Exploring biological shape analysis through topology, geometry and statistics

Ph. D. summer school: Biomedical image analysis, 2024/03/20

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AI is pretty good at segmenting stuff, what's next?

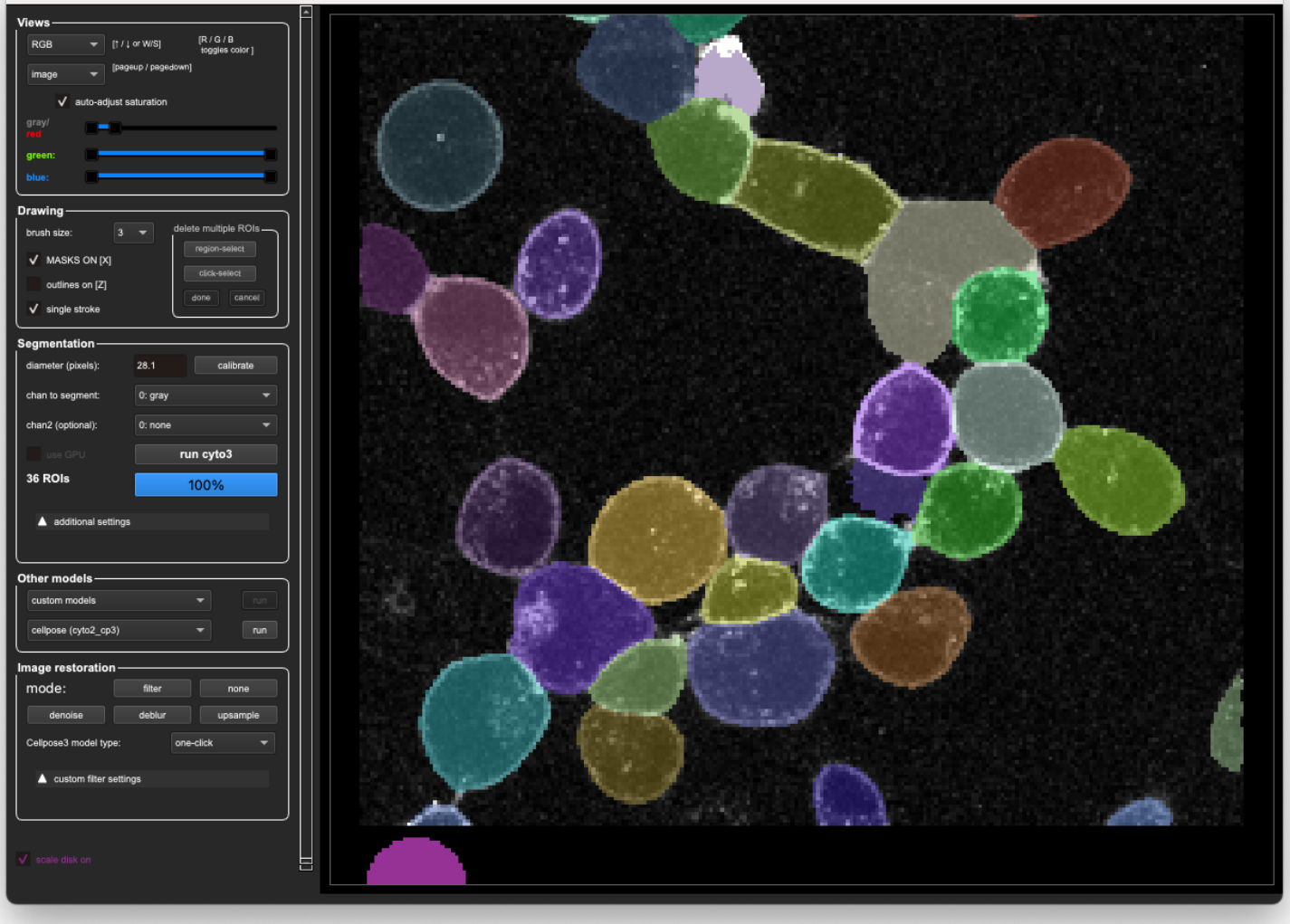
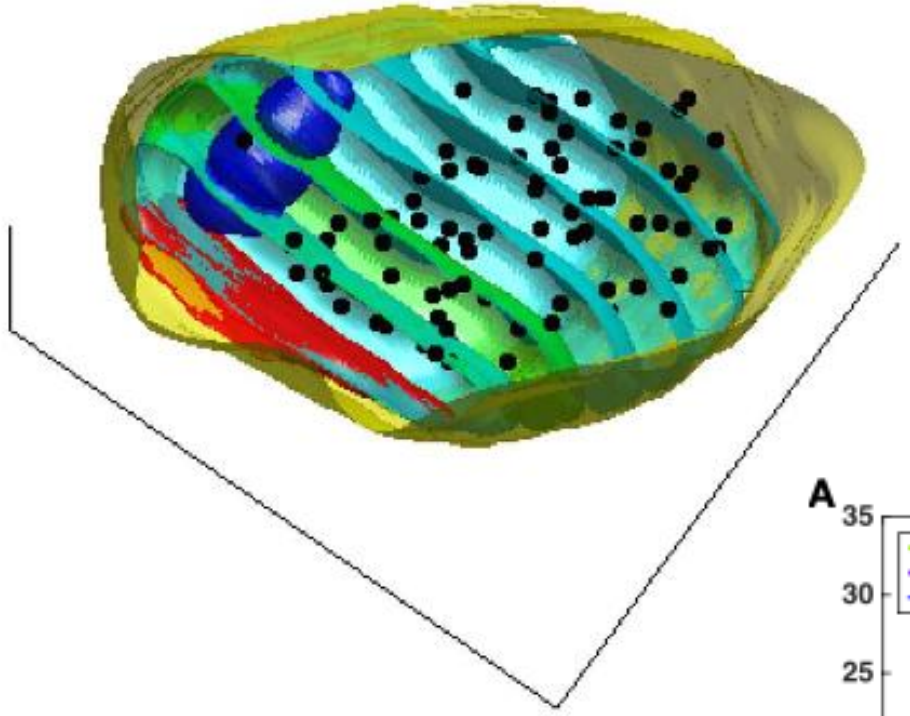
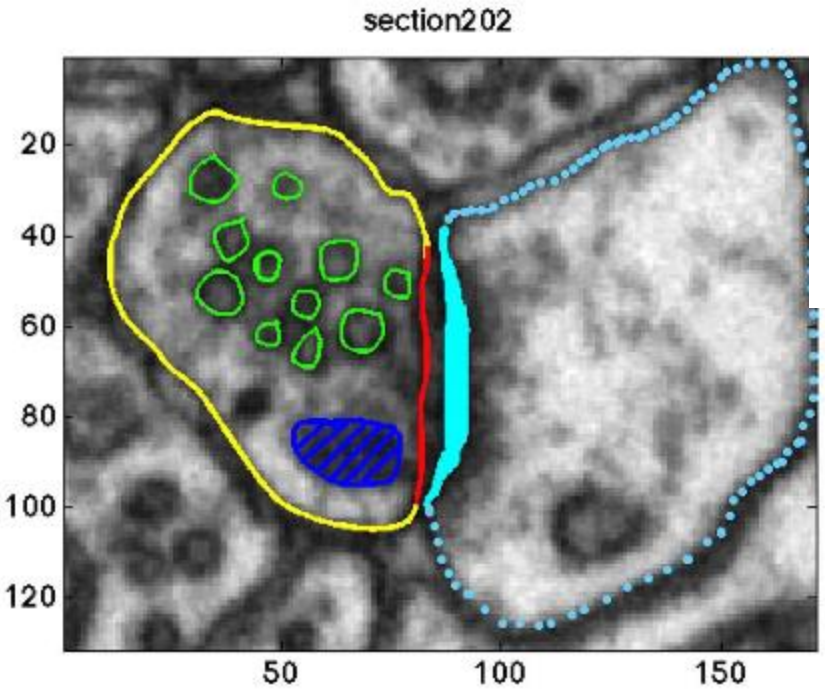
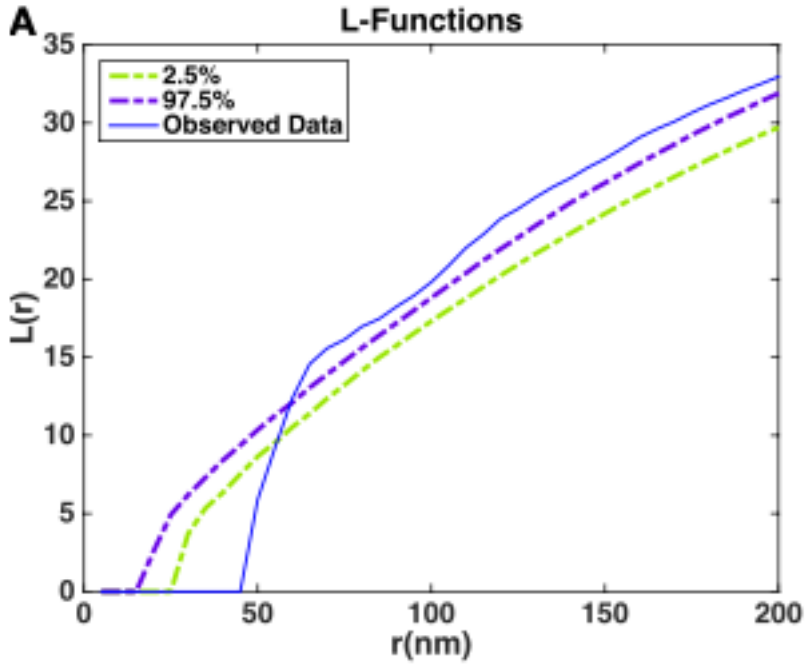


