INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Proposal for a new course in CAI

1	Name of the Department	**Centre for Artificial Intelligence**
2	Name of the Subject	**Mathematical Foundations for AI and ML**
3	LTP and Credit	**3+1+0 (4 credit)**
4	Status of the subject	
	(a) Specify the Session, Semester, from which the subject is going to be offered (b) Please Specify the Level of the Subject (c) Whether the subject will be offered as compulsory or elective (d) The semester in which the subject will be offered (e) Name(s) of the Programme(s) in whose curricula this subject will be included	Autumn 2019-2020 **B. Tech, M. Tech, Integrated M.Sc, Ph. D**. **Elective** **To be decided** (1) Elective course for 5 year integrated M.Sc./B. Tech./Dual Degree students (2) Research scholars from CAI/CS/MA (3) Elective course for of CS/MA M.Tech. students
5	Prerequisite(s) for the subject, if any (Please give the subject numbers and names)	
6	Objective and Contents	
	(a) Objective	Linear algebra, optimization techniques and statistical methods together form essential tools for most of the algorithms in artificial intelligence and machine learning. In this course, we propose to build some background in these mathematical foundations and prepare students to take on advanced study or research in the field of AI and ML. The objective of this course is to be familiarize the students with the important concepts and computational techniques in linear algebra and optimization useful for AI and ML applications. The unique objective of this course and the distinguishing point from the existing courses on the similar topics would be illustration of application of these concepts to many real life problems in AI and ML.

	(b) Contents (in 100 to 150 words)	Part 0: (Motivation) Netflix problem, Google page ranking algorithm, Portfolio optimization, Construction of Markov chains from the given stationary distribution, AR model order selection, Approximate GCD of polynomials and image deblurring Part 1: (Revision) linear functions, inner product, orthogonality, norm and distance, concept of similarity in AI/ML applications, K means clustering application on feature vectors, system of linear equations (square, underdetermined and overdetermined), existence and uniqueness of solutions, inverse and pseudo inverse of matrices, simple and multiple regression problems Part 2: least squares solution, parameter estimation problems, concept of cost function and relation to parameter estimation, constrained least squares, multiobjective least squares, applications to portfolio optimization, linear quadratic control and linear quadratic state estimation problems, eigenvalue eigenvector decomposition of square matrices, spectral theorem for symmetric matrices, SVD, multicollinearity problem and applications to principal component analysis (PCA) and dimensionality reduction, power method, application to Google page ranking algorithm, inverse eigenvalue problem, construction of Markov chains from the given stationary distribution, low rank approximation and structured low rank approximation problem, Autoregressive model order selection using Hankel SLRA, approximate GCD computation and application to image deblurring Part 3: (Optimization) Convex optimality conditions: differentiable and non-differentiable cases, application to multivariate Gaussian parameter estimation, gradient descent, projected gradient methods,

		accelerated gradient methods, coordinate descent, biconvexity and alternating minimization, low rank approximation problem using alternating minimization, missing data and matrix completion problem, formulation of the matrix completion problem using nuclear norm, Netflix prize problem and recommender systems
7	Names of the faculty members of the Department/Centers/School who have the necessary expertise and will be the willing to teach the subject (Minimum two faculty members should be willing to teach the subject)	1. Swanand Khare 2. Adway Mitra
8	Do the contents of the subject have an overlap with any other subject offered in the Institute?	Yes (up to 20%) The first part of the course (revision) has overlap with a basic courses offered by the Department of Mathematics on linear algebra and matrix algebra (MA20105, MA20107, MA30003)
9	Recommended Text books/References	
	a) Theory (Text Books)	1. *Introduction to Applied Linear Algebra: Vectors, Matrices and Least Squares*, Stephan Boyd, Lieven Vandenberghe, Cambridge University Press, 2018. 2. *Linear Algebra and Learning from Data*, Gilbert Strang, Wellesley Cambridge Press, 2019. 3. *Statistical Learning with Sparsity: the LASSO and generalizations*, Trevor Hastie, Robert Tibshirani, Martin Wainwright, CRC Press 2017.
	b) References (Literature)	1. *Matrix Computations*, G. Golub and C. Van Loan, Hindustan Book Agency, 2007. 2. *Numerical Linear Algebra*, L. Trefethen and D. Bau, SIAM, 1997. 3. *Fundamentals of Matrix Computations*, David Watkins, Wiley Interscience, 2002.

		4. *Numerical Linear Algebra and Applications*, B. N. Datta, Prentice Hall of India, 2010.
10	Names of Departments/Centers/Schools/Programmes whose students are expected to register for this subject	Centre for Artificial Intelligence (CAI), Mathematics (MA), Computer Science (CS)

Lecture-wise Topics:

Number of Lectures	Topics	Tutorial (in python with AWS support)
3 Lectures Week 1	Motivation: Real life problems which require linear algebra, Netflix problem, Google page ranking algorithm, Portfolio optimization, Construction of Markov chains from the given stationary distribution, AR model order selection, Approximate GCD of polynomials and image deblurring	Demonstration in the class
3 Lectures Week 2	vector spaces, linear dependence, independence, linear span, basis and dimension, inner product	problems to be solved in the class
3 Lectures Week 3	norm and distance, linear functions, system of linear equations (square, underdetermined and overdetermined)	concept of similarity, K means clustering application on feature vectors (concept of norm and distance)
3 Lectures Week 4	QR decomposition, existence and uniqueness of solutions, inverse and pseudo inverse of matrices	Numpy functions and concepts
3 Lectures Week 5	least squares solution	House price prediction problem,
3 Lectures Week 6	constrained least squares	portfolio optimization, linear quadratic control, linear quadratic state estimation
3 Lectures Week 7	eigenvalue eigenvector decomposition of square matrices, spectral theorem for symmetric matrices, SVD	multicollinearity problem, PCA, dimensionality reduction
3 Lectures Week 8	power method for dominant eigenvector computation, inverse power method	Google page ranking algorithm
3 Lectures Week 9	inverse eigenvalue problem, problem formulation, existence, uniqueness, structure	construction of Markov chains from the given

	constraints, applications	stationary distribution
3 Lectures Week 10	low rank approximation and structured low rank approximation problem, formulation, solution using SVD, Hankel structured low rank approximation	Autoregressive model order selection using Hankel SLRA, approximate GCD computation and application to image deblurring
3 Lectures Week 11	convex optimality conditions: differentiable and non-differentiable cases, gradient descent	simulation case study: multivariate Gaussian parameter estimation
3 Lectures Week 12	projected gradient methods, accelerated gradient methods, coordinate descent, biconvexity and alternating minimization	low rank approximation problem using alternating minimization
3 Lectures Week 13	missing data and matrix completion problem, formulation of the matrix completion problem using nuclear norm	Netflix prize problem

Total: 39 Lectures + 13 Tutorial classes