1. INTRODUCTION

Artificial Neural Networks (ANNs) are computational models inspired by the structure and functioning of the human brain. Just as biological neurons process and transmit information through electrical and chemical signals, artificial neurons process numerical inputs and produce outputs based on mathematical operations. ANNs are widely used in fields such as pattern recognition, image classification, speech processing, medical diagnosis, and predictive analytics.

Artificial Neural Networks mimic the learning and processing behavior of the human brain. Their functioning is fundamentally governed by activation dynamics, which determine how neurons compute outputs, and synaptic dynamics, which define how learning occurs through weight adjustments. Together, these two mechanisms enable ANNs to process complex patterns, adapt to data, and solve real-world problems efficiently.

- **Activation dynamics** describe how neuron states (activations) evolve with time for a fixed input.
- **Synaptic dynamics** describe how connection weights change during learning. Mathematically, both are represented using first-order differential equations, which model the rate of change of activation or weight with respect to time.

2. Passive Decay Model

$$\frac{d\boldsymbol{x_i(t)}}{\boldsymbol{dt}} = -\boldsymbol{A_i x_i(t)}$$

Symbols

- $x_i(t)$: activation (state) of neuron indexed by $i$at time t
- i: index identifying the neuron
- t: time variable
- $\frac{dx_i(t)}{dt}$: derivative of $x_i(t)$with respect to time (rate of change)
- A_i: positive decay (leakage) constant for neuron i
- $-$: indicates reduction (decay) of activation

3. Modified Passive Decay Model

$$\frac{d\boldsymbol{x_i(t)}}{\boldsymbol{dt}} = -\frac{\boldsymbol{A_i}}{\boldsymbol{C_i}}\boldsymbol{x_i(t)}$$

Symbols

- $x_i(t)$: activation of neuron $i$at time t

- t: time
- $\frac{dx_i(t)}{dt}$: rate of change of activation
- A_i: decay constant
- C_i: membrane capacitance of neuron i
- $\frac{A_i}{C_i}$: effective decay rate
- $-$: decay effect

4. Non-Zero Resting Potential Model

$$\boldsymbol{\frac{dx_i(t)}{dt} = -A_i x_i(t) + P_i}$$

Symbols

- $x_i(t)$: activation of neuron $i$at time t
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change of activation
- A_i: decay constant
- P_i: constant input (resting potential or bias)
- $+$: additive effect

5. External Input Activation Model

$$\boldsymbol{\frac{dx_i(t)}{dt} = -A_i x_i(t) + B_i I_i}$$

Symbols

- $x_i(t)$: activation of neuron $i$at time t
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change of activation
- A_i: decay constant
- B_i: input gain (scaling factor)

- I_i: external input signal applied to neuron i
- $+$: additive contribution

6. Additive Autoassociative Model

$$\frac{dx_i(t)}{dt} = -A_i x_i(t) + B_i I_i + \sum_{j=1}^{N} W_{ij} f_j(x_j(t))$$

Symbols

- $x_i(t)$: activation of neuron $i$at time t
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change of activation
- A_i: decay constant
- B_i: input gain
- I_i: external input to neuron i
- $\sum_{j=1}^{N}$: summation over index $j$from 1 to N
- j: index of presynaptic neuron
- N: total number of neurons
- W_{ij}: synaptic weight from neuron $j$to neuron i
- $f_j(\cdot)$: output function of neuron j
- $x_j(t)$: activation of neuron $j$at time t

7. Inhibitory Feedback Model

$$\frac{dx_i(t)}{dt} = -A_i x_i(t) - B_i I_i - \sum_{j=1}^{N} W_{ij} f_j(x_j(t))$$

Symbols

- $x_i(t)$: activation of neuron $i$at time t
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change

- A_i: decay constant
- B_i: scaling constant
- I_i: external input
- $\sum_{j=1}^{N}$: summation over neurons
- j: neuron index
- N: total neurons
- W_{ij}: weight from neuron $j$to neuron i
- $f_j(x_j(t))$: output of neuron j

8. Perkel's Model

$$\frac{\boldsymbol{dx_i(t)}}{\boldsymbol{dt}} = \frac{\mathbf{1}}{\boldsymbol{R_i}}\boldsymbol{x_i(t)} + \sum_{\boldsymbol{j=1}}^{\boldsymbol{N}} \frac{\mathbf{1}}{\boldsymbol{R_{ij}}}\boldsymbol{\phi_j(x_j(t))}$$

Symbols

- $x_i(t)$: activation of neuron i
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change
- R_i: resistance of neuron i
- $\frac{1}{R_i}$: conductance
- $\sum_{j=1}^{N}$: summation over neurons
- j: neuron index
- N: total neurons
- R_{ij}: resistance between neurons $i$and j
- $\phi_j(\cdot)$: output function
- $x_j(t)$: activation of neuron j

9. Hopfield Model

$$\frac{d\boldsymbol{x}_i(\boldsymbol{t})}{d\boldsymbol{t}} = -\boldsymbol{A}_i\boldsymbol{x}_i(\boldsymbol{t}) + \sum_{j=1}^{N} \boldsymbol{W}_{ij}\, \boldsymbol{f}(\boldsymbol{x}_j(\boldsymbol{t})) + \boldsymbol{I}_i$$

