School of Electronic Engineering

CB54: Machine Learning Algorithms for EM Wave Scattering Problems

Project Research Log

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MEng in Electronic and Computer Engineering

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Please read before making entries in this log

The purpose of this Project Research Log is to capture concise, focused summaries of research materials you read, as you progress through your project. The emphasis is to record (i) how the material you have read will determine or influence your project solution approach and (ii) your assessment of the key strengths and weaknesses of the solutions, methods, technologies, etc. proposed in the material you have read.

In the first stage of your project, the literature review, use the Log to capture this information for the key papers you have read (for example, the three most important papers of your 10 literature review references). As your project progresses into the design and implementation phases, you will need to continue to search the literature so you can review, revise and refine your initial thinking and the details of your approach to a project solution. Use this Research Log to capture your continued research reading and its influence on your project design and implementation.

Be selective about what you record in this log. Do not use it as an informal notebook while you are reading a new paper. Only make an entry after you have read a paper that you consider important to the development of your project solution. It is expected that, by the end of the project, you will have made between 10 and 20 entries (20 maximum). Share your log with your supervisor for viewing throughout the project. You will submit the final version of the log for grading, at the end of the project implementation period. It will be assessed on the basis of how well you have used your analysis of the literature to inform your project design, implementation and the evaluation of your project results. The Research Log contributes 5% to the overall project mark.

Note: All entries you make in this log must use the prescribed format shown on the next page. You will maintain other notes as you progress through your project but they should not be recorded here. Fill in the details where the *** signs are.

Contents

Log Entry 00: 2022/11/28

Statement of project problem / research question (maximum 200 words)

This statement should be periodically reviewed and updated, as necessary, as your project progresses and you gain further insight into the detailed project challenges, requirements and objectives as your project work moves from background reading, literature review, initial project design planning and detailed design and implementation. Initially, start by stating your current understanding of the project objectives. After each meeting with your supervisor, review and refine your project problem statement, as required.

THIS IS JUST TO RECORD THE INITIAL PROJECT STARTING POINT

"When an electromagnetic wave encounters an object it scatters, with some energy being transmitted into the object and the rest propagating in a variety of directions depending on the material composition and local geometry. A precise knowledge of the scattering phenomenon is desirable for a variety of applications, such as medical imaging, radar and wireless communications. Numerical techniques such as the method of moments give highly accurate results, but are computationally expensive. An emerging alternative is the use of machine learning tools that can be trained using a training set of data covering a sufficiently wide feature set (i.e. problem geometry, material, frequency etc). This project will use an in-house, Matlab-based, implementation of the method of moments to train an artificial neural network to solve the problem of EM scattering from convex dielectric bodies."

A complete reference for the paper

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Summary of paper (maximum 100 words)

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How is this paper relevant to solving your project problem or addressing your research question? (maximum 100 words)

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What are the strengths and weaknesses of the solutions/methods/technologies proposed in this paper? (maximum 100 words)

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Log Entry 01: 2022/12/31

Statement of project problem / research question (maximum 200 words)

"Can the manner in which deep learning has been shown to solve two-dimensional, forward electromagnetic scatting problems applied to the problem of predicting EM wave propagation over rural terrain, namely emulation, be expanded or improved upon?"

A complete reference for the paper

[1] , please refer to the final bibliography.

Summary of paper (maximum 100 words)

This paper claims that the sub-algorithmic infusion of a deep learning model into an iterative solver, essentially replacing steps in the iterative solver at every iteration, can accelerate the realization of a solution to the VEFIE formulation of the electromagnetic scattering problem.

How is this paper relevant to solving your project problem or addressing your research question? (maximum 100 words)

It is extremely relevant to the project as it deals directly with the problem domain and takes the outlook of a hybrid methodology where a conventional approach is enhanced with a new machine learning technique. The paper describes how the solver algorithms are augmented and includes diagrams of deep learning architecture designs.

