

# Abstract

Interspecific competition plays an important role in structuring communities but few studies have been done on herbivores. We present a competition model that predicts the outcome of competition between herbivore species competing for plants. Our model imbeds well-known concepts of the resource-ratio theory, such as the minimum level of resources, consumption vectors, and the quotas of resources required in the biomass. However, unlike traditional approaches that focused on plants as resources, we suggest that chemical elements and energy bounded in plant biomass represent the ultimate resources that herbivores compete for. Our model shows that the outcomes of competition between herbivores result from two main processes: the minimal requirement of resource for herbivores ( $R^*$ ), and spatial segregation of resources embedded into different plants. The first process (minimal requirement) follows the classical  $R^*$  rule and determines the competitive exclusion principle. On the other hand, foraging strategy of herbivores can allow coexistence because resources are spatially segregated. Put together, these two processes determine how herbivore species can coexist, since resources are bounded. Hence, a plant community rich in one resource should support an herbivore community rich in the same resource, leading to a bottom-up effect.

# Introduction

Interspecific competition for resources is thought to play an important role in structuring communities (Gause, 1934; Tilman, 1987). Modelling approaches have proven helpful to predict competitive outcomes, when resources are well identified and the number of different resources is limited. Hence, the theory is particularly well suited for autotrophic organisms (e.g., algae or plants) competing for essential nutrients (Tilman, 1982). Indeed, nutrients (i.e., chemical elements constituting biomass) are well-identified non-substitutable resources, and usually, competition is acute for only a few limiting nutrients, such as, for instance, phosphorous (P), nitrogen (N), or potassium (K). In short, models predict that niche segregation along resource ratios should promote species coexistence. For example, plants with low N:P requirement are more likely to coexist with plants with high N:P requirement. Tilman (1980) provided a graphical representation of these competition-coexistence processes based on Zero-Net-Growth Isoclines and consumption vectors, usually referred to as the resource-ratio theory. In the last few decades, the resource-ratio theory has helped popularize the use of competition models to predict competitive outcomes in experimental set ups and in semi-natural conditions.

However, the transfer of such a theory to higher trophic levels is not obvious. The main reason is that for heterotrophic organisms, resources are not easy to characterize. Consider for instance the case of herbivores. If one considers that plant species are the resources herbivores compete for, questions arises: to what extent does a given herbivore require more of a given plant species than another? Should plant species be considered essential non-substitutable resources, or substitutable resources? For example, the diet of the red deer (*Cervus elaphus*) in Europe includes

145 different plant species (Gebert and Verheyden-Tixier, 2001). How many of these  
species are really essential for red deer, and how many of the essential resources  
are substitutable? This question is a research topic in itself. As a consequence,  
predicting competitive outcomes in heterotrophic communities with simple models  
remains extremely challenging and restricted to very specific cases (Murray and  
Baird, 2008). This limitation may prevent significant progress in our understanding  
of community structure at higher trophic levels.

In accordance with the theory of biological stoichiometry (Sternern and Elser,  
2002), we argue here that nutrients and energy contained in plant biomass rather  
than the plants themselves are the resources which limit the growth rate of herbivores  
and for which herbivores compete for. In this case, most of the upper-cited challenges  
disappear. Indeed, nutrients and energy clearly are essential non-substitutable re-  
sources for heterotrophs. For example, nitrogen is required in proteins and nucleic  
acids, and phosphorous is required in phospholipids, nucleic acids, or bones. The  
number of essential nutrients required for heterotrophic organism biomass of het-  
erotrophs does not exceed 26 (Sternern and Elser, 2002). The fact that herbivore's  
growth rate may be limited by food quality (e.g., the level of nitrogen or phosphorus  
in plant biomass) rather than food quantity has been widely documented (Sternern  
et al., 1992; Hessen, 1992; Urabe and Watanabe, 1992). Moreover, it is possible to de-  
termine the requirements of a given herbivore for a given element, using metabolism  
studies (Mould and Robbins, 1981). In addition, it is clear that the ratios of nutri-  
ents required in the biomass vary across herbivore species. For example, the C:N:P  
ratio markedly varies among zooplanktonic herbivores (Andersen and Hessen, 1991;  
Sternern et al., 1992). In table 1, we show that such a variation is observed as well  
in terrestrial herbivores. Based on these observations, a competition model based  
on niche segregation across herbivore species along energy and nutrient axes appears  
perfectly relevant. All these arguments support the idea that the resource ratio the-  
ory could be applied to herbivores if one considers that just like plants, herbivores  
are ultimately limited by energy and nutrients (figure 1).

Yet, a major challenge remains: while plants take up energy and different nu-  
trients separately, herbivores consume these resources already packaged within the  
plant biomass they ingest. This constraint precludes a straightforward use of the re-  
source ratio theory. In this paper, we introduce a theoretical framework that adapts  
the classical resource-ratio theory to the specific case of herbivores consuming nutri-  
ents and energy that are bounded in fixed ratios in plant biomass. We show that it  
is possible to predict the competitive outcomes based on plant stoichiometry, and on  
herbivore feeding strategies.

## The model

Simulations and graphics were performed with the R software (R Development Core  
Team, 2013). Equations for isoclines and consumption vector slopes were found  
analytically, as well as stability of equilibrium points. Results were also tested with  
simulations using Runge-Kutta 4 approximation, from package deSolve (Soetaert  
et al., 2010).

86 The model includes  $p$  herbivore species competing for  $k$  resources embedded into  
 87  $n$  plant species. For each resource  $u$ , each herbivore has its own specific require-  
 88 ment, and each plant contains its own resource availability. These requirements and  
 availabilities are represented by quotas (i.e., quantity of resource per biomass unit).  
 90 Hence, for an herbivore  $i$ , the biomass dynamic writes:

$$\frac{dH_i}{dt} = \left( \text{Min} \left\{ \frac{\sum_{j=1}^n g_{ij} V_j Q_{vjR1}}{Q_{hiR1}}, \dots, \frac{\sum_{j=1}^n g_{ij} V_j Q_{vjRk}}{Q_{hiRk}} \right\} - m_i \right) H_i \quad (1)$$

Where  $H_i$  is herbivore  $i$  biomass,  $V_j$  is plant  $j$  biomass,  $g_{ij}$  is consumption rate of  
 92 plant  $j$  by herbivore  $i$ ,  $m_i$  is mortality rate of herbivore  $i$ ,  $Q_{vjRk}$  is the quota of  
 resource  $k$  into plant  $j$  biomass (for example: g  $R_k$ /kg of plant  $j$ ), and  $Q_{hiRk}$  is  
 94 quota of resource  $k$  into herbivore  $i$  biomass (for example: g  $R_k$ /kg of herbivore  $i$ ).  
 The ratio between herbivore and plant quotas sets influence of this chemical element  
 96 on growth limitation for this herbivore. According to Liebig's law of minimum, the  
 least available resource relative to herbivore requirements is assumed to be growth  
 98 limiting.

Plant biomass is:

$$\frac{dV_j}{dt} = S_j - a_j V_j - \sum_{i=1}^p g_{ij} V_j H_i \quad (2)$$

100 Where  $S_j$  is a function representing the increase of plant biomass (gross supply),  $a_j$   
 is intrinsic loss per capita, such as senescence, the last term on the right represents  
 102 consumption by herbivores.

Resource availability is described as the plant biomass multiplied by the plant quota  
 104 of the considered element.

$$R_u = \sum_{j=1}^n V_j Q_{vjRu} \quad (3)$$

Table 2 represents a review of the state variables and the parameters used in the  
 106 model. Indeed, each plant provides several resources, and resources are provided by  
 several plants. The first case is a stoichiometric balance issue between plant and  
 108 herbivore, while the second case is a spatial segregation of resources.

