Intraindividual Dynamic Network of Affects: Linear Autoregressive Mixed-Effects Models for Ecological Momentary Assessment

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Abstract

The use of advanced mathematical modeling and data analysis in clinical psychology is increasingly growing. An interesting recent development is the discovery that mental state can be modeled as networks of temporally interacting symptoms. External factors, such as stressors or treatments, can affect the network by enhancing or dampening the activation of one or more symptoms.

In this work we consider very simple directed intraindividual networks comprising two symptom nodes and one node for external factors. The two symptom nodes interact by enhancing or dampening each other and the external factors are assumed to be acting on both nodes in a one-way fashion, i.e. the possible effects of a symptom on the perception of external factors are neglected. Assuming intensive longitudinal data through ecological momentary assessment, we then formalize the mathematical representation of such networks by exogenous linear autoregressive mixed-effects models

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such that the autoregressive coefficients represent the interactions and the external factors enter the model as exogenous covariates.

We let every parameter in the model to have fixed and random components aiming at networks that are allowed to have variable structures over reasonable units of time like days or weeks depending on the study design. Given the fact that a rich theoretical and computational literature furnishes classic linear mixed-effects models, we transform the autoregressive formulation to a classical one and use the already developed methods and tools. Then assuming our model is the true data generating process, we simulate data using a predefined set of parameters and investigate the performance and feasibility of this approach in delivering reliable estimates for different choices of the number of observations and the intensity of noise.

1 Introduction

One of the most interesting recent developments in clinical psychology and psychiatry is the discovery that mental state can be represented and modeled as a network of symptoms that interact, i.e., causally affect each other via biological, psychological, and social mechanisms (Kalisch et al., 2019; Borsboom, 2017; Kendler, Zachar, & Craver, 2011; Cramer, Waldorp, van der Maas, & Borsboom, 2010). Symptom interactions include mechanisms where activation of one ”node” influences activation of other nodes. For example, frequent hypervigilance and anxiety may lead to high levels of stress hormones; these may impair prefrontal function (biological mechanism of interaction), including executive control, which in turn may lead to problems in professional performance or in social interaction. Poor performance or social conflicts may lead to a generalized negative appraisal of one’s abilities and competencies (a psychological mechanism), which may induce feelings of despair or hopelessness. Poor performance or social conflicts may also lead to a lack of social support from others (a social mechanism), which in turn can induce anxiety and stress.

As such, emotional fluctuations are believed to shape the daily experiences, behaviors, and decisions of a person to a great deal (Kuppens, 2015) and the evolution of affective state across daily life, as a system, can be modeled as a network of interacting emotional variables to predict the psychological well-being (Bringmann et al., 2013; Bishop, 2012; Vanhasbroeck, Ariens, Tuerlinckx, & Loossens, 2021).

The dynamics of such networks can be modeled using time-series data. This has been facilitated by technological improvements in recent years and the fact that smartphones and wearable devices are inseparable from the daily lives of many people. It is now a well-established practice to acquired data by experience sampling methods (Myin-Germeys & Kuppens, 2021) where individuals are observed frequently within short periods of time, every day for a couple of weeks resulting in the so-called ecological momentary assessment (EMA) data. As the acquisition of EMA data is made easier, single-subject analysis is also gaining momentum where constructing individual dynamic networks might be helpful in understanding the mental state of a specific person and this, in turn, might promote personal interventions (Bringmann, 2021; Frumkin, Piccirillo, Beck, Grossman, & Rodebaugh, 2021).

In building such networks when one variable is correlated to the other, the corresponding network nodes (S1 and S2 in Figure 1) can be seen as being connected with a weight according to the type (positive or negative) and the intensity of the correlation (line in Figure 1a). A connection may also be of some causal nature such that one variable, S2, at time $t+1$ is highly correlated (positive or negative) with the other, S1, at time t. In this case, we draw an arrow from S1 to S2 which represents the Granger-causality of the source node on its target. The relation might be reciprocal as well, i.e., poor performance increasing despair, which reduces performance even more, arrows in Figure 1b.

Having two-way causal relations in this simple network with two variables may result in the presence of a positive feedback loop in the network. In a more complex network this is also possible with only one-way connections, e.g., socially induced anxiety increasing stress hormone levels, thus reducing prefrontal executive function, thus reducing performance, thus reducing support, thus increasing anxiety and stress. At a certain level of activation, these characteristic network features can promote a self-sustaining state of general high activation of several strongly interconnected variables, called a mental disorder. Every variable might also have a different degree of serial correlation in terms of how its value at time $t+1$ is influenced by its previous values. In network representation, such a relation is shown by looped arrows on variables. External factors can also affect the network state by enhancing or dampening certain variables or even their interaction. For instance, stressors might have enhancing effects on symptoms or their interactions and any protective factor of biological, psychological, or social nature might have dampening effects (Kalisch et al., 2019; Cramer et al., 2016). In this work we only consider the contemporaneous effects of external factors, i.e., the effects of E at time t on other variables also at time t. The extension of these networks to include more nodes is straightforward.

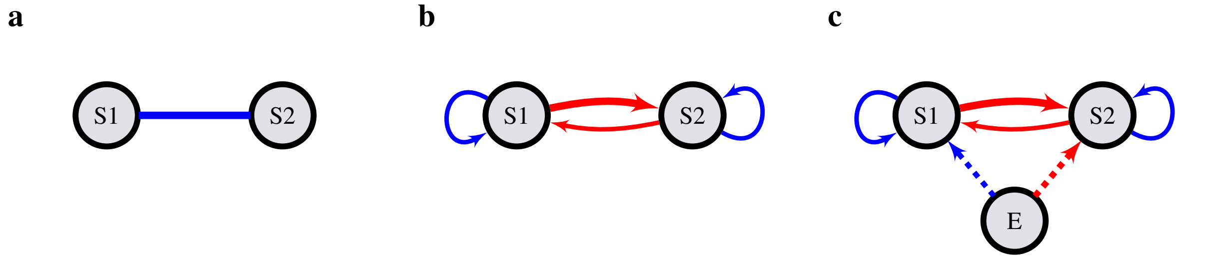

Figure 1: Variables, $S1$ and $S2$, interact with each other leading to meaningful contemporaneous correlation between them. This is shown by a solid line (a). Solid arrows represent temporal correlation meaning that the value of the source variable (node) at time t has an impact on the value of its target at time $t + 1$. Looped arrows indicate autoregressive effects (b). Exogenous factors act on moods at the same time in a one-way fashion (c). Blue and red arrows indicate activation and suppression respectively. Different thicknesses stand for distinct interaction strengths. Dashed arrows indicate contemporaneous effects while solid ones are indicative of possible temporal causalities.

In building the model, we restrict ourselves to a very minimal network comprising two interacting nodes and one node for external stressors. This way, we avoid the complexity related to the number of parameters in the model as it grows quadratically by the number of variables in an autoregressive setting.

The evolution of states in an autoregressive model usually does not explicitly depend on time and it is assumed that the observations are made one unit of time apart in a consecutive manner. However, ecological momentary assessment violates this assumption by randomizing the observation time points which is a well-accepted practice in psychology. It also halts the observation overnights which is the only feasible way of acquiring data. This latter issue leaves EMA with inherent missing data, not at random.

Many researchers agree that randomized time points are acceptable for psychological data to be modeled through VAR processes and some ignore the missing points at nights, pool data over days as a time series, and apply autoregressive methods directly (Epskamp, 2020; de Haan-Rietdijk, Voelkle, Keijsers, & Hamaker, 2017). Others consider performing some preprocessing prior to employing VAR and deal with the irregular time stamps and jump overnights by smoothing data, e.g. by interpolation (van der Krieke et al., 2015; Fisher, Reeves, Lawyer, Medaglia, & Rubel, 2017).