What are the strengths and weaknesses of the solutions/methods/technologies proposed in this paper? (maximum 100 words)

The paper gives a lot of detail to each element of the problem and even gives information about the experimental apparatus. The student remains sceptical that the approach can be implemented to yield consistent results as the deep learning model should give a single guess upon which every successive completion of the iterative solver would beat in terms of minimising the residual vector. The student feels that the deep learning model would hinder, rather than assist, the iterative solver after the initial guess. The paper is also advanced in terms of implementation requirements to achieve a duplication of the paper so the idea may not be suitable to approach in the time frame of the project. Also the hardware used in the paper is far beyond the budget deemed reasonable by the student to achieve experimental results as the price for the GPU at time of writing is over $10, 000.

Log Entry 02: 2023/01/11

Statement of project problem / research question (maximum 200 words)
"Can the manner in which deep learning has been shown to solve two-dimensional, forward electromagnetic scatting problems applied to the problem of predicting EM wave propagation over rural terrain, namely emulation, be expanded or improved upon?"

A complete reference for the paper
[2], please refer to the final bibliography.

Summary of paper (maximum 100 words)
The paper describes a deep learning architecture that can search for suitable problem-specific architectures as it is training on the domain data. The general problem areas that the paper discusses covers ten scientific simulation topics that exist at a variety of contrasting physical scales. While electromagnetic scattering is not specifically mentioned, the approach of the paper is highly-likely to be transferrable to the domain.

How is this paper relevant to solving your project problem or addressing your research question? (maximum 100 words)

All material referenced in the literature review relies on pre-existing deep learning architectures, such as U-net, that were not initially developed with the project topic problem at the core of their inception. The student believes that a new architecture should be developed or sought-out that deals with the intricacies of simulating electromagnetic scattering. For example, max-pooling may cause unacceptable domain border errors when considering a hybrid conventional/deep learning solver design. The requirements in terms of layer count and parameters for electromagnetic scattering problems is also extremely vague in the literature reviewed so far by the student. Using a neural network meta-architecture may assist in developing a model that can be trained faster that existing architectures and may require less training data simulations, thus reducing the major bottleneck in deploying machine learning algorithms in the electromagnetic domain.

What are the strengths and weaknesses of the solutions/methods/technologies proposed in this paper? (maximum 100 words)
A weakness of the paper is that it claims the benefits of emulators while it is actually dealing with a search meta-architecture that only implicitly acts as an emulator. The student found this slightly misleading upon first review. From the perspective of the student's project aim, the paper is also limited in that the main signals it handles are 1-dimensional. However, the general idea of the paper and the description of the "super-architecture" as visualised in Figure 1. make this paper a stimulating read that may lead to a direct contribution to the student's final emulator design. The inclusion of zero-layers in the architecture is something that the student had never considered or even heard about in previous reading or course materials.

Log Entry 03: 2023/05/16

Statement of project problem / research question (maximum 200 words)

"Can the manner in which deep learning has been shown to solve two-dimensional, forward electromagnetic scatting problems applied to the problem of predicting EM wave propagation over rural terrain, namely emulation, be expanded or improved upon?"

As already reported in the Literature Review, various sources have described using deep learning to tackle forward electromagnetic scattering problems, however, to the knowledge of the student, none have provided a public, reproducible, open-source workflow or a model to the research community. The student proposes to approach the integration of the developed deep learning model, Prescient2DL, into SolverEMF2 through the use of Prescient2DL to generate initial guesses for the Krylov Iterative Solver. By establishing the SolverEMF2 workflow, this primary research question will be approached through the investigation of simulations with several segmented statistical hypothesis tests in lieu of qualitive mathematical proofs. Implicitly, the primary aim of the project is to implement a solver with a deep learning model that optimally shifts calculation metrics to towards the lower left corner of the Residual Error versus Iterations/Time graph when solving permittivity contrast source only Volume Electric Field Integral Equations.