## Bottom-up stoichiometry

110 Herbivores might compete for one or several resources. First of all, consider a simple  
 model with  $p$  herbivore species feeding on one plant species embedding a single  
 112 resource. In case of one resource involved, this resource ( $R$ ) is assumed to be growth  
 limiting.

114 For a matter of simplicity, in case of one plant and one resource, resource quota  
 in plant biomass ( $Q_{vjRk}$ ) is written  $Q_{vR}$ , and resource quota in herbivore  $i$  biomass  
 116 ( $Q_{hiRk}$ ) is written  $Q_{hiR}$ . At steady state, for any herbivore  $i$ , the model leads to the  
 following equations (see appendix 1 for details):

$$\bar{H}_i = \frac{g_i Q_{vR} S - a Q_{hiR} m}{g_i Q_{hiR} m} \quad (4)$$

Therefore, at equilibrium, herbivore biomass increases with plant availability (supply) and resource quota in plant biomass ( $Q_{vR}$ ), while it decreases with herbivore mortality and resource quota in herbivore biomass ( $Q_{hiR}$ ). Resource availability at steady state is:

$$\bar{R}_{hi} = \bar{V} Q_{vR} = \frac{S}{a + g_i \bar{H}_i} Q_{vR} \quad (5)$$

This  $\bar{R}_{hi}$  represents level of resource remaining available for another herbivore, it also represents the minimal threshold of resource availability for herbivore species  $i$  (i.e., if resource availability is below this threshold this herbivore species cannot maintain a population at steady state). Indeed, it can be assimilated to Tilman's  $R^*$ . Therefore, considering two herbivores (species 1 and species 2), if  $g_1 \bar{H}_1 > g_2 \bar{H}_2$ , species 1 depletes resource below the threshold of species 2, and species 2 is excluded. In other words, the competitive exclusion principles holds, that is, no more than one herbivore species can persist on one limiting resource if this resource is provided by a single plant. In other words, plant stoichiometry has an impact herbivore competition, which is a bottom-up stoichiometric constraint.

According to the classical resource-ratio theory (Tilman, 1982), increasing the number of resources considered should promote coexistence between consumers. Consider  $p$  herbivores feeding on one plant species embedding  $k$  resources. According to the Liebig's law of the minimum expressed by the minimum function in equation 1, for each herbivore  $i$  the interplay between resource requirement and resource availability will determine which resource is the most limiting for growth (Grover, 1997). Depending on their requirements, and on the ratios of resources available in plant biomass, two herbivores feeding on a single plant species may not be limited by the same resource. Thus, requirement differences among herbivores can be interpreted as niche segregation.

For a single herbivore  $i$ , feeding on the plant, the level of limiting resource  $R_u$  (with  $u \in [1, k]$ ) in plant biomass at equilibrium writes (see appendix 3 for details):

$$\bar{R}_{uhi} = \bar{V}|_{R_u} Q_{vRu} = \frac{m_i}{g_i} Q_{hiRu} \quad (6)$$

where  $\bar{V}|_{R_u}$  is plant biomass at steady state when  $R_u$  is limiting. Similarly to former cases, this level represents the level of resource available to another herbivore and is therefore a key driver of competitive outcomes. It can be graphically represented by a zero net growth isocline (ZNGI) in the phase space of the  $k$  resources in plant biomass. One can also represent how resources are consumed by the herbivore with a consumption vector in this phase space (Tilman, 1980). As for the classical resource-ratio theory (Tilman, 1982), we will show here that this graphical representation is a valuable tool to address the competitive outcomes between herbivore species.

For simplicity, consider two herbivore species ( $H_1$  and  $H_2$ ) competing for two resources ( $R_1$  and  $R_2$ ) embedded into a single plant species ( $V$ ). According to equation 6, the slopes of the ZNGIs only depend on herbivore parameters, which are constant.

Therefore, the ZNGIs are parallel to the axis on the phase plan  $\{R_1, R_2\}$ . According to the classical resource-ratio theory (Tilman, 1982), a first necessary condition for coexistence at equilibrium is that the ZNGIs of the two herbivores cross one another. A second necessary condition relies on the difference between the slopes of the consumption vectors of the competitors (Tilman, 1982). The consumption vectors graphically illustrate how resource consumption drives the levels of resources from resource supply to equilibrium levels. In the phase plan, resource supply is represented by the supply point  $S$ , that is, the total amount of resources at equilibrium in absence of consumption:

$$S_{R_1} = Q_{vR_1}(S - a\bar{V}) \quad (7)$$

$$S_{R_2} = Q_{vR_2}(S - a\bar{V}) \quad (8)$$

For a given herbivore  $i$ , the consumption vector  $\vec{C}_i$  writes:

$$\vec{C}_i = \begin{pmatrix} g_i \bar{V} \bar{H}_i Q_{vR_1} \\ g_i \bar{V} \bar{H}_i Q_{vR_2} \end{pmatrix} = g_i \bar{V} \bar{H}_i \begin{pmatrix} Q_{vR_1} \\ Q_{vR_2} \end{pmatrix} \quad (9)$$

The vectors slope is determined by the ratios of resources embedded in plant biomass. An important consequence is that all herbivores have the same slope of consumption vector when feeding on a single plant. This constraint precludes coexistence, and the resource-ratio embedded in plant biomass determines which of the competitor will displace the other (figure 2). Hence, despite niche segregation on the requirement axe, two herbivores cannot coexist on two resources. This result is in contrast with the classical resource-ratio theory, which stipulates that two consumer may coexist on two resources. The reason is that classically, consumers are assumed to control the slope of their consumption vector (Tilman, 1982). This assumption is valid for primary producers, which take up essential nutrients independently in their environment, in the form of dissolved chemical molecules, but not for herbivores, which take up essential nutrients already bounded in plant biomass. Therefore, niche segregation alone cannot promote coexistence. Only herbivore that can match plant stoichiometry the best can persist and outcompete other herbivores. This leads to a strong bottom-up stoichiometric constraint.

## Spatial segregation of resources

However, consider now that a limiting resource ( $R$ ) is provided by  $n$  plants. Herbivores have access to the resource through the consumption of several species of plants, depending on their feeding strategy. At steady state, it is possible to calculate  $\bar{R}_{hi}$  for each herbivore  $i$  (see appendix 2 for details):

$$\bar{R}_{hi} = \sum_{j=1}^n \frac{S_j}{a_j + g_{ij} \bar{H}_i} Q_{v1R} \quad (10)$$

For simplicity, consider the case where two herbivore species ( $H_1$  and  $H_2$ ) compete for one resource ( $R$ ) embedded into two plants species ( $V_1$  and  $V_2$ ). Extending equation

188 10, resource availabilities for each herbivore species at steady state are:

$$\bar{R}_{h1} = \frac{S_1}{a_1 + g_{11}\bar{H}_1} Q_{v1R} + \frac{S_2}{a_2 + g_{12}\bar{H}_1} Q_{v2R} \quad (11)$$

and

$$\bar{R}_{h2} = \frac{S_1}{a_1 + g_{21}\bar{H}_2} Q_{v1R} + \frac{S_2}{a_2 + g_{22}\bar{H}_2} Q_{v2R} \quad (12)$$

190 Clearly, the competition outcome depends on the foraging strategies of the com-  
 192 petitors, more specifically, on the efficiencies of the consumption functions ( $g_{ij}$ ). If  
 $g_{11} > g_{21}$  and  $g_{12} > g_{22}$ , herbivore 1 consumes both plants more efficiently than  
 its competitor. Thus, the competitive exclusion principle holds, and herbivore 2 is  
 194 excluded. The reverse situation ( $g_{11} < g_{21}$  and  $g_{12} < g_{22}$ ) leads to the exclusion of  
 herbivore 1. However, if the two herbivores are specialized on different plant species  
 196  $g_{11} > g_{21}$  and  $g_{12} < g_{22}$ , or if  $g_{11} < g_{21}$  and  $g_{12} > g_{22}$ , coexistence is possible. In other  
 words, despite the fact that herbivores compete for a single resource, the competitive  
 198 exclusion principle does not necessarily hold. Indeed, plants species create a spatial  
 segregation of the resource, and the specialization of herbivores on different plant  
 200 species, similar to niche segregation in space, makes coexistence possible because  
 herbivores can overcome the bottom-up stoichiometric constraint.