Nevertheless, by pooling data and using VAR one implicitly assumes that the network structure is fixed over time, and should be considered as a trait rather than a state. In this work, we focus on individual dynamic networks with potential time-varying structure and further elaborate on using multilevel methods for EMA data by employing an Exogenous Linear Autoregressive Mixed-Effects model (LARMEx) which is an extension of the classic linear mixed models for normally distributed (continuous) responses (de Haan-Rietdijk, Kuppens, & Hamaker, 2016; Schuurman, Ferrer, de Boer-Sonnenschein, & Hamaker, 2016). It treats the dependence on lagged values as well as covariates in one formulation and lets the parameters vary over levels of data as random effects. This way, we assume that the underlying data generating process for EMA data is a Vector Autoregressive (VAR) process which is initiated every day in the morning from an initial value independent from the last measurement of the previous day, and evolves through a day potentially with slightly different parameters compared to other days. By this formulation we let the AR coefficients vary over days (state) around a subject-specific set of values (trait), and this time-variant property is captured by random effects. The subject-specific parameters are the fixed-effects component of the model which represents the overall characteristics associated with a person, i.e., the state.

This formulation is capable of representing other multilevel designs whenever there is evidence that pooling data over specific units of observations is feasible. Assuming that the network structure is stable over weeks one could consider a week as the unit of observation and pool daily data. Another possibility would be the assumption of a stable network for the whole population of respondents and allowing individuals to have slightly different parameters for which one could aggregate all data from a single person. In this work, however, we focus on single-subject analysis and assume that the observations are nested in days which in turn are nested in a respondent.

2 Mathematical Representation

Linear autoregressive (AR) processes are used to model stochastic dynamical systems of discrete data through difference equations. That is, the current state of the system, s_t, is assumed to depend linearly on the preceding states, $s_{t-1}, \ldots, s_{t-p}$ and a constant term analogous to the intercept in linear regression. All the unknown factors which might affect the system are encapsulated in the error term ϵ_t and is usually assumed to be white noise with zero mean (Hamilton, 1994),

$$s_t = \sum_{l=1}^{p} \beta_l s_{t-l} + \beta_0 + \epsilon_t. \tag{1}$$

In this formulation, s is usually supposed to be a time series comprising sufficient observations over time which makes it possible to detect its temporal dynamics. The autoregressive coefficients β_i determine how the current value of S is affected by its preceding values by p steps back in time. In the simplest form of an AR process, the dependence on previous states is only taken one step back and the so-called *Lag(1)* processes are formulated by $s_t = \beta_1 s_{t-1} + \beta_0 + \epsilon_t$. In the absence of noise, this process has a fixed point of $\frac{\beta_0}{1-\beta_1}$. If such a process reaches its fixed point it will stay there forever. Otherwise, such a point determines the asymptotic behavior of the process in long run. In this work we only consider stable systems that will eventually converge to fixed points and require that $|\beta_1| < 1$ for a single variable *Lag(1)* process.

The immediate generalization of AR processes to accommodate multiple states is a common practice whenever one needs to study the dynamics of several variables simultaneously. Exogenous covariates might also be included in a linear fashion to account for the effects of known external factors.

LARMEx models the mean response as a combination of trait-like characteristics of an individual as fixed effects and state-like features varying over days (weeks) as random effects. Needless to say, existing techniques allow for the estimation of fixed effects and the variances of random effects, and the values of random effects are predicted using these estimations. This way, the model accounts for the dependency in different levels of data, e.g. during a day, and at the same time compensates for the lack of statistical power due to few observations on the first level (Funatogawa & Funatogawa, 2018; Fitzmaurice, Laird, & Ware, 2011).

We build a dynamic affect network comprising two moods variables, positive affect (PA) and negative affect (NA), and an exogenous factor (E) representing the external factors that may be considered as the input for the dynamical system.

2.1 Mood Networks

Let $\boldsymbol{s}_{i,t} = [s_1, s_2]^T_{i,t}$ be the 2×1 vector corresponding to the two mood values from the ith measurment day at any observation occasion $t = 1, 2, \ldots, n_i$ where n_i is the number of observations per day and T denotes the transpose of a matrix. Note that n_i might differ for every i despite the fact that it is planned to collect the same amount of data every day. In the absence of external driving forces like stressors and noise, the mood network is represented by a first-order autoregressive system

$$\boldsymbol{s}_{i,t} = \boldsymbol{B}_i^{ar} \boldsymbol{s}_{i,t-1} + \boldsymbol{B}_i^{c}, \tag{2}$$

in which $\boldsymbol{B}_i^{ar} = \boldsymbol{\beta}^{ar} + \boldsymbol{b}_i^{ar}$ are 2×2 matrices of auto- and cross-lagged regression coefficients, and $\boldsymbol{B}_i^{c} = \boldsymbol{\beta}^{c} + \boldsymbol{b}_i^{c}$ are 2×1 vectors of constant terms corresponding to the two moods. Every coefficient matrix and constant term is a sum of fixed and random-effects terms representing the subject, $\boldsymbol{\beta}$, and day specific, $\boldsymbol{b}_i$, components. The random-effects are assumed to be normally distributed with the mean zero and a certain variance-covariance, $\boldsymbol{b} \sim N(0, \boldsymbol{G})$. Different structures of $\boldsymbol{G}$ are used to account for possible scenarios representing the dependencies amongst random-effects. We restrict ourselves to a compound symmetric (CS) type where all random-effects share the same variance, the diagonal elements of $\boldsymbol{G}$, and the autoregressive and exogenous components are assumed to be independent. For more structures we refer the reader to (Funatogawa & Funatogawa, 2018) and references therein.

To avoid the complexity of restricting moods to take only positive values, we assume that they are centered and can take any value. Nevertheless, we set the initial values, constant terms and the range of exogenous factors in a way that for reasonably low noise levels, most of the trajectories remain in the interval $[-1, 1]$. The dynamics represented by Equation 2 is asymptotically stable given that all the eigenvalues of $\boldsymbol{B}_i^{ar}$ have modulus less than one, Figure 2b, (Lütkepohl, 2005). The asymptotic behavior is determined by the autoregressive coefficients and constant terms. Note that the coefficients take positive and negative values so the moods interact through activating and suppressing each other although in a more complex networks there might be only positive or negative interactions. We also assume that the constant term has a zero fixed effect component to reduce the number of parameters. In dealing with real data we put no constraint on the coefficients to account for any possibility.

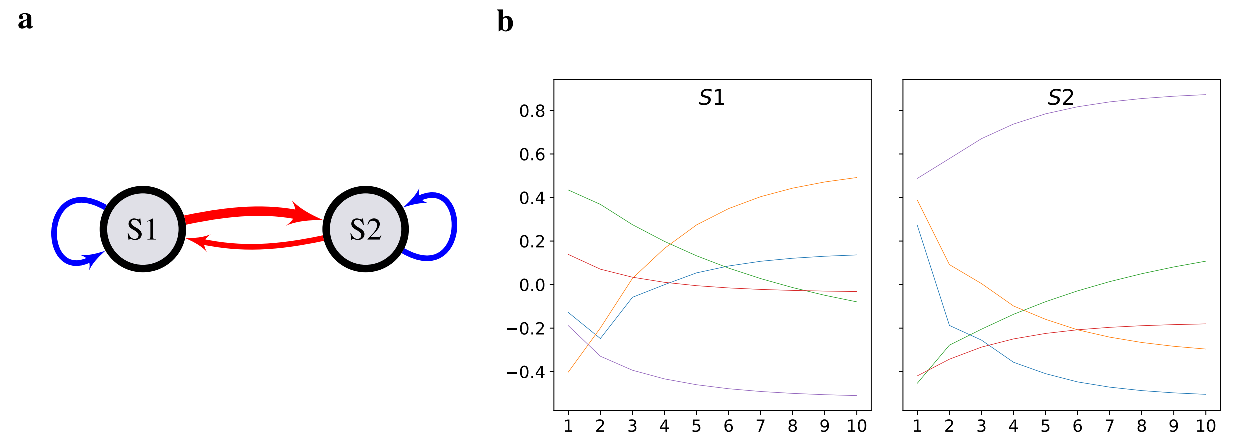

Figure 2: Network of two nodes without external forces (a), realizations for 5 simulated days of 10 observation each (b). Data is generated using Equation 2 with initial values for different days sampled uniformly from the interval $[-0.5, 0.5]$.

In Figure 2b, depending on the magnitudes of coefficients, different patterns of decay are seen. For larger values, variables take a longer time to relax toward the asymptotic level. Moreover, the initial values have a stronger influence on the early next steps until the trajectories approach the asymptotes. Therefore, in generating data a couple of first steps are discarded to avoid any artifact caused by initial values.

