18/20

well done!
just one graph missing!

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Section: 4
Lab: 3
Date: October 4 2024

Distribution Functions (Part 1)

Introduction:

In this experiment, we study the Poisson and Gaussian distribution functions. To do this, we measure and analyze the radioactive decay of cesium-137, varying how long we measure in a single sample of decay events. We analyze the data in MATLAB to compare how Poisson and Gaussian distributions describe the data, especially the standard deviation and its relationship to the uncertainty in our measurements.

Experimental Methods:

We measure the decay events of a sample of cesium-137 mounted on an aluminum plate by placing the sample under a geiger tube which is connected to a computer (Figure 1). Each time a nuclear decay event occurs and enters the geiger tube, it generates a pulse of current, which is transmitted to the computer and recorded by software called Logger Pro. In Logger Pro, we create a .txt file containing the data generated for each set of measurements described below. The text file is exported and the data analyzed in MATLAB. The analytical methods are described in the Analysis section and the analysis MATLAB file is included in the lab folder for reference.

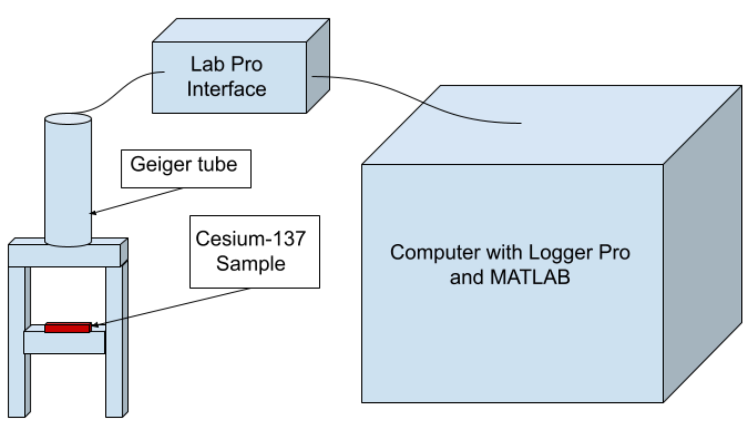

Figure 1: Experimental Setup

Procedure:

Definitions

The measurements we take are intended to analyze the distribution of nuclear decay events N in a "run," or set of runs. We define a run as a set of sample measurements of equal intervals. A sample is a set of measurements N during a predetermined time interval Δt, and $\overline{N}$ is the mean of the measurements. The set of total measurements n is the total set of samples divided by the time interval.

For a single run, the collection of decay counts N represent represent the measurements of the unknown true count N_0. With this, we define our measurements of the decay rate R as a representation of the true decay rate R_0. For the analysis in which we use the Gaussian distribution, see *equation 2.* For the analysis in which the Poisson distribution is used, see *equation 3*.

$$R = N / \Delta t \tag{1}$$

$$P_G(N) = \frac{1}{\sqrt{2\sigma^2\pi}} e^{-\frac{(N-\mu)^2}{2\sigma^2}} \tag{2}$$

$$P_P(N) = \frac{\mu^N}{N!} e^{-\mu} \tag{3}$$

Distribution of Decay Counts (N) in a Single Run

First we measure N for a single run. We expect the distribution of N to be Gaussian and present the results in the Analysis. We set the measuring interval Δt in Logger Pro as ½ second and total run time of 30 seconds, giving 60 total samples.

The first run giving only 60 total samples may not closely resemble a Gaussian distribution, so we conduct two more runs, each with ½ second time intervals per sample. The first runs for 5 minutes to produce a collection of 300 samples and the second runs for 10 minutes to produce 600 samples. All three runs are exported as text files for analysis in MATLAB.

Distribution of Decay Counts (N) with smaller time intervals

To study the Poisson distribution, we take measurements at small time intervals. We want $\bar{N}$ to take on values between 0.5 and 5, so it was necessary to flip the cesium-137 plate over to reduce the decay counts. We allow Logger Pro to collect measurements for 1 minute at $\Delta t = 1/20\ s$. We then take measurements where the celsius-137 is moved progressively further from the geiger tube. We set Logger Pro to run for 20 seconds with $\Delta t = 1/10\ s$ and measure 3 runs. The distances from the geiger tube's sensor and the cesium-137 are 100 mm, 120 mm, and 140 mm +/- 1 mm. Each of these four runs are exported as text files for analysis in MATLAB.

