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Notes on the Pumping Lemma by TA Robert Lee

The Generalized Pumping Lemma for Regular Languages

If L is a regular language, then there exists a constant n (the number of states in the DFA for L) such that

1. <u>for all</u> strings s in L such that $|s| \geq n$, and
2. <u>for all</u> ways of partitioning s as $s = xyz$ such that $|y| \geq n$,
3. <u>there exists</u> a division of the substring y into three substrings u, v, w such that $y = uvw$, where $|v| > 0$ and $|uv| \leq n$, and
4. <u>for all</u> $i \geq 0$, xuv^iwz **is** in L.

How to Use the Pumping Lemma

We use the Pumping Lemma to prove that a language L is NOT regular.

Proof by Contradiction. Assume that L is regular; then the constant n (the number of states in the DFA for L) exists.

To contradict the property of L given by the Pumping Lemma, we must show that

1. <u>there exists</u> a string s in L such that $|s| \geq n$, and
2. <u>there exists</u> a way of partitioning s as $s = xyz$, where $|y| \geq n$, so that
3. <u>for all</u> divisions of the substring y into three substrings u, v, w such that $y = uvw$, where $|v| > 0$ and $|uv| \leq n$,
4. <u>there exists</u> a value of $i \geq 0$ such that xuv^iwz **is not** in L.

When we have shown that the string s exists, we have contradicted the property of L given by the Pumping Lemma. Therefore we conclude that L is not regular.

An Example of Using the Pumping Lemma

Consider the language $L = \{a^k b^k \mid k \geq 0\}$. Often it helps to think about what kind of strings are in the language; in this language, the strings consist of zero or more a's followed by an equal number of b's.

Proof by Contradiction. Assume that L is regular; then the constant n (the number of states in the DFA for L) exists.

1. Choose a string s in L such that $|s| \geq n$. We will pick the string $s = a^n b^n$. Notice that $|s| = 2n \geq n$, so the length requirement is satisfied.

2. Choose a partitioning of s into substrings xyz, where $|y| \geq n$. We choose the partitioning $x = \epsilon$, $y = a^n$, $z = b^n$. Notice that $|y| = n \geq n$, so the length requirement is satisfied.

3. Consider all possible divisions of y into uvw such that $|v| > 0$ and $|uv| \leq n$. Because y consists only of a's, no matter how we divide y into substrings uvw, we must have $v = a^j$, where $0 < j \leq n$.

4. Choose the value $i = 2$. The string $xuv^i wz = a^{n+j} b^n$. Since $j > 0$, we have $n + j \neq n$; thus the string $xuv^i wz$ is not in L.

Therefore we have a contradiction, and we conclude that the language L is not regular.

Pumping Lemma Practice Problems

Use the Generalized Pumping Lemma for Regular Languages to prove that the following languages are not regular.

1. $L = \{a^k b^r \mid k \neq r\}$. We solved this at the review session. Try to recreate the proof. Think about why the factorial is necessary.
2. $L = \{a^k b^r c^r \mid k > 0, r > 0\}$.
3. $L = \{a^k \mid k$ is a prime number$\}$. Hint: what does it mean for a number not to be prime?
4. $L = \{a^k \mid k$ is a perfect cube; that is, you can write $k = m^3$ for some integer $m \geq 0\}$.
5. $L = \{ww^R,$ where $w \in \{a, b\}^*$, and w^R denotes the string w in reverse$\}$.
6. $L =$ the set of palindromes of even length in $\{a, b\}^*$. A string is a palindrome if it is the same whether read forwards or backwards; for example, "racecar" and "kayak" are palindromes in English.