

Spruce Budworm Work in Progress

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1 General framework of a model

We can use a circle map approach. Each life stage has a temperature-dependent development rate. Let's define R_j as the j th life stage, r_j as the instantaneous development rate of this life stage, and T as temperature.

$$R_j(t) = \int_{t_{j-1}}^{t_j} r_j(T_s) ds \quad (1)$$

where $R_j(t_{j-1}) = 0$ and $R_j(t_j) = 1$ (full development of the life stage). The model determines t^* when the solution reaches $R_j(t^*) = 1$, which is the completion of the life stage.

Former studies used the circle map to investigate existence of steady states in life cycles like (Powell and Logan, 2005). However, the purpose was to determine steady states in the life cycle, not the population dynamics. Adding temperature dependence to population dynamics has been used for spruce budworms (Régnière et al., 2012), but not over a whole life cycle.

Our model should consider such temperature-dependence of development, which is an individual-based process, and population dynamics. The equilibrium point of such a system is more complex since it needs to find the day of the year that allows steady states of both life stage and population density. In other words, if we define for example, the start of the life cycle as the oviposition day, the real equilibrium is when there is a day in the year that will allow oviposition to occur the same day of the following year, with the same population density.

2 More detailed framework

The starting point of the life stage is arbitrary. Let's begin it with the date of oviposition. Thus, R_0 is the egg stage, R_1 is the first instar, and so on. The completion of the final life stage (N) in year y is also the beginning of the following life cycle for the following year $y + 1$.

$$t_N^y = t_0^{y+1} \quad (2)$$

The dynamics between t_0 and t_1 (e.g. egg life stage) can be described by a set of two equations.

$$\begin{cases} \dot{R}_0 = r_0 \\ \dot{N}_0 = -\mu_0 N_0 \end{cases} \quad (3)$$

where $R_0(t_0) = 0$, and $R_0(t_1) = 1$. The first equation of the system gives the time from t_0 to t_1 (time needed for eggs to hatch). It is also the time during which the second equation (egg population density) will run. Each life stage has a mortality rate (μ_j). There is no population growth since all individuals are produced once a year during oviposition. Each life stage dynamics is a survival of the starting population. It is only at t_0 at the beginning of each new year that new individuals will be added.

The following life stage (first instar) will then be:

$$\begin{cases} \dot{R}_1 = r_1 \\ \dot{N}_1 = -\mu_1 N_1 \end{cases} \quad (4)$$

where $R_1(t_1) = 0$, $R_1(t_2) = 1$, and $N_1(t_1) = N_0(t_1)$. This approach works well for the first stages because their development relies on energy stored into the eggs from the start. Hence, their development is independent of food availability from the trees.

The problem comes from later stages because they feed on trees. We therefore need at least, a third equation for food availability. For example, for third instar:

$$\begin{cases} \dot{R}_3 = r_3 \\ \dot{N}_3 = -\mu_3(F)N_3 \\ \dot{F} = Prod(t,T) - \alpha_3 N_3 \end{cases} \quad (5)$$

where F is food availability. Food production is time-dependent. It is probably temperature-dependent. This food is consumed by larvae at a rate α

which is a *per capita* consumption rate of a given instar. Mortality (survival) of a given larvae population depends on food availability ($\mu(F)$). It is quite obvious that the model can quickly become very complex.

3 Questions

Some questions raised.

3.1 How does r_j depend on food?

Is development rate only varies with temperature? It is likely that it also varies with food availability. At least, we may imagine that $ropt_j$ is an optimal rate. If food availability is above a given threshold, then $r_j = ropt_j$. But if food availability is below this threshold, then $r_j << ropt_j$.

3.2 What are the sources of mortality?

Mortality (μ) can have multiple sources (e.g. natural death, enemies). How many of sources should we consider? Can we merge them into a single parameter?

3.3 What determines the end of the diapause?

Emergence from diapause is a critical moment. What determines this moment? Is it only temperature?

3.4 How many variables (and equations) are needed for the trees?

This last question is probably the most important one. Hence, tree density is probably not enough. Leaf area is also relevant because it is the real food source. The age of trees should also matter. In Jones (1976)' model, there are 75 different ages for trees, which is probably too much, but we may need to include some age structure. Hence, a fourth equation may be added to the system (5). The tree population may increase (new trees available) or decrease (trees that are cut or that die from different causes). The number of trees will change the leaf area available for the budworm.

4 Perspective

Before going into a fully detailed and complex model, we may begin with a very simple model with only one life stage that feeds on leaves (equation 5). Four equations (if we consider tree population and leaf area) stay tractable. An analysis of this simplified system will give some clue about possible behaviour of the full model.

I need to check the literature to see if some studies investigated both phenology and dynamics (as we want to do) already, and how they did it.

References

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