Modern Introduction to Flavor Physics and Weak & Strong CP Violation

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Abstract

This review provides a modern overview of flavor physics and weak & strong CP violation, highlighting their structure within and beyond the Standard Model (SM) and their connection to the precision frontier of ultralight dark matter. In the SM, flavor mixing and CP violation arise uniquely from the Yukawa couplings through the Cabibbo–Kobayashi–Maskawa framework. Using a symmetry-based and spurion-oriented approach, the SM's predictive flavor structure and the suppression of flavor-changing neutral currents via the Glashow–Iliopoulos–Maiani mechanism are clarified. Precision measurements from kaon and B-meson systems, together with LHC results, confirm that all observed weak CP violation is consistent with a single CKM phase. The LHC has also provide direct evidences that the Higgs mechanism generates fermion masses, linking electroweak symmetry breaking to the origin of flavor. Beyond the SM, effective field theory analyses show that flavor data place stringent constraints on new physics, and discussing viable frameworks such as minimal flavor violation. The review concludes with the strong CP problem and its relation to axions and ultralight dark matter, discussing how new experimental approaches—ranging from atomic clocks to quantum sensors—extend the search for new physics across the frontiers of flavor, CP violation, and the precision frontiers.

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1 Flavor physics & weak CP violation

1.1 Prologue, structure of matter an existential question

For thousands of years humans were interested in the question "what are we made of ?", or perhaps rephrasing it to a more modern form asking what are the building block of nature? For laymen (not familiar with the wonders of the relativistic-quantum microscopic world) it sounds as if these questions are equivalent to ask what are the smallest particles currently known. However, in the context of quantum mechanics, and quantum field theory, this question might be a bit subtle. We know, for instance, that the size of atoms is quantised, and has negligible dependence on the size of the nuclei. Atom size is basically inversely proportional to the mass of the electron. Therefore, it is the electron mass, in addition to the rules of quantum mechanics, that essentially controls all form of elements known to us. Furthermore, the mass of the nuclei, protons and neutrons, and their size, cannot be simply obtained by "adding up" the "smaller" degrees of freedom that is it made form. It is rather dictated by the fact that the coupling constant of the strong force changes with energy scale and at some specific low energy scale, commonly denoted as $\Lambda_{\rm QCD}$, it becomes so big such that, the force between two particle becomes approximately independent of their distance, inducing confinement. Therefore, the size of the nuclei (say protons and neutrons) is basically inversely proportional to $\Lambda_{\rm QCD}$, which is fully dictated by the physics of quantum chromo dynamics (QCD) and the gluons, the messengers of the strong force.

The above description might give the impression that all that is required to understand the basic structure of the elements is to understand the origin of the electron mass and to study QCD which controls the mass and size of the proton and neutrons. This misses an essential aspects of the story as the whole structure of the existing elements is associated with the structure of the nucleus, consist of the protons and neutrons, the light long-lived/stable baryons. These light baryons, from low energy perspective, are the lightest solitonic-excitations of the pion field (see for instance [1]), which are constructed only from the ultra-light up and down quarks, (u and d), and perhaps also strange one s. These states are protected by a U(1) symmetry and therefore can be a result of a net-baryonic charge created at the early universe. Naively this concludes the discussion and suggests that to understand the structure of matter it is sufficient just to divide them to the heavy and light ones. This, however, would be wrong as to get the rich abundance of stable elements, requires subtle balance among the different components of nuclear physics. One might be surprised that nuclear dynamics has a strong sensitivity to the light quark masses. After all, the up and down quark masses are more than an order of magnitude below the QCD scale. However, the nuclear forces, spectrum, and binding energies are in general highly non-trivial functions of the QCD parameters. For instance, increasing the up-down quark mass difference by less than 10 MeV (that is, on the order of 1% of the proton mass) would make hydrogen and its isotopes unstable [2, 3, 4]. To demonstrate this we show on in Fig. 1 (based on [3]) the regions where elements similar to the ones in our universe can be formed (in green), with two stable baryons participating in nuclei, regions which do not allow for elements to be formed (in red) and regions where one cannot make a clear statement in white. The plot, the sum of light quark masses, $m_T = m_u + m_d + m_s$, is varied together with $\Lambda_{\rm QCD}$ such that the average mass of the two lightest baryons is held fixed to its observed value (940 MeV). For a fixed value of m_T , light quark masses may be represented by the points in the interior of an equilateral triangle with altitude of m_T . As we know, we have six quarks (three up-type and three

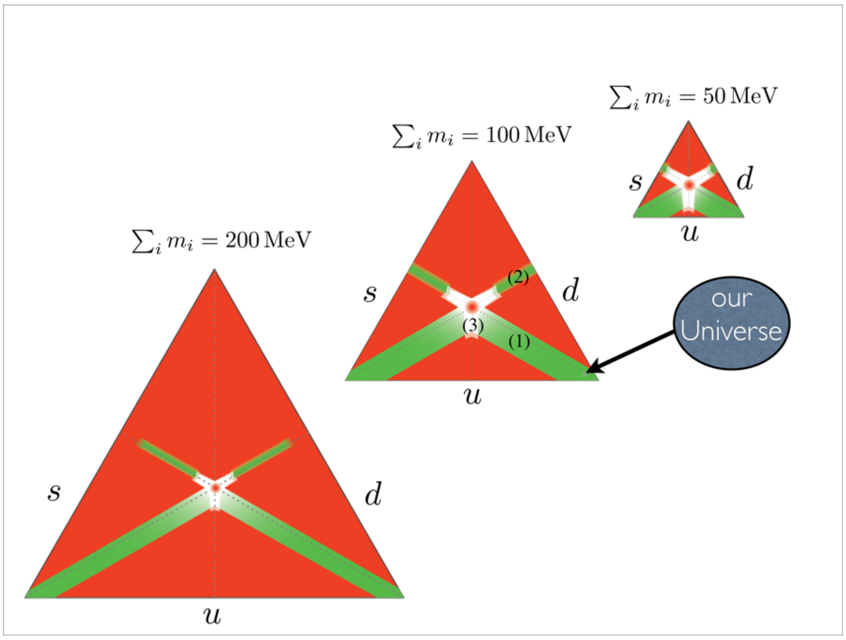

Figure 1: Graphical presentation of the parameter space examined by Jaffe, Jenkins, and Kimchij [**?**,] the triangle altitude corresponds to the sum of quark masses $m_T = \sum_{i=u,d,s} m_i$. The quark mass is given by the perpendicular distance from the corresponding side. Our Universe corresponds to the point marked by the arrow on the middle triangle. The green (red) bands represent congenial (uncongenial) worlds and the white region stands for region where the uncertainty is too large to draw conclusions.

down-type), making more quarks lighter render a too complicated system to analyse but surely would result in a structure different then the observed one in an essential manner.

Now that the readers are hopefully convinced that understanding the structure of masses of the elementary particles is an existentialistic issue we are motivated to understand what is the origin of the fermion masses. What does it have to do with the discovery of the Higgs and the confirmation of it ruling in electroweak symmetry breaking (EWSB)? Well, in principle, not much, if you look at technicolor models, where symmetry breaking is induced by strong dynamics, or if you think about the canonical description of superconductivity where effectively electromagnetism is broken by a condensate of Cooper-pairs, there is no clear linkage between the source of symmetry breaking and the induction of fundamental fermion masses. However, in the case of the SM Higgs mechanism one can make an economical choice, and couple the Higgs to the fundamental fermion and via this linking the field behind EWSB to the origin of fermion masses. Within the Standard Model (SM) of elementary particles and fundamental interactions, in its most minimalistic version, this implies switching-on at least[1] nine coupling three for the up-type quarks (u, c, t), three for the down-type quarks (d, s, b), and additional three for the charged leptons (e, μ, τ), nine couplings of the three generation, the flavor sector of the SM. Each coupling is associated with linear, Yukawa, coupling of the Higgs to two fermions

Currently the LHC experiments have managed to established that the Higgs mechanism is behind the the origin of masses of all the third generation fermions, in addition to the muon. While the above result is very significant, it tells us very little about the origin

[1]in principle, the charged-fermion sector contains at least four more physical parameters associated with quark-mixing and CP violation.

of the matter in the universe which is almost exclusively made out of fermions of the first generation (with some possible contributions from the strange quark), which are associated with extremely small coupling to the Higgs. On this front we are amidst of reaching another milestone as follows. We have indirect information that link the origin of mass of the first and second generations. Thus, collecting evidences that the Higgs mechanism is also behind the second generation masses is another important step in our understanding of the origin of masses.The above discussion was regarding the direct information that we have obtained regarding the Higgs mechanism and the origin of masses.

We end this part by going back to discuss existentialism, now in the context of the Higgs coupling to matter. There are two interesting aspects of the Higgs Yukawa couplings related to our existence. The first, associated with the first generation masses, as was pointed in [2], is that increasing the Higgs VEV (or its negative mass) by a factor of few would render all nuclei unstable preventing the existing of structure. This was used to claim that the negative Higgs mass is bounded from above addressing the hierarchy problem [5] (however it was argued in [6, 7] that probably this argument is not robust). The second, associated with the top quark, is that apparently the SM top-Yukawa is close to its maximal size, and increasing it by order 10% would render our universe unstable [8].

We note that all the above discussion was focused about our direct knowledge regarding the origin of flavor within the SM Higgs mechanism, indirectly, however, we have gathered huge body of information from low energy measurements, supporting the SM picture of the origin of masses, mixing and CP violation, which is rather unique. In order to understand this better we move now to discuss flavor physics within the SM.

1.2 Introduction, flavor physics

Flavors are replications of states with identical quantum numbers. The standard model (SM) consists of three such replications of the five fermionic representations of the SM gauge group. Flavor physics describes the non-trivial spectrum and interactions of the flavor sector. What makes this field particularly interesting is that the SM flavor sector is rather unique, and its special characteristics make it testable and predictive.[2] Let us list few of the SM unique flavor predictions:

- Predicts liner relation between the Higgs couplings and the charged fermion masses (with the slope predicted by the Higgs mechanism).
- It contains a single CP violating parameter.[3]
- Flavor conversion is driven by three mixing angles.
- To leading order, flavor conversion proceeds through weak charged current interactions.
- To leading order, flavor conversion involves left handed (LH) currents.
- CP violating processes must involve all three generations.

[2]This set of lectures discusses the strong sector sector only. Many of the concepts that are explained here can be directly applied to the lepton sector.

[3]We further discuss below the other source of CP violation within the SM, the strong CP phase.

- The dominant flavor breaking is due to the top Yukawa coupling, hence the SM possesses a large approximate global flavor symmetry (as shown below, technically it is given by $U(2)_Q \times U(2)_U \times U(1)_t \times U(3)_D$).

In the last four decades or so, a huge effort was invested towards testing the SM predictions related to its flavor sector. Due to the success of the B factories, the field of flavor physics has made a dramatic progress, culminated in Kobayashi and Maskawa winning the Nobel prize. It is now established that the SM contributions drive the observed flavor and CP violation (CPV) in nature, via the Cabibbo-Kobayashi-Maskawa (CKM) [9, 10] description. In addition, within the neutral D meson system the SM contribution to the CP violation in the mixing amplitude is expected to be below the permil level, which was verified experimentally. Furthermore, as already mentioned above, the LHC experiments have provided us with direct information related to the Higgs Yukawa interactions giving a strong direct support for the SM flavor picture.

Well, we have just given several rather solid arguments for the validity of the SM flavor description. What else is there to say then? Could this be the end of the story? We have several important reasons to think that flavor physics will continue to play a significant role in our understanding of microscopical physics at and beyond the reach of current colliders. Let us first mention a few examples that demonstrate the role that flavor precision tests played in the past:

- The smallness of $\Gamma(K_L \to \mu^+\mu^-)/\Gamma(K^+ \to \mu^+\nu)$ led to predicting a fourth quark (the charm) via the discovery of the GIM mechanism.
- The size of the mass difference in the neutral Kaon system, Δm_K, led to a successful prediction of the charm mass.
- The size of Δm_B led to a successful prediction of the top mass (for a review see [11] and refs. therein).
- The mass of the top was linked to bracketing the Higgs mass-range.

This partial list demonstrates the power of flavor precision tests in terms of being sensitive to short distance dynamics. Even in view of the SM successful flavor story, it is likely that there are missing experimental and theoretical ingredients, as follows:

- Within the SM, as mentioned, there is a single CP violating parameter. We shall see that the unique structure of the SM flavor sector implies that CP violating phenomena are highly suppressed. Baryogenesis, which requires a sizable CP violating source, therefore cannot be accounted for by the SM CKM phase. Measurements of CPV in flavor changing processes might provide evidence for additional sources coming from short distance physics.
- The SM flavor parameters are hierarchical, and most of them are small (excluding the top Yukawa and the CKM phase), which is denoted as the flavor puzzle. This peculiarity might stem from unknown flavor dynamics. Though it might be related to very short distance physics, we can still get indirect information about its nature via combinations of flavor precision and high p_T measurements.
- The SM fine tuning problem, which is related to the quadratic divergence of the Higgs mass, generically requires new physics at, or below, the TeV scale. If such new

physics has a generic flavor structure, it would contribute to flavor changing neutral current (FCNC) processes orders of magnitude above the observed rates. Putting it differently, the flavor scale at which NP is allowed to have a generic flavor structure is required to be larger than $\mathcal{O}(10^5)$ TeV, in order to be consistent with flavor precision tests. Since this is well above the electroweak symmetry breaking scale, it implies an "intermediate" hierarchy puzzle (*cf.* the little hierarchy [12] problem). We use the term "puzzle" and not "problem" since in general, the smallness of the flavor parameters, even within NP models, implies the presence of approximate symmetries. One can imagine, for instance, a situation where the suppression of the NP contributions to FCNC processes is linked with the SM small mixing angles and small light quark Yukawas [13]. In such a case, this intermediate hierarchy is resolved in a technically natural way, or radiatively stable manner, and no fine tuning is required.

