Data Analysis &
Machine Learning

Systematic Errors

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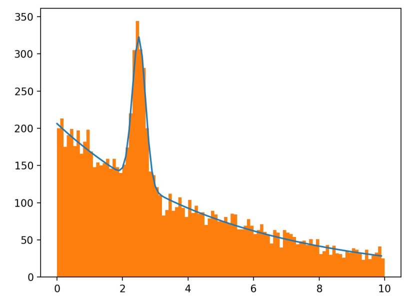
Aim:

- To introduce the idea that there are more sources of uncertainty than the pure statistical error on a measurement - we call these systematic errors.
- To show several different ways to estimate systematic errors.
- To show how to add systematic errors in quadrature to the statistical error.

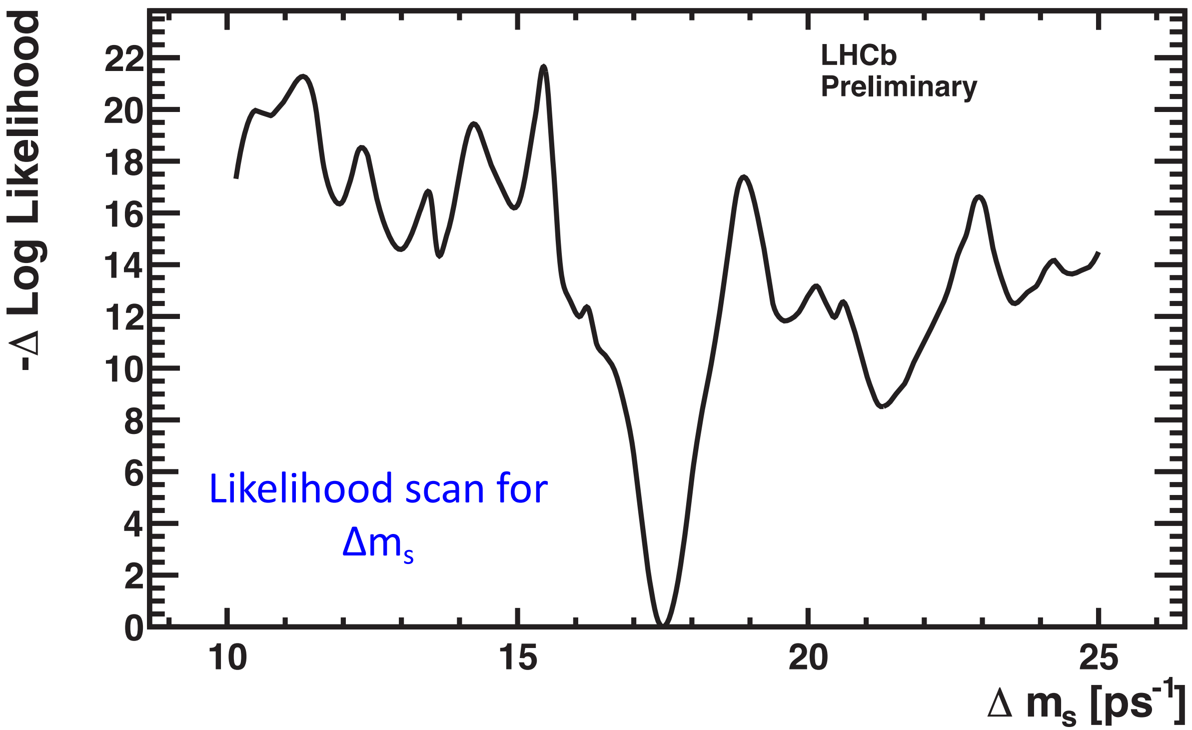
Recap of parameter estimation

- You learned how to estimate parameters by minimising some quantity



- You minimised a χ^2 for a straight line fit
- You minimised a negative-log-likelihood for more realistic fits to data to a mass distribution.
- This involved
  - A PDF (Probability Density Function)
  - A combination PDF used to model this complex shape
- The parameters were
  - Lifetime of exponential for background
  - Mean of Gaussian for signal
  - Width of Gaussian (but we fixed this)
  - Fraction of exponential
- You also learned how to obtain statistical errors from the fit –
  - By varying χ^2 by 1 unit
  - By varying NLL by 0.5 units

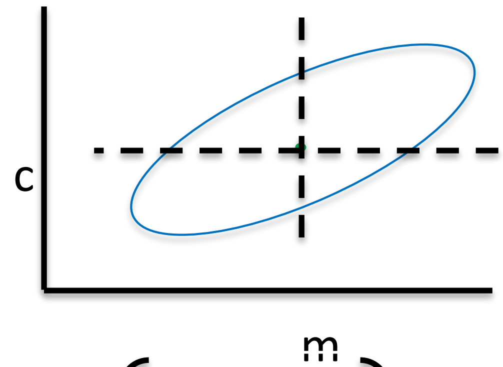
Digression 1: here is a NLL plot about a minimum, which is not parabolic



- We can still use it as it is ~ parabolic around the minimum up to NLL+8
- We observe a central value Δm_s= 17.50 ± 0.15 (stat) ps^{-1}

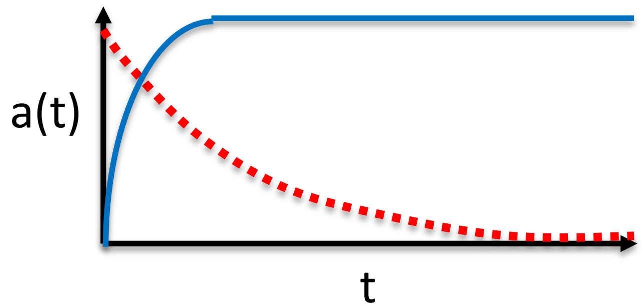
Recap of parameter estimation

- ❑ You also learned about correlations and correlation matrices



$$\begin{pmatrix} 1 & 0.3 \\ 0.3 & 1 \end{pmatrix}$$

- ❑ You learned about how to include a time acceptance function



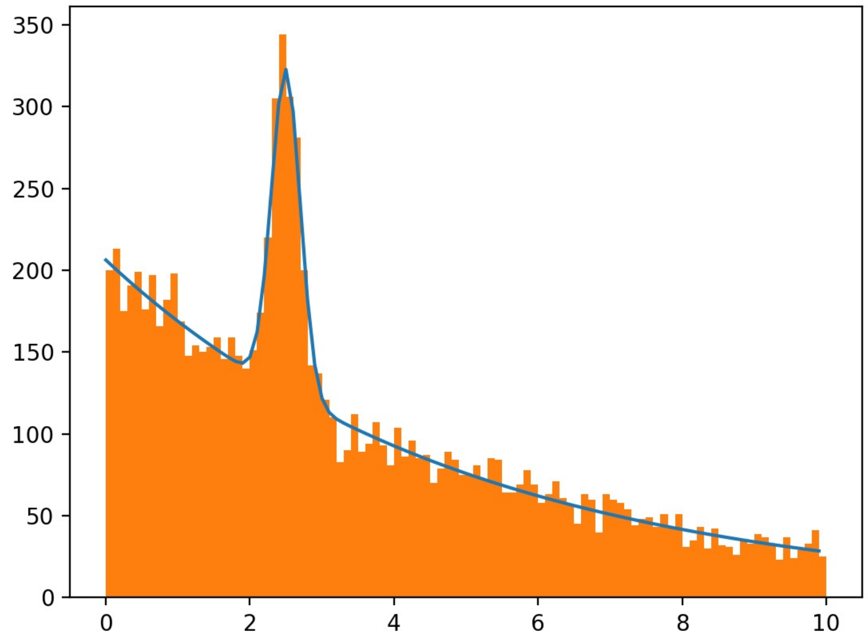
- ❑ You learned how to include backgrounds

Systematic errors

- ❑ Everything done so far assumes the ideal (perfect) case

    - The PDF is the correct PDF for the data
    - You know your time acceptance perfectly
    - You understand your background perfectly
    - The data is measured perfectly (with perfect resolution)
    - …….

- ❑ None of these statements are ever likely to be true !



