Voting Methods

Preference Ballots

Ballot 1	Ballot 2	Ballot 3	Ballot 4	Ballot 5	Ballot 6	Ballot 7	Ballot 8	Ballot 9	Ballot 10	Ballot 11
1st P	1st S	1st R	1st P	1st R	1st S	1st R	1st S	1st P	1st S	1st P
2nd R	2nd R	2nd T	2nd R	2nd T	2nd R	2nd T	2nd R	2nd R	2nd R	2nd R
3rd S	3rd T	3rd S	3rd S	3rd S	3rd T	3rd S	3rd T	3rd S	3rd T	3rd S
4th T	4th P	4th P	4th T	4th P	4th P	4th P	4th P	4th T	4th P	4th T

Ballot 12	Ballot 13	Ballot 14	Ballot 15	Ballot 16	Ballot 17	Ballot 18	Ballot 19	Ballot 20	Ballot 21	Ballot 22	Ballot 23	Ballot 24
1st T	1st P	1st T	1st S	1st P	1st T	1st R	1st P	1st S	1st P	1st P	1st T	1st P
2nd S	2nd R	2nd S	2nd R	2nd R	2nd S	2nd T	2nd R	2nd T	2nd R	2nd R	2nd S	2nd R
3rd R	3rd S	3rd R	3rd T	3rd S	3rd R	3rd S	3rd S	3rd R	3rd S	3rd S	3rd R	3rd S
4th P	4th T	4th P	4th P	4th T	4th P	4th P	4th T	4th P	4th T	4th T	4th P	4th T

Ballot 25	Ballot 26	Ballot 27	Ballot 28	Ballot 29	Ballot 30	Ballot 31	Ballot 32	Ballot 33	Ballot 34	Ballot 35	Ballot 36	Ballot 37
1st T	1st P	1st P	1st S	1st P	1st S	1st T	1st S	1st P	1st T	1st T	1st S	1st S
2nd S	2nd R	2nd R	2nd R	2nd R	2nd R	2nd S	2nd R	2nd R	2nd S	2nd S	2nd R	2nd R
3rd R	3rd S	3rd S	3rd T	3rd S	3rd T	3rd R	3rd T	3rd S	3rd R	3rd R	3rd T	3rd T
4th P	4th T	4th T	4th P	4th T	4th P	4th P	4th P	4th T	4th P	4th P	4th P	4th P

Preference table

TABLE 13.1 Preference Table for the Student Film Institute Election

Number of Votes	**14**	**10**	**8**	**4**	**1**
First Choice	P	S	T	R	S
Second Choice	R	R	S	T	T
Third Choice	S	T	R	S	R
Fourth Choice	T	P	P	P	P

Plurality voting

Candidate with the most first place votes wins.

TABLE 13.1 Preference Table for the Student Film Institute Election

Number of Votes	**14**	**10**	**8**	**4**	**1**
First Choice	P	S	T	R	S
Second Choice	R	R	S	T	T
Third Choice	S	T	R	S	R
Fourth Choice	T	P	P	P	P

Who wins with plurality voting?

TABLE 13.2 Preference Table for the Smallville Mayoral Election

Number of Votes	**130**	**120**	**100**	**150**
First Choice	A	D	D	C
Second Choice	B	B	B	B
Third Choice	C	C	A	A
Fourth Choice	D	A	C	D

Borda Count

- Each last place vote gets 1 point
- Each next to last place vote gets 2 points
- Each next place vote gets 3 points

⋮

Used for Heisman Trophy

Borda Count

TABLE 13.1 Preference Table for the Student Film Institute Election

Number of Votes	**14**	**10**	**8**	**4**	**1**
First Choice	P	S	T	R	S
Second Choice	R	R	S	T	T
Third Choice	S	T	R	S	R
Fourth Choice	T	P	P	P	P

Who wins with Borda Count?

