Constructing a time-series of physical input-output tables for Australia

Jacob Fry,[1] Manfred Lenzen, Timothy Baynes, James West and Arne Geschke

Abstract

Physical input-output tables provide a complete map of physical flows within economies and more broadly between the economy and natural environment. Physical input-output tables can assist with environmental impact analysis and assessing progress towards the 'circular economy' by, for example, mapping material cycles within economies. However, producing an account that maps all physical flows can be difficult since generally physical data describing the economy are less detailed than monetary data. In addition, physical production data is often limited in the number of products covered. Also, time-series information can contain gaps, and variations in data classification. In this study, we construct a time-series of physical input-output tables (PIOTs) for Australia. A new RAS variant tailored to the use of qualitative priors is employed to construct reconciled PIOTs from a variety of incomplete and unharmonised data sources. The tables are constructed for the years 1985 to 2012 at a resolution of 150 products/industries. The PIOT construction method described in this paper is applicable to constructing PIOTs for other countries where the physical data is similarly disparate, patchy and unharmonised. The usefulness of these PIOTs is demonstrated through a case study focusing on material flow in the construction industries.

Keywords: circular economy, material flow, RAS, PIOT, input-output analysis

1 Introduction

1.1 Motivation

The 'circular economy' has been suggested as a method to reduce the material throughput of industrialised societies (Ellen McArthur Foundation, 2013; Hislop and Hill, 2011). In general, circular economy principles involve achieving a high degree of material efficiency using strategies such as recycling, remanufacturing and reuse (Allwood et al., 2011). These principles are analogous to strategies employed in the field of industrial ecology, whereby industrial systems are designed to more closely resemble, and do less damage to, the natural ecologies in which they are contained (Allenby and Cooper, 1994; Lifset and Graedel, 2002).

Of particular relevance is the concept of 'industrial symbiosis', the process by which waste products from one industry are used as inputs to the production processes of other industries (Ehrenfeld and Gertler, 1997). In general, these approaches reduce the extraction and use of raw materials from nature by increasing the use-intensity of materials already in the human economy. Reducing the use of raw materials decreases direct environmental impacts caused by the extractive industries, and can also reduce the impacts from processing and manufacturing (for example, producing glass and aluminium using a portion of recycled content reduces direct manufacturing energy use).

In this study we construct physical input-output tables for Australia (PIOTs). PIOTs are a detailed representation of material flows between entities in the economy and can therefore be used as an aid exploring the circular economy. For example, technical coefficients can be derived from input-output tables that represent an industry's 'production recipe'. These coefficients can be analysed over time to determine whether changes in production technology are occurring (Deutsch and Syrquin, 1989; Wood and Lenzen, 2009), such as material substitution (Allwood et al., 2011).

[1] Corresponding author: jacob.fry@sydney.edu.au, ISA, School of Physics A28, The University of Sydney, NSW, Australia

In the Australian economy, the construction industries are a significant driver of material consumption. If the magnitude and composition of these material flows were known, future resource requirements for building stock growth could be predicted. In addition, the availability of waste products from the dismantling of building stocks for use as inputs to new structures could be estimated. The capability of existing domestic recycling infrastructure to recover these wastes, as well as the existence of markets for recycled products, could also be assessed.

1.2 Accounting for physical flows

Measuring the 'degree-of-circularity' that exists within an economy is challenging due to the complex web of interactions and material exchanges that occur along product supply chains. Input-output analysis (IOA) is a useful tool for conducting supply-chain analysis and has been applied to many environmental-economic problems to-date, for example: calculating national greenhouse gas footprints that incorporate global trade flows (Peters et al., 2012; Wiebe et al., 2012), and accounting for the effect of international trade on biodiversity threats (Lenzen et al., 2012b). IOA has also been applied specifically to the problem of tracing physical flows in economies (Wiedmann et al., 2015; Wood et al., 2009).

Traditional monetary input-output tables account for the financial transactions between economic sectors. However, there are inherent problems in using traditional monetary input-output (MIO) frameworks to account for physical flows of material within economies. In standard MIO models, the trade of physical quantities between industries is assumed to be proportional to the magnitude of the monetary transactions, however this results in inaccurate allocation when the price paid for physical products varies between receiving sectors (Lenzen, 2000). Also, product categories may be a heterogeneous mix of materials, with varying weight-to-value ratios (Giljum et al., 2015; Schoer et al., 2012). In addition, production output measured in monetary terms can be influenced by economies of scale, however physical production output may be less affected by scale (Bullard and Herendeen, 1975).

The difficulty of inferring physical flows from monetary transactions is particularly apparent in exchanges involving waste products. The value of waste products is highly variable and depends upon both the type of waste and the context, for example the waste may have:

(i) negative value (e.g. there are costs associated with safely and legally disposing of the waste)

(ii) positive value (e.g. the waste has market value)

(iii) neutral or zero value (e.g. the value of the waste is approximately equal to the disposal costs)

The monetary transactions are also comprised of freight costs, labour costs (e.g. sorting and dismantling) and tariffs (e.g. landfill gate-fees). The combined uncertainty in these components makes it difficult to infer the magnitude and direction of physical flows from the monetary transactions.

1.3 Physical input-output tables

One method to account for waste flows in the economy is to employ physical input-output tables (PIOTs), in which transactions are accounted for using physical units (often tonnes). Once constructed, PIOTs are useful from both accounting and modelling perspectives (Hoekstra and van den Bergh, 2006). The PIOT framework provides a platform in which many different physical and environmental data can be warehoused and aligned. By using comparable classification schemes to economic data, PIOTs can then contribute to environmental-economic analyses.

PIOTs have been constructed for a number of countries, including The Netherlands (Konijn et al., 1997), West Germany (Stahmer et al., 1998), Denmark (Pedersen, 1999), Italy (Nebbia, 2000), Japan (Moriguchi et al., 2000), Finland (Maenpaa and Muukkonen, 2001), the EU (Giljum and Hubacek, 2001) and China (Xu and Zhang, 2008).[2] PIOTs have also been constructed to investigate specific sectors and scenarios, for example the New Zealand dairy industry (Lenzen and Lundie, 2012), energy footprints in a Spanish subregion (Carballo Penela and Sebastián Villasante, 2008), paper recycling in China (Liang et al., 2012) and wood and paper flows in Germany (Bösch et al., 2015). Physical flows in ecological systems have also been studied, see for example Isard (1969) and Lenzen (2007). PIOTs have also been used to study 'interindustry linkages' (Strassert, 2001), a concept important to the circular economy.

Constructing PIOTs can be difficult for a number of reasons. Firstly, while constructing PIOTs requires significant amounts of physical data, in general published physical data are less detailed than economic data. Sourcing physical data that does exist and aligning it with the structure of the PIOT can be particularly labour intensive (Altimiras-Martin, 2014). In addition, it is not possible to simply convert a monetary IO table into a physical table, even if detailed price information exists, since the monetary structure of the economy may be very different to the physical structure (Dietzenbacher, 2005; Giljum and Hubacek, 2004; Hubacek and Giljum, 2003). While the System of Environmental-Economic Accounting (SEEA) (UN et al., 2014) defines a standardised structure for PIOTs, there has been much discussion about different approaches (Dietzenbacher, 2005; Giljum and Hubacek, 2004; Giljum et al., 2004; Suh, 2004; Weisz and Duchin, 2006). These barriers to PIOT construction have most likely contributed to the fewer number of tables constructed, and the relative lower resolution of PIOTs, compared to monetary tables.