This means that just publishing

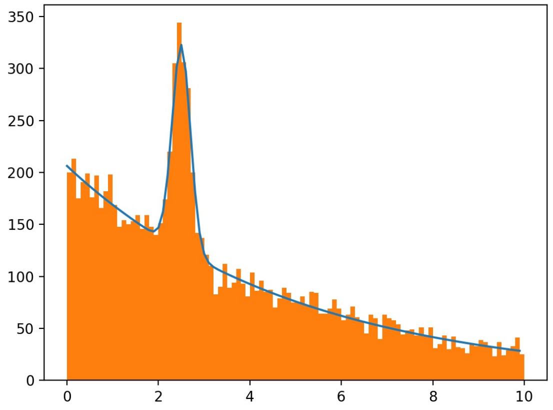
Tau = 5.3 +- 0.1
Mean = 2.5 +- 0.2
Fraction = 0.9 +- 0.15

Where the errors are just statistical is incomplete, and does not <u>fully</u> represent the error on the knowledge of the parameters

Systematic errors

- We say that there are additional errors coming from our lack of knowledge of many things.
- We call these “systematic errors” which we add in addition to the statistical error
- The “art” of estimating systematic errors is vital to any measurement
- Without systematic errors you formally do not have a meaningful measurement.
- Even if you conclude there are no systematic errors, you have to do the study to know this and then state it.
- We are going to look at some ways of estimating systematic errors

Systematic error on the fitting model



m (mass in some units)

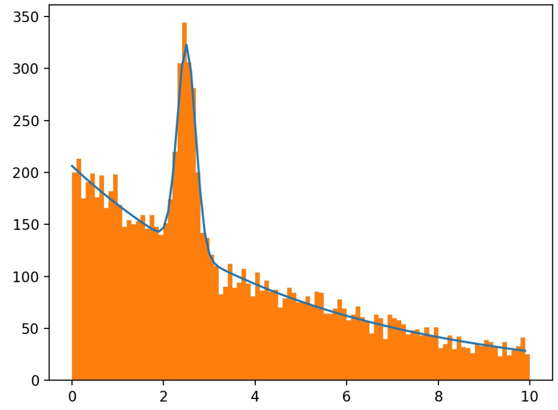
- Take these data to represent a mass distribution
- So they look like a particle peak on top of some background

❑ The fit model is embodied in the PDF -> this is what we think the physics is

❑ The PDF used was
- An exponential for the background, with parameter L : exp(-mL)
- A Gaussian for the signal with mean M and width W
- A fraction of the exponential, F

❑ How do we know that the signal is a properly modelled by a Gaussian ?
- It could be an asymmetric Gaussian
- It could be another sort of function (e.g. a so called Crystal Ball function)
- It could be……..

Systematic error on the fitting model



m (mass in some units)

- Lets take these data to represent a MASS distribution
- So they look like a particle peak on top of some background

❑ We can get a gauge of the error that this lack of knowledge introduces by re-doing the fit using different models for the signal

❑ Supposing we do the fit with three different models:

➢ Gaussian → gives X +-.....
➢ Asymmetric Gaussian → gives Y +-
➢ Crystal Ball → gives Z +- ....

❑ Then we may take the largest difference between X/Y/Z as some gauge of our systematic error due to signal fit model. [Note we are only interested in the change of best fit value.]

❑ We will label this as E_{model} meaning *"systematic error due to mass model"*

How do you add a systematic error on to the statistical error ?

- If the primary result with value central V is written as:

$$V \pm E_{stat}$$

- To show the systematic error separately we quote

$$V \pm E_{stat} \pm E_{model}$$

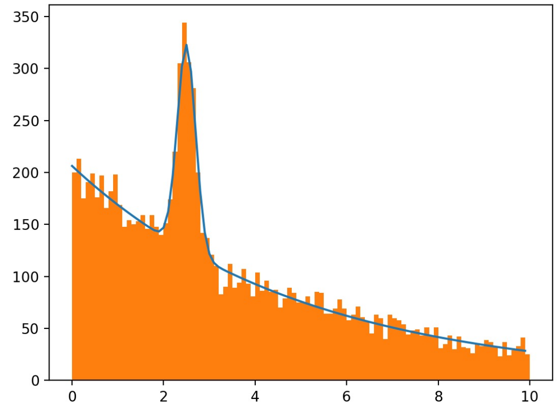
- To combine errors we add the new systematic error in QUADRATURE

$$E_{total} = \mathrm{sqrt}(E_{stat}^2 + E_{model}^2)$$

- We then can quote

$$V \pm E_{total}$$

Systematic error on the background



m (mass in some units)

- Similarly we may have imperfect knowledge of our background model

- Possibilities include
  - Exponential
  - Straight falling line
  - Polynomial
  - .....

- You will try these out for the checkpoint

- We will label this systematic error as E_{bck}

Continue adding in quadrature

- We can quote

$$V \pm E_{stat} \pm E_{model} \pm E_{bck}$$

- To combine errors we add the new systematic error in QUADRATURE

$$E_{total} = \text{sqrt}(E_{stat}^2 + E_{model}^2 + E_{bck}^2)$$

- We then quote

$$V \pm E_{total}$$

Learn that when we say

$$E_{stat} \pm E_{model} \pm E_{bck}$$

we mean adding in quadrature, i.e:

$$\text{sqrt}(E_{stat}^2 + E_{model}^2 + E_{bck}^2)$$

A better way if you have the (CPU) time

- The method just introduced is the simplest. Generically it is:

    - Vary some component of the fit according to some sensible variations which encompass your lack of knowledge
    - Measure the change this makes to the best fit value of a parameter
    - Take the difference to the nominal best fit value as your systematic error (the shift induced in best fit value)

- This might be all you have sometimes, but is pretty simplistic and may give you an unlucky over estimate or underestimate. This arises as you are only doing it once.

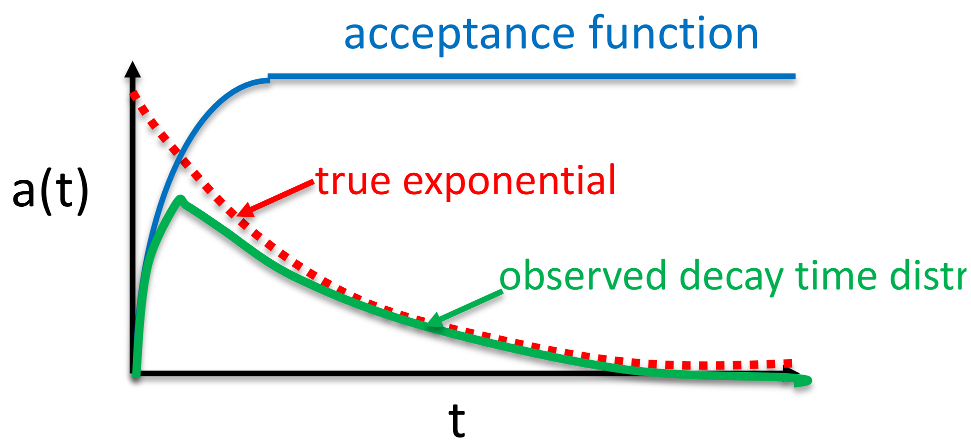
- A better way is to use the toy Monte-Carlo method: If you can simulate the experiment then you can measure this shift many times and plot it

    - You generate a data set - and fit it with the variations, and measure the shift
    - Repeat this N times and plot the N shifts → and it will be a Gaussian
    - Take the mean or width of the Gaussian as a systematic error, depending on what you see.

This prescription is often got wrong – you must fit twice to each toy data set –
i.e <u>fit the same data twice</u>
The common mistake is to fit to one toy data set with one model and another toy data set with the varied model. This is no good as it introduces a statistical error as well.

