CSCI 4448 Robots

In Class Day 7

2.3. R_x (translation) (vector)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos & -\sin & 0 \\ 0 & \sin & \cos & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -a \\ 0 & 1 & 0 & -b \\ 0 & 0 & 1 & -c \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

R_y (or) R_z

$$\begin{bmatrix} \cos & 0 & \sin & 0 \\ 0 & 1 & 0 & 0 \\ -\sin & 0 & \cos & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ (or) } \begin{bmatrix} \cos & -\sin & 0 & 0 \\ \sin & \cos & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4.1.1 origin / vector (normal vector)

.2 $\min \begin{bmatrix} x_1 & y_1 \\ \vdots & \vdots \\ x_n & y_n \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ $\quad \min \sum_{i=1}^{n} \left((r_i - \mu) \cdot \hat{N} \right)^2$

3 no constraints, decision vars - origin/vectors, linear

4 nonlinear unconstrained

5 find eigen vectors of correlation matrix

6 PCA, use SVD on mcd to find eigenvectors of correlation matrix

4.2.7 center, radius $x(t)$, $y(t)$

$(x-h)^2 + (y-k)^2 = R^2$

.8 center, radius

.9 $\min \sum_{i=1}^{n} \left(\|r_i - c\| - R \right)^2$, nonlinear

.10 $R \geq 0$, linear

.11 nonlinear constrained

.12

22 $U = \int_0^z mg \, dp = mgz$

23 $U = \int_{|r|}^{\infty} \frac{GMm}{p^2} dp = -\frac{GMm}{|r|}$

24 $U = \int_{\infty}^{|r|} \frac{kQq}{p^2} dp = -\frac{kQq}{|r|}$

25 $-Gp \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{d\lambda}{|\vec{r} - \vec{r}_m|}$

$s(t) = \begin{bmatrix} x_2 - x_1 \\ y_2 - y_1 \end{bmatrix} t + \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$

26 $-Gp \int_0^1 \frac{s'(t)\,dt}{|\vec{r} - s(t)|}$

27 $-Gp \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{dx\,dy}{|\vec{v} - \langle x, y \rangle|}$

28 $-Gp \int_{-R}^{R} \int_{-\sqrt{R^2 - x^2}}^{\sqrt{R^2 - x^2}} \frac{dy\,dx}{|\vec{r} - \langle x, y \rangle|}$ (Cartesian)

$-Gp \int_0^{2\pi} \int_0^1 \frac{r\,dr\,d\theta}{|\vec{r} - \langle r\cos\theta, r\sin\theta \rangle|}$