2.2 Network with Exogenous Factors

Assuming that a collection of external factors, E, acts on mood nodes and its effect could vary over days, the network of moods as discussed above could be extended in two ways. On the one hand, we could assume that on every day a subject is exposed to a baseline level of input, e.g. stress, which might be different for every day. This has already been treated by adding a constant term, $\boldsymbol{B}_i^c = \boldsymbol{\beta}^c + \boldsymbol{b}_i^c$ as fixed and random intercepts for day i, Equation 2. On the other hand, one could assume that E has a time dependent component as well which is measured along with the moods. This term enters as a time varying covariate to the formulation in Equation 2 and forms an exogenous autoregressive process of

$$\boldsymbol{s}_{i,t} = \boldsymbol{B}_i^{ar} \boldsymbol{s}_{i,t-1} + \boldsymbol{B}_i^e \boldsymbol{e}_{i,t} + \boldsymbol{B}_i^c. \tag{3}$$

In this formulation, $\boldsymbol{B}_i^e$ could be considered as a measure of contemporaneous stress reactivity in the long run and short periods of time like days. The term $\boldsymbol{B}_i^e \boldsymbol{e}_{i,t} + \boldsymbol{B}_i^c$ determines how the asymptotic values of moods are affected by external amplifiers. Since E varies over time the asymptotic levels are changing from time to time and therefore a fluctuating pattern emerges over the mood trajectories. In this way, the differences in stressor exposure are accounted for. The stressor is measured longitudinally alongside moods. In Figure 3b below, we included E as an impulse, i.e., it has a nonzero value at $t = 5$ and is zero elsewhere. This is done only for presentation purposes and in practice, it might take any value at different points. Opposite perturbations in the mood levels are visible compared to Figure 2b.

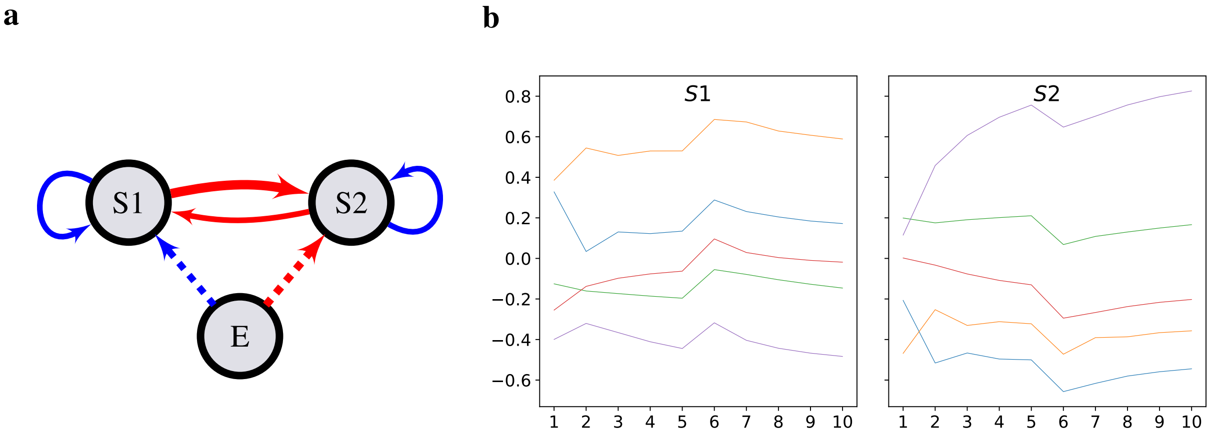

Figure 3: Network of two nodes with a fixed amount of external stressors E acting on both from time point 5 (a), realizations for 5 simulated days of 10 observation each (b).

In a more realistic scenario, one would introduce a positive random value at each time point with a non-negative value as the fixed effect and generate random-effects from a truncated normal distribution to keep the term $\boldsymbol{B}_i^e \boldsymbol{e}_{i,t}$ positive. If this was the case an overall the upward (downward) profile would be visible in data indicating the exciting (inhibiting) effect of the stressors on NA (PA).

2.3 Unknown Effects as Noise

So far we considered only deterministic models where the dynamics of the system is fully known once the parameters are fixed. However, our knowledge of real systems is usually imperfect either under the influence of measurement errors or lack of information about unknown influences on the dynamics. To account for all the uncertainties due to unknown factors, but not measurement errors, we add a white noise element, $\boldsymbol{\epsilon}_{i,t}$, to Equation 3 ending up at the full model of mood networks in the presence of external factors as an exogenous linear autoregressive mixed-effects model,

$$\boldsymbol{s}_{i,t} = \boldsymbol{B}_i^{ar} \boldsymbol{s}_{i,t-1} + \boldsymbol{B}_i^e \boldsymbol{e}_{i,t} + \boldsymbol{B}_i^c + \boldsymbol{\epsilon}_{i,t} \tag{4}$$

in which $\boldsymbol{\epsilon}_{i,t} \sim N(\boldsymbol{0}, \sigma^2 \boldsymbol{I}_{2\times 2})$. Treating measurement errors adds another layer of complexity to the model and hence is avoided here for the sake of simplicity. Taken E into account, there might be an increasing variance and divergent profiles up- and downwards. Adding noise to the system results in Figure 4.

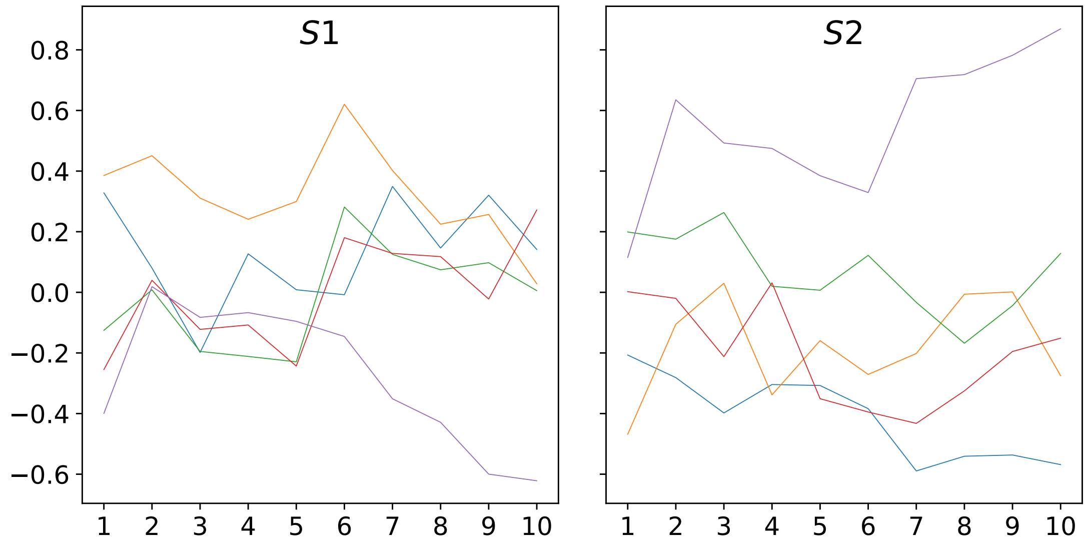

Figure 4: Realizations from a network of two nodes for 5 simulated days of 10 observation each. External factors act on both nodes from time points 5 in the presence of noise.

3 Parameter Estimation

In classic linear models, the expected value of the response variable is parameterized by fixed effects and the only source of variation comes from error terms which are usually considered uncorrelated and independently distributed. This distribution gives rise to the likelihood function and the estimation is carried out by maximizing it over the parameters. In mixed-effects models, however, the correlation structure of the response variable is addressed by including random-effects which induces further sources of variability in addition to the error terms. Assuming normality of errors and random-effects, the ordinary least squares method of classic linear theory is extended to be used in estimating the parameters (Laird & Ware, 1982; Diggle, Heagerty, Liang, & Zeger, 2013; Bates, Mächler, Bolker, & Walker, 2015). Recently it has been further generalized through the inclusion of autoregressive coefficient matrices as fixed-effects to account for the overall trend in data exhibited through patterns of profiles approaching an asymptote (Funatogawa & Funatogawa, 2018). We modify this approach by including a random-effects component to address the dynamic interactions for individuals by subject-specific autoregressive coefficients.

In this section, we transform the model into a simple matrix form and review the estimation procedure by formulating the maximum likelihood and restricted maximum likelihood methods.