Systematic Error on Acceptance

$a(t) = 1 - \exp(-t/G) \rightarrow$



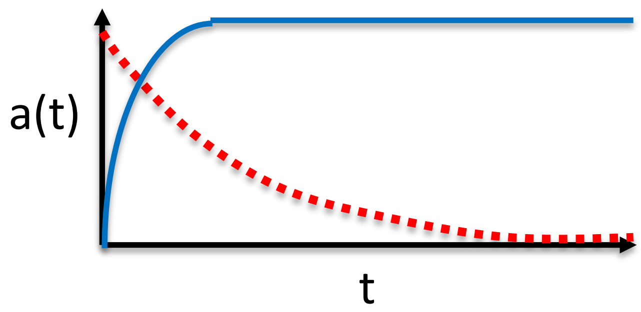
- ❑ Here is the time acceptance we used earlier on

- ❑ In general you vary the time acceptance model in some way
  - Maybe a different functional form (same method as just shown for model and background)
  - Or if you measured it from some data it will likely have parameters with statistical errors of their own – so vary these within their errors.
  - In this case suppose the parameter G is known with some error $G \pm E_G$

- ❑ There are (at least) three methods to deal with this

Systematic Error on Acceptance

$a(t) = 1 - \exp(-t/G)$ →



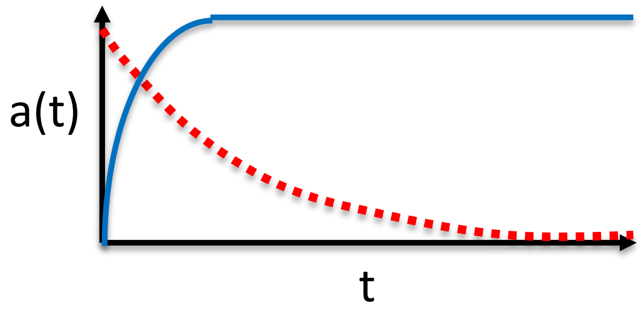
G is known with some error $G \pm E_G$

- ❑ Method 1 (simplistic method)

- Change G to
  $G \rightarrow G + E_G$
- Then re-run your fit and measure the shift it induces on the best fit physics parameters as usual
- Take the shift as your systematic error to add in quadrature.
- You might also repeat for $G \rightarrow G - E_G$ and take average

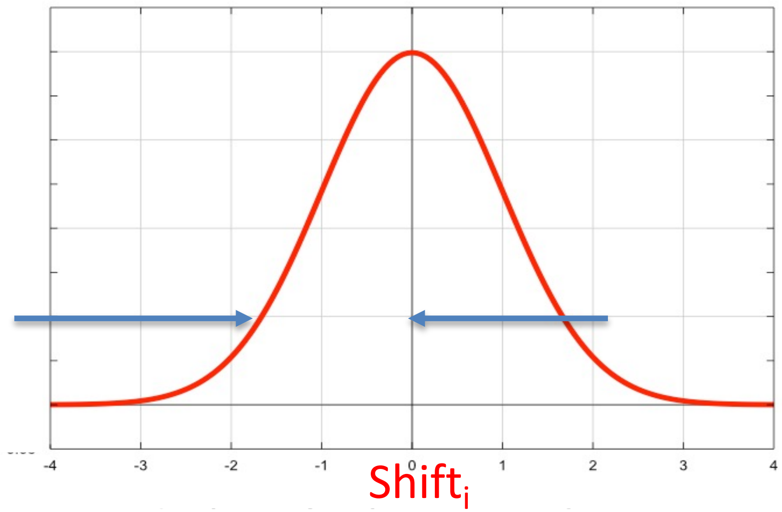
Systematic Error on Acceptance

$a(t) = 1 - \exp(-t/G) \rightarrow$



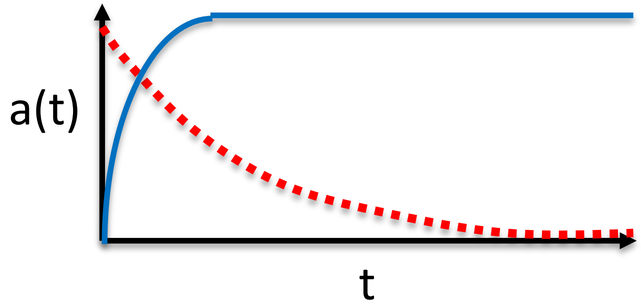
❑ Method 2

- Pick random values of G_r from a Gaussian distribution centred on G with width E_G
- Then re-run your fit with each G_r and measure the set of shifts it induces on the best fit physics parameters as usual. Call these $Shift_i$
- Plot this set of $Shift_i$
- This should give a Gaussian distribution
- Take the standard deviation as the systematic error



Systematic Error on Acceptance

$a(t) = 1 - \exp(-t/G) \rightarrow$



- ❑ Method 3 : Adding a Gaussian constraint (this will take a lot more thinking about)

- We will include the measured value of $G \pm E_G$ as a "constrained parameter" in the fit itself

- To do this we add G_{fit} as an extra fit parameter (often called a nuisance parameter)

- We also add a term to the likelihood which makes it likely to be close to G, and applies a penalty when it moves away.

- This term is simply a Gaussian PDF
  $$P(G_{fit} ; G) = \exp(-(G_{fit} - G)^2 / 2 E_g^2))$$

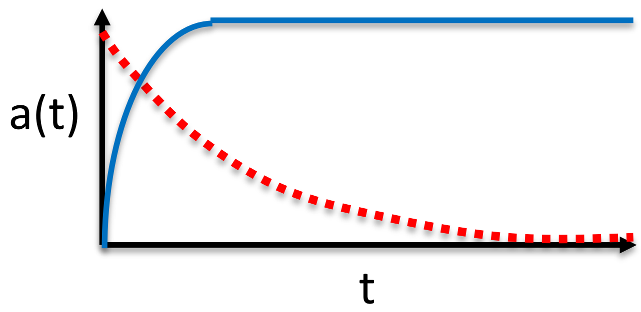
- In practice you take the negative log and add it to the overall NLL that you minimise

  - $NLL \rightarrow NLL + (G_{fit} - G)^2 / (2 E_g^2)$

- Now run the fit as normal.

Systematic Error on Acceptance

$a(t) = 1 - \exp(-t/G) \rightarrow$

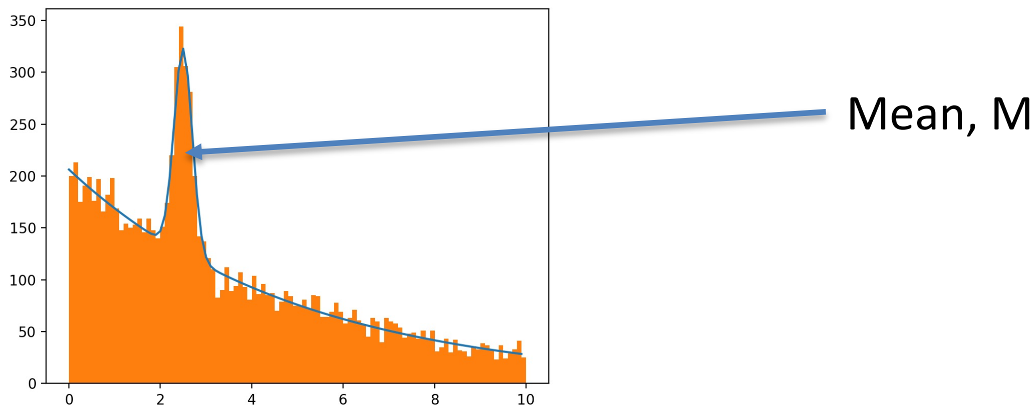


- ❑ Method 3: Adding a Gaussian constraint (continued)

- What happens is that the fit will produce larger statistical errors for the best fit values of all physical parameters
- This is because the NLL + 0.5 point will get "stretched" (flattened) a bit if it can do a better fit by changing G_{fit} a little bit
- This then incorporates the systematic error due to the measurement of G into the reported statistical errors of the results
- You can't then separate them – but this is often done when some well measured parameter is input to the fit. We say we introduce it with a Gaussian constraint.

Systematic error on mass measurement scale

- The mass on the x-axis was measured somehow



m (mass in some units)

- In general this has some limited precision - lets say you know it to $\pm$ 10% of any measurement

- In this case it is clear that this translates into a 10% error on the measurement of the mean, M, of the Gaussian

- So you can calculate this as $E_{mass\text{-}scale} = 0.1M$

$$\text{Value of Mean} = M \pm E_{stat} \pm E_{mass\text{-}scale}$$

Systematic error from fitting bias

- You want to know whether your basic fitting method is inherently biased, i.e. if you could repeat the experiment many times and average, then would the result trend towards the true value ? Or to some shifted (biased) value

- We tend to test this using Monte Carlo methods.

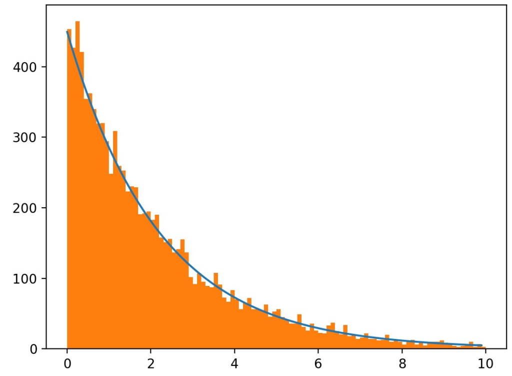
- In some cases we can learn from a toy MC study, where we simulate and fit many times. You did this in the first CP

- Recall the second CP where you fitted to the exponential distribution

  - The true value was 2.2

  - Those of you who DID NOT normalise your PDF properly got 2.17

  - Hence your fit was biased by 0.03 due to forgetting something.