1.3 The standard model flavor sector

The SM quarks furnish three representations of the SM gauge group, $SU(3) \times SU(2) \times U(1)$: $Q(3,2)_{\frac{1}{6}} \times U(3,1)_{\frac{2}{3}} \times D(3,1)_{-\frac{1}{3}}$, where Q, U, D stand for $SU(2)$ weak doublet, up type and down type singlet quarks, respectively. Flavor physics is related to the fact that the SM consists of three replications/generations/flavors of these three representations. The flavor sector of the SM is described via the following part of the SM Lagrangian

$$\mathcal{L}^{\rm F} = \overline{q^i} \not{D}\, q^j \delta_{ij} + (Y_U)_{ij} \overline{Q^i} U^j H_U + (Y_D)_{ij} \overline{Q^i} D^j H_D + \text{h.c.}\,, \tag{1}$$

where $\not{D} \equiv D_\mu \gamma^\mu$ with D_μ being a covariant derivative, $q = Q, U, D$, within the SM with a single Higgs $H_U = i\sigma_2 H_D^*$ (however, the reader should keep in mind that at present, the nature and content of the SM Higgs sector is unknown) and $i, j = 1, 2, 3$ are flavor indices.

If we switch off the Yukawa interactions, the SM would possess a large global flavor symmetry, $\mathcal{G}^{\rm SM}$,[4]

$$\mathcal{G}^{\rm SM} = U(3)_Q \times U(3)_U \times U(3)_D\,. \tag{2}$$

Inspecting Eq. (1) shows that the only non-trivial flavor dependence in the Lagrangian is in the form of the Yukawa interactions. It is encoded in a pair of 3×3 complex matrices, $Y_{U,D}$.

1.3.1 The SM quark flavor parameters

Naively one might think that the number of the SM flavor parameters is given by $2 \times 9 = 18$ real numbers and $2 \times 9 = 18$ imaginary ones, the elements of $Y_{U,D}$. However, some of the parameters which appear in the Yukawa matrices are unphysical. A simple way to see that (see *e.g.* [14] and refs. therein) is to use the fact that a flavor basis transformation,

$$Q \to V_Q Q\,, \qquad U \to V_U U\,, \qquad D \to V_D D\,, \tag{3}$$

leaves the SM Lagrangian invariant, apart from redefinition of the Yukawas,

$$Y_U \to V_Q Y_U V_U^\dagger\,, \qquad Y_D \to V_Q Y_D V_D^\dagger\,, \tag{4}$$

[4] At the quantum level, a linear combination of the diagonal $U(1)$'s inside the $U(3)$'s, which corresponds to the axial current, is anomalous.

where V_i is a 3×3 unitary rotation matrix. Each of the three rotation matrices $V_{Q,U,D}$ contains three real parameters and six imaginary ones (the former ones correspond to the three generators of the $SO(3)$ group, and the latter correspond to the remaining six generators of the $U(3)$ group). We know, however, that physical observables do not depend on our choice of basis. Hence, we can use these rotations to eliminate unphysical flavor parameters from $Y_{U,D}$. Out of the 18 real parameters, we can remove 9 (3×3) ones. Out of the 18 imaginary parameters, we can remove 17 ($3 \times 6 - 1$) ones. We cannot remove all the imaginary parameters, due to the fact that the SM Lagrangian conserves a $U(1)_B$ symmetry.[5] Thus, there is a linear combination of the diagonal generators of $\mathcal{G}^{\rm SM}$ which is unbroken even in the presence of the Yukawa matrices, and hence cannot be used in order to remove the extra imaginary parameter.

An explicit calculation shows that the 9 real parameters correspond to 6 masses and 3 CKM mixing angles, while the imaginary parameter corresponds to the CKM celebrated CPV phase. To see that, we can define a mass basis where $Y_{U,D}$ are both diagonal. This can be achieved by applying a bi-unitary transformation on each of the Yukawas:

$$Q^{u,d} \to V_{Q^{u,d}} Q^{u,d}\,, \qquad U \to V_U U\,, \qquad D \to V_D D\,, \tag{5}$$

which leaves the SM Lagrangian invariant, apart from redefinition of the Yukawas,

$$Y_U \to V_{Q^u} Y_U V_U^\dagger\,, \qquad Y_D \to V_{Q^d} Y_D V_D^\dagger\,. \tag{6}$$

The difference between the transformations used in Eqs. (3) and (4) and the ones above (5,6), is in the fact that each component of the $SU(2)$ weak doublets (denoted as $Q^u \equiv U_L$ and $Q^d \equiv D_L$) transforms independently. This manifestly breaks the $SU(2)$ gauge invariance, hence such a transformation makes sense only for a theory in which the electroweak symmetry is broken. This is precisely the case for the SM, where the masses are induced by spontaneous electroweak symmetry breaking via the Higgs mechanism. Applying the above transformation amounts to "moving" to the mass basis. The SM flavor Lagrangian, in the mass basis, is given by (in a unitary gauge),

$$\begin{aligned}\mathcal{L}_m^{\rm F} = \left(\overline{q^i} D\!\!\!/ q^j \delta_{ij}\right)_{\rm NC} + \left(\overline{u_L}\,\overline{c_L}\,\overline{t_L}\right) \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix} \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} (v+h) + (u,c,t) \leftrightarrow (d,s,b) \\ + \frac{g_2}{\sqrt{2}} \overline{u_{Li}} \gamma^\mu V_{ij}^{\rm CKM} d_{Lj} W_\mu^+ + \text{h.c.},\end{aligned} \tag{7}$$

where the subscript NC stands for neutral current interaction for the gluons, the photon and the Z gauge bosons, $W^\pm$ stands for the charged electroweak gauge bosons, h is the physical Higgs field, $v \sim 176\,\text{GeV}$, $m_i = y_i v$ and $V^{\rm CKM}$ is the CKM matrix

$$V^{\rm CKM} = V_{Q^u} V_{Q^d}^\dagger\,. \tag{8}$$

In general, the CKM is a 3×3 unitary matrix, with 6 imaginary parameters. However, as evident from Eq. (7), the charged current interactions are the only terms which are not invariant under individual quark vectorial $U(1)^6$ field redefinitions,

$$u_i, d_j \to e^{i\theta_{u_i,d_j}}\,. \tag{9}$$

[5] More precisely, only the combination $U(1)_{B-L}$ is non-anomalous.

The diagonal part of this transformation corresponds to the classically conserved baryon current, while the non-diagonal, $U(1)^5$, part of the transformation can be used to remove 5 out of the 6 phases, leaving the CKM matrix with a single physical phase. Notice also that a possible permutation ambiguity for ordering the CKM entries is removed, given that we have ordered the fields in Eq. (7) according to their masses, light fields first. This exercise of explicitly identifying the mass basis rotation is quite instructive, and we have already learned several important issues regarding how flavor is broken within the SM (we shall derive the same conclusions using a spurion analysis in a symmetry oriented manner in Sec. 1.4):

- Flavor conversions only proceed via the three CKM mixing angles.
- Flavor conversion is mediated via the charged current electroweak interactions.
- The charge current interactions only involve LH fields.

Even after removing all the unphysical parameters, there are various possible forms for the CKM matrix. For example, a parameterization used by the particle data group [15], is given by

$$V^{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta^{\rm KM}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta^{\rm KM}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta^{\rm KM}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta^{\rm KM}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta^{\rm KM}} & c_{23}c_{13} \end{pmatrix}, \tag{10}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. The three $\sin\theta_{ij}$ are the three real mixing parameters, while $\delta^{\rm KM}$ is the Kobayashi-Maskawa phase.

1.3.2 Weak CP violation

The SM predictive power picks up once CPV is considered. We have already proven that the SM flavor sector contains a single CP violating parameter. Once presented with a SM Lagrangian where the Yukawa matrices are given in a generic basis, it is not trivial to determine whether CP is violated or not. This is even more challenging when discussing beyond the SM dynamics, where new CP violating sources might be present. A brute force way to establish that CP is violated would be to show that no field redefinitions would render a Lagrangian real. For example, consider a Lagrangian with a single Yukawa matrix,

$$\mathcal{L}^Y = Y_{ij}\overline{\psi^i_L}\phi\psi^j_R + Y^*_{ij}\overline{\psi^j_R}\phi^\dagger\psi^i_L, \tag{11}$$

where ϕ is a scalar and ψ^i_X is a fermion field. A CP transformation exchanges the operators

$$\overline{\psi^i_L}\phi\psi^j_R \leftrightarrow \overline{\psi^j_R}\phi^\dagger\psi^i_L, \tag{12}$$

but leaves their coefficients, Y_{ij} and Y^*_{ij}, unchanged, since CP is a linear unitary non-anomalous transformation. This means that CP is conserved if

$$Y_{ij} = Y^*_{ij}\,. \tag{13}$$

This is, however, not a basis independent statement. Since physical observables do no depend on a specific basis choice, it is enough to find a basis in which the above relation holds.[6]

[6]It is easy to show that in this example, in fact, CP is not violated for any number of generations.

Sometimes the brute force way is tedious and might be rather complicated. A more systematic approach would be to identify a phase reparameterization invariant or basis independent quantity, that vanishes in the CP conserving limit. For the SM, one can define the following quantity

$$C^{\rm SM} = \det[Y_D Y_D^\dagger, Y_U Y_U^\dagger]\,, \tag{14}$$

and the SM is CP violating if and only if

$$\mathrm{Im}\left(C^{\rm SM}\right) \neq 0. \tag{15}$$

It is trivial to prove that only if the number of generations is three or more, then CP is violated. Hence, within the SM, where CP is broken explicitly in the flavor sector, any CP violating process must involve all three generations. This is an important condition, which implies strong predictive power. Furthermore, all the CPV observables are correlated, since they are all proportional to a single CP violating parameter, $\delta^{\rm KM}$. Finally, it is worth mentioning that CPV observables are related to interference between different processes, and hence are measurements of amplitude ratios. Thus, in various known cases, they turn out to be cleaner and easier to interpret theoretically.

1.3.3 The flavor puzzle

Now that we have precisely identified the SM physical flavor parameters, it is interesting to ask what is their experimental value (using $\overline{\rm MS}$) [15]:

$$\begin{aligned}
m_u &= 1.5..3.3\,{\rm MeV}\,,\ m_d = 3.5..6.0\,{\rm MeV}\,,\ m_s = 150^{+30}_{-40}\,{\rm MeV}\,,\\
m_c &= 1.3\,{\rm GeV}\,,\ m_b = 4.2\,{\rm GeV}\,,\ m_t = 170\,{\rm GeV}\,,\\
\left|V_{ud}^{\rm CKM}\right| &= 0.97\,,\ \left|V_{us}^{\rm CKM}\right| = 0.23\,,\ \left|V_{ub}^{\rm CKM}\right| = 3.8\times 10^{-3}\,,\\
\left|V_{cd}^{\rm CKM}\right| &= 0.23\,,\ \left|V_{cs}^{\rm CKM}\right| = 0.97\,,\ \left|V_{cb}^{\rm CKM}\right| = 41\times 10^{-3}\,,\\
\left|V_{td}^{\rm CKM}\right| &= 8.6\times 10^{-3}\,,\ \left|V_{ts}^{\rm CKM}\right| = 41\times 10^{-3}\,,\ \left|V_{tb}^{\rm CKM}\right| = 1.0\,,\ \delta^{\rm KM} = 1.47\pm 0.026\,,
\end{aligned}$$

where $V_{ij}^{\rm CKM}$ corresponds to the magnitude of the ij entry in the CKM matrix, $\delta^{\rm KM}$ is the CKM phase, only uncertainties bigger than 10% are shown, numbers are shown to a 2-digit precision and the $V_{ti}^{\rm CKM}$ entries involve indirect information (a detailed description and refs. can be found in [15]).

Inspecting the actual numerical values for the flavor parameters given in Eq. (16), shows a peculiar structure. Most of the parameters, apart from the top mass and the CKM phase, are small and hierarchical. The amount of hierarchy can be characterized by looking at two different classes of observables:

- Hierarchies between the masses, which are not related to flavor converting processes – as a measure of these hierarchies, we can just estimate what is the size of the product of the Yukawa coupling square differences (in the mass basis)

$$2^6\,\frac{\left(m_t^2-m_c^2\right)\left(m_t^2-m_u^2\right)\left(m_c^2-m_u^2\right)\left(m_b^2-m_s^2\right)\left(m_b^2-m_d^2\right)\left(m_s^2-m_d^2\right)}{v^{12}} = \mathcal{O}\left(10^{-17}\right)\,, \tag{16}$$

with v being the Higgs VEV.

- Hierarchies in the mixing which mediate flavor conversion – this is related to the tiny misalignment between the up and down Yukawas; one can quantify this effect in a basis independent fashion as follows. A CP violating quantity, associated with $V^{\rm CKM}$, that is independent of parametrization, $J_{\rm KM}$, is defined through

$$\begin{aligned} &\mathrm{Im}\big[V_{ij}^{\rm CKM}V_{kl}^{\rm CKM}\big(V_{il}^{\rm CKM}\big)^*\big(V_{kj}^{\rm CKM}\big)^*\big] = J_{\rm KM}\sum_{m,n=1}^{3}\epsilon_{ikm}\epsilon_{jln} = \\ &= c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13}\sin\delta^{\rm KM} \simeq \lambda^6 A^2\eta = \mathcal{O}\left(10^{-5}\right), \end{aligned} \tag{17}$$

  where $i,j,k,l = 1,2,3$. We see that even though $\delta^{\rm KM}$ is of order unity, the resulting CP violating parameter is small, as it is "screened" by small mixing angles. If any of the mixing angles is a multiple of $\pi/2$, then the SM Lagrangian becomes real. Another explicit way to see that Y_U and Y_D are quasi aligned is via the Wolfenstein parametrization of the CKM matrix, where the four mixing parameters are (λ, A, ρ, η), with $\lambda = |V_{us}| = 0.23$ playing the role of an expansion parameter:

$$V^{\rm CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)\,, \tag{18}$$

  where for consistency of the order of expansion to order λ^4, we shall use below, $\bar\rho, \bar\eta$ which are obtained upon the relation $(\bar\rho + i\bar\eta) = (\rho + i\eta)(1-\lambda^2/2) + \mathcal{O}\left(\lambda^4\right)$.