3.1 Matrix Form

The simplest way of presenting the estimation procedure is transforming the autoregressive mixed effects form to a general one by treating the lagged variables as covariates and grouping the fixed- and random-effects separately as follows (Jones, 1993; Laird & Ware, 1982). For any time point, t, of every day, i, one has

$$\boldsymbol{s}_{i,t} = (\boldsymbol{\beta}^{ar} + \boldsymbol{b}_i^{ar})\boldsymbol{s}_{i,t-1} + (\boldsymbol{\beta}^{e} + \boldsymbol{b}_i^{e})\boldsymbol{e}_{i,t} + (\boldsymbol{\beta}^{c} + \boldsymbol{b}_i^{c}) + \boldsymbol{\epsilon}_{i,t},$$

which takes the following matrix form with separated fixed and random effects.

$$\begin{aligned}\begin{bmatrix} s_1 \\ s_2 \end{bmatrix}_{i,t} = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}_{i,t-1} &+ \begin{bmatrix} \beta_1^e \\ \beta_2^e \end{bmatrix} e_t + \begin{bmatrix} \beta_1^c \\ \beta_2^c \end{bmatrix} + \\ \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}_i \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}_{i,t-1} &+ \begin{bmatrix} b_1^e \\ b_2^e \end{bmatrix}_i e_t + \begin{bmatrix} b_1^c \\ b_2^c \end{bmatrix}_i + \begin{bmatrix} \boldsymbol{\epsilon}_{1i} \\ \boldsymbol{\epsilon}_{2i} \end{bmatrix}_t .\end{aligned}$$

Given that the above equation is true for every time point, we stack the mood values, parameters and covariates related to the day i by defining

$$\boldsymbol{y}_i = \begin{bmatrix} s_{1,1} & s_{1,2} & \dots & s_{1,n_i} & s_{2,1} & s_{2,2} & \dots & s_{2,n_i} \end{bmatrix}_i^T$$

and

$$\boldsymbol{\beta} = \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{21} & \beta_{22} & \beta_1^e & \beta_2^e & \beta_1^c & \beta_2^c \end{bmatrix}^T .$$

Consequently by defining $\boldsymbol{b}_i$ and $\boldsymbol{\epsilon}_i$ in a similar way, the matrix representation for every day becomes

$$\boldsymbol{y}_i = \boldsymbol{X}_i\boldsymbol{\beta} + \boldsymbol{Z}_i\boldsymbol{b}_i + \boldsymbol{\epsilon}_i, \tag{5}$$

in which $\boldsymbol{X}_i$ and $\boldsymbol{Z}_i$ are design matrices for fixed and random effects of the form (Funatogawa & Funatogawa, 2018)

$$\begin{bmatrix} s_{1,0} & s_{2,0} & 0 & 0 & 1 & e_1 & 0 & 0 \\ s_{1,1} & s_{2,1} & 0 & 0 & 1 & e_2 & 0 & 0 \\ \vdots & \vdots & & & & \vdots & & \\ s_{1,n_i-1} & s_{2,n_i-1} & 0 & 0 & 1 & e_{n_i} & 0 & 0 \\ 0 & 0 & s_{1,0} & s_{2,0} & 0 & 0 & 1 & e_1 \\ 0 & 0 & s_{1,1} & s_{2,1} & 0 & 0 & 1 & e_2 \\ \vdots & \vdots & \vdots & \vdots & & & & \vdots \\ 0 & 0 & s_{1,n_i-1} & s_{2,n_i-1} & 0 & 0 & 1 & e_{n_i} \end{bmatrix}_i .$$

In its general form, Equation 5 for k moods, $\boldsymbol{y}_i$ is a $k(n_i - 1) \times 1$ vector of mood values; $\boldsymbol{\beta}$ is a $(k^2 + 2k) \times 1$ vector of fixed-effects; $\boldsymbol{b}_i$ is a $(k^2 + 2k) \times 1$ vector of random-effects; $\boldsymbol{X}_i$ and $\boldsymbol{Z}_i$ are $k(n_i - 1) \times (k^2 + 2k)$ design matrices. The random-effects $\boldsymbol{b}_i$ and residuals $\boldsymbol{\epsilon}_i$ are assumed to be independent with a multivariate normal (MVN) distribution of

$$\begin{bmatrix} \boldsymbol{b}_i \\ \boldsymbol{\epsilon}_i \end{bmatrix} \sim MVN\left(\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \boldsymbol{G} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_i \end{bmatrix} \right).$$

3.2 Maximum Likelihood

Any inference in this setting, Equation 5, involves the estimation of fixed effects, $\boldsymbol{\beta}$, and covariance components, $\boldsymbol{G}$, and $\boldsymbol{\Sigma}_i$, and then the prediction of random-effects since $\boldsymbol{b}_i$ are considered as random variables and not parameters. The distributional assumptions on the residual and random effects lead to some restrictions on the observations which at the same time help to formulate the likelihood of data given the parameters and hence establish the likelihood-based estimation methods. The observations in this formulation have the marginal distribution given by

$$\boldsymbol{y}_i \sim MVN(\boldsymbol{X}_i\boldsymbol{\beta}, \boldsymbol{Z}_i\boldsymbol{G}\boldsymbol{Z}_i^T + \boldsymbol{\Sigma}_i).$$

Assuming $\boldsymbol{V}_i = \boldsymbol{Z}_i\boldsymbol{G}\boldsymbol{Z}_i^T + \boldsymbol{\Sigma}_i$ the likelihood function for a single day reads as

$$p(\boldsymbol{y}_i|\boldsymbol{\beta}, \boldsymbol{V}_i) = 2\pi^{-\frac{n}{2}}|\boldsymbol{V}|^{-\frac{1}{2}} \exp\left\{ -\frac{1}{2}(\boldsymbol{y}_i - \boldsymbol{X}_i\boldsymbol{\beta})^T\boldsymbol{V}^{-1}(\boldsymbol{y}_i - \boldsymbol{X}_i\boldsymbol{\beta}) \right\}.$$

In practice, data from all days are pooled and $-2 \times \ln p$ is used to make the estimation procedure feasible,

$$l(\boldsymbol{\beta}, \theta) = \sum_{i=1}^{N} \ln|\boldsymbol{V}_i| + \sum_{i=1}^{N}(\boldsymbol{y}_i - \boldsymbol{X}_i\boldsymbol{\beta})^T\boldsymbol{V}(\theta)^{-1}(\boldsymbol{y}_i - \boldsymbol{X}_i\boldsymbol{\beta}),$$

in which constants are excluded. The covariance matrix is usually parameterized and factorized as $\boldsymbol{V}_i = \boldsymbol{V}_i(\theta) = L^T L$, where L is an upper triangular matrix. This guarantees that the estimated covariance is a positive definite matrix and at the same time it overcomes some computational difficulties as the determinant and inverse is used in the likelihood function.

If the covariance is known, the generalized least square estimator of $\boldsymbol{\beta}$ that minimizes $l(\boldsymbol{\beta}, \theta)$ is

$$\hat{\boldsymbol{\beta}} = \left[\sum_{i=1}^{N} \boldsymbol{X}_i^T\boldsymbol{V}_i^{-1}\boldsymbol{X}_i \right]^{-1} \sum_{i=1}^{N} \boldsymbol{X}_i^T\boldsymbol{V}_i^{-1}\boldsymbol{y}_i,$$

in which a generalized inverse is used if the sum is not invertible. This estimator is asymptotically unbiased, i.e., $E(\hat{\boldsymbol{\beta}}) = \boldsymbol{\beta}$ and follows a multivariate normal distribution if the response variable has a conditional normal distribution (Fitzmaurice et al., 2011), with

$$Cov(\hat{\boldsymbol{\beta}}) = \left[\sum_{i=1}^{N} \boldsymbol{X}_i^T \boldsymbol{V}_i^{-1} \boldsymbol{X}_i \right]^{-1}.$$