- You would easily see this bias from a toy MC study with a few hundred toys.

- If it is actually an error then you fix it - i.e. this can be a good way to find bugs in your fit

- But if it is an inherent bias of the method then you can either
  - correct the result for the bias if you know it well enough,
  - or treat the bias as an additional systematic error

Some published results : 1
LHCb CP Violation analysis

arXiv:1411.3104v1 [hep-ex] 12 Nov 2014

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH (CERN)

CERN-PH-EP-2014-271
LHCb-PAPER-2014-059
November 11, 2014

Precision measurement of *CP* violation in $B_s^0 \to J/\psi K^+K^-$ decays

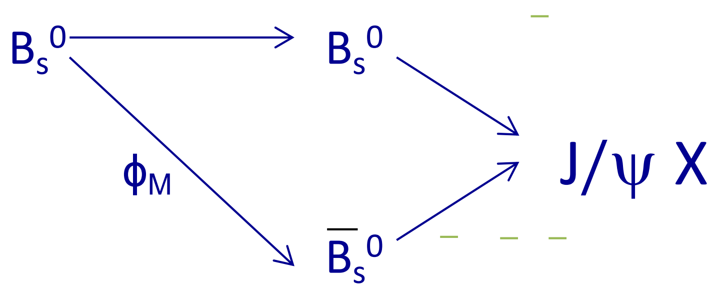
The LHCb collaboration†

Abstract

The time-dependent *CP* asymmetry in $B_s^0 \to J/\psi K^+K^-$ decays is measured using pp collision data, corresponding to an integrated luminosity of $3.0\,\text{fb}^{-1}$, collected with the LHCb detector at centre-of-mass energies of 7 and 8 TeV. In a sample of 96 000 $B_s^0 \to J/\psi K^+K^-$ decays, the *CP*-violating phase ϕ_s is measured, as well as the decay widths $\Gamma_{\rm L}$ and $\Gamma_{\rm H}$ of the light and heavy mass eigenstates of the B_s^0–$\overline{B}_s^0$ system. The values obtained are $\phi_s = -0.058 \pm 0.049 \pm 0.006\,\text{rad}$, $\Gamma_s \equiv (\Gamma_{\rm L}+\Gamma_{\rm H})/2 = 0.6603 \pm 0.0027 \pm 0.0015\,\text{ps}^{-1}$, and $\Delta\Gamma_s \equiv \Gamma_{\rm L} - \Gamma_{\rm H} = 0.0805 \pm 0.0091 \pm 0.0032\ \text{ps}^{-1}$, where the first uncertainty is statistical and the second systematic. These are the most precise single measurements of those quantities to date. A combined analysis with $B_s^0 \to J/\psi \pi^+\pi^-$ decays gives $\phi_s = -0.010 \pm 0.039\,\text{rad}$. All measurements are in agreement with the Standard Model predictions. For the first time the phase ϕ_s is measured independently for each polarisation state of the K^+K^- system and shows no evidence for polarisation dependence.

Submitted to Phys. Rev. Lett.

- We are measuring these decays in order to understand the different behaviour of matter and anti-matter

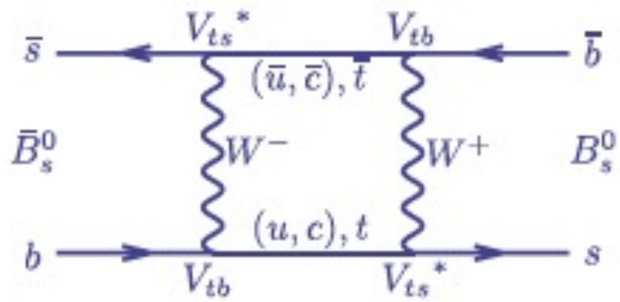


- The B_s meson can decay directly - or it can mutate into its anti-particle and then decay.

- By quantum mechanics the two decays interfere with each other

- By measuring this interference we learn about the mechanisms causing the matter anti-matter imbalance

- The bottom picture shows how we might see this : through new particles appearing in these so called loops which cause the B_s meson to turn into its anti-particle

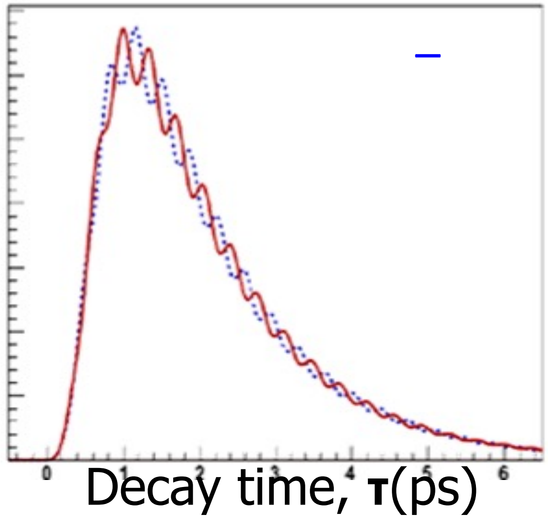


- The theory predicts the angular distribution and the time distribution

- As example the decay time distribution is the form of an exponential but with a sinusoid superimposed on top of it

- The amplitude of this sinusoid measures directly the amount of matter / anti-matter asymmetry

- The theory depends on about 20 parameters - here are some of them:



Parameter	Value
Γ_s [ps^{-1}]	
$\Delta\Gamma_s$ [ps^{-1}]	
$\|A_\perp\|^2$	
$\|A_0\|^2$	
$\delta_\parallel$ [rad]	
$\delta_\perp$ [rad]	
ϕ_s [rad]	
$\|\lambda\|$	
Δm_s [ps^{-1}]	

- Here are the final results from the fit

Parameter	Value
Γ_s [ps^{-1}]	0.6603 ± 0.0027
$\Delta\Gamma_s$ [ps^{-1}]	0.0805 ± 0.0091
$\lvert A_\perp\rvert^2$	0.2504 ± 0.0049
$\lvert A_0\rvert^2$	0.5241 ± 0.0034
$\delta_\parallel$ [rad]	$3.26\ ^{+0.10}_{-0.17}$
$\delta_\perp$ [rad]	$3.08\ ^{+0.14}_{-0.15}$
ϕ_s [rad]	-0.058 ± 0.049
$\lvert\lambda\rvert$	0.964 ± 0.019
Δm_s [ps^{-1}]	$17.711\ ^{+0.055}_{-0.057}$

- And here is the correlation matrix given out by MINUIT

Table 5: Statistical correlation matrix from the polarisation-independent fit.

	Γ_s	$\Delta\Gamma_s$	$\lvert A_\perp\rvert^2$	$\lvert A_0\rvert^2$	$\delta_\parallel$	$\delta_\perp$	ϕ_s	$\lvert\lambda\rvert$	Δm_s
Γ_s	$+1.00$	-0.45	$+0.39$	-0.31	-0.07	-0.02	$+0.01$	-0.01	$+0.01$
$\Delta\Gamma_s$		$+1.00$	-0.69	$+0.65$	$+0.02$	-0.03	-0.08	$+0.02$	-0.03
$\lvert A_\perp\rvert^2$			$+1.00$	-0.59	-0.29	-0.10	$+0.04$	-0.03	$+0.00$
$\lvert A_0\rvert^2$				$+1.00$	-0.02	-0.04	-0.03	$+0.02$	-0.03
$\delta_\parallel$					$+1.00$	$+0.42$	$+0.01$	$+0.05$	$+0.05$
$\delta_\perp$						$+1.00$	$+0.14$	-0.17	$+0.67$
ϕ_s							$+1.00$	-0.02	$+0.09$
$\lvert\lambda\rvert$								$+1.00$	-0.21
Δm_s									$+1.00$

- Here how we show the systematic errors

You "care" if the syst error ~ stat error

Parameter	Value
Γ_s [ps^{-1}]	$0.6603 \pm 0.0027 \pm 0.0015$
$\Delta\Gamma_s$ [ps^{-1}]	$0.0805 \pm 0.0091 \pm 0.0032$
$\lvert A_\perp\rvert^2$	$0.2504 \pm 0.0049 \pm 0.0036$
$\lvert A_0\rvert^2$	$0.5241 \pm 0.0034 \pm 0.0067$
$\delta_\parallel$ [rad]	$3.26\ ^{+0.10}_{-0.17}\ ^{+0.06}_{-0.07}$
$\delta_\perp$ [rad]	$3.08\ ^{+0.14}_{-0.15} \pm 0.06$
ϕ_s [rad]	$-0.058 \pm 0.049 \pm 0.006$
$\lvert\lambda\rvert$	$0.964 \pm 0.019 \pm 0.007$
Δm_s [ps^{-1}]	$17.711\ ^{+0.055}_{-0.057} \pm 0.011$

Table 3: Statistical and systematic uncertainties for the polarisation-independent result.