In Australia, physical data are usually limited to economy-wide production totals plus imports and exports. While high-detail information exists for industry production inputs in national IO tables (ABS, 2012), industry-level physical consumption data is typically not available. Time-series data for physical quantities often contains gaps and in some cases the publication ceases entirely and updated information is not available, see for example: ABS (1994, 2004). Published physical waste data can contain large uncertainties (Hyder Consulting, 2012; Netbalance, 2009) and usually do not contain information on the use of recycled waste products by industry.

1.4 A brief overview of IO table construction using the RAS optimisation method

A tool that can be used to construct balanced PIOTs that incorporate as much information as possible from these scarce physical data is the RAS algorithm. RAS was first developed by Stone (1961) and is a procedure whereby the elements of a contingency table are iteratively adjusted using row and column scaling factors. While there are a number of other methods to reconcile contingency tables, RAS is often employed because of its simplicity. See Lahr and Mesnard (2004) for a comprehensive review of the application of RAS, and other related algorithms, to constructing input-output tables. A common application of RAS is to 'balance' an input-output table, so that the sum of inputs into each sector equals the sum of outputs. A number of RAS variants have been developed, for example Junius and Oosterhaven (2003) propose the GRAS method to handle negative matrix entries. Gilchrist and Louis (1999) introduce the TRAS method which allows information in addition to marginal row and column sums to be used in the solution. Lenzen et al. (2009) describe a RAS variant that handles matrix balancing with non-unity constraint relations and under conflicting information (KRAS).

[2] See Hoekstra and van den Bergh (2006) and Altimiras-Martin (2014) for a detailed review of these frameworks.

1.5 Aim and contribution of our study

In this study, we construct a time-series of physical input-output tables from 1985-2012 for Australia. A new RAS variant tailored to the use of qualitative priors is employed to construct reconciled PIOTs from a variety of incomplete and unharmonised data sources. We extend the RAS method developed by Lenzen et al. (2006), which allows data of arbitrary shapes and sizes to be included in the solution, to also handle ratio constraints. In addition, we implement a procedure of forward- and backward-sweeps to progressively build the time series and to estimate missing time-series values. To the authors' knowledge, the PIOTs constructed in this study are the first for Australia. The table construction process is highly automated, allowing new data to be incorporated easily once it becomes available. We construct the tables with high detail for the construction sectors in order to demonstrate the application of the PIOTs with respect to material recycling in these sectors.

2 Methods

Our new approach of constructing PIOTs involves 4 main steps:

(i) A qualitative prior **Q** is defined as an initial table estimate.

(ii) Physical data is extracted from numerous sources and stored in a constraints matrix **c** and an address matrix **G**.

(iii) A RAS routine tailored to the use of qualitative priors is implemented to realise these constraints in the PIOTs. A variety of constraint types are used, including point, summation, ratio and balancing constraints.

(iv) Forward- and backward-sweeps are used to progressively improve the estimate of the time-series.

These four steps will be described in detail in the following subsections.

2.1 Defining the qualitative prior

We use a qualitative prior **Q** to define the physical production structure in the PIOT (Lenzen and Lundie, 2012; Müller et al., 2009). **Q** is a binary matrix where engineering knowledge is used to define links between entities: a value of 1 defines a link and 0 otherwise (for example, iron ore into basic iron and steel production). Table 1 shows an aggregated version of **Q**, the full size of this matrix is 156 by 156.[3]

Table 1: Physical input-output table structure. Table 1 shows an aggregated version of **Q**, the full matrix is given in Appendix E. **T** = interindustry transactions matrix, **y** = final demand matrix, **v** = primary inputs matrix, RoW = rest of the world, RoE = rest of the economy

		T					**y**		
		Basic commodities	Manufactured products	Construction services	Waste treatment	RoE	RoW (exports)	Nature	Stock growth
T	Basic commodities		1	1		1	1	1	
	Manufactured products		1	1	1	1	1		
	Construction services			1	1			1	1
	Waste treatment		1	1	1	1	1	1	
	RoE		1		1	1	1	1	1
v	RoW (imports)	1	1		1	1			
	Nature	1				1			
	Stock decline			1	1	1			

[3]See appendix A for a list of sector names and appendix E for the complete qualitative prior

In this model, basic commodities enter the economy from nature and rest of the world (RoW) (basic commodities include mined ores and forestry products). Once in the economy, these commodities are used to produce manufactured products. In addition, a proportion of the total material returns directly to nature (e.g. waste from ore refining processes). Manufactured products then flow to construction services, other manufactured products, RoE (rest of economy), RoW or to waste treatment industries. The construction services sector embodies the material in building stock while also generating waste products during the construction process. The decline in building stock, through demolitions, provides an alternative supply of physical material back into the economy, see figure 1.

The waste treatment industries are distinguished by the type of materials they process and the method employed (e.g. landfill or recycling). The landfill sectors output this waste to nature, which is considered an exogenous sink, however if 'landfill mining' (Krook et al., 2012) became an available technique in the future this sink could be endogenised to allow the material to flow back into the economy. The waste recycling sectors return material to the manufacturing sectors (e.g. metal scrap), construction services (e.g. crushed concrete for road base) and exports. Waste may also flow directly between industries and bypass the waste treatment industries, for example fabrication scrap from metal product manufacturing can flow directly back into basic metal production. Direct reuse of waste materials is also permitted, for example clay bricks recovered by the demolition sector can be used directly in new construction. We impose a material balance constraint on the tables so that all material inputs to the economy are accounted for either in buildings, other structures or waste products (Ayres and Warr, 2010).

A simplified diagram of material flows as they are accounted for in the PIOTs is shown in figure 1. The model constructed in this study does not account for the total stock of buildings, only flows to-and-from a 'virtual' stock account. Such a stock account is exogenous to the PIOT and acts as a 'sink' for material that flows into the *construction* sectors, and a 'source' of material flowing into the *demolition* sectors. This separation between stocks and flows is illustrated in figure 1.