The estimation of $\boldsymbol{\beta}$ is substituted in the likelihood function and the result is maximized with respect to $\boldsymbol{V}_i(\theta)$ to give the covariance estimator. This procedure is usually accomplished using iterative methods such as Newton-Raphson or Expectation-Maximization algorithms (Lindstrom & Bates, 1988). It is well known that the maximum likelihood estimators might be biased in finite samples. Therefore restricted maximum likelihood is used to correct for the loss of degrees of freedom resulting from the estimation of $\boldsymbol{\beta}$. After estimating these parameters, the prediction of random-effects follows from their conditional mean given the responses, $\boldsymbol{y}_i$,

$$E(\boldsymbol{b}_i|\boldsymbol{y}_i) = \boldsymbol{G}\boldsymbol{Z}_i^T \boldsymbol{V}_i^{-1}(\boldsymbol{y}_i - \boldsymbol{X}_i\hat{\boldsymbol{\beta}}),$$

and consequently, the so-called best linear unbiased predictor (BLUP) is (Bates et al., 2015; Pinheiro, Bates, DebRoy, & Sarkar, 2013)

$$\hat{\boldsymbol{b}}_i = \hat{\boldsymbol{G}}\boldsymbol{Z}_i^T \hat{\boldsymbol{V}}_i^{-1}(\boldsymbol{y}_i - \boldsymbol{X}_i\hat{\boldsymbol{\beta}}). \tag{6}$$

4 Simulations

In this section, assuming that the mixed-effects system in Equation 4 is the underlying true data generating process, we generate data using preset fixed effects, the variance of random effects, and the number of observations per day. Different combination of noise intensities and the total number of observations in terms of the different number of days is used to cover common scenarios for real-world data sets; the known parameters are called ”true” values. Then using the Julia package *MixedModels.jl* and the R package *brms* whenever credible intervals are needed, we fit LARMEx to these data sets which results in the ”estimated” parameters. In all cases, we simulate realizations from a network of two moods and one node representing external factors and investigate the feasibility and efficacy of this approach in terms of how well the true parameters are recovered.

4.1 Data Generating Process

Let the fixed effects including the matrix of autoregressive coefficients, the vector of coefficients corresponding to the exogenous variables and the constant terms be

$$\boldsymbol{\beta}^{ar} = \begin{bmatrix} 0.3 & -0.3 \\ -0.3 & 0.3 \end{bmatrix}, \quad \boldsymbol{\beta}^{e} = \begin{bmatrix} 0.3 \\ -0.3 \end{bmatrix}, \quad \boldsymbol{\beta}^{c} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{7}$$

which represent a moderately connected mood network of a hypothetical person, and is assumed to be of a trait-like nature meaning that this network structure is preserved over long periods of time like weeks or months. We choose the effects of the exogenous factors, $\boldsymbol{\beta}^e$, to be of the same order of magnitude as the autoregressive coefficients. To keep the trajectories mostly in $[-1, 1]$, the initial values and the intensity of external factors are uniformly chosen from $[-0.5, 0.5]$ and $[0, 0.5]$ respectively. Random effects are chosen to be normally distributed with a mean zero and independent from each other.

To guarantee the stability of autoregressive processes, the elements of $\boldsymbol{\beta}^{ar} + \boldsymbol{b}_i^{ar}$ should be such that all the eigenvalues are smaller than one. This is achieved by generating a large sample compared to the number of days, 20000 in this case, and then retaining all sets for which the eigenvalues meet the stability condition. This step could already be a challenging task in that, likely, the resulting random effects are no longer normally distributed. This becomes even more demanding when we force the network to assume a theoretically more meaningful structure of negative values for edges between opposing nodes, PA and NA, and positive self-loops, and leaves a narrow range for random effects ending up in low variances. Therefore, we relax the latter requirement and allow some edges in the networks to take values of the opposite sign than seen in Figure 3a. The distributions of the sum of fixed and random effects are shown in Figure 5 for which the covariance of random effects at the population level is an 8×8 diagonal matrix, $G = 0.03 \times I_{8\times 8}$, where I is the identity matrix.

With these considerations in 78% of the cases the structure of the network is preserved as Figure 3a, i.e., with the same sign of edges as illustrated by different colors. The noise is assumed to be of the form $\sigma^2 I$ where we run the simulations for low, moderate, and high noise intensities characterized by the variance of noise and signal-to-noise ratio (SNR) as $(\sigma^2, SNR) \in \{(0.01, 12), (0.02, 6), (0.06, 2)\}$ which is valid for the above choice of G. One could get higher SNR values by increasing the variance of random effects, and hence we fix this G throughout the paper. In each case, we calculate SNR as the ratio of the variance of the noiseless signal to that of the noise (Hastie, Tibshirani, & Friedman, 2009). That is, for Equation 4,

$$SNR = \frac{\mathrm{VAR}(\boldsymbol{B}_i^{ar}\boldsymbol{s}_{i,t-1} + \boldsymbol{B}_i^{e}\boldsymbol{e}_{i,t} + \boldsymbol{B}_i^{c})}{\sigma^2}.$$

We avoid transforming the SNR to dB (decibel) units, as it is common in signal processing, because of its unfamiliarity in the field.

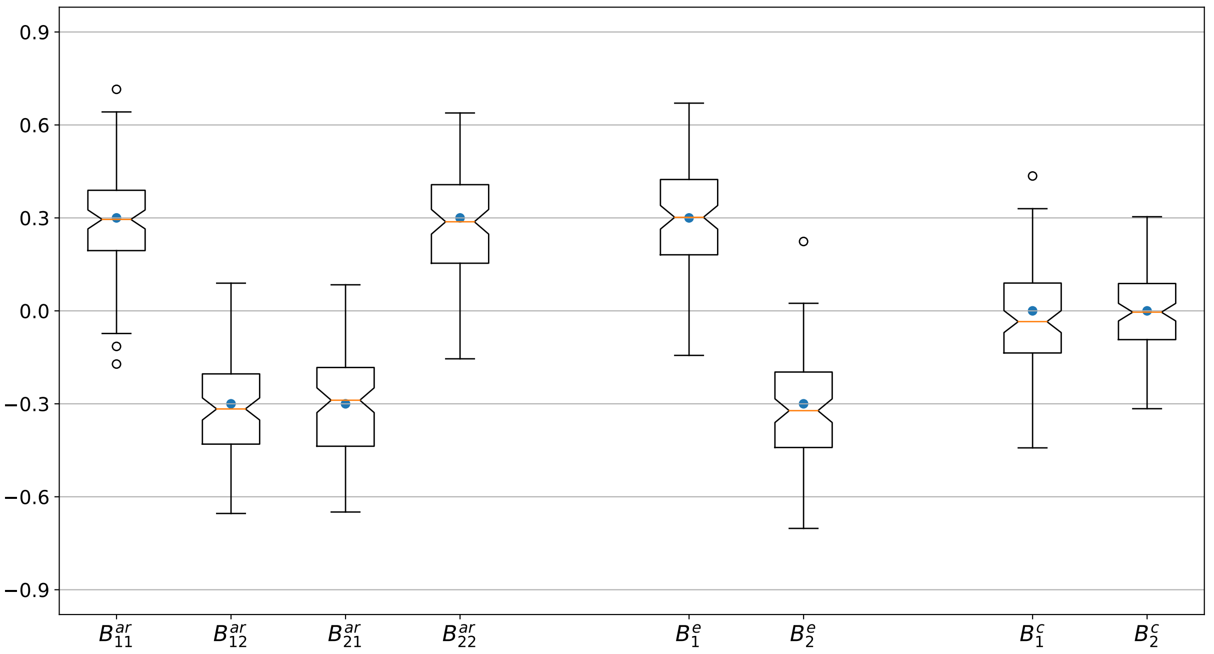

Figure 5: Distributions of the parameters, $\boldsymbol{B}^{\cdots}_{\cdots} = \boldsymbol{\beta}^{\cdots}_{\cdots} + \boldsymbol{b}^{\cdots}_{\cdots}$, sum of fixed and random effects for 200 simulated days. Each boxplot shows the interquartile range (IQR) notched over the median and whiskers extending 1.5IQR beyond the first and third quartiles. Fixed effects are shown by dark points.

This work is part of an ongoing study that aims at collecting EMA data from 250 subjects, 10 observations per day over the first week of six consecutive months [to be cited: Study protocol description: Dynamic Modelling of Resilience - Observational Study (DynaM-OBS)]. Therefore we generate data by choosing the number of days, N, from the set $\{4, 6, \dots, 36\}$ to study the effect of adding more days of EMA in parameter estimation. The mean number of longitudinal observations per day, $\bar{n} = \sum_{i=1}^{N} n_i/N$ has a lower bound to have identifiable random effects. Given the network of k moods and one node representing external factors, one has to predict a total number of $(k^2 + 2k)N$ random effects while there are $k\sum_{i=1}^{N}(n_i - 1)$ observations. A rule-of-thumb identifiability criterion for mixed-effects is that the number of observations should be bigger than the number of random effects (Bates et al., 2015). Consequently, the number of moods in the aforementioned network should be strictly less than $\bar{n} - 3$.

