Review of JAE-2021-00431

I have read and reviewed the manuscript titled "A temperature-driven model of phenological mismatch provides insights into the potential impacts of climate change on consumer-resource interactions" (JAE-2021-00431). The authors develop theoretical tools to understand the impact of changing temperatures on the phenology of a consumer and its resource. They further apply these tools to the spruce budworm - balsam fir system and predict the possible changes in their phenology and the resulting mismatch that might occur.

The manuscript takes concrete steps towards solving an important issue – prediction of changing phenology in the context of anthropogenic climate change. The theoretical tools developed by the authors are agnostic to the accumulation function used and are thus widely applicable. The authors develop these tools for couple of different scenarios of temperature increase, which is useful. The application to the empirical system is tightly interwoven with the theory and provides a nice demonstration of the tools developed.

Writing comments

While I think the manuscript makes significant contributions, it would benefit from better development and perhaps some restructuring. The manuscript is currently written as an empirical paper (with Introduction, Methods, Results and Discussion). However, this causes the authors to jump back and forth between the definitions of the functions and their contributions which can be a bit hard to follow. For instance, Section 2.1 is titled Theoretical Development, where the authors develop their main equation (Eq 1) and provide background on the subject. However, the equation is solved under two different scenarios in Sections 3.1.1 and 3.1.2. These solutions enable the application to data and are thus the main contribution of the paper. As it currently stands, the significance of this gets lost a bit on the reader.

To fix this issue, I would suggest restructuring the manuscript so that the theoretical development (the equation and the solutions under constant increase in temperature and a warm spell) comes immediately after the introduction. This will allow for the development of the theoretical tools and the authors can use this opportunity to emphasize the generality of their tools. After the theory has been developed, the authors can then use the budworm-fir system as an example to showcase these tools. In this section, the accumulation functions can be specified and predictions specific to this system can be developed (Figs. 3, 4 and 5). I would strongly suggest adding an illustrative figure here to demonstrate the life cycles of the budworm and the fir with an annual timeline including the stages of diapause and quiescence. This will make the phenology mismatches clearer and easier to describe. Overall, the new structure will help the flow significantly and will highlight the novel contributions better.

Scientific comments

Apart from the comments on the structure of the manuscript, I have some scientific comments as well. I list these below:

- 1. The authors base their entire analysis of the impact of rising temperatures on consumer-resource systems on the heat accumulation function. While heat accumulation is crucial in how temperature might impact consumer-resource systems, temperature is also expected to impact physiological processes in other life stages. Moreover, the model completely ignores ecological and evolutionary dynamics of consumer-resource systems. Therefore, it is important that the authors explicitly address these limitations in the text and place their findings in context.
- 2. The authors use a specific function as R(x(t)) for the budworm. The study that the authors reference also lists a couple of other formulations. It might be useful for the readers to know why that specific function was chosen as opposed to others.
- 3. It isn't immediately clear whether the generated temperature series under different warming scenarios (in Fig. 5) are analyzed using the constant difference equation (Eq. 5) or the warm spell equation (Eq. 6). While I assume it is the former, it will be helpful to explicitly state that when discussing those results.
- 4. In Section 3.1.2, it might be beneficial to explore different combinations of sensitivity and phenology between the consumer and the resource in a systematic way. Right now, the authors focus on the case where the resource emerges before the consumer and is more sensitive at lower temperatures. It might be worthwhile to examine other scenarios here as well consumer emerges first, consumer has higher sensitivity etc. If pressed for space, the authors can focus on the cases they think are most biologically relevant. This analysis will show that the tools are potentially applicable in situations which might be significantly different from the budworm-fir system.

Minor Comments

There are a few relatively minor issues that need to be fixed in the manuscript. I list some here for the authors' convenience. These mostly pertain to reproducibility and typesetting issues and should be relatively easier to address.

- 1. Currently, the theoretical figures lack parameter values. Even though some of the figures are illustrative, it is useful to provide the parameter values for the purposes of reproducibility and further analyses. They can be added to the figure captions or be provided in the Supplementary Information.
- 2. In Fig. 1D, the dark dashed scenario is not shown. Further, it might be helpful to mark t_b^* and t_e^* on this figure. The mismatch can thus be clearly stated as $t_e^* t_b^*$, which will correspond to the text.
- 3. For reproducibility, the authors should also list the tools used for the analyses programming language, any relevant software packages etc.
- 4. There are some typesetting issues (e.g., Eq. 3) and some minor notation errors (e.g., Eq. 7 should say x_i and not x_1) in the manuscript. I would suggest a careful read of the manuscript with these in mind at the end.

In summary, the advances made in the manuscript are interesting and useful. However, the manuscripts will benefit from some restructuring and some additional elaboration of the theoretical tools.