A complete reference for the paper

[3], please refer to the final bibliography.

Summary of paper (maximum 100 words)

This is a new book (2022) dealing with the application of deep learning to electromagnetic problems that the student did not know existed until 2023/05/08 well after the literature review was submitted. The student has read the relevant chapters 1, 2 and 13 of this book and it conforms with the student's literature review with strong overlaps in the references covered. The student views this an as independent confirmation that their research to date and literature review reflects much of the current research energy in the project domain.

How is this paper relevant to solving your project problem or addressing your research question? (maximum 100 words)

In Chapter 13 there is a section dealing with the pros and cons of using DL in the domain whose synthesis would be helpful in the final project portfolio.

Chapter 13 also raises some problems that may be faced in the project. There is a lack of transparency and understanding of the inner workings of the DL architectures. The student believes there are developments in ML space that are working on reducing this lack of transparency, for example, Professor Paul Whelan's visualization methodology for the various layers in the Computer Vision module assignment and the student's understanding that a recent new research domain of explainability in DL may have yielded recent breakthroughs.

What are the strengths and weaknesses of the solutions/methods/technologies proposed in this paper? (maximum 100 words)

One difference to the literature review was that there seems to be a greater consideration given to the sub-algorithmic approaches (references 126-130) that the student had partially avoided. As a result the student may reconsider these approaches and consider their inclusion. The sources not previously considered in the literature review have been recorded in the student's Zotero database for future consideration. In terms of downsides of the book, there was nothing that the student hadn't previously considered or covered in the literature review already.

Log Entry 04: 2023/05/17

Statement of project problem / research question (maximum 200 words)

"Can the manner in which deep learning has been shown to solve two-dimensional, forward electromagnetic scatting problems applied to the problem of predicting EM wave propagation over rural terrain, namely emulation, be expanded or improved upon?"

A complete reference for the paper

[4], please refer to the final bibliography.

Summary of paper (maximum 100 words)

This is a book so new recent that no copy is available at time of writing, however, the student was able to consult the table of contents. The book is concerned with recent advancements in deep learning with application to electromagnetics and is part of the IEEE Press Series on Electromagnetic Wave Theory. This series was also includes [5], as recommended by the supervisor, and has a wide range of titles that concern the general project domain.

How is this paper relevant to solving your project problem or addressing your research question? (maximum 100 words)

The student reviewed the table of contents to check for any forward problem developments that might be useful or prompt queries at the final stage of the project implementation. The vast bulk of the book seems to be concerned with solving inverse problems, such as design optimisation. The only chapter that looks pertinent is the short chapter "Machine Learning Advances in Computational Electromagnetics". Most of the sub-headers have already been covered in the literature review and those that are not obvious to the student, such as "Deep Surrogate Solvers Trained with Physical Regularization" seem to have less than a page of material.

What are the strengths and weaknesses of the solutions/methods/technologies proposed in this paper? (maximum 100 words)

The student does not feel that a review of the table of contents is enough to pass comment on the strengths/weaknesses etc. of the text except for the fact that the overwhelming majority of the book is focused on inverse, rather than forward, problems. If the text becomes available within the time limit of the project, the student will review Chapter 7 in case there are ideas that can be easily incorporated into the project.

Log Entry 05: 2023/06/06

Statement of project problem / research question (maximum 200 words)
"Can the manner in which deep learning has been shown to solve two-dimensional, forward electromagnetic scatting problems applied to the problem of predicting EM wave propagation over rural terrain, namely emulation, be expanded or improved upon?"

A complete reference for the paper
[6] , please refer to the final bibliography.

Summary of paper (maximum 100 words)
The paper discusses a hybrid approach that incorporates neural networks into a finite element method (FEM) solver. The approach is to calculate a residual from the finite element method and a custom loss function from the deep learning model to form a new solver algorithm. The idea is to create surrogate models that can be more generalisable and wrapped in a conventional solver. The applications are not in the domain of electromagnetics.

