Documentation for the General QMC code

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1 Theory part

1.1 Definition of the physical Hamiltonian and its implementation

The physical Hamiltonians that we can simulate have the general form:

$$\mathcal{H} = \sum_{s=1}^{N_{fl}} \sum_{x,y} c_{xs}^{\dagger} T_{xy} c_{ys} - \sum_{k=1}^{M} U_k \left[\sum_{s=1}^{N_{fl}} \sum_{x,y} \left(c_{xs}^{\dagger} V_{xy}^{(k)} c_{ys} - \alpha_k \right) \right]^2 . \tag{1}$$

Note, that we introduced two different labels to address spin degrees of freedom:

- The number of spin flavours is set by N_{fl} .
- The number N_{sun} also labels spin states and specifially sets the symmetry group of the fermions, namely the dimension of the special unitary group $SU(N_{sun})$.

And the following labels specify the lattice sites:

• The indices x, y are multi-indices that label degrees of freedom, containing lattice sites and spin states: $x = (i, \sigma)$, where $i = 1, \dots, N_{sites}$ and $\sigma = 1, \dots, N_{sun}$, so

$$\sum_{x,y} \equiv \sum_{i=1,j=1}^{N_{sites}} \sum_{\sigma=1,\sigma'=1}^{N_{sun}} . \tag{2}$$

- The number of correlated sites which is a subset of all sites, is labelled by M ($M \le N_{sites}$).
- Let us further define $N_{dim} = N_{sun}N_{sites}$ such that the matrices T and $V^{(k)}$ are of dimension $N_{dim} \times N_{dim}$
- N_{sites} is the total number of spacial vertices: $N_{sites} = N_{unit\ cell}N_{orbital}$, where $N_{unit\ cell}$ is the number of unit cells of the underlying Bravais lattice and $N_{orbital}$ is the number of (spacial) orbitals per unit cell Check the definition of $N_{orbital}$ in the code.

1.2 Structure of the matrices T and $V^{(k)}$ and their implementation

In general, the matrices \mathbf{T} and $\mathbf{V}^{(k)}$ are sparse matrices. This property is used to minimize computational cost and storage. In the following, we discuss the implementation of the single-particle matrix representation $\mathbf{V}^{(k)}$ of the interaction operator. The same logic applies for the hopping matrix \mathbf{T} .

We denote a subset of N_{eff} (in the code, N_{eff} is called just N) degrees of freedom by the set $[z_1, \dots z_{N_{eff}}]$ and define it to contain only vertices for which an interaction term is defined:

$$V_{xy}^{(k)} \neq 0$$
 only if $x, y \in [z_1^{(k)}, \dots z_{N_{eff}^{(k)}}^{(k)}]$. (3)

We define the projection matrices $\mathbf{P}_{V}^{(k)}$ of dimension $N_{eff}^{(k)} \times N_{dim}$:

$$(P_V^{(k)})_{i,z} = \delta_{z^{(k)},z}, \tag{4}$$

where $i \in [1, \dots, N_{eff}^{(k)}]$ and $z \in [1, \dots, N_{dim}]$. The matrix operator $\mathbf{P}_V^{(k)}$ picks out the non-vanishing entries of $\mathbf{V}^{(k)}$, which are contained in the $(N_{eff}^{(k)} \times N_{eff}^{(k)})$ - dimensional matrix $\mathbf{O}_V^{(k)}$:

$$V^{(k)} = P_V^{(k)T} O_V^{(k)} P_V^{(k)} , (5)$$

and

$$V_{xy}^{(k)} = (P_V^{(k)})_{ix} \left[O_V^{(k)} \right]_{ij} (P_V^{(k)})_{jy} = \sum_{i,j}^{N_{eff}^{(k)}} \delta_{z_i^{(k)},x} \left[O_V^{(k)} \right]_{ij} \delta_{z_j^{(k)},y} . \tag{6}$$

Comment that the P matrices have only one non-vanishing entry per column. To set the interaction part, we therefore have to specify the following:

- \bullet the matrix elements $\left[O_V^{(k)}\right]_{ij}$
- the set $[z_1^{(k)}, \cdots z_{N_{eff}^{(k)}}^{(k)}]$
- the interaction strengths U_k
- the numbers α_k .