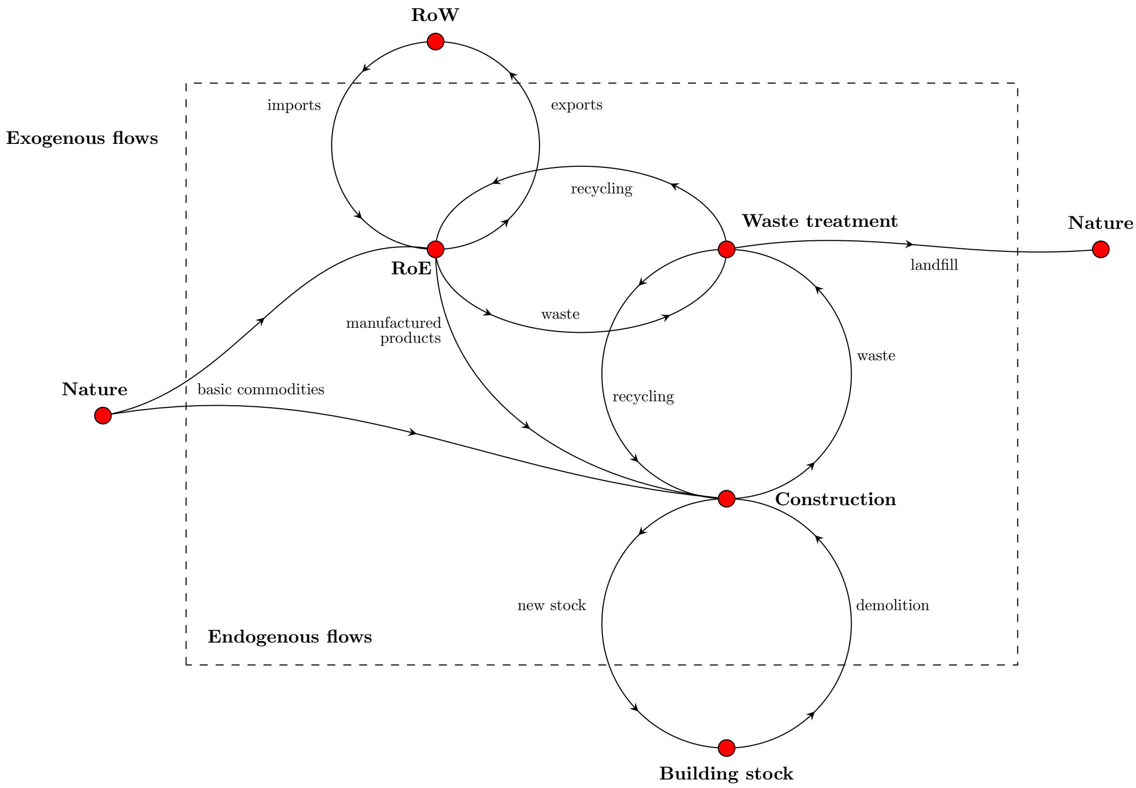

Figure 1: A simplified diagram of material flows as they are accounted for in the PIOTs. The capital account is exogenous to the PIOT while interracting with it by acting as a 'sink' for materials in new construction and a 'source' of material from demolitions. (RoW: rest of the world, RoE: rest of the economy)

2.2 Data sources

The PIOTs constructed in the study are built using published data sources only. The method presented in this study is also capable of incorporating detailed process data, however that is beyond the scope of this study. The framework is constructed using a number of data sources. Information on bulk commodity production, including ores, other mining outputs and forestry products, was found in ABARES (2014a,b); OCE (2014); USGS (2013). Manufacturing production data, for products such as bricks, concrete and wood products, were sourced from ABS (2013a,b). Data on the primary and secondary production of metals were found in Corder et al. (2015); Pauliuk et al. (2013). Sea freight data were sourced from BITRE (2014). Waste production and treatment data were extracted from ABS (2014); Crowther (2000); Fry et al. (2016); Hyder Consulting (2012). Water consumption data was provided in ABS (2015).

In some cases monetary data was used to apportion physical inputs between sectors where no suitable physical data are available. For example, total production of *sand* is known, but there is no information about how the total quantity is split between the inputs of the different construction industry types (residential: e.g. houses and apartments, commercial: e.g. offices and retail, non-building: e.g. factories and industrial facilities, and infrastructure: e.g. roads and bridges).[4] In these cases, monetary data are used from historical input-output tables (Wood, 2011) and other sources (ABS, 2016) to apportion the physical quantities between these sectors.

[4]The ratios for selected construction materials are given in appendix F.

2.3 Implementing the RAS routine

A RAS routine is implemented to realise the constraint values in the PIOTs.[5] The routine can handle a number of different constraint types: point, summation, ratio, marginal sum and balancing constraints. Approximately 8,500 constraints were written and applied in total. For each time step t, the constraint set is applied iteratively until the maximum error is less than ϵ or a maximum of n iterations has occurred.

The information contained in each data source is processed and stored in $\mathbf{c}$ and $\mathbf{G}$, where $\mathbf{c}$ is an $(n \times 1)$ vector containing the constraint values and $\mathbf{G}$ is an $(n \times m)$ matrix containing the addresses of the elements which these constraints refer to in $\mathbf{A}$.[6] In practice, $\mathbf{G}$ is a vectorised array. $\mathbf{A}$ is a 3-dimensional matrix that holds the progressive estimates of the 2-dimensional PIOT table for each year t.[7] $\mathbf{A}_{0,t}$ is used to denote the initial estimate of $\mathbf{A}_t$, that is, before any constraints are applied. $\mathbf{A}_{f,t}$ is the final estimate of $\mathbf{A}_t$ for the current sweep.

Formulating the initial estimate at each time step

Before any constraints are applied to the table in year t, an initial estimate $\mathbf{A}_{0,t}$ is formed by taking the table from the previous year and multiplying it by the gross output in year t. In this context 'previous year' can mean either $(t+1)$ or $(t-1)$, depending on the current sweep direction. For each year, the initial estimate is calculated according to:

$$\beta = \frac{\sum \mathbf{x}_t}{\sum\sum \mathbf{A}_{f,t-m}} \tag{1}$$

$$\mathbf{A}_{0,t} = \beta \mathbf{A}_{f,t-m} \tag{2}$$

where $m = 1$ during a forward sweep and $m = -1$ during a backward sweep. $\mathbf{A}_{f,t-m}$ is the final estimate of $\mathbf{A}$ in the year $t - m$. To calculate the initial estimate for year 1 in the first sweep, the binary qualitative prior $\mathbf{Q}$ is used in place of $\mathbf{A}_{f,t-m}$ in the denominator of equation 2.

Point and summation constraints

A summation constraint sets the sum over any combination of matrix elements. A point constraint is a special case where the sum is over only 1 element. This type of constraint is required, for example, to set the total output of *iron ore mining.*

Let $\mathbf{c}_k$ be the value of constraint k in the matrix $\mathbf{c}$. If C is the number of elements addressed by c_k and the address of element e in constraint c is $(i_{c,e}, j_{c,e}, t_{c,e})$, then each element in $\mathbf{A}_{i,j,t}$ is updated according to:

$$\beta = \frac{c_k}{\sum\limits_{e=1}^{C} \mathbf{A}_{0,i_{c,e}}} \tag{3}$$

$$\mathbf{A}_{1,i_{c,e},j_{c,e}} = \beta \mathbf{A}_{0,i_{c,e},j_{c,e}} \tag{4}$$

Where $\mathbf{A}_{1,t}$ is the new estimate of $\mathbf{A}_t$ and assuming all e exist in the same year t.

Ratio constraints

A ratio constraint imposes defined proportions between elements in the table. Ratio constraints do not affect the absolute magnitude of the addressed elements, rather their

[5]See appendix B for an outline of the 'standard' RAS procedure

[6]This notation is also used by Lenzen et al. (2013) and Lenzen et al. (2012a)

[7]In RAS literature, it is common to use $\mathbf{A}$ to denote the contingency table on which the routine is acting, however this is not to be confused with the coefficients matrix that is commonly denoted $\mathbf{A}$ in input-output analysis literature. In this text, $\mathbf{A}$ is the matrix containing the entire IO system ($\mathbf{v}$,$\mathbf{T}$,$\mathbf{y}$ and a SAM of all zeros).

relative sizes. This type of constraint is required, for example, to impose material input ratio between the various construction sector types, e.g. the input of steel into commercial building construction may be twice that of residential building construction.