Image courtesy: Karen Martinez & Gabriella von Scheel von Rosing; AI: http://www.cellpose.org/

Point models





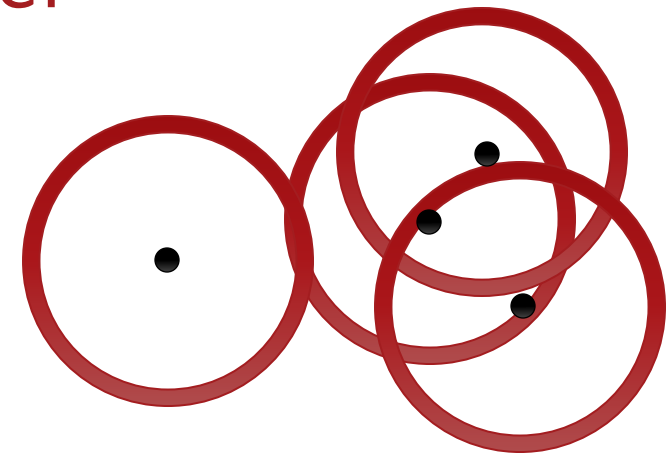


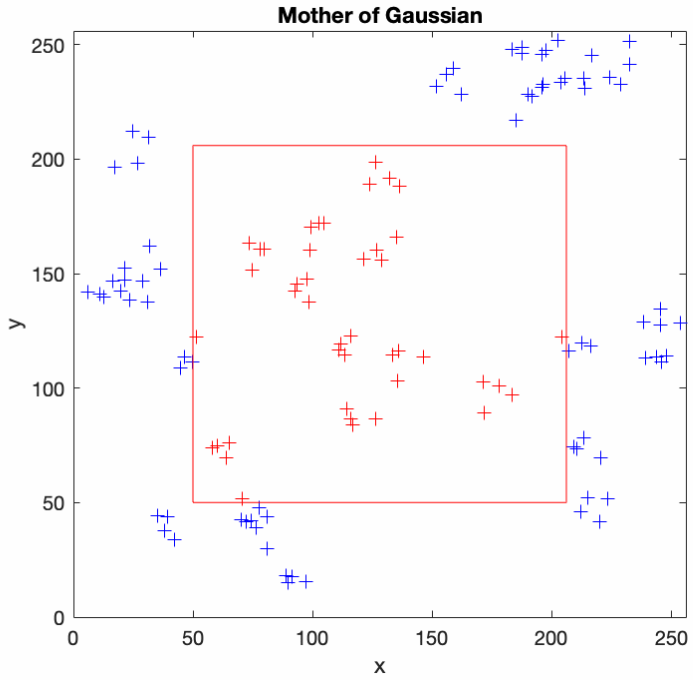
Analysis of shape and spatial interaction of synaptic vesicles using data from focused ion beam scanning electron microscopy (FIB-SEM); M Khanmohammadi, RP Waagepetersen & J Sporring, Frontiers in Neuroanatomy, 2015