Be more specific here what really has to specified in the actual code. The same logic also applies to the implementation of the hopping interaction be more specific.

1.3 The Hubbard-Stratonovich decomposition

1.4 Implementation: the Operator variable

In the code implementation, we define a structure called Operator. This structure variable Operator bundles several components that are needed to define and use an operator matrix in the program. In Fortran a structure variable like this is called a derived type. The components it contains are:

- the projector P_V ,
- the matrix O_V ,
- the effective dimension N_{eff} ,
- and a couple of auxiliary matrices and scalars.

In general, we will not only have one structure variable Operator, but a whole array of these structures.

2 Tutorial to set up the SU(2)-Hubbard model on a square lattice

The SU(2) symmetric Hubbard model is given by

$$\mathcal{H} = -t \sum_{\langle i,j \rangle, \sigma} \left(c_{i,\sigma}^{\dagger} c_{j,\sigma} + \text{H.c.} \right) + \frac{U}{2} \sum_{i} \left[\sum_{\sigma} \left(c_{i\sigma}^{\dagger} c_{i\sigma} - 1/2 \right) \right]^{2} . \tag{7}$$

Name of variable in the code	Description
$\mathrm{Op_{-}V\%N}$	effective dimension N_{eff}
$0p_V$ %0	matrix \mathbf{O}_V
$0p_V$ VU	matrix containing the eigenvectors of \mathbf{O}_V
$0p_V\%E$	eigenvalues of \mathbf{O}_V
Op_V%P	projection matrix \mathbf{P}_V
Op_V%N_non_zero	number of non-vanishing eigenvalues of \mathbf{O}_V
Op_V%g	coupling strength in Hubbard-Stratonovich transformation
Op_V%alpha	constant, to set particle-hole symmetry correct?

Table 1: Components of the Operator structure variable Op_V.

To bring Eq. (7) in the general form (1), we set

$$N_{fl} = 1$$

$$N_{sun} = 2$$

$$T_{xy} = -t\delta_{\langle i,j\rangle}\delta_{\sigma,\sigma'}$$

$$M = N_{sites}$$

$$U_k = -U/2$$

$$V_{xy}^{(k)} = \delta_{x,y}\delta_{i,k} = \delta_{i,j}\delta_{\sigma,\sigma'}\delta_{i,k}$$

$$\alpha_k = 1/(N_{sites}N_{sun})^2 = 1/(2N_{sites})^2.$$
(8)

In the following, we skip the N_{sun} -spin degree of freedom which is present in the multi-indices x, y of the matrices T and $V^{(k)}$. So $N_{dim} = N_{sites}$.

What is the role of the N_{sun} index in the code?

- it appears in the coupling g in the Operator structure.
- it appears as a normalization constant in the definition of observables
- it appears as exponent in the calculation of the phase factor and the update ratio

Is the code limited to SU(N) symmetric models with respect to the N_{sun} degree of freedom?

Note that in this example $N_{dim} = Latt\%N$ since there is only one spacial orbital per unit cell of the underlying Bravais lattice.

2.1 Hopping term

The hopping matrix is implemented as follows. We allocate an array of dimension 1×1 , called Op_T. It therefore contains only a single Operator structure. We set the effective dimension (here, $N_{eff} = N_{dim}$), and allocate and initialize this structure by a single call to the subroutine Op_make:

Since the effective dimension is identical to the total dimension, it follows trivially, that $P_T = 1$ and $O_T = T$. Although a checkerboard decomposition is not yet used for the Hubbard model, in principle it can be implemented.

2.2 Interaction term

To implement this interaction, we allocate an array of Operator structures. The array is called Op_V and has dimensions $N_{dim} \times N_{fl} = N_{sites} \times 1$. We set the effective dimension, $N_{eff} = 1$, and allocate and initialize this array of structures by repeatedly calling the subroutine Op_make:

```
N_sites = Latt%N
N_fl = 1
N_eff = 1

do nf = 1, N_FL
do i = 1, Latt%N
call Op_make(Op_V(i,nf),N_eff)
enddo
enddo
```

For each lattice site i, the projection matrices $P_V^{(i)}$ are of dimension $1 \times N_{dim}$ and have one non-vanishing entry: $(P_V^{(i)})_{1j} = \delta_{ij}$. The effective matrices are again trivial: $O_V^{(i)} = 1$.