Beginning with the following generalised ratio constraint equation:

$$ax_1 - bx_2 - cx_3 = \rho \tag{5}$$

where ρ is the realisation or current value of the elements x_1,x_2,x_3. An adjustment α is introduced:

$$ax_1(1-\alpha) - bx_2(1+\alpha) - cx_3(1+\alpha) = 0 \tag{6}$$

Expanding:

$$ax_1 - ax_1\alpha - bx_2 - bx_2\alpha - cx_3 - cx_3\alpha = 0 \tag{7}$$

Substituting 5 into 7:

$$\rho - \alpha(ax_1 + bx_2 + cx_3) = 0 \tag{8}$$

$$\alpha = \frac{\rho}{ax_1 + bx_2 + cx_3} \tag{9}$$

The table element x_1 is then adjusted by a factor of $(1-\alpha)$ and the elements x_2,x_3 adjusted by factors of $(1+\alpha)$.

$$\beta_1 = \frac{x_1(1-\alpha)}{x_1} \tag{10}$$

See Appendix C for more detail on the formulation and application of ratio constraints.

Marginal row- and column-sum constraints

Marginal row- and column sum- constraints set the row and column sums of sector j to a known value. The marginal row- and column sums are discovered by searching the constraints matrix $\mathbf{G}$ for constraints that address an entire column or row for sector j in year t. If found, the gross output vector $\mathbf{x}_{j,t}$ is updated with this value. For some physical quantities, the raw data is aggregated with respect to the PIOT classification scheme, for example:

$$\text{Hardwood logs} + \text{Softwood logs} = z \ \ \text{tonnes} \tag{11}$$

In this case $\mathbf{x}_{j,t}$ is *not* updated using this constraint data since the constraint addresses more than one sector, and the proportion that applies to sector j is unknown. If the vector $\mathbf{x}_j \forall t$ is partially incomplete (contains $\leq 25\%$ zeros) the missing values are estimated using a mixture of linear interpolation and nearest-neighbor-averaging. If a value exists in x_j, t, the marginal sum constraint is applied.

The constraint is applied by multiplying each element in row i by the scale factor β

$$\beta = \frac{\mathbf{x}_{j,t=1}}{\sum \mathbf{A}_{0,j}} \tag{12}$$

$$\mathbf{A}_{1,j} = \beta \mathbf{A}_{0,j} \tag{13}$$

Balancing

Balancing constraints are applied to ensure that the sum of inputs into each sector equals the sum of outputs from that sector. In physical terms, this is the equivalent of imposing a mass balance on the system. The rows are balanced by multiplying each element in row i with the scale factor β.

$$\beta = \frac{\sum \mathbf{A}_{0,j=i}}{\sum \mathbf{A}_{0,i}} \tag{14}$$

$$\mathbf{A}_{1,i} = \beta \mathbf{A}_{0,i} \tag{15}$$

where i includes only the endogenous sectors. Column balancing is achieved by swapping the i and j indices in the above.

3 Analysis and results

3.1 Construction sector material consumption

Figure 2 shows the cumulative net additions to building stock (additions to stock less demolitions) by construction industry type between 1985 and 2012. Additions to stock include both new construction and increases occurring through alterations and additions. The largest growth has been in infrastructure stock (total growth of 1.2 MT) followed by residential housing (total growth of 1.0 MT). The largest period of growth occurred between 2008 and 2012 which collectively account for $\approx$ 25% of the total growth.

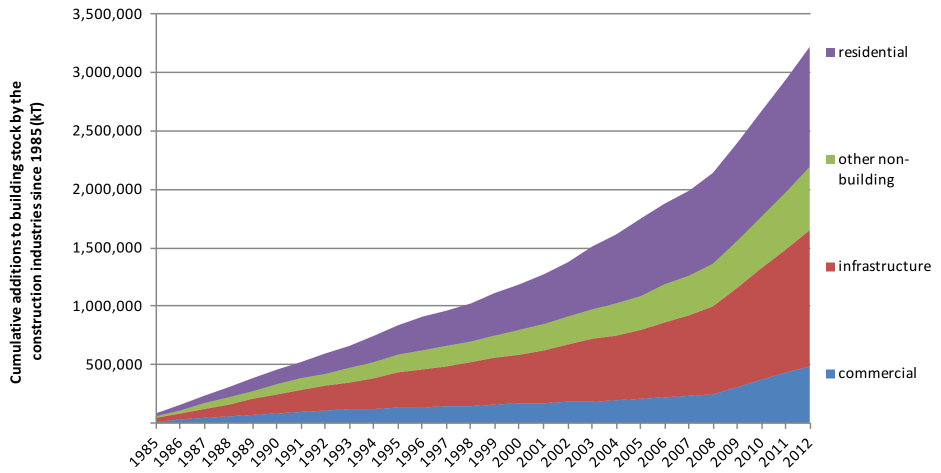

Figure 2: Cumulative net additions to building stock (kT) (additions to stock less demolitions)

Figure 3 shows the direct material consumption for all construction industries in kilotonnes. The largest flows by mass are pre-mixed concrete ($\approx$ 40%), stone and rock ($\approx$ 26%) and sand and gravel ($\approx$ 13%).

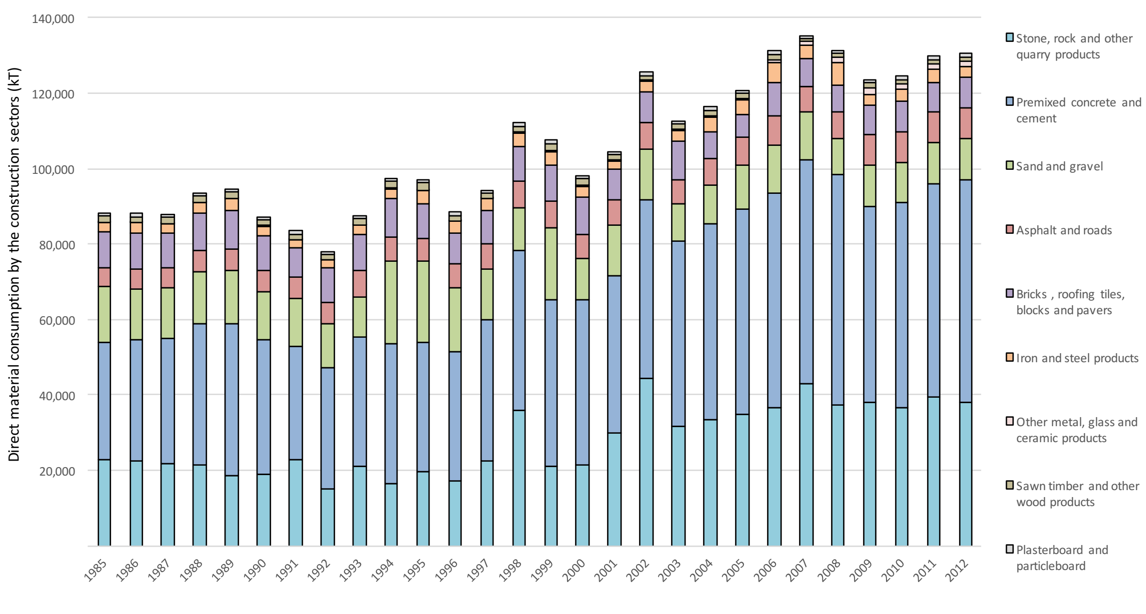

Figure 3: Direct material consumption of the construction industries (kT)

3.2 Secondary inputs

The constructed PIOTS were used to calculate the proportion of the inputs to production which come from secondary sources. Secondary sources refers to both fabrication scrap and waste products. In some cases multiple PIOT categories were aggregated together. The flows of recycled material are dominated by the flows of concrete, masonry, rock and rubble. The materials with the highest fraction of secondary inputs are metals, such as lead and copper, and also glass. The proportion of recycled plasterboard increases significantly from 0.14 in 1985 to 0.23 in 2012