## 202 Competition for several plant species and several resources

In this section, we present the general case of the model, where herbivores compete  
 204 for multiple resources embedded into multiple plants. Assuming that nutrient quo-  
 tas differ among plant species, the consumption of a given plant is not equivalent  
 206 to consumption of another plant. Due to spatial segregation among resources, her-  
 bivores may control their diet by allocating more time/energy to feed on specific  
 208 plants rather than others. Therefore, each herbivore may find a specific pathway to  
 collect resources (Simpson and Raubenheimer, 1995; Raubenheimer and Simpson,  
 210 1999). The foraging strategy of a given herbivore is graphically represented by the  
 consumption vector, which is the combination of the consumption vectors for all  
 212 the plant species consumed. As a consequence, the slope of the consumption vector  
 depends on the foraging strategy of the herbivore, which can adjust the slope by  
 214 consuming more efficiently a given species versus the others. Thus, niche segregation  
 among herbivore species is a combination of spatial segregation and segregation on  
 216 the requirements. As we shall see, this niche segregation allows for coexistence of  
 multiple herbivores on multiple resources.

218 To simplify, we consider here the case of two herbivores competing for two plants  
 embedding two resources (see appendix 4 for details). The resource supply is repre-  
 220 sented in the phase plan by the supply point S, which is the total amount of resources  
 at equilibrium in plants 1 and 2, in absence of consumption:

$$S_{R1} = Q_{v1R1}(S_{v1} - a\bar{V}_1) + Q_{v2R1}(S_{v2} - a\bar{V}_2) \quad (13)$$

$$S_{R2} = Q_{v1R2}(S_{v1} - a\bar{V}_1) + Q_{v2R2}(S_{v2} - a\bar{V}_2) \quad (14)$$

For a given herbivore  $i$ , the ZNGI slope ( $\alpha|_{Ru}$ ) for each resource ( $R_u$ ) is:

$$\alpha|_{Ru} = \frac{\bar{V}_1|_{Ru} Q_{v1R2} + \bar{V}_2|_{Ru} Q_{v2R2}}{\bar{V}_1|_{Ru} Q_{v1R1} + \bar{V}_2|_{Ru} Q_{v2R1}} \quad (15)$$

224 Note that the slope depends on  $\{\bar{V}_1, \bar{V}_2\}$ , which themselves depend both on the  
herbivore and the plant parameters (including the supply parameters). Hence, the  
226 ZNGI slope is not parallel to the axis.

However, it is possible to define boundary ZNGI. Considering that herbivore  $i$  can  
228 consume both plants, its foraging strategy will lie between exclusive consumption of  
plant 1, on one side, and exclusive consumption of plant 2, on the other side. Thus,  
230 boundary ZNGI slope writes (see appendix 4 for details):

$$\bar{R}_u = \frac{Q_{hiRu} m_i}{g_{ij}} \quad (16)$$

This isocline  $\bar{R}_u$  represents the ZNGI in the case where herbivore  $i$  consumes only  
232 plant  $j$  and is limited by nutrient  $u$ . In case of two plants consumed, the real  $\bar{R}_u$  will  
lie between two boundary ZNGI (one for each plant).

234 By looking at boundary ZNGI equation (16), it appears that ZNGI slopes depend  
only on herbivore constant parameters. Therefore, it is parallel to the axis. Moreover,  
236 two herbivores having similar quotas but different feeding strategies would have dif-  
ferent ZNGI. Thus, if boundary ZNGI of two herbivores cross each other, coexistence  
238 might be possible.

Next, competition outcome will depend on the relative orientation of the con-  
240 sumption vector of herbivores (Tilman, 1980). However, vector slope calculation is  
uneasy in the case of packaged resources because vector slope depends on plant sup-  
242 ply points. Hence, for a given herbivore, vectors do not have a constant slope valid  
for all supply conditions. This is why instead of consumption vectors, the model  
244 allows the calculation of boundary vectors that determine an area in the phase plan  
where each herbivore species could coexist with an other one. It is possible to deter-  
246 mine a boundary relation between plant supply points ( $S_1$  and  $S_2$ ) that allows each  
herbivore to consume both resources when one resource is limiting, and leads to  $\bar{V}_1$   
248 and  $\bar{V}_2$  at steady state. These boundary supply points are  $S_{1hi}$  and  $S_{2hi}$  for plant 1  
and 2 respectively (see appendix 4). Hence, boundary vector slopes are:

$$\vec{C}_{h1} = \begin{pmatrix} \bar{S}_{1h1} Q_{v1R1} + \bar{S}_{2h1} Q_{v2R1} \\ \bar{S}_{1h1} Q_{v1R2} + \bar{S}_{2h1} Q_{v2R2} \end{pmatrix} \quad (17)$$

250

$$\vec{C}_{h2} = \begin{pmatrix} \bar{S}_{1h2} Q_{v1R1} + \bar{S}_{2h2} Q_{v2R1} \\ \bar{S}_{1h2} Q_{v1R2} + \bar{S}_{2h2} Q_{v2R2} \end{pmatrix} \quad (18)$$

These two boundary vectors play a similar role as consumption vectors in Tilman's  
252 model. However, as resources are packaged into plants, vector slopes depend on the  
way herbivore species feed on plants containing resources. Hence, assuming plant 1  
254 is richer in resource 1 ( $Q_{v1R1} > Q_{v2R1}$ ), and plant 2 is richer in resource 2 ( $Q_{v1R2} <$   
 $Q_{v2R2}$ ), and assuming herbivore 1 is more limited by resource 1 ( $Q_{h1R1} > Q_{h1R2}$ ), and  
256 herbivore 2 is more limited by resource 2 ( $Q_{h2R1} < Q_{h2R2}$ ), two general strategies can

be considered. The first one occurs when each herbivore species consumes preferentially the plant species which gives the greater quantity of the most limiting resource for this herbivore species, and which is less limiting for its competitor (i.e.,  $g_{11} > g_{12}$  and  $g_{21} < g_{22}$ ). In that case, if we consider the boundary vectors for herbivore 1 and for herbivore 2, we can define several zones on the phase plan (see figure 3). The zone between the two boundary vectors represents the supply conditions allowing stable coexistence of the two herbivore species. Thus, each herbivore consumes the most profitable plant (i.e., quotas and consumption functions follow a similar trend), but each herbivore is specialized (more or less) on a different plant than its competitor, which promotes coexistence.