Name of variable in the code	Description
Ndim	Spacial dimension of the lattice (total number of sites)
	what about the N_{sun} ?
Latt%N	Number of unit cells of the underlying Bravais lattice
$0p_{-}T$	Array of structure variables that bundles all variables
	needed to define the hopping operator.
$0p_{-}V$	Array of structure variables that bundles all variables
	needed to define the two-particle interaction operator.
N_sun	Number of spin states of the $SU(N_{sun})$ -symmetric fermions
N_{-} fl	Number of spin flavors

Table 2: Common variables that are set in the Hamiltonian, operator and lattice modules of the code.

2.3 Definition of the lattice

This is set in the subroutine Ham_latt. The square lattice is already implemented. In principle, one can specify other lattice geometries and use them by specifying the keyword Lattice_type in the parameter file.

2.4 Observables for the Hubbard model

To do next:

- dicuss the measurements: what observables exit and how do I add a new one?
- discuss the implementation of the lattice.
- discuss the Hubbard-Stratonovich decompositions (this is related to the coupling in the operator structure), discuss also the spin-symmetry-breaking HS-decomposition for the Hubbard model.

3 Tutorial to set up a lattice

4 Tutorial to define observables

5 Input and output of the code

5.1 The parameter file

6 Using the code

Example simulation, tutorial: where to find and how to start

6.1 Parameter files

describe the input parameters, give sample values for the stabilization parameters

6.2 Analysis files

 $how\ the\ analysis\ of\ Monte\ Carlo\ data\ is\ done$

7 List of files

all files that constitute the code, with a brief description

 $7.1 \quad {\rm cgr1.f90} \ \& \ {\rm cgr2.f90}$

Stable computation of the physical single-particle equal time Green function $G(\tau)$.

7.2 control_mod.f90

Includes a set of auxiliary routines, regarding the flow of the simulation. Examples are initialization of performance variables, precision tests and controlled termination of the code.

7.3 gperp.f90

7.4 Hamiltonian_Hub.f90

Here, the physical simulation parameters (the model parameters) and the lattice parameters are read in. The lattice, the non-interacting and the interacting part of the Hubbard Hamiltonian are set according to the parameters and the chosen Hubbard-Stratonovich decomposition.

 $7.5 \quad Hop_mod.f90$

7.6 inconfc.f90

The auxiliary-field QMC method is based on a Hubbard-Stratonovich decomposition of the interaction term. This decomposition introduces a space-time array of (discrete) configurations of auxiliary fields, i.e. Ising spins. In this routine, an existing configuration is read in, checks on its dimensionality are made and, in case no prior configuration exists, a random configuration of Ising spins is set up.

7.7 main.f90

Top-level part of the program. Here, the program flow which consists of initialization, sweeps through the space-time lattice, and finalizing the program, is coded.

7.8 nranf.f90

Auxiliary routine controlling the evaluation of random numbers.

7.9 Operator.f90

The algorithms is centered around evaluation of single-particle operators, represented as square matrices. In this routine, the abstract type Operator is defined, including information on the coupling strength, the sites that participate in the single-particle hopping process, and the type of Hubbard-Stratonovich transformation. This routine collects all program relevant operations that are applied to the type Operator, like initializations or multiplications.

7.10 outconfc.f90

Description in plain text:

At the end of the simulation, the last configuration of Hubbard-Stratonovich variables, together with the last set of random numbers is written to the file confout. Prior to the start of a new simulation of the identical space-time dimesion, one can (manually) copy the file confout to the file confin and make the new run use the old configuration.

Doing this saves warmup time compared to a complete random (unphysical) configuration.