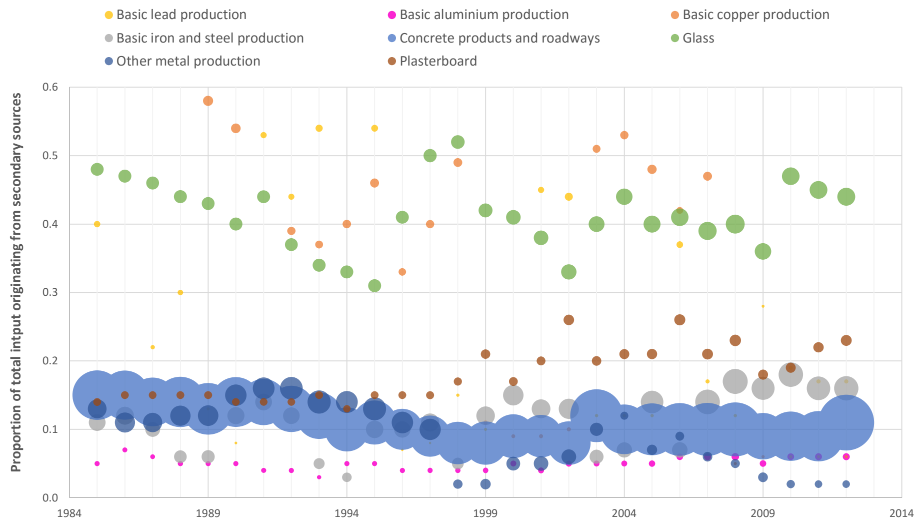

Figure 4: A time-series showing the proportion of the inputs to production which come from secondary sources. Secondary sources refers to both fabrication scrap and waste products The size of the bubble scales with the total production output of the specific product.

3.3 Constraint adherence

During table construction, the RAS routine seeks to formulate a solution that realises all of the constraint values **c**. However, violations where the constraints are not realised in the solution may exist for a number of reasons. For example, there may be inconsistencies in the raw data, such as separate data sources that contradict one another. Conflict between constraints can also be created when the prior information is too restrictive (for example, interlinkages have not been permitted in the qualitative prior where they are required). While some conflicts can be resolved by manually adjusting or selecting raw data to avoid conflict, other conflicts are not solvable using RAS. These 'RAS-infeasible' problems (Möhr et al., 1987) can be solved using more advanced methods, see for example Lenzen et al. (2009) and Geschke (2012). To determine how well the constructed PIOTs adhere to the raw data, the constraints **c** are compared with their realisations in the table **GP**. The total violation ϵ for a given PIOT and constraint set is calculated according to:

$$\epsilon = \sqrt{\sum_n (GP_n - c_n)^2} \tag{16}$$

where c_n is the value of constraint n and GP_n is the realisation in the table.

Figure 5 shows the constraint violations ϵ for each year in the time-series. The total violation between the initial estimate $\mathbf{A}_{0,t}$ and the solution $\mathbf{A}_{f,t}$ is compared. The gap between the initial estimate and the solution is the equivalent of the work done by the routine to realise the constraints. Also plotted is the residual violation as a percentage of the intital estimate violation. In general, the routine is better able to realise larger constraints than the smaller ones. This effect is demonstrated in figure 6

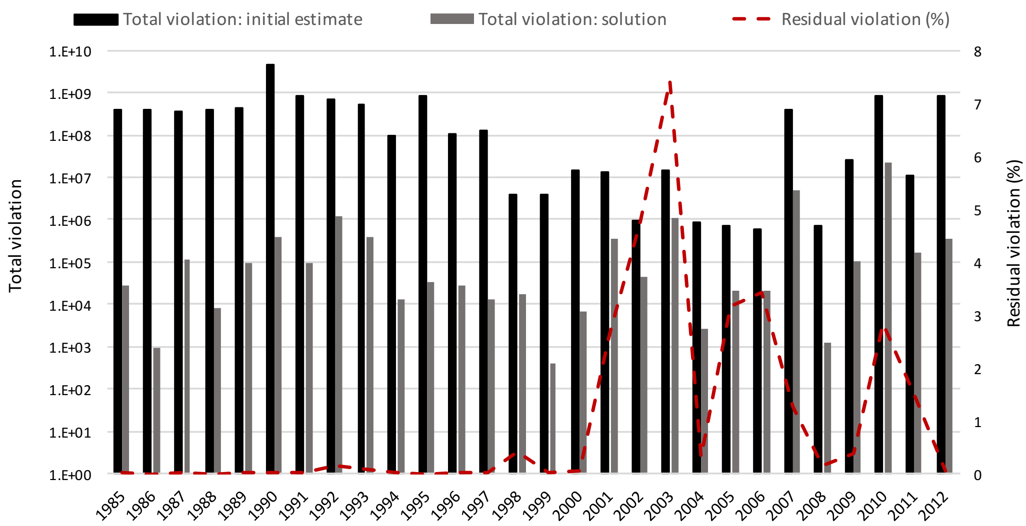

Figure 5: Constraint violation statistics for the final RAS sweep during PIOT construction shown for both the initial estimate and the solution (a smaller violation is desirable). Also shown is the residual violation as a percentage of the intital estimate violation.

where the constraint values are plotted against their realisations in units of megatonnes (MT). It can be seen that smaller values (bottom left) are less likely to adhere to the constraints than larger values (top right).

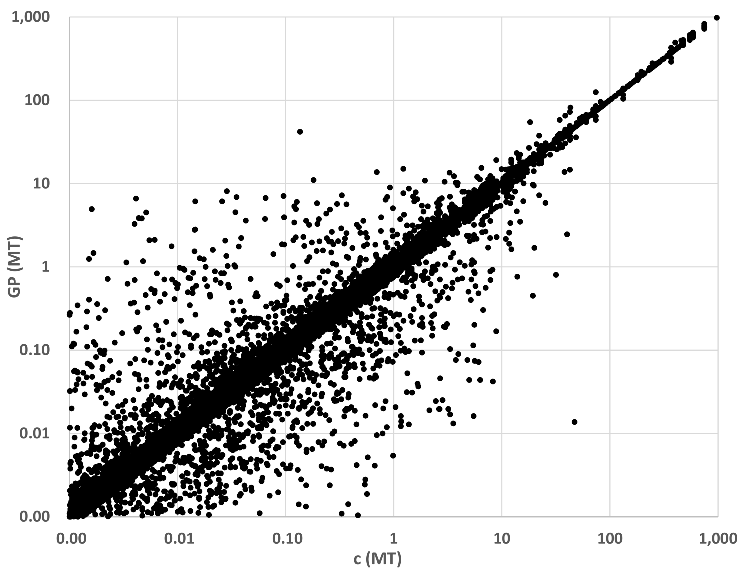

Figure 6: A comparison between the constraints **c** and the realisations **GP** in megatonnes (MT). Larger constraints are more likely to be accurately realised than smaller ones.