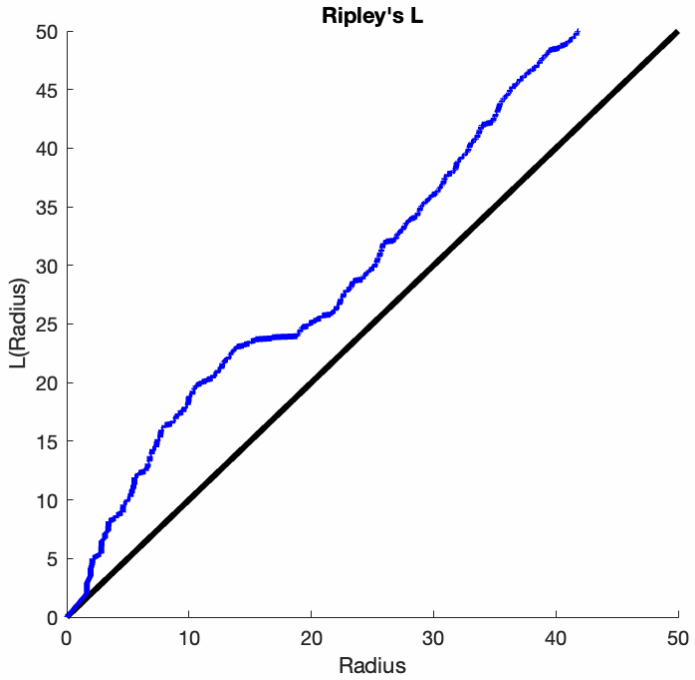
Ripley's K- and L-functions: expected number of neighboring points by radius

$$K(r) = \frac{1}{\lambda}\mathbb{E}\left[I\left(d_{ij} < r\right)\right]$$

$$L = \sqrt{\frac{K}{\pi}}$$

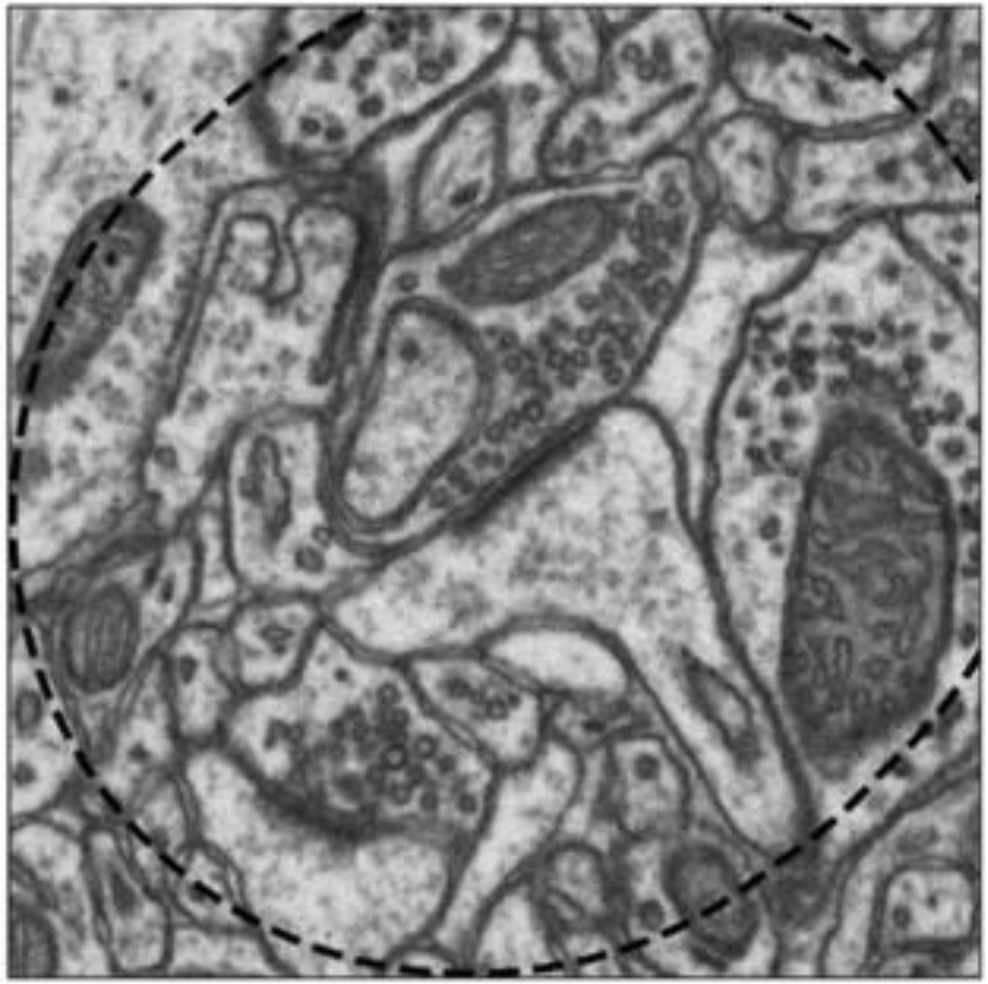




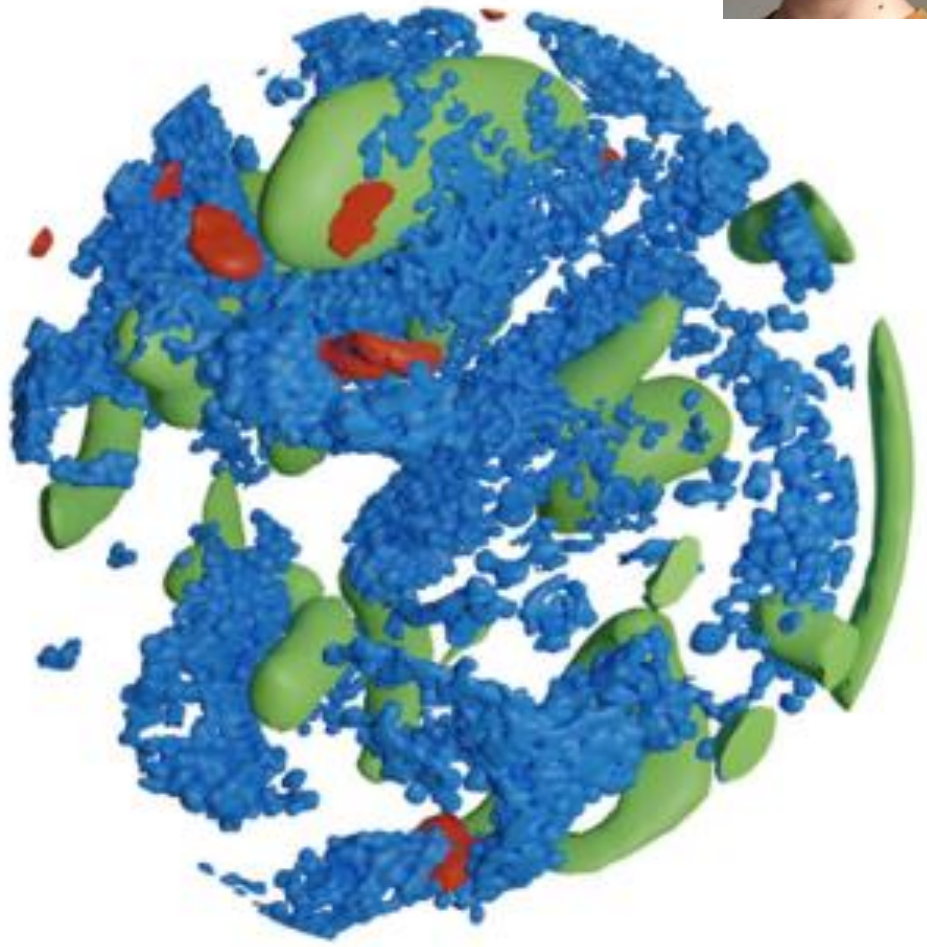


Real structures are not points, small structures are difficult to separate

Graham Knott and Marco Cantoni. Electron microscopy dataset. https://cvlab.epfl.ch/data/ data-em/