29 potential at any point is sum of potentials at that point

30 same as before

33 Global min $(-1, -1)$

$(2, 2)$ escapes

34 still escapes @ -3

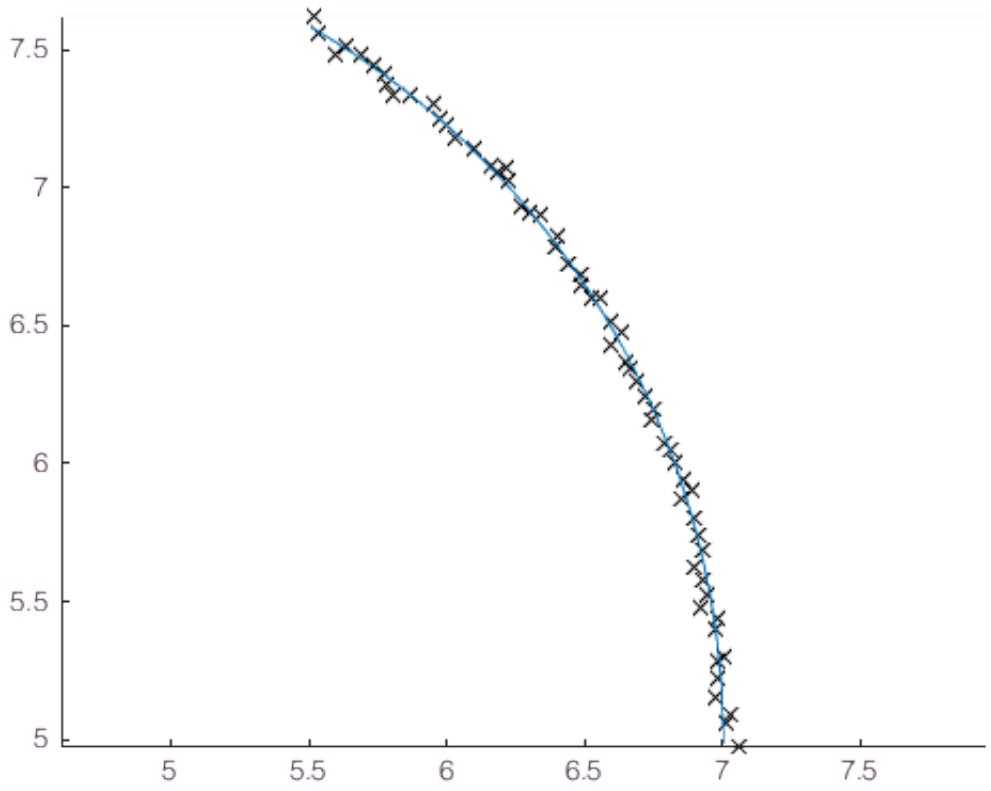
-10 world work

35 still escaped

```
r = 3;
x0 = 4;
y0 = 5;
dev = .02;
t = linspace(0,deg2rad(60),60)';
npts = length(t);
x = cos(t)*r+x0+randn(npts,1)*dev;
y = sin(t)*r+y0+randn(npts,1)*dev;
[cent_found,r_found] = findCenter(x,y)
```

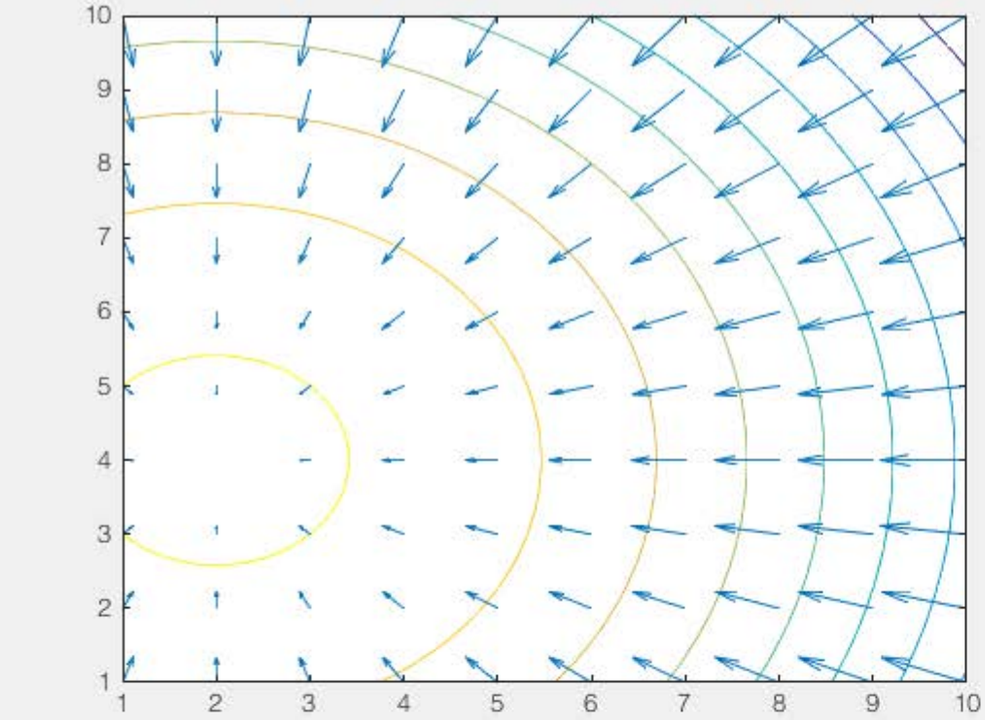
```
w =

   -8.0157
   -9.9799
   31.9943
cent_found =

    4.0079    4.9899
r_found = 2.9947
```

```
figure
clf
hold on
plot(x,y,'kx')
plot(cos(t)*r_found+cent_found(1),sin(t)*r_found+cent_found(2))
axis equal
hold off
```



```
function [cent, r] = findCenter(x ,y)
    nPoints = length(x);
    A = [x y ones(nPoints,1)];
    b = -x.^2-y.^2;
    w = A\b
    cent = [-w(1)/2 -w(2)/2];
    r = sqrt(cent(1)^2 + cent(2)^2 -w(3));
%     nPoints = length(x);
%     bestCost = Inf;
%     bestCent = [0;0];
%     bestR = 0;
%     XY = [x y];
%     for succ = 1:nTries
%         i=0;
%         j=0;
%         k=0;
%         while(i==j || i==k || j==k)
%             i = randi(nPoints);
%             j = randi(nPoints);
%             k = randi(nPoints);
%         end
%         a = [x(i);y(i)];
%         b = [x(j);y(j)];
%         c = [x(k);y(k)];
%
%         abCent = mean([a b],2);
%         acCent = mean([a c],2);
%         ab = b-a;
%         ac = c-a;
%         nab = [-ab(2);ab(1)];
%         nac = [-ac(2);ac(1)];
%
%         ut = linsolve([nab,-nac],acCent-abCent);
%
%         cent = abCent + ut(1)*nab;
%         A = XY - cent';
%         %r = mean(sqrt(diag(A*A')));
%         r = norm(a-cent);
%
% %         abc = [a b c];
% %         cents = [abCent, acCent];
%
%
%
%         cost = sum(circleCost(x, y, cent, r));
%
% %         hold on
% %         plot(x,y,'ks');
% %         plot(abc(1,:), abc(2,:), 'go');
% %         plot(cents(1,:), cents(2,:), 'ro');
% %         quiver(abCent(1), abCent(2), nab(1), nab(2));
% %         quiver(acCent(1), acCent(2), nac(1), nac(2));
% %         plot(cent(1),cent(2),'m*');
% %         t = linspace(0,2*pi);
% %         plot(cos(t)*r+cent(1), sin(t)*r+cent(2));
% %         axis equal
% %         hold off
% %         figure
%         if(cost < bestCost)
```

```
%             bestCost = cost;
%             bestCent = cent;
%             bestR = r;
%         end
%     end
end
```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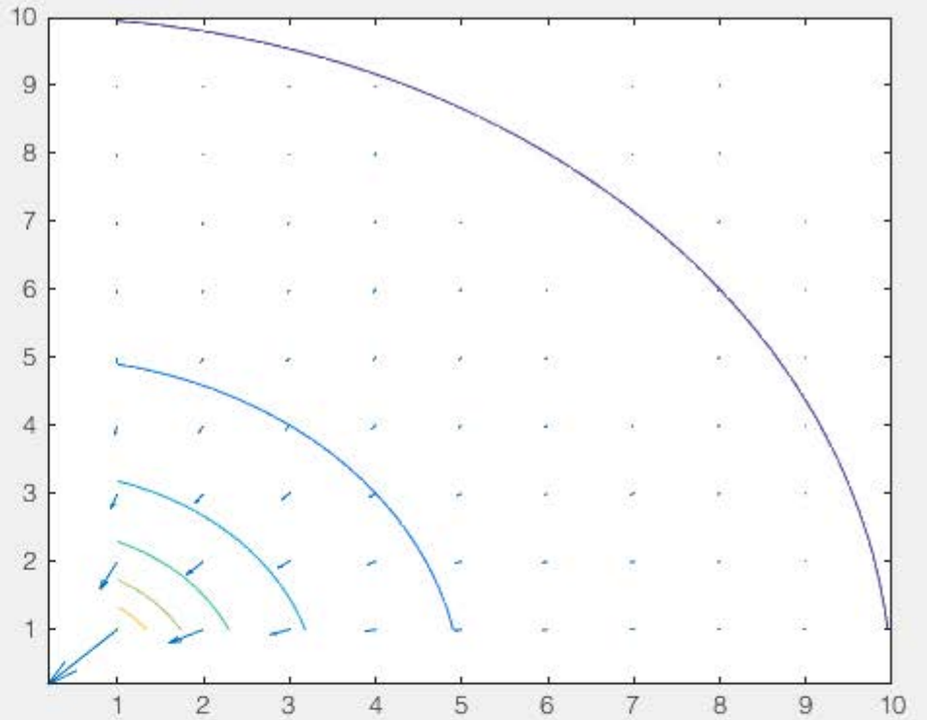
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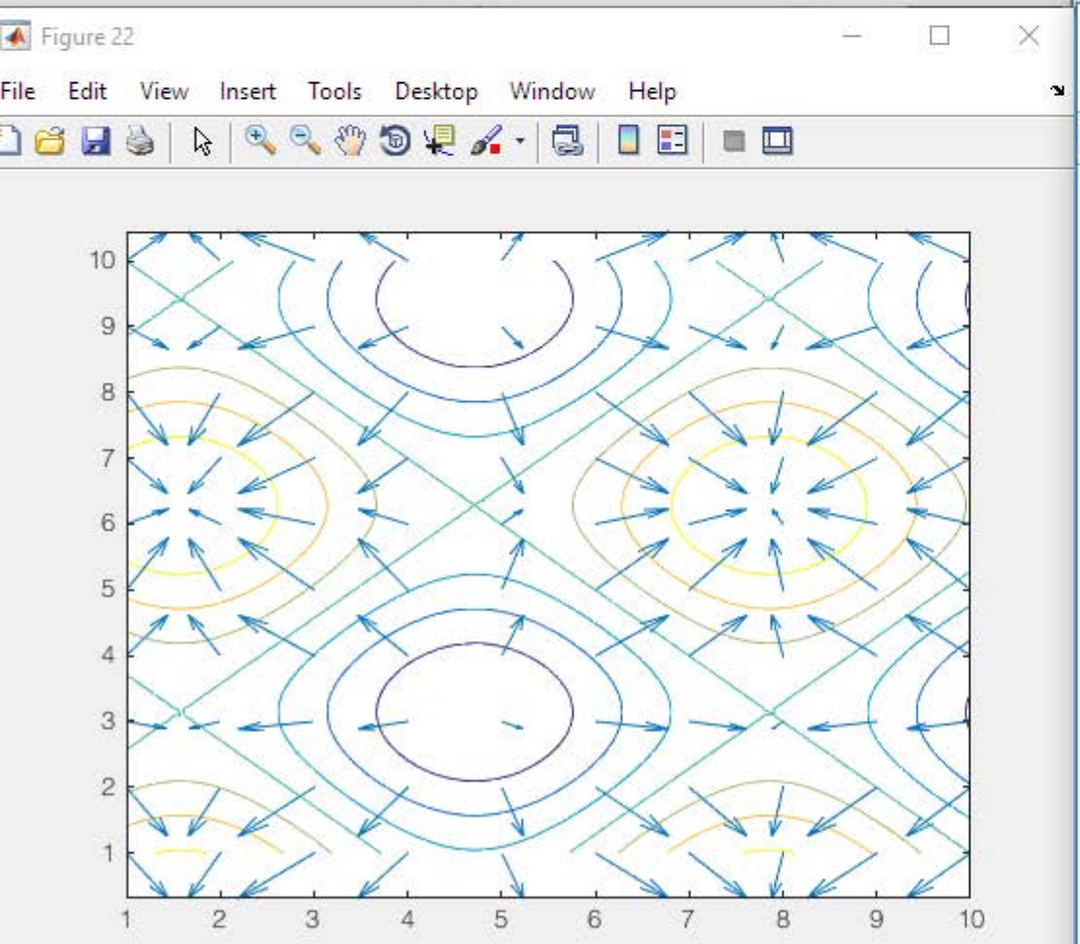
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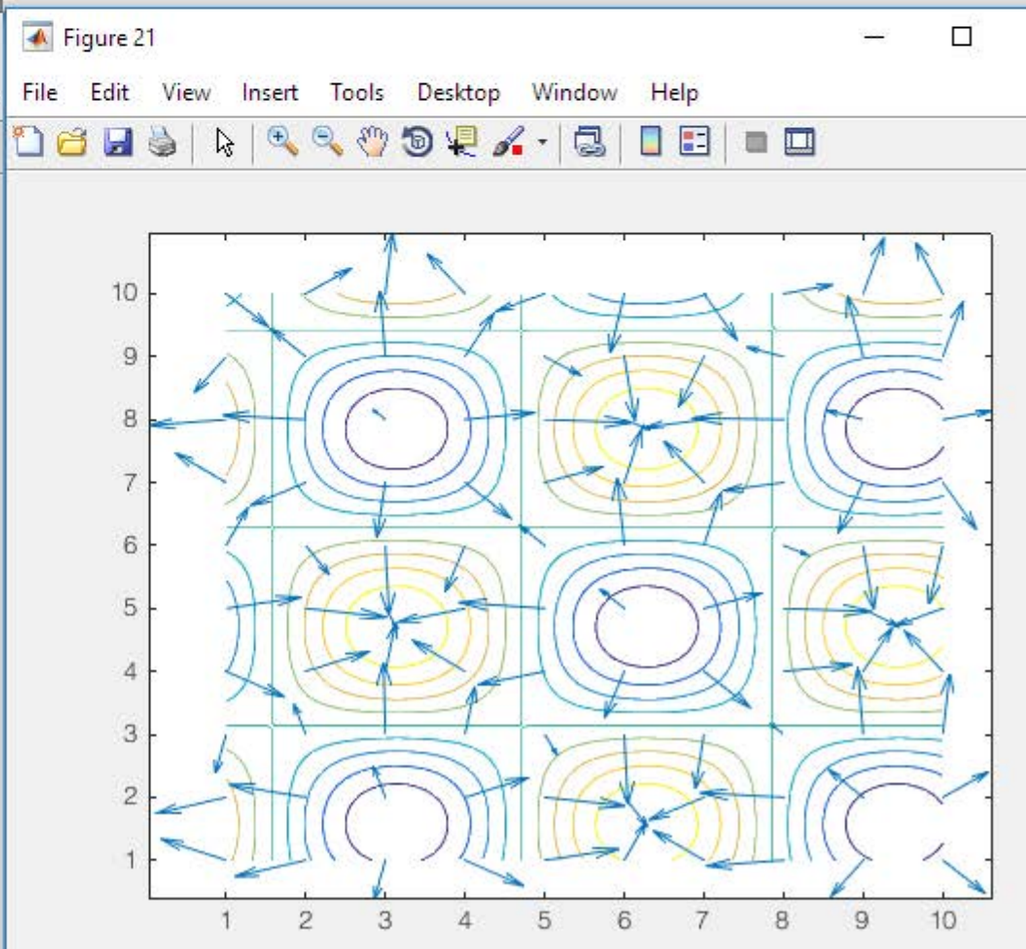
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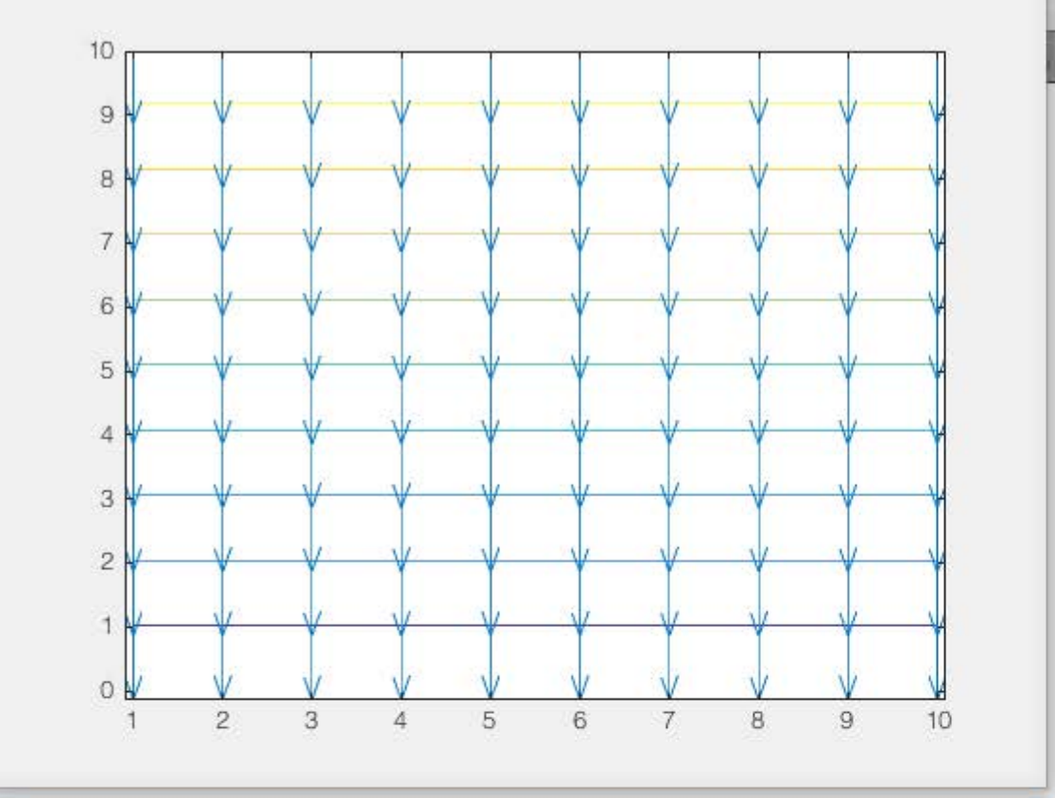
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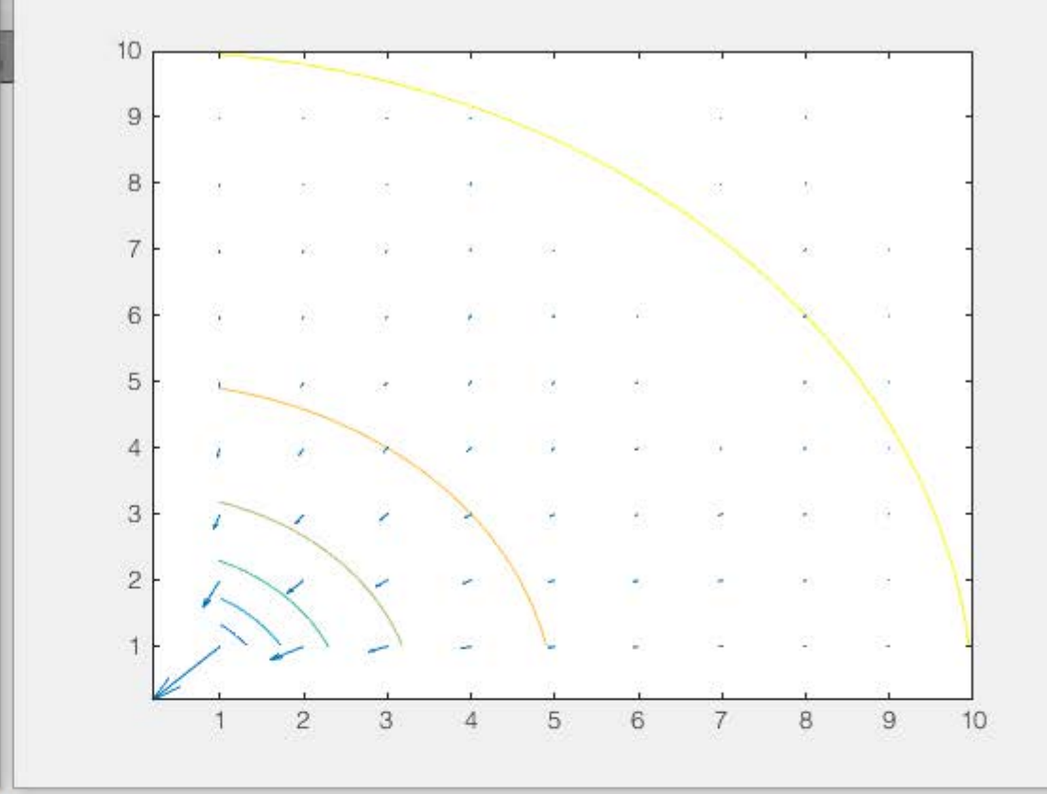
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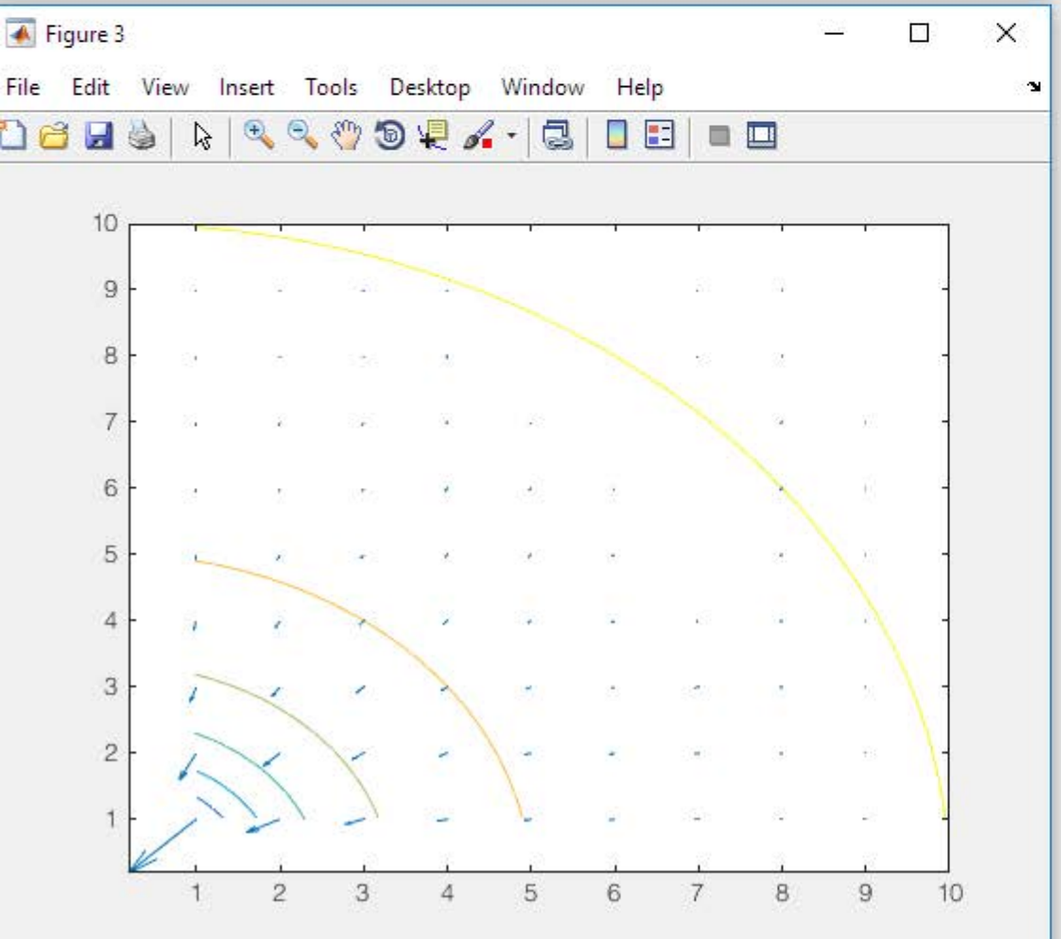
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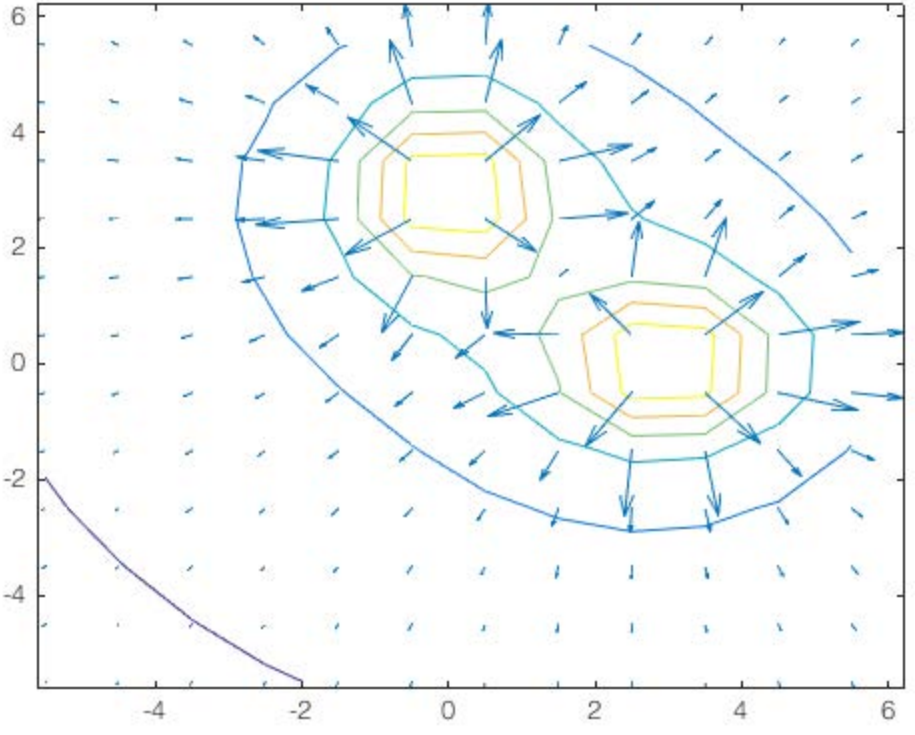
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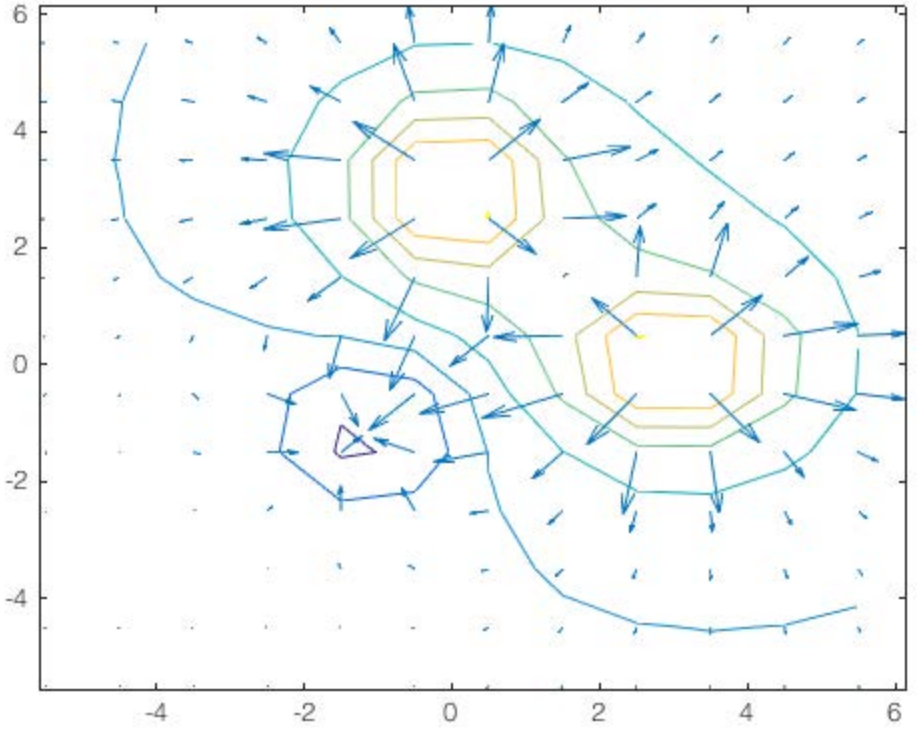
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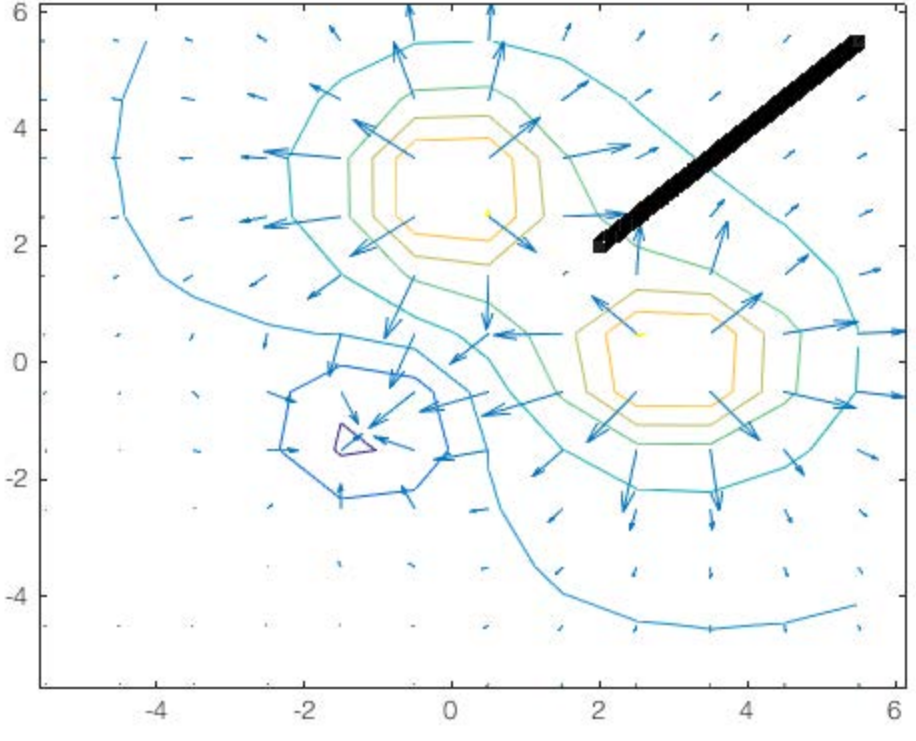
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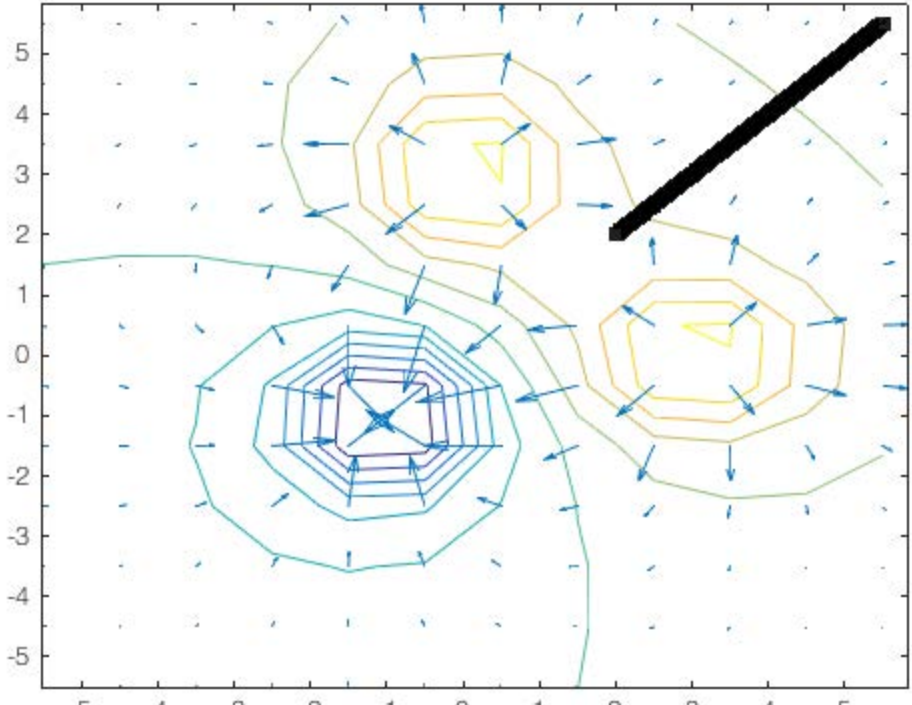
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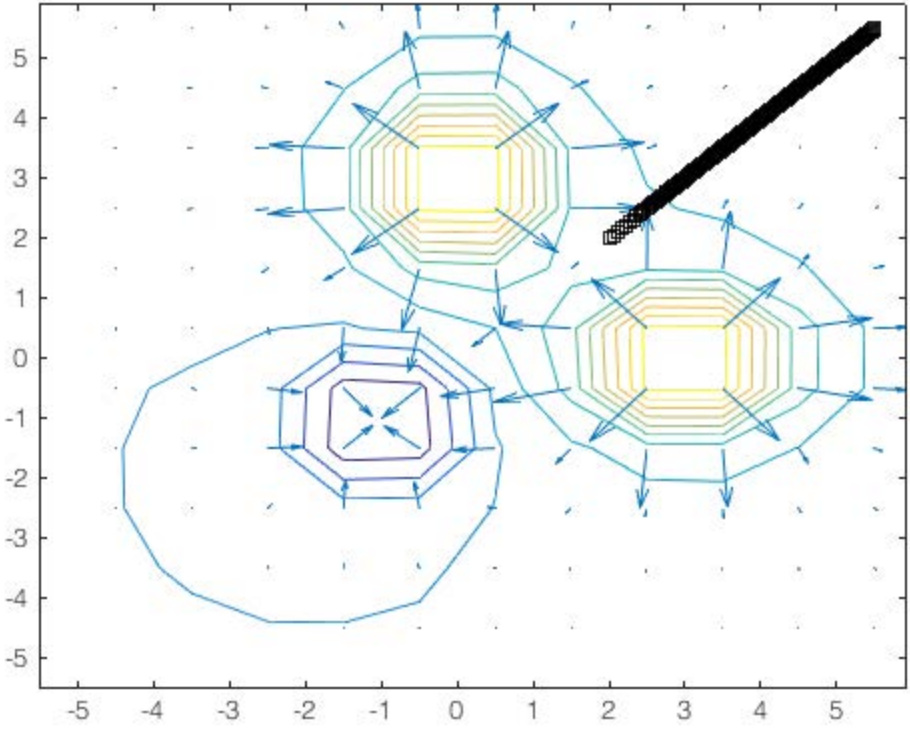
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