3.4 Data providence

In general, RAS techniques are effective in finding a solution to under-determined systems (systems where there are many more unknown than known elements). Once constructed, the end-user of an IO table may wish to know which table elements are supported by external data in order to estimate reliability. To aid these users, we have constructed data providence maps for each PIOT in the time-series. These maps show what constraint types were used to inform each element. Some elements are influenced by multiple constraints, e.g. with both summation and ratio constraints, others with only one type. The 'estimated' elements were affected by balancing constraints only. In figure 7, the data providence map for the year 2009 is compared with 2010.[8]

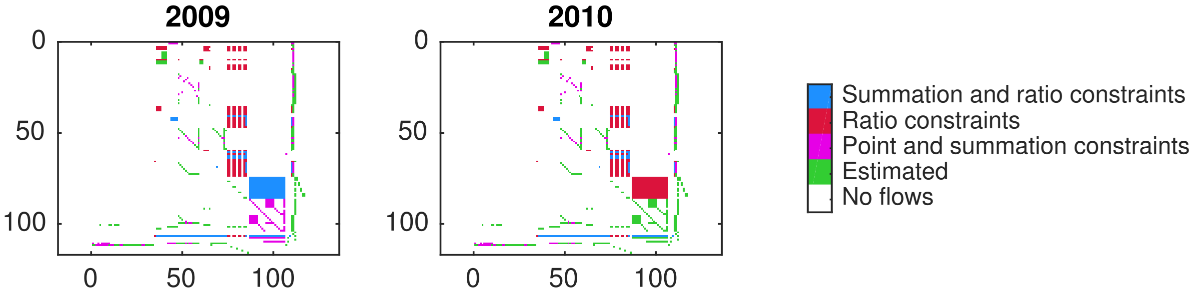

Figure 7: Data providence maps for 2009 and 2010, where the horizontal and vertical axes indicate the sector number and match table 2 in appendix A. The colours indicate which combination of constraints were used to inform each element. 'Estimated' refers to elements affected by balancing constraints only.

4 Discussion

The PIOTs constructed in this study were built using published physical data only. The accuracy and detail of the tables could be improved by including process information. This further detail could be incorporated using ratio constraints formulated from stoichiometric relationships, see for example Ayres and Ayres (1997). Furthermore, prices ($/tonne) can be used to inform the table as well as linking the physical and monetary tables (Többen, 2017).

In this study, material flows to new building stocks and the flows from deconstructed (demolished) building stocks were estimated. At the time of writing, no better information about these flows existed, however this estimation could be improved through linkage with a building stock model. In general, such stock models contain information about the composition and location of existing building stock, as well as embodied materials. While several of these models do exist for Australia, they do not include physical embodied material data and are not complete with respect to building types, see for example Baynes (2014) and Dunford et al. (2014). Stock models have been used previously to inform input-output models (Pauliuk et al., 2015) as well as to investigate waste and secondary resource flows (Hashimoto et al., 2009). The PIOTs produced in this study could be improved through linkage with such a building stock model as the estimation of flows between the exogenous capital account and the PIOT would be improved.

The PIOT structure described in this article considers waste a by-product of industrial production and final consumption. Once produced, waste can flow as an input to new production and in doing so, it is argued that an environmental benefit occurs since the use of waste negates virgin material extraction from nature. In other words, the waste and virgin material are substitutable. A variety of conditions affect whether, and to what extent, this substitution occurs, including the relative costs of production inputs (Duchin and Levine, 2011). There are also technical limits to substitution, for example a high-quality and uniform glass waste stream is required for use in float-glass manufacturing (Isa, 2008; Letcher and Vallero, 2011). In addition, domestic recycling infrastructure must exist for the waste material, otherwise the waste may be exported for recycling internationally or sent to landfill. For example in Australia, there is currently

[8]The maps for every year in the time-series are given in Appendix D.

no suitable smelters for metal recovery from electrical and electronic equipment (e-waste) (Khaliq et al., 2014) so this waste is exported internationally.

Ecological systems and economic-social systems are fundamentally interlinked (Isard et al., 1968). Once extracted from nature, some materials flow through the human-economy very quickly while others reside for extended periods, for example they are trapped in structures. In this study, building stocks are considered exogenous to the annual physical input-output tables. However in the long-term, these materials embodied in building stock are returned to nature. On a geological time-scale, materials used by the human-economy are endognised to it and will eventually flow back to the natural environment.

5 Conclusion

In this study we demonstrate a technique for constructing a time-series of physical input-output tables. In general, constructing such tables is difficult because physical data is limited and contained within a variety of disparate data sources. To overcome this, a RAS procedure is implemented that produces materially balanced tables using information from all these data sources. This procedure utilised a 'qualitative prior' that allows expert knowledge about interlinkages in the economy to be defined. We demonstrate how these physical input-output tables can be used to quantify progress towards the circular economy by detailing the proportion of production inputs that are sourced from waste and secondary products. Physical input-output tables provide a structure in which theories and ideas on the circular economy can tested as the flow of waste products back into new production can be explicitly accounted for.

Acknowledgements

Thank you to Michael Menon, for advice on the Australian construction and recyling industries, John Gallagher for his guidance on sea freight statistics, and Scott Horsnell for his assistance obtaining glass production data. Figure 1 was constructed based on an example by Pascal Seppecher (`http://www.texample.net/tikz/examples/interaction-diagram/`). Thank you also to the anonymous reviewers' comments which improved an earlier version of this manuscript.

About the authors

Jacob Fry is a PhD candidate, Manfred Lenzen is a professor, and Arne Geschke a postdoctoral researcher, at ISA, University of Sydney. Timothy Baynes is a research scientist at the Commonwealth Scientific and Industrial Research Organisation (CSIRO) and adjunct associate professor and the Australian National University. James West is a systems modeller at CSIRO.

References

ABARES (2014a). Australian Commodities. *Australian Bureau of Agricultural and Resource Economics (ABARES), Australian Government Department of Agriculture.*

ABARES (2014b). Australian forest and wood products statistics, September and December quarters 2013. *Australian Bureau of Agricultural and Resource Economics and Sciences, Canberra.*

ABS (1994). Manufacturing production: Australia: Wood and wood products. *Australian Bureau of Statistics (ABS) Catalogue No. 8369.0.*

ABS (2004). Manufacturing production: Australia. *Australian Bureau of Statistics (ABS) Catalogue No. 8301.0.*

ABS (2012). Australian National Accounts: Input-Output tables, 2008-09. *Australian Bureau of Statistics (ABS) Catalogue No. 5209.0.55.001.*

ABS (2013a). Manufacturing Production,Australia. *Australian Bureau of Statistics (ABS) Catalogue No. 8301.0.*

ABS (2013b). Production of Selected Construction Materials. *Australian Bureau of Statistics (ABS) Catalogue No. 8301.0.*

ABS (2014). Waste Account, Australia. *Australian Bureau of Statistics (ABS) Catalogue No. 4602.0.55.006.*

ABS (2015). Water account Australia, 2013-14. *Australian Bureau of Statistics (ABS) Catalogue No. 4610.0.*

ABS (2016). Engineering construction activity, Australia. *Australian Bureau of Statistics (ABS) Catalogue No. 8762.0.*

Allenby, B. R. and Cooper, W. E. (1994). Understanding industrial ecology from a biological systems perspective. *Environmental Quality Management*, 3(3):343–354.

Allwood, J. M., Ashby, M. F., Gutowski, T. G., and Worrell, E. (2011). Material efficiency: A white paper. *Resources, Conservation and Recycling*, 55(3):362–381.

Altimiras-Martin, A. (2014). Analysing the Structure of the Economy Using Physical Input-Output Tables. *Economic Systems Research*, 26(4):463–485.

Ayres, R. U. and Ayres, L. W. (1997). Use of materials balances to estimate aggregate waste generation in the United States. *Measures of environmental performance and ecosystem condition*, pages 96–156.

