University of Central Florida
School of Computer Science
COT 4210 Fall 2004

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Test 2 Solutions (by TA Robert Lee)

1. Design a DFA to recognize the language consisting of strings over $\{a, b\}$ that do not contain the substring abb.
   Solution See page 3 for state diagram.

2. Let $|w|_x$ denote the number of occurrences of the symbol x in the string w. Let L be the language consisting of strings over $\{a, b\}$ for which either $|w|_a = 3|w|_b$ or $|w|_b > |w|_a$. Prove or disprove that L is regular.
   Solution
   Proof by contradiction. Assume L is a regular language. By the Pumping Lemma, there is a constant n associated with L.

   (a) Choose the string $s = a^{3n}b^n$. Note that $s \in L$ because $|w|_a = 3|w|_b$, and $|s| = 4n \geq n$.
   (b) Choose the partition $s = xyz$ such that $x = \epsilon$, $y = a^n$, $z = a^{2n}b^n$. Note that $|y| = n \geq n$.
   (c) In any possible division $y = uvw$, we must have $v = a^m$, where $0 < m \leq n$.
   (d) Choose $i = 2$. Then $xuv^iwz = xuv^2wz = a^{3n+m}b^n$. Because
   $|xuv^2wz|_a = 3n + m > 3n = 3|xuv^2wz|_b$ and
   $|xuv^2wz|_b = n < 3n + m = |xuv^2wz|_a$,
   we have $|xuv^2wz|_a \neq 3|xuv^2wz|_b$ and $|xuv^2wz|_b < |xuv^2wz|_a$. Thus $xuv^2wz \notin L$. This is a contradiction. Therefore L is not a regular language.

3. Construct an NFA that recognizes the language $(ab \cup (aa)^*bb)^*$.
   Solution See page 3 for state diagram.

4. DFA to regular expression conversion.
   Solution $10^*1(00^*1 \cup 1(0 \cup 10^*1))^*$. See page 4 for diagrams of the conversion procedure.

5. NFA to DFA construction.

   Solution See page 3 for state diagrams.

   The NFA is $(\{A, B, C, D\}, \{0, 1\}, \delta_{NFA}, D, \{C\})$, where δ_{NFA} is given by

δ_{NFA}	0	1	ϵ
A	$\{A\}$	$\{D\}$	$\emptyset$
B	$\{C\}$	$\{B\}$	$\{A\}$
C	$\{B\}$	$\emptyset$	$\emptyset$
D	$\emptyset$	$\{A, C\}$	$\emptyset$

Using the construction in Theorem 1.19, the constructed DFA is

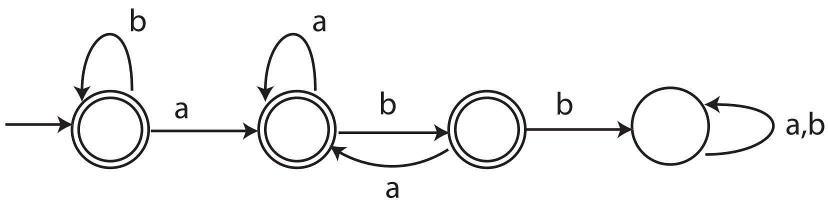
$$(\mathcal{P}(\{A, B, C, D\}), \{0, 1\}, \delta_{DFA}, \{D\}, \{S \subseteq \{A, B, C, D\} \mid C \in S\}),$$

