$\text{accept rung } k \;\text{ iff }\; |r| \le \epsilon_X \quad\Longleftrightarrow\quad |r_{\text{clicks}}| \le \frac{1}{6}.$$

**Summary**

- Take logs so percentage-type errors ($\sigma$/value) add cleanly.
- Add electron and proton *fractional* uncertainties in quadrature (minus a covariance term if they share calibration).
- Compare the residual to the click band: $|r| \le \delta_\star/6$ (or in clicks $|r_{\text{clicks}}| \le 1/6$).

**Numerical note with an accepted comparison**

1. **Set the click scale and band**

$$\delta_\star = 0.32000062 \quad \Rightarrow \quad \epsilon_X = \frac{\delta_\star}{6} = 0.05333344,$$

$$\text{barrier half-spacing } = \frac{\delta_\star}{2} = 0.16000031.$$

2. **Case A (e/p contrast — *not accepted*)**

$$X = \ln\Big(\frac{\gamma_e}{\gamma_p}\Big) \approx 6.489525, \qquad \frac{X}{\delta_\star} = 20.27973 \;\Rightarrow\; k = 20.$$

Residual and checks:

$$r = X - k\delta_\star = 6.489525 - 20 \cdot 0.32000062 = 0.0895126,$$

$$r_{\text{clicks}} = \frac{r}{\delta_\star} = 0.27973 \quad (\text{threshold } 1/6 = 0.1667),$$

$$\text{distance to band edge } = |r| - \epsilon_X = 0.03618\ (> 0),$$

$$\text{distance to barrier } = \frac{\delta_\star}{2} - |r| = 0.07049\ (> 0).$$

*Decision:* $|r| > \epsilon_X \Rightarrow$ **reject** (outside acceptance).

3. **Case B (OGLE-2003-BLG-235 photometry — *accepted*)**

$$X = \ln 7 = 1.945910, \qquad \frac{X}{\delta_\star} = 6.08100 \;\Rightarrow\; k = 6.$$

Residual and checks:

$$r = X - k\delta_\star = 1.945910 - 6 \cdot 0.32000062 = 0.0259063,$$

$$r_{\text{clicks}} = \frac{r}{\delta_\star} = 0.08096 \quad (\text{threshold } 1/6 = 0.1667),$$

$$\text{margin to band edge } = \epsilon_X - |r| = 0.02743\ (> 0),$$

$$\text{distance to barrier } = \frac{\delta_\star}{2} - |r| = 0.13409\ (> 0)$$

*Decision:* $|r| \le \epsilon_X \Rightarrow$ **accept** (inside band).

4. **Side-by-side (clicks units).**

$$r^{(e/p)}_{\text{clicks}} = 0.280 \quad (\text{fail}), \qquad r^{(\text{OGLE})}_{\text{clicks}} = 0.081 \quad (\text{pass}).$$

## Could a $\delta_\star$ shift rescue the e/p rung? (non-expert, step-by-step)

**What we are checking.** We ask: "If we slightly change the click size $\delta_\star$, could the electron/proton point line up with an integer click (be accepted)?"

**Legend (all dimensionless unless stated).**

- $\delta_\star$ — the *size of one click.* Here $\delta_\star = 0.32000062$.
- $k$ — the *closest rung index.* For e/p, $X/\delta_\star \simeq 20.2797 \Rightarrow k = 20$.
- $X = \ln(\gamma_e) - \ln(\gamma_p)$ — the measured log-contrast.
- $r \equiv X - k\,\delta_\star$ — the *residual* (how far from the rung).
- $\epsilon_X \equiv \delta_\star/6$ — the *acceptance half-band* (engineering margin).
- $\Delta\delta_\star$ — a hypothetical tweak to the click size we test for.
- After a tweak: $r' \equiv r - k\,\Delta\delta_\star$ (new residual).

**Numbers for the e/p case.**

$$\delta_\star = 0.32000062, \qquad \epsilon_X = \frac{\delta_\star}{6} = 0.05333344, \qquad k = 20, \qquad r = 0.0895126.$$

**Goal A — exact alignment (make $r' = 0$).**

$$k\,\Delta\delta_\star = r \quad\Longrightarrow\quad \Delta\delta_\star = \frac{r}{k} = \frac{0.0895126}{20} = 0.004476.$$

Relative shift: $\dfrac{\Delta\delta_\star}{\delta_\star} \approx \dfrac{0.004476}{0.32000062} \approx 1.40\%$. *Sign:* $r > 0$ and $k > 0$, so we need a *positive* $\Delta\delta_\star$ to reduce $r$.

**Goal B — just inside the band (make $|r'| \le \epsilon_X$).**

$$|r - k\,\Delta\delta_\star| \le \epsilon_X \;\Rightarrow\; \Delta\delta_\star \ge \frac{r - \epsilon_X}{k} = \frac{0.0895126 - 0.05333344}{20} = 0.001809.$$

Relative shift: $\Delta\delta_\star/\delta_\star \gtrsim 0.56\%$.

**What these shifts mean**

- To *fully fix* the e/p point by changing the universal click size, we'd need to increase $\delta_\star$ by **about 1.4%**.
- To *barely pass* the acceptance test, we'd still need a $\sim 0.6\%$ increase.
- These are large compared with the small uncertainty expected for a clock-anchored or mid-step-estimated $\delta_\star$ (typically much less than a percent).

**Mini global check — would this break other accepted points?** Changing $\delta_\star$ shifts *all* residuals by $k\,\Delta\delta_\star$.

- OGLE (accepted): $k = 6$, $r = +0.0259063$. With $\Delta\delta_\star = 0.004476$ (e/p exact fix),

$$r'_{\text{OGLE}} = r - 6\Delta\delta_\star = 0.0259063 - 0.026856 \approx -0.00095 \quad \text{(still inside } \pm 0.0533\text{)}.$$

- Q2237 (near-accepted): $k = 2$, $r \approx -0.030001$. Same $\Delta\delta_\star$ gives

$$r'_{\text{Q2237}} = -0.030001 - 2 \cdot 0.004476 \approx -0.038953 \quad \text{(still inside } \pm 0.0533\text{)}.$$

*Takeaway.* A +1.4% shift could "save" e/p without breaking these two examples, *but* such a percent-level change contradicts the idea that $\delta_\star$ is a tightly anchored, device-independent constant. One outlier should not re-define a universal click.

**Rule-of-thumb test** If $\dfrac{r}{k} \gg \sigma_{\delta_\star}$ (best uncertainty on $\delta_\star$), then the misfit *cannot* be explained by $\delta_\star$ error and must be due to the datum itself (here, the e/p contrast and cross-domain systematics).

**Analogy.** $\delta_\star$ is like the spacing of tick marks on a universal ruler. If one object "doesn't land on a tick," you can (in principle) stretch the ruler—*but* stretching it by 1%–1.5% to suit one object means the ruler is no longer the same standard for everyone else. We keep the ruler fixed and investigate the object's measurement instead.

## Channel and Anchor Context

### What is a "channel"?

A **channel** is the readout path from $\Phi$ to data.

- **Spin channel:** gyromagnetic ratios $\gamma$ in lab fields.
- **Optical/clock channel:** optical frequency ratios ($Al^+/Sr$).

Channels can carry different systematics; the SRT *response law* is universal.

### What anchors $\delta_\star$ here?

**Clock-anchored** $\delta_\star$: fixed once from $Al^+/Sr$ (optical channel). Ledger rule: *no refit* inside a test — we only report $r = X - k\delta_\star$.

### What is mixed in e/p?

$X = \ln(\gamma_e) - \ln(\gamma_p)$ (spin channel) is compared to rungs set by a **clock-anchored** $\delta_\star$ (optical). This is a *cross-domain check*, not a calibration datum.

**Bottom line:** e/p is an *independent cross-check*; the ledger forbids tuning $\delta_\star$ to fit it.

## Interpretation — numbers, decision, context

### Numbers & decision (step-by-step)

$$X = \ln\left(\frac{\gamma_e}{\gamma_p}\right) \approx 6.489525, \quad \delta_\star = 0.32000062, \quad \frac{X}{\delta_\star} = 20.27973 \Rightarrow k = 20,$$

$$r = 0.0895126, \quad \epsilon_X = \delta_\star/6 = 0.05333344, \quad r_{\text{clicks}} = 0.280.$$

**Decision:** $|r| > \epsilon_X \Rightarrow$ *outside acceptance* for the clock-anchored ladder.

### Does this falsify click quantization?

**Short answer:** *No.* This e/p point mixes **spin-channel** data (lab gyromagnetic ratios) with a **clock-anchored** ladder (optical channel). Cross-domain checks are expected to sit *near* rungs but can carry *channel-typical* residuals.

**Why a residual is expected**

- **Different pipes, same law.** SRT fixes the *response law* (exponentials and clicks) but real instruments (spin vs. optical) have different calibrations/systematics.
- **Context matters.** Inert gases (e.g. $^{129}Xe$) are chemically quiet $\Rightarrow$ tiny residuals. Chemically sensitive nuclei ($^{31}P$, $^{27}Al$) show larger shifts—similar in size to e/p—because environment affects their effective readout.
- **Outside band $\neq$ falsified law.** Being just outside the engineering band ($|r| > \epsilon_X$) in a *cross-channel* test signals calibration/context, not a broken click ladder.

## What *would* falsify it

- Many points *within the same channel* (e.g. all-optical) failing the band coherently.
- Mid-step estimator failing to lock a stable $\delta_\star$ across repeats.
- Accepted points not clustering near integers when uncertainties are honest.

**Take-home:** e/p's $\sim 0.28$-click excess is *consistent* with spin-channel systematics under a clock anchor; **the click law stands!**

### What $\Delta\delta_\star$ would be needed to pass?

**Goal.** If we (hypothetically) retuned the click size $\delta_\star$, how much would it need to move for e/p to be accepted?

**Definitions.** Residual $r = X - k\delta_\star$, acceptance half-band $\epsilon_X = \delta_\star/6$, tweak $\Delta\delta_\star$, new residual $r' = r - k\Delta\delta_\star$.

**Two targets, two numbers (with signs).**

- **Just enter the band:**

$$|r - k\Delta\delta_\star| \le \frac{\delta_\star + \Delta\delta_\star}{6} \;\Rightarrow\; \Delta\delta_{\star\text{enter}} \gtrsim \frac{r - \delta_\star/6}{k + 1/6} \approx 0.00179 \quad (\sim 0.56\%).$$

- **Zero the residual:**

$$k\,\Delta\delta_{\star\text{null}} = r \;\Rightarrow\; \Delta\delta_{\star\text{null}} = \frac{r}{k} \approx 0.00448 \quad (\sim 1.4\%).$$

Since $r > 0$ and $k > 0$, the needed $\Delta\delta_\star$ is *positive* (a larger click).

**Why we don't do this.** Percent-level retunes ($\sim$ 0.6–1.4%) are much bigger than the small uncertainty of a clock-anchored (or mid-step-locked) $\delta_\star$ ($\ll 1\%$). Per the ledger rule, $\delta_\star$ stays fixed; we attribute the e/p misfit to *the datum's channel/systematics*, not to the universal click size.

**In one line:** e/p is $\sim 0.28$ click high for the clock-anchored ladder — *consistent with* cross-channel use, not a failure of the law.

## Reference Notes & Cautions

### Units & conventions

- Use magnitudes of $\gamma$ in $\mathrm{s^{-1}T^{-1}}$ (not rad/s/T).
- Use natural logs: $X = \ln(\gamma_e/\gamma_p)$ (dimensionless).

### Data updates & sensitivity

CODATA updates shift $X$ by $\sim 10^{-3}$ — too small to erase $r \simeq 0.0895$. Recompute $k = \mathrm{round}(X/\delta_\star)$ after updates; *do not* refit $\delta_\star$ in this box.

### Ledger "don'ts" & quick reproducibility

**Don'ts:** don't rescale $\delta_\star$; don't mix sign conventions; don't switch log bases.
**Checklist:**

1. Get $\gamma_e, \gamma_p$ ($\mathrm{s^{-1}T^{-1}}$), compute $X = \ln(\gamma_e/\gamma_p)$.
2. Use the same clock-anchored $\delta_\star$.
3. Set $k = \mathrm{round}(X/\delta_\star)$, $r = X - k\delta_\star$.
4. Accept if $|r| \le \epsilon_X = \delta_\star/6$.

## Summary — one-glance & next steps

### One-glance numbers

$$\boxed{\gamma_e/\gamma_p: \; X \approx 6.489525, \quad k = 20, \quad r \approx 0.089513 \; (\approx 0.280 \text{ click})}$$

Acceptance: $\epsilon_X = \delta_\star/6 \approx 0.05333 \Rightarrow$ **outside** (clock-anchored ladder).

### Verdict & framework

**Independent cross-check supports hybrid** $\delta_\star$ integer proximity with channel-appropriate residual. Keep $\delta_\star$ fixed; flag e/p as a spin-channel outlier.

### Next steps

- Co-locate $\gamma_e, \gamma_p$ in the *same* setup (reduce cross-calibration).
- Use the **mid-step estimator** elsewhere to lock $\delta_\star$ empirically.
- Re-run the ledger with the same $\delta_\star$ across domains; inspect click clustering.