Ayres, R. U. and Warr, B. (2010). *The Economic Growth Engine: How Energy and Work Drive Material Prosperity*. Edward Elgar Publishing.

Baynes, T. (2014). Technology based physical stocks and flows model (MEFISTO). *CSIRO Ecosystem Sciences.*

BITRE (2014). Australian sea freight 2012-2013. *Bureau of Infrastructure, Transport and Regional Economics (BITRE).*

Bullard, C. W. and Herendeen, R. A. (1975). The energy cost of goods and services. *Energy Policy*, 3(4):268–278.

Bösch, M., Jochem, D., Weimar, H., and Dieter, M. (2015). Physical input-output accounting of the wood and paper flow in Germany. *Resources, Conservation and Recycling*, 94:99–109.

Carballo Penela, A. and Sebastián Villasante, C. (2008). Applying physical input-output tables of energy to estimate the energy ecological footprint (EEF) of Galicia (NW Spain). *Energy Policy*, 36(3):1148–1163.

Corder, G. D., Golev, A., and Giurco, D. (2015). "Wealth from metal waste": Translating global knowledge on industrial ecology to metals recycling in Australia. *Minerals Engineering*, 76:2–9.

Crowther, P. (2000). Building deconstruction in Australia.

Deutsch, J. and Syrquin, M. (1989). Economic Development and the Structure of Production. *Economic Systems Research*, 1(4):447–464.

Dietzenbacher, E. (2005). Waste treatment in physical input–output analysis. *Ecological Economics*, 55(1):11–23.

Duchin, F. and Levine, S. H. (2011). Sectors may use multiple technologies simultaneously: The rectangular choice-of-technology model with binding factor constraints. *Economic Systems Research*, 23(3):281–302.

Dunford, M., Power, L., and Cook, B. (2014). National Exposure Information System (NEXIS) Building Exposure - Local Government Area (LGA). *Geoscience Australia, Canberra.*

Ehrenfeld, J. and Gertler, N. (1997). Industrial ecology in practice. *Journal of industrial Ecology*, 1(1):67–79.

Ellen McArthur Foundation (2013). *Towards the circular economy.* In: Economic and Business Rationale for an Accelerated Transition, vol. 1. Ellen MacArthur Foun- dation.

Fry, J., Lenzen, M., Giurco, D., and Pauliuk, S. (2016). An Australian Multi-Regional Waste Supply-Use Framework. *Journal of Industrial Ecology*, 20(6):1295–1305.

Geschke, A. (2012). AISHA: A Tool to Construct a Series of Large-scale Contingency Tables. In *The 20th International Input-Output Conference, Bratislava.* University of Sydney.

Gilchrist, D. A. and Louis, L. V. S. (1999). Completing Input–Output Tables using Partial Information, with an Application to Canadian Data. *Economic Systems Research*, 11(2):185–194.

Giljum, S., Bruckner, M., and Martinez, A. (2015). Material Footprint Assessment in a Global Input-Output Framework. *Journal of Industrial Ecology*, 19(5):792–804.

Giljum, S. and Hubacek, K. (2001). International trade, material flows and land use: developing a physical trade balance for the European Union. *Interims Report*, (01-059).

Giljum, S. and Hubacek, K. (2004). Alternative Approaches of Physical Input–Output Analysis to Estimate Primary Material Inputs of Production and Consumption Activities. *Economic Systems Research*, 16(3):301–310.

Giljum, S., Hubacek, K., and Sun, L. (2004). Beyond the simple material balance: a reply to Sangwon Suh's note on physical input–output analysis. *Ecological Economics*, 48(1):19–22.

Hashimoto, S., Tanikawa, H., and Moriguchi, Y. (2009). Framework for estimating potential wastes and secondary resources accumulated within an economy – A case study of construction minerals in Japan. *Waste Management*, 29(11):2859–2866.

Hislop, H. and Hill, J. (2011). *Reinventing the wheel: a circular economy for resource security.* Green Alliance.

Hoekstra, R. and van den Bergh, J. C. J. M. (2006). Constructing physical Input-Output tables for environmental modeling and accounting: Framework and illustrations. *Ecological Economics*, 59(3):375–393.

Hubacek, K. and Giljum, S. (2003). Applying physical input–output analysis to estimate land appropriation (ecological footprints) of international trade activities. *Ecological Economics*, 44(1):137–151.

Hyder Consulting (2012). Waste and recycling in Australia 2011: Incorporating a revised method for compiling waste and recycling data: The Method Report. *Department of Sustainability, Environment, Water, Population and Communities.*

Isa, H. (2008). The need for waste management in the glass industries: A review. *Scientific Research and Essays*, 7(3):276–279.

Isard, W. (1969). Some Notes on the Linkage of the Ecologic and Economic Systems. *Papers in Regional Science*, 22(1):85–96.

Isard, W., Bassett, K., Choguill, C., Furtado, J., Izumita, R., Kissin, J., Romanoff, E., Seyfarth, R., and Tatlock, R. (1968). On the Linkage of Socio-Economic and Ecologic Systems. *Papers in Regional Science*, 21(1):79–99.

Junius, T. and Oosterhaven, J. (2003). The Solution of Updating or Regionalizing a Matrix with both Positive and Negative Entries. *Economic Systems Research*, 15(1):87–96.

Khaliq, A., Rhamdhani, M. A., Brooks, G., and Masood, S. (2014). Metal Extraction Processes for Electronic Waste and Existing Industrial Routes: A Review and Australian Perspective. *Resources*, 3(1):152–179.

Konijn, P., de Boer, S., and van Dalen, J. (1997). Input-output analysis of material flows with application to iron, steel and zinc. *Structural Change and Economic Dynamics*, 8(1):129–153.

Krook, J., Svensson, N., and Eklund, M. (2012). Landfill mining: A critical review of two decades of research. *Waste Management*, 32(3):513–520.

Lahr, M. and Mesnard, L. d. (2004). Biproportional Techniques in Input-Output Analysis: Table Updating and Structural Analysis. *Economic Systems Research*, 16(2):115–134.

Lenzen, M. (2000). Errors in Conventional and Input-Output—based Life—Cycle Inventories. *Journal of Industrial Ecology*, 4(4):127–148.

Lenzen, M. (2007). Structural path analysis of ecosystem networks. *Ecological Modelling*, 200(3–4):334–342.

Lenzen, M., Gallego, B., and Wood, R. (2006). A flexible approach to matrix balancing under partial information. *Journal of Applied Input-Output Analysis*, 11(12):1–24.

Lenzen, M., Gallego, B., and Wood, R. (2009). Matrix Balancing Under Conflicting Information. *Economic Systems Research*, 21(1):23–44.

Lenzen, M., Kanemoto, K., Moran, D., and Geschke, A. (2012a). Mapping the structure of the world economy. *Environmental Science & Technology*, 46(15):8374–8381, http://dx.doi.org/10.1021/es300171x.

Lenzen, M. and Lundie, S. (2012). Constructing enterprise input-output tables - a case study of New Zealand dairy products. *Journal of Economic Structures*, 1(1):1–15.

Lenzen, M., Moran, D., Kanemoto, K., Foran, B., Lobefaro, L., and Geschke, A. (2012b). International trade drives biodiversity threats in developing nations. *Nature*, 486(7401):109–112.

Lenzen, M., Moran, D., Kanemoto, K., and Geschke, A. (2013). Building Eora: A global multi-region input-output database at high country and sector resolution. *Economic Systems Research*, 25(1):20–49.