As we shall discuss further below, both kinds of hierarchies described in the bullets lead to suppression of CPV. Thus, a nice way to quantify the amount of hierarchies, both in masses and mixing angles, is to compute the value of the reparameterization invariant measure of CPV introduced in Eq. (14)

$$C^{\rm SM} = J_{\rm KM}\, 2^6\, \frac{\left(m_t^2-m_c^2\right)\left(m_t^2-m_u^2\right)\left(m_c^2-m_u^2\right)\left(m_b^2-m_s^2\right)\left(m_b^2-m_d^2\right)\left(m_s^2-m_d^2\right)}{v^{12}} = \mathcal{O}\big(10^{-22}\big). \tag{19}$$

This tiny value of $C^{\rm SM}$ that characterizes the flavor hierarchy in nature would be of order 10% in theories where $Y_{U,D}$ are generic order one complex matrices. The smallness of $C^{\rm SM}$ is something that many flavor models beyond the SM try to address. Furthermore, SM extensions that have new sources of CPV tend not to have the SM built-in CP screening mechanism. As a result, they give too large contributions to the various observables that are sensitive to CP breaking. Therefore, these models are usually excluded by the data, which is, as mentioned, consistent with the SM predictions.

Why we associate the hierarchical structure of the SM flavor sector with a puzzle rather than a problem? This is related to the idea of naturalness, or more precisely technical-naturalness as formulated by 't-Hooft [16]. This is an important concept in QFT and EFT, which is not the focus of these lectures but yet plays role in our understanding of flavor and CP violation, in the context of the SM Higgs mechanism and BSM, and latter we come back to it when we describe the strong CP problem. Thus, we briefly describe it here, a parameter is technically natural if when you set it to zero the symmetry of the theory is enhanced. Technically natural parameters are subject to multiplicative running and therefore admit a simple relation between the UV/short-distance/microscopic description of the theory and its low energy description that is typically the only part which is accessible to us as observers in experiments. This is in contrast to unnatural parameters that are subject to additive running and therefore obscure the relation between the UV and IR of the theory. In a sense as 't-Hooft mention only natural theories allow us to have a simple relation between the UV of the theory (where we would like to define our model) and the IR (where we test it), and therefore generalize the concept of causality. In general, it is also quite simple to build models that address the smallness of natural parameters and thus they seem to be generic, while to build a model which account for the smallness of unnatural parameters is very challenging. In the SM we have two (possibly three including the strong CP problem, see later), unnatural parameters, the mass of the Higgs (or the scale of electroweak symmetry breaking) and the cosmological constant. For the former we have only two microscopic theoretical frameworks that explains their smallness (SUSY and composite-Higgs and maybe possibly the relaxion) while for the latter we basically have none apart from Weinberg's anthropic principle. As we already discuss in the SM the flavor parameters are related to breaking of the flavor symmetries, already mentioned, and thus being technically natural parameters. The flavor puzzle can be address in variety ways for instance using the Froggatt-Nielsen mechanism [17] or the idea of split-fermions in extra dimension, warped [18, 13] or not [19], which is probably dual to solving it using strong dynamics.

1.4 Spurion analysis of the SM flavor sector

In this part we shall try to be more systematic in understanding the way flavor is broken within the SM. We shall develop a spurion, symmetry-oriented description for the SM flavor structure, and also generalize it to NP models with similar flavor structure, that goes under the name minimal flavor violation (MFV).

1.4.1 Understanding the SM flavor breaking

It is clear that if we set the Yukawa couplings of the SM to zero, we restore the full global flavor group, $\mathcal{G}^{\rm SM} = U(3)_Q \times U(3)_U \times U(3)_D$. In order to be able to better understand the nature of flavor and CPV within the SM, in the presence of the Yukawa terms, we can use a spurion analysis as follows. Let us formally promote the Yukawa matrices to spurion fields, which transform under $\mathcal{G}^{\rm SM}$ in a manner that makes the SM invariant under the full flavor group (see *e.g.* [20, 21, 22] and refs. therein). From the flavor transformation given in Eqs. (3,4), we can read the representation of the various fields under $\mathcal{G}^{\rm SM}$ (see illustration in Fig. 2)

$$\begin{aligned} \text{Fields}: &\ \ Q(\mathbf{3},1,1),\ U(1,\mathbf{3},1),\ D(1,1,\mathbf{3})\,; \\ \text{Spurions}: &\ \ Y_U(\mathbf{3},\bar{\mathbf{3}},1),\ Y_D(\mathbf{3},1,\bar{\mathbf{3}})\,. \end{aligned} \tag{20}$$

The flavor group is broken by the "background" value of the spurions $Y_{U,D}$, which are bi-fundamentals of $\mathcal{G}^{\rm SM}$. It is instructive to consider the breaking of the different flavor groups separately (since $Y_{U,D}$ are bi-fundamentals, the breaking of quark doublet and singlet flavor groups are linked together, so this analysis only gives partial information to be completed below). Consider the quark singlet flavor group, $U(3)_U \times U(3)_D$, first. We can construct a polynomial of the Yukawas with simple transformation properties under the flavor group. For instance, consider the objects

$$A_{U,D} \equiv Y_{U,D}^\dagger Y_{U,D} - \frac{1}{3}{\rm tr}\left(Y_{U,D}^\dagger Y_{U,D}\right) \mathbb{1}_3\,. \tag{21}$$

Under the flavor group $A_{U,D}$ transform as

$$A_{U,D} \to V_{U,D} A_{U,D} V^\dagger_{U,D} \,. \tag{22}$$

Thus, $A_{U,D}$ are adjoints of $U(3)_{U,D}$ and singlets of the rest of the flavor group [while $\mathrm{tr}(Y^\dagger_{U,D} Y_{U,D})$ are flavor singlets]. Via similarity transformation, we can bring $A_{U,D}$ to a diagonal form, simultaneously. Thus, we learn that the background value of each of the Yukawa matrices separately breaks the $U(3)_{U,D}$ down to a residual $U(1)^3_{U,D}$ group, as illustrated in Fig. 3.

$$\mathcal{G}^{\rm SM} = U(3)_Q \times U(3)_U \times U(3)_D$$

$$Y_U(\mathbf{3},\bar{\mathbf{3}},1)\,, \qquad Y_D(\mathbf{3},1,\bar{\mathbf{3}})$$

Figure 2: The SM flavor symmetry breaking by the Yukawa matrices.

$$U(3)_U \overset{Y_U^\dagger Y_U}{\Rightarrow} U(1)^3_U \qquad\qquad U(3)_D \overset{Y_D^\dagger Y_D}{\Rightarrow} U(1)^3_D$$

Figure 3: Breaking of the $U(3)_{U,D}$ groups by the Yukawa matrices, which form an appropriate LH (RH) flavor group singlet (adjoint+singlet).

Let us now discuss the breaking of the LH flavor group. We can, in principle, apply the same analysis for the LH flavor group, $U(3)_Q$, via defining the adjoints (in this case we have two independent ones),

$$A_{Q^u,Q^d} \equiv Y_{U,D} Y^\dagger_{U,D} - \frac{1}{3}\mathrm{tr}\left(Y_{U,D} Y^\dagger_{U,D}\right) \mathbb{1}_3 \,. \tag{23}$$

However, in this case the breaking is more involved, since $A_{Q^{u,d}}$ are adjoints of the same flavor group. This is a direct consequence of the $SU(2)$ weak gauge interaction, which relates the two components of the $SU(2)$ doublets. This actually motivates one to extend the global flavor group as follows. If we switch off the electroweak interactions, the SM global flavor group is actually enlarged to [23]

$$\mathcal{G}^{\rm SM}_{\rm weakless} = U(6)_Q \times U(3)_U \times U(3)_D \,, \tag{24}$$

since now each $SU(2)$ doublet, Q_i , can be split into two independent flavors, $Q_i^{u,d}$, with identical $SU(3) \times U(1)$ gauge quantum numbers [6]. This limit, however, is not very illuminating, since it does not allow for flavor violation at all. To make a progress, it is instructive to distinguish the W^3 neutral current interactions from the $W^\pm$ charged current ones, as follows: The W^3 couplings are flavor universal, which, however, couple up and down quarks separately. The $W^\pm$ couplings, $g_2^\pm$, link between the up and down LH quarks. In the presence of only W^3 couplings, the residual flavor group is given by[7]

$$\mathcal{G}^{\rm SM}_{\rm exten} = U(3)_{Q^u} \times U(3)_{Q^d} \times U(3)_U \times U(3)_D \,. \tag{25}$$

In this limit, even in the presence of the Yukawa matrices, flavor conversion is forbidden. We have already seen explicitly that only the charged currents link between different flavors (see Eq. (7)). It is thus evident that to formally characterize flavor violation, we can extend the flavor group from $\mathcal{G}^{\rm SM} \to \mathcal{G}^{\rm SM}_{\rm exten}$, where now we break the quark doublets to their isospin components, U_L, D_L, and add another spurion, $g_2^\pm$

$$\begin{aligned} \text{Fields}: &\ \ U_L(\mathbf{3},1,1,1),\ D_L(1,\mathbf{3},1,1),\ U(1,1,\mathbf{3},1),\ D(1,1,1,\mathbf{3}) \\ \text{Spurions}: &\ \ g_2^\pm(\mathbf{3},\bar{\mathbf{3}},1,1),\ Y_U(\mathbf{3},1,\bar{\mathbf{3}},1),\ Y_D(1,\mathbf{3},1,\bar{\mathbf{3}})\,. \end{aligned} \tag{26}$$

Flavor breaking within the SM occurs only when $\mathcal{G}^{\rm SM}_{\rm exten}$ is fully broken via the Yukawa background values, but also due to the fact that $g_2^\pm$ has a background value. Unlike $Y_{U,D}$, $g_2^\pm$ is a special spurion in the sense that its eigenvalues are degenerate, as required by the weak gauge symmetry. Hence, it breaks the $U(3)_{Q^u} \times U(3)_{Q^d}$ down to a diagonal group, which is nothing but $U(3)_Q$. We can identify two bases where $g_2^\pm$ has an interesting background value: The weak interaction basis, in which the background value of $g_2^\pm$ is simply a unit matrix[8]

$$\left(g_2^\pm\right)_{\rm int} \propto \mathbb{1}_3 \,, \tag{27}$$

and the mass basis, where (after removing all unphysical parameters) the background value of $g_2^\pm$ is the CKM matrix

$$\left(g_2^\pm\right)_{\rm mass} \propto V^{\rm CKM} \,. \tag{28}$$

Now we are in a position to understand the way flavor conversion is obtained in the SM. Three spurions must participate in the breaking: $Y_{U,D}$ and $g_2^\pm$. Since $g_2^\pm$ is involved, it is clear that generation transitions must involve LH charged current interactions. These transitions are mediated by the spurion backgrounds, A_{Q^u,Q^d} (see Eq. (23)), which characterize the breaking of the individual LH flavor symmetries,

$$U(3)_{Q^u} \times U(3)_{Q^d} \to U(1)^3_{Q^u} \times U(1)^3_{Q^d} \,. \tag{29}$$

Flavor conversion occurs because of the fact that in general we cannot diagonalize simultaneously A_{Q^u,Q^d} and $g_2^\pm$, where the misalignment between A_{Q^u} and A_{Q^d} is precisely characterized by the CKM matrix. This is illustrated in Fig. 4, where it is shown that the flavor breaking within the SM goes through collective breaking [21] – a term often used in the context of little Higgs models (see *e.g.* [24] and refs. therein). We can now combine the LH and RH quark flavor symmetry breaking to obtain the complete picture of how flavor is broken within the SM. As we saw, the breaking of the quark singlet groups is rather trivial. It is, however, linked to the more involved LH flavor breaking, since the Yukawa matrices are bi-fundamentals – the LH and RH flavor breaking are tied together. The full breaking is illustrated in Fig. 5.

[7] To get to this limit formally, one can think of a model where the Higgs field is an adjoint of $SU(2)$ and a singlet of color and hypercharge. In this case the Higgs vacuum expectation value (VEV) preserves a $U(1)$ gauge symmetry, and the W^3 would therefore remain massless. However, the $W^\pm$ will acquire masses of the order of the Higgs VEV, and therefore charged current interactions would be suppressed.

[8] Note that the interaction basis is not unique, given that $g_2^\pm$ is invariant under a flavor transformation where Q^u and Q^d are rotated by the same amount – see more in the following.

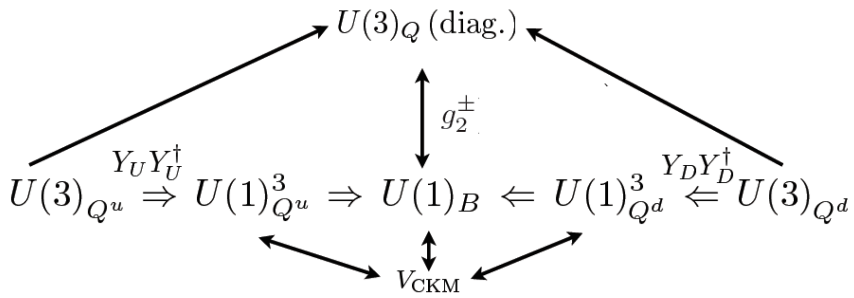

Figure 4: $U(3)_{Q^u,Q^d}$ breaking by A_{Q^u,Q^d} and $g_2^\pm$.

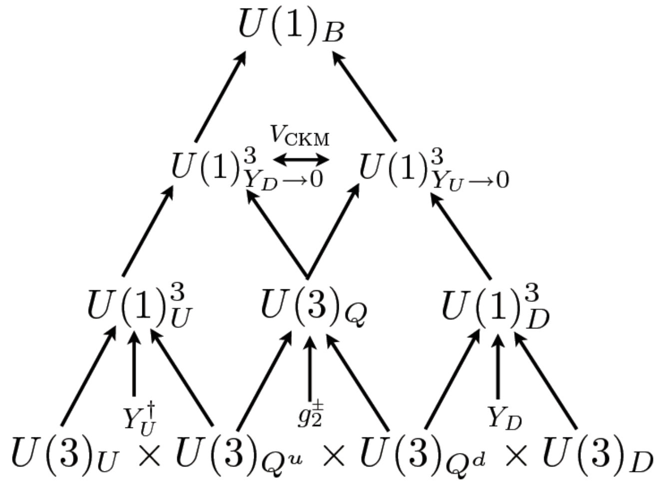

Figure 5: The schematic structure of the various ingredients that mediate flavor breaking within the SM.

