## Course title: COMBINATORICS AND COMPUTING

**Credit requirement:** (L-T-P: 3-1-0, Credit: 4)

**Committee for approval:** PGPEC

**Name of the Dept.**: Computer Science & Engineering

**Level of the subject:** PG Level

**Compulsory or Elective:** Elective

**Prerequisite:** Nil

**Description & Objective:** Most of the mathematics used by computer scientists is discrete in nature and combinatorics concerns the study of existence, enumeration, and optimization of countable discrete structures. Combinatorics is used frequently in computer science to obtain formulas and estimates in the analysis of exact, randomized, and approximate algorithms. Classical techniques in combinatorics like **counting, pigeon-hole principle** etc. that are widely used in this regard are covered in detail in the first part of this course. In the second part, we study **extremal set theory**, a study of how large or how small a family of sets that satisfies certain desired properties can be. Next, with a brief introduction to some of the essential concepts of **linear algebra**, we take a look into its application **in combinatorics**. Finally, we learn **the probabilistic method** – one of the most powerful and most widely used tools in combinatorics. The basic probabilistic method can be described as follows: In order to prove the existence of a combinatorial structure with certain properties, we construct an appropriate probability space and show that a randomly chosen element in this space has the desired properties with positive probability. Many a times, this yields good randomized (and sometimes, after derandomization, efficient deterministic) algorithms for finding such a combinatorial structure and is therefore very useful to Theoretical Computer Scientists (TCSs). Some generic results in the probabilistic method like **the Local Lemma** (due to Erdos and Lovasz, the last topic in syllabus given below) are equally popular amongst TCSs and combinatorialists.

## Syllabus:

**THE CLASSICS: Counting:** The binomial theorem, selections with repetitions, partitions, double counting, the averaging principles. **Advanced counting:** Bounds on intersection size, Zarankiewicz's problem, Density of 0-1 matrices. **Inclusion and Exclusion principle:** The number of derangements. **The pigeon-hole principle:** The Erdos-Szekeres theorem, Mantel's theorem, Turan's theorem, Dirichlet's theorem.

**EXTREMAL SET THEORY: Intersecting families:** The sunflower lemma, The Erdos-Ko-Rado theorem. **Chains and antichains**: Dilworth's theorem, Sperner's theorem, Bollobas's theorem.

**THE LINEAR ALGEBRA METHOD:** A short introduction to some basic concepts of linear algebra. **The basic method (using linear independence):** Fisher's inequality, polynomial technique, Frankl-Wilson theorem, another proof of Bollobas's theorem. **Orthogonality and rank arguments:** Orthogonal coding, a bribery party, balanced families.

**THE PROBABILISTIC METHOD: Basic tools. Counting sieve:** Ramsey numbers, Van der Waerden's theorem, Tournaments, Kleitman-Spencer theorem. **The Lovasz sieve:** The local lemma and some of its applications.

## Textbooks:

1. Extremal combinatorics, by Stasys Jukna.
2. Handbook of combinatorics, by R. L. Graham, M. Grotschel, and L. Lovasz.
3. The probabilistic method, by Noga Alon and Joel Spencer.
4. Linear algebra methods in combinatorics with applications in geometry and computer science, by Laszlo Babai and Peter Frankl.

## References:

1. Katona, Gyula OH. "A simple proof of the ErdÃ¶sChao KoRado theorem." Journal of Combinatorial Theory, Series B 13.2 (1972): 183184.
2. B. Bollobas. "On generalized graphs." Acta Math. Acad. Sci. Hungar. 16 (1965) 447452.

## Content:

## Lectures (total: 40 hours):

1. The Classics (10 hours)
    a. Counting (2 hours)
    b. Advanced counting (3 hours)
    c. Inclusion and Exclusion principle (2 hours)
    d. Pigeon-hole principle (3 hours)
2. Extremal set theory (10 hours)
    a. Intersecting families (5 hours)
    b. Chains and Antichains (5 hours)
3. Linear algebra method (12 hours)
    a. Introducing some of the basic concepts of linear algebra (4 hours)
    b. The basic method (using linear independence argument) (4 hours)
    c. Rank and orthogonality arguments (4 hours)
4. Probabilistic method (8 hours)
    a. Basic tools (1.5 hours)
    b. Counting sieve (4 hours)
    c. Lovasz sieve (2.5 hours)

## Tutorials (total: 10 hours):

1. Explaining solutions of difficult exercise problems from Chapters 1, 2, 3, 4, 7, 8, 9, 14, 15, 17, 18, and 19 of Extremal combinatorics (by Stasys Jukna) (8 hours)
2. Setting up linear algebra background. (1 hour)
3. Refreshing basic concepts of probability. (1 hour)