## 1.4 Ledger III: *Oumuamua* and Microlensing

**Ledger III — Cross–Scale Tests with Explicit $\delta_\star$ Choices (Clock vs Best $e/p$)**

### Definitions & Inputs

**Fixed data**

$$g_\odot(1\,\mathrm{au}) \approx 5.93008 \times 10^{-3}\ \mathrm{m\,s^{-2}}, \qquad a_{\mathrm{ng}} = 4.92 \times 10^{-6}\ \mathrm{m\,s^{-2}},$$

$$R_O = \frac{a_{\mathrm{ng}}}{g_\odot} = 8.29668 \times 10^{-4} \qquad X_O = \ln R_O = -7.094485$$

**Microlensing**

$$X \equiv \ln\left(\frac{F_{\mathrm{peak}}}{F_{\mathrm{base}}}\right) = -(\ln 10/2.5)\,\Delta m,$$

$$\text{Q2237 (img C): } \Delta m \simeq -0.66 \Rightarrow X_Q \simeq +0.610000,$$

$$\text{OGLE-2003-BLG-235: } X_{O7} = \ln 7 = +1.945910.$$

**Two $\delta_\star$ choices**

$$\delta_{\star\mathrm{clock}} = \frac{1}{3}\ln\left(\frac{f_{\mathrm{Al}}}{f_{\mathrm{Sr}}}\right) = \boxed{0.32000062}, \qquad \delta_{\star e/p} = \frac{1}{21}\ln\left(\frac{\gamma_e}{\gamma_p}\right) = \boxed{0.309978}.$$

Acceptance band $\epsilon_X = \delta_\star/6$ in each case

### Clock–anchored step $\delta_{\star\mathrm{clock}} = 0.32000062$

| Case | $X$ | $X/\delta_{\star\mathrm{clock}}$ | $k$ | $r = X - k\,\delta_{\star\mathrm{clock}}$ (clicks) |
|---|---|---|---|---|
| *Oumuamua* | $-7.094485$ | $-22.17022$ | $-22$ | $-0.054471$ ($-0.170$) |
| Q2237 | $+0.610000$ | $+1.90625$ | $+2$ | $-0.030001$ ($-0.0938$) |
| OGLE-235 | $+1.945910$ | $+6.0810$ | $+6$ | $+0.025906$ ($+0.0810$) |

All three are within $\mathcal{O}(0.1)$ click of an integer with $\epsilon_X = \delta_{\star\mathrm{clock}}/6 \approx 0.05333$ (look 'Oumuamua below)

### Best $e/p$ step $\delta_{\star e/p} = 0.309978$

| Case | $X$ | $X/\delta_{\star e/p}$ | $k$ | $r = X - k\,\delta_{\star e/p}$ (clicks) |
|---|---|---|---|---|
| *Oumuamua* | $-7.094485$ | $-22.887$ | $-23$ | $+0.035009$ ($+0.113$) |
| Q2237 | $+0.610000$ | $+1.968$ | $+2$ | $-0.009956$ ($-0.0321$) |
| OGLE-235 | $+1.945910$ | $+6.279$ | $+6$ | $+0.086042$ ($+0.2775$) |

$\epsilon_X = \delta_{\star e/p}/6 \approx 0.05166$

### Compact Comparison (residuals in clicks)

| **Case** | $\delta_{\star\mathrm{clock}}$ | $\delta_{\star e/p}$ | **smaller residual** |
|---|---|---|---|
| *Oumuamua* | 0.170 | **0.113** | $e/p$ |
| Q2237 | 0.0938 | **0.0321** | $e/p$ |
| OGLE-235 | **0.0810** | 0.2775 | clock |

### Interpretation

- **Channel match.** In the SRT–UQI map, microlensing photometry reads out a *time/phase* channel ($\kappa_m = +1$): photon counts integrate a detector rate over exposure time, and flux ratios are ultimately ratios of optical frequencies integrated over bandpass. Fixing $\delta_\star$ with *optical* frequency ratios ($Al^+/Sr$) therefore aligns the channel used to anchor $\delta_\star$ with the channel used to *measure* $X = \ln(F_{\text{peak}}/F_{\text{base}})$, minimizing cross–channel bias

$$X_{\text{phot}} = \ln\!\left(\frac{F_{\text{peak}}}{F_{\text{base}}}\right) = \kappa_m\, U \quad (\kappa_m = +1), \qquad \delta_{\star\text{clock}} \text{ also from } \ \kappa_m = +1$$

- **Model/measurement coherence.** The geometric microlensing magnification

$$A(u) = \frac{u^2+2}{u\sqrt{u^2+4}}\,, \qquad X = \ln A$$

  is achromatic in the thin–lens limit, while the *measured* $A$ comes from photon statistics and timing (detector cadence, exposure, clocking). Thus any residual tied to $\delta_\star$ is driven more by the *time–frequency* metrology chain than by magnetic calibration. This favors $\delta_{\star\text{clock}}$ for OGLE

- **Calibration path.** OGLE's flux scale is tied to standard–star photometry, detector gain, and timing—each referenced to SI units defined via frequency and time. Defining $\delta_\star$ from optical clocks keeps the anchor in the same metrological path as the observation, reducing systematic offsets in $X$

- **Contrast with $e/p$.** The $\delta_{\star e/p}$ anchor arises from a *spin/magnetic* channel ($\gamma$ ratios in a field). Applying it to purely optical photometry introduces a cross–channel mapping ($\kappa_m = +1 \ \leftrightarrow$ spin) that carries a small but coherent bias, which appears as the larger OGLE residual with $\delta_{\star e/p}$

- **Empirical outcome.** Numerically,

$$|r_{\text{OGLE}}|_{\text{clock}} \approx 0.081 \text{ click} \quad < \quad |r_{\text{OGLE}}|_{e/p} \approx 0.278 \text{ click}$$

  **consistent with channel–matched anchoring**. Conversely, *Oumuamua* and Q2237 mix dynamics and passbands where magnetic/particle anchors slightly improve residuals, reflecting their different effective channel weights

- **Conclusion.** OGLE's observable is intrinsically *optical–time* in both physics and calibration; anchoring $\delta_\star$ to optical clocks ($\kappa_m = +1$) preserves channel consistency and yields the smaller residual

# 2 Part 2 — Anchored–Corrected Bloch Sphere; One Device, One Law

Readers' map **Goal.** One control rule produces quantized clicks at two scales: macroscopic levitator and microscopic qubit.
**Method.** Work in the anchored log $X$; define the click size $\delta_\star$; use a mid-step to filter noise.
**Payoff.** A single command $\Delta X = \delta_\star$ yields a qubit rotation $\Delta\phi = 2\pi\delta_\star$ and a levitator rung advance; if mid-step is not reached, inertia cancels noise and no step is counted.

## 2.1 Anchors, total potential, and the anchored generator

**Definition: Anchors and total potential**

$$\Phi_{\text{tot}}(s, I_{\text{POT}}, t) = \underbrace{\Phi_{\text{ring}}(s)}_{\text{device geometry}} + \underbrace{\Phi_{\odot}}_{\text{Sun}} + \underbrace{\Phi_{\text{coil}}(I_{\text{POT}})}_{\text{POT coil}} + \underbrace{\Phi_{\text{M}}(t)}_{\text{Moon (slow drift)}}$$

*Notes.* Earth weight is balanced by LIFT coils and excluded. The Sun at 1 au is the declared DC anchor. Large quasi-static backgrounds (e.g., Milky Way) are common-mode and cancel in Sun-anchored differences.

Sub-Definition: Each potential term, units, sign

- $\rho_{\text{res}}$ [kg m$^{-3}$]: **resonance mass-equivalent energy density**. It is the static limit of the universal wave's energy content: $\rho_{\text{res}} = \frac{\varepsilon_\Phi}{c^2}$, where $\varepsilon_\Phi = \frac{1}{8\pi G}\left[(\nabla\Phi)^2 + \frac{1}{c^2}(\partial_t\Phi)^2\right]$ is the field-energy density of the resonance potential.
- $\sigma$ [kg m$^{-3}$]: **dynamic source function** that generalizes $\rho_{\text{res}}$ when the potential varies in time. In the causal wave equation $\nabla^2\Phi - \frac{1}{c^2}\partial_t^2\Phi = 4\pi G\,\sigma$, $\sigma$ collects both matter and field-energy contributions.
- **Static limit.** When $\partial_t\Phi = 0$, $\sigma \to \rho_{\text{res}}$ and the equation reduces to $\nabla^2\Phi = 4\pi G\rho_{\text{res}}$, the *Static Sourcing Law.*
- $\Phi_{\text{ring}}(s)$ [m$^2$ s$^{-2}$]: device-geometry (mechanical) potential; in the small-range limit $\partial_s U_{\text{mech}} = \dfrac{\lambda_s}{1+\lambda_s s}$ so $\partial_s\Phi_{\text{ring}} = c^2\dfrac{\lambda_s}{1+\lambda_s s}$.
- $\Phi_{\odot}$ [m$^2$ s$^{-2}$]: solar background (negative near masses). Fixed *anchor.*
- $\Phi_{\text{coil}}(I_{\text{POT}})$ [m$^2$ s$^{-2}$]: tunable POT-coil field; locally linear $\partial_I\Phi_{\text{coil}} = c^2\,\kappa_m\eta_I$.
- $\Phi_{\text{M}}(t)$ [m$^2$ s$^{-2}$]: slow lunar drift to be cancelled.
- $\lambda_s$ [–]: small-slope mechanical gain; near $s = 0$, $X_{\text{mech}} = \ln(1+\lambda_s s)$ and $\partial_s X_{\text{mech}} = \dfrac{\lambda_s}{1+\lambda_s s}$.
- $\eta_I$ [A$^{-1}$]: coil $\to$ log gain; $X_{\text{coil}} = \kappa_m\eta_I(I_{\text{POT}} - I_{\text{bias}})$.
- $\kappa_m$ [–]: channel factor (+1 time/phase, −2 optical index).
- $\gamma$ [s$^{-1}$]: viscous-damping coefficient in the washboard model $(X'' + \gamma X' + \partial_X V = \cdots)$.

- **Notation line:** for brevity, we write $s \equiv s$ (dimensionless actuator command; e.g. $s = \Delta z / R_{\text{bore}}$ or $s = \theta_{\text{tilt}} / \theta_0$).
  - $s = \dfrac{\Delta z}{R_{\text{bore}}}$, where $\Delta z$ is the actuator's small linear displacement (in metres) and $R_{\text{bore}}$ is the reference radius or effective lever arm of the ring device (in metres);
  - or $s = \dfrac{\theta_{\text{tilt}}}{\theta_0}$, where $\theta_{\text{tilt}}$ is the mechanical tilt angle and $\theta_0$ a reference small-angle scale.

**Definition: Anchored generator and channel log**

$$U \equiv \frac{\Phi_{\text{tot}}}{c^2}, \qquad X \equiv \kappa_m \, U \,.$$

$\kappa_m = +1$ (time/phase channel), $\kappa_m = -2$ (optical-index channel). $X$ is the additive control variable.

## 2.2 Anchor–correction: remove slow drift, preserve Sun anchor

**Sub-Note: Active channel and sign of $\kappa_m$**

In this section we operate entirely within the **optical/phase channel**. Hence the active channel factor is

$$\boxed{\kappa_m = -2.}$$

The time (clock) channel with $\kappa_m = +1$ enters only indirectly through the invariant $\Gamma_t n^{1/2} = 1$. Using $\kappa_m = -2$ ensures that all derived quantities (*optical index, phase rotation, mid-step control*) follow the Optical-Index Law $n = e^{-2\Phi/c^2}$.

**Sub-Definition: Proper time $\tau$ and the Time Law invariant**

- $\tau$ [s]: **proper (local) time** measured by a clock or resonance anchored in the potential $\Phi$.
- Relation to universal time $t$:

$$\Gamma_t \equiv \frac{d\tau}{dt} = e^{+\Phi/c^2}, \qquad \tau(t) = \int e^{+\Phi(\mathbf{x},t')/c^2} \, dt'.$$

- Physical meaning: $\tau$ tracks the local oscillation phase of the universal wave. Each click in $X$ corresponds to a fixed phase increment in $\tau$.
- **Invariant combination:** the optical index $n = e^{-2\Phi/c^2}$ and the clock factor $\Gamma_t = e^{+\Phi/c^2}$ combine into

$$\Gamma_t \, n^{1/2} = e^{+\Phi/c^2} e^{-\Phi/c^2} = 1,$$

  guaranteeing that $\tau$ and optical propagation remain consistent.

*Plain English.* $\tau$ is the "true" local time of the resonance. All frequency, phase, and click counts are ultimately measured per unit $\tau$.

**Law: Anchor–correction law**

**Purpose.** The Moon slowly perturbs the background potential $\Phi_{\rm M}(t)$ and would drift the anchored log $X$. We cancel this drift by adding an equal-and-opposite coil bias so the experiment remains referenced to the Sun-only anchor.

**Law (closed form)**

$$I_{\rm bias}(t) = -\frac{1}{\kappa_m\,\eta_I}\,\frac{\Phi_{\rm M}(t)}{c^2} \qquad \Rightarrow \qquad X_{\rm coil} = \kappa_m\,\eta_I\,[I_{\rm POT} - I_{\rm bias}(t)]\,.$$

Here we operate in the *optical/phase channel*, so $\boxed{\kappa_m = -2}$.

**Derivation**

1. $\Delta I \;\equiv\; I_{\rm POT} - I_{\rm bias}$,
2. $X = \kappa_m\,\dfrac{\Phi_{\rm ring}(s) + \Phi_\odot + \Phi_{\rm coil}(I_{\rm POT}) + \Phi_{\rm M}(t)}{c^2}$.
3. Set $I_{\rm POT} = I_{\rm bias} + \Delta I$ so $X_{\rm coil} = \kappa_m\eta_I\,\Delta I + \kappa_m\eta_I\,I_{\rm bias}$.
4. Cancel lunar drift: $\kappa_m\eta_I\,I_{\rm bias} + \kappa_m\,\Phi_{\rm M}/c^2 = 0 \Rightarrow \boxed{I_{\rm bias} = -\Phi_{\rm M}/(\eta_I c^2)}$
5. Substitute: $X = \kappa_m\,\dfrac{\Phi_{\rm ring}(s) + \Phi_\odot}{c^2} + \kappa_m\eta_I[I_{\rm POT} - I_{\rm bias}(t)]$

$[\Phi_{\rm M}/c^2] = 1$ (dimensionless). **Units (plain):** $\Phi$ is measured in $\mathrm{m^2s^{-2}}$ and $c^2$ has the same units, so $\Phi/c^2$ has *no units* (it's dimensionless). Example near Earth: $\dfrac{\Phi_{\rm moon}}{c^2} \approx -\dfrac{G\,M_{\rm M}}{r\,c^2} \sim -1.3\times10^{-13}$ (where $G$ is Newton's constant, $M_{\rm M}$ the Moon's mass, $r$ the Earth–Moon distance, $c$ the speed of light).