1.4.2 A comment on description of flavor conversion in physical processes

The above spurion structure allows us to describe SM flavor converting processes. However, the reader might be confused, since we have argued above that flavor converting processes must involve the three spurions, $A_{Q^{u,d}}$ and $g_2^\pm$. It is well known that the rates for charge current processes, which are described via conversion of down quark to an up one (and vise a versa), say like beta decay or $b \to u$ transitions, are only suppressed by the corresponding CKM entry, or $g_2^\pm$. What happened to the dependence on $A_{Q^{u,d}}$? The key point here is that in a typical flavor precision measurement, the experimentalists produce mass eigenstate (for example a neutron or a B meson), and thus the fields involved are chosen to be in the mass basis. For example, a $b \to c$ process is characterized by producing a B meson which decays into a charmed one. Hence, both A_{Q^u} and A_{Q^d} participate, being forced to be diagonal, but in a nonlinear way. Physically, we can characterize it by writing an operator

$$\mathcal{O}_{b\to c} = \bar{c}_{\rm mass} \left(g_2^\pm\right)^{cb}_{\rm mass} b_{\rm mass}\,, \tag{30}$$

where both the $b_{\rm mass}$ and $c_{\rm mass}$ quarks are mass eigenstate. Note that this is consistent with the transformation rules for the extended gauge group, $\mathcal{G}^{\rm SM}_{\rm exten}$, given in Eqs. (25) and (26), where the fields involved belong to different representations of the extended flavor group.

The situation is different when FCNC processes are considered. In such a case, a typical measurement involves mass eigenstate quarks belonging to the same representation of $\mathcal{G}^{\rm SM}_{\rm exten}$. For example, processes that mediate $B^0_d - \overline{B}^0_d$ oscillation due to the tiny mass difference Δm_{B_d} between the two mass eigenstates (which was first measured by the ARGUS experiment), are described via the following operator, omitting the spurion structure for simplicity,

$$\mathcal{O}_{\Delta m_{B_d}} = \left(\bar{b}_{\rm mass}\, d_{\rm mass}\right)^2 \,. \tag{31}$$

Obviously, this operator cannot be generated by SM processes, as it is violates the $\mathcal{G}^{\rm SM}_{\rm exten}$ symmetry explicitly. Since it involves flavor conversion (it violates b number by two units, hence denoted as $\Delta b = 2$ and belongs to $\Delta F = 2$ class of FCNC processes), it must have some power of $g_2^{\pm}$. A single power of $g_2^{\pm}$ connects a LH down quark to a LH up one, so the leading contribution should go like $\bar{D}^i_L \left(g_2^{\pm}\right)^{ik} \left(g_2^{\pm *}\right)^{kj} D^j_L$ $(i,k,j = 1,2,3)$. Hence, as expected, this process is mediated at least via one loop. This would not work as well, since we can always rotate the down quark fields into the mass basis, and simultaneously rotate also the up type quarks (away from their mass basis) so that $g_2^{\pm} \propto \mathbb{1}_3$. These manipulations define the interaction basis, which is not unique (see Eq. (27)). Therefore, the leading flavor invariant spurion that mediates FCNC transition would have to involve the up type Yukawa spurion as well. A naive guess would be

$$\begin{aligned} \mathcal{O}_{\Delta m_{B_d}} &\propto \left[\bar{b}_{\rm mass} \left(g_2^{\pm}\right)^{bk}_{\rm mass} \left(A_{Q^u}\right)_{kl} \left(g_2^{\pm *}\right)^{ld}_{\rm mass} d_{\rm mass}\right]^2 \\ &\sim \left\{\bar{b}_{\rm mass} \left[m_t^2 V^{\rm CKM}_{tb} \left(V^{\rm CKM}_{td}\right)^* + m_c^2 V^{\rm CKM}_{cb} \left(V^{\rm CKM}_{cd}\right)^*\right] d_{\rm mass}\right\}^2 \,, \end{aligned} \tag{32}$$

where it is understood that $\left(A_{Q^u}\right)_{kl}$ is evaluated in the down quark mass basis (tiny corrections of order m_u^2 are neglected in the above). This expression captures the right flavor structure, and is correct for a sizeable class of SM extensions. However, it is actually incorrect in the SM case. The reason is that within the SM, the flavor symmetries are strongly broken by the large top quark mass [21]. The SM corresponding amplitude consists of a rather non-trivial and non-linear function of A_{Q^u}, instead of the above naive expression (see *e.g.* [25] and refs. therein), which assumes only the simplest polynomial dependence of the spurions. The SM amplitude for Δm_{B_d} is described via a box diagram, and two out of the four powers of masses are canceled, since they appear in the propagators.

1.4.3 The SM approximate symmetry structure

In the above we have considered the most general breaking pattern. However, as discussed, the essence of the flavor puzzle is the large hierarchies in the quark masses, the eigenvalues of $Y_{U,D}$ and their approximate alignment. Going back to the spurions that mediate the SM flavor conversions defined in Eqs. (21) and (23), we can write them as

$$\begin{aligned} A_{U,D} &= {\rm diag}\left(0,0,y^2_{t,b}\right) - \frac{y^2_{t,b}}{3}\mathbb{1}_3 + \mathcal{O}\left(\frac{m^2_{c,s}}{m^2_{t,b}}\right) \,, \\ A_{Q^u,Q^d} &= {\rm diag}\left(0,0,y^2_{t,b}\right) - \frac{y^2_{t,b}}{3}\mathbb{1}_3 + \mathcal{O}\left(\frac{m^2_{c,s}}{m^2_{t,b}}\right) + \mathcal{O}\left(\lambda^2\right) \,, \end{aligned} \tag{33}$$

where in the above we took advantage of the fact that $m^2_{c,s}/m^2_{t,b}, \lambda^2 = \mathcal{O}\left(10^{-5,-4,-2}\right)$ are small. The hierarchies in the quark masses are translated to an approximate residual RH

$U(2)_U \times U(2)_D$ flavor group (see Fig. 6), implying that RH currents which involve light quarks are very small.

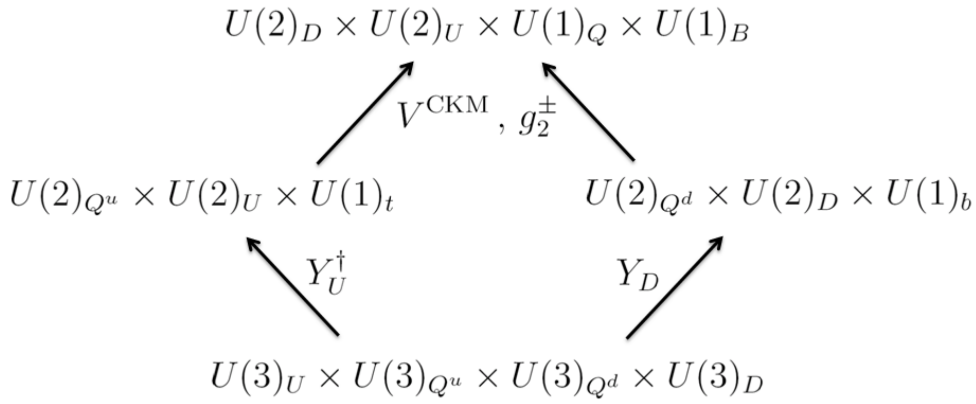

Figure 6: The approximate flavor symmetry breaking pattern. Note that there is also a residual $U(1)_Q$ symmetry, as explained in Sec. **??**.

We have so far only briefly discussed the role of FCNCs. In the above we have argued, both based on an explicit calculation and in terms of a spurion analysis, that at tree level there are no flavor violating neutral currents, since they must be mediated through the $W^{\pm}$ couplings or $g_2^{\pm}$. In fact, this situation, which is nothing but the celebrated GIM mechanism, goes beyond the SM to all models in which all LH quarks are $SU(2)$ doublets and all RH ones are singlets. The Z boson might have flavor changing couplings in models where this is not the case.

Can we guess what is the leading spurion structure that induces FCNC within the SM, say which mediates the $b \to d\nu\bar{\nu}$ decay process via an operator $O_{b\to d\nu\bar{\nu}}$? The process changes b quark number by one unit (belongs to $\Delta F = 1$ class of FCNC transitions). It clearly has to contain down type LH quark fields (let us ignore the lepton current, which is flavor-trivial). Therefore, using the argument presented when discussing Δm_{B_d} (see Eq. (32)), the leading flavor invariant spurion that mediates FCNC would have to involve the up type Yukawa spurion as well

$$O_{b\to d\nu\bar{\nu}} \propto \bar{D}^i_L g^{\pm}_{2\;ik} \left(A_{Q^u}\right)_{kl} g^{\pm *}_{2\;lj} D^j_L \times \bar{\nu}\nu \,. \tag{34}$$

The above considerations demonstrate how the GIM mechanism removes the SM divergencies from various one loop FCNC processes, which are naively expected to be log divergent. The reason is that the insertion of A_{Q^u} is translated to quark mass difference insertion. It means that the relevant one loop diagram has to be proportional to $m_i^2 - m_j^2$ $(i \neq j)$. Thus, the superficial degree of divergency is lowered by two units, which renders the amplitude finite.[9] Furthermore, as explained above (see also Eq. (37)), we can use the fact that the top contribution dominates the flavor violation to simplify the form of $O_{b\to d\nu\bar{\nu}}$

$$O_{b\to d\nu\bar{\nu}} \sim \frac{g_2^4}{16\pi^2 M_W^2}\, \bar{b}_L V^{\rm CKM}_{tb} \left(V^{\rm CKM}_{td}\right)^* d_L \times \bar{\nu}\nu \,, \tag{35}$$

[9]For simplicity, we only consider cases with hard GIM, in which the dependence on mass differences is polynomial. There is a large class of amplitudes, for example processes that are mediated via penguin diagrams with gluon or photon lines, where the quark mass dependence is more complicated, and may involve logarithms. The suppression of the corresponding amplitudes goes under the name soft GIM [25].

where we have added a one loop suppression factor and an expected weak scale suppression. This rough estimation actually reproduces the SM result up to a factor of about 1.5 (see *e.g.* [25]).

We thus find that down quark FCNC amplitudes are expected to be highly suppressed due to the smallness of the top off-diagonal entries of the CKM matrix. Parameterically, we find the following suppression factor for transition between the ith and jth generations:

$$\begin{aligned} b \to s &\propto \left|V_{tb}^{\rm CKM} V_{ts}^{\rm CKM}\right| \sim \lambda^2 \,, \\ b \to d &\propto \left|V_{tb}^{\rm CKM} V_{td}^{\rm CKM}\right| \sim \lambda^3 \,, \\ s \to d &\propto \left|V_{td}^{\rm CKM} V_{ts}^{\rm CKM}\right| \sim \lambda^5 \,, \end{aligned} \tag{36}$$

where for the $\Delta F = 2$ case one needs to simply square the parametric suppression factors. This simple exercise illustrates how powerful is the SM FCNC suppression mechanism. The gist of it is that the rate of SM FCNC processes is small, since they occur at one loop, and more importantly due to the fact that they are suppressed by the top CKM off-diagonal entries, which are very small. Furthermore, since

$$\left|V_{ts,td}^{\rm CKM}\right| \gg \frac{m_{c,u}^2}{m_t^2} \,, \tag{37}$$

in most cases the dominant flavor conversion effects are expected to be mediated via the top Yukawa coupling.[10]

We can now understand how the SM uniqueness related to suppression of flavor converting processes arises:

- RH currents for light quarks are suppressed due to their small Yukawa couplings (them being light).
- Flavor transition occurs to leading order only via LH charged current interactions.
- To leading order, flavor conversion is only due to the large top Yukawa coupling.

1.5 The Success of the SM-CKM picture of flavor and CP violation

In this part we briefly describe how the B-factories have done the work to establish the validity of the CKM description. We begin by describing briefly how the matching form the UV, say weak-scale perturbative scale is done down to the hadronic scale where the measurement are done, and then we proceed to describe the unitary triangle, the test associated to it and show the current experimental status.

1.5.1 From short distance physics to observables

In order to derive bounds on the microscopic dynamics, say in this section of the SM weak Lagrangian, one needs to take into account the fact that the experimental input is usually given at the energy scale in which the measurement is performed, while the bound is

[10]This is definitely correct for CP violating processes, or any ones which involve the third generation quarks. It also generically holds for new physics MFV models. Within the SM, for CP conserving processes which involve only the first two generations, one can find exceptions, for instance when considering the Kaon and D meson mass differences, $\Delta m_{D,K}$.

presented at some other scale (say at the weak scale). Moreover, the contributing higher dimension operators mix, in general. Finally, all such processes include long distance contributions (that is, interactions at the hadronic level) in actual experiments. Therefore, a careful treatment of all these effects is required. We do not attempt to provide a comprehensive and detailed description of the whole flavor and CP violating interactions arising from the SM weak interaction (for this we refer the readers to other reviews, see for instance [25]), but rather focus on one concrete sector of four-fermion operators violating the flavor charge by two units. These are useful both to understand how the SM flavor picture is tested and in order to obtain some of the most important constraints on BSM physics.

A complete set of four quark operators relevant for $\Delta F = 2$ transitions is given by

$$\begin{aligned} Q_1^{q_i q_j} &= \bar{q}_{jL}^{\alpha} \gamma_\mu q_{iL}^{\alpha} \bar{q}_{jL}^{\beta} \gamma_\mu q_{iL}^{\beta} \,, \\ Q_2^{q_i q_j} &= \bar{q}_{jR}^{\alpha} q_{iL}^{\alpha} \bar{q}_{jR}^{\beta} q_{iL}^{\beta} \,, \\ Q_3^{q_i q_j} &= \bar{q}_{jR}^{\alpha} q_{iL}^{\beta} \bar{q}_{jR}^{\beta} q_{iL}^{\alpha} \,, \\ Q_4^{q_i q_j} &= \bar{q}_{jR}^{\alpha} q_{iL}^{\alpha} \bar{q}_{jL}^{\beta} q_{iR}^{\beta} \,, \\ Q_5^{q_i q_j} &= \bar{q}_{jR}^{\alpha} q_{iL}^{\beta} \bar{q}_{jL}^{\beta} q_{iR}^{\alpha} \,, \end{aligned} \tag{38}$$

where i, j are generation indices and α, β are color indices. There are also operators $\tilde{Q}_{1,2,3}^{q_i q_j}$, which are obtained from $Q_{1,2,3}^{q_i q_j}$ by the exchange $L \leftrightarrow R$, and the results given for the latter apply to the former as well.