Analysis:

<u>Analysis of Distribution of Decay Counts (N) for 60, 300, and 600 Samples</u>

The first run was a collection of 60 samples (30 seconds, ½ second intervals). To analyze the

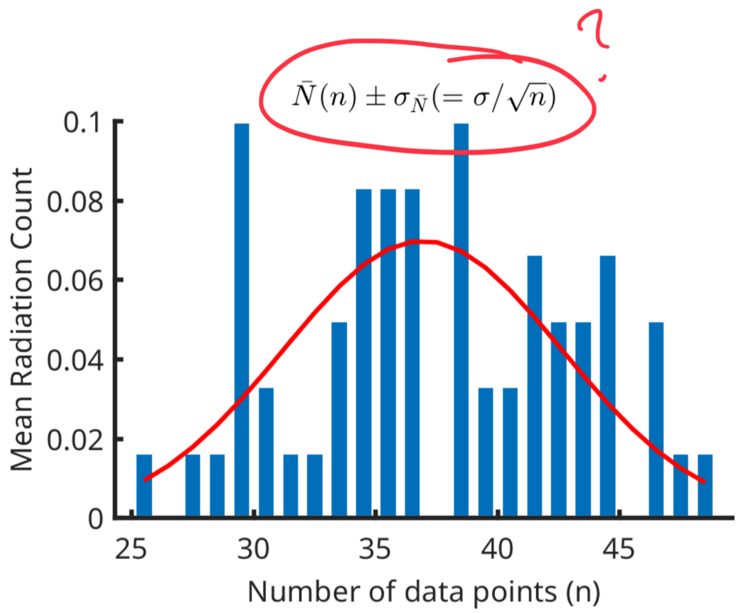

Figure 2: Histogram 30 seconds nuclear decay count

distribution of nuclear radiation events N, and to try and determine the true value of the rate of decay R_0 , we plot a normalized histogram of the mean radiation count $\bar{N}$ vs. the number of data points n in ***Figure 2*** and overlaid a Gaussian distribution curve with the MATLAB function "normpdf" that is based on *equation (1)*. We found the best estimate of R_0 for each run, see *Table 1.*

Table 1: Best estimates R_0 by total sample time		
Run Time (seconds)		R_0 estimate (decay counts)
30		74 +/- 1
300		73.2 +/- 0.5
600		73.0 +/- 0.3

What is the number?

-0.5

One can hardly recognize the histogram in ***Figure 2*** as a Gaussian distribution without the Gaussian curve (red line) overlay. For this reason, which was expected, we conducted two additional runs with longer run times, shown in ***Figure 3*** and ***Figure 4***. *Table 1* shows how the increase in total sample size improves the uncertainty in measuring R_0.

We used the data from the second run (5 minutes) to create a histogram divided into 3 “bins.” The central bin corresponds to plus or minus the standard deviation sigma, the outer bins represent counts that fall within plus or minus two times the standard deviation. The results, which can be seen in ***Figure 5***, show that, as predicted, about sixty eight percent of the counts fall into the central bin.

Next, we studied the distribution of N in the 10 minute run. The prediction for this data set was that the much longer run length would tend to smooth out the histogram’s curve, more closely aligning with the theoretical prediction of the Gaussian distribution. This is exactly the result, as can be seen in ***Figure 4***.

