Mini course on algebraic analysis of dynamical systems

Syllabus

Lecture 1
Fundamentals of commutative algebra: rings, ideals, maximal and prime ideals, quotient ring. Ring homomorphisms, first isomorphism theorem. Examples. Modules, module homomorphisms. Examples.

Lecture 2
Systems of linear constant coefficient partial differential/difference equations (PDEs/pdes). Behaviours. Operator modules, Malgrange's Theorem. Oberst's Theorem.

Lecture 3
Solving ODEs: the case of a single equation. GCD. Euclidean division algorithm (EDA).

Lecture 4
EDA used for solving a scalar system of ODEs. The quotient ring, the companion matrix, finding out remainder after division by matrix computations.

Lecture 5
The first order representation of a system of ODEs. Construction. Proof of equivalence.

Lecture 6
Torsion module, torsion-free module. Controllability. Serre conjecture, strong controllability. Observability. The special case of ODEs.

Lecture 7
Solving PDEs. Gröbner basis. Oberst-Riquier algorithm.

Lecture 8
Systems of partial difference equations: discrete n-D systems. Characteristic sets. Special case: characteristic cones. Algebraic characterization of characteristic cones.

Lecture 9
Semi-group ring. Gordan's Lemma. Algorithm for checking characteristic cones.

Lab session 1
Euclidean division with remainder using the companion matrix using Matlab/Scilab/Octave. Solving ODEs by EDA. First order representation.

Lab session 2
Obtainging Gröbner basis over $\mathbb{Q}[x_1, \ldots, x_n]$ using Sagemath. The special case of zero dimensional rings (Artinian). Understanding Grothendieck Schemes by companion matrices. The Stetter-Möller theory.

Lab session 3
Oberst-Riquier algorithm. Handling Laurent polynomial rings in Sagemath. Checking characteristic cones in Sagemath.

Textbooks and references

- **D. Eisenbud**, *Commutative Algebra with a View Toward Algebraic Geometry*, Springer, 1995.
- **M. Atiyah and I. MacDonald**, *Commutative Algebra*, Addison-Wesley, 1969.
- **E. Miller and B. Sturmfels**, *Combinatorial Commutative Algebra*, Springer, 2004.
- **B. Sturmfels**, *Solving Systems of Polynomial Equations*, American Mathematical Society with support from the National Science Foundation, 2002.

- **H.K. Pillai and S. Shankar**, A behavioral approach to control of distributed systems, *SIAM Journal on Control and Optimization*, 37(2):388408, 1998.

- **D. Pal and H.K. Pillai**, Representation formulae for discrete 2D autonomous systems, *SIAM Journal on Control and Optimization*, 51(3):24062441, 2013.

- **D. Pal**, Every discrete 2D autonomous system admits a finite union of parallel lines as a characteristic set. Multidimensional Systems and Signal Processing, 28:1, 49-73, 2017.

- **M. Mukherjee and D. Pal**, On characteristic cones of discrete nD autonomous systems: theory and algorithm. *Multidimensional Systems and Signal Processing*, 30:611-644, 2019.