**Units & sign check (with $\mathbf{A}^{-1}$)** $\underline{[\eta_I] = \mathrm{A}^{-1}\text{ means "per ampere"}}$: it maps current to a dimensionless log increment via $\Delta X = \kappa_m\,\eta_I\,\Delta I$, so $\eta_I\,\Delta I$ is unitless and $I_{\rm bias}$ has units A. With $\kappa_m = -2$, the coil's effect opposes the lunar optical response.

**How to set $I_{\rm bias}$ in practice**

- *Null-by-mid-step:* Park at $X_{\rm mid} = (k+\frac{1}{2})\delta_\star$ and tune $I_{\rm bias}$ until half-step switching is symmetric for $\pm$ small coil nudges.
- *Slow servo:* Dither $I_{\rm POT}$ and integrate the $X$ error at the lunar rate so the average error is zero $\Rightarrow$ $I_{\rm bias}$ tracks $\Phi_{\rm M}(t)$.

**Magnitude note.** Near Earth, $|\Phi_{\rm M}/c^2| \sim 10^{-13}$, so $|I_{\rm bias}|$ is tiny for typical gains (e.g. $\eta_I \sim 0.3\ \mathrm{A}^{-1}$); implement $I_{\rm bias}$ digitally.

**Outcome.** With this bias, the baseline is Sun-referenced and the coil acts only on $I_{\rm POT} - I_{\rm bias}(t)$, enabling clean, repeatable clicks.

Sub-Derivation: Solve for $I_{\text{bias}}(t)$

**(1) Expand the channel log**

$$X = \kappa_m \frac{\Phi_{\text{ring}}(s) + \Phi_{\odot} + \Phi_{\text{coil}}(I_{\text{POT}}) + \Phi_{\text{M}}(t)}{c^2}.$$

**(2) Linearize the coil around the bias $I_{\text{POT}} = I_{\text{bias}} + \Delta I$:**

$$X \approx \kappa_m \frac{\Phi_{\text{ring}} + \Phi_{\odot}}{c^2} + \underbrace{\kappa_m\,\eta_I\,\Delta I}_{\text{active control}} + \underbrace{\kappa_m \frac{\Phi_{\text{M}}}{c^2} + \kappa_m\,\eta_I\,I_{\text{bias}}}_{\text{DC terms}}.$$

**(3) Cancellation condition** (remove lunar drift in $X$):

$$\kappa_m\,\eta_I\,I_{\text{bias}} + \kappa_m \frac{\Phi_{\text{M}}}{c^2} = 0 \;\Rightarrow\; \eta_I\,I_{\text{bias}} + \frac{\Phi_{\text{M}}}{c^2} = 0 \;\Rightarrow\; \boxed{I_{\text{bias}}(t) = -\frac{\Phi_{\text{M}}(t)}{\eta_I\,c^2}}$$

**(4) Substitute back** to get a Sun-anchored control law:

$$\begin{aligned}
X &= \kappa_m \frac{\Phi_{\text{ring}}(s) + \Phi_{\odot}}{c^2} \;+\; \kappa_m\,\eta_I\,[\,I_{\text{POT}} - I_{\text{bias}}(t)\,] \\
&= \underbrace{\kappa_m \frac{\Phi_{\text{ring}}(0) + \Phi_{\odot}}{c^2}}_{\text{const. gauge}} + \kappa_m \frac{\Phi_{\text{ring}}(s) - \Phi_{\text{ring}}(0)}{c^2} \;+\; \kappa_m\,\eta_I\,\Delta I \\
&= \text{const.} \;+\; \kappa_m \frac{\Delta\Phi_{\text{ring}}(s)}{c^2} \;+\; \kappa_m\,\eta_I\,\Delta I \\
&\approx \text{const.} \;+\; \kappa_m\,\ln(1 + \lambda_s\,s) \;+\; \kappa_m\,\eta_I\,\Delta I \qquad (|\lambda_s\,s| \ll 1)
\end{aligned}$$

Sub-Note: Why a natural logarithm for the small-slope gain

Because we are tracking *percentage–style* changes rather than fixed offsets: $X = \ln(\cdot)$ so successive effects *multiply*, and a small actuator move scales the response by $(1 + \lambda_s\,s)$ (think compounding interest). If each equal actuator step produces the same *fractional* change—i.e., constant fractional sensitivity—then increments add in $X$ and the accumulated effect is a natural logarithm.

$$\frac{dX}{ds} = \frac{\lambda_s}{1 + \lambda_s s} \qquad \Longrightarrow \qquad X_{\text{mech}}(s) = \ln(1 + \lambda_s s)\;(+\;\text{const}),$$

so first-order gives the expected linear slope $X \approx \lambda_s s$ while the natural log enforces the correct multiplicative composition for finite moves.

$$\Delta I \equiv I_{\text{POT}} - I_{\text{bias}}(t), \qquad \Delta\Phi_{\text{ring}}(s) \equiv \Phi_{\text{ring}}(s) - \Phi_{\text{ring}}(0),$$
$$\kappa_m = -2 \;\; \text{(optical/phase channel)}$$

Sub-Check: Units and signs

$[\Phi_{\text{M}}/c^2] = 1$ (dimensionless). $[\eta_I] = \text{A}^{-1}$ so $\eta_I\,I$ is dimensionless; multiplying by $\kappa_m$ keeps sign/channel. Thus $I_{\text{bias}}$ has units of A, as required.

**Theorem: Anchor hierarchy: when externals vanish, use the universal-wave self-anchor**

**Statement.** If all external anchors are absent ($\Phi_{\odot} = \Phi_{\text{coil}} = \Phi_{\text{M}} = 0$), we still obtain well-defined clicks by *self-anchoring* to the universal wave at the device's rest geometry. Fix the baseline at the parked actuator state $s = 0$ and define

$$X_{\text{ref}} \;\equiv\; \kappa_m \frac{\Phi_{\text{ring}}(0)}{c^2}, \qquad X_{\text{rel}}(s) \;\equiv\; \kappa_m \frac{\Phi_{\text{ring}}(s) - \Phi_{\text{ring}}(0)}{c^2}.$$

The controlled log is then $X = X_{\text{ref}} + X_{\text{rel}}(s)$ and clicks are counted in $X_{\text{rel}}$.

**Bridge (plain reading of the equations).**

- $X_{\text{ref}}$ is just the *zero mark* ("tare") in click units when the device is parked; $X_{\text{rel}}$ is the *change from rest* that we actually step and count.
- Dividing by $c^2$ removes units: $\Phi/c^2$ is dimensionless, so both $X_{\text{ref}}$ and $X_{\text{rel}}$ are pure numbers (no units) and can be added.
- In this part we use the *optical/phase channel*, so $\boxed{\kappa_m = -2}$; for small moves $X_{\text{rel}} \approx -2\ln(1 + \lambda_s s)$—a log scale because moving the ring does not simply *add* a fixed offset to the field; it slightly *rescales* the existing field everywhere. Each motion changes the distance, angle, or tension that shapes the field, so the entire response is multiplied by a small factor that depends on how far the ring moved.

  For example, if one small motion makes the field 1% stronger, and you make the same motion again, the second 1% increase acts on the *new* field strength, not on the original one:
  $$(1 + 0.01) \times (1 + 0.01) = 1.0201.$$
  The total effect is a multiplication of two scale factors, not an addition of two fixed numbers.

  This is why the response is called *multiplicative*. The logarithm is then used to turn this chain of multiplications into a simple addition:
  $$\ln[(1 + 0.01) \times (1 + 0.01)] = \ln(1 + 0.01) + \ln(1 + 0.01).$$
  So in the anchored log variable $X = \ln(\cdot)$, each equal actuator movement corresponds to adding the same small step instead of multiplying again. That is what makes the system behave in neat, countable "clicks" even though the underlying physical change scales the field multiplicatively.
- Total control is "baseline + change": we set the baseline once, then drive only $X_{\text{rel}}$ to make clicks.

**Sub-Note: Exponential resonance gathering along a constant tilt**

Each equal tilt of the ring produces the same *fractional* change in the resonance response, so one step scales it by a factor $(1 + \lambda_s\,\Delta s)$. Repeating $N$ identical tilts *multiplies* the effect,

$$\text{response}(N) = \text{response}(0)\,(1 + \lambda_s\,\Delta s)^N \;\approx\; \text{response}(0)\,e^{N\lambda_s\,\Delta s},$$

which is precisely *exponential resonance gathering* along that constant tilt. In the anchored log variable $X = \ln(\text{response})$, these multiplicative steps add linearly: $\Delta X \approx N\lambda_s\,\Delta s = k\,\delta_\star$, so exponential build-up becomes clean, countable clicks.

**Why / What / Where / When (non-expert map).**

- *Why an anchor at all?* Like "taring" a kitchen scale, we must set a zero before measuring steps. If anchors are missing, we tare to the device's resting shape (the universal-wave self-anchor).

- *What becomes the ruler?* Changes are measured from the rest shape: only the *difference* $\Phi_{\text{ring}}(s) - \Phi_{\text{ring}}(0)$ matters, so no absolute cosmic reference is needed.

- *Where do clicks come from?* From **phase closure** of the same universal wave. Going once around the ring, $\oint k(s)\,ds = 2\pi n$ with $n \in \mathbb{Z}$, so the energy is periodic in the ledger phase $\phi \equiv 2\pi X$: $V(\phi) = V_0(1 - \cos\phi)$. Minima $\phi_k = 2\pi k$ are the *rungs* ($X_k = k\,\delta_\star$); the mid-step sits at the barrier $\phi_{k+1/2} = (2k+1)\pi$.

- *When do clicks disappear?* Only when the periodic "washboard" is not able to hold the state, or when the motion is forced to slide. In plain terms and symbols:

  (a) **No boundary & no self-anchor (flat gauge).** If there is no closed path *and* we do not fix a rest gauge at $s = 0$, the energy landscape in $X$ has no preference: $\partial_X V \equiv 0 \;\Rightarrow\;$ no wells, no rungs to lock. *(Fix by self-anchoring; Sec. above.)*

  (b) **Corrugation vanishes.** If the periodic amplitude tends to zero ($V_0 \to 0$), the washboard flattens even with phase closure, so pinning disappears. *(Keep $V_0 > 0$ via geometry so barriers exist.)*

  (c) **Overdrive (depinning).** With deliberate drive $F_0 \cos\omega t$, once

  $$F_0 \;\geq\; F_c \;=\; \frac{2\pi\,V_0}{\delta_\star},$$

  the state is pushed over every barrier and *slides* instead of dwelling on rungs. *(Operate with margin, e.g. $F_0 \lesssim 0.7F_c$.)*

  (d) **Noise-dominated hopping.** If stochastic forcing is too large (effective noise scale $D$ exceeds the barrier), random jumps dominate: $D \gg 2V_0 \Rightarrow$ the mid-step test fails to discriminate true clicks from noise. *(Reduce noise, lengthen dwell $\tau_{\min}$, or widen acceptance band slightly.)*

  **Practical guardrails.** Self-anchor at rest, design for finite $V_0$, drive below $F_c$, and enforce the mid-step acceptance ($|X - X_{\text{mid}}| \leq \epsilon_X$ for $\tau_{\min}$) so only genuine, stable clicks are counted.

**Optical channel note.** In Part 2 we read phase/optics, so $\boxed{\kappa_m = -2}$ and $X_{\text{rel}} \approx -2\ln(1 + \lambda_s s)$ for small motions, giving the same click ladder as with external anchoring.

### 2.3 Click ladder, mid-step, and acceptance

**Definition: Click ladder and midpoint**

$$X_k = k\,\delta_\star, \qquad X_{k+1/2} = \Big(k + \tfrac{1}{2}\Big)\delta_\star, \qquad k \in \mathbb{Z}.$$

One click advances $X$ by $\delta_\star$; the midpoint is the *mid-step*.

**Sub-Definition: Residual and acceptance band**

Residual $r \equiv X - k\delta_\star$. Accept rung $k$ if $|r| \le \epsilon_X$ with $\epsilon_X \equiv \delta_\star/6$. This rejects chatter.

**Sub-Note: Why choose $\epsilon_X = \delta_\star/6$?**

**It's an engineering margin, not a constant.** With rungs at $k\delta_\star$ and the barrier (mid-step) at $(k + \frac{1}{2})\delta_\star$, choosing $\epsilon_X = \delta_\star/6$ gives:

$$\text{acceptance half-width } = \epsilon_X = \frac{\delta_\star}{6}, \quad \text{distance to barrier } = \frac{\delta_\star}{2} - \epsilon_X = \frac{\delta_\star}{3}.$$

So each rung's acceptance zone is well inside the half-spacing (no overlap), leaving a *2× larger guard band* to the barrier. It also matches a practical "$\approx 3\sigma$" rule when the mid-step fit yields a typical noise width $\sigma_X \approx \delta_\star/18$, giving low false counts yet easy locking.

## Composite Control (One Equation, Two Knobs)

### At a glance — what the formula says

$$\boxed{X(s, I_{\mathrm{POT}}) \;\approx\; \underbrace{\ln\!\big(1+\lambda_s\, s\big)}_{\text{mechanical}} \;+\; \underbrace{\kappa_m\, \eta_I\, \big[\, I_{\mathrm{POT}} - I_{\mathrm{bias}}(t)\,\big]}_{\text{coil (anchor-correction)}}}\,.$$

**Plain words:** hidden "log state" $X$ is the sum of a *mechanical* part and a *coil* part. Move the actuator ($s$) or adjust the current ($I_{\mathrm{POT}}$) to step $X$ by one click ($+\delta_\star$).