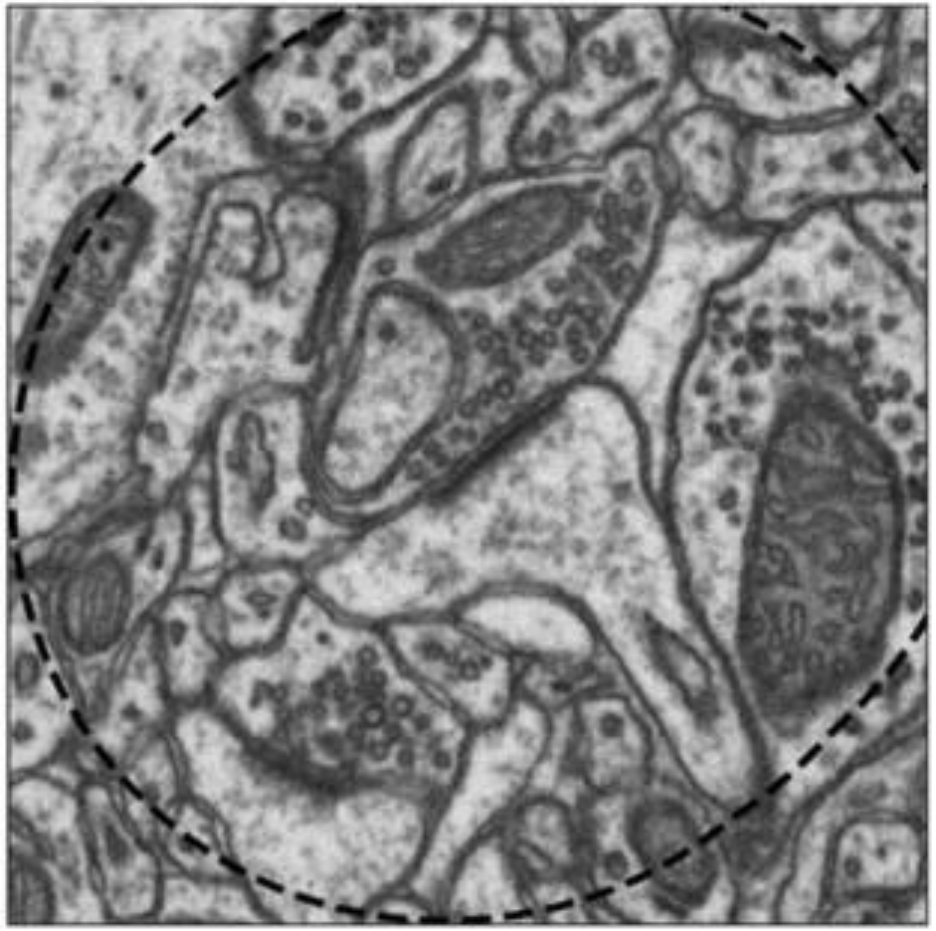


Deep Learning

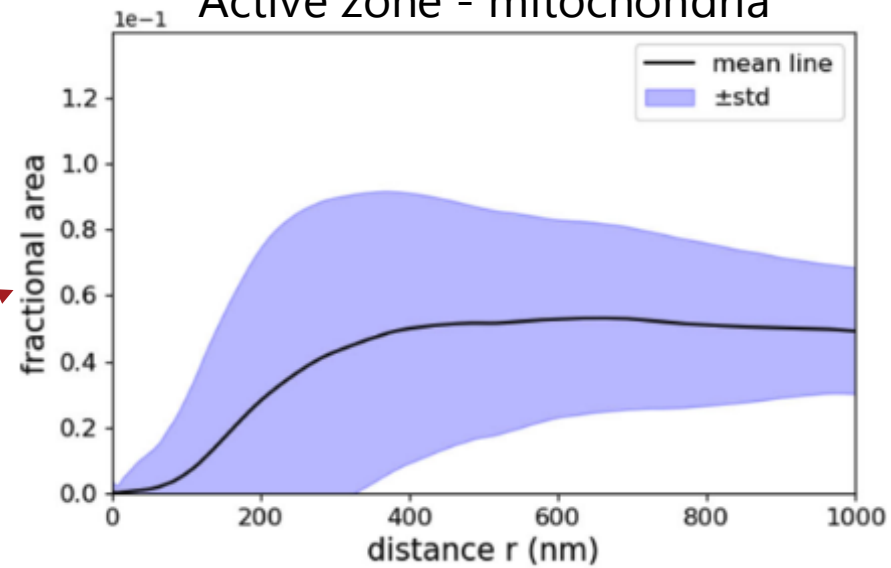


Measuring Shape Relations Using r-Parallel Sets; HJT Stephensen, AM Svane, CB Villanueva, SA Goldman, & J Sporring; Journal of mathematical imaging and vision, 2021

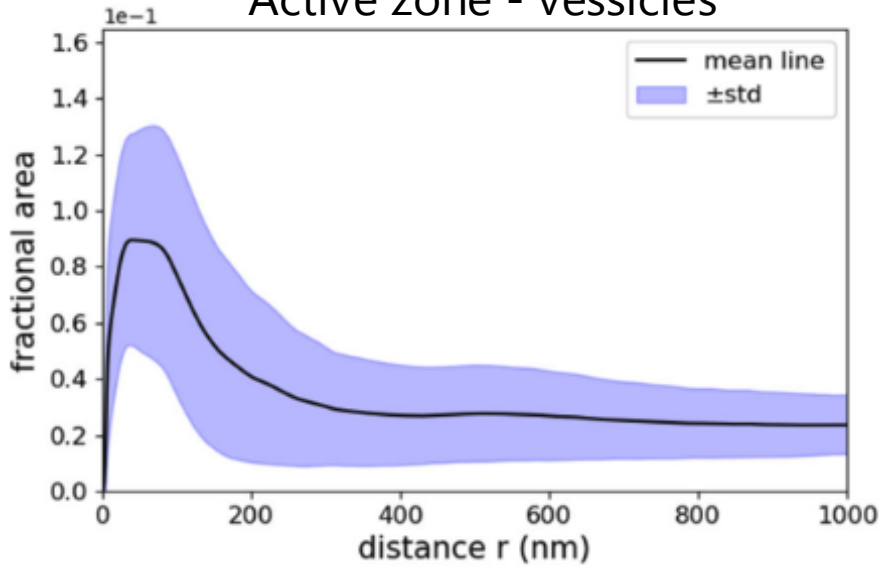
Shape relations for statistical summary of families of shapes and their relations