Input/output variables

in:
inout:
inout:
out:

Dependencies include: mpif.h modules: Hamiltonian

interfaces:

global variables: ntrot, nsigma

subroutines: MPI_COMM_RANK, MPI_COMM_SIZE, Get_seed_Len, Ranget

Things to check:

Rename subroutine to confout.f90 for consistency

7.11 print_bin_mod.f90

Description in plain text: Here the way to write the measure bins to the respective output files is coded. A bin is an average over many individual measurements. The bin defines the unit of Monte Carlo time. Dependencies include: modules: interfaces: Print_bin Contains subroutines: Print_bin_c, Print_bin_r, Print_scal, Print_bin_tau Things to check: 7.11.1 Print_bin_C Description in plain text: Input/output variables in: Latt, type(Lattice) in: Phase_bin_tmp, complex in: File_pr, character(Len=64) in: nobs, integer inout: Dat_eq,(:,:,:), complex inout: Dat_eq0(:), complex out: Dependencies include: mpif.h modules: Lattices_v3 interfaces: global variables: TYPE(LATTICE), N, listk, b1_p, b2_p subroutines: MPI_COMM_RANK, MPI_COMM_SIZE, MPI_REDUCE, Fourier_R_to_k Things to check: STATUS(MPI_STATUS_SIZE), integer, needed? $7.11.2 \quad Print_bin_R$ Description in plain text: Input/output variables in: Latt, type(Lattice) in: Phase_bin_tmp, complex in: File_pr, character(Len=64) in: nobs, integer inout: Dat_eq,(:,:,:), real inout: Dat_eq0(:), real

include: mpif.h

Dependencies

out:

modules: Lattices_v3 interfaces: global variables: TYPE(LATTICE), N, listk, b1_p, b2_p subroutines: MPI_COMM_RANK, MPI_COMM_SIZE, MPI_REDUCE, Fourier_R_to_k Things to check: STATUS(MPI_STATUS_SIZE), integer, needed? 7.11.3 Print_scal Description in plain text: Input/output variables in: File_pr, character(Len=64) in: nobs, integer inout: Obs,(:), complex out: Dependencies include: mpif.h interfaces: global variables: subroutines: MPI_COMM_RANK, MPI_COMM_SIZE, MPI_REDUCE Things to check: STATUS(MPI_STATUS_SIZE), integer, needed? change subroutine name to print_bin_scal for consistency 7.11.4 Print_bin_tau Description in plain text: Input/output variables in: Latt, type(Lattice) in: Phase_bin, complex in: File_pr, character(Len=64) in: nobs, integer in: dtau, real inout: Dat_tau,(:,:,:), complex inout, optional: Dat0_tau(:), complex out: Dependencies include: mpif.h modules: Lattices_v3 interfaces: global variables: TYPE(LATTICE), N, listk, b1_p, b2_p subroutines: MPI_COMM_RANK, MPI_COMM_SIZE, MPI_REDUCE, Fourier_R_to_k

Things to check:

STATUS(MPI_STATUS_SIZE), integer, needed?

```
Description in plain text:
module tau_m_mod, with several subroutines
Dependencies
include:
modules: Hamiltonian, Operator_mod, Precdef, Control, Hop_mod
Contains
subroutines: tau_m, propr, proprm1
Things to check:
change file name to tau_m_mod.f90 for consistency
7.12.1 TAU_M
Description in plain text:
Input/output variables
in: nstm, integer
in: nwrap, integer
in: ust(ndim,ndim; nstm,n_fl), complex
in: vst(ndim,ndim,nstm,n_fl), complex
in: dst(ndim,nstm,n_fl), complex
in: GR(ndim,ndim,n_fl), complex
in: phase, complex
in: stab_nt(0:nstm), integer
inout:
inout:
out:
Dependencies
include:
modules:
interfaces: wrapul, cgr2_1, cgr2_2, cgr2
global variables: ndim, n_fl, cone, ltrot
subroutines: obsert, initd, propr, proprm1, wrapur, cgr2_2, Control_Precision
Things to check:
cone?
7.12.2 propr
Description in plain text:
Input/output variables
in: nt
inout: Ain(ndim,ndim,n_fl)
inout:
out:
Dependencies
include:
```