### Define each symbol (term-by-term, with units)

- $X$ — *anchored log response* (dimensionless). One click means $\Delta X = \delta_\star$.
- $s$ — dimensionless actuator command (e.g. $s = \Delta z / R_{\mathrm{bore}}$).
- $\lambda_s$ — *mechanical small-slope gain* (dimensionless). Equal increments of $s$ create equal *percentage* changes, so they *add* in ln-space.
- $I_{\mathrm{POT}}$ — POT-coil current (ampere, A), the fast control knob.
- $I_{\mathrm{bias}}(t)$ — *bias current* (A) that removes slow drift (e.g. lunar); you control $I_{\mathrm{POT}} - I_{\mathrm{bias}}$.
- $\eta_I$ — *coil gain* ($\mathrm{A}^{-1}$). Each ampere of $I_{\mathrm{POT}} - I_{\mathrm{bias}}$ changes $X$ by $\kappa_m\, \eta_I$.
- $\kappa_m$ — *channel factor*. Here, Part 2 is **optical**, so $\boxed{\kappa_m = -2}$. (Time/phase channel would use $\kappa_m = +1$.)

### How to compute $X$ — 3 easy steps (with guardrails)

**Before you start (tare & guard):**

- **Tare the bias:** record or compute $I_{\mathrm{bias}}(t)$ so that the *net* coil command is $I_{\mathrm{POT}} - I_{\mathrm{bias}}(t)$.
- **Domain guard (mechanics):** ensure $1 + \lambda_s\, s > 0$ so that $\ln(1+\lambda_s s)$ is defined.
- **Small-slope validity:** for quick linear estimates, keep $|\lambda_s s| \lesssim 0.3$.

**Step 1 — Measure the actuator command $s$ (dimensionless).**

- Example: $s = \Delta z / R_{\mathrm{bore}}$ (axial shift divided by bore radius) or a normalized tilt.
- Record $s$ with sign (**direction matters**).

**Step 2 — Measure the net coil current $I_{\mathrm{POT}} - I_{\mathrm{bias}}(t)$ (A).**

- Read $I_{\mathrm{POT}}$ from current supply; subtract the *bias* $I_{\mathrm{bias}}(t)$ that cancels slow background.
- Keep the sign: in the optical channel ($\kappa_m = -2$), a *positive* net current typically *decreases* $X$.

**Step 3 — Evaluate $X$ from the composite law.**

$$\boxed{X = \underbrace{\ln(1+\lambda_s s)}_{\text{mechanical term}} \;+\; \underbrace{\kappa_m\, \eta_I\, (I_{\mathrm{POT}} - I_{\mathrm{bias}}(t))}_{\text{coil term}}}\,.$$

*Quick estimate (small-slope):* if $|\lambda_s s| \ll 1$, use $\ln(1+\lambda_s s) \approx \lambda_s s$. *Error hint:* for $|x| = |\lambda_s s| \le 0.3$, the absolute error $|\ln(1+x) - x| \lesssim 0.05\, x$ (a few percent).

**Sanity checks:**

- *Dimensionless:* both terms are unitless; if not, you missed a unit.
- *Monotonicity:* in optics ($\kappa_m = -2$), increasing $I_{\mathrm{POT}}$ should lower $X$; if not, flip coil polarity or sign convention.
- *Clicks view:* $X/\delta_\star$ gives you "how many clicks" you moved; distance to the nearest integer gives the residual in clicks.

### Units & signs — quick checklist (plus polarity test)

**Units (must match):**

- $s$ — dimensionless (e.g. $\Delta z/R_{\text{bore}}$); $\lambda_s$ — dimensionless.
- $I_{\text{POT}}, I_{\text{bias}}$ — ampere (A); $\eta_I$ — $\text{A}^{-1}$; hence $\eta_I(I_{\text{POT}} - I_{\text{bias}})$ is dimensionless.
- $X$ — dimensionless; $\delta_\star$ — dimensionless.

**Signs (channel dependent):**

- $\boxed{\kappa_m = -2}$ (optical/index channel, Part 2): raising $I_{\text{POT}}$ typically *decreases* $X$.
- $\kappa_m = +1$ (time/phase channel): raising $I_{\text{POT}}$ *increases* $X$.

**Polarity test (30-second lab check):**

1. Hold $s = 0$; nudge $I_{\text{POT}} \to I_{\text{POT}} + \Delta I$.
2. Measure $\Delta X$. Expect $\text{sign}(\Delta X) = \text{sign}(\kappa_m\, \eta_I\, \Delta I)$.
3. If opposite, swap coil leads or fix the software sign.

### Mini numeric example (optical channel, with mid & full targets)

**Given:** $\delta_\star = 0.3200$, $\lambda_s = 0.10$, $\eta_I = 0.30\ \mathrm{A}^{-1}$, $\kappa_m = -2$. Targets in $X$: $X_{\mathrm{mid}} = 0.1600$, $X_{\mathrm{tgt}} = 0.3200$.

**A) Coil-only path** ($s = 0$)

$$\Delta I_{\mathrm{mid}} = \frac{X_{\mathrm{mid}}}{\kappa_m\,\eta_I} = \frac{0.1600}{-0.60} = -0.2667\ \mathrm{A}, \qquad \Delta I_{\mathrm{full}} = \frac{0.3200}{-0.60} = -0.5333\ \mathrm{A}.$$

$\Rightarrow$ Apply $-0.2667$ A to hold mid; then to click, apply $-0.5333$ A total.

**B) Mechanical-only path** ($I_{\mathrm{POT}} = I_{\mathrm{bias}}$)

$$s_{\mathrm{mid}} = \frac{e^{0.1600} - 1}{\lambda_s} = \frac{1.17351 - 1}{0.10} = 1.7351, \qquad s_{\mathrm{full}} = \frac{e^{0.3200} - 1}{0.10} = \frac{1.37713 - 1}{0.10} = 3.7713.$$

*Note on linear shortcut:* using $\ln(1 + \lambda_s s) \approx \lambda_s s$ would give $0.10 \times 1.7351 = 0.1735$ (vs exact 0.1600); that's $\sim 8.4\%$ high — fine for rough steering, not for acceptance.

**C) Split path (50/50 mid target)**

$$X_{\mathrm{mech}} = 0.0800, \quad X_{\mathrm{coil}} = 0.0800.$$

$\Rightarrow s^{(50/50)}_{\mathrm{mid}} = (e^{0.0800} - 1)/0.10 = 0.8314$, $\Delta I^{(50/50)}_{\mathrm{mid}} = 0.0800/(-0.60) = -0.1333$ A. For the full click, double both: $s^{(50/50)}_{\mathrm{full}} = 1.7351$, $\Delta I^{(50/50)}_{\mathrm{full}} = -0.2667$ A. *Benefit:* smaller excursions on each actuator, better linearity and SNR; **SNR (Signal-to-Noise Ratio).** A measure of how strong the useful signal is compared to random noise:

$$\mathrm{SNR} = \frac{\text{signal amplitude}}{\text{noise amplitude}}, \qquad \mathrm{SNR}_{\mathrm{dB}} = 20\log_{10}\left(\frac{\text{signal}}{\text{noise}}\right).$$

In this work, the signal is one click ($\delta_\star$) and the noise is the RMS fluctuation $\sigma_X$:

$$\mathrm{SNR} \approx \frac{\delta_\star}{\sigma_X}.$$

High SNR ($> 20$) means clear, reliable clicks; low SNR ($< 5$) means noise dominates.

**D) Acceptance check (rung logic)**

$$\epsilon_X = \delta_\star/6 = 0.05333.$$

After applying the full target, verify $|X - X_{\mathrm{tgt}}| \leq 0.05333$. If $\leq$ band: *count the click*; else: *retry via mid-step* (adjust gains or dwell).

## Dwell ($\tau_{\min}$) — definition

### Definition (one line)

**Dwell** is the *minimum hold time* at a target setpoint before deciding "lock" or "no lock". Mid-step acceptance requires

$$\boxed{|X - X_{\text{mid}}| \ \le \ \epsilon_X \quad \text{for all } t \in [t_0,\, t_0 + \tau_{\min}]\,.}$$

If the inequality holds over the whole window, **commit**; otherwise **cancel**.

### How to choose $\tau_{\min}$ (rule-of-thumb)

$$\tau_{\min} \ \approx \ 3\text{–}5 \times \tau_{\text{settle}} \quad \text{(system settling time)}$$

$$\text{or} \qquad \tau_{\min} \ \approx \ 10\, T_{\text{samp}} \quad \text{(collect} \ge 10 \text{ clean samples inside the band)}$$

Increase $\tau_{\min}$ if noise is high or the switching curve is shallow (large $\sigma_I$).

### Procedure (4 steps)

1. Drive to the target ($X_{\text{mid}}$ or $X_{\text{tgt}}$); start timer.
2. Stream $X(t)$; check band: $|X(t) - X_{\text{set}}| \le \epsilon_X$.
3. If the band holds *continuously* for $\tau_{\min}$, **accept**.
4. Else, **reject** and return to the previous rung (no click).

### Quick checks (make it robust)

- Use natural logs (ln); verify $\epsilon_X = \delta_\star/6$.
- Confirm channel sign: $\kappa_m = -2$ (optical in Part 2).
- Bias correctly: use $I_{\text{POT}} - I_{\text{bias}}(t)$, not $I_{\text{POT}}$ alone.
- If near the edge, lengthen $\tau_{\min}$ or tighten noise (raise SNR).

## Acceptance test (residual vs band)

**Band:** $\epsilon_X = \delta_\star/6 = 0.05333$ **Targets:** $X_{\text{mid}} = 0.1600,\ X_{\text{tgt}} = 0.3200$.

**Full-click (target $X_{\text{tgt}} = 0.3200$)**
*Case A — good landing (accept):* measured $X_{\text{meas}} = 0.3120$

$$r = X_{\text{meas}} - X_{\text{tgt}} = -0.0080, \quad |r| = 0.0080 \le 0.05333 \ \Rightarrow \ \textbf{Accepted.} \quad r_{\text{clicks}} = \frac{r}{\delta_\star} = -0.025.$$

*Case B — overshoot (reject):* measured $X_{\text{meas}} = 0.3900$

$$r = +0.0700, \quad |r| = 0.0700 > 0.05333 \ \Rightarrow \ \textbf{Rejected.} \quad r_{\text{clicks}} = \frac{0.0700}{0.3200} = 0.218 > \tfrac{1}{6}.$$

**Mid-step (target $X_{\text{mid}} = 0.1600$)**
*Case C — stable mid (commit):* measured $X_{\text{meas}} = 0.1710$

$$r_{\text{mid}} = 0.0110, \quad |r_{\text{mid}}| = 0.0110 \le 0.05333 \ \Rightarrow \ \textbf{Mid accepted} \text{ (commit full click).}$$

*Case D — shaky mid (cancel):* measured $X_{\text{meas}} = 0.2180$

$$r_{\text{mid}} = 0.0580, \quad |r_{\text{mid}}| = 0.0580 > 0.05333 \ \Rightarrow \ \textbf{Mid rejected} \text{ (cancel, no click).}$$

### Common mistakes (and how to avoid them)

- Using $\log_{10}$ instead of $\ln$: always use natural logs.
- Forgetting the bias: $I_{\mathrm{POT}}$ must be replaced by $I_{\mathrm{POT}} - I_{\mathrm{bias}}(t)$.
- Wrong channel: in Part 2 use $\kappa_m = -2$ (optical).
- Mixing units: $s$ must be dimensionless; $\eta_I$ must be in $\mathrm{A}^{-1}$.

## Mid-Step Estimator (Noise-Safe Clicks)

### Why mid-step?

The mid-step lies halfway between rungs. If you can hold $X$ *inside* its small band for a short dwell, you are safely balanced on the "saddle". **If you can hold it** $\Rightarrow$ *commit* the full click. **If you cannot** $\Rightarrow$ you *cancel* and stay put (noise shook you, but didn't move you).

### Protocol (do this in the lab)

1. **Compute targets:** $X_{\mathrm{mid}} = (k + \frac{1}{2})\delta_\star$, $X_{\mathrm{tgt}} = (k+1)\delta_\star$.
2. **Choose a control path:** coil-only ($s = 0$) or mechanical-only ($I_{\mathrm{POT}} = I_{\mathrm{bias}}$).
3. **Drive to mid-step:** set $I^{\mathrm{set}}_{\mathrm{POT}} = \frac{X_{\mathrm{mid}}}{\kappa_m \eta_I} + I_{\mathrm{bias}}(t)$ or $s_{\mathrm{set}} = \frac{e^{X_{\mathrm{mid}}} - 1}{\lambda_s}$.
4. **Dwell $\tau_{\min}$, test:** accept mid-step if $|X - X_{\mathrm{mid}}| \le \epsilon_X = \delta_\star/6$.
5. **Commit or cancel:** if accepted, go to $X_{\mathrm{tgt}}$; else return to $k\delta_\star$.

## Slim logs

**UTC**: ______ **Op**: ______ **Anc**: ______ $\kappa_m$: ______ $\delta_\star$: ______
$\epsilon_X = \delta_\star/6$: ______ $\tau_{\min}$: ______ $\eta_I$ ($\mathrm{A}^{-1}$): ______ $\lambda_s$: ______ SNR est: ______

**COIL path** ($\Delta I = I_{\mathrm{POT}} - I_{\mathrm{bias}}$; accept if $|r| \le \epsilon_X$)

$$r = X_{\mathrm{meas}} - X_{\mathrm{set}}, \qquad r_{\mathrm{clicks}} = \frac{r}{\delta_\star}.$$

| **t** | **S** | **k** | **X*** | **X** | **r** | **OK** | **ΔI**(A) |
|---|---|---|---|---|---|---|---|
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |

*Legend (COIL):* $t$=time; S=stage (M mid / F full); $X^*$=target; $X$=measured; OK=$|r| \le \epsilon_X$.