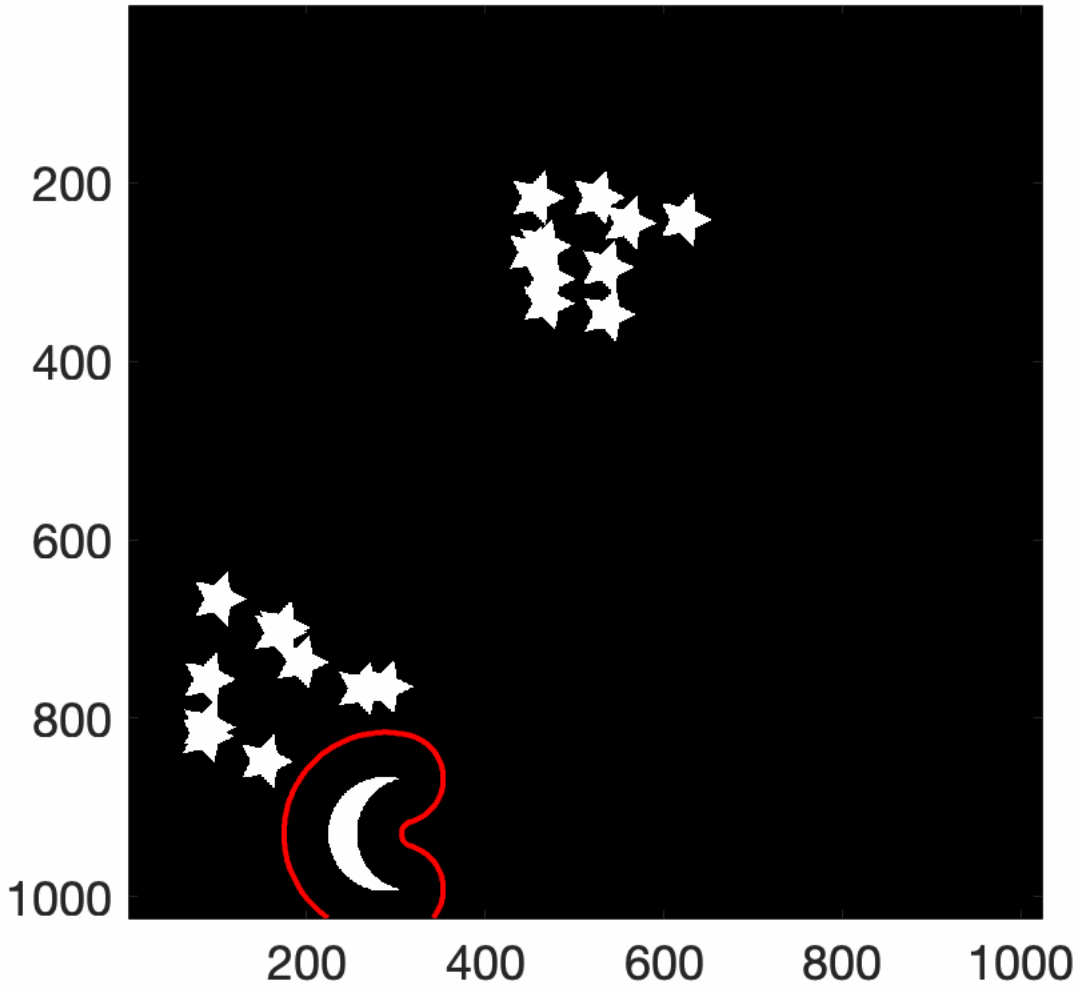
Active zone - mitochondria

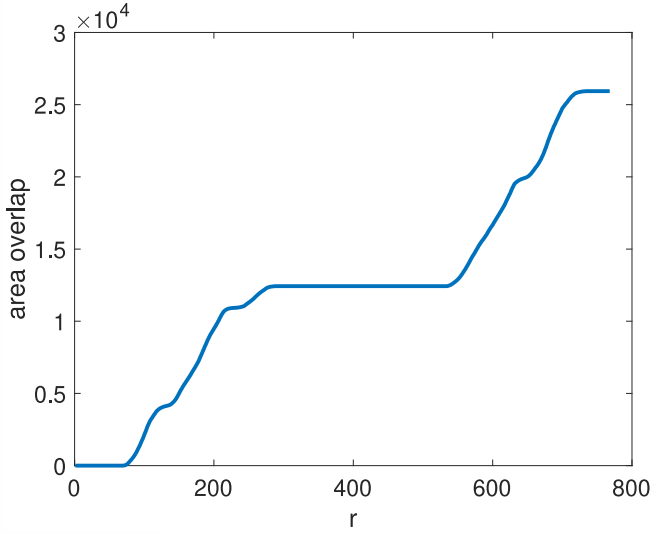


Active zone - vessicles



Shape relation measures: K-functions for objects

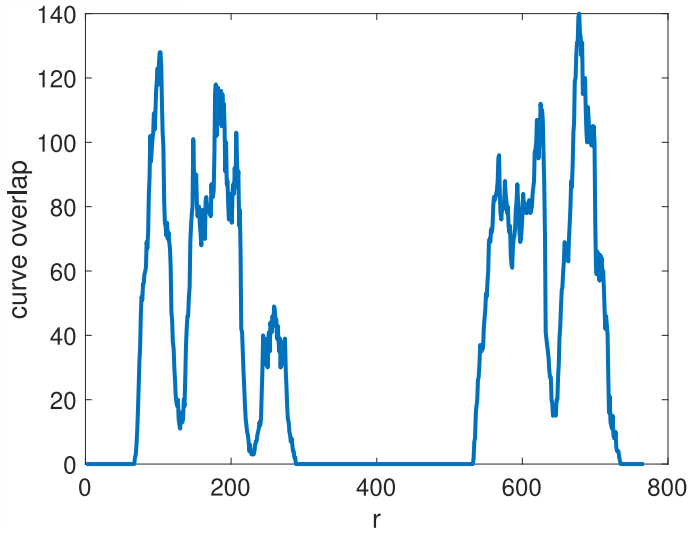


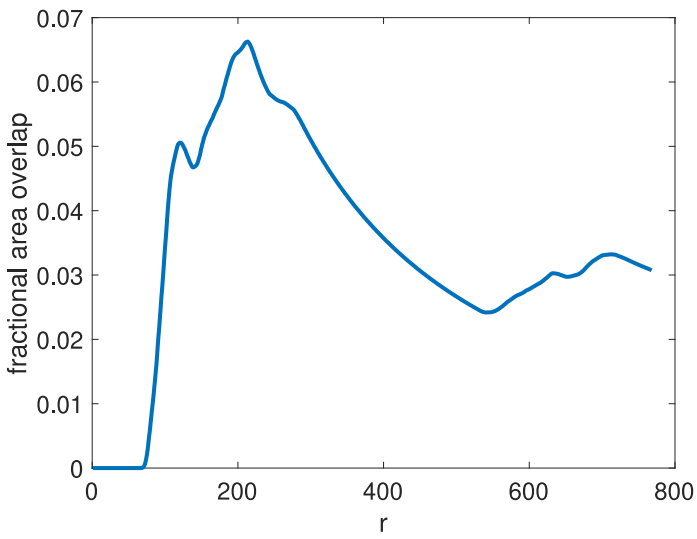


$$\mu_{00}(r) = \mathcal{H}(X \cap Y^r)$$