The second case represents the reverse situation, where each herbivore species consumes preferentially the plant species which provides greater quantity of the most limiting resource for its competitor (i.e.,  $g_{11} < g_{12}$  and  $g_{21} > g_{22}$ ). Hence, the zone between the boundary vectors does not allow coexistence (see figure 4), and in many cases equation 32A does not have any realistic solution (with both herbivore species persisting). Thus, competitive exclusion is the general outcome.

Spatial segregation of resources embedded in plants and herbivore response to bottom-up stoichiometry are the two key points. If resources are not spatially decoupled or if herbivores do not have a feeding strategy that can compensate for the bottom-up stoichiometry imbalance, coexistence is impossible between herbivore species. The following cases illustrate this argument. A first case occurs when herbivore species are not selective at all: they consume plant species according to their respective biomass. In that case, both herbivores consume resources in the same way (i.e.,  $g_{11} = g_{21}$  and  $g_{12} = g_{22}$ ). Hence, there is no segregation in herbivore consumption: all herbivore species have the same consumption vector, which is similar to the case where only one plant species is present (see figure 2). Therefore, the trajectory will either cross herbivore 1 ZNGI first, and this herbivore will be excluded, or the trajectory will cross herbivore 2 ZNGI first, and this herbivore will be excluded. Coexistence will be unlikely because even if resources are spatially segregated, herbivores are constrained by plant stoichiometry.

A second case occurs when the previous assumptions on herbivore and plant quotas (i.e., inequality for herbivore requirements, and inequality for plant profitability) are released. Therefore, both herbivores can have the same quotas (i.e.,  $Q_{h1R1} = Q_{h2R1}$  and  $Q_{h1R2} = Q_{h2R2}$ ) but different consumption functions. In that case, one plant will be more interesting for both herbivores (i.e., the plant that is richer for the most limiting resource). Hence, the herbivore that can consume this plant the most efficiently will exclude its competitor. Again, bottom-up stoichiometry constrained herbivore coexistence. Another case can occur when both plants have the same quotas (i.e.,  $Q_{v1R1} = Q_{v2R1}$  and  $Q_{v1R2} = Q_{v2R2}$ ). In that case, the system is similar to the case where only one plant is present, which leads to competitive exclusion. Here, spatial segregation does not occur.

Last, one can notice that plant stoichiometry constrains the supply point because of resource packaging into plants. Resource ratio within each plant will define a slope:

$$\alpha_{vj} = \frac{Q_{vjR2}}{Q_{vjR1}} \quad (19)$$



In case of two plants, supply point will lie between the two extreme slopes ( $\alpha_{v1}$  and  $\alpha_{v2}$ ) defining a feasibility cone. Therefore, the whole phase plan is not available. Resource supply occurs only within this feasibility cone, which adds another constraint on herbivore persistence and competition.

## Discussion

Unlike classical studies on herbivore competition we consider here that the resource limiting herbivores growth is not plant biomass but rather, the essential nutrients contained in plant biomass.

According to our results, the fundamental difference between herbivore competition and plant competition is that resources are not independent, which leads to specific constraints for herbivores. Thus, several differences exist between classical resource-ratio theory of competition usually applied to plants (Tilman, 1980) and our model for herbivores. First, if only one plant species is available, coexistence is almost impossible even if herbivore compete for several resources. Second, ZNGI can be represented with boundary ZNGI delimiting resource levels at steady state.

Third, consumption vectors are not informative because their slope varies with supply points. However, boundary vectors can be calculated, and they play a similar role as usual consumption vectors. Last, due to this packaging effect of resources and to plant quotas, resource availability itself is constrained. Hence, part of the phase plan might be unavailable. This is a major difference with models assuming a total independence between resources (such as N and P for plants) within which availabilities of resources can vary independently from one another. When resources are packaged, they are not independent. Thus, some supply couples (e.g., large quantity of  $R_1$  and almost no  $R_2$  available) is unlikely because plants will provide both resources (according to their own ratio). Hence, parameter space allowing herbivore persistence can be narrower than predicted by its ZNGI and vector.

More generally, existing theories about coexistence between herbivores are based on niche segregation. This segregation may depend on space utilization or on relationship between body size and metabolism (Owen-Smith and Novellie, 1982). Our approach is different and allows us to disentangle the different mechanisms of niche segregation. We argue that coexistence between herbivores can occur by two ways, which are diversity between foraging strategies as well as stoichiometric diversity between niches. Diversity between foraging strategies is a spatially niche segregation within which each herbivore species consumes one plant species more than others, assuming that this plant species has the best profitability for this herbivore species. If this spatial segregation is total, with each herbivore species specialized on one plant species which is different from other competitors, coexistence occurs without other constraint than herbivore species-specific persistence. The second way is niche segregation based on resource ratios (i.e., a bottom-up stoichiometric component). Although the packaging effect creates a supplementary level of complexity, divergence with classic resource-ratio models should not impede the rising of a clear conclusion: coexistence is favoured by requirement segregation as well as consumption segregation; only one of them (i.e., different requirements and similar consumptions, or

similar requirements and different consumptions) is not enough. We retrieve here  
344 the two classical components of the niche theory (Chase and Leibold, 2003): species  
requirement (Hutchinson, 1957) and species impact (Elton, 1927; Macarthur and  
346 Levins, 1967). However, while these two components are clearly separated for com-  
petition between plants (i.e., ZNGI represent species requirements, and vectors rep-  
348 resent species impact), these two components are merged for herbivore competition  
(boundary ZNGIs and boundary vectors involve both aspects).

350 Nonetheless, our model allows us to disentangle the relative effects of the bottom-  
up stoichiometric component (i.e., plant and herbivore resource quotas) and the non-  
352 stoichiometric component (i.e., spatial segregation) of the competitive interaction.  
Indeed, the foraging strategy of the competing herbivores will mainly determine  
354 the competitive outcome. Non-selective herbivores (representing case 2 and case  
4 with non-selective behavior) will show a strong bottom-up stoichiometric effect.  
356 Therefore, coexistence is unlikely. For example, non-selective zooplanktonic filters,  
such as cladocerans (limited by P) and copepods (limited by N), usually do not  
358 coexist: either one group or the other dominates according the N:P ratio of the  
consumed algae (Andersen and Hessen, 1991; Hessen, 1992; Sterner et al., 1992;  
360 Elser et al., 1996; Koski, 1999). A similar assumption can be made for non-selective  
terrestrial grazer herbivores (Albon and Langvatn, 1992) for which coexistence would  
362 be unlikely. On the other hand, selective herbivores, specialized on a few plants,  
would show a strong spatial component and a weaker stoichiometric component of  
364 the interaction (representing case 2 and case 4 with selective behavior). Therefore,  
they should be more prone to coexist.

366 Concerning the stoichiometric component, our assumption is that a stoichiometric  
diversity between niches exists among herbivore species, and field studies tend to  
368 show it. For aquatic herbivores, such as zooplankton, it seems that grazers (especially  
*Daphnia sp.*) are P-limited, while copepods seem to be N-limited (see above). This  
370 stoichiometric diversity exists for terrestrial herbivores either (see table 1). Chemical  
requirements for wild herbivores are mostly unknown, but for mammal herbivores,  
372 numerous studies have used data from cattle as proxy for diet requirement for wild  
species (Voeten and Prins, 1999). Nevertheless, data begins to be available. For  
374 example, N requirements have been estimated for fawns (Smith et al., 1975), yearlings  
(Holter et al., 1979) and adults (Asleson et al., 1996) of white-tailed deer (*Odocoileus*  
376 *virginianus*), as well as P requirement for white-tailed deer (Grasman and Hellgren,  
1993) and moose (Schwartz et al., 1987). Moreover, males from big species should  
378 be more concerned by P-limitation (Grasman and Hellgren, 1993). More generally,  
it seems that nutrient requirement and absorption constraint depend on body size  
380 and digestive system (Janis, 1976).