Source	Γ_s [ps^{-1}]	$\Delta\Gamma_s$ [ps^{-1}]	$\lvert A_\perp\rvert^2$	$\lvert A_0\rvert^2$	$\delta_\parallel$ [rad]	$\delta_\perp$ [rad]	ϕ_s [rad]	$\lvert\lambda\rvert$	Δm_s [ps^{-1}]
Total stat. uncertainty	0.0027	0.0091	0.0049	0.0034	$^{+0.10}_{-0.17}$	$^{+0.14}_{-0.15}$	0.049	0.019	$^{+0.055}_{-0.057}$
Mass factorisation	–	0.0007	0.0031	0.0064	0.05	0.05	0.002	0.001	0.004
Signal weights (stat.)	0.0001	0.0001	–	0.0001	–	–	–	–	–
b-hadron background	0.0001	0.0004	0.0004	0.0002	0.02	0.02	0.002	0.003	0.001
B_c^+ feed-down	0.0005	–	–	–	–	–	–	–	–
Angular resolution bias	–	–	0.0006	0.0001	$^{+0.02}_{-0.03}$	0.01	–	–	–
Ang. efficiency (reweighting)	0.0001	–	0.0011	0.0020	0.01	–	0.001	0.005	0.002
Ang. efficiency (stat.)	0.0001	0.0002	0.0011	0.0004	0.02	0.01	0.004	0.002	0.001
Decay-time resolution	–	–	–	–	–	0.01	0.002	0.001	0.005
Trigger efficiency (stat.)	0.0011	0.0009	–	–	–	–	–	–	–
Track reconstruction (simul.)	0.0007	0.0029	0.0005	0.0006	$^{+0.01}_{-0.02}$	0.002	0.001	0.001	0.006
Track reconstruction (stat.)	0.0005	0.0002	–	–	–	–	–	–	0.001
Length and momentum scales	0.0002	–	–	–	–	–	–	–	0.005
S-P coupling factors	–	–	–	–	0.01	0.01	–	0.001	0.002
Fit bias	–	–	0.0005	–	–	0.01	–	0.001	–
Quadratic sum of syst.	0.0015	0.0032	0.0036	0.0067	$^{+0.06}_{-0.07}$	0.06	0.006	0.007	0.011

These ones are all ones we have covered

Varying the signal mass model

Efficiency

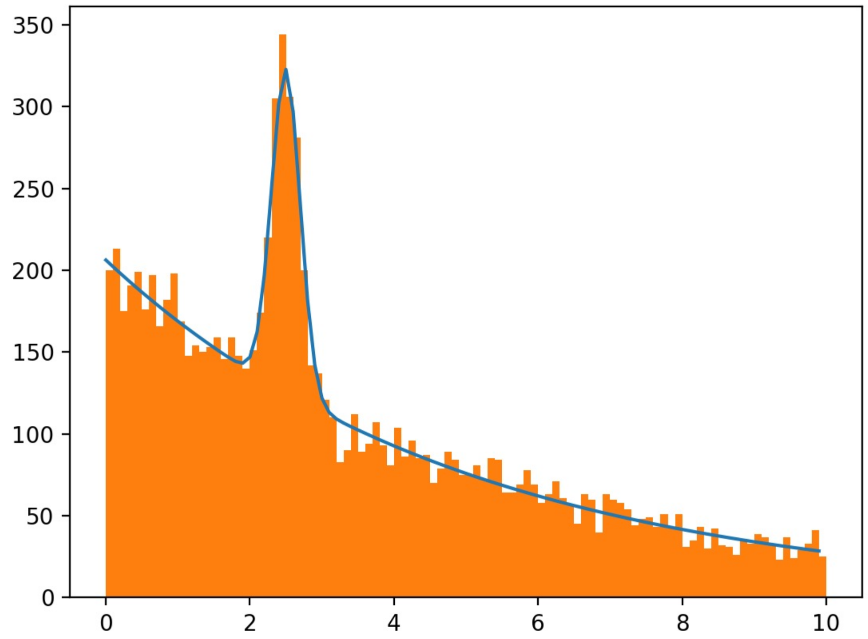
Overall scales

Bias of the fit process

Source	Γ_s [ps^{-1}]	$\Delta\Gamma_s$ [ps^{-1}]	$\lvert A_\perp\rvert^2$	$\lvert A_0\rvert^2$	$\delta_\parallel$ [rad]	$\delta_\perp$ [rad]	ϕ_s [rad]	$\lvert\lambda\rvert$	Δm_s [ps^{-1}]
Total stat. uncertainty	0.0027	0.0091	0.0049	0.0034	$^{+0.10}_{-0.17}$	$^{+0.14}_{-0.15}$	0.049	0.019	$^{+0.055}_{-0.057}$
Mass factorisation	–	0.0007	0.0031	0.0064	0.05	0.05	0.002	0.001	0.004
Signal weights (stat.)	0.0001	0.0001	–	0.0001	–	–	–	–	–
b-hadron background	0.0001	0.0004	0.0004	0.0002	0.02	0.02	0.002	0.003	0.001
B_c^+ feed-down	0.0005	–	–	–	–	–	–	–	–
Angular resolution bias	–	–	0.0006	0.0001	$^{+0.02}_{-0.03}$	0.01	–	–	–
Ang. efficiency (reweighting)	0.0001	–	0.0011	0.0020	0.01	–	0.001	0.005	0.002
Ang. efficiency (stat.)	0.0001	0.0002	0.0011	0.0004	0.02	0.01	0.004	0.002	0.001
Decay-time resolution	–	–	–	–	–	0.01	0.002	0.001	0.005
Trigger efficiency (stat.)	0.0011	0.0009	–	–	–	–	–	–	–
Track reconstruction (simul.)	0.0007	0.0029	0.0005	0.0006	$^{+0.01}_{-0.02}$	0.002	0.001	0.001	0.006
Track reconstruction (stat.)	0.0005	0.0002	–	–	–	–	–	–	0.001
Length and momentum scales	0.0002	–	–	–	–	–	–	–	0.005
S-P coupling factors	–	–	–	–	0.01	0.01	–	0.001	0.002
Fit bias	–	–	0.0005	–	–	0.01	–	0.001	–
Quadratic sum of syst.	0.0015	0.0032	0.0036	0.0067	$^{+0.06}_{-0.07}$	0.06	0.006	0.007	0.011

Systematic errors

- Everything done so far assumes the ideal (perfect) case
  - The PDF is the correct PDF for the data
  - You know your time acceptance perfectly
  - You understand your background perfectly
  - The data is measured perfectly (with perfect resolution)
  - …….
- None of these statements are ever likely to be true !



So you perform lots of systematic studies and then quote:

Tau = 5.3 +- 0.1 +- 0.03
Mean = 2.5 +- 0.2 +- 0.1
Fraction = 0.9 +- 0.15 +- 0.05

Where the first errors is statistical and the second is systematic

The systematic error is always the quadrature sum of lots of individual systematic errors.