Letcher, T. M. and Vallero, D. (2011). *Waste: A Handbook for Management*. Academic Press.

Liang, S., Zhang, T., and Xu, Y. (2012). Comparisons of four categories of waste recycling in China's paper industry based on physical input–output life-cycle assessment model. *Waste Management*, 32(3):603–612.

Lifset, R. and Graedel, T. (2002). Industrial ecology: goals and definitions Reid Lifset and Thomas E. In *A Handbook of Industrial Ecology*. Edward Elgar Publishing.

Maenpaa, I. and Muukkonen, J. (2001). Physical input-output tables in Finland: methods, preliminary results and tasks ahead. Stockholm: Conference on economic growth, material flows and environmental pressure.

Moriguchi, Y., Matsui, S., and Saito, A. (2000). 3-Dimensional Physical Input Output Tables for Environmental and Resources Analysis. In *11th Conference Reports of Pan-Pacific Association of Input-Output Studies*, Sendai, Japan.

Möhr, M., Crown, W. H., and Polenske, K. R. (1987). A Linear Programming Approach to Solving Infeasible Ras Problems. *Journal of Regional Science*, 27(4):587–603.

Müller, M., Djanibekov, N., and others (2009). Calibration of an agricultural sector model for the region Khorezm (Uzbekistan) based on survey data. In *27th international IAAE conference, the new landscape of global agriculture, Beijing, China. International Association of Agricultural Economists (IAAE), Milwaukee. http://ageconsearch. umn. edu/bitstream/50354/2/533. pdf.*

Nebbia, G. (2000). Contabilità monetaria e contabilità ambientale. *Economia pubblica.*

Netbalance (2009). National waste data system requirements study. *Prepared for: Department of Evironment, Water, Heritage and the Arts.*

OCE (2014). Resources and energy statistics 2014. *Office of the Chief Economist (OCE), Department of Industry.*

Pauliuk, S., Wang, T., and Müller, D. B. (2013). Steel all over the world: Estimating in-use stocks of iron for 200 countries. *Resources, Conservation and Recycling*, 71:22–30.

Pauliuk, S., Wood, R., and Hertwich, E. G. (2015). Dynamic Models of Fixed Capital Stocks and Their Application in Industrial Ecology. *Journal of Industrial Ecology*, 19(1):104–116.

Pedersen, O. G. a. (1999). *Physical Input-output Tables for Denmark: Products and Materials 1990: Air Emissions 1990-92*. Danmarks Statistik.

Peters, G. P., Davis, S. J., and Andrew, R. (2012). A synthesis of carbon in international trade. *Biogeosciences*, 9(8):3247–3276.

Schoer, K., Weinzettel, J., Kovanda, J., Giegrich, J., and Lauwigi, C. (2012). Raw Material Consumption of the European Union – Concept, Calculation Method, and Results. *Environmental Science & Technology*, 46(16):8903–8909.

Stahmer, C., Kuhn, M., and Braun, N. (1998). Physical input-output tables for Germany, 1990. *Eurostat Working*.

Stone, R. (1961). *Input - output and national accounts.* [s.n.], Paris.

Strassert, G. (2001). Interindustry Linkages: The Flow Network of a Physical Input-Output Table (PIOT): Theory and Application. In Lahr, M. and Dietzenbacher, E., editors, *Input-output analysis: Frontiers and extensions*. Palgrave, Basingstoke, England.

Suh, S. (2004). A note on the calculus for physical input–output analysis and its application to land appropriation of international trade activities. *Ecological Economics*, 48(1):9–17.

Többen, J. (2017). On the simultaneous estimation of physical and monetary commodity flows. *Economic Systems Research*, 29(1):1–24.

UN, European Union, Food and Agriculture Organization, International Monetary Fund, Organisation for Economic Co-operation and Development, and The World Bank (2014). The System of Environmental-Economic Accounting (SEEA) Central Framework. *United Nations Publications*.

USGS (2013). Minerals Yearbook 2012. *Volume III, Area Reports—International, United States Geological Survey, http://minerals.usgs.gov/minerals/pubs/myb.html.*

Weisz, H. and Duchin, F. (2006). Physical and monetary input–output analysis: What makes the difference? *Ecological Economics*, 57(3):534–541.

Wiebe, K. S., Bruckner, M., Giljum, S., and Lutz, C. (2012). Calculating Energy-Related (CO2) Emissions Embodied in International Trade Using a Global Input-Output Model. *Economic Systems Research*, 24(2):113–139.

Wiedmann, T. O., Schandl, H., Lenzen, M., Moran, D., Suh, S., West, J., and Kanemoto, K. (2015). The material footprint of nations. *Proceedings of the National Academy of Sciences*, 112(20):6271–6276.

Wood, R. (2011). Construction, stability and predictability of an input-output time-series for Australia. *Economic Systems Research*, 23(2):175–211.

Wood, R. and Lenzen, M. (2009). Aggregate Measures of Complex Economic Structure and Evolution. *Journal of Industrial Ecology*, 13(2):264–283.

Wood, R., Lenzen, M., and Foran, B. (2009). A Material History of Australia. *Journal of Industrial Ecology*, 13(6):847–862.

Xu, Y. and Zhang, T. (2008). Regional metabolism analysis model based on three dimensional PIOT and its preliminary application. In *The 2008 International Input-Output Meeting: Input-output & environment*.

A Sector names

Table 2: Names of sectors contained in the PIOTs

#	Name	#	Name
1	Cereals	79	Plasterboard
2	Other crops	80	Premixed concrete
3	Roots and tubers	81	Concrete Brick, Block and Pavers
4	SugarCrops	82	Concrete roofing tiles
5	Pulses	83	Asphalt
6	Nuts	84	Brown coal
7	Oil bearing crops	85	Concrete Brick, Block and Pavers
8	Vegetables	86	Concrete roofing tiles
9	Fruits	87	Asphalt
10	Fibres	88	Brown coal
11	Crop residues used	89	Hard coal
12	Grazed biomass	90	Peat
13	Hardwood logs	91	Crude oil and natural gas liquids
14	Softwood logs	92	Natural gas
15	Wood fuel and other extraction	93	Other oil and petroleum products
16	Livestock	94	Iron and steel products
17	Other agricultural products	95	Copper products
18	Sand	96	Aluminium products
19	Silica	97	Zinc products
20	Gravel	98	Lead products
21	Bentonite clay	99	Nickel products
22	Kaolin clay	100	Other metal products
23	Refactory clay	101	Insulated wire and cabling
24	Other clay	102	Copper scrap
25	Limestone	103	Fertilizers
26	Gypsum	104	Chemicals and related products
27	Dolomite and chalk	105	Manufactured food products
28	Dimension stone	106	Animal and vegetable fats and oils
29	Crushed broken stone	107	Beverages and tobacco
30	Other minerals, mining and quarry products	108	All other manufacturing
31	Chemical and fertilizer minerals	109	Machinery and transport equipment
32	Lithium	110	Residential construction (new building)
33	Salt	111	Residential construction (repair, rennovation, alteration)
34	Iron ores	112	Residential demolition
35	Copper ores	113	Commercial construction (new building)
36	Nickel ores	114	Commercial construction (repair, rennovation, alteration)
37	Lead ores	115	Commercial demolition
38	Zinc ores	116	Other non-building construction (building)
39	Tin ores	117	Other non-building construction (repair, rennovation, alteration)
40	Gold ores	118	Other non-building demolition
41	Silver ores	119	Infrastructure construction (new)
42	Other precious metal ores	120	