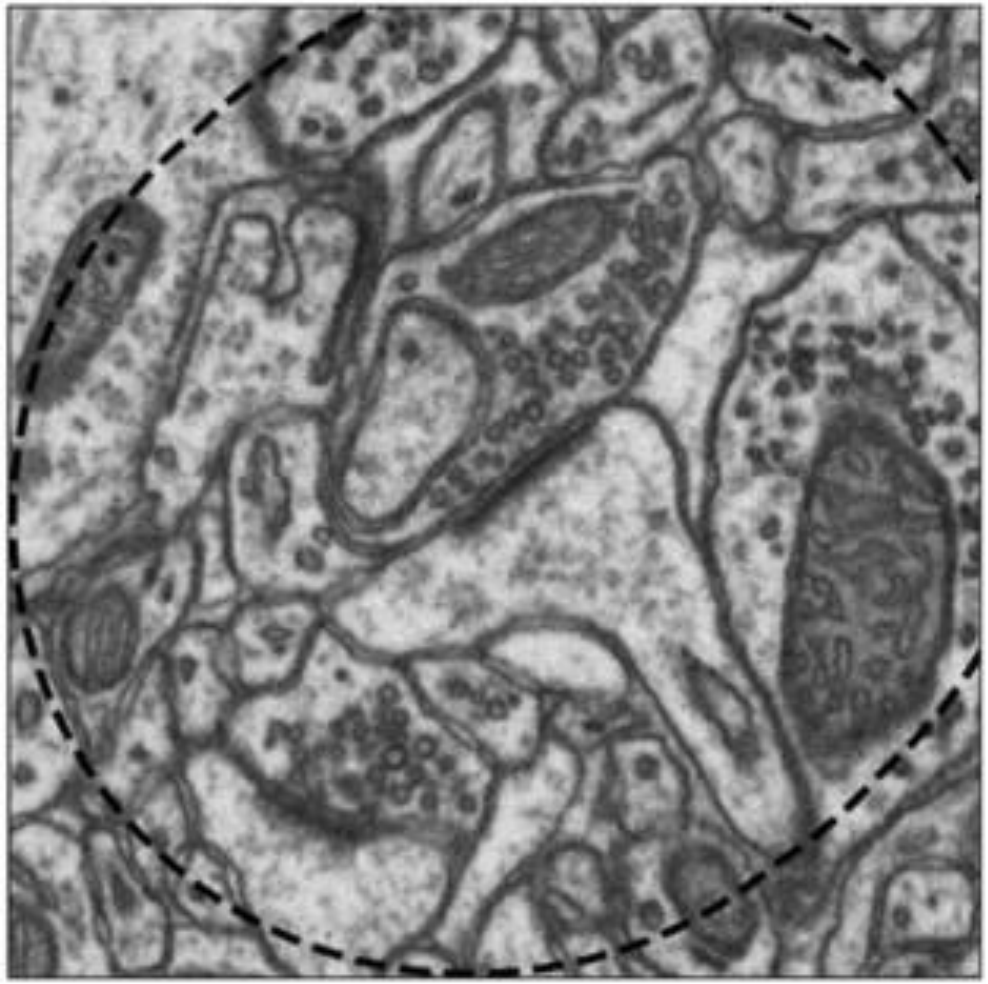
$$g_{00}(r) = \frac{d\mu_{00}(r)}{dr}$$

$$f_{00}(r) = \frac{\mu_{00}(r)}{\mathcal{H}(Y^r)}$$

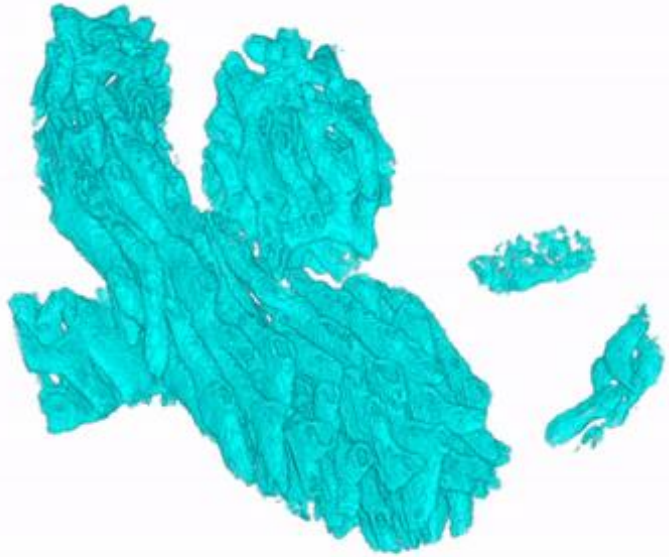




Analyzing cristae membranes in Mitochondria

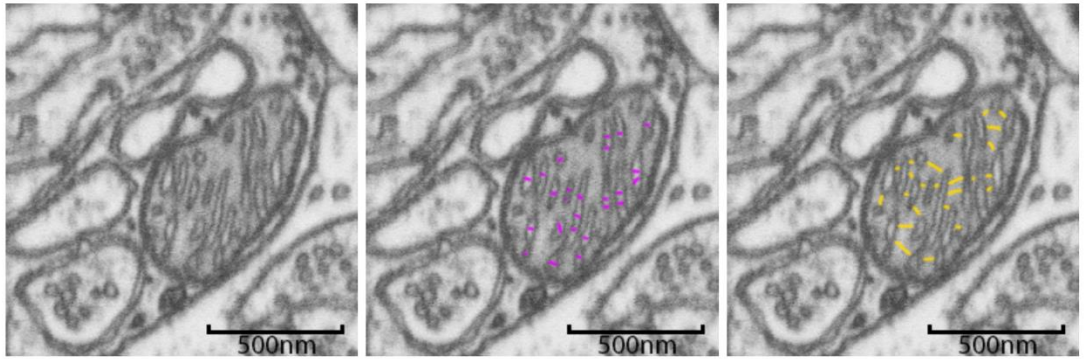


Deep Learning

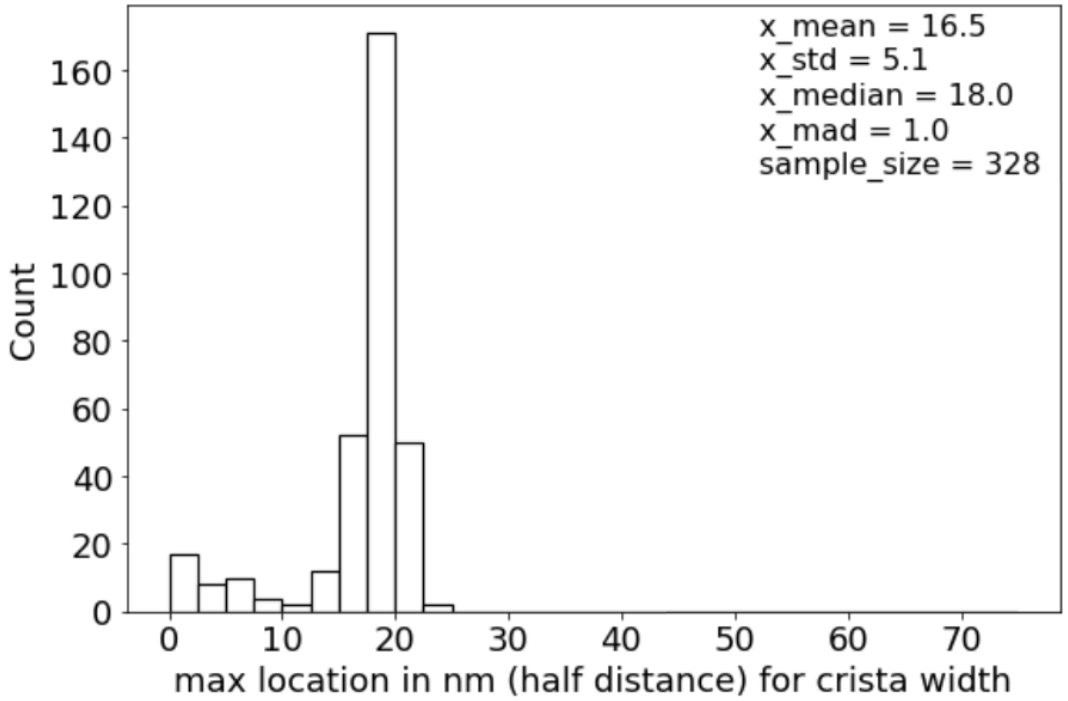