The result is now complete and usable

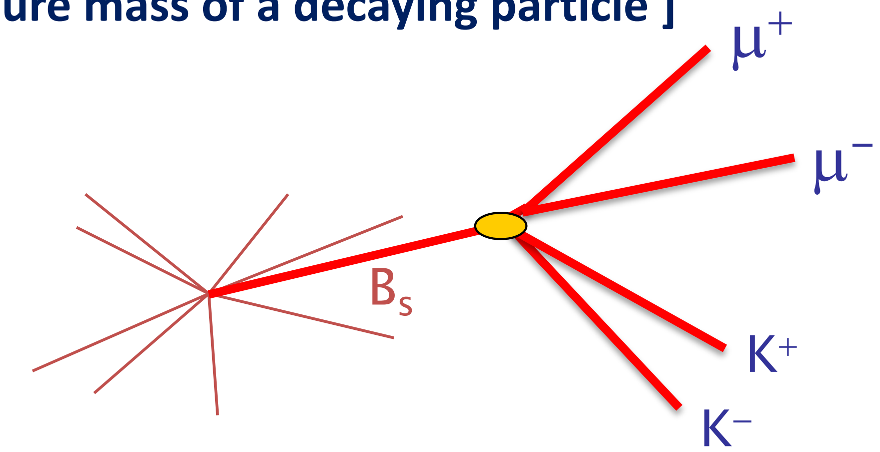
Summary:

- That there are many sources of systematic error on any measurement
- How to estimate systematic errors in different ways
- How to add systematic errors in quadrature to the statistical error to obtain the final result X +- E_{stat} +- E_{syst}

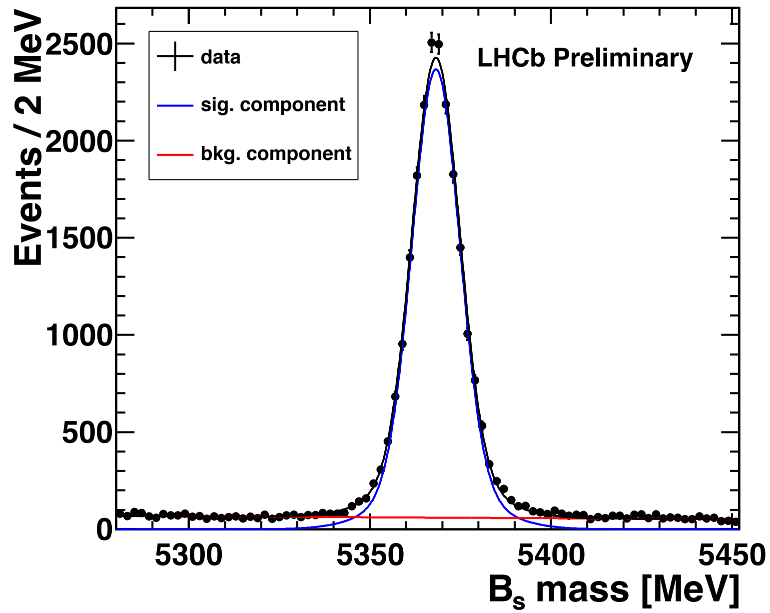
Extra slides for information only

[Digression 2: how do you measure mass of a decaying particle]

- Here is a decay of a Bs meson in LHCb
- The original particle is a Bs
- It decays to 4 particles as shown



- To reconstruct the mass of the Bs (which we cant observe directly)
  - Measure the 4 vectors of each of the decaying particles,
    $P_i^u = [E, px, py, pz]$
  - Add all 4 together to get the 4 vector of the Bs
    $P_B^u = Sum_i\{ P_i^u \}$
  - Take the "invariant mass" of P_B^u
    $M = sqrt(E^2 – px^2 - py^2 - pz^2)$
  - Plot M ➔



Some published results : 2
ATLAS Higgs Measurement

PHYSICAL REVIEW D **90,** 052004 (2014)

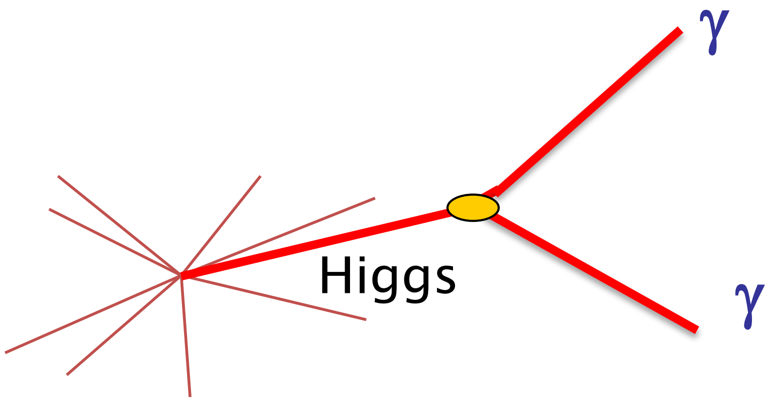
Measurement of the Higgs boson mass from the $H \to \gamma\gamma$ and $H \to ZZ^* \to 4\ell$ channels in pp collisions at center-of-mass energies of 7 and 8 TeV with the ATLAS detector

G. Aad *et al.*[*]
(ATLAS Collaboration)
(Received 17 June 2014; published 9 September 2014)

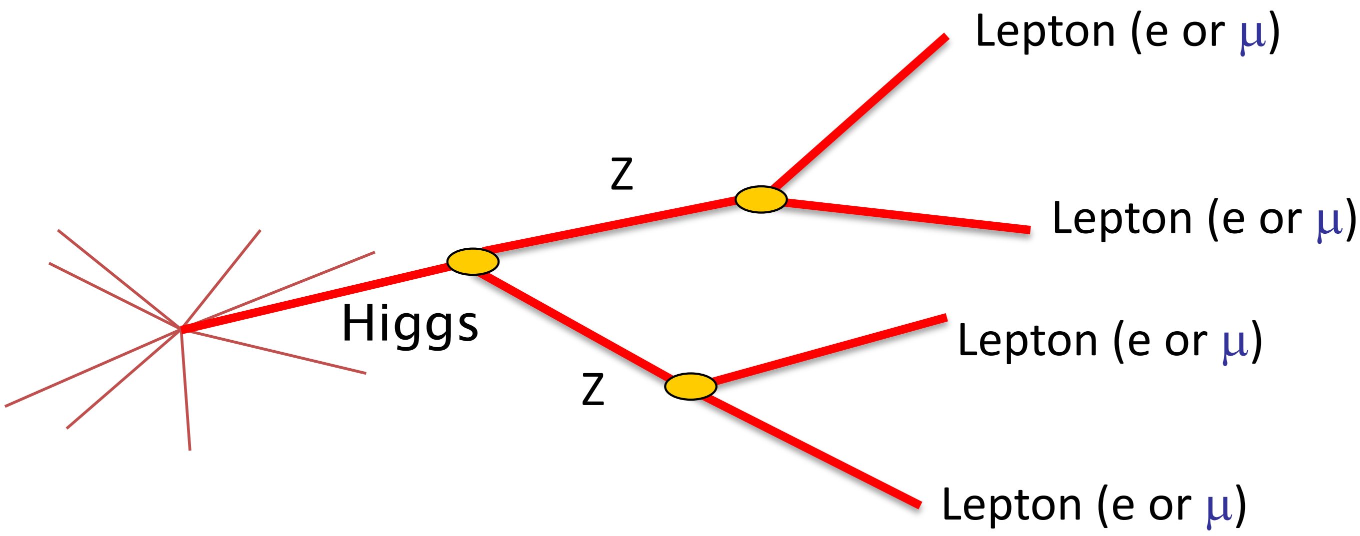
An improved measurement of the mass of the Higgs boson is derived from a combined fit to the reconstructed invariant mass spectra of the decay channels $H \to \gamma\gamma$ and $H \to ZZ^* \to 4\ell$. The analysis uses the pp collision data sample recorded by the ATLAS experiment at the CERN Large Hadron Collider at center-of-mass energies of 7 TeV and 8 TeV, corresponding to an integrated luminosity of 25 fb^{-1}. The measured value of the Higgs boson mass is $m_H = 125.36 \pm 0.37(\mathrm{stat}) \pm 0.18(\mathrm{syst})$ GeV. This result is based on improved energy-scale calibrations for photons, electrons, and muons as well as other analysis improvements, and supersedes the previous result from ATLAS. Upper limits on the total width of the Higgs boson are derived from fits to the invariant mass spectra of the $H \to \gamma\gamma$ and $H \to ZZ^* \to 4\ell$ decay channels.