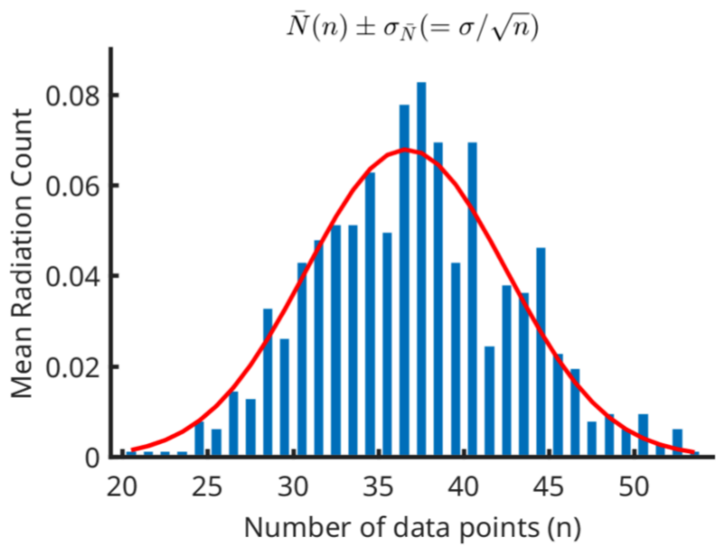

Figure 3: Histogram 5 minutes nuclear decay count

Finally, we compare the distribution of our data with theoretical predictions of an ideal Gaussian distribution by creating a table giving the percentage of intervals where N falls within $\pm\sigma_N$, $\pm 2\sigma_N$, $\pm 3\sigma_N$. The percentage of intervals matched very closely with theoretical values, as seen in *Table 2*.

-0.5

2.

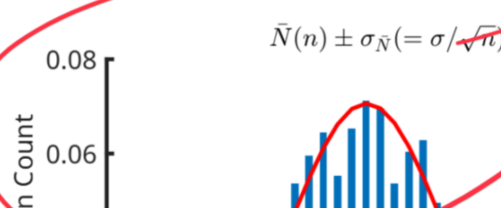

Table 2: N distribution comparison (experiment data vs theory)		
Sigma Multiple (+/-)	Data Percent	Theory Percent
1	71.67	68.27
2	95.17	95.45
3	99.83	99.73

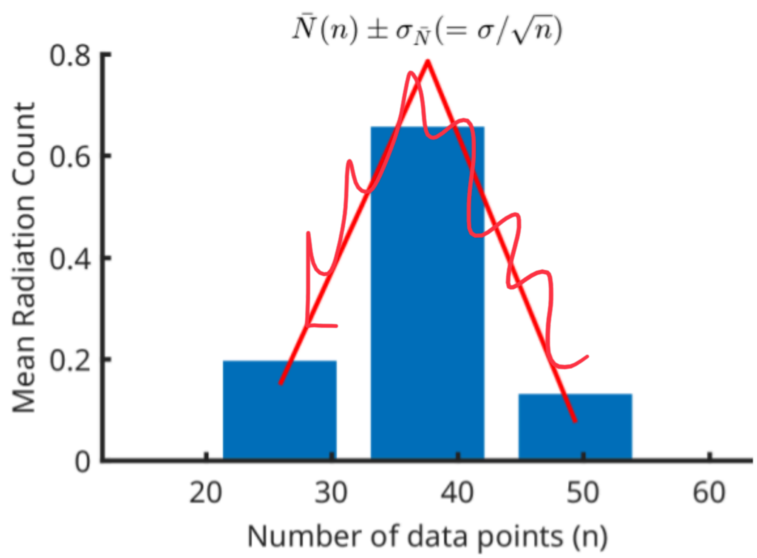

Figure 5: Gaussian over histogram with sigma, 2sigma, 3sigma distribution bins

Analysis of Decay Count Distribution Small Average N

In the previous runs, we used sample sizes with $\bar{N}$ in the range of about 40 per ½ second sample. In this section we study Poisson distribution which is more appropriate when $\bar{N}$ becomes small. The measurements we took next had a time interval of 1/20 second and ran for 1 minute.