Concerning the spatial component and the foraging strategy, several studies have  
382 been done, mainly on terrestrial herbivore species. It seems that most of migrated  
species adopt an extraction maximizing strategy: through seasons, they move from  
384 places to places that have a large amount of nutritive quality elements (Albon and  
Langvatn, 1992). There, they can find plant communities with a high proportion of  
386 nutritional plant species. They adopt this strategy rather than selecting nutritious  
species within communities (Ben-Shahar and Coe, 1992). On the other hand, resident  
388 species mostly adopt a demand minimizing strategy: sometimes, they have a low

metabolic rate, and flexible breeding period, which allow them to decrease demand  
 390 in energy during dry period (Murray, 1991). Moreover, for continental herbivore  
 species, the diet quality decreases when body size increases, especially during dry  
 392 season (Codron et al., 2007).

However, there are some limits to this model. We pay only attention to the  
 394 requirement for a given herbivore species. But we do not consider what happens in  
 case of an overconsumption of a non-limiting nutrient. In fact, we can have a more  
 396 physiological approach. Considering the case where an herbivore species consuming  
 a plant species that gives a small amount of a limiting nutrient and a large amount  
 398 of a non-limiting nutrient, one can argue that an excretion cost might exist for this  
 nutrient. This can limit the amount of plant consumed by this herbivore species.

Finally, this model gives an interesting conclusion. The stoichiometric constraints  
 400 should go up through the trophic chain. It means that, on a soil that is poor in a  
 given nutrient (e.g., nitrogen), we should find plant species that can survive with a  
 402 poor availability for this nutrient. Then, they contain a small amount of this nutrient  
 and they will sustain herbivore species that are poor in this nutrient, leading to a  
 404 strong bottom-up effect.

## 406 Appendix 3

When  $p$  herbivores compete for  $k$  resources embedded into 1 plant, equations 1, 2  
 408 and 3 respectively become:

$$\frac{dH_i}{dt} = \left( \text{Min} \left\{ \frac{Q_{vR1}}{Q_{hiR1}}, \dots, \frac{Q_{vRk}}{Q_{hiRk}} \right\} g_i V - m_i \right) H_i \quad (20A)$$

$$\frac{dV}{dt} = S - aV - \sum_{i=1}^p g_i V H_i \quad (21A)$$

$$R_k = V Q_{vRk} \quad (22A)$$

At steady state, it is possible to determine resource availability for each herbivore  $i$ .  
 412 Assuming that  $R_u$  is a limiting resource, with  $u \in [1, k]$ .

$$\bar{V}|_{R_u} = \frac{m_i}{g_i} \frac{Q_{hiRu}}{Q_{vRu}} \quad (23A)$$

Where  $\bar{V}|_{R_u}$  is the steady state plant biomass when  $R_u$  is limiting.  $\bar{R}_u$  is:

$$\bar{R}_u = \bar{V}|_{R_u} Q_{vRu} = \frac{m_i}{g_i} Q_{hiRu} \quad (24A)$$

## 414 Appendix 4

In a case of two herbivore species ( $H_1$  and  $H_2$ ) competing for two resources ( $R_1$  and  
 416  $R_2$ ) embedded into two plant species ( $V_1$  and  $V_2$ ), the system considered becomes:

$$\frac{dH_i}{dt} = \left( \text{Min} \left\{ \frac{g_{i1} V_1 Q_{v1R1} + g_{i2} V_2 Q_{v2R1}}{Q_{hiR1}}, \frac{g_{i1} V_1 Q_{v1R2} + g_{i2} V_2 Q_{v2R2}}{Q_{hiR2}} \right\} - m_i \right) H_i \quad (25A)$$

$$\frac{dV_j}{dt} = S_j - a_j V_j - g_{1j} V_j H_1 - g_{2j} V_j H_2 \quad (26A)$$

418

$$R_1 = q_{v1R1} V_1 + q_{v2R1} V_2 \quad (27A)$$

$$R_2 = q_{v1R2} V_1 + q_{v2R2} V_2 \quad (28A)$$

420 Each herbivore has two zero net growth isoclines (ZNGI), but their calculation is  
 422 more complicate because of multiple sources for  $R_1$  and  $R_2$ . However, it is possible  
 424 to define boundary ZNGI. Considering that herbivore  $i$  can consume both plants,  
 its foraging strategy will lie between exclusive consumption of plant 1, on one side,  
 426 and exclusive consumption of plant 2, on the other side. Thus, for an herbivore  
 $i$  consuming each plant  $j$  (with  $j \in [1, 2]$ ) and limited by each resource  $u$  (with  
 $u \in [1, 2]$ ), equation 25A becomes

$$\frac{dH_i}{dt} = \left( \frac{g_{ij} V_j Q_{vjRu}}{Q_{hiRu}} - m_i \right) H_i \quad (29A)$$

At steady state,  $\bar{R}_u = \bar{V}_j * Q_{vjRu}$ . Thus,

$$\frac{g_{ij} \bar{V}_j Q_{vjRu}}{Q_{hiRu}} - m_i = \frac{g_{ij} \bar{R}_u}{Q_{hiRu}} - m_i = 0 \quad (30A)$$

428 which leads to

$$\bar{R}_u = \frac{Q_{hiRu} m_i}{g_{ij}} \quad (31A)$$

$\bar{R}_u$  represents the slope of the boundary ZNGI for herbivore  $i$  consuming plant  $j$  and  
 430 limited by resource  $u$ . In case of two plants and two resources, each herbivore will  
 have two boundary ZNGI for each resources. The real value of  $\bar{R}_u$  will lie between  
 432 the two boundary ZNGI.

It is possible to calculate  $\bar{V}_1$  and  $\bar{V}_2$  that are the equilibrium biomass of plant 1  
 434 and 2 respectively and are solutions of the following system:

$$\begin{cases} g_{11} \bar{V}_1 Q_{v1R1} + g_{12} \bar{V}_2 Q_{v2R1} = m_1 Q_{h1R1} \\ g_{21} \bar{V}_1 Q_{v1R2} + g_{22} \bar{V}_2 Q_{v2R2} = m_2 Q_{h2R2} \end{cases} \quad (32A)$$

If system (32A) has two realistic solutions (i.e.,  $\bar{V}_1 > 0$  and  $\bar{V}_2 > 0$ ), and if  $\bar{H}_1 > 0$   
 436 and  $\bar{H}_2 > 0$ , which means that coexistence occurs at that point, then this point is  
 an equilibrium point. Knowing  $\bar{V}_1$  and  $\bar{V}_2$  at this equilibrium point, it is possible to  
 438 determine a boundary relation between plant supply points ( $S_1$  and  $S_2$ ) that allows  
 herbivore species 1 to consume both resources when  $R_1$  is limiting, and leads to  $\bar{V}_1$   
 440 and  $\bar{V}_2$  at steady state. Similarly, we can determine a relation that allows herbivore  
 2 to consume both resources when  $R_2$  is limiting, and leads to  $\bar{V}_1$  and  $\bar{V}_2$ . The  
 442 relations are:

$$\bar{S}_{2h1} = (\bar{S}_{1h1} - a_1) \frac{g_{12} \bar{V}_2}{g_{11} \bar{V}_1} + a_2 \bar{V}_2 \quad (33A)$$

$$\bar{S}_{2h2} = (\bar{S}_{1h2} - a_1) \frac{g_{22}\bar{V}_2}{g_{21}\bar{V}_1} + a_2\bar{V}_2 \quad (34A)$$

444 Where  $S_{1h1}$  and  $S_{2h1}$  are the boundary supply points for plant 1 and 2 respectively  
for herbivore 1. Similarly,  $S_{1h2}$  and  $S_{2h2}$  are boundary supply points for herbivore 2.

