Simple model for spruce budworms

1 Features

The model should include a diapause. Emergence depends on temperature, bu may also depends on internal energy (to be discussed). In a first step, the model includes only two stages. L1 and L2 stages are lumped together (i.e. Winter stage). Similarly, L3-L6 are lumped as well (i.e. Summer stage). Budworms emerge in Spring, they develop during Summer. Then they lay eggs that hatch into a Winter stage that goes into diapause.

Hence, the model considers a so-called Winter stage population (L_w) that runs from egg stage until the end of the diapause. Then, a Summer stage population (L_s) runs from the end of the diapause until reproduction.

L_w individuals do not feed and rely on internal energy provided by the former generation. L_s individuals need food for survival. Forage during Summer also translates into internal energy for the future generation.

2 Model

2.1 Winter population (L_w)

$$\left\{\begin{array}{lcl} \dot{R_w} & = & r_w(T(t)) \\ \dot{L_w} & = & -m_w(\alpha_w(t))L_w \\ \dot{\alpha_w} & = & -\alpha_1 T^{\alpha_2}(t) \end{array}\right. \tag{1}$$

The first equation is larval development, where R_w is the Winter life stage, r_w is the development rate that depends on temperature (T) at any time (t). $R_w(t_0) = 0$ and development ends when $R_w(t) = 1$. It gives the duration of the life stage, and the time during which the two following equations run.

The second equation is larval survival. Larvae die at a rate m_w that depends on internal energy available (α_w). The third equation represents the decay of internal energy through time that increases with temperature, where α_1 and α_2 are data-derived parameters.

It is possible to write explicit solutions for the second and third equations. Winter population writes

$$L_w(t) = e^{-\int m_w(\alpha_w(t))\mathrm{d}t} L_w(t_0) \tag{2}$$

Moreover, m_w is monotone decreasing in α_w (i.e. the more energy larvae get, the less they are likely to die). Available energy writes

$$\alpha_w(t) = \alpha_w(t_0) - \int_{t_0}^{t} \alpha_1 T^{\alpha_2}(t)\mathrm{d}t \tag{3}$$

A question remains: if larvae run out of energy before the end of the diapause, do they emerge anyway even if the development is not fully completed, or do they die?

2.2 Summer population (L_s)

$$\begin{cases} \dot{R}_s &= r_s(T(t)) \\ \dot{L}_s &= -m_s(F)L_s \\ \dot{F} &= P(t) - dF - \beta F L_s \\ \dot{\alpha}_s &= c\beta F L_s - m_s(F)L_s\alpha_i \end{cases} \tag{4}$$

The first equation represents development during Summer stage. $R_s(t_0) = 0$ and development ends when $R_s(t) = 1$. The second equation is larval survival during Summer. Death rate (m_s) is now food-dependent.

The third equation is food (leaves) availability. Leaves are produced at a rate P that varies through time. There is a natural decay (d) due to senescence and other sources of loss not related to budworms. Last, leaves are consumed by budworms at a rate β.

The last equation is energy storage (α_s) made by Summer stages and that will be given to Winter stages via eggs. Energy comes from consumed food, with a conversion efficiency (c). When a given individual dies during Summer time (i.e. before it reproduces), all its energy is lost: α_i represents individual energy storage.

For simplicity, we assume no tree mortality (i.e. the number of trees is constant). For a longer trend, this parameter will have to vary as well.

2.3 Reproduction

At the end of the Summer (once Summer stage is fully developed), reproduction occurs.

$$\begin{cases} L_w(t_n) &= g_i L_s(t_n) \\ \alpha_w(t_n) &= \dfrac{\alpha_s(t_n)}{L_w(t_n)} \end{cases} \tag{5}$$

Remaining Summer stage individuals produce a given number of offspring (g_i) *per capita*. Each of these individuals will receive energy which is the total amount of energy stored divided by the number of offspring.

These new offspring are the next generation:

$$\begin{cases} L_w(t_0) & = & L_w(t_n) \\ \alpha_w(t_0) & = & \alpha_w(t_n) \end{cases} \tag{6}$$

Two questions remain.

1) Should we make food production (P) dependent on temperature, or photoperiod, or both?

2) What would be the most relevant metrics for food (F)? Should we consider leaf area, leaf biomass ...?