The Wilson coefficients of the above operators, $C_i(\Lambda)$, are obtained in principle by integrating out all new particles at the NP scale. Then they have to be evolved down to the hadronic scales. We denote the Wilson coefficients at the relevant hadronic scale, which are the measured observables, as

$$M_{12} = \langle \overline{M} | \mathcal{L}_{\text{eff}} | M \rangle \,,$$

where M represents a meson (note that amplitude or the matrix element, M_{12}, has dimension of [mass]). These should be functions of the Wilson coefficients at the NP scale, $C_i(\Lambda)$, the running of α_s between the NP and the hadronic scales and the hadronic matrix elements of the meson. In all the three accessible short distance amplitudes (K^0–$\overline{K^0}$, B_d–$\overline{B}_d$, and B_s–$\overline{B}_s$) the magnitude of the NP amplitude cannot exceed the SM short distance contribution. The latter is suppressed by both the GIM mechanism and the hierarchical structure of the CKM matrix,

$$\mathcal{A}_{\text{SM}}^{\Delta F=2} \approx \frac{G_F^2 m_t^2}{16\pi^2} \left[\left(V_{ti}^{\text{CKM}}\right)^* V_{tj}^{\text{CKM}} \right]^2 \times \langle \overline{M} | (\overline{Q}_{Li} \gamma^\mu Q_{Lj})^2 | M \rangle \times F\left(\frac{M_W^2}{m_t^2}\right) , \tag{39}$$

where F is a loop function of $\mathcal{O}(1)$. For more details we refer the dedicated readers, say to [22] and Refs. therein.

1.5.2 CP violation

First we notice that because the SM is a chiral theory then C (and P) can not really be thought of symmetries moving between particles and their anti-particles because along the way they change the representation of the fields under the weak interactions. This makes CPV special because it only changes the electric (hyper) charge but not the representation under the weak force, and thus form a true manifestation of particle-to-anti-particle transformation. In order to obtain CP violation one should find a way to create or define a process consist of an asymmetry between particles and anti-particles. Naively one might think that it is enough to have complex parameters in the action to create CPV observables or asymmetries but in reality as we have already discussed above it is more subtle. To demonstrate this point let us consider for example a case in which we would like to form a (decay) asymmetry between a particle A decaying some particles that we denote collectively as B (characterize by a differential matrix element M_{AB}) and its CP conjugate process in which a particle $\bar{A}$ decay into its conjugate particle $\bar{B}$ ((characterize by a matrix element $M_{\bar{A}\bar{B}}$)). Let assume that M_{AB} is complex and that it is given by (or it is proportional to) a single complex number such that $M_{\bar{A}\bar{B}} = M^*_{AB}$. Then a CP asymmetry can be obtain by considering the difference between the decay process and its CP conjugate process (say normalized by the sum), which is proportional to:

$$\frac{|M_{AB}|^2 - |M_{\bar{A}\bar{B}}|^2}{|M_{AB}|^2 + |M_{\bar{A}\bar{B}}|^2}\,, \tag{40}$$

which is easy to show that is vanishes. In order to obtain a non-zero result one require interference between two contributions, which have a relative complex phase between them, but even this is not sufficient. Consider for instance the case where the amplitude is a sum of two contributions with a relative weak CP phase:

$$M_{AB} = M^1_{AB} + \exp{(i\theta_{\rm CP})} M^2_{AB} \;\;\&\;\; M_{\bar{A}\bar{B}} = M^1_{AB} + \exp{(-i\theta_{\rm CP})} M^2_{AB}\,, \tag{41}$$

well it is easy to see that even in this case the above asymmetry would vanish. A non-zero asymmetry is found only in the case where the two amplitude differ by two phases, one is usually denoted as a weak phase which flips its sign between a process and its CP conjugate one, the other is denoted as a "strong" phase (has nothing to do with the strong CP phase - it is just a jargon) which does not change its sign (it could arise in cases where for instance the process is time dependent or involved an intermediate on-shell states):

$$M_{AB} = M^1_{AB} + \exp{(i\theta_{\rm CP} + i\sigma)} M^2_{AB} \;\;\&\;\; M_{\bar{A}\bar{B}} = M^1_{AB} + \exp{(-i\theta_{\rm CP} + i\sigma)} M^2_{AB}\,, \tag{42}$$

with σ being the "strong" phase. *Show that in this case the asymmetry is indeed non-zero and discuss the origin of strong phases.* This is just the tip of the iceberg, in some more details there are three main type of CP asymmetries that participate in measuring the SM flavor parameters and constraining BSM, CPV in decay; in mixing; and in interference between mixing with and without decays; and in some cases A or B can be CP eigenstates, but in the interest of time and space we refer the readers for instance to [14] for more information.

1.5.3 The standard CKM unitarity triangle

All flavour transitions in the SM depend on only 4 fundamental parameters, λ, A, ρ, and η. We can test the Kobayashi-Maskawa mechanism by making many measurements, over-constraining the system. One way to visualize a subset of experimental constraints is through the standard CKM unitarity triangle, which tests one out of nine unitarity equations, $V_{\rm CKM} V^\dagger_{\rm CKM} = 1$. The standard CKM unitarity triangle is obtained from a product of the first and the third column of the CKM matrix

$$V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0, \tag{43}$$

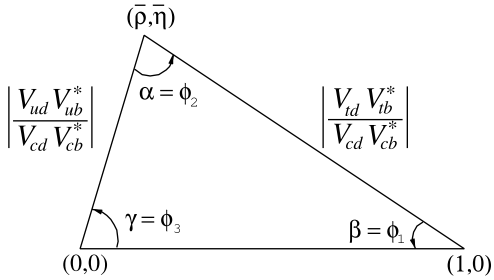

Figure 7: The standard CKM unitarity triangle (from [26]).

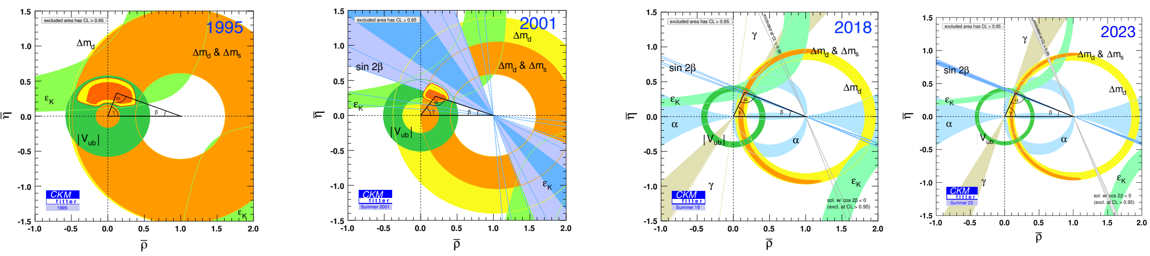

Figure 8: The evolution of the constraints in the standard CKM unitarity triangle plane from (left-to-right) 1995, to recently (2023). Taken from the ckmfitter website [27].

which we can rewrite as

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0. \tag{44}$$

In terms of the Wolfenstein parameters this sum rule is

$$-\big(\bar\rho + i\bar\eta\big) + 1 + \big(-1 + \bar\rho + i\bar\eta\big) = 0. \tag{45}$$

The relation (44) can be interpreted as a sum of three complex numbers that are the sides of a triangle, shown in Fig. 7. There are two common notations for the angles of the standard CKM unitarity triangle: either α, β, γ or ϕ_1, ϕ_2, ϕ_3, used by the two B-factories, BaBar and Belle, respectively. The Belle experiment (1999-2010) at KEK, Japan produced about $\sim 1.5 \times 10^9$ B mesons, while BaBar experiment 1999-2008) at SLAC, USA collected about $\sim 0.9 \times 10^9$ B mesons. The two experiments established that the KM mechanism is the main source of CP violation in the SM. The progression of constraints in the CKM unitarity triangle plane is shown in Fig. 8. We see that there was a big qualitative jump after the start of the B factories, and a very impressive set of improvements in the constraints since then.

The constraints on the standard CKM unitarity triangle are coming from several different meson systems, the B_d^0, B^+ mesons from measurements at Belle, BaBar and LHCb, the B_s meson and Λ_b baryon from measurements at LHCb, and the kaon physics experiments. Beautiful representation of the different processes participating in constraining the SM flavor parameters, was compiled by Zupan [28], using the CKM unitarity triangle plane, shown in Fig. 9, together with the relevant SM diagrams. The upshot of these results is that the KM mechanism is the dominant origin of CPV. The measurements point to a consistent picture of flavour violation, described by four parameters, A, λ, $\bar\rho$, $\bar\eta$, with the values can be extracted from the CKM elements shown when have discussed the flavor puzzle in Eq. (16), and are given by [15]

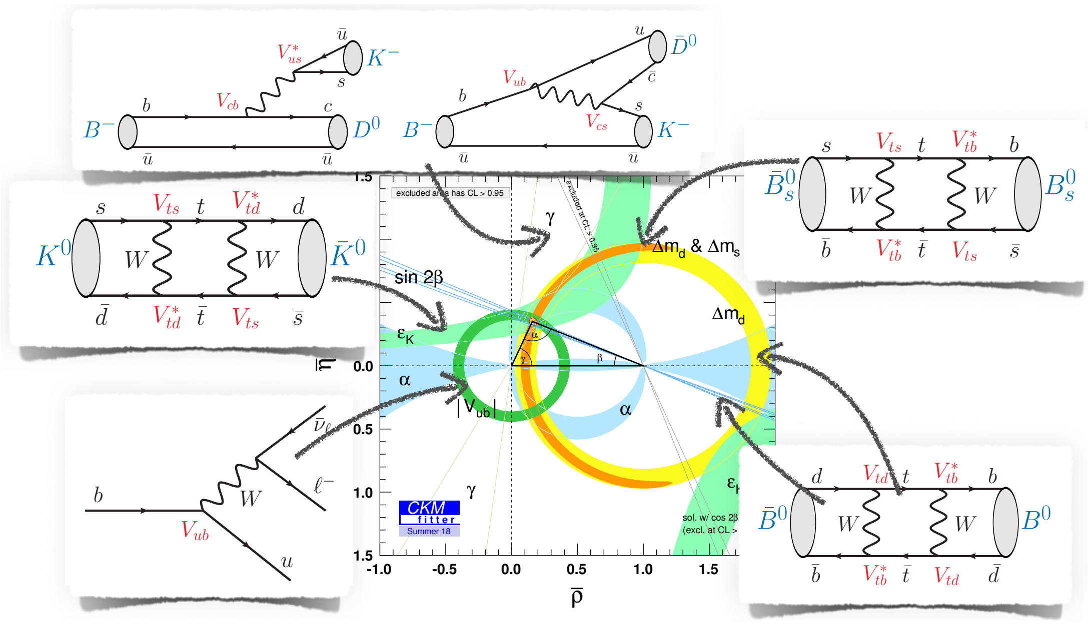

Figure 9: Illustration, some of the processes involved in the determination of the CKM elements, taken from [28].

$$\lambda = 0.22501 \pm 0.00068 \,, A = 0.826 \pm 0.016 \,, \bar{\rho} = 0.1591 \pm 0.0094 \,, \bar{\eta} = 0.3523 \pm 0.0072 \,. \quad (46)$$

Since $\bar{\rho} \sim \bar{\eta}$ the CKM phase is sizable, it is related to

$$\gamma = \arctan\left(\bar{\eta}/\bar{\rho}\right) , \quad (47)$$

experimentally [15],

$$\gamma = (65.7 \pm 3.0)^{\circ}, \quad (48)$$

so that the weak phase is indeed $\mathcal{O}(1)$.

The field is undergoing a big upgrade in available statistics. The successor to Belle experiment, called Belle II, is ramping up right now, aiming to collect about 10^{11} B mesons, roughly 50× more than Belle did. The LHCb experiment also has ambitious upgrade plans [29]. After the end of Upgrade II in 2035 it may have the statistics that corresponds to roughly $\sim 10^{11}$ or more useful B's (because of hadronic environment this number fluctuates from channel to channel), as well as B_s mesons and heavy baryons, which are also produced in the pp collisions. The constraints on the elements of the CKM matrix are thus set to become much more precise in the future.

1.6 Model independent bounds

In order to describe NP effects in flavor physics, we can follow two main strategies: (i) build an explicit ultraviolet completion of the model, and specify which are the new fields beyond the SM, or (ii) analyze the NP effects using a generic effective theory approach, by integrating out the new heavy fields. The first approach is more predictive, but also more model dependent. In this and the next section we adopt the second strategy, which is less predictive but also more general.

Assuming the new degrees of freedom to be heavier than SM fields, we can integrate them out and describe NP effects by means of a generalization of the Fermi Theory. The SM Lagrangian becomes the renormalizable part of a more general local Lagrangian. This Lagrangian includes an infinite tower of operators with dimension $d > 4$, constructed in terms of SM fields and suppressed by inverse powers of an effective scale $\Lambda > M_W$:

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \sum \frac{C_i^{(d)}}{\Lambda^{(d-4)}}\, O_i^{(d)}(\text{SM fields}). \tag{49}$$

This general bottom-up approach allows us to analyze all realistic extensions of the SM in terms of a limited number of parameters (the coefficients of the higher dimensional operators). The drawback of this method is the impossibility to establish correlations of NP effects at low and high energies – the scale Λ defines the cutoff of the effective theory. However, correlations among different low energy processes can still be established implementing specific symmetry properties, such as the MFV hypothesis (Sec. **??**). The experimental tests of such correlations allow us to test/establish general features of the new theory, which hold independently of the dynamical details of the model. In particular, B, D and K decays are extremely useful in determining the flavor symmetry breaking pattern of the NP model.

