Practice Problems

Problem 12.11:

A manufacturer of laundry detergent was interested in testing a new product prior to market release. One area of concern was the relationship among the height of the detergent suds in a washing machine as a function of the amount of detergent added and the degree of agitation in the wash cycle (measured in minutes). The complete data is in Table 12.11.

Table 12.11

Height (Y)	Agitation (X1)	Amount (X2)	Interaction	Square of Agitation	Square of Amount
28.1	1	6	6	1	36
32.3	1	7	7	1	49
34.8	1	8	8	1	64
38.2	1	9	9	1	81
43.5	1	10	10	1	100
60.3	2	6	12	4	36
63.7	2	7	14	4	49
65.4	2	8	16	4	64
69.2	2	9	18	4	81
72.9	2	10	20	4	100
88.2	3	6	18	9	36
89.3	3	7	21	9	49
94.1	3	8	24	9	64
95.7	3	9	27	9	81
100.6	3	10	30	9	100

SAS Printout for Problem 12.11

Model: EQ1
Dependent Variable: Y Height (Y)

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	2	8792.16533	4396.08267	2569.306	0.0001
Error	12	20.53200	1.71100		
C Total	14	8812.69733			

Root MSE	1.30805	R-square	0.9977
Dep Mean	65.08667	Adj R-sq	0.9973
C.V.	2.00971		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	-19.406667	2.10917045	-9.201	0.0001
X1	1	29.100000	0.41364236	70.351	0.0001
X2	1	3.286667	0.23881653	13.762	0.0001

Variable	DF	Variable Label
INTERCEP	1	Intercept
X1	1	Agitation (X1)
X2	1	Amount (X2)

SAS Printout for Problem 12.11

Model: EQ2
Dependent Variable: Y Height (Y)

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	3	8793.67783	2931.22594	1695.286	0.0001
Error	11	19.01950	1.72905		
C Total	14	8812.69733			

Root MSE	1.31493	R-square	0.9978
Dep Mean	65.08667	Adj R-sq	0.9973
C.V.	2.02028		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	-23.806667	5.16016473	-4.614	0.0007
X1	1	31.300000	2.38869211	13.103	0.0001
X2	1	3.836667	0.63517237	6.040	0.0001
X12	1	-0.275000	0.29402767	-0.935	0.3697

Variable	DF	Variable Label
INTERCEP	1	Intercept
X1	1	Agitation (X1)
X2	1	Amount (X2)
X12	1	Interaction

SAS Printout for Problem 12.11

Model: EQ3
Dependent Variable: Y Height (Y)

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	5	8807.19631	1761.43926	2881.819	0.0001
Error	9	5.50102	0.61122		
C Total	14	8812.69733			

Root MSE	0.78181	R-square	0.9994
Dep Mean	65.08667	Adj R-sq	0.9990
C.V.	1.20118		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	-14.816190	8.20925743	-1.805	0.1046
X1	1	38.580000	2.22506661	17.339	0.0001
X2	1	-0.049048	1.96676898	-0.025	0.9806
X12	1	-0.275000	0.17481774	-1.573	0.1502
X1SQ	1	-1.820000	0.42821427	-4.250	0.0021
X2SQ	1	0.242857	0.12063570	2.013	0.0749

Variable	DF	Variable Label
INTERCEP	1	Intercept
X1	1	Agitation (X1)
X2	1	Amount (X2)
X12	1	Interaction
X1SQ	1	Square of Agitation
X2SQ	1	Square of Amount

(a) What is the predicted equation of the second-order interaction model?

(b) Conduct an *F* test to investigate the overall usefulness of this model. ($\alpha = 0.05$)

(c) Conduct a *t* test to investigate the interaction term in the model. ($\alpha = 0.05$)

(d) Based on the results in part (c) and (b), do you think that you will use the first order model or the second order interaction model in the prediction?

(e) Conduct an *F* test to investigate the overall usefulness of the complete second-order model. ($\alpha = 0.05$)

Problem 12.12:

An experiment was conducted to examine the effect of temperature (x1) and month (x2) on the amount of paint exposed in the air. The sample data are in Table 12.12.

Table 12.12

Paint (Y)	Temperature (X1)	month (X2)	Interaction	Square of Temperature	Square of month
120	-10	1	-10	100	1
101	-10	3	-30	100	9
110	0	2	0	0	4
105	0	2	0	0	4
92	10	1	10	100	1
130	10	3	30	100	9

SAS Printout for Problem 12.12

Model: EQ1
Dependent Variable: Y Paint (Y)

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	2	90.50000	45.25000	0.162	0.8575
Error	3	838.83333	279.61111		
C Total	5	929.33333			

Root MSE	16.72158	R-square	0.0974
Dep Mean	109.66667	Adj R-sq	-0.5044
C.V.	15.24764		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	100.166667	18.06136659	5.546	0.0116
X1	1	0.025000	0.83607881	0.030	0.9780
X2	1	4.750000	8.36078811	0.568	0.6097

Variable	DF	Variable Label
INTERCEP	1	Intercept
X1	1	Temperature (X1)
X2	1	month (X2)

SAS Printout for Problem 12.12

Model: EQ2
Dependent Variable: Y Paint (Y)

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	3	902.75000	300.91667	22.639	0.0426
Error	2	26.58333	13.29167		
C Total	5	929.33333			

Root MSE	3.64577	R-square	0.9714
Dep Mean	109.66667	Adj R-sq	0.9285
C.V.	3.32441		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	100.166667	3.93788578	25.437	0.0015
X1	1	-2.825000	0.40760990	-6.931	0.0202
X2	1	4.750000	1.82288690	2.606	0.1211
X12	1	1.425000	0.18228869	7.817	0.0160

Variable	DF	Variable Label
INTERCEP	1	Intercept
X1	1	Temperature (X1)
X2	1	month (X2)
X12	1	Interaction

(a) What is the predicted equation for the second-order interaction model?

(b) Conduct an *F* test to investigate the overall usefulness of this model. (α = 0.05)

(c) Conduct a *t* test to investigate the interaction term in the model. (α = 0.05)

(d) Based on the result in part (c) and (b), do you think that you will use the first order model or the second order interaction model in the prediction?

(e) Is β_2 important in the second-order interaction model?