Extracting Mitochondrial Cristae Characteristics from 3D Focused Ion Beam Scanning Electron Microscopy Data, C Wang, L Østergaard, S Hasselholt, & J Sporring, to appear in Communications Biology, 2024

Persistent homology: Statistical measures on H_0



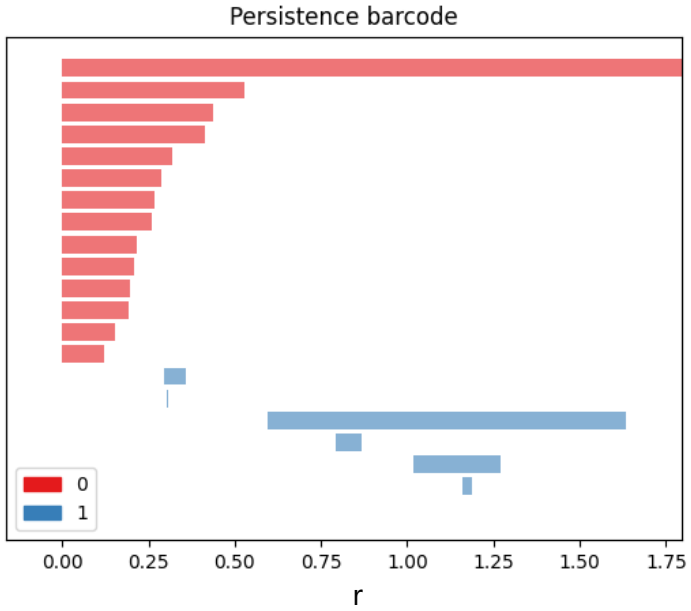
(a) (b) (c)