1.6.1 $\Delta F = 2$ transitions

The starting point for this analysis is the observation that in several realistic NP models, we can neglect non-standard effects in all cases where the corresponding effective operator is generated at tree level within the SM. This general assumption implies that the experimental determination of the CKM matrix via tree level processes is free from the contamination of NP contributions. Using this determination, we can unambiguously predict meson-antimeson mixing and FCNC amplitudes within the SM and compare it with data, constraining the couplings of the $\Delta F = 2$ operators in Eq. (49).

The mixing of neutral mesons has provided severe constraints on new degrees of freedom at high energies: since measurements of mixing and CP violation in neutral kaons in the 1960s, it has provided precious information on charm and top quarks before their discovery. The hypothesis of Kobayashi–Maskawa for the origin of CP violation [10] observed in kaons was only tested experimentally when BaBar and Belle around 2003–2004 established CP violation in good agreement with the predictions of the standard model (SM) [27, 30]. These B-factory results showed that the standard model (SM) source of CP violation in the flavor sector was the dominant part. However, even after BaBar and Belle, and the LHCb results of the last decade, new physics (NP) is still allowed to contribute at the 10–20% level, compared to the SM, in flavor-changing neutral current (FCNC) processes [31].

Since neutral-meson mixings are FCNC processes which are suppressed in the SM, they provide strong constraints on new physics. This led to the development of numerous mechanisms to suppress such contributions, should NP exist at the TeV scale. Low-energy supersymmetry is one example, where the ansatz of degeneracy or alignment were both motivated by constraints from neutral meson mixing and other FCNC processes. In a large class of NP models the unitarity of the CKM matrix is maintained, and the most significant NP effects occur in observables that vanish at tree level in the SM [32, 33, 34, 35]. In such scenarios, which encompass a large class of models, possible effects of heavy particles in each neutral meson system can be described by two real parameters,

$$M_{12} = \left(M_{12}\right)_{\rm SM} \times \left(1 + h_{d,s}\, e^{2i\sigma_{d,s}}\right), \tag{50}$$

where M_{12} relates to the time evolution of the two-state neutral meson system (for a review, see [36]). However, the extraction of NP contribution to meson mixing is entangled with the determination of the SM parameters, namely the CKM elements. It is not enough to measure the mixing amplitude itself, only the combination of many measurements can reveal a deviation from the SM. In the SM CKM fit [37, 27], the constraints come from $\Delta F = 1$ processes dominated by tree-level charged-current interactions, and $\Delta F = 2$ meson mixing processes, which first arise at one-loop level. We can modify the CKM fit to constrain new physics in $\Delta F = 2$ processes, under the assumption that it does not significantly affect the SM tree-level charged-current interactions.

The parameterization in Eq. (50) is convenient because any NP contribution to M_{12} is additive, so it is easy to read off from a fit the bounds on the magnitude and the phase of the NP contribution, or to convert the result to bounds on SMEFT operators [38, 39]. In particular, for the NP contribution to the mixing of a meson with $q_i\bar{q}_j$ flavor quantum numbers, due to the operator

$$\frac{C_{ij}^2}{\Lambda^2}\left(\bar{q}_{i,L}\gamma_\mu q_{j,L}\right)^2, \tag{51}$$

where C_{ij} is related to the flavour dependence and Λ to the NP energy scale, one finds [40]

$$\begin{aligned} h &\simeq 1.5\,\frac{|C_{ij}|^2}{|\lambda_{ij}^t|^2}\,\frac{(4\pi)^2}{G_F\Lambda^2} \simeq \frac{|C_{ij}|^2}{|\lambda_{ij}^t|^2}\left(\frac{4\ \text{TeV}}{\Lambda}\right)^2, \\ \sigma &= \arg\!\big(C_{ij}\,\lambda_{ij}^{t*}\big), \end{aligned} \tag{52}$$

where $\lambda_{ij}^t = V_{ti}^*\,V_{tj}$ and V is the CKM matrix. Operators of different chiralities have conversion factors differing by $\mathcal{O}(1)$ factors [41]. Minimal flavor violation (MFV), where the NP contributions are aligned with the SM ones, correspond to $\sigma = 0$ (mod $\pi/2$).

1.6.2 Constraining BSM physics with $\Delta F = 2$ processes

To set explicit bounds, let us consider for instance the LH $\Delta F = 2$ operator Q_1 from Eq. (38), and rewrite it as

$$\sum_{i\neq j}\frac{c_{ij}}{\Lambda^2}(\overline{Q}_{Li}\gamma^\mu Q_{Lj})^2\ , \tag{53}$$

where the c_{ij} are dimensionless couplings. The condition $|\mathcal{A}_{\text{NP}}^{\Delta F=2}| < 0.1|\mathcal{A}_{\text{SM}}^{\Delta F=2}|$ implies [42, 31]

$$\Lambda > \frac{4.4\ \text{TeV}}{|\,(V_{ti}^{\text{CKM}})^*\,V_{tj}^{\text{CKM}}|/|c_{ij}|^{1/2}} \sim \begin{cases} 3\times 10^4\ \text{TeV}\times|c_{sd}|^{1/2} \\ 1\times 10^3\ \text{TeV}\times|c_{bd}|^{1/2} \\ 2\times 10^2\ \text{TeV}\times|c_{bs}|^{1/2} \end{cases} \tag{54}$$

Analyzing these results a few points are in order:

(i) The strong bounds on Λ for generic c_{ij} of order 1 is a manifestation of what in many specific frameworks (supersymmetry, technicolor, etc.) goes by the name of the *flavor problem*: if we insist that the new physics emerges in the TeV region, then it must possess a highly non-generic flavor structure.

(ii) In the case of $B_{d,s}$–$\overline{B}_{d,s}$ and K^0–$\overline{K^0}$ mixing, where both CP conserving and CP violating observables are measured with excellent accuracy, there is still room for a sizable NP contribution (relative to the SM one), provided that it is, to a good extent, aligned in phase with the SM amplitude ($\mathcal{O}\,(0.01)$ for the K system and $\mathcal{O}\,(0.1-0.2)$ for the $B_{d,s}$

systems). This is because the theoretical errors in the observables used to constrain, say the phases, $S_{B_d \to \psi K}$ and ϵ_K, are smaller with respect to the theoretical uncertainties in Δm_{B_d} and Δm_K, which constrain the magnitude of the mixing amplitudes.

(iii) The corresponding constraints, associated with $\Delta c = 2$ processes, in the D system are only second to those from ϵ_K, leading to new physics bound of around $\Lambda > 10^3$ TeV $\times |c_{uc}|^{1/2}$. However, in this case the SM contributions are unknown (see for instance [22] and Refs. therein), and the only robust SM prediction is the absence of CPV.

1.7 Higgs and flavor physics

As we have seen above, within the SM (minimal) Higgs-mechanism of electroweak symmetry breaking (EWSB), the physics of flavor is all encoded within the Higgs interactions. As we discuss in the following section this is a highly non-trivial statement, which in fact was not yet directly fully tested.

1.8 Higgs mechanism & elementary masses

Before we discuss the details of the status of the experimental tests of the Higgs mechanism of fermion masses let us start with a more basic question. What does the origin of elementary masses have to do with the the discovery of the Higgs and the confirmation of it ruling in electroweak symmetry breaking (EWSB)? Well, in principle, not much, if you look at technicolor models, where symmetry breaking is induced by strong dynamics, or if you think about the canonical description of superconductivity where effectively electromagnetism is broken by a condensate of Cooper-pairs, there is no clear linkage between the source of symmetry breaking and the induction of fundamental fermion masses. However, in the case of the SM Higgs mechanism one can make an economical choice, and couple the Higgs to the fundamental fermion and via this linking the field behind EWSB to the origin of fermion masses. Within the Standard Model (SM) of elementary particles and fundamental interactions, in its most minimalistic version, this implies switching-on at least[11] nine coupling three for the up-type quarks (u, c, t), three for the down-type quarks (d, s, b), and additional three for the charged leptons (e, μ, τ), nine couplings of the three generation, the flavor sector of the SM. Each coupling is associated with linear, Yukawa, coupling of the Higgs to two fermions that can be written as

$$y_f \bar{F} f H \,, \tag{55}$$

with H being the Higgs field (or an SU(2) weak-interaction conjugate of it), F (f) being a fermion weak-doublet (singlet). It implies that all the masses of the charged fermions, with their existential importance, are dictated by the strength of the Higgs coupling to the fermion, through the following relation

$$y_f^{\rm SM} = \frac{\sqrt{2} m_f}{v} \,, \tag{56}$$

with $v \simeq 246\,$GeV being the Higgs vacuum expectation value (VEV) and m_f corresponds to the mass of a fermion f . The relation in Eq. (56) shows that there is a linear relation

[11]in principle, the charged-fermion sector contains at least four more physical parameters associated with quark-mixing and CP violation.

between the Yukawa coupling and the fermion masses with the Higgs vacuum expectation being the slope of corresponding line. It also tells us that all the structure of observed fermion masses, which is quite non-trivial and span some five orders of magnitude is all encoded in the Yukawa coupling, on other words the fermion-mass hierarchies are directly translated to Yukawa hierarchies. Within the SM, the Yukawa couplings (evaluated at the Higgs mass scale) are given by [43],

$$\begin{aligned} y^{\rm SM}_{e,u,d}(m_h) &\simeq 2.0\times 10^{-6}\,,\quad 5.4\times 10^{-6}\,,\quad 1.1\times 10^{-5}\;. \\ y^{\rm SM}_{\mu,c,s}(m_h) &\simeq 5.9\times 10^{-4}\,,\quad 3.6\times 10^{-3}\,,\quad 3.2\times 10^{-4}\;. \\ y^{\rm SM}_{\tau,t,b}(m_h) &\simeq 1.0\times 10^{-2}\,,\quad 0.96\,,\quad 1.6\times 10^{-2}\,, \end{aligned} \tag{57}$$

where the coupling can be represented as points on the orange diagonal line of Fig. 10, which shows the relation between the fermion-Yukawas and their masses. We learn from it that apart from the case of some of the members of the third generation, t, b, τ, the Yukawa couplings are extremely small.

Could there be other options, as a matter of principle? We have already hinted above that there could be cases where the above theory is modified. We shall describe below two such options that to some extent describe two opposite extremes. In the first, as proposed in [44] one can use an horizontal Pecci-Quinn [45] symmetry to explain the flavor mass hierarchies without imposing the hierarchies on the Yukawa couplings. The basic idea is to use the symmetry such that the lighter particles are not allowed to have linear coupling to the Higgs but rather arise at higher dimension operators, which effectively would modify Eq. (55) to $\bar{F} f H \left(H^\dagger H/\Lambda^2\right)^n$. Thus, assuming that $v^2/\Lambda^2 \ll 1$, and assigning n of order few-ten but with some hierarchy among the different flavors can naturally accounts for the mass hierarchies instead of the Yukawas. In this case the effective light-fermion-Higgs couplings would be enhanced by n ,. This case is illustrated by the upper-blue curve of Fig. 10. The other extreme possibility discussed in [46] is where the mass of the light fermions is instead coming with coupling to a sequestered and subdominant source of EWSB, which yield an alternative insight for the flavor hierarchies. This result with vanishing light-fermion-Higgs couplings, as illustrated by the bottom-red curve of Fig. 10. The main conclusion of this discussion is that, in principle, the Yukawa couplings of the fermions to the Higgs, might differ from the SM prediction and therefore measuring it leads to a significant improvement in our understanding of the origin of masses in nature. Both cases lead to an alternative understanding of the flavor puzzle and to the establishment of new physics.

We end this part by going back to discuss existentialism, now in the context of the Higgs coupling to matter. There are two interesting aspects of the Higgs Yukawa couplings related to our existence. The first, associated with the first generation masses, as was pointed in [2], is that increasing the Higgs VEV (or its negative mass) by a factor of few would render all nuclei unstable preventing the existing of structure. This was used to claim that the negative Higgs mass is bounded from above addressing the hierarchy problem [5] (however it was argued in [6, 7] that probably this argument is not robust). The second, associated with the top quark, is that apparently the SM top-Yukawa is close to its maximal size, and increasing it by order 10% would render our universe unstable [8].

1.9 Experimental challenges & legacy

To make a progress on the question related to the origin of masses the experimentalists need to weigh-in and measuring the Higgs coupling to the matter field comparing them

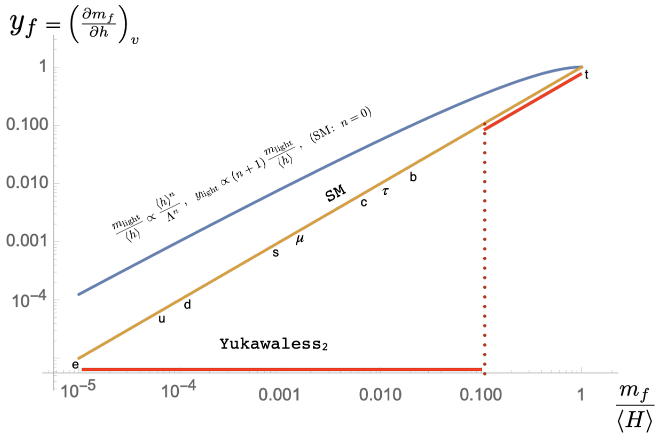

Figure 10: Schematic of the relation between the fundamental masses and their Yukawa to the Higgs field. The diagonal orange line describe the SM case, the upper-blue curve corresponds to a Yukawa-full model [44] and the lower-bottom-red curve corresponds to a Yukawaless model for the light generations [46], see text for more details.

to the relation given in Eq. (57). It took a few year after the Higgs discovery for the first breakthrough on that front and by 2018 it was established that indeed all the SM Higgs mechanism is behind the third generation fermion masses [47, 48, 49, 50, 51, 52]:

$$\mu_{t\bar{t}h} = 1.29 \pm 0.18,\ \mu_b = 0.0.98 \pm 0.20,\ \mu_\tau = 1.09 \pm 0.23\,, \tag{58}$$

with roughly $\mu_f \equiv \frac{\sigma}{\sigma_{\rm SM}} \frac{{\rm BR}_{f\bar{f}}}{{\rm BR}^{\rm SM}_{f\bar{f}}}$. This is a legacy result from the LHC experiment that sometimes is overlook by our community, and consequently, by general public as well [53].

