

## ***Practice Problems***

### **Problem 12.13:**

The following model was used to relate  $E(y)$  to a single qualitative variable with four levels:

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$$

where

$$x_1 = \begin{cases} 1 & \text{if the first level} \\ 0 & \text{if the other level} \end{cases}$$

$$x_2 = \begin{cases} 1 & \text{if the second level} \\ 0 & \text{if the other level} \end{cases}$$

$$x_3 = \begin{cases} 1 & \text{if the third level} \\ 0 & \text{if the other level} \end{cases}$$

This model fits to  $n = 40$  observations and the regression prediction equation is  $\hat{y} = 8 + 3x_1 + 5x_2 + 7x_3$ .

(a) Use the least squares prediction equation to find the estimate of  $E(y)$  for each level of the qualitative independent variable.

(b) Specify the null and alternative hypotheses you would use to test whether  $E(y)$  is the same for all four levels of the dependent variable.

### Problem 12.14:

A psychologist is interested in examining the effects of sleep description on a person's ability to perform simple arithmetic tasks. Each individual selected is randomly assigned to one of five groups,

Group1: 0 hours of sleep

Group2: 2 hours of sleep

Group3: 4 hours of sleep

Group4: 6 hours of sleep

Group5: 8 hours of sleep

The data for this experiment are in Table 12.14.

**Table 12.14 Data for Problem 12.14**

Group 1	Group 2	Group 3	Group 4	Group 5
39	25	10	4	6
33	29	18	6	0
41	34	14	-1	-3
40	26	17	9	-8

Although you want to use regression to analyze the data and define the dummy variable as follows:

$$x_1 = \begin{cases} 1 & \text{if the first group} \\ 0 & \text{if the other group} \end{cases}$$

$$x_2 = \begin{cases} 1 & \text{if the second group} \\ 0 & \text{if the other group} \end{cases}$$

$$x_3 = \begin{cases} 1 & \text{if the third group} \\ 0 & \text{if the other group} \end{cases}$$

$$x_4 = \begin{cases} 1 & \text{if the fourth group} \\ 0 & \text{if the other group} \end{cases}$$

you only have the SAS Printout that analyzed the data with complete randomized experiment. Based on this SAS printout, answer the following questions.

# SAS Printout Analysis with Complete Randomized Design

## Analysis of Variance Procedure

Dependent Variable: Y

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	4312.7000000	1078.1750000	57.30	0.0001
Error	15	282.2500000	18.8166667		
Corrected Total	19	4594.9500000			

  

R-Square	C.V.	Root MSE	Y Mean
0.938574	25.59185	4.3378182	16.950000

Source	DF	Anova SS	Mean Square	F Value	Pr > F
GROUP	4	4312.7000000	1078.1750000	57.30	0.0001

Level of GROUP	N	Mean	SD
1	4	38.2500000	3.59397644
2	4	28.5000000	4.04145188
3	4	14.7500000	3.59397644
4	4	4.5000000	4.20317340
5	4	-1.2500000	5.85234996

(a) Find the least squares estimators for  $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3$ , and  $\hat{\beta}_4$ .

(b) Report the least squares prediction equation.

(c) What null and alternative hypotheses are tested by the global  $F$ -test for this model? Interpret the hypotheses both in terms of the  $\beta$  coefficients and the group means.

(d) Test the hypotheses in part (c) at  $\alpha = 0.05$ .

(e) Place a 95% confidence interval on the difference between the mean scores of Group 1 and Group 4.