where δ_{DFA} is given by

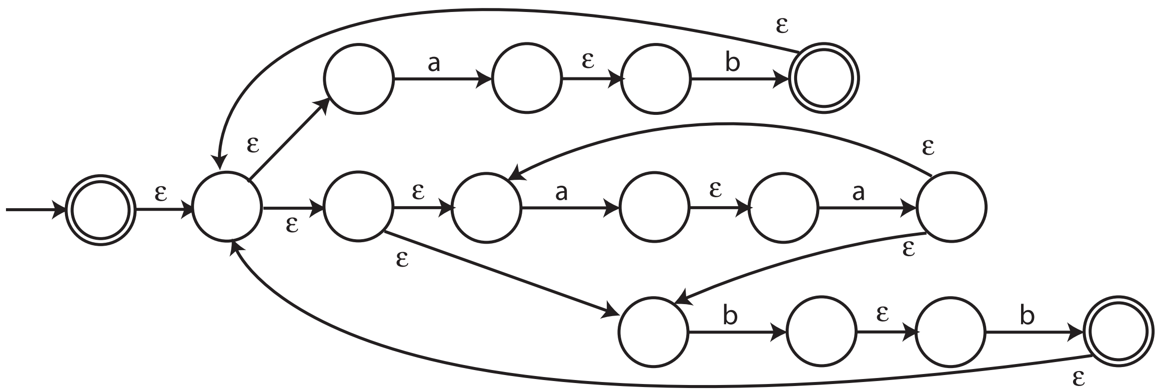
δ_{DFA}	0	1
$\{D\}$	$\emptyset$	$\{A, C\}$
$\{A, C\}$	$\{A, B\}$	$\{D\}$
$\{A, B\}$	$\{A, C\}$	$\{A, B, D\}$
$\{A, B, D\}$	$\{A, C\}$	$\{A, B, C, D\}$
$\{A, B, C, D\}$	$\{A, B, C\}$	$\{A, B, C, D\}$
$\{A, B, C\}$	$\{A, B, C\}$	$\{A, B, D\}$

(Note that, for the sake of brevity, unreachable states and sink states are left out of this table and the corresponding state diagram.)

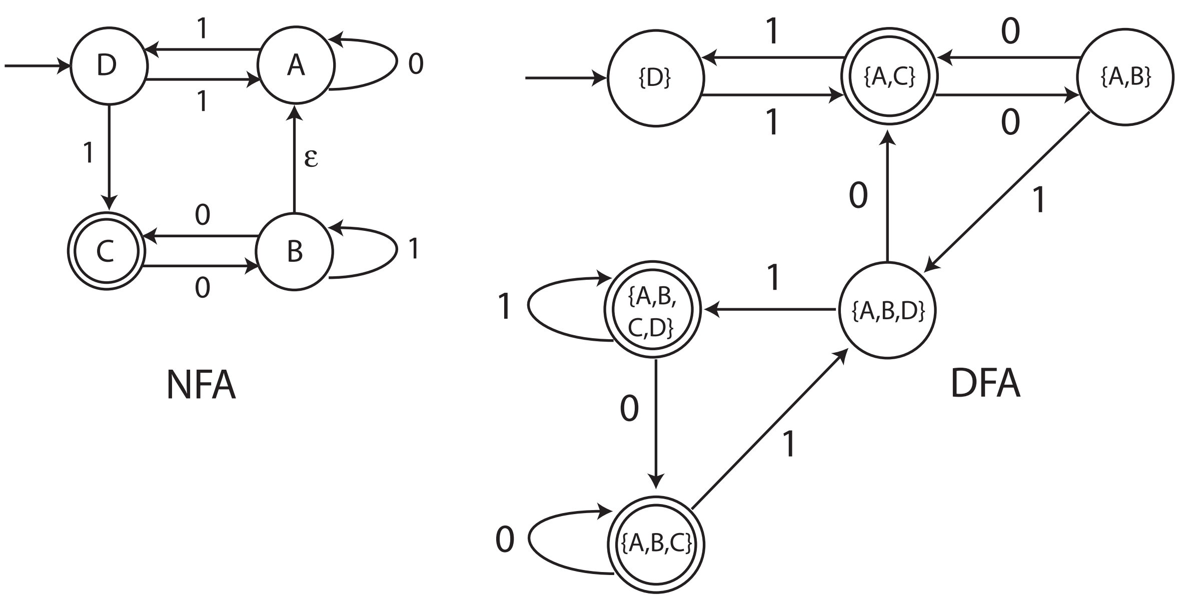
1. The strings over {a,b} that do not contain the substring abb.



3. NFA that recognizes the language (ab U (aa)*(bb))*.



5. NFA to DFA construction.



4. DFA to regular expression conversion.