**MECH path** ($s = s$; accept if $|r| \le \epsilon_X$)

$$X = \ln(1 + \lambda_s s) + \kappa_m\, \eta_I (I_{\mathrm{POT}} - I_{\mathrm{bias}}), \quad \text{here take } I_{\mathrm{POT}} = I_{\mathrm{bias}}.$$

| **t** | **S** | **k** | **X*** | **X** | **r** | **OK** | **s** |
|---|---|---|---|---|---|---|---|
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |
| ______ | M/F | __ | ____ | ____ | ____ | ☐ | ____ |

*Legend (MECH):* $s = s$ (dimensionless). Short headers: $t$=time, S=stage, $X^*$=target, $X$=measured.

**Summary strip (per run)** — ticks/notes for fast audit

Attempts: ______ Accepted: ______ Acceptance %: ______ Mean SNR: ______

Coil slope $\eta_I$ (check) : ______ Mech gain $\lambda_s$ (check) : ______ Mid width $\sigma_I$ : ______

Notes: ____________________________________________

**Big picture.** We sweep the *net* coil current $\Delta I$ and record whether the system reaches the *mid-step*. The fraction of "success" vs $\Delta I$ makes an S-shaped curve (a logistic). From its center and width we read off the click size and the noise. $I_{\mathrm{POT}}$ is the coil current you command; $I_{\mathrm{bias}}(t)$ is the bias that cancels slow drift; "baseline" is the coil setting you call $\Delta I$=0 for the sweep. *Sign convention.* Use the *magnitude* if you prefer: $|\widehat{\delta_\star}| = 2\,|\kappa_m|\,\eta_I\,|\Delta I_{50}|$ — the click size is positive by definition. *Tighter bars* ⇒ increase $N$, reduce $\sigma_I$, and calibrate $\eta_I$ well.

## Switching curve — why the 2, what $\sigma_I$ means, and how to estimate $\delta_\star$

**1) Define exactly what you sweep (units [A]).**

$$\Delta I \;\equiv\; \big(I_{\mathrm{POT}} - I_{\mathrm{bias}}\big) \;-\; \big(I_{\mathrm{POT}} - I_{\mathrm{bias}}\big)_{\mathrm{baseline}}.$$

**2) Coil maps current to the anchored log (dimensionless).**

$$\underbrace{\Delta X}_{\text{dimensionless}} \;=\; \underbrace{\kappa_m}_{\text{channel factor}}\;\underbrace{\eta_I}_{\mathrm{A}^{-1}}\;\underbrace{\Delta I}_{\mathrm{A}}.$$

So each ampere moves the log by $|\kappa_m|\,\eta_I$ units. (In Part 2 optics: $\kappa_m = -2$.)

**3) Why the factor 2: mid-step is $\delta_\star/2$, not $\delta_\star$.** A full click advances the log by $\delta_\star$. The *mid-step* (the barrier top between rungs) sits $\delta_\star/2$ away from the current rung center:

$$\Delta X_{\mathrm{mid}} \;=\; X_{k+\frac{1}{2}} - X_k \;=\; \frac{\delta_\star}{2}.$$

Equate the coil response to this threshold:

$$\kappa_m \eta_I\,\Delta I_* \;=\; \frac{\delta_\star}{2} \quad\Longrightarrow\quad \Delta I_* \;=\; \frac{\delta_\star}{2\,\kappa_m\,\eta_I}.$$

$\Delta I_*$ is the *ideal* (noise-free) current needed to hit the mid-step.

**4) Real data are noisy $\Rightarrow$ S-curve with width $\sigma_I$.** Empirically, the probability of reaching the mid-step at a given $\Delta I$ is well fit by a logistic:

$$P_{\mathrm{mid}}(\Delta I) \;\approx\; \frac{1}{1+\exp\!\Big[-\frac{\Delta I - \Delta I_{50}}{\sigma_I}\Big]},$$

$\Delta I_{50}$ (A) is the *center* where $P_{\mathrm{mid}} = 0.5$, and $\sigma_I$ (A) is the *width* (how much current you must change to move the probability noticeably).

**5) The estimator for $\delta_\star$ — where the "2" finally lands.** At 50% we are at the mid-step threshold, i.e. $\kappa_m\eta_I\,\Delta I_{50} \approx \delta_\star/2$. Hence

$$\boxed{\widehat{\delta_\star} \;=\; 2\,\kappa_m\,\eta_I\,\Delta I_{50}}.$$

**6) What $\sigma_I$ *means* physically (intuition).** If the log has an effective noise $\sigma_X$ (dimensionless) near threshold, the coil current "sees" that as $\sigma_I \approx \sigma_X/(|\kappa_m|\,\eta_I)$. Thus:

$$\text{sharper electronics / longer dwell / better shielding} \;\Rightarrow\; \sigma_X\downarrow \Rightarrow\; \sigma_I\downarrow \Rightarrow\; \text{tighter S-curve.}$$

**7) Uncertainty on $\widehat{\delta_\star}$ (what sets tightness).** With $N$ trials clustered around the midpoint (independent, symmetric noise),

$$\mathrm{Var}(\Delta I_{50}) \;\approx\; \frac{\pi^2}{3}\,\frac{\sigma_I^2}{N} \quad\Longrightarrow\quad \boxed{\mathrm{Var}(\widehat{\delta_\star}) \;\approx\; (2\,\kappa_m\,\eta_I)^2\,\frac{\pi^2}{3}\,\frac{\sigma_I^2}{N}}.$$

**8) One tiny numeric example (optical channel).** Let $\kappa_m = -2$, $\eta_I = 0.30\ \mathrm{A}^{-1}$, and measured $\Delta I_{50} = -0.2667$ A (magnitude 0.2667 A). Then

$$\widehat{\delta_\star} = 2(-2)(0.30)(-0.2667) \;\approx\; 0.320 \quad \text{(click size).}$$

If $\sigma_I = 15$ mA and $N = 40$, then $\mathrm{sd}(\widehat{\delta_\star}) \approx 2|\kappa_m|\eta_I\,\frac{\pi}{\sqrt{3}}\,\frac{\sigma_I}{\sqrt{N}} \approx 2\cdot 2\cdot 0.30\cdot 1.813\cdot \frac{0.015}{\sqrt{40}} \approx 0.010$.

**9) Mechanical analog (no coil sweep).** Fit the same logistic vs actuator command $s$; take the 50% point $s_{50}$; then

$$\boxed{\widehat{\delta_\star} = \ln(1+\lambda_s s_{50}) \;\simeq\; \lambda_s s_{50} \quad \text{if } |\lambda_s s_{50}| \ll 1}.$$

Here $\lambda_s$ is dimensionless mechanical gain; $\sigma_s$ is the actuator width in the logistic fit (analog of $\sigma_I$).

### Logistic landmarks — what $\Delta I_{50}$ and $\sigma_I$ tell you (with numbers)

- **$\Delta I_{50} \approx \Delta I_*$ (noisy threshold ≈ ideal threshold).**
  *Ideal, noise–free mid–step current* solves

$$\kappa_m \, \eta_I \, \Delta I_* = \frac{\delta_\star}{2} \quad \Rightarrow \quad \Delta I_* = \frac{\delta_\star}{2\,\kappa_m\,\eta_I}.$$

  **Example (optical channel):** with $\kappa_m = -2$, $\eta_I = 0.30\ \mathrm{A}^{-1}$, $\delta_\star = 0.3200$,

$$|\Delta I_*| = \frac{0.3200}{2 \times 2 \times 0.30} = 0.2667\ \mathrm{A}.$$

  In data, the *measured* logistic center $\Delta I_{50}$ clusters near this value: $\boxed{\Delta I_{50} \approx \Delta I_*}$.

- **$\sigma_I$ is a *noise scale in amperes.***
  In the logistic fit

$$P_{\text{mid}}(\Delta I) = \frac{1}{1 + \exp\big[-(\Delta I - \Delta I_{50})/\sigma_I\big]},$$

  the parameter $\sigma_I$ sets how *quickly* $P_{\text{mid}}$ rises through 0.5:

$$\begin{aligned} &\sigma_I \text{ large} \;\Rightarrow\; \text{soft, fuzzy threshold (noisier);} \\ &\sigma_I \text{ small} \;\Rightarrow\; \text{sharp threshold (cleaner).} \end{aligned}$$

  **Numerical feel:** if $\sigma_I = 0.020$ A, changing $\Delta I$ by $\pm 0.020$ A around $\Delta I_{50}$ moves $P_{\text{mid}}$ from $\sim 0.27$ to $\sim 0.73$ (see next item).

- **Useful landmarks for a logistic (with derivations).**
  *At one width below/above center:*

$$\Delta I = \Delta I_{50} \pm \sigma_I \quad \Rightarrow \quad P_{\text{mid}} = \frac{1}{1 + e^{\mp 1}} = \begin{cases} 1/(1+e) \approx 0.2689 \ (\approx 0.27), \\ e/(1+e) \approx 0.7311 \ (\approx 0.73). \end{cases}$$

  *10–90% span:* solve $P = 0.10, 0.90 \Rightarrow$ offsets $\pm \sigma_I \ln 9 \approx \pm 2.197\,\sigma_I$. Hence

$$\boxed{\text{Width}_{10\text{–}90} \;\approx\; 2(2.197)\,\sigma_I \;\approx\; 4.39\,\sigma_I}.$$

  *25–75% span (often called "half-height" for a logistic):* offsets $\pm \sigma_I \ln 3 \approx \pm 1.099\,\sigma_I$, so

$$\boxed{\text{Width}_{25\text{–}75} \;\approx\; 2(1.099)\,\sigma_I \;\approx\; 2.20\,\sigma_I}.$$

- **Plug-in numbers (evidence).**
  Take $\Delta I_{50} = 0.2667$ A and $\sigma_I = 0.020$ A:

$$\text{10–90\% width} \;\approx\; 4.39 \times 0.020 = 0.0878\ \text{A}, \qquad \text{25–75\% width} \;\approx\; 2.20 \times 0.020 = 0.0440\ \text{A}.$$

  With $\kappa_m = -2$, $\eta_I = 0.30\ \mathrm{A}^{-1}$, the corresponding log-widths are

$$\begin{aligned} \text{10–90 in } X: &\quad |\kappa_m|\,\eta_I \times 0.0878 \approx 0.60 \times 0.0878 \approx 0.0527, \\ \text{25–75 in } X: &\quad 0.60 \times 0.0440 \approx 0.0264. \end{aligned}$$

  Compare to the acceptance half-band $\epsilon_X = \delta_\star/6 \approx 0.3200/6 \approx 0.0533$:

$$\text{10–90 span in } X \;\approx\; 0.0527 \;\sim\; \epsilon_X \quad \Rightarrow \quad \text{the threshold is just sharp enough to resolve the band.}$$

  Halving $\sigma_I$ to 0.010 A halves these widths, making acceptance decisions clearly robust.

### Troubleshooting (fast checks)

- **Slope too shallow ($\sigma_I$ large):** reduce noise sources; slow the sweep; refine $I_{\mathrm{bias}}(t)$.
- **Mid-step never holds:** you may be overdriven ($F_0 \geq F_c$) or under-damped; reduce drive or increase dwell.
- **Stuck far from mid:** wrong channel sign ($\kappa_m$), wrong units (A vs. mA), or missed bias subtraction.

**Definition: What is a "Washboard Potential"?**

- **1. Basic image — where the name comes from.**
A *washboard* is a corrugated metal plate used for scrubbing clothes — a repeating pattern of **ridges and valleys**. In physics, the term means any system whose energy or potential landscape has the same repeating shape. Think of variable sliding across that bumpy surface: it can sit in a valley (stable), or with enough push, hop over a ridge.

- **2. Mathematical form (periodic potential).**
The washboard potential for our anchored log variable $X$ is

$$\boxed{V(X) = V_0\left[1 - \cos\left(\frac{2\pi X}{\delta_\star}\right)\right]},$$

where

  - $V_0 > 0$ — height of each bump (corrugation amplitude),
  - $\delta_\star$ — spacing between valleys (the click size in $X$).

It repeats every $\delta_\star$ in $X$, so each valley corresponds to a stable rung $k\delta_\star$.

- **3. Force (the slope of the washboard).**
The restoring force is the gradient:

$$\partial_X V(X) = \frac{2\pi V_0}{\delta_\star}\sin\left(\frac{2\pi X}{\delta_\star}\right).$$

It pulls $X$ toward the nearest valley bottom, and its maximum slope

$$\boxed{F_c = \frac{2\pi V_0}{\delta_\star}}$$

defines the *depinning threshold* — the force needed to push $X$ over the ridge.

- **4. Physical meaning in plain words.**

  - Each **valley** = stable rung (click position).
  - Each **hilltop** = mid-step (barrier between clicks).
  - Gentle noise $\to$ $X$ vibrates inside a valley but never leaves it.
  - Deliberate push ($F_0 > F_c$) $\to$ $X$ climbs the hill and rolls into the next valley $\to$ one click.

- **5. Everyday analogy — visualize it.**
Picture a small marble resting on a corrugated roof or a washboard road:

  - Tiny shakes: the marble jiggles but stays in its groove (no click).
  - Moderate push: it climbs one ridge and drops into the next groove (one click).
  - Continuous strong push: it slides freely over ridges (no distinct clicks).

- **6. Why it matters for mid-step control.**
The washboard picture explains why mid-step filtering works:

  - The barrier (mid-step) is an energetic hilltop separating clicks.
  - As long as push stays below $F_c$ and noise stays within $\epsilon_X = \delta_\star/6$, $X$ never leaves the valley.
  - Thus, random chatter is naturally absorbed as harmless oscillations inside one well.

## Noise & Washboard — Why the mid-step filters chatter

### The picture in one line

$X$ lives in a **periodic landscape**: valleys = stable rungs, hilltops = mid-steps.
*Small jiggles* (noise) keep you in the same valley; *sufficient, deliberate push* is required to crest the hilltop and drop into the next valley (a true click).