How is this paper relevant to solving your project problem or addressing your research question? (maximum 100 words)

The paper reaffirms the student's project objective of testing the ability of deep learning models to enhance conventional forward problem solvers. The paper does not totally abandon the conventional solver but instead finds a way to integrate the new machine learning approach with the more established FEM methodology. The benefits of this framework have already been expounded in the student's literature review.

What are the strengths and weaknesses of the solutions/methods/technologies proposed in this paper? (maximum 100 words)
The paper lays out the algorithm for integrating the neural network into the FEM solver in a clear fashion. Such visual description would be a useful addition to the student's own write-up towards the end of the project. Although the student's domain knowledge of the applications, and indeed FEM, tackled in the paper are limited, the two case studies are nicely detailed and give toy examples that may be useful in future work if problems beyond electromagnetics were to be developed. The paper also highlights some of the issues in generating the deep learning surrogates which is useful towards planning the student's project implementation. The paper is lacking detail on contrasting the computational improvement of the new hybrid solver and focuses on the accuracy of the predictions.

Log Entry 06: 2023/07/13

Statement of project problem / research question (maximum 200 words)

"Can the manner in which deep learning has been shown to solve two-dimensional, forward electromagnetic scatting problems applied to the problem of predicting EM wave propagation over rural terrain, namely emulation, be expanded or improved upon?"

A complete reference for the paper

[7]–[9], please refer to the final bibliography.

Summary of paper (maximum 100 words)

This log entry deals with references concerning the domain of application for solver, namely biomedical. The references consulted were:

- Section 5.4 of "Case Study: Scattering from Red Blood Cells" of [7];
- Section 2.5.1 & Section 6 of [8];
- Table 1 of [9] titled "Microwave parameters of three breast tissue types at low (0.5 GHz), middle (2 GHz, 4 GHz, 6 GHz), and high (8 GHz) frequencies".

How is this paper relevant to solving your project problem or addressing your research question? (maximum 100 words)

These references were consulted when considering topic and parameter selection. The setting of carrier wave incident frequency, geometric scale and discretization outputs matter in sizing the data inputs for the deep learning model.

What are the strengths and weaknesses of the solutions/methods/technologies proposed in this paper? (maximum 100 words)

- [7] indicated that a 474 THz incident wave would be required, with a red blood cell having a length of roughly 7.7 micrometers. According to the text, such scales lead to matrix equations with dimensions of over 200,000. Using such large arrays for building ML models is not suitable with current resources and even generating a dataset with solved fields is far beyond what the remaining project time would allow.
- In [8], a model of the relative complex permittivity of human muscle tissue is described in Section 2.5.1. This is the basis for an illustration of Deep Regional Hyperthermia Treatment Planning in Section 6. The example is in the time domain and is too computationally intensive as it depends on three dimensions with multiple incident waves in the 90 MHz range. The main reason for setting aside this source is that, due to time constraints, the incident wave is fixed for all simulations. Creating a sophisticated look-up table for a set of relative permittivities and conductivities based on the carrier incident wave frequency would be wasteful.
- Leading on from [8], the student found Table 1 in [9]. It gives a description of the effective dielectric permittivity and conductivity for normal, benign tumor and cancer cell tissues in the GHz range. This allows for discretization in the scale of interest of 128 and 256 which are more easily accommodated in deep learning architectures.

Log Entry 07: 2023/07/15

Statement of project problem / research question (maximum 200 words)

"Can the manner in which deep learning has been shown to solve two-dimensional, forward electromagnetic scatting problems applied to the problem of predicting EM wave propagation over rural terrain, namely emulation, be expanded or improved upon?"

A complete reference for the paper

[10], please refer to the final bibliography.