7.12 tau_m.f90

modules:

```
interfaces:
global variables: ndim, n_fl, op_v, nsigma, Phi, type
subroutines: Hop_mod_mmthr, Op_mmultR
Things to check:
7.12.3 proprm1
Description in plain text:
Input/output variables
in: nt
inout: Ain(ndim,ndim,n_fl)
inout:
out:
Dependencies
include:
modules:
interfaces:
global variables: ndim, n_fl, op_v, nsigma, Phi, type
subroutines: Hop_mod_mmthl, Op_mmultL
```

7.13 UDV_WRAP.f90

modules: interfaces:

global variables:

Description in plain text: UDV_Wrap_mod is a module file, containing subroutines on the stabilization of matrix computations. Dependencies modules: MyMats, Files_mod Contains subroutines: UDV_Wrap_Pivot, UDV_Wrap Things to check: change file name to UDV_Wrap_mod.f90 for consistency 7.13.1 UDV_Wrap_Pivot Description in plain text: Input/output variables in: A(:,:), complex in: ncon, integer in: n1, integer in: n2, integer inout: U(:,:), complex inout: V(:,:), complex inout: D(:), complex out: Dependencies include: modules: interfaces: global variables: subroutines: UDV_Wrap, MMULT, Compare Things to check: 7.13.2 UDV_Wrap Description in plain text: Input/output variables in: A(:,:), complex in: ncon, integer inout: U(:,:), complex inout: V(:,:), complex inout: D(:), complex out: Dependencies include: mpif.h

subroutines: MPI_COMM_SIZE, MPI_COMM_RANK, QR, SVD, MMULT

Things to check:

STATUS(MPI_STATUS_SIZE), integer: not used

7.14 upgrade.f90

Description in plain text:

alpha: both a local and a global variable. CHECK!!

The update of the Hubbard-Stratonovich configuration is done sequentially for each point in the space-time lattice, i.e. one Hubbard-Stratonovich Ising spin after the other. In this routine, an update (i.e. a spin flip) is accepted or rejected. The decision is made using the Metropolis method of importance sampling. Input/output variables in: N_op, integer in: nt, integer in: OP_dim, integer inout: GR(ndim,ndim,n_fl), complex inout: Phase, complex out: Dependencies include: modules: Hamiltonian, Random_wrap, Control, Precdef interfaces: global variables: ndim, n_fl, op_v, nflipl, Phi, n_non_zero, Gaml, P, nsigma, g, alpha, type, E subroutines: zgemm, control_upgrade Things to check: nranf, integer, external (where is external fct. defined) log, logical (reserved name)

7.15 wrapgrdo.f90

Description in plain text:

Single-particle equal-time Green functions are the central object of the code.

The physical single-particle equal-time Green function $G(\tau)$ is updated in wrapgrdo.f90 (down propagation, from τ).

The update is sequentially, over all (interacting) lattice sites or lattice bonds.

Input/output variables

in: ntau, integer

inout: gr (ndim,ndim,n_fl), complex

inout: phase, complex

out:

Dependencies

include:

modules: Hamiltonian, MyMats, Hop_mod

interfaces: upgrade

global variables: op_v, phi, nsigma, ndim, n_fl

subroutines: Hop_mod_mmthl, Hop_mod_mmthr_m1, Op_Wrapdo, Upgrade

7.16 wrapgrup.f90

Description in plain text:

Single-particle equal-time Green functions are the central object of the code.

The physical single-particle equal-time Green function $G(\tau)$ is updated in wrapgrup.f90 (up propagation, from $\tau = 0$ to τ).

The update is sequentially, over all (interacting) lattice sites or lattice bonds.