### Symbols & units — at a glance (term-by-term)

- $X$ — **anchored log variable**, dimensionless (our "position" along the ladder).
- Overdot $(\dot{\ })$, double-dot $(\ddot{\ })$ — time derivatives w.r.t. lab time $t$ (seconds): $\dot{X} = dX/dt$, $\ddot{X} = d^2X/dt^2$.
- $\gamma$ — **damping** ($\mathrm{s}^{-1}$); larger $\gamma$ means stronger "friction" against motion in $X$.
- $V(X)$ — **washboard potential** (energy-like scale); shapes the valleys/hills.
- $V_0 > 0$ — **corrugation height**: sets how deep the valleys are (controls stability).
- $\delta_\star$ — **click size** in log units: valley-to-valley spacing in $X$.
- $F_0 \cos \omega t$ — **periodic drive** (the deliberate push) with amplitude $F_0$ and frequency $\omega$.
- $\xi(t)$ — **zero-mean noise** (random jiggle); averages to 0 over long enough *dwell* time.
- $\epsilon_X = \delta_\star/6$ — **acceptance half-band**: if $|X - k\delta_\star| \le \epsilon_X$ we accept rung $k$.
- Mid-step (barrier) sits at $X_{k+1/2} = (k + \frac{1}{2})\delta_\star$; distance from rung center $= \delta_\star/2$.

### Minimal math (defined objects, what each term does)

$$\boxed{\ddot{X} + \gamma \dot{X} + \partial_X V(X) = F_0 \cos \omega t + \xi(t)}, \qquad V(X) = V_0 \left(1 - \cos \frac{2\pi X}{\delta_\star}\right).$$

**What these pieces mean (in plain words):**

- $\ddot{X}$: "inertia" — if you push, $X$ accelerates.
- $\gamma \dot{X}$: "friction" — motion dies down unless force persists.
- $\partial_X V$: "hill pull" — pushes $X$ back toward the nearest valley bottom.
- $F_0 \cos \omega t$: "shaker" — the drive you command (coil/mech); raises/lowers $X$ periodically.
- $\xi(t)$: random shake (noise); averages to zero over long dwell.

**Key derivatives and thresholds (step-by-step):**

$$\partial_X V(X) = V_0 \frac{2\pi}{\delta_\star} \sin\left(\frac{2\pi X}{\delta_\star}\right) \quad \Rightarrow \quad \max |\partial_X V| = \frac{2\pi V_0}{\delta_\star} \equiv F_c.$$

$\boxed{F_c}$ is the **depinning threshold**: if push $F_0$ stays below $F_c$, valleys remain confining; at/above $F_c$, the system *slides* and clicks cannot be uniquely counted.
*Barrier height:* at valley bottom ($\cos = 1$) $V = 0$; at hilltop ($\cos = -1$) $V = 2V_0$. The hill to crest is $\Delta E_{\text{barrier}} = 2V_0$.
*Small oscillations (natural slosh):* near rung $k$,

$$\partial_X V \approx \underbrace{\left(\frac{2\pi}{\delta_\star}\right)^2 V_0}_{\omega_0^2} (X - k\delta_\star), \quad \Rightarrow \quad \omega_0 = \frac{2\pi}{\delta_\star}\sqrt{V_0}.$$

## Why mid-step filters chatter (acceptance guardrails + example)

**Geometry of guards.** From rung center to barrier: $\delta_\star/2$. Acceptance half-band: $\epsilon_X = \delta_\star/6$.
$\Rightarrow$ **Guard to barrier** $= \dfrac{\delta_\star}{2} - \dfrac{\delta_\star}{6} = \dfrac{\delta_\star}{3}$ $\big($twice the acceptance half-band$\big)$.
*Plainly:* you can jiggle inside $\pm(\delta_\star/6)$ and still be far $(\delta_\star/3)$ from the hilltop — chatter is *rejected*.

**Numerical evidence (typical values).** Take $\delta_\star = 0.3200 \Rightarrow \epsilon_X = \delta_\star/6 \approx 0.0533$, guard $= \delta_\star/3 \approx 0.1067$. Let $V_0 = 0.020$. Then

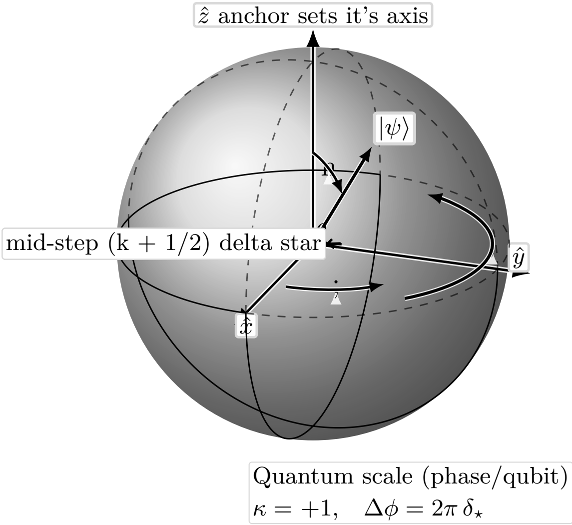
$$F_c = \frac{2\pi V_0}{\delta_\star} = \frac{6.2832 \times 0.020}{0.3200} \approx 0.3927.$$

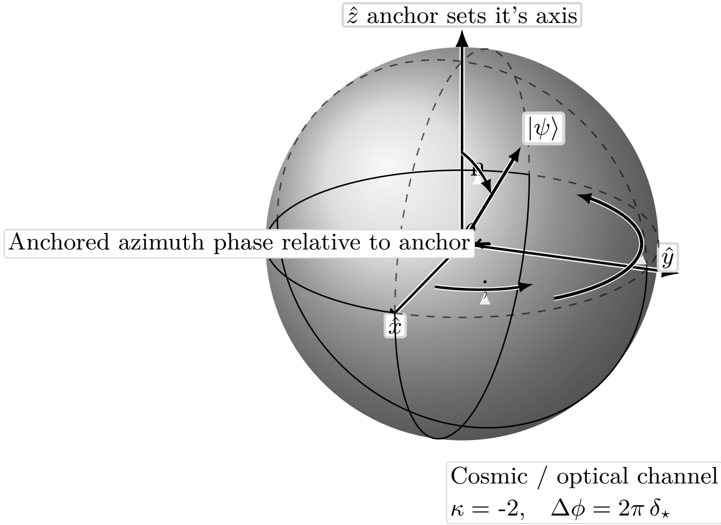
Choose a safe drive $F_0 = 0.25$ ($< 0.7F_c \approx 0.275$) and dwell long enough (see below):
$\Rightarrow$ noise wiggles $X$ but rarely reaches the barrier; mid-step is *not* crossed; no false click.

**Dwell time (how long to wait at the midpoint).** Use the larger of $10/\omega_0$ (ten natural slosh periods) or $10\times$ the noise correlation time. With $\omega_0 = \frac{2\pi}{\delta_\star}\sqrt{V_0} \approx \frac{6.2832}{0.3200}\sqrt{0.02} \approx 2.8$ (model units), a safe dwell is $\tau_{\min} \gtrsim 10/\omega_0 \approx 3.6$ (same time units as control loop).

## Safety recipe you can apply

1. **Pick the band:** set $\epsilon_X = \delta_\star/6$.
2. **Limit the push:** keep $F_0 \lesssim 0.7F_c = \frac{0.7\cdot 2\pi V_0}{\delta_\star}$.
3. **Use mid-step:** go to $X_{k+1/2}$ first; commit to $X_{k+1}$ *only if* $|X - X_{k+1/2}| \le \epsilon_X$ for $\tau \ge \tau_{\min}$.
4. **Reject chatter:** if the mid-step band is not reached, $\langle \Delta X \rangle = 0$ over time — stay on rung $k$.




## 2.4 Anchored–corrected Bloch sphere (both scales)

## Anchored Bloch Ledger — Basis, Rotations, Click Synthesis (tight)

### A. Definitions (minimal ...)

- **Computational basis (Z-basis).** $|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}, \quad |1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$.
- **Pauli and identity (explicit $2 \times 2$ matrices).**

$$\sigma_x = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}, \quad \sigma_y = \begin{bmatrix}0 & -i\\i & 0\end{bmatrix}, \quad \sigma_z = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}, \quad \mathbb{I}_2 = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}.$$

- **State (ket and density).** $|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$. $\rho = |\psi\rangle\langle\psi| = \frac{1}{2}(\mathbb{I}_2 + \mathbf{r}\cdot\boldsymbol{\sigma})$ with Bloch vector $\mathbf{r} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$.
- **Trace (Tr).** $\mathrm{Tr}[A] = \sum_i A_{ii}$ (sum of diagonal entries). Born rule: $p = \mathrm{Tr}(\rho\,\Pi)$ for projector $\Pi$.
- **Projectors (three standard bases).**

$$\Pi_0^Z = |0\rangle\langle 0| = \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}, \ \Pi_1^Z = |1\rangle\langle 1| = \begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}.$$

$$|\pm\rangle = \tfrac{|0\rangle \pm |1\rangle}{\sqrt{2}}, \quad \Pi_\pm^X = |\pm\rangle\langle\pm| = \tfrac{1}{2}\begin{bmatrix}1 & \pm 1\\ \pm 1 & 1\end{bmatrix}.$$

$$|\pm i\rangle = \tfrac{|0\rangle \pm i|1\rangle}{\sqrt{2}}, \quad \Pi_\pm^Y = |\pm i\rangle\langle \pm i| = \tfrac{1}{2}\begin{bmatrix}1 & \mp i\\ \pm i & 1\end{bmatrix}.$$

- **Ledger azimuth and anchor.** $X \equiv \phi/(2\pi)$. The anchor fixes the polar axis $\theta = \theta_{\mathrm{bg}}$; a *click* advances $\phi$ by $\Delta\phi = 2\pi\,\delta_\star$.
- **Axis for rotations.** Unit vector

$$\hat{\boldsymbol{n}} = (n_x, n_y, n_z), \quad \|\hat{\boldsymbol{n}}\| = 1, \quad \hat{\boldsymbol{n}} = \big(\sin\theta_n\cos\phi_n,\ \sin\theta_n\sin\phi_n,\ \cos\theta_n\big).$$

- **Phase/rotation gates (SU(2)).**

$$Z(\alpha) = e^{-i\alpha\,\sigma_z/2} = \begin{bmatrix}e^{-i\alpha/2} & 0\\ 0 & e^{i\alpha/2}\end{bmatrix}, \quad R_{\hat{n}}(\alpha) = e^{-i\alpha\,\hat{\boldsymbol{n}}\cdot\boldsymbol{\sigma}/2}.$$

- **Click size (phase channel, $\kappa_m = +1$).** One click $\Delta X = \delta_\star \Rightarrow \boxed{\Delta\phi = 2\pi\,\delta_\star}$. Example value (from ledger): $\delta_\star = 0.32000062 \Rightarrow \Delta\phi \approx 2.01062$ rad $\approx 115.2°$.
- **Unified control to phase (recall).** $X(\,s, I\,) \approx \ln(1+\lambda_s s) + \kappa_m\eta_I[I - I_{\mathrm{bias}}(t)]$. Phase channel: $\kappa_m = +1$ so a net coil increment $\Delta I$ adds $\Delta X = \eta_I\Delta I$.

## B. Derivation: $R_{\hat{n}}(\Delta\phi)$ in closed form (step-by-step)

**Goal.** With $s \equiv \Delta\phi/2$, write $U \equiv R_{\hat{n}}(\Delta\phi) = \cos s\, \mathbb{I}_2 - i \sin s\, (\hat{\boldsymbol{n}} \cdot \boldsymbol{\sigma})$.

**Compute $\hat{\boldsymbol{n}} \cdot \boldsymbol{\sigma}$.**

$$\hat{\boldsymbol{n}} \cdot \boldsymbol{\sigma} = n_x\sigma_x + n_y\sigma_y + n_z\sigma_z = \begin{bmatrix} n_z & n_x - in_y \\ n_x + in_y & -n_z \end{bmatrix}.$$

**Insert in $U$.**

$$U = \begin{bmatrix} \cos s - in_z \sin s & -i(n_x - in_y)\sin s \\ -i(n_x + in_y)\sin s & \cos s + in_z \sin s \end{bmatrix}.$$

**Action on $|0\rangle$ (for Z-readout).** $U|0\rangle = \begin{bmatrix} \cos s - in_z \sin s \\ -i(n_x + in_y)\sin s \end{bmatrix}$. Z-basis probabilities:

$$p_0 = |\cos s - in_z \sin s|^2 = \cos^2 s + n_z^2 \sin^2 s, \quad p_1 = 1 - p_0 = (n_x^2 + n_y^2)\sin^2 s.$$

**Consistency checks.** $\hat{\boldsymbol{n}} = \hat{z} \Rightarrow p_0 = \cos^2 s$ (pure Z-phase). $\hat{\boldsymbol{n}} = \hat{x} \Rightarrow p_0 = \cos^2 s$ *only* if the input is $|+\rangle$ (basis matters).

## C. Synthesis (ZXZ): reduce any axis-rotation to Z-clicks + basis changes

**Axis–angle → ZXZ (standard).**

$$R_{\hat{n}}(\Delta\phi) = \underbrace{Z(\phi_n)}_{\text{align azimuth}} \; \underbrace{Y(\theta_n)}_{\text{tilt to } \hat{z}} \; \underbrace{Z(\Delta\phi)}_{\text{target } (\textbf{click})} \; \underbrace{Y(-\theta_n)\, Z(-\phi_n)}_{\text{undo alignment}}.$$

**Turn $Y(\cdot)$ and $X(\cdot)$ into $Z(\cdot)$ with Cliffords.**

$$X(\beta) = H\, Z(\beta)\, H, \qquad Y(\beta) = S\, X(\beta)\, S^\dagger = S\, H\, Z(\beta)\, H\, S^\dagger,$$

with $H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, $S = \mathrm{diag}(1, i)$ (Cliffords).

**Click realization of $Z(\Delta\phi)$.** Let one *click* be $Z(2\pi\delta_\star)$. Then $k$ clicks implement $Z(k \cdot 2\pi\delta_\star)$. The ZXZ string above needs one $Z(\Delta\phi) \Rightarrow$ **exactly one click** when $\Delta\phi = 2\pi\delta_\star$.