Summary of paper (maximum 100 words)

Chapter 6 of this thesis describes an accelerated implementation the Volume Electric Field Integral Equations. Accompanying MATLAB code was sent by the supervisor.

How is this paper relevant to solving your project problem or addressing your research question? (maximum 100 words)

The student was able to match the equations from the derivations in this thesis, culminating with Equation 6.5, to the derivation in [11] for the scalar scenario in Chapter 1, specifically Equation 1.43. The texts use different conventions and approaches to deriving the VEFIE. Initially the student hoped to adapt this MATLAB code to python in order to generate a dataset. The dataset would then be used to train a deep learning model. After enhancing the code parameters to accept a Debeye material model for common building materials, unfortunately, the student realised that the code did not produce scattering simulations due to a possible error in the way the contrast was assigned to the domain of interest.

What are the strengths and weaknesses of the solutions/methods/technologies proposed in this paper? (maximum 100 words)

The focus of the thesis is on conventional approaches to solving the forward problem of electromagnetic scattering. The convention is different to [11], which is the main reference for the student's project. The common complaint, at least to the electromagnetics community, of a lack of code in the body or appendices of the text that reflects the experimental findings of the thesis arises here.

Log Entry 08: 2023/07/28

Statement of project problem / research question (maximum 200 words)
"Can the manner in which deep learning has been shown to solve two-dimensional, forward electromagnetic scatting problems applied to the problem of predicting EM wave propagation over rural terrain, namely emulation, be expanded or improved upon?"

A complete reference for the paper
[12], please refer to the final bibliography.

Summary of paper (maximum 100 words)
This paper was published soon after the completion of the literature review stage of the project. The paper discusses the use of U-net architecture and physics-informed loss functions to predict nonlinear optical scattering problems and the solution to an inverse design problem. Both TE and TM problems are referenced. The paper is accompanied by a supplementary materials appendix. The simulations are generated using finite difference schemes.

How is this paper relevant to solving your project problem or addressing your research question? (maximum 100 words)

The paper provides supplementary material that describes elements of the deep learning design that may be relevant to the student's project. Aside from tackling both Transverse Electric and Transverse Magnetic problems, elements of the U-net architecture used in the deep learning model is described as well as the loss function that tries to embed physics properties arising from Maxwell's equations . The paper highlights the increased difficulty of attempting to train a model in the TM scenario.

What are the strengths and weaknesses of the solutions/methods/technologies proposed in this paper? (maximum 100 words)
The paper presents material useful to researchers trying to prioritise and implement approaches to using deep learning to solve scattering problems. Evidence of training equipment and time requirements are welcome as such information is lacking in the general literature. The paper has made the student reconsider the urgency of using physics-based loss functions due to the large amounts of time required to achieve modest results. A major weakness of the paper is the lack of computer code or dataset available to the reader at the time of writing. It is difficult to assess the diagrams and the layers in the U-net architecture are not clear to the reader.

Log Entry 09: 2023/08/05

Statement of project problem / research question (maximum 200 words)
"Can the manner in which deep learning has been shown to solve two-dimensional, forward electromagnetic scatting problems applied to the problem of predicting EM wave propagation over rural terrain, namely emulation, be expanded or improved upon?"

A complete reference for the paper
[13], please refer to the final bibliography.

Summary of paper (maximum 100 words)
The paper describes a developed conventional approach to reducing the complexity of solving low frequency, high-contrast problems in the domain of non-destructive biomedical evaluation, in particular magnetic induction tomography (MIT). Although the paper is following a conventional, forward-problem methodology, it outlines the difficulties in electromagnetic scattering simulation that are relevant to the masters project. The paper also makes multiple references to the work of Peter van den Berg, a key reference in the project.