## 446 References

- Albon, S. D., and R. Langvatn. 1992. Plant phenology and the benefits of migration  
448 in a temperate ungulate. *Oikos* 65:502–513.
- Andersen, T., and D. O. Hessen. 1991. Carbon, nitrogen, and phosphorus content of  
450 freshwater zooplankton. *Limnology and Oceanography* 36:807–814.
- ARC. 1980. The nutrient requirements of ruminant livestock. Agricultural Research  
452 Council. CBA, Slough, UK .
- Asleson, M. A., E. C. Hellgren, and L. W. Varner. 1996. Nitrogen requirements for  
454 antler growth and maintenance in white-tailed deer. *Journal of Wildlife Manage-  
ment* 60:744–752.
- 456 Ben-Shahar, R., and M. J. Coe. 1992. The relationships between soil factors, grass  
nutrients and the foraging behavior of wildbeest and zebra. *Oecologia* 90:422–428.
- 458 Chase, J., and M. Leibold. 2003. Ecological niches: linking classical and contempo-  
rary approaches. University of Chicago Press.
- 460 Codron, D., J. A. Lee-Thorp, M. Sponheimer, J. Codron, D. De Ruiter, and J. S.  
Brink. 2007. Significance of diet type and diet quality for ecological diversity of  
462 african ungulates. *Journal of Animal Ecology* 76:526–537.
- Elser, J. J., D. R. Dobberfuhl, N. A. MacKay, and J. H. Schampel. 1996. Organism  
464 size, life history, and n:p stoichiometry. *Bioscience* 46:674–684.
- Elton, C. 1927. Animal ecology. University of Chicago Press.
- 466 Gause, G. 1934. The Struggle for Existence. Williams and Wilkins ed., Baltimore,  
Maryland. 163 pp.
- 468 Gebert, C., and H. Verheyden-Tixier. 2001. Variations of diet composition of red  
deer (*cervus elaphus* l.) in europe. *Mammal Review* 31:189–201.
- 470 Grasman, B. T., and E. C. Hellgren. 1993. Phosphorus nutrition in white-tailed deer:  
Nutrient balance, physiological responses, and antler growth. *Ecology* 74:2279–  
472 2296.
- Grover, J. 1997. Resource Competition. Chapman & Hall ed., London. 332 pp.
- 474 Hessen, D. O. 1992. Nutrient element limitation of zooplankton production. *Ameri-  
can Naturalist* 140:799–814.

- 476 Holter, J. B., H. H. Hayes, and S. H. Smith. 1979. Protein requirement of yearling  
white-tailed deer. *Journal of Wildlife Management* 43:872–879.
- 478 Hutchinson, G. E. 1957. Population studies - animal ecology and demography - con-  
cluding remarks. *Cold Spring Harbor Symposia on Quantitative Biology* 22:415–  
480 427.
- Janis, C. 1976. The evolutionary strategy of equidae and origins of rumen and cecal  
482 digestion. *Evolution* 30:757–774.
- Koski, M. 1999. Carbon : Nitrogen ratios of baltic sea copepods - indication of  
484 mineral limitation? *Journal of Plankton Research* 21:1565–1573.
- Ludwig, F., T. E. Dawson, H. de Kroon, F. Berendse, and H. H. T. Prins. 2003.  
486 Hydraulic lift in acacia tortilis trees on an east african savanna. *Oecologia* 134:293–  
300.
- 488 Macarthur, R., and R. Levins. 1967. Limiting similarity convergence and divergence  
of coexisting species. *American Naturalist* 101:377–385.
- 490 Menard, C., P. Duncan, G. Fleurance, J. Y. Georges, and M. Lila. 2002. Compara-  
tive foraging and nutrition of horses and cattle in european wetlands. *Journal of*  
492 *Applied Ecology* 39:120–133.
- Mould, E. D., and C. T. Robbins. 1981. Nitrogen metabolism in elk. *Journal of*  
494 *Wildlife Management* 45:323–334.
- Murray, M. G. 1991. Maximizing energy retention in grazing ruminants. *Journal of*  
496 *Animal Ecology* 60:1029–1045.
- Murray, M. G., and D. R. Baird. 2008. Resource-ratio theory applied to large herbi-  
498 vores. *Ecology* 89:1445–1456.
- Owen-Smith, N., and P. Novellie. 1982. What should a clever ungulate eat. *American*  
500 *Naturalist* 119:151–178.
- Raubenheimer, D., and S. J. Simpson. 1999. Integrating nutrition: a geometrical  
502 approach. *Entomologia Experimentalis Et Applicata* 91:67–82.
- Schwartz, C. C., W. L. Regelin, and A. W. Franzmann. 1987. Protein digestion in  
504 moose. *Journal of Wildlife Management* 51:352–357.
- Simpson, S. J., and D. Raubenheimer. 1995. The geometric analasis of feeding and  
506 nutrition: a user's guide. *Journal of Insect Physiology* 41:545–553.
- Smith, S. H., J. B. Holter, H. H. Hayes, and H. Silver. 1975. Protein requirement of  
508 white-tailed deer fawns. *Journal of Wildlife Management* 39:582–589.
- Soetaert, K., T. Petzoldt, and R. W. Setzer. 2010. Solving differential equations in  
510 r: Package desolve. *Journal of Statistical Software* 33:1–25.

- 512 Sterner, R., and J. Elser. 2002. Ecological stoichiometry: the biology of elements  
from molecules to the biosphere. Princeton Univ Press, New Yersey, USA. 584 pp.
- 514 Sterner, R. W., J. J. Elser, and D. O. Hessen. 1992. Stoichiometric relationships  
among producers, consumers and nutrient cycling in pelagic ecosystems. *Biogeo-*  
*chemistry* 17:49–67.
- 516 Tilman, D. 1980. Resources - a graphical-mechanistic approach to competition and  
predation. *American Naturalist* 116:362–393.
- 518 ———. 1982. *Resource Competition and Community Structure*, vol. 17. Princeton  
Univ Press, New Jersey, USA. 296 pp.
- 520 ———. 1987. The importance of the mechanisms of interspecific competition. *Amer-*  
*ican Naturalist* 129:769–774.
- 522 Treydte, A. C., I. M. A. Heitkonig, and F. Ludwig. 2009. Modelling ungulate depen-  
dence on higher quality forage under large trees in african savannahs. *Basic and*  
524 *Applied Ecology* 10:161–169.
- Urabe, J., and Y. Watanabe. 1992. Possibility of n or p limitation for planktonic  
526 cladocerans: an experimental test. *Limnology and Oceanography* 37:244–251.
- Voeten, M., and H. Prins. 1999. Resource partitioning between sympatric wild and  
528 domestic herbivores in the tarangire region of tanzania. *Oecologia* 120:287–294.