4.2 Parameter Recovery

In order to investigate how well one can recover the parameters, we fit LARMEx to data generated with different combinations of parameters mentioned earlier.

First, we construct bootstrap confidence intervals which are used to quantify the sampling distribution of the estimates. A large number of independent data sets are generated and parameter estimation is performed in every case using *MixedModels.jl* package in Julia. Figure 6 and 7 depict such confidence intervals for fixed effects and variances of random effects respectively. A visual inspection reveals that by adding more days of observations the confidence intervals shrink and their widths decrease for lower noise intensities. It is worth noting that for these levels of SNR they do not differ drastically. Moreover, the parameters corresponding to the exogenous variable are less precise compared to others. One could also use this method for conducting a power analysis by setting an appropriate null hypothesis, e.g. in terms of a cutoff for a certain parameter. Then the percent of the truly rejected null hypothesis, when it is indeed false, would give an approximate power (Kumle, Võ, & Draschkow, 2021).

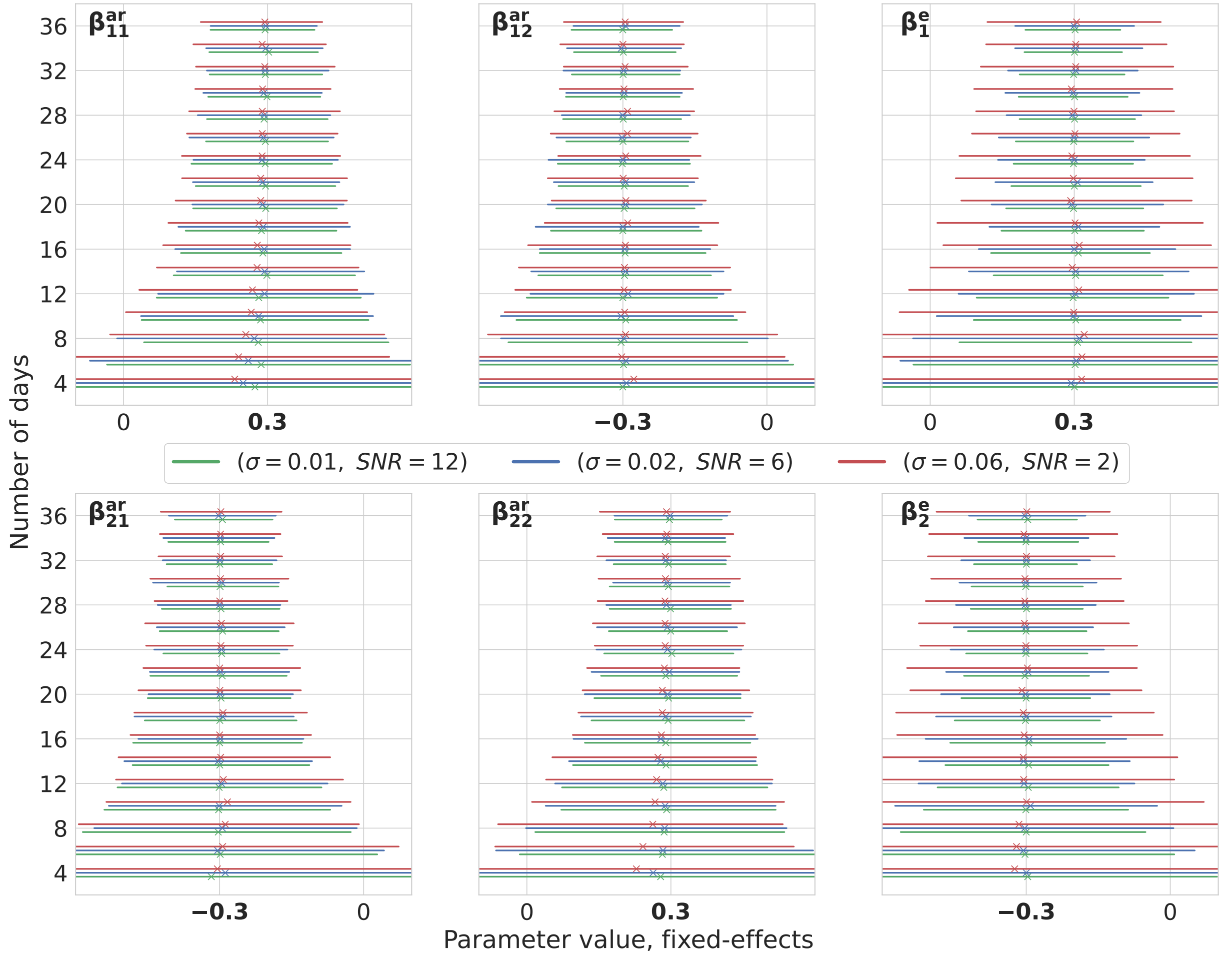

Figure 6: Estimations of fixed effects for different levels of noise intensity and number of days. The number of observations per day is 10. True values of parameters are highlighted bold on x axes. For every number of days at the horizontal axes, three lines are drawn representing the bootstrap 95% confidence intervals around the median depicted by cross signs. Every line is color-coded to demonstrate one noise intensity. Data has been generated for one simulated subject, 1000 times repeatedly, with $(\sigma^2, \text{SNR}) \in \{(0.01, 12), (0.02, 6), (0.06, 2)\}$ and estimations are done using *MixedModels.jl* package in Julia.

