University of Central Florida
School of Computer Science
COT 4210 Fall 2004

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T1

1. Prove the following by induction

$$2\sum_{i=1}^{n} i = n(n+1)$$

for all $n \geq 1$.

2. For each part, give a relation on the set $A = \{1, 2, 3\}$ that satisfies the condition.

   (a) Reflexive and symmetric but not transitive

   (b) Reflexive and transitive but not symmetric

   (c) Transitive and symmetric but not reflexive

3. Answer true or false and briefly justify your answer.

   (a) The set of finite subsets of the natural numbers is countably infinite.

   (b) Every graph with five nodes must have either a complete subgraph of size three or an independent set [1] of size three.

   (c) All bipartite graphs must have an even number of vertices.

   (d) $10^{93} \equiv_{11} 10$.

   (e) Suppose there exists an island where if you pick any set of two horses, they have the same color. Then in this island all horses have the same color.

   (f) If $x \in \{1, 2, 3, 4\}$ then there exists y such that $xy \equiv_5 1$.

   (g) If $x \in \{1, 2, 3, 4, 5\}$ then there exists y such that $xy \equiv_6 1$.

[1] An independent set is a set of vertices with no edges between them.