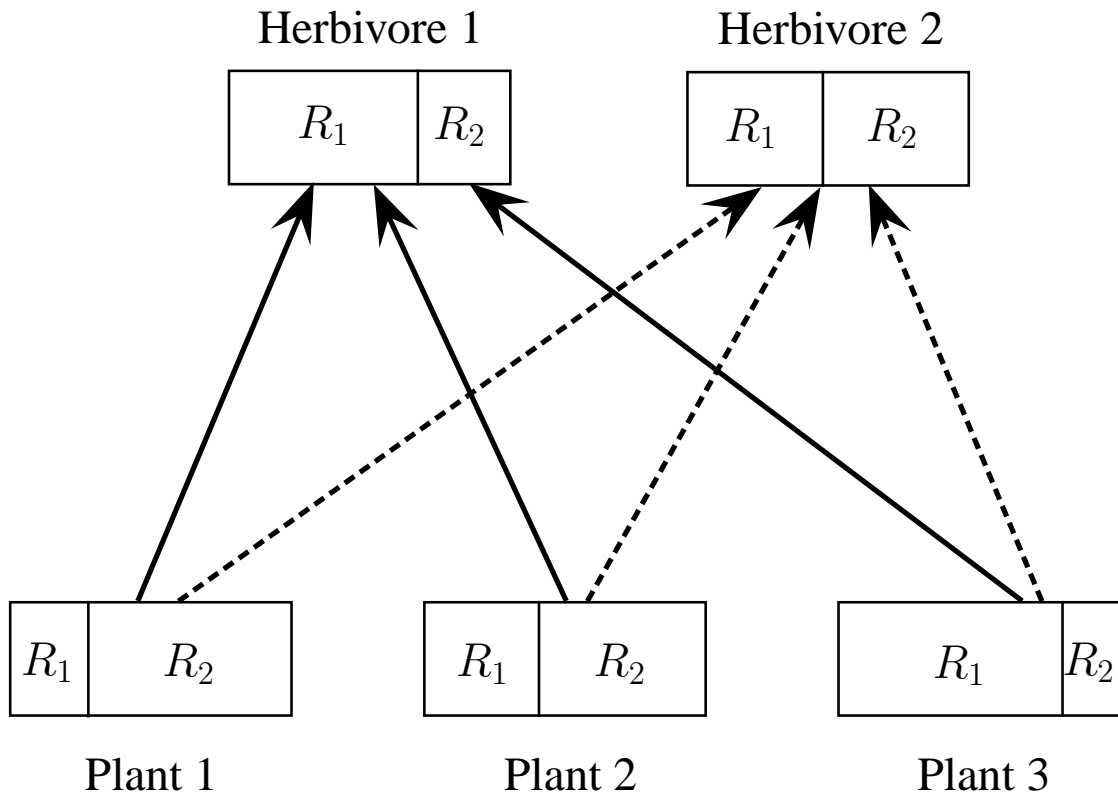
**Table 1:** Minimum daily intake requirements of digestible protein (DP) and phosphorus (P) for four grazer species of different body weight (BW in kg). Modified from Treydte et al. (2009).

Intake (mg/kg BW/day)	Model scenario source	Warthog 83 kg BW	Wildebeest 143 kg BW	Zebra 271 kg BW	Buffalo 481 kg BW
DProtein	ARC (1980)	730	550	430	380
	Menard et al. (2002)	970	850	720	630
	Ludwig et al. (2003)	1040	910	770	680
P	ARC (1980)	9	14	18	20
	Menard et al. (2002)	13	27	37	42
DProtein:P ratio		83	38	23	18

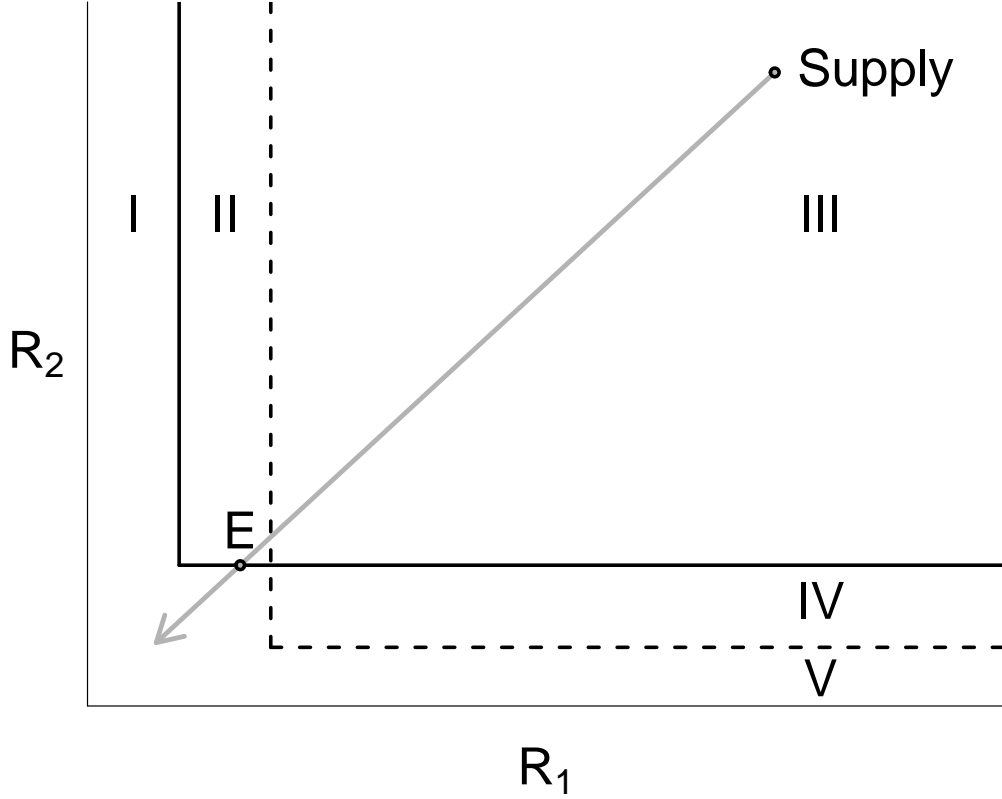
**Table 2:** State variables and parameters used in the model. Dimensions for each variable and parameter are based on mass (M) and time (T).

Symbols	Definitions	Dimensions
<b>State Variables</b>		
$H_i$	Herbivore $i$ biomass	M
$V_j$	Plant $j$ biomass	M
$R_k$	Resource $k$ mass	M
<b>Parameters</b>		
$g_{ij}$	plant $j$ consumption rate by herbivore $i$	$T^{-1}.M^{-1}$
$m_i$	herbivore $i$ biomass mortality rate <i>per capita</i>	$T^{-1}$
$S_j$	plant $j$ biomass supply	$M.T^{-1}$
$a_j$	plant $j$ biomass natural loss rate <i>per capita</i>	$T^{-1}$
$Q_{hiRu}$	resource $u$ quota for herbivore $i$	dimensionless
$Q_{vjRu}$	resource $u$ quota for plant $j$	dimensionless

