University of Central Florida
School of Computer Science
COT 4210 Fall 2004

Prof. Rene Peralta
Homework 2

Due date: Sept. 17, in class

1. A correspondence between $\mathbb{N}$ and $\mathbb{Z}$ is defined by ordering the latter thus

   $$0, 1, -1, 2, -2, 3, -3, \ldots$$

   Give a mathematical formula for this correspondence. (i.e. find $f : \mathbb{N} \to \mathbb{Z}$ which is one-to-one and onto).

   Hint: $(-1)^i$ can be used to alternate positive and negative signs. You might also want to use the "floor" and/or "ceiling" integer functions.

2. (from Test 1) Describe a way to list the set of finite subsets of the natural numbers.

   Hint: recall the technique I used in lecture 4 to show that the set of languages over the binary alphabet is not countable.

3. Denote by $\neg u$ the negation of the Boolean variable u. Here is a proof that

   $$\neg(x \vee y) \Leftrightarrow (\neg x \wedge \neg y)$$

   proof: If $x = T$ then

   $$\begin{aligned} \neg(x \vee y) &\Leftrightarrow \neg(T \vee y) \\ &\Leftrightarrow \neg T \\ &\Leftrightarrow F \end{aligned}$$

   and

   $$\begin{aligned} (\neg x \wedge \neg y) &\Leftrightarrow (\neg T \wedge \neg y) \\ &\Leftrightarrow (F \wedge \neg y) \\ &\Leftrightarrow F. \end{aligned}$$

Since the formula is symmetric in x, y, the claim is also true if $y = T$. The remaining case is $X = Y = F$. In this case

$$\begin{aligned} \neg(x \vee y) &\Leftrightarrow \neg(F \vee F) \\ &\Leftrightarrow \neg F \\ &\Leftrightarrow T \end{aligned}$$

and

$$\begin{aligned} (\neg x \wedge \neg y) &\Leftrightarrow (\neg F \wedge \neg F) \\ &\Leftrightarrow (T \wedge T) \\ &\Leftrightarrow T. \end{aligned}$$

Give an analogous proof that

$$\neg(x \wedge y) \Leftrightarrow (\neg x \vee \neg y).$$

4. Exercise 1.2, page 83 of text.

5. Exercise 1.4, page 84 of text.