2 The strong CP problem & the physics of ultralight DM

2.1 introduction

The origin of dark matter (DM) remains one of the most significant mysteries in physics. While we know it exists and surrounds us, we have no understanding of its microscopic nature (see [54] for a recent review). The search for dark matter has entered a new era of precision and creativity. While decades of experiments have probed the weak-scale frontier with ever-increasing sensitivity, the lack of detections has motivated a shift toward ultralight dark matter (ULDM) candidates—bosonic fields with masses well below the eV scale, whose collective wave-like behavior can imprint subtle, coherent signatures across astrophysical systems and laboratory observables. As we shall discuss models of ULDM could be extremely simple and thus quite compelling theoretically, basically requiring the presence of a single (free or almost-free?) ULDM field. It is sufficient for the ULDM to have even extremely feeble interactions with the SM fields (maybe even with super-Planckian strength), leading to minute but potentially detectable modulations in atomic transition frequencies, electromagnetic signals, mechanical forces or magneto-spin type of interactions. The physics of ultralight dark matter thus bridges cosmology, field theory, and quantum measurement.

A central theoretical motivation for such particles arises from the strong CP problem—the puzzling smallness of CPV within the pure strong sector, as we discuss below might give us a hint for BSM physics. Arguebly most compelling resolution introduces a new global U(1) symmetry, spontaneously broken to produce a pseudo–Nambu–Goldstone boson: the axion. Originally proposed to dynamically cancel the CP-violating term in QCD, the axion also naturally behaves as a cold dark matter candidate through nonthermal production in the early universe. This deep theoretical connection between strong-interaction symmetry and cosmological structure has made the axion, and its broader class of axion-like particles, a central focus for experimental exploration.

The rapid progress of quantum science has opened a new landscape for detecting such elusive fields. Quantum sensors—ranging from optical and atomic clocks to superconducting qubits, magnetometers, and optomechanical devices—achieve sensitivities approaching fundamental quantum limits. Their coherence and precision make them uniquely suited to search for the tiny, coherent effects predicted by ultralight dark matter or axion couplings. By leveraging entanglement, squeezing, and novel measurement protocols, these instruments extend the reach of particle physics far beyond traditional collider or detector paradigms. In this lecture, we will explore how these developments are reshaping the experimental frontier, uniting particle physics and quantum metrology in the pursuit of answers to the strong CP problem and the nature of dark matter.

2.2 The strong CP problem from EFT perspective

From a modern effective field theory (EFT) perspective of quantum field theories (QFTs), the strong CP problem can be understood in four different levels of sophistication as follows:
The first would say that the strong CP problem is associated with the smallness of the strong CP phase,

$$\bar{\theta} = \theta - \arg\det(Y_d Y_u)\,, \tag{59}$$

where the 'bare' θ parameterizes the super-selected sector of QCD (to be discussed in detail below) and Y_u, Y_d are the Yukawa coupling matrices of the Standard Model (SM). However, it is not obvious at all that it is problematic for a parameter appearing in the theory to be small, thus we continue to the following level.

The second level would say that it is associated with the hierarchy between the CKM phase,

$$\theta_{\rm CKM} = \arg\left\{\det[Y_u Y_u^\dagger, Y_d Y_d^\dagger]\right\} \equiv \arg\left(J_{\rm CP}\right)\,, \tag{60}$$

which was measured to be of order one [15] and the strong CP one, whose magnitude is bounded to be smaller than 10^{-10} [15]. To this formulation of the problem we reply by asking how different this is from the flavor puzzle, dealing with the hierarchical structure of the masses and mixing? While it sounds like a real problem, in the context of the flavor hierarchies we denote it as a puzzle. The reason to that stems from the observation that the flavor parameters are technical natural parameters (see 't-Hooft's [16] for definition). It implies that the flavor hierarchies can be result of some unspecified new physics with no low energy consequences. Or one might even solve the flavor puzzle using spontaneously broken global symmetry, giving rise to a pseudo-Nambu-Goldstone boson (pNGB), with a shift symmetry only broken softly by a mass term giving rise to a viable model of an axion-like paricle (ALP), which can serve as ultralight dark matter (UDM). In this sense it seems that the strong CP problem with the axion solution is on par with the flavor puzzle.

The third level raises the possibility that $\bar{\theta}$ is in fact not a technical natural parameter, noticing that the strong CP phase, at 7-loop order [55][12], is expected to suffer from a logarithmic divergent proportional to the CKM phase. However, thus far, no one has been able to calculate this contribution, yet we can use spurion analysis to estimate the size of this potential additive log-divergent contributions to the strong phase [56],

$$\delta\bar{\theta} \sim \frac{1}{(16\pi^2)^7} g'^2 [Y^2(u_R) - Y^2(d_R)] \mathrm{Im}\,(J_{\mathrm{CP}}) \log \Lambda_{\mathrm{UV}} \sim 10^{-40} \log \frac{\Lambda_{\mathrm{UV}}}{m_W}\,, \tag{61}$$

where g' is the $U(1)_Y$ gauge coupling, $Y(u_R)$ and $Y(d_R)$ are hyper-charge of right hand quarks. Thus, it makes this contribution negligibly small, even for a cutoff of the order of the Planck scale. Below we further discuss the issues associated with the UV sensitivity of $\bar{\theta}$.

The fourth level goes beyond the framework of conventional EFT. It is possible that once adding gravitational interaction, this picture is changed as has been emphasized in [57]. Since this goes beyond the scope of EFT, we will not further comment on it. Furthermore, in Ref. [58] it was suggested that the strong CP phase is not a fundamental parameter of the theory but rather related to a choice of the relevant sector of the theory. Using the analogy of how Bloch states are defined, it was further asserted that as all possible choices of θ are equal and in particular cannot be associated with any fundamental symmetry of the action, the observed value in our universe seems suspiciously small. Thus, only models where it is dynamically driven to zero can account for the problem, similar to the argument made in [57], however, without direct reference to gravitational interactions.

Within EFT framework there are three possible resolution to the strong CP problem. The first, as briefly mentioned above, is that it is not a problem at all – rather it is just a puzzle associated with a technical natural parameter, similar to the flavor puzzle. Namely the CP problem is actually associated with radiatively stable initial conditions, which require no further explanation, because the strong CP phase behaves effectively as a technically natural parameter following 't-Hooft. One can, rephrase the argument made in [57, 58] as questioning this approach, claiming that microscopically all values of the strong CP phase are equivalent, as they are determined by selection of particular state out of infinite set of equivalent states. Thus, the strong CP problem becomes similar to other (fine-tuning) problems associated with selection of very unique initial/boundary conditions such as baryogenesis or inflation. We further discuss this issue below and question this argument by heuristic consideration of string theory, using the notion that "beauty is attractive" [59], complemented by the intuition provided for discrete symmetries from Vafa-Witten [60] (which is obviously the heart of the PQ [45, 61, 62] solution but also generalization of it see for instance [63, 64]). We also provide a toy model for the landscape within QFT that shows how one can generate the boundary conditions required for this setup at microscopic scales that are not relevant for low energy observers.

The second solution, proposed in [45, 61, 62] is the axionic solution to the problem, which is well established. As discussed below our only comment on this solution is that in general it does not come with a definite scale, which in the context of EFT might lead to some conceptual issues.

[12]The argument given in [56] to understand the seven loops suppression of the RGE contribution goes as follows: i) J_{CP} is the first flavor singlet which can contribute to the log-divergent part of $\bar{\theta}$ and $J_{CP} \sim y^{12}$, so we need to close the 6 Higgs loops. ii) Swap between u and d implies $J_{CP} \to -J_{CP}$ while $\bar{\theta}$ is left unchanged. Therefore, we need to break this symmetry with an extra gauge loop which distinguishes up and down quarks in order to get the first non-zero contribution to $\bar{\theta}$.

The third possibility that we discuss in this work is that of spontaneous breaking of CP, such as the Nelson-Barr solution [65, 66, 67] (for earlier attempts see [68, 69], see also [70, 71] for a different but related approach).

2.3 Briefly, the physics of ULDM from misalignment

In this section we provide a minimal and brief review of the misalignment mechanism of ULDM. The purpose of this subsection apart from being self explanatory is to demonstrate how simple is it to write a model of ULDM, in fact we will see that a model of a single real free scalar field, given certain initial conditions, is sufficient to provide us with a viable model of DM, for more information see *e.g.* [72] and Refs. therein. The Lagrangian of the field is just given by that of a free scalar

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi \partial \nu_\phi g^{\mu\nu} - m^2\phi^2\right) . \tag{62}$$

with the information about the cosmology is basically encoded in the metric, g. Using the FRW metric, and assuming that at early time the universe and the field are homogeneous, the scalar field equation of motion is:

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0 , \tag{63}$$

with H being the scale factor, where for radiation domination it is given by, $H \sim T^2/M_{\rm Pl}$, with T being the temperature and $M_{\rm Pl}$ being the Planck mass. This is the equation of a dumped harmonic oscillator, where for early time, $H \gg m$ the field is over-dumped and is basically frozen, and for late time, $m \gg H$ the solution is given by

$$\phi(t) \approx \phi_0 \left(\frac{a_{\rm osc}}{a}\right)^3 \cos\left(mt\right) , \tag{64}$$

with a being the scale factor, $a_{\rm osc}$ is define as the scale factor when the oscillation has begun, and ϕ_0 is the amplitude of the field at this time, either when $m \sim H$ or possibly just after reheating (see below). The background energy density and pressure of the axion field are:

$$\bar{\rho}_a = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m_a^2\phi^2 , \tag{65}$$

$$\bar{P}_a = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m_a^2\phi^2 . \tag{66}$$

Using the solution of Eq. (64) we find that

$$\bar{\rho}_a = \frac{1}{2}\left(\frac{a_{\rm osc}}{a}\right)^3 m^2\phi_0^2 , \tag{67}$$

$$\bar{P}_a = 0 , \tag{68}$$

These precisely describe the solution of a non-interacting matter background, namely describing dark matter cosmology. We can evaluate the relation between the initial field value, ϕ_0, or if it is a compact field we can convert it to an angle using the relation $\theta = \phi/f$, and the resulting DM density requiring that we obtain the correct observed DM relic abundance, as a function of the ULDM mass,

$$\phi_0 \equiv \theta f \ (f_{\rm min}) = \begin{cases} 10^{18}\,\text{GeV}\left(\frac{10^{-27}\,\text{eV}}{m_\phi}\right)^{\frac{1}{4}} & m_\phi \lesssim 10^{-15}\,\text{eV} \\ 10^{15}\,\text{GeV}\left(\frac{10^{-15}\,\text{eV}}{m_\phi}\right) & m_\phi \gtrsim 10^{-15}\,\text{eV} , \end{cases} \tag{69}$$

where in the second line we used the maximally optimistic case, to amplify the DM density, where the reheating scale occurs just before BBN time ($T \sim$MeV), after the time where $m = H$ and thus the dumping is delayed. We also note that that the DM oscillate with frequency of the order of its mass (up to corrections of order of its velocity square, β^2)

$$w \sim \text{Hz} \times \frac{m_\phi}{10^{-15}\,\text{eV}}\,.$$

Our last point is associated with the ULDM occupation number per phase-space volume,

$$N_\phi^{\text{occup}} \sim 10^3 \times \left(\frac{\text{eV}}{m}\right)^4 \times \left(\frac{10^{-3}}{\beta}\right)^3\,,$$

where β stands for the ULDM's typical velocity around the solar system.

2.4 Model independent searches of a free ULDM (?)

So far, above, our ULDM field has no coupling to us (apart from gravitational), therefore, how can we search for it? A minimal plausible assumption is that it'd couple to us suppressed by some very high scale (Planck suppressed?), which are extremely weak. Let us consider for instance the case where the ULDM is either a scalar or a pseudo scalar (axion) couples to the strong sector,

$$\mathcal{L}_{\text{Pl}} \in d_g \frac{\alpha_s}{\pi}\frac{\phi}{M_{\text{Pl}}}\,GG + \frac{a}{32\pi^2 f}\,G\tilde{G} \quad\Longrightarrow\quad d_g \frac{m_n}{M_{\text{Pl}}}\,\phi\,\bar{n}n + \frac{m_n}{f}\,a\,\bar{n}\gamma_5 n\,, \tag{70}$$

where the arrow corresponds to the effective coupling in the low energy theory, below the QCD scale (not being careful about order one coefficients), and notice that for the axion case we use a notation where f corresponds to the inverse scale in which it couples to the SM fields, and for the scalar $d_g = 1$ corresponds to the extremely-weak gravitational-strength coupling. In Fig. 11 we show the comparison between bounds on the two couplings, which shows quite clearly that current bounds on scalar ULDM are so strong such that the are sensitive to super Planckian physics, as for the axion case the constraints are significantly weaker. Maybe we should accept that probing axions is work in progress and we are currently still long way from constraining interesting models of minimal misalignment (shown on the RH panel of Fig. 11 as dashed blue, according to Eq. (69)) and even more so reaching the Planck scale. It is interesting to ask the question whether we can use the sensitivity to scalar interaction, which is much stronger to search for the axion. Axion models do predict quadratic scalar coupling that are suppressed, however, by m^2/f^2 and thus seems hopeless to be probed [73]. Yet, as we shall briefly discuss below in the case of QCD-like-axion models the the suppression factor is much smaller, which might be probed in the future in (nuclear or molecular) clocks [74, 75, 76, 77, 78].