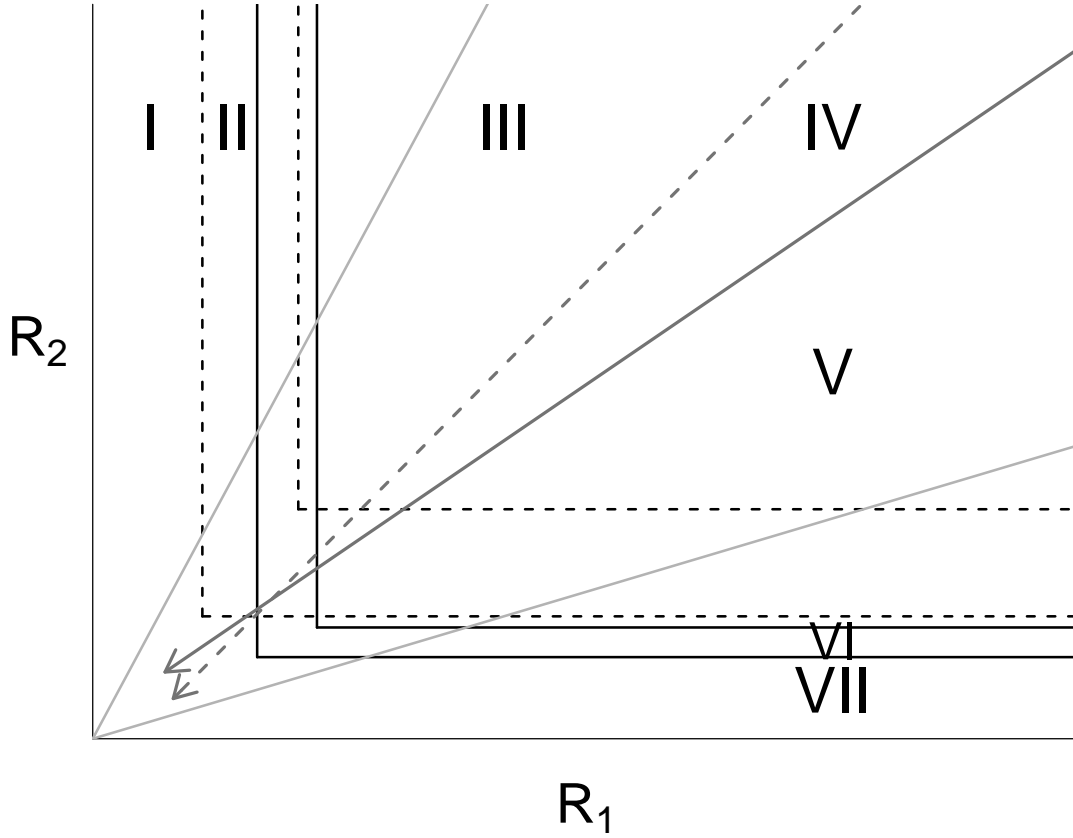




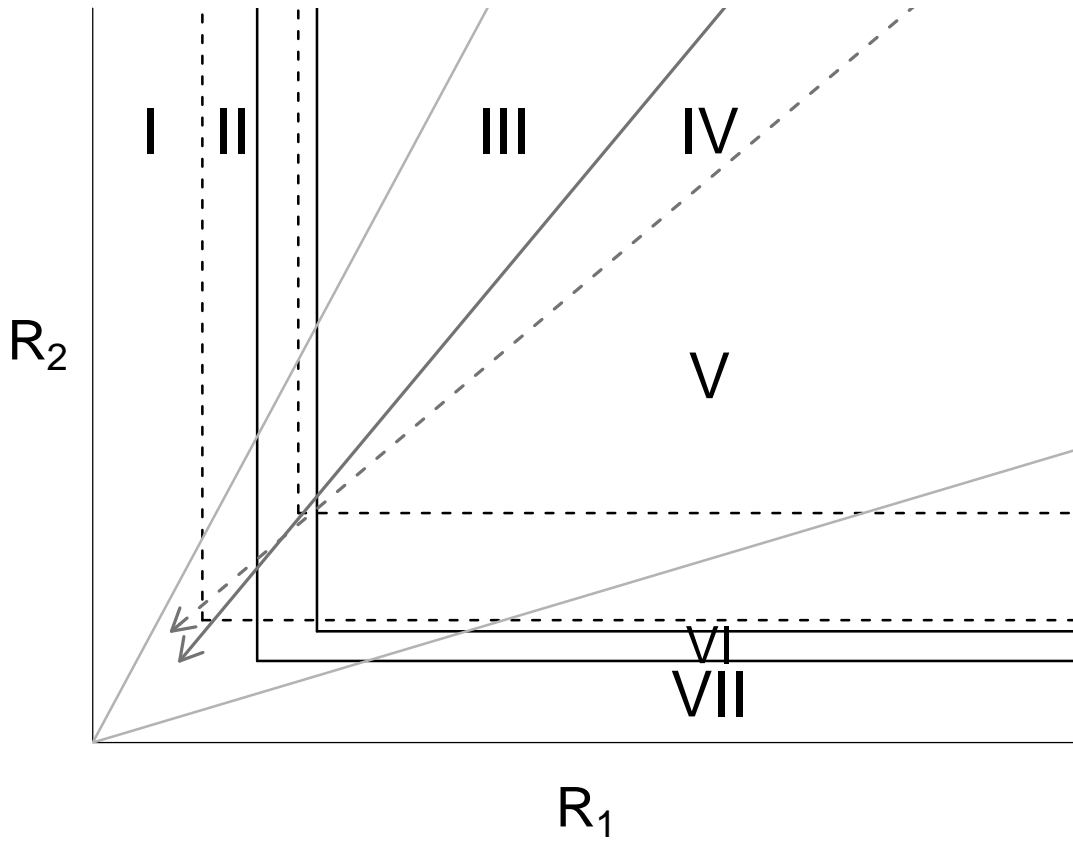
**Figure 1:** Each herbivore species has its own resource ratio between resource 1 ( $R_1$ ) and resource 2 ( $R_2$ ). This species feeds on different plant species with different resource ratios. Herbivore requirements as well as their resource consumptions are key factors for their persistence.



**Figure 2:** Phase plan with two herbivore species feeding on one plant species. Black lines are herbivore 1 ZNGI, dotted lines are herbivore 2 ZNGI, and grey arrow is the common consumption vector. Interpretations are quite similar with those from Tilman's model (Tilman, 1982). Roman numbers represent different zones for supply point position. Zone I and zone V do not provide sufficient amounts in  $R_1$  and  $R_2$  respectively. None herbivore species can live in these conditions. Zone II contains enough quantities of  $R_1$  for herbivore 1, but not for herbivore 2. Zone III potentially has enough quantities of both resources for both herbivore species. In zone IV, only herbivore 2 can survive because there is not enough  $R_2$  for herbivore 1. Supply point represents total amount of resources due to plant biomass rebuilding. Both herbivore species sample resources through the same consumption vector. In this example, the trajectory cross herbivore 2 ZNGY first and then herbivore 1 ZNGI. The crossing point with herbivore 1 ZNGI becomes an equilibrium point (E). Herbivore 1 is limited by  $R_2$ . Herbivore 2 is excluded.



**Figure 3:** Phase plan for two coexisting herbivores. Lines represent boundary ZNGI, arrows are boundary vectors, and grey lines bound feasible supply conditions according to plants considered (i.e., feasibility cone). Continuous ZNGI and vector belong to herbivore 1, and dotted ZNGI and vector belong to herbivore 2. Zone I and zone VII contain not enough  $R_1$  and  $R_2$  respectively. Neither herbivore 1 nor herbivore 2 can survive. Zone II contains not enough  $R_1$  for herbivore 2. Zone III does not allow herbivore 2 to survive if herbivore 1 is present. Zone V does not allow herbivore 1 to survive if herbivore 2 is present. Zone VI contains not enough  $R_2$  for herbivore 1. If the supply point is within zone IV both herbivore can survive together: it is coexistence. Then, the cross zone of the two ZNGIs is a stable equilibrium point (SE).



**Figure 4:** Phase plan for competitive exclusion. Representation of ZNGI and vectors is similar to figure 3, except for zone IV. This zone generally leads to exclusion of herbivore 1 or 2. The crossing zone of ZNGIs either can be a non stable equilibrium (NSE) or does not allow coexistence of the two herbivore species.