TABLE 13.2 Preference Table for the Smallville Mayoral Election

Number of Votes	**130**	**120**	**100**	**150**
First Choice	A	D	D	C
Second Choice	B	B	B	B
Third Choice	C	C	A	A
Fourth Choice	D	A	C	D

Plurality with elimination

- If there is a candidate with a majority of the first place votes, that candidate wins.
- Otherwise, eliminate the candidate with the fewest first place votes and start over.

Use in San Francisco and a few other cities.

Common for committee elections.

Plurality with elimination

TABLE 13.1 Preference Table for the Student Film Institute Election

Number of Votes	**14**	**10**	**8**	**4**	**1**
First Choice	P	S	T	R	S
Second Choice	R	R	S	T	T
Third Choice	S	T	R	S	R
Fourth Choice	T	P	P	P	P

Plurality with elimination?

TABLE 13.2 Preference Table for the Smallville Mayoral Election

Number of Votes	**130**	**120**	**100**	**150**
First Choice	A	D	D	C
Second Choice	B	B	B	B
Third Choice	C	C	A	A
Fourth Choice	D	A	C	D

Pairwise comparison

- Candidates are compared with each other two at a time.
- If a majority of voters prefer X to Y, then X gets one point.
- If X and Y are tied among the voters, then each gets $\frac{1}{2}$ point.

Pairwise Comparison

TABLE 13.1 Preference Table for the Student Film Institute Election

Number of Votes	**14**	**10**	**8**	**4**	**1**
First Choice	P	S	T	R	S
Second Choice	R	R	S	T	T
Third Choice	S	T	R	S	R
Fourth Choice	T	P	P	P	P

Pairwise Comparison?

TABLE 13.2 Preference Table for the Smallville Mayoral Election

Number of Votes	130	120	100	150
First Choice	A	D	D	C
Second Choice	B	B	B	B
Third Choice	C	C	A	A
Fourth Choice	D	A	C	D

TABLE 13.7 Summary of Voting Methods

Voting Method	How the Winning Candidate Is Determined
Plurality Method	The candidate with the most first-place votes is the winner.
Borda Count Method	Voters rank all candidates from the most favorable to the least favorable. Each last-place vote receives 1 point, each next-to-last-place vote 2 points, and so on. The candidate with the most points is the winner.
Plurality-with-Elimination Method	The candidate with the majority (over 50%) of first-place votes is the winner. If no candidate receives a majority, eliminate the candidate with the fewest first-place votes. Either hold another election or adjust the preference table. Continue this process until a candidate receives a majority of first-place votes. That candidate is the winner.
Pairwise Comparison Method	Voters rank all the candidates. A series of comparisons is made in which each candidate is compared to each of the other candidates. The preferred candidate in each comparison receives 1 point; in case of a tie, each receives $\frac{1}{2}$ point. The candidate with the most points is the winner.

Which system is best?

Criterion	Description
Majority Criterion	If a candidate receives a majority of first-place votes in an election, then that candidate should win the election.
Head-to-Head Criterion	If a candidate is favored when compared head-to-head with every other candidate, then that candidate should win the election.
Monotonicity Criterion	If a candidate wins an election and, in a reelection, the only changes are changes that favor the candidate, then that candidate should win the reelection.
Irrelevant Alternatives Criterion	If a candidate wins an election and, in a recount, the only changes are that one or more of the other candidates are removed from the ballot, then that candidate should still win the election.

THE MAJORITY CRITERION

If a candidate receives a majority of first-place votes in an election, then that candidate should win the election.

The Borda Count can fail the Majority Criterion

TABLE 13.8 Preference Table for Selecting a New College President

Number of Votes	6	3	2
First Choice	E	G	F
Second Choice	F	H	G
Third Choice	G	F	H
Fourth Choice	H	E	E

THE HEAD-TO-HEAD CRITERION

If a candidate is favored when compared separately—that is, head-to-head—with every other candidate, then that candidate should win the election.