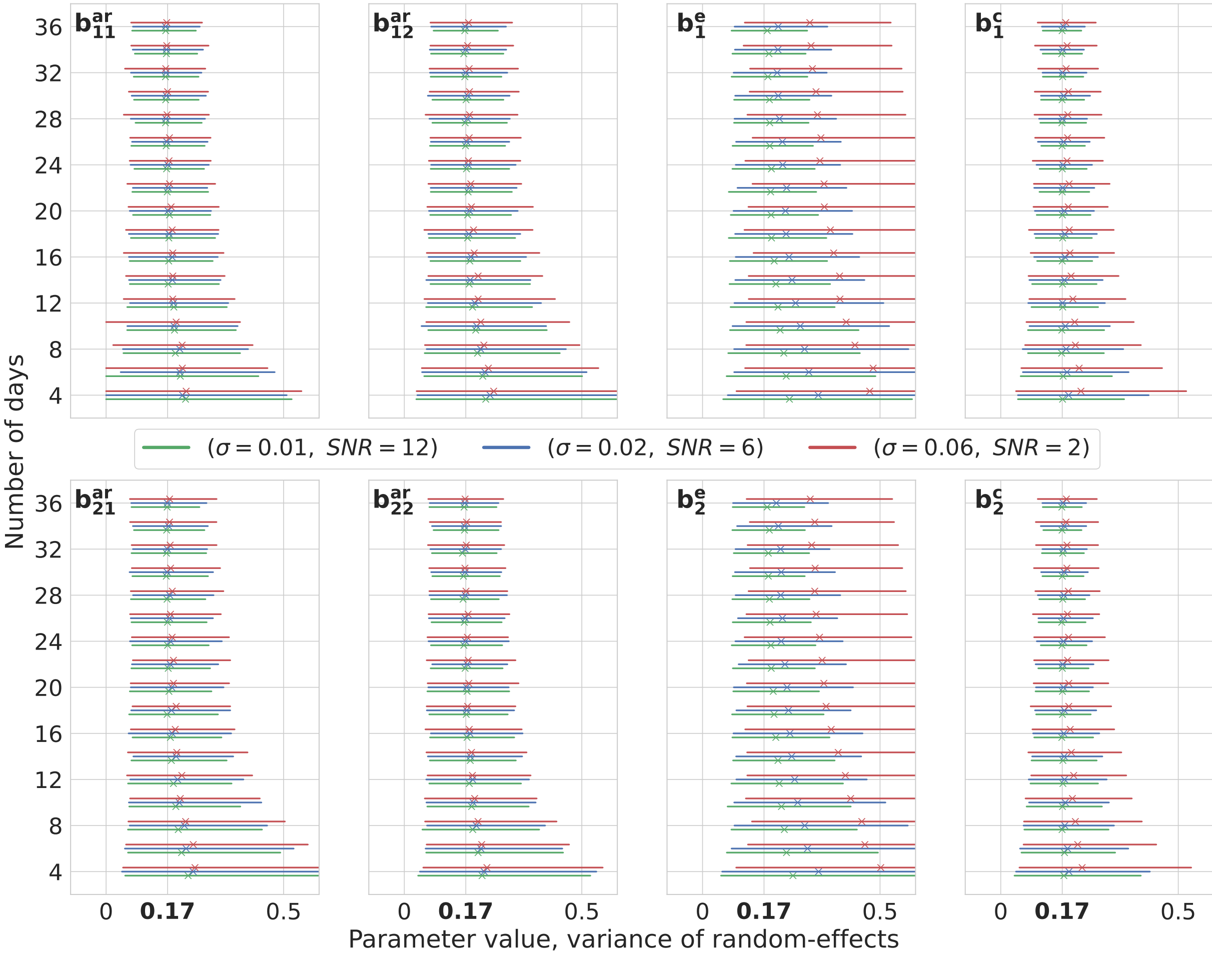

Figure 7: Estimations of the variance of random effects for different levels of noise intensity and number of days. The number of observations per day is 10. True values of parameters are highlighted bold on x axes, $\sqrt{0.03} \approx 0.17$. For every number of days at the vertical axes, three lines are drawn representing the bootstrap 95% confidence intervals around the median depicted by cross signs. Every line is color-coded to demonstrate one noise intensity. Data has been generated for one simulated subject, 1000 times repeatedly, with $(\sigma^2, \text{SNR}) \in \{(0.01, 12), (0.02, 6), (0.06, 2)\}$ and estimations are done using *MixedModels.jl* package in Julia.

Second, we calculate the relative estimation error, δ, in the aforementioned cases as $\delta_\theta = |\hat{\theta} - \theta|/|\theta|$ in which θ and $\hat{\theta}$ are the true and estimated parameters respectively. To avoid computational difficulties only random effects larger than 0.02 are considered in these analyses. We also report only the coefficients present in a network representation, i.e., $\boldsymbol{\beta}^{ar}$, $\boldsymbol{\beta}^{e}$, $\boldsymbol{b}^{ar}$ and $\boldsymbol{b}^{e}$. This is to show that in a mixed effects model the prediction of random effects is not as reliable as fixed effects in terms of being able to recover the parameters varying over days in our case. The reason is that only the variance of random effects are present in the likelihood function and individual values are not estimated directly but only up to their variance. More precisely what one gets as the output of software packages like *MixedModels.jl* in Julia or *lme4* in R holds only for the expected values, Equation **??**, and should not be considered as single parameter estimations. In Figure 8 it is clearly seen that the median relative estimation error for random effects approaches to about 40% as the number of days grows and the change is minimal after 20 days. Whereas, for the fixed effects, the value is below 20% which indicates that with a reasonable amount of data one could hope for even better estimates for the fixed effects but this is still far from being optimal. The two separate solid and dashed lines correspond to the coefficients of the exogenous factors which have poor estimations compared to others, Figures 6 and 7.

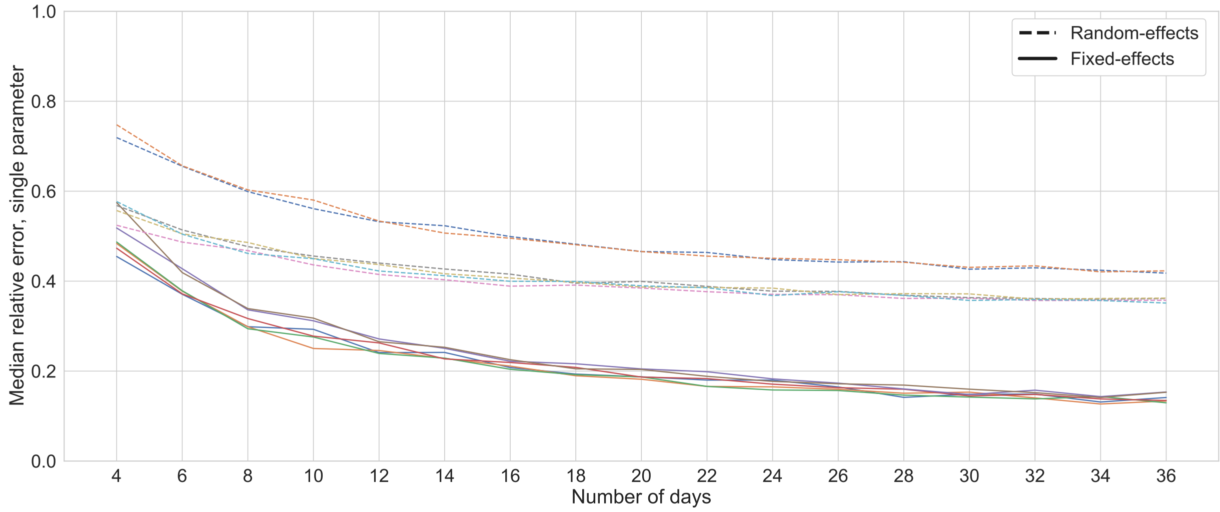

Figure 8: *Mean relative estimation errors.* Solid and dashed lines represent the mean relative estimation errors for the six parameters of a network as in Figure 1c fixed. Data is generated for one simulated subject, 1000 times repeatedly, with $(\sigma^2, \text{SNR}) = (0.02, 6)$ and estimations are done using *MixedModels.jl* package in Julia. The two lines with higher relative errors correspond to exogenous factors.

Third, one data set is generated and the posterior distribution of the parameters are approximated using the *brms* package in R which employs Markov chain Monte Carlo methods and provides point estimates as posterior means together with credible intervals. This way we get an approximate picture of possible results for a specific data set. As one can see in Figure 9 although the credible intervals shrink as more days of observation are added, the point estimates exhibit distorted patterns even for a reasonably large number of days. The lack of convergence is better seen when we summarize relative estimation errors for a single subject in Figure 10.