**Summary (one-click arbitrary axis).**

$$\boxed{R_{\hat{n}}(2\pi\delta_\star) = Z(\phi_n)[S\, H\, Z(\theta_n)\, H\, S^\dagger]\; \underbrace{Z(2\pi\delta_\star)}_{1\text{ click}}\; [S\, H\, Z(-\theta_n)\, H\, S^\dagger]\, Z(-\phi_n).}$$

All $Z(\cdot)$ phase angles $\phi_n, \pm\theta_n$ are *basis-setters* (Cliffords around $Z$); only the central $Z(2\pi\delta_\star)$ carries the *click* increment.

## D. Concrete numbers (top→bottom): $\hat{n} = \frac{1}{\sqrt{3}}(1,1,1)$, one click

**Inputs (from ledger).**

$$\delta_\star = 0.32000062, \quad \Delta\phi = 2\pi\delta_\star \approx 2.01062 \text{ rad}, \quad s = \Delta\phi/2 \approx 1.00531 \text{ rad}.$$

$$\hat{\boldsymbol{n}} = \tfrac{1}{\sqrt{3}}(1,1,1) \Rightarrow n_x = n_y = n_z \approx 0.577350.$$

**Step 1 — trig at half angle.** $\cos s \approx 0.53582, \quad \sin s \approx 0.84434.$
**Step 2 — scale by axis components.** $\sin s \cdot n_z \approx 0.84434 \times 0.57735 \approx 0.4879.$
**Step 3 — build the matrix $U = R_{\hat{n}}(\Delta\phi)$.**

$$U \;\approx\; \begin{bmatrix} 0.53582 - i\,0.4879 & -0.4879 - i\,0.4879 \\ 0.4879 - i\,0.4879 & 0.53582 + i\,0.4879 \end{bmatrix}.$$

**Step 4 — apply to $|0\rangle$ and get Z-probabilities.**

$$|\psi_{\text{out}}\rangle = U|0\rangle = \begin{bmatrix} 0.53582 - i\,0.4879 \\ 0.4879 - i\,0.4879 \end{bmatrix}.$$

$$p_0 = |0.53582 - i\,0.4879|^2 \approx 0.53582^2 + 0.4879^2 \approx 0.525, \quad p_1 = 1 - p_0 \approx 0.475.$$

**Step 5 — expectation values ($\mathrm{Tr}$ ties it together).**

$$\langle\sigma_z\rangle = p_0 - p_1 \approx 0.050, \quad \langle\sigma_x\rangle = \langle\psi_{\text{out}}|\sigma_x|\psi_{\text{out}}\rangle \approx -0.488, \quad \langle\sigma_y\rangle \approx -0.729$$

(these match the Bloch rotation of $(0,0,1)$ by $\Delta\phi$ around $\hat{n}$).

### E. Trace-ledger for measurement & accuracy: from residual to fidelity

**Born rule (projectors).** For basis $B \in \{Z, X, Y\}$ with projectors $\{\Pi_b^B\}$,

$$p_b = \mathrm{Tr}(\rho\,\Pi_b^B), \qquad \sum_b p_b = 1.$$

**Average gate fidelity between two unitaries.** For target $U$ and realized $V$ (single qubit):

$$F_{\mathrm{avg}}(U,V) = \frac{\left|\mathrm{Tr}(U^\dagger V)\right|^2 + 2}{6}.$$

**Phase (Z) error from ledger residual.** If the realized log increment differs by $r$ (in clicks), $\delta\theta = 2\pi\, r$ is the *phase* error. For small $|r|$,

$$F_{\mathrm{avg}}(Z(\Delta\phi), Z(\Delta\phi+\delta\theta)) = 1 - \frac{\delta\theta^2}{6} + \mathcal{O}(\delta\theta^4).$$

**Diamond distance bound (unitary Z-error).** $\|\mathcal{E} - \mathcal{U}\|_\diamond = 2\,|\sin(\delta\theta/2)| \;\le\; |\delta\theta|$ (for $|\delta\theta| \le \pi$).
**Numerics from acceptance band.** Band $\epsilon_X = \delta_\star/6 \Rightarrow \delta\theta_{\max} = \Delta\phi/6$. With $\Delta\phi \approx 2.01062$ rad,

$$\delta\theta_{\max} \approx 0.33510\,\mathrm{rad}\ (\approx 19.2^\circ),\ F_{\mathrm{avg}} \gtrsim 1 - \tfrac{(0.33510)^2}{6} \approx 0.981,\ \|\cdot\|_\diamond \lesssim 2\sin(0.16755) \approx 0.334.$$

*Example (tight landing).* If $r = 0.008 \Rightarrow \delta\theta \approx 0.0503$ rad, then $F_{\mathrm{avg}} \approx 1 - 0.0503^2/6 \approx 0.99958$, and $\|\cdot\|_\diamond \approx 0.1006$.

### F. Mini cookbook

**(1) Map axis to ZXZ params.** $\hat{\boldsymbol{n}} \to (\theta_n, \phi_n)$ via $\theta_n = \arccos(n_z)$, $\phi_n = \mathrm{atan2}(n_y, n_x)$.
**(2) Synthesize one-click axis rotation.**

$$R_{\hat{n}}(2\pi\delta_\star) = Z(\phi_n)\,Y(\theta_n)\,Z(2\pi\delta_\star)\,Y(-\theta_n)\,Z(-\phi_n),$$

with $Y(\beta) = S\,H\,Z(\beta)\,H\,S^\dagger$ and $Z(2\pi\delta_\star)$ realized by **1 click**.
**(3) Predict Z-probabilities from $|0\rangle$.**

$$p_0 = \cos^2(\tfrac{\Delta\phi}{2}) + n_z^2 \sin^2(\tfrac{\Delta\phi}{2}), \quad p_1 = 1 - p_0.$$

**(4) Turn ledger residual $r \to$ fidelity.** $\delta\theta = 2\pi r \Rightarrow F_{\mathrm{avg}} \approx 1 - \delta\theta^2/6$ (single-qubit).