The plurality method can fail the Head to Head Criterion

TABLE 13.11 Preference Table for Three Brands of Tuna Fish

Number of Votes	**8**	**6**	**4**	**4**
First Choice	A	C	C	B
Second Choice	B	B	A	A
Third Choice	C	A	B	C

THE MONOTONICITY CRITERION

If a candidate wins an election and, in a reelection, the only changes are changes that favor the candidate, then that candidate should win the reelection.

Plurality with elimination can fail the Monotonicity Criterion

TABLE 13.13 Preference Table for the Straw Vote

Number of Votes	20	16	14	8
First Choice	W	S	G	G
Second Choice	G	W	S	W
Third Choice	S	G	W	S

TABLE 13.14 Preference Table for the Second Election

Number of Votes	28	16	14
First Choice	W	S	G
Second Choice	G	W	S
Third Choice	S	G	W

THE IRRELEVANT ALTERNATIVES CRITERION

If a candidate wins an election and, in a recount, the only changes are that one or more of the other candidates are removed from the ballot, then that candidate should still win the election.

Pairwise comparison can fail the Irrelevant Alternatives Criterion

TABLE 13.19 Preference Table for the Mayor of Bolinas (repeated)

Number of Votes	160	100	80	20
First Choice	E	G	H	H
Second Choice	F	F	E	E
Third Choice	G	H	G	F
Fourth Choice	H	E	F	G

TABLE 13.20 Preference Table for the Mayor of Bolinas with F and G Removed

Number of Votes	160	100	80	20
First choice	E	H	H	H
Second choice	H	E	E	E

TABLE 13.23 Voting Methods and Whether They Satisfy the Fairness Criteria

	Voting Method			
Fairness Criteria	**Plurality Method**	**Borda Count Method**	**Plurality-with-Elimination Method**	**Pairwise Comparison Method**
Majority Criterion	Always satisfies	May not satisfy	Always satisfies	Always satisfies
Head-to-Head Criterion	May not satisfy	May not satisfy	May not satisfy	Always satisfies
Monotonicity Criterion	Always satisfies	May not satisfy	May not satisfy	May not satisfy
Irrelevant Alternatives Criterion	May not satisfy	May not satisfy	May not satisfy	May not satisfy

Is there a system that satisfies all of the criteria?

ARROW'S IMPOSSIBILITY THEOREM

It is mathematically impossible for any democratic voting system to satisfy each of the four fairness criteria.

Jeremy will buy either the red pillow or the blue pillow. The blue pillow is quite comfortable. If he buys it, he will sleep in and miss his meeting with the President. If Jeremy misses the meeting with the President, the Cabinet will be misled by Jeremy's adversaries. Either the Cabinet will not be misled, or we will go to war. Therefore, either Jeremy will buy the red pillow or we will go to war.

If Paul buys one more piece of candy, he will not be able to afford to buy both his math book and his history book. Paul must major in either math or government or he will lose his scholarship. If Paul loses his scholarship, he will drop out of college and will never become a lawyer or a doctor. If Paul does not buy his math book, he will fail math and will not be able to major in math. If he does not buy his history book, he will not be able to major in government. Therefore, if Paul does not buy one more piece of candy, he will either become a lawyer or a doctor.

Sue is good at basket weaving and at logic. If she majors in basket weaving, she will always be able to find a job in a remote village of South America. If she majors in logic, she will at least be able to work at Wal-Mart. Since Sue is good at logic, she is not good at basket weaving. Therefore, if Sue majors in basket weaving and in logic, then she will one day sell her baskets in a Wal-Mart in South America.

Either fuzzy glow worms eat yellow leaves, or they eat green leaves. Either fuzzy glow worms are green, or the leaves they eat are green. If fuzzy glow worms eat yellow leaves, then they turn orange. Either fuzzy glow worms are not orange, or they are eaten by large purple birds. If large purple birds eat fuzzy glow worms, then large purple birds exist. Therefore, either large purple birds exist, or fuzzy glow worms eat green leaves.