2.4.1 Naturalness

As we are dealing with very light spin-0 fields, with some couplings to the SM fields, it is interesting to ask whether these coupling are natural. It is easy to see that for the scalar coupling the following relation holds

$$\Delta m_\phi^2 \sim d_g^2\,\Lambda_{\text{UV}}^4/(4\pi M_{\text{Pl}})^2 \lesssim m_\phi^2 \quad\Rightarrow\quad d_g \lesssim 10^{-5}\,\frac{m_\phi}{\text{Hz}}\,\frac{\Lambda_{\text{UV}}^2}{\text{GeV}^2}\,, \tag{71}$$

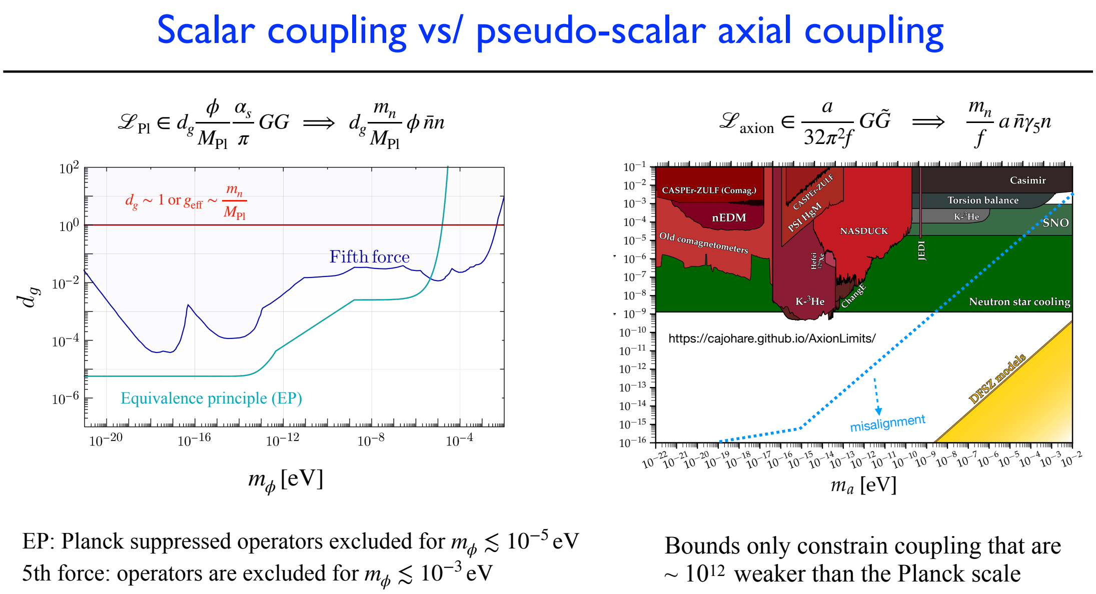

Figure 11: Scalar vs. axion coupling for more details see text around Eq. (70).

this assumes, say, a GeV mirror sector for the $\Lambda_{\rm UV}$ which is very hard to achieve for dilaton ULDM [79], however, not impossible for quadratically coupled ULDM [80].

What about the axion couplings? So far we have only looked at the $aG\tilde{G}$ coupling, and it's a bit subtle to analyze. This term is associated with an anomaly and is generating a mass term through instanton effects that are exponentially suppressed and within the induce mass (potential and quadratic coupling) for the axion below the confinement scale. Therefore, by itself this term is not associated with naturalness issue. In more simplistic terms, similar to an EFT type of consideration, one can view the QCD axion as a theory with a cutoff of the order of $\Lambda_{\rm QCD}$. Therefore, within conventional QCD axion models we expect the axion mass to be of the order of

$$m^2 \gtrsim \Lambda_{\rm QCD}^4/(4\pi f)^2\,, \tag{72}$$

where for conventional axion models the axion mass is close to saturating this naturalness bound (see [81] for a way out of this naturalness bound, which requires n-copies of the SM strong sector).

2.5 Axions or ALPs

As we saw above the sensitivity to scalar ULDM of experiments is very strong resulting with very strong bounds, in addition to the fact that they are susceptible to severe naturalness problems. ALPs on the other hand are much more protected from these issues, because of the shift symmetry mentioned above. This is basically an outcome of the Goldstone's theorem and non-linear realization of (a spontaneously broken) U(1) symmetry. In case you have never analyzed it in a concrete model it is instructive to build up a linear toy model of how axions arises. Consider for instance the following Lagrangian

$$\mathcal{L}_{\rm kin} + y\Phi\bar{\psi}\psi + h.c. + \left(\Phi^\dagger\Phi - f^2\right)^2\,, \tag{73}$$

with ψ being a fermion, y being a Yukawa coupling to Φ a complex scalar. This Lagrangian describe a theory invariant under a U(1) symmetry, which is broken spontaneously. To understand the low energy limit of this theory it is useful to use non-linear redefinition of the scalar degrees of freedom,

$$\Phi = (\rho + f/\sqrt{2}) \exp(ia/f) \,,$$

where it is easy to see that ρ acquire mass of the order of f the fermions acquire a mass of order $y\,f$ while a, the celebrated axion field, remains massless. It is also useful to use a local axial fermionic transformation to remove a from the Yukawa interaction, which will only appear now in the kinetic term as ∂a. As expected from the Goldstone theorem a cannot have a potential and it is massless but it does acquire derivative coupling. We basically see that the system has a "residual memory" of the U(1) symmetry under which the axion, which take the role of the phase of the field, linearly shift under the U(1) transformation, and basically the symmetry is still non linearly realized using this (initially weird-looking) exponential parameterization of the fields. In fact one can easily generalized the above model and conclude that the way a will appear in the kinetic terms is always in the form

$$\partial^\mu a J^5_\mu / f \,,$$

with J^5_μ corresponds to the axial current that is spontaneously broken. One can also show that there are higher order terms of the form $(\partial^\mu a)^2 m_\psi \bar{\psi}\psi/f^2$, which in the presence of (a soft breaking) mass term for a would lead to (a typically tiny) quadratic coupling of the form $m^2 a^2 m_\psi \bar{\psi}\psi/f^2$, mentioned above [73].

The above discussion holds for the case in which the U(1) symmetry is non anomalous, typically because the symmetry acts on multiple generation and the sum of axial charges vanishes (as could happen in the case of flavor horizontal symmetry, a la Froggatt-Nielsen [17]). However, the situation is more involved and interesting when the U(1) symmetry is anomalous such as in the case of the QCD axion and the symmetry is the celebrated Peccei-Quinn one [45], leading to a solution of the strong CP problem. Before we describe QCD axion framework we shall briefly describe the relevant implication for the anomalous symmetry. In our context, an anomalous symmetry refers to a conserved global U(1) classical symmetry that is violated at the quantum level. In this case basically the Noether current is not conserved and one can instead write the following relation:

$$\partial^\mu J^5_\mu = \frac{g^2}{16\pi^2} G\tilde{G} \,, \tag{74}$$

where in the presence of a mass the current is subjects to explicit breaking:

$$\partial^\mu J^5_\mu = im\bar{\psi}\gamma^5\psi + \frac{g^2}{16\pi^2} G\tilde{G} \,, \tag{75}$$

which is associated with Eq. (59), that says that only the shift to the mass and the coefficient of the $G\tilde{G}$ form a physical phase or CP violating invariant combination, as due to the axial anomaly they both tranforms under the U(1) axial phase transformation.

To conclude we have learned that generic axions (alps) associated with spontaneous breaking of a U(1) symmetry have to leading order derivative coupling to the fields of the form $\partial^\mu a J^5_\mu/f$, and in the presence of a mass term also usually small quadratic coupling to scalar operators, further suppressed by m^2/f^2. If those axion are related to anomalous symmetry they could also acquire coupling to $G\tilde{G}$ (and similarly to the other gauge fields), this coupling is also suppressed by $1/f$ as we one discuss for the QCD axion.

2.6 The QCD axion

The original idea of using an axion to solve the strong CP problem is due to Peccei and Quinn [45]. They have realized that if they would promote θ (or $\bar{\theta}$ more precisely) to a dynamical field whose minimum is at $\bar{\theta} = 0$ and it can be driven there dynamically independent of the initial conditions then it would solve the problem. Their theoretical framework, which we now can understand using our derivation in the previous section, simply assumes that the SM is extended to include a sector which is invariant under a spontaneously-broken global axial symmetry (usually denoted as Peccei-Quinn or just PQ symmetry), which is anomalous in the presence of QCD. In the absence of the anomaly the axion is a derivatively-coupled, parity-odd, massless field, however, due to the anomaly it acquire coupling to $G\tilde{G}$. Then Vafa and Witten showed that the axion has a potential which dynamically push it to $\theta_{\text{QCD}} = 0$ via QCD non-perturbative effects (instantons) [60]. To learn its properties and learn how the QCD axion couples to the SM field, up to order one factors, we need to follow two steps. The first is easy, namely in the absence of the anomaly all that we need to know is what is the PQ current and the axion will couple to it according to $\partial^\mu a J^5_\mu/f$, which lead to one class of observables. Furthermore, due to the anomaly and the instanton effects, the axion would also acquire non-derivative coupling and potential. To obtain these, one can use a shortcut and match the above theory with the $aG\tilde{G}$ coupling to the chiral Lagrangian, which at lowest order consists of only the pions and the axion field (and in principle also the η'), see [82, 83] for more details. It leads to the following final expression for the axion-dependent pion mass and the axion potential:

$$V = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u+m_d)^2}\sin^2\frac{\tilde{\theta}}{2}}\,, \quad m_\pi^2 = a f_\pi \sqrt{m_u^2 + m_d^2 + 2m_u m_d \cos\tilde{\theta}}\,, \tag{76}$$

with $\tilde{\theta} \equiv \bar{\theta} + a/f$. Studying these two relation we learn several things. First as anticipated for the axion the minimum of the potential is at $\tilde{\theta} = 0$, which solves the strong CP problem. Furthermore we find, by expanding the pion mass dependence to leading order in the axion value, that the pion acquire quadratic coupling to the axion and as a result also it couples quadratically to the proton and neutrons, in a coupling which is much larger than that of a pure ALP [75], and is given instead roughly by,

$$\mathcal{L} \in \frac{m_u}{\Lambda_{\text{QCD}}}\frac{a^2}{f^2} m_{n,p}(\bar{n}n, \bar{p}p)\,, \tag{77}$$

with n (p) stands for a neutron (proton). What makes the above framework exciting is that the axion field could form a viable ULDM candidate, with basically no extra ingredient added.

This solution however is not free of theoretical concerns due to what is denoted the axion quality problem, as it relies on very precise global U(1) PQ symmetry, which may not be respected by quantum gravity. Thus higher dimension operator suppressed by the Planck scale might completely disrupt the structure of the above potential, pushing the axion away from the CP conserving VEV invalidating the whole idea.

2.7 Solving the strong CP problem using spontaneous CP breaking

A completely different approach to the problem could be formulated if one assumes that at some microscopic scale the theory is fully CP symmetric. It implies that both the strong and weak phases vanish, which is inconsistent with observations. Thus, one needs to assume that at some lower scale CP will be spontaneously broken in a controlled manner. In the sense that a special structure for the Yukawa matrices is obtained such that the CKM phase is order one and the strong phase is negligibly small. It could arise for instance if the Yukawa matrices are Hermitians or in other structure associated with special textures, all that can be obtained naturally. This class of theory is more involved on the one hand but on the other hand it link s the two hierarchical sources of CPV together. In addition, it typically suffers from a less severe quality problem and in addition was recently shown to also lead to a viable source of ULDM.

2.8 ULDM terrestrial searches

In this part we briefly describe different strategies used to search for ULDM using quantum sensors. First we consider how the axion field change the EM dynamics due to the presence of the term $aF\tilde{F}/f$ (or $aE\dot{B}/f$). This term modifies Maxwell's equations:

$$\nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0} - \frac{1}{\varepsilon_0}\mathbf{B}\cdot\nabla a/f, \tag{78}$$

$$\nabla\cdot\mathbf{B} = 0, \tag{79}$$

$$\nabla\times\mathbf{E} + \frac{\partial\mathbf{B}}{\partial t} = 0, \tag{80}$$

$$\nabla\times\mathbf{B} - \mu_0\varepsilon_0\frac{\partial\mathbf{E}}{\partial t} = \mu_0\mathbf{J} + \mu_0\left(\mathbf{B}\cdot\frac{\partial a}{\partial t}/f + \mathbf{E}\times\nabla a/f\right), \tag{81}$$

The above modification of Maxwell's equations is known as 'axion electrodynamics'. The axion field a appears on the right hand side of the sourced Maxwell equations, like a new type of 'dark' charge or current density. The axion field only appears with derivatives, due to the shift symmetry, and it is multiplied by $\mathbf{E}$ or $\mathbf{B}$. Namely, gradients of the axion field can act as sources for electric and magnetic fields, in other words, the axion always sources an $\mathbf{E}$ (anti)parallel to an applied $\mathbf{B}$ and vice versa. This last fact is in distinction to an electromagnetic wave, in which electric and magnetic fields are perpendicular to one another, where the strength of the axion interactions is inversely proportional to f.

The second class of interactions is related to the axion couplings to the fermions that arises from the term $\partial^\mu a J_5^\mu/f$. Expanding it to leading order in the non-relativistic approximation leads to the following coupling in the Hamiltonian,

$$S_f\cdot\nabla a\,, \tag{82}$$

which implies that the gradient of the axion field behaves as an effective magnetic, which couples to the spin of the fermions, S_f, leading to an effective force inducing precision in the spin that can be searched for.

Lastly, for the QCD axion an important effect arises due to the term $aG\tilde{G}/f$ which induces oscillating EDM signal that can be searched for in various experiments looking for time-varying dipole moments.

As we have discussed it also lead to time dependent neutron and proton masses, from the quadratic axion-coupling, that can be searched for in various clock experiment. Clocks are also very sensitive to models where the ULDM is a scalar which couples to scalar operators which lead to oscillatory behavior of variety of would be constants of nature.

Finally, so far we have discussed searches that focus on the oscillatory nature of the ULDM, however, one could also search it based on the fact that it is very light and thus mediate either effects associated with the violation of the equivalence principle or signals for 5th forces, for the case of scalar ULDM, or long range spin-dependent forces for the case of ALPs.

3 Conclusions

The field of flavor physics is now amidst a new era where the LHC measurements together with the results from the flavor factories have established both the Higgs mechanism as the origin of the SM masses, and also the CKM mechanism of flavor and CP violation. This implies a very strong sensitivity to physics BSM. On another frontier using the exponential progress made in quantum science and theoretical progress, the understanding of models of ULDM as well as their phenomenology and their detection have seen a significant advances, with current searches probing viable models, and in the future even some of the region related to the celebrated QCD axion will be reached.

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