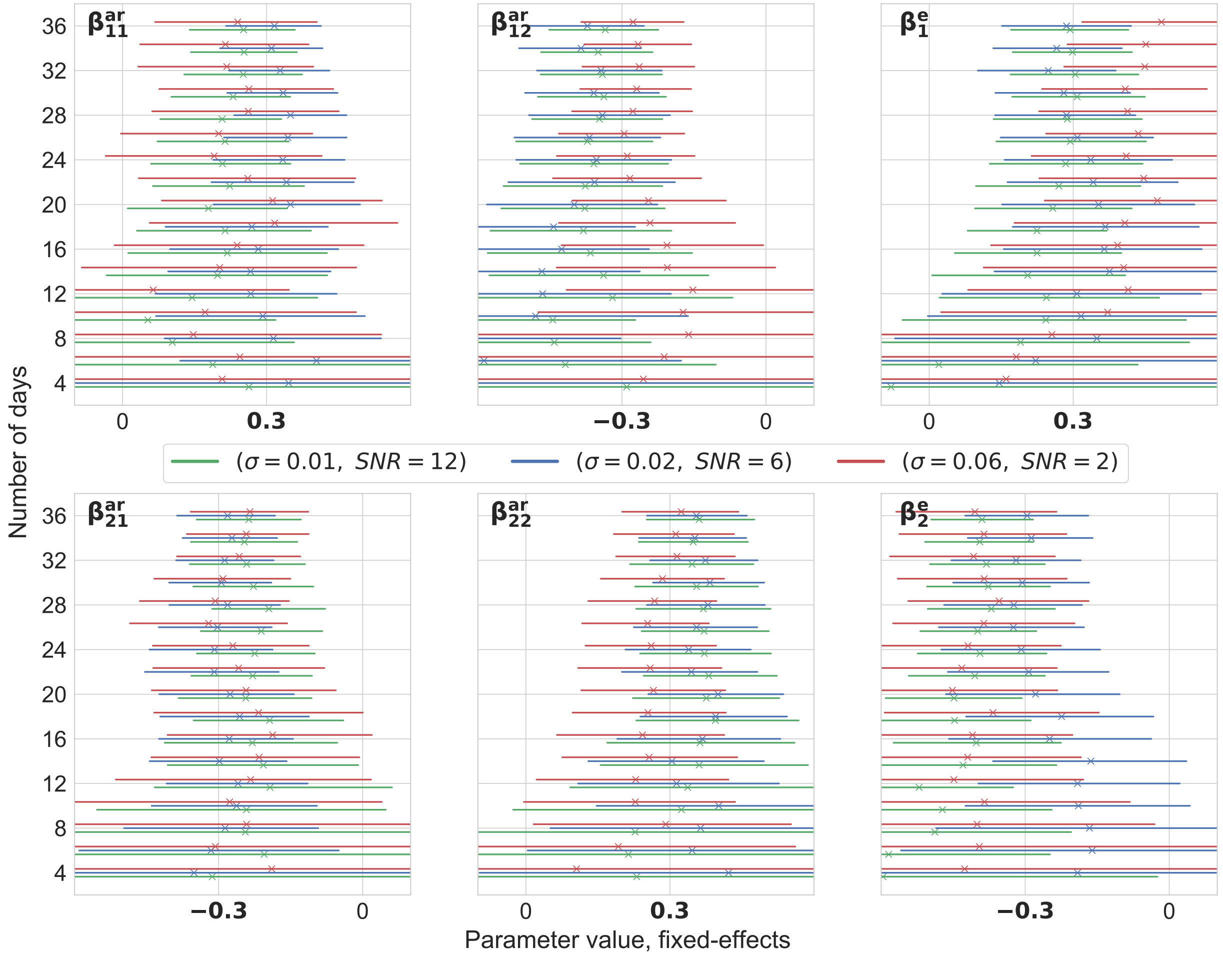

Figure 9: Estimations of fixed effects for different levels of noise intensity and the number of days. The number of observations per day is 10. True values of parameters are highlighted in bold on the horizontal axes. For every number of days at the horizontal axes, three lines are drawn representing the 95% credible intervals around the point estimates, posterior means, depicted by cross signs. Every line is color-coded to demonstrate one noise intensity. Data has been generated for one simulated subject with $(\sigma^2, \text{SNR}) \in \{(0.01, 12), (0.02, 6), (0.06, 2)\}$ and estimations are done using *brms* package in R.

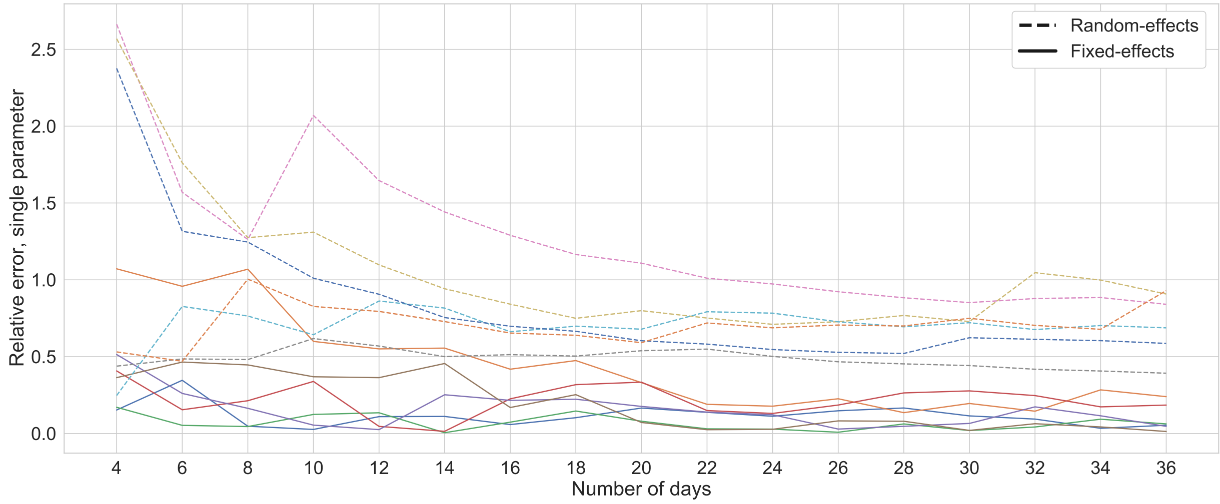

Figure 10: *Relative estimation errors (REE)*. Solid lines are REE for fixed effects and dashed lines represent the mean REE random effects for the six parameters of a network as in Figure 1c. Data is generated for one simulated subject with $(\sigma^2, \mathrm{SNR}) = (0.02, 6)$ and estimations are done using *brms* package in R.

5 Conclusion

In this work, we studied an extension of linear mixed-effects models by adding an autoregressive component to model the network representation of mental state which is suitable to model intensive longitudinal data, the so-called ecological momentary assessments acquired by experience sampling methods. Specifically, we assumed the simplest possible network of two causally interacting state nodes under the influence of external factors represented by a node acting on both states in a contemporaneous fashion.

We showed how this representation is mathematically formulated and discussed its components by generating data in a step-by-step manner. Then it was transformed to a classic linear mixed-effects form and the foundations of parameter estimation by likelihood function were summarized.

This extension has been introduced before as a multilevel level model with daily observations nested in days, days nested in respondents, and respondents nested in a sample of subjects (de Haan-Rietdijk et al., 2016; Schuurman et al., 2016). Given that our target was building individualized networks, we based our study on a two-level model with daily observations nested in days for a specific respondent. Using simulated data, we constructed bootstrap confidence intervals for the fixed effect parameters and showed what one could expect from this model as more days are added to the observation. We also showed the posterior probability distributions of the parameters for data from a single respondent with point estimates and credible intervals. We argued that random effects are not directly estimated in this procedure and highlighted the difference between the relative estimation errors for fixed and random effects which showed that in a mixed model random effects are identified less precisely and as the number of observations increases the gain in precision is very little. Therefore, in order to infer individual networks from ecological momentary assessments, one should build a two-level model with daily observations nested in days for a single respondent.

We mainly considered one aspect of study design related to sample size and varied the number of observations per subject by adding more days. We also showed how sensitive such a model might be to the intensity of noise in data. In a nutshell, although one could represent mental state networks by autoregressive mixed-effects models, estimating individualized parameters seems to remain challenging with common longitudinal designs such that the required number of observations per individual to achieve an acceptable accuracy might not be feasible. Furthermore, autoregressive models are unable to reveal complex characteristics of a dynamical system such as bistable points which are required to study the transition of mental state to and from different stable states like mental disorder and health (Haslbeck & Ryan, 2021; Kalisch et al., 2019).

In this work, we presumed that networks have time-varying characteristics but this time dependence was not formulated explicitly, rather up to their variance. Nevertheless, it is legitimate to assume that the interactions or the way external forces act on states, change over time in a specific way. A simple time dependency might be achieved by adding slopes to the model which capture the linear time trend of the corresponding parameter, $[\beta_{...} + b_{...}]_{slo}$. Another possibility is that one lets the parameters follow a random walk $\boldsymbol{\theta}_t = \boldsymbol{\theta}_{t-1} + \eta_t$, or the variance-covariance structures to be time-dependent, $\Sigma_t = F_t D_t F_t'$. Monte Carlo Markov Chain or state-space techniques are usually used for estimating such models (Primiceri, 2005; Sommerlade et al., 2012).

In a more realistic setting, one could assume that external forces act not only on states but also on their interactions in an autoregressive way. This assumption results in having arrows pointing to some edges in the network representation. The following equation then needs to be added to Equation 4, resulting in a nonlinear system,

$$\boldsymbol{B}^{ar}_{i,t} = \boldsymbol{\alpha}^{ar}_i \boldsymbol{B}^{ar}_{i,t-1} + \boldsymbol{\alpha}^{e}_i \boldsymbol{e}_{i,t} + \boldsymbol{\alpha}^{c}_i + \boldsymbol{\eta}_{i,t},$$

in which $\boldsymbol{\eta}_i$ are assumed to be white noises and mutually uncorrelated at all leads and lags to other sources of variability in the model. This formulation implies time-varying autoregressive coefficients. All these methods as treated in the literature require long time series to achieve acceptable results and the feasibility of mixed-effects modeling needs further work to be addressed.

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