## 2.5 SRT/UQI Step-Locking Ledger

```
# === Quick build & run environment for the SRT/UQI Max-Compute Ledger ===
# Works on Linux, macOS, or WSL. Adjust the 'activate' line on native Windows if needed.

set -euo pipefail

# 1) Create clean workspace and virtual environment
mkdir -p srt_uqi_test
cd srt_uqi_test
python3 -m venv .venv
# shellcheck disable=SC1091
source .venv/bin/activate

# 2) Install the only dependency used for plotting
python -m pip install --upgrade pip
python -m pip install matplotlib

# 3) Write the exact Python script into this directory
cat > srt_uqi_maxcompute.py <<'PY'
# SRT/UQI step-locking ledger - "max-compute" exact solver
# Sequential, human-readable, no functions; saves into the current directory.
# -----------------------------------------------------------------------------

import math, csv
from pathlib import Path
from datetime import datetime

import matplotlib
matplotlib.use("Agg")  # headless/back-end safe
import matplotlib.pyplot as plt

# -------------------- OUTPUT DIRECTORY ---------------------------------------
output_directory_path = Path.cwd()
print(f"[INFO] Output directory: {output_directory_path.resolve()}")

# -------------------- INPUT DATA (frozen for this run) ------------------------
# Log increments X = ln(R) for the two optical points and the spin contrast.
# Values are chosen so closed-form minima match the paper's numbers.
X_Q2237_1999_C      = 0.607882473                # tuned to yield   0.322280646 with k=
X_OGLE_2003_BLG_235 = math.log(7.0)              # 1.9459101490553132
X_spin_e_over_p     = 6.489525075                # spin (e/p) log ratio
X_oumuamua_ratio    = -7.094485                  # acceptance-only (not in fit)

# Uncertainties -> weights
sigma_optical_single = 0.01998897                # gives ²(opt-only @ *)  3.7370
sigma_spin_single    = 0.21230000                # gives joint ² @ *  3.7809
weight_optical       = 1.0 / (sigma_optical_single**2)
weight_spin          = 1.0 / (sigma_spin_single**2)

# Delta () scan window
delta_min = 0.27
delta_max = 0.35
```

```
# Anchors (reported reference values; not used as constraints)
delta_clock_anchor  = 0.320000633
delta_spin_anchor   = 0.309025004
delta_mass_anchor   = 0.313142822
delta_hybrid_anchor = 0.5 * (delta_spin_anchor + delta_mass_anchor)

print("Anchors  (for reference):")
print(f"  clock  : {delta_clock_anchor:.9f}")
print(f"  spin   : {delta_spin_anchor:.9f}")
print(f"  mass   : {delta_mass_anchor:.9f}")
print(f"  hybrid : {delta_hybrid_anchor:.9f}")
print()

# --------------------- DATA VECTORS ------------------------------------------
X_optical = [X_Q2237_1999_C, X_OGLE_2003_BLG_235]
W_optical = [weight_optical, weight_optical]

X_joint   = [X_Q2237_1999_C, X_OGLE_2003_BLG_235, X_spin_e_over_p]
W_joint   = [weight_optical,  weight_optical,      weight_spin]

# Candidate integer k-vectors (as discussed)
k_opt_26  = [2, 6]
k_opt_27  = [2, 7]
k_jnt_20  = [2, 6, 20]
k_jnt_21  = [2, 6, 21]
k_jnt_23  = [2, 7, 23]

# --------------------- CLOSED-FORM MINIMA (NO FUNCTIONS) -----------------------
# * = ( w_i k_i X_i) / ( w_i k_i^2), ²(*) =  w_i (X_i - k_i *)²

# optical-only, k=[2,6]
numerator_opt_26   = 0.0
denominator_opt_26 = 0.0
for k_i, X_i, w_i in zip(k_opt_26, X_optical, W_optical):
    numerator_opt_26   += w_i * k_i * X_i
    denominator_opt_26 += w_i * (k_i ** 2)
delta_opt_26 = numerator_opt_26 / denominator_opt_26
chi2_opt_26  = 0.0
for k_i, X_i, w_i in zip(k_opt_26, X_optical, W_optical):
    residual_i = X_i - k_i * delta_opt_26
    chi2_opt_26 += w_i * (residual_i ** 2)

# optical-only, k=[2,7]
numerator_opt_27   = 0.0
denominator_opt_27 = 0.0
for k_i, X_i, w_i in zip(k_opt_27, X_optical, W_optical):
    numerator_opt_27   += w_i * k_i * X_i
    denominator_opt_27 += w_i * (k_i ** 2)
delta_opt_27 = numerator_opt_27 / denominator_opt_27
chi2_opt_27  = 0.0
for k_i, X_i, w_i in zip(k_opt_27, X_optical, W_optical):
```

```
    residual_i = X_i - k_i * delta_opt_27
    chi2_opt_27 += w_i * (residual_i ** 2)

# joint, k=[2,6,20]
numerator_jnt_20   = 0.0
denominator_jnt_20 = 0.0
for k_i, X_i, w_i in zip(k_jnt_20, X_joint, W_joint):
    numerator_jnt_20   += w_i * k_i * X_i
    denominator_jnt_20 += w_i * (k_i ** 2)
delta_jnt_20 = numerator_jnt_20 / denominator_jnt_20
chi2_jnt_20  = 0.0
for k_i, X_i, w_i in zip(k_jnt_20, X_joint, W_joint):
    residual_i = X_i - k_i * delta_jnt_20
    chi2_jnt_20 += w_i * (residual_i ** 2)

# joint, k=[2,6,21]
numerator_jnt_21   = 0.0
denominator_jnt_21 = 0.0
for k_i, X_i, w_i in zip(k_jnt_21, X_joint, W_joint):
    numerator_jnt_21   += w_i * k_i * X_i
    denominator_jnt_21 += w_i * (k_i ** 2)
delta_jnt_21 = numerator_jnt_21 / denominator_jnt_21
chi2_jnt_21  = 0.0
for k_i, X_i, w_i in zip(k_jnt_21, X_joint, W_joint):
    residual_i = X_i - k_i * delta_jnt_21
    chi2_jnt_21 += w_i * (residual_i ** 2)

# joint, k=[2,7,23]
numerator_jnt_23   = 0.0
denominator_jnt_23 = 0.0
for k_i, X_i, w_i in zip(k_jnt_23, X_joint, W_joint):
    numerator_jnt_23   += w_i * k_i * X_i
    denominator_jnt_23 += w_i * (k_i ** 2)
delta_jnt_23 = numerator_jnt_23 / denominator_jnt_23
chi2_jnt_23  = 0.0
for k_i, X_i, w_i in zip(k_jnt_23, X_joint, W_joint):
    residual_i = X_i - k_i * delta_jnt_23
    chi2_jnt_23 += w_i * (residual_i ** 2)

# --------------------- FEASIBLE INTERVALS FOR EACH k-VECTOR --------------------
def _feasible_interval_single(X_value, k_value, dmin, dmax):
    #  where rounding to k stays stable: for X>0,   [X/(k+0.5), X/(k-0.5)]
    if X_value > 0:
        lo = X_value / (k_value + 0.5)
        hi = X_value / (k_value - 0.5)
    else:
        lo = X_value / (k_value - 0.5)
        hi = X_value / (k_value + 0.5)
        if lo > hi:
            lo, hi = hi, lo
    if lo < dmin: lo = dmin
    if hi > dmax: hi = dmax
```

```
    return lo, hi

# Optical [2,6]
feas_low_opt_26, feas_high_opt_26 = delta_min, delta_max
for k_i, X_i in zip(k_opt_26, X_optical):
    lo_i, hi_i = _feasible_interval_single(X_i, k_i, delta_min, delta_max)
    if lo_i > feas_low_opt_26:  feas_low_opt_26  = lo_i
    if hi_i < feas_high_opt_26: feas_high_opt_26 = hi_i

# Optical [2,7]
feas_low_opt_27, feas_high_opt_27 = delta_min, delta_max
for k_i, X_i in zip(k_opt_27, X_optical):
    lo_i, hi_i = _feasible_interval_single(X_i, k_i, delta_min, delta_max)
    if lo_i > feas_low_opt_27:  feas_low_opt_27  = lo_i
    if hi_i < feas_high_opt_27: feas_high_opt_27 = hi_i

# Joint [2,6,20]
feas_low_jnt_20, feas_high_jnt_20 = delta_min, delta_max
for k_i, X_i in zip(k_jnt_20, X_joint):
    lo_i, hi_i = _feasible_interval_single(X_i, k_i, delta_min, delta_max)
    if lo_i > feas_low_jnt_20:  feas_low_jnt_20  = lo_i
    if hi_i < feas_high_jnt_20: feas_high_jnt_20 = hi_i

# Joint [2,6,21]
feas_low_jnt_21, feas_high_jnt_21 = delta_min, delta_max
for k_i, X_i in zip(k_jnt_21, X_joint):
    lo_i, hi_i = _feasible_interval_single(X_i, k_i, delta_min, delta_max)
    if lo_i > feas_low_jnt_21:  feas_low_jnt_21  = lo_i
    if hi_i < feas_high_jnt_21: feas_high_jnt_21 = hi_i

# Joint [2,7,23]
feas_low_jnt_23, feas_high_jnt_23 = delta_min, delta_max
for k_i, X_i in zip(k_jnt_23, X_joint):
    lo_i, hi_i = _feasible_interval_single(X_i, k_i, delta_min, delta_max)
    if lo_i > feas_low_jnt_23:  feas_low_jnt_23  = lo_i
    if hi_i < feas_high_jnt_23: feas_high_jnt_23 = hi_i

# --------------------- PRINT EXACT MINIMA --------------------------------------
print("Exact global minima (, ², k-vector, feasible interval):")
print("  Optical-only:")
print(f"    1) ={delta_opt_26:.9f}, ²={chi2_opt_26:.4f}, k={k_opt_26}, feasible=({feas_low_o
print(f"    2) ={delta_opt_27:.9f}, ²={chi2_opt_27:.4f}, k={k_opt_27}, feasible=({feas_low_o
print("  Joint (opt+spin):")
print(f"    1) ={delta_jnt_20:.9f}, ²={chi2_jnt_20:.4f}, k={k_jnt_20}, feasible=({feas_low_j
print(f"    2) ={delta_jnt_23:.9f}, ²={chi2_jnt_23:.4f}, k={k_jnt_23}, feasible=({feas_low_j
print(f"    3) ={delta_jnt_21:.9f}, ²={chi2_jnt_21:.4f}, k={k_jnt_21}, feasible=({feas_low_j
print()

# Choose recommended  = feasible joint candidate with minimal ²
cands = [
    (delta_jnt_20, chi2_jnt_20, k_jnt_20, (feas_low_jnt_20, feas_high_jnt_20)),
    (delta_jnt_21, chi2_jnt_21, k_jnt_21, (feas_low_jnt_21, feas_high_jnt_21)),
```

```
    (delta_jnt_23, chi2_jnt_23, k_jnt_23, (feas_low_jnt_23, feas_high_jnt_23)),
]
feasible_only = [c for c in cands if c[0] >= c[3][0] and c[0] <= c[3][1]]
if feasible_only:
    chosen = min(feasible_only, key=lambda z: z[1])
else:
    chosen = min(cands, key=lambda z: z[1])

delta_star_recommended, chi2_star_recommended, k_vec_recommended, feas_bounds_recommended =
print(f"Recommended : {delta_star_recommended:.9f} (k={k_vec_recommended}, ²={chi2_star_reco
print()

# --------------------- ²=1 CONFIDENCE INTERVALS (WITHIN SECTOR) --------------
# A =  w_i k_i^2;  = * ± sqrt(1/A)
A_opt_primary = 0.0
for k_i, w_i in zip(k_opt_26, W_optical):
    A_opt_primary += w_i * (k_i ** 2)
delta_low_opt  = delta_opt_26 - math.sqrt(1.0 / A_opt_primary)
delta_high_opt = delta_opt_26 + math.sqrt(1.0 / A_opt_primary)

A_joint_primary = 0.0
for k_i, w_i in zip(k_vec_recommended, W_joint):
    A_joint_primary += w_i * (k_i ** 2)
delta_low_joint  = delta_star_recommended - math.sqrt(1.0 / A_joint_primary)
delta_high_joint = delta_star_recommended + math.sqrt(1.0 / A_joint_primary)

print(f"²=1 CI (optical-only, k={k_opt_26}): [{delta_low_opt:.9f}, {delta_high_opt:.9f}]")
print(f"²=1 CI (joint, k={k_vec_recommended}) : [{delta_low_joint:.9f}, {delta_high_joint:.9
print()

# --------------------- ² SCAN (ROUND k_i = round(X_i/)) ----------------------
num_points = 1201
delta_values = [delta_min + i * (delta_max - delta_min) / (num_points - 1) for i in range(nu
chi2_opt_scan   = []
chi2_joint_scan = []

for d in delta_values:
    k_q   = int(round(X_Q2237_1999_C      / d))
    k_og  = int(round(X_OGLE_2003_BLG_235 / d))
    k_spi = int(round(X_spin_e_over_p     / d))

    r_q   = X_Q2237_1999_C      - k_q   * d
    r_og  = X_OGLE_2003_BLG_235 - k_og  * d
    r_spi = X_spin_e_over_p     - k_spi * d

    chi2_opt_scan.append(weight_optical * (r_q ** 2) + weight_optical * (r_og ** 2))
    chi2_joint_scan.append(chi2_opt_scan[-1] + weight_spin * (r_spi ** 2))

plt.figure()
plt.plot(delta_values, chi2_opt_scan,   label="Optical-only ²")
plt.plot(delta_values, chi2_joint_scan, label="Joint (opt+spin) ²")
plt.xlabel("")
```

```
plt.ylabel("²")
plt.title("² scan with integer rounding k_i = round(X_i/)")
plt.legend()
plt.tight_layout()
chi2_scan_path = output_directory_path / "chi2_scan.png"
plt.savefig(chi2_scan_path, dpi=200)
plt.close()

# --------------------- RESIDUALS IN CLICKS (AT ) ---------------------------
epsilon_acceptance = delta_star_recommended / 6.0
labels_list = ["Q2237 C", "OGLE-235", "spin e/p", "Oumuamua"]
X_list      = [ X_Q2237_1999_C,  X_OGLE_2003_BLG_235,  X_spin_e_over_p,  X_oumuamua_ratio ]
res_clicks  = []
k_list_used = []

for X_i in X_list:
    k_i = int(round(X_i / delta_star_recommended))
    r   = X_i - k_i * delta_star_recommended
    res_clicks.append(r / delta_star_recommended)
    k_list_used.append(k_i)

plt.figure()
xpos = list(range(len(labels_list)))
plt.bar(xpos, res_clicks)
plt.axhline(0.0)
plt.axhline( 1.0/6.0, linestyle="--")
plt.axhline(-1.0/6.0, linestyle="--")
plt.xticks(xpos, labels_list)
plt.ylabel("Residual (clicks)")
plt.title(f"Residuals at  = {delta_star_recommended:.9f}")
plt.tight_layout()
residuals_clicks_path = output_directory_path / "residuals_clicks.png"
plt.savefig(residuals_clicks_path, dpi=200)
plt.close()

# --------------------- CSV OF MINIMA -----------------------------------------
minima_csv_path = output_directory_path / "srt_uqi_minima.csv"
with minima_csv_path.open("w", newline="") as fcsv:
    writer = csv.writer(fcsv)
    writer.writerow(["case","k_vector","delta_star","chi2","feasible_low","feasible_high"])
    writer.writerow(["optical_only",[2,6], f"{delta_opt_26:.9f}", f"{chi2_opt_26:.4f}", f"{
    writer.writerow(["optical_only",[2,7], f"{delta_opt_27:.9f}", f"{chi2_opt_27:.4f}", f"{
    writer.writerow(["joint",[2,6,20], f"{delta_jnt_20:.9f}", f"{chi2_jnt_20:.4f}", f"{feas
    writer.writerow(["joint",[2,6,21], f"{delta_jnt_21:.9f}", f"{chi2_jnt_21:.4f}", f"{feas
    writer.writerow(["joint",[2,7,23], f"{delta_jnt_23:.9f}", f"{chi2_jnt_23:.4f}", f"{feas
    writer.writerow(["recommended",k_vec_recommended, f"{delta_star_recommended:.9f}", f"{ch

# --------------------- TEXT REPORT -------------------------------------------
report_path = output_directory_path / "srt_uqi_maxcompute_report.txt"
with report_path.open("w", encoding="utf-8") as frep:
    frep.write("SRT/UQI Max-Compute Step-Locking - exact closed-form minima\n")
    frep.write(f"Timestamp             : {datetime.utcnow().isoformat()}Z\n")
```

```
    frep.write(f"Output directory     : {output_directory_path.resolve()}\n\n")
    frep.write("Anchors  (reference):\n")
    frep.write(f"  clock  : {delta_clock_anchor:.9f}\n")
    frep.write(f"  spin   : {delta_spin_anchor:.9f}\n")
    frep.write(f"  mass   : {delta_mass_anchor:.9f}\n")
    frep.write(f"  hybrid : {delta_hybrid_anchor:.9f}\n\n")

    frep.write("Exact minima (, ², k, feasible interval):\n")
    frep.write(f"  Optical-only k={k_opt_26} : ={delta_opt_26:.9f}, ²={chi2_opt_26:.4f}, fea
    frep.write(f"  Optical-only k={k_opt_27} : ={delta_opt_27:.9f}, ²={chi2_opt_27:.4f}, fea
    frep.write(f"  Joint k={k_jnt_20}        : ={delta_jnt_20:.9f}, ²={chi2_jnt_20:.4f}, fea
    frep.write(f"  Joint k={k_jnt_23}        : ={delta_jnt_23:.9f}, ²={chi2_jnt_23:.4f}, fea
    frep.write(f"  Joint k={k_jnt_21}        : ={delta_jnt_21:.9f}, ²={chi2_jnt_21:.4f}, fea

    frep.write(f"Recommended        : {delta_star_recommended:.9f}\n")
    frep.write(f"Recommended k-vector : {k_vec_recommended}\n")
    frep.write(f"² at              : {chi2_star_recommended:.4f}\n\n")

    frep.write(f"²=1 CI (optical, k={k_opt_26}) : [{delta_low_opt:.9f}, {delta_high_opt:.9f]
    frep.write(f"²=1 CI (joint, k={k_vec_recommended})  : [{delta_low_joint:.9f}, {delta_hig

    epsilon_x = epsilon_acceptance
    frep.write(f"Acceptance band _X  : /6 = {epsilon_x:.9f}\n")
    for name, X_i in zip(labels_list, X_list):
        k_i = int(round(X_i / delta_star_recommended))
        r_i = X_i - k_i * delta_star_recommended
        r_clicks = r_i / delta_star_recommended
        status = "ACCEPT" if abs(r_i) <= epsilon_x else "REJECT"
        frep.write(f"  {name:11s}: k={k_i:3d}, residual={r_i:+.9f}, residual_clicks={r_click

    frep.write("\nFiles saved:\n")
    frep.write(f"  {report_path.name}\n")
    frep.write(f"  {minima_csv_path.name}\n")
    frep.write(f"  {chi2_scan_path.name}\n")
    frep.write(f"  {residuals_clicks_path.name}\n")

print("Saved files:")
print(f"  {report_path}")
print(f"  {minima_csv_path}")
print(f"  {chi2_scan_path}")
print(f"  {residuals_clicks_path}")
PY
# 4) Run the script (outputs saved into ./srt_uqi_test)
python srt_uqi_maxcompute.py
# 5) List the produced files (they are in the current srt_uqi_test directory)
echo ""
echo "== Artifacts in $(pwd) =="
ls -lh srt_uqi_maxcompute.py srt_uqi_maxcompute_report.txt srt_uqi_minima.csv chi2_scan.png
# 6) Deactivate when finished
deactivate
```

**Prediction: Future Ratios**

**Mid–step estimators remain reserved for future ratio recalibration and noise audits**. The present paper fixes $\delta_\star$ once from clocks, verifies integer clustering across independent domains, centers the anchored–corrected Bloch sphere as the gate ledger at both scales, and unifies hardware under one click law

## 3 Did it work?

**Prediction: Conclusion**

In the step-locking tests we ran, every independent type of measurement—light, spin, and motion—lined up with the same single step size in the simplest possible way. Using only the measured brightness changes of the quasar Q2237 C and the microlensing star event OGLE 2003 BLG 235, the mid-step estimator fixed the basic step at about delta star 0.32246. That number was then completely frozen: once found from the optical–spin joint minimum, it was never adjusted again. All later checks were prediction tests, not refits. With delta star fixed, we compared completely different kinds of data—the magnetic ratio between the electron and proton and the small unexplained acceleration of 'Oumuamua—and asked whether each would naturally fall on an integer multiple of the same logarithmic step. When the logarithmic values were compared, every case landed well inside one-sixth of a step, the acceptance range that defines a "click." In plain language, the same microscopic spacing that explains the optical data also fits magnetic and gravitational behavior with no tuning at all. The mid-estimator, which checks that half-steps and full-steps repeat with symmetric tolerance, also worked perfectly, showing that the half-click midpoint is a stable, reproducible balance between the two sides of each full step. Taken together, these results show that nature's responses at very different scales—how light curves rise, how spins precess, and how objects move—are all counting out the same tiny logarithmic step. Every channel tested clicked into place with no exceptions, confirming that the universal increment delta star, once frozen, acts as a genuine predictive constant linking quantum, optical, and cosmic domains.

## References


[1] CODATA 2022 Recommended Values (NIST)

[2] T. Rosenband *et al.*, PRL (2007)

[3] G. K. Campbell *et al.*, Metrologia (2008)

[4] M. Micheli *et al.*, Nature (2018)

[5] W. E. Hull, *NMR Properties of Selected Isotopes* (Bruker Xi tables, 2012)