Input/output variables

in: ntau, integer

inout: gr (ndim,ndim,n_fl), complex

inout: phase, complex

out:

Dependencies

include:

modules: Hamiltonian, Hop_mod

interfaces: upgrade

global variables: op_v, phi, nsigma, ndim, n_fl

subroutines: Hop_mod_mmthr, Hop_mod_mmthl_m1, Op_Wrapup, Upgrade

7.17 wrapul.f90

Input/output variables

Description in plain text: To stabilize the simulation at the time slice $\hat \xi_2=i n_{stab}$, the Green function has to be recomputed regularly, based on the stable matrices at an earlier stabilization point, $\hat \xi_1=(i-1) n_{stab}$. These stable matrices result from a singular-value-decomposition of the propagation matrix. They are computed in wrapul.f90 (down propagation).

in: ntau1, integer
in: ntau, integer
inout: ulup (ndim,ndim,n_fl), complex
inout: dlup (ndim,n_fl), complex
inout: vlup (ndim,ndim,n_fl), complex
out:

 ${\tt Dependencies}$

include:

modules: Hamiltonian, Hop_mod, UDV_Wrap_mod

interfaces:

global variables: ndim, n_fl, Op_V, Phi, nsigma,

subroutines: initd, Op_mmultL, Hop_mod_mmthl, mmult, UDV_Wrap

7.18 wrapur.f90

```
Description in plain text: To stabilize the simulation at the time slice \hat{2}=i n_{stab}, the Green function has to be recomputed regularly, based on the stable matrices at an earlier stabilization point, \hat{1}=(i-1) n_{stab}. These stable matrices result from a singular-value-decomposition of the propagation matrix. They are computed in wrapur.f90 (up propagation).
```

```
Input/output variables
in: ntau1, integer
in: ntau, integer
inout: ur (ndim,ndim,n_f1), complex
inout: dr (ndim,n_f1), complex
inout: vr (ndim,ndim,n_f1), complex
out:
```

Dependencies

include:

modules: Hamiltonian, Hop_mod, UDV_Wrap_mod

interfaces:

global variables: ndim, n_fl, Op_V, Phi, nsigma,

subroutines: initd, Op_mmultR, Hop_mod_mmthr, mmult, UDV_Wrap

```
Description in plain text:

Input/output variables
in:
inout:
inout:
out:

Dependencies
include:
modules:
interfaces:
global variables:
subroutines:

Things to check:
```

8 Module Hamiltonian

Detailed description of the module Hamiltonian since it will be modified by the users The module contains the following subroutines:

8.1 ham_set

It calls the subroutines

- ham_latt
- ham_hop
- ham_v

It reads in the file

• parameters

It sets the variables ltrot,n_fl,n_sun. If compiled as a MPI-program, it broadcasts all variables that define the lattice, the model and the simulation process.

8.2 ham latt

It sets the lattice, by calling the subroutine

• make_lattice(l1_p, l2_p, a1_p, a2_p, latt)

8.3 ham_hop

Setup of the hopping amplitudes between the vertices of the graph (lattice sites and unit cell orbitals). It calls the subroutines

- op_make(op_t(nc,n),ndim
- $op_set(op_t(nc,n))$

8.4 ham_v

It calls the subroutines

- $op_make(op_v(i,nf),1)$
- op_set(op_v(nc,nf))

$8.5 \, \text{so}(\text{n,nt})$

It is defined as s0(n, nt) = 1.d0. Why? It is superfluous.

8.6 alloc_obs(ltau)

Allocation of equal time and time-resolved quantities.

8.7 init_obs(ltau)

Initializes equal time and time-resolved quantities with zero.

8.8 obser(gr,phase,ntau)

Includes the definition of all equal-time observables (scalars and correlation functions) that are built from the single-particle Green function based on Wick's theorem.

```
8.9 pr_obs(ltau)
```

Output (print) of the observables.

8.10 obsert(nt,gt0,g0t,g00,gtt,phase)

Includes the definition of time-resolved observables that are built from the time-resolved single-particle Green function based on Wick's theorem.

9 Installation

9.1 Dependencies

which software and libraries are needed and which version

- libraries: LAPACK, BLAS, EISPACK, NAG They are included in the package, but NAG is not public-domain (?)
- tools: cmake
- compiler: gfortran or ifort

9.2 Build the GQMC program from source code

 $configuration,\ compile\ and\ Installation$ In the top level directory, where the README file resides, do:

```
mkdir build
cd build
cmake ..
make
```

10 Reference manual

11 License

Use of the GQMC code requires citation of the paper \dots The GQMC code is available for academic and non-commercial use under the terms of the license \dots For commercial licenses, please contact the GQMC development team.

12 ideas

FAQ, walkthroughs,