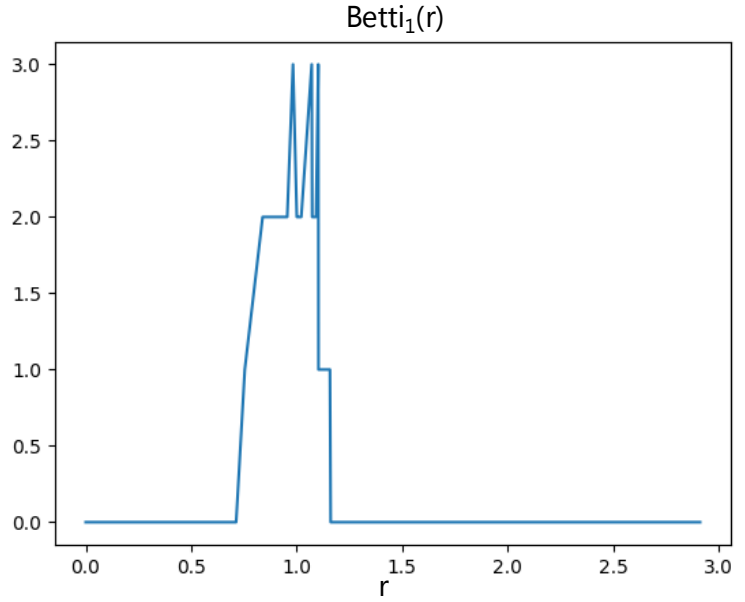


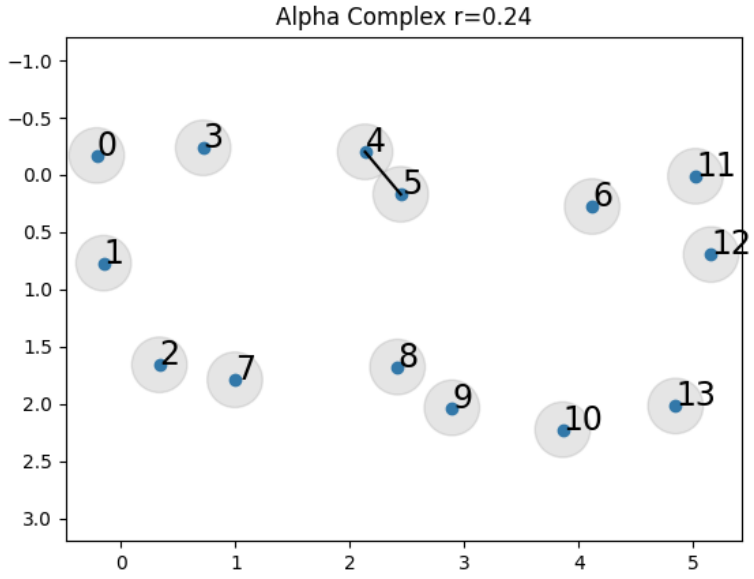
Persistent homology and bar codes

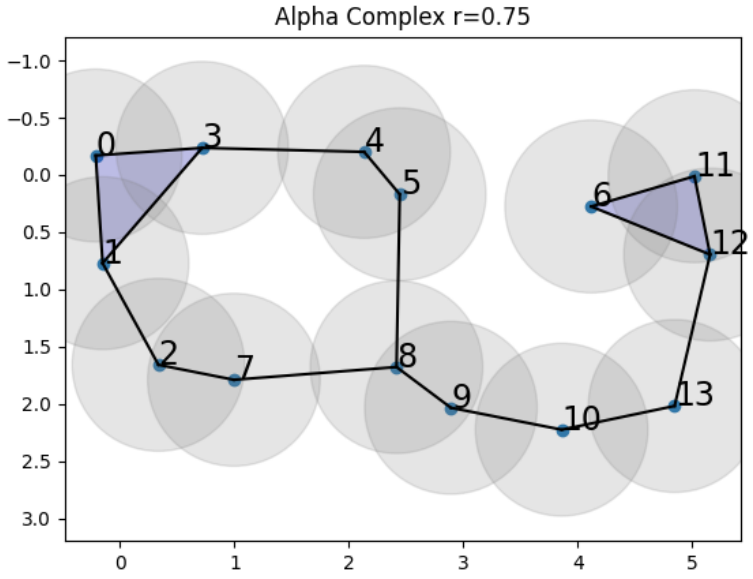
Simplex $\sigma = [x_0, x_1, \ldots x_k]$ is in the alpha complex if

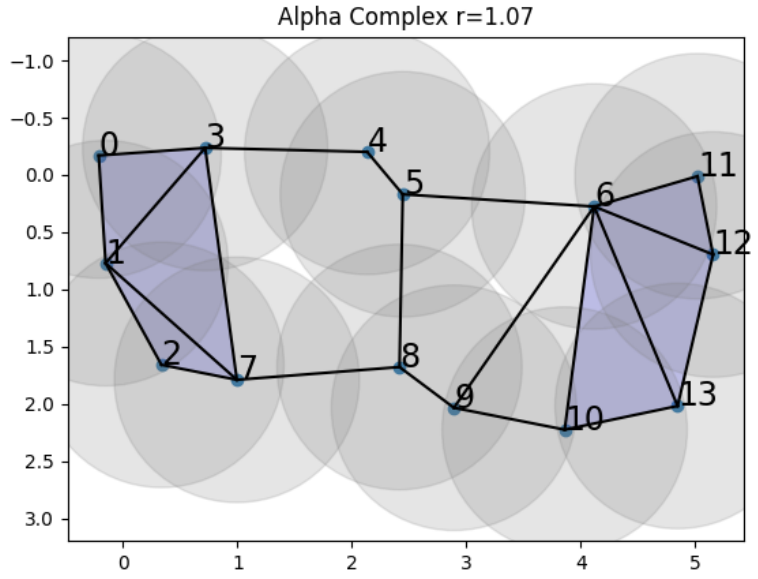
$$\bigcap_{x_i \in \sigma} B(x_i, r) \neq \emptyset$$





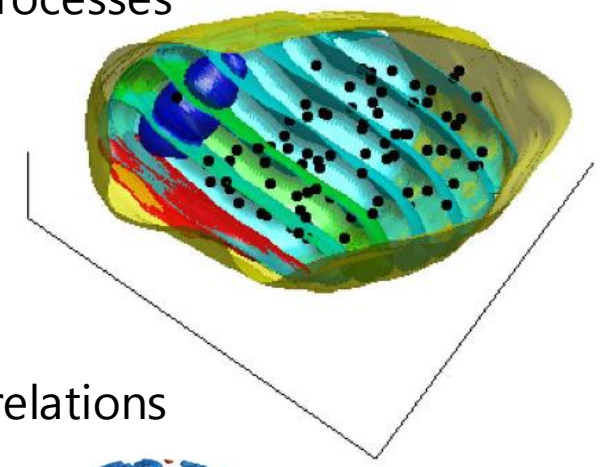






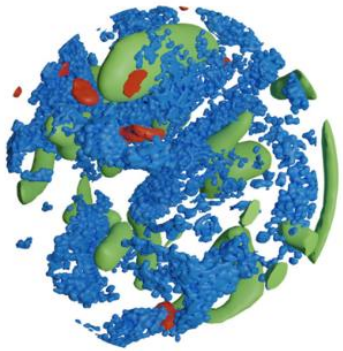
Statistical summary of object collections

Point processes



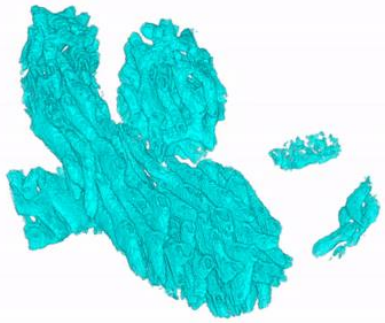
Pair correlation and Ripley's K functions summarizes 1st order point relations – e.g., do the vessicles cluster?

Shape relations



Hausdorf measures on overlaping sets extends notion of points to shapes – e.g., are mitochondria seen close to the synapse?

Persistent homology



Filtrations brings topological concepts to measurements - e.g., what is the average tubular radius of complicated objects