Symbols

- $x_i(t)$: activation of neuron i
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change
- A_i: decay constant
- $\sum_{j=1}^{N}$: summation
- j: neuron index
- N: total neurons
- W_{ij}: symmetric connection weight
- $f(\cdot)$: bounded activation function
- $x_j(t)$: activation of neuron j
- I_i: external input

10. Heteroassociative Network

Layer-1

$$\frac{d\boldsymbol{x}_i(\boldsymbol{t})}{d\boldsymbol{t}} = -\boldsymbol{A}_i\boldsymbol{x}_i(\boldsymbol{t}) + \sum_{j=1}^{M} \boldsymbol{f}_j\,(\boldsymbol{y}_j(\boldsymbol{t}))\boldsymbol{V}_{ji} + \boldsymbol{I}_i$$

Symbols

- $x_i(t)$: activation of neuron $i$in layer-1
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change
- A_i: decay constant
- $\sum_{j=1}^{M}$: summation over layer-2 neurons
- j: index of neuron in layer-2

- M: number of neurons in layer-2
- $f_j(\cdot)$: output function of neuron j
- $y_j(t)$: activation of neuron $j$in layer-2
- V_{ji}: weight from neuron $j$to neuron i
- I_i: external input

Layer-2

$$\frac{d\boldsymbol{y_j(t)}}{\boldsymbol{dt}} = -\boldsymbol{B_j y_j(t)} + \sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} \boldsymbol{g_i}\,(\boldsymbol{x_i(t)})\boldsymbol{W_{ij}} + \boldsymbol{J_j}$$

Symbols

- $y_j(t)$: activation of neuron $j$in layer-2
- t: time
- $\frac{dy_j(t)}{dt}$: rate of change
- B_j: decay constant
- $\sum_{i=1}^{N}$: summation over layer-1 neurons
- i: neuron index in layer-1
- N: number of neurons in layer-1
- $g_i(\cdot)$: output function of neuron i
- $x_i(t)$: activation of neuron i
- W_{ij}: weight from neuron $i$to neuron j
- J_j: external input

11. Bidirectional Associative Memory (BAM)

Same equations as heteroassociative network with

- $V = W^T$: weight matrix Vequals transpose of weight matrix W

12. Basic Shunting Model

$$\frac{d\boldsymbol{x}_i(\boldsymbol{t})}{d\boldsymbol{t}} = -\boldsymbol{A}_i\boldsymbol{x}_i(\boldsymbol{t}) + (\boldsymbol{B}_i - \boldsymbol{x}_i(\boldsymbol{t}))\boldsymbol{I}_i$$

Symbols

- $x_i(t)$: activation
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change
- A_i: decay constant
- B_i: upper saturation limit
- I_i: external input
- $(B_i - x_i(t))$: remaining activation capacity

13. On-Center Off-Surround Model

$$\frac{d\boldsymbol{x}_i(\boldsymbol{t})}{d\boldsymbol{t}} = -\boldsymbol{A}_i\boldsymbol{x}_i(\boldsymbol{t}) + (\boldsymbol{B}_i - \boldsymbol{x}_i(\boldsymbol{t}))\boldsymbol{I}_i - \boldsymbol{x}_i(\boldsymbol{t})\sum_{\boldsymbol{j}\neq\boldsymbol{i}}\boldsymbol{I}_j$$

Symbols

- $x_i(t)$: activation
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change
- A_i: decay constant
- B_i: upper bound
- I_i: excitatory input
- $\sum_{j\neq i}$: summation over all neurons except i
- $j \neq i$: index condition
- I_j: inhibitory inputs

14. Modified Shunting Model

$$\frac{dx_i(t)}{dt} = -A_i x_i(t) + (B_i - x_i(t))I_i - (E_i + x_i(t)) \sum_{j \neq i} I_j$$

Symbols

- $x_i(t)$: activation
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change
- A_i: decay constant
- B_i: upper bound
- E_i: lower bound parameter
- I_i: excitatory input
- $\sum_{j \neq i}$: summation excluding neuron i
- I_j: inhibitory inputs

15. Shunting Model with Feedback

$$\frac{dx_i(t)}{dt} = -A_i x_i + (B_i - C_i x_i)[I_i + f_i(x_i)] - (E_i + D_i x_i)[J_i + \sum_{j \neq i} f_j\,(x_j) w_{ji}]$$

Symbols

- x_i: activation of neuron i
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change
- A_i: decay constant
- B_i: upper bound
- C_i: scaling constant for excitatory term
- I_i: external excitatory input
- $f_i(x_i)$: feedback function of neuron i

- E_i: lower bound parameter
- D_i: scaling constant for inhibitory term
- J_i: external inhibitory input
- $\sum_{j \neq i}$: summation over neurons except i
- $f_j(x_j)$: output of neuron j
- w_{ji}: feedback weight from neuron $j$to neuron i

<u>16. Synaptic Dynamics Model</u>

$$\boldsymbol{\frac{dw_{ij}(t)}{dt} = -w_{ij}(t) + f_i(x_i(t))f_j(x_j(t))}$$

Symbols

- $w_{ij}(t)$: synaptic weight from neuron $j$to neuron $i$at time t
- $\frac{dw_{ij}(t)}{dt}$: rate of change of weight
- t: time
- $f_i(x_i(t))$: output of neuron i
- $x_i(t)$: activation of neuron i
- $f_j(x_j(t))$: output of neuron j
- $x_j(t)$: activation of neuron j