DOI: 10.1103/PhysRevD.90.052004 PACS numbers: 14.80.Bn

- This paper measures Higgs bosons in two decay modes

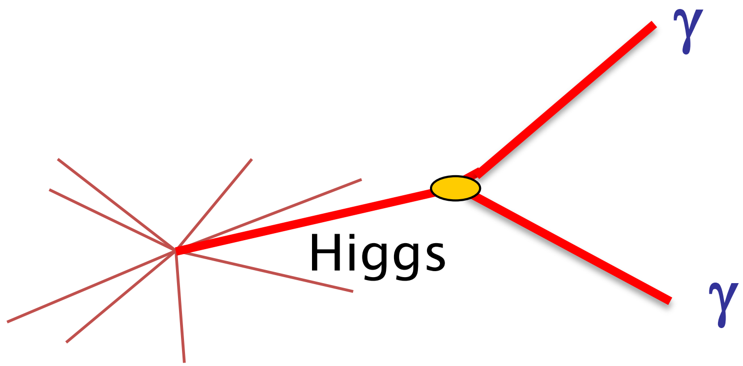


Higgs decays to two photons directly

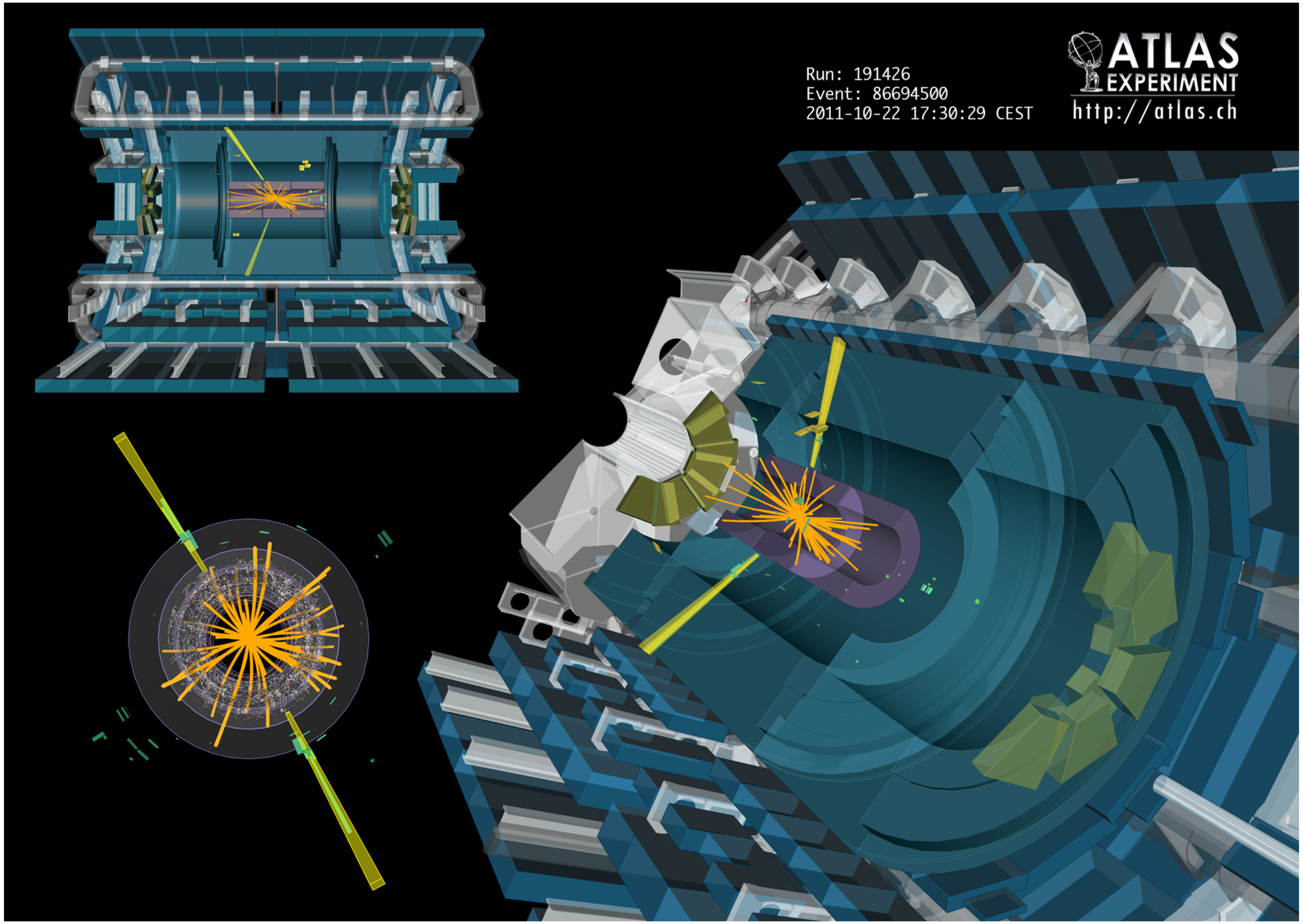


Higgs decays to four leptons indirectly via ZZ

- ❑ But note that this:



- ❑ Is hidden in this:



- ❑ For the 2 photon decay
- ❑ Lots of background
- ❑ Very small signal component

[Do you recognise why I did the CP2]