How is this paper relevant to solving your project problem or addressing your research question? (maximum 100 words)
This paper grounds the project domain in a viable, applied research area concerning magnetic induction tomography (MIT). The preferred carrier frequencies used in MIT are in the 10 MHz region, the same as used to develop the deep learning dataset. There are major difficulties with modelling biological tissue in this frequency range since permittivity values present extreme contrast values. The paper points to future applications and project developments that could lead to medical applications. Unfortunately, the paper deals with a three-dimensional scenario which is beyond the scope of the student's current interest.

What are the strengths and weaknesses of the solutions/methods/technologies proposed in this paper? (maximum 100 words)
In relation to the project that the student is undertaking, the paper provides the existential reason to simulate 10 MHz carrier frequency and allows the student to establish a raison d'être for attempting to use innovative deep learning techniques in this frequency range. Lack of code is a major weakness with regard to this paper.

Final Information as per guidelines

The reasons why you selected the papers that you have entered in your research log.

I have tried to select papers that touch on all aspects of the project workflow to gain insights into as many subparts of the process as possible. The area is relatively new no source consulted contained enough information in a standalone manner to offer a complete solution to the project problem. I also wanted to ground the project in a real-world application, such as the biomedical domain, to increase the usefulness of any insights gained during the project development process.

How you have used the literature that you have read to guide your project plan and implementation.

I used the papers and books to inform choices in the deep learning architecture, establish expected behaviours of the solvers and to avoid following routes that were beyond the time scope of the project in terms of complexity. The books were largely used as a springboard to find related papers in the domain. One major regret was running out of time and not being able to investigate the skip layer idea in [2]. I really wanted to test this idea as a route to creating bespoke architectures specific to the electromagnetic scattering domain. If I was to extend the research time I would prioritise this step immediately before trying to do anything else. The literature, and associated videos on the internet, also directed me away from trying to implement physics embedded loss functions.

How you compared your implementation and results to previous outputs described in the selected papers.

In order to avoid directly copying existing work, I used the literature to rule out some design implementations that have been already completed. For example, the image-to-image approach is the main implementation of the emulator design so I routed towards keeping the problem as an array instead of an image array. In many cases though, due to the scant literature availble on the project topic and lack of transparency in many cases with regard to results and design choices, I tried to use the literature review to set default values in the design choices.

Describe the value of your continued reading of literature relating to your project.

There were a number of texts I found after the literature review that reiterated the lack of development in this area compared to the solutions developed for the inverse problem. This reiterates the difficulty surrounding this topic and that trying to minimialise every single aspect of the toy problem was the correct thing to do in terms of achieving any results at all. I continued to seek out new material all the way until the final week of the project, including trying to infuse second stage DnCNN denoising models to improve the performance without success.

Briefly describe any other impacts that literature had on your project.

In hindsight, reviewing literature constantly actually was a hinderance in terms of achieving results and experimentation. Instead of reading about existing literature, after the exams in May, I should have just tried to build a basic implementation of the U-net architecture. I also feel that reading so much about the electromagnetic components of the data generation was wasted. Early on in my literature review I found a number of key texts, such as [11] and [14]. I should have drawn a line at these texts and proceeded with an attempt to replicate their findings instead of aiming to be comprehensive in literature review, project presentation and project proposal. I would have achieved much more by learning as I built than trying to plan ahead so much. I also took a lot of books out of the library and this biased my reading towards the electromagnetic end of the project since most of the relevant books available were to do with scattering simulation rather than deep learning. Another major negative impact was the late realisation that, although the Unet architecture originated out of biomedical segmentation problems, the problem at hand was closer to a denoising/generative formulation. I had referenced and read [15] in the literature review and although an investigation into GANs would be well beyond the time limits for this project, I feel I might have avoided spending so much time looking at PINNs now that I appreciate the intended function of the Unet in these problems.

Complete Bibliography

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[14] Q. Ren, Y. Wang, Y. Li, and S. Qi, *Sophisticated Electromagnetic Forward Scattering Solver via Deep Learning*. Singapore: Springer, 2022. doi: 10.1007/978-981-16-6261-4.

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