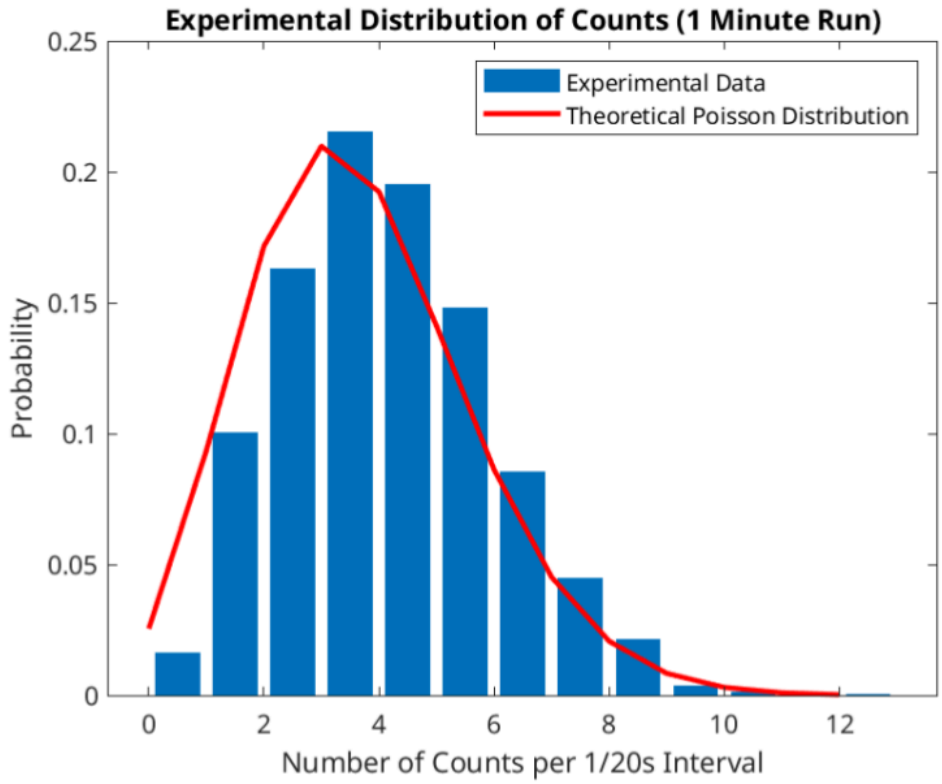

Figure 6: 1/20 sec intervals run time of 1 minute

Comparing the histogram in ***Figure 6*** with the overlay of the theoretical Poisson distribution shows that for very small intervals, the Poisson does a great job of showing the distribution.

We then analyzed the data from the last 3 runs, each moving the radioactive source further from the geiger tube. The average counts become fewer and fewer as the cesium-137 is moved further away.

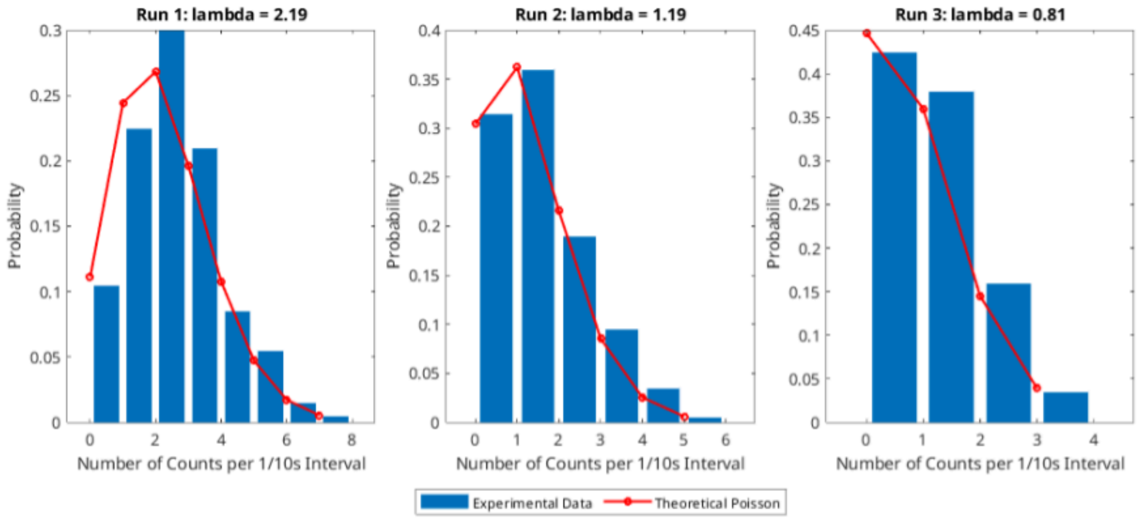

Figure 7: 1/10 sec intervals with 20 second run times at progressively further distance from geiger tube: approximately 100mm, 120mm, 140mm

In histograms with the theoretical Poisson line overlaid in ***Figure 7***, we can see that the mu (average count of N) moves along with the peak. This is not consistent with what the lab guide predicts and the reason is not known to the author at this time.

Conclusion:

The experiment analyzed the relationship between the Gaussian distribution and the Poisson distribution, showing a clear distinction between how Poisson and Gaussian distributions describe the data. The distribution of N about the mean becomes smaller is well described by the Poisson, with theoretical values very close to those in our data. In contrast, as the mean becomes larger, the Gaussian distribution almost exactly matches theory to experiment, showing that the Gaussian is ideal in these circumstances.