% Math 244: MATLAB Assignment 7

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clear;

clc;

f = @(x,y) y;

g = @(x,y) -sin(4\*x) - 0.5\*y;

F = @(t, x) [f(x(1), x(2)); g(x(1), x(2))];

[t,x] = ode45(F, [0, 12], [0;3]);

xSol1 = x(:, 1);

ySol1 = x(:, 2);

figure();

set(gcf, 'Position', [100, 100, 800, 800]);

subplot(2,1,1)

plot(t, xSol1);

axis([0, 12, -4, 6]);

subplot(2, 1, 2)

plot(t, ySol1);

axis([0, 12, -4, 6]);

% x(t) goes to 4.6 as t goes to infinity;

% y(t) goes to -0.1 as t approaches infinity.

f = @(x,y) y;

g = @(x,y) -sin(4\*x) - 0.5\*y;

F = @(t, x) [f(x(1), x(2)); g(x(1), x(2))];

[t,x] = ode45(F, [0, 12], [-1;3]);

xSol2 = x(:, 1);

ySol2 = x(:, 2);

figure();

set(gcf, 'Position', [100, 100, 800, 800]);

subplot(2,1,1)

plot(t, xSol2);

axis([0, 12, -4, 6]);

subplot(2, 1, 2)

plot(t, ySol2);

axis([0, 12, -4, 6]);

[t,x] = ode45(F, [0, 12], [0;-2]);

xSol3 = x(:, 1);

ySol3 = x(:, 2);

figure();

set(gcf, 'Position', [100, 100, 800, 800]);

subplot(2,1,1)

plot(t, xSol3);

axis([0, 12, -4, 6]);

subplot(2, 1, 2)

plot(t, ySol3);

axis([0, 12, -4, 6]);

% For (-1,3), y(t) approaches -0.15 and x(t) approaches 3.15.

% For (0,-2), x(t) approaches -3.2 and y(t) approaches 0.

% Based on these, it seems like the solutions depend on both initial

% conditions and are not independent.

figure();

hold on;

plot(xSol1, ySol1, 'b');

plot(xSol2, ySol2, 'r');

plot(xSol3, ySol3, 'g');

hold off;

figure();

hold on;

phasePortrait244(f, g, -4, 8, -3, 3, 0, 10, [1], [3]);

phasePortrait244(f, g, -4, 8, -3, 3, 0, 10, [-1,0], [3,-2]);

hold off;

figure();

set(gcf, 'Position', [100, 100, 800, 600]);

phasePortrait244(f, g, -4, 6, -3, 3, 0, 10, ...

[-4:0.2:6, -4:0.2:6], [3\*ones(1, 51), -3\*ones(1, 51)]);

figure();

set(gcf, 'Position', [100, 100, 800, 600]);

phasePortrait244(f, g, -4, 6, -3, 3, 0, 10, [-4:0.2:6, -4:0.2:6], ...

[3\*ones(1, 51), -3\*ones(1, 51)]);

xlim([-0.41 1.39])

ylim([-0.54 0.54])

% After completing the lab, I went back and zoomed in (made another plot

% with smaller limits) on this phase portrait. This second plot really

% shows the different behavior at the points discussed in step 7. Namely,

% you can see, side by side, the converging behavior around (0,0) and the

% diverging behavior around (pi/4 , 0). I didn't really understand the

% point of step 7 until I came back and examined this plot closely as

% shown.

% (a)

f = @(x,y) 3\*x - y;

g = @(x,y) -4\*x + 3\*y;

figure();

%set(gcf, 'Position', [100, 100, 800, 600]);

phasePortrait244(f, g, -3, 3, -3, 3, 0, 1.1, ...

[-3:0.5:3, -3:0.5:3], [1\*ones(1,13), -1\*ones(1,13)]);

% (b)

f = @(x,y) 2\*x - 3\*y;

g = @(x,y) -4\*y;

figure();

%set(gcf, 'Position', [100, 100, 800, 800]);

phasePortrait244(f, g, -3, 3, -3, 3, 0, 0.8, ...

[-3:0.5:3, -3:0.5:3], [1\*ones(1,13), -1\*ones(1,13)]);

% (c)

f = @(x,y) 3\*x + 5\*y;

g = @(x,y) -x + y;

figure();

%set(gcf, 'Position', [100, 100, 800, 800]);

phasePortrait244(f, g, -3, 3, -3, 3, 0, 0.6, ...

[-3:0.5:3, -3:0.5:3], [1\*ones(1,13), -1\*ones(1,13)]);

% (d)

f = @(x,y) 7\*x + 3\*y;

g = @(x,y) -3\*x + y;

figure();

%set(gcf, 'Position', [100, 100, 800, 800]);

phasePortrait244(f, g, -3, 3, -3, 3, 0, 0.9, ...

[-3:0.5:3, -3:0.5:3], [1\*ones(1,13), -1\*ones(1,13)]);

% (a) Point: (0,0)

f = @(x,y) y;

g = @(x,y) -4\*x + 0.5\*y;

figure();

set(gcf, 'Position', [100, 100, 800, 600]);

phasePortrait244(f, g, -2, 2, -2, 2, 0, 10, ...

[-3:0.25:3, -3:0.25:3], [1\*ones(1,25), -1\*ones(1,25)]);

% (b) Point (, 0)

f = @(x,y) y;

g = @(x,y) 4\*x + 0.5\*y;

figure();

set(gcf, 'Position', [100, 100, 800, 600]);

phasePortrait244(f, g, -1, 1, -1, 1, 0, 10, ...

[-3:0.1:3, -3:0.1:3], [1\*ones(1,61), -1\*ones(1,61)]);

% Comment:

% These portraits do indeed show the behavior around the critical points

% (0,0) and (pi/4 , 0). As discussed below the plot in step 5, one can

% see this behavior in the "fuller" plot, where this converging critical

% point transitions to a diverging critical point in a cycle of pi/4.