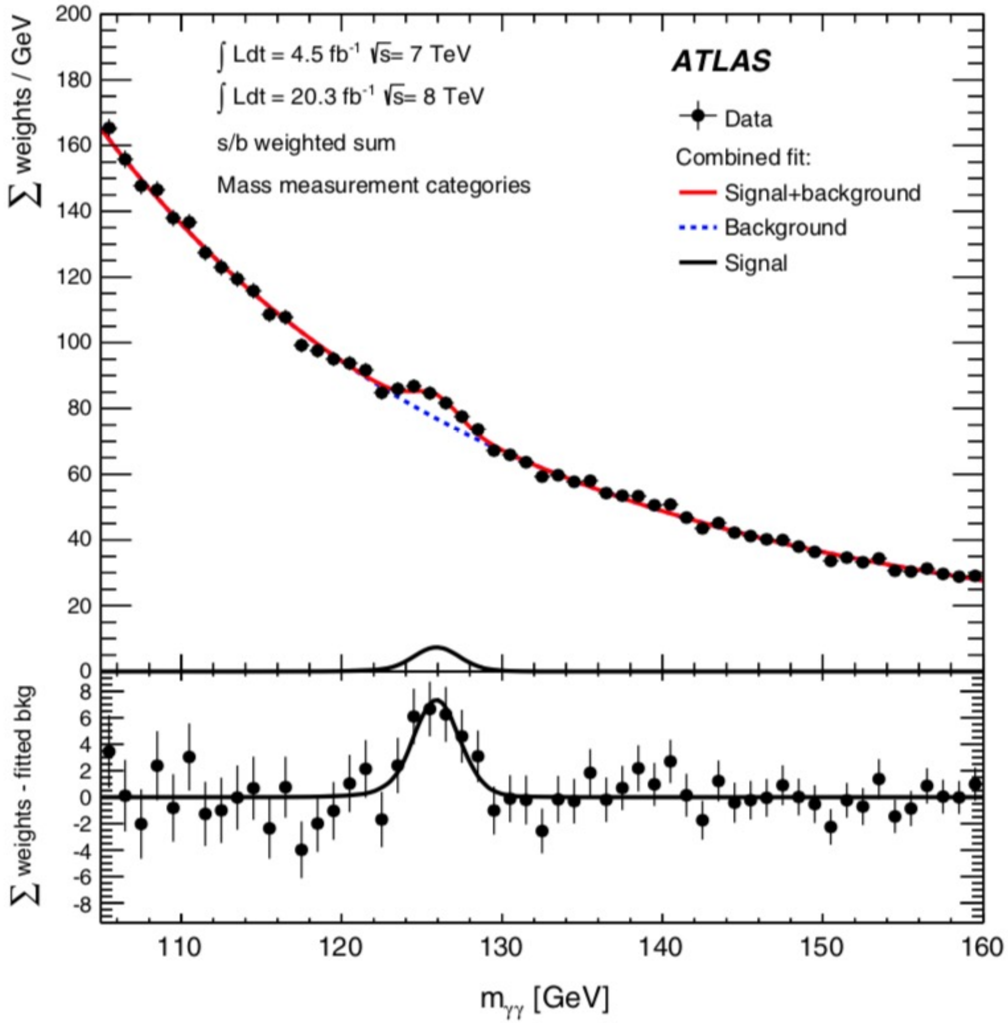


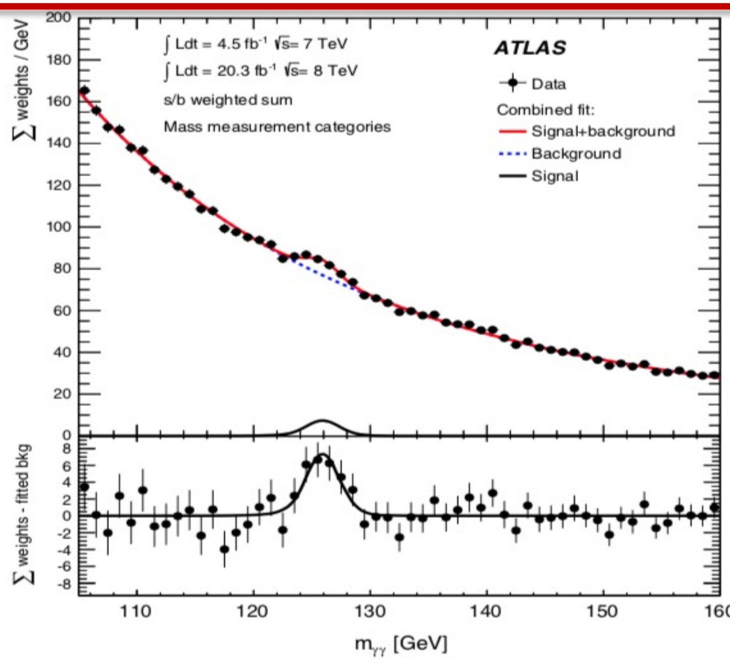
FIG. 4 (color online). Invariant mass distribution in the $H \to \gamma\gamma$ analysis for data (7 TeV and 8 TeV samples combined), showing weighted data points with errors, and the result of the simultaneous fit to all categories. The fitted signal plus background is shown, along with the background-only component of this fit. The different categories are summed together with a weight given by the s/b ratio in each category. The bottom plot shows the difference between the summed weights and the background component of the fit.

Figure 4 shows the result of the simultaneous fit to the data over all categories. For illustration, all categories are summed together, with a weight given by the signal-to-background (s/b) ratio in each category.

F. Systematic uncertainties

The dominant systematic uncertainties on the mass measurement arise from uncertainties on the photon energy scale. These uncertainties, discussed in Sec. II, are propagated to the diphoton mass measurement in each of the ten categories, by modifying the peak of the Crystal Ball function and the average of the Gaussian function describing the signal mass spectrum. The total uncertainty on the mass measurement from the photon energy scale uncertainties ranges from 0.17% to 0.57% depending on the category. The category with the lowest systematic uncertainty is the low p_{Tt} central converted category, for which the energy scale extrapolation from $Z \to e^+e^-$ events is the smallest.

Systematic uncertainties related to the reconstruction of the diphoton primary vertex are investigated using $Z \to e^+e^-$ events reweighted to match the transverse momentum distribution of the Higgs boson and the η distribution of the decay products. The primary vertex is



reconstructed using the same technique as for diphoton events, ignoring the tracks associated with the electrons, and treating them as unconverted photons. When this procedure is applied to simulated samples, the efficiency to reconstruct the primary vertex is the same in $Z \to e^+e^-$ events and $H \to \gamma\gamma$ events [17]. The dielectron invariant mass is then computed in the same way as the diphoton invariant mass. Comparing the results of this procedure in data and simulation leads to an uncertainty of 0.03% on the position of the peak of the reconstructed invariant mass.

Systematic uncertainties related to the modeling of the background are estimated by performing signal-plus-background fits to samples containing large numbers of simulated background events plus the expected signal at various assumed Higgs boson masses. The signal is injected using the same functional form used in the fit, so the fitted Higgs boson mass is sensitive only to the accuracy of the background modeling. The maximum difference between the fitted Higgs boson mass and the input mass over the tested mass range is assigned as a systematic uncertainty on the mass measurement. This uncertainty varies from 0.05% to 0.20% depending on the category. The uncertainties in the different categories are taken as uncorrelated. As a cross-check, to investigate the impact of a background shape in data different than in the large statistics simulated background sample, signal-plus-background pseudo-experiments are generated using a functional form for the background with one more degree of freedom than the nominal background model used in the fit: for the four high p_{Tt} categories, a second-order Bernstein polynomial or the exponential of a second-order polynomial is used; for the six other categories, a third-order Bernstein polynomial is used. The parameters of the functional form used to generate these pseudo-experiments are determined from the data. These pseudo-experiments are then fitted using the nominal background model. This procedure leads to an uncertainty on the mass measurement between 0.01% and 0.05% depending on the category, and smaller than the uncertainties derived from the baseline method using the large sample of simulated background events.

Systematic uncertainties on the diphoton mass resolution due to uncertainties on the energy resolution vary between 9% and 16% depending on the category and have a negligible impact on the mass measurement.

Systematic uncertainties affecting the relative signal yield in each category arise from uncertainties on the

- For the 2 photon decay
- Lots of background
- Very small signal component

[Do you recognise why I did the CP2]

G. Result

The measured Higgs boson mass in the $H \rightarrow \gamma\gamma$ decay channel is

$$m_H = 125.98 \pm 0.42(\text{stat}) \pm 0.28(\text{syst})\ \text{GeV}$$
$$= 125.98 \pm 0.50\ \text{GeV} \quad (1)$$

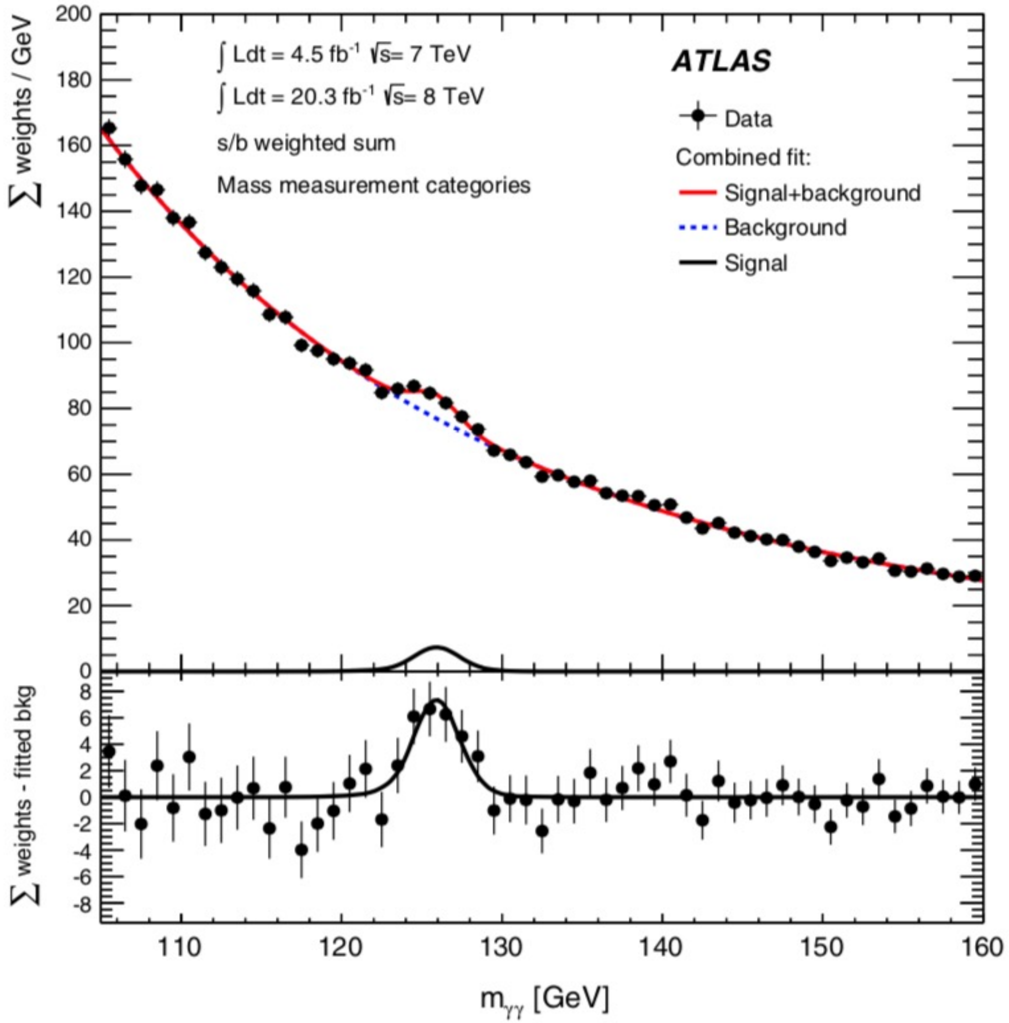


FIG. 4 (color online). Invariant mass distribution in the $H \rightarrow \gamma\gamma$ analysis for data (7 TeV and 8 TeV samples combined), showing weighted data points with errors, and the result of the simultaneous fit to all categories. The fitted signal plus background is shown, along with the background-only component of this fit. The different categories are summed together with a weight given by the s/b ratio in each category. The bottom plot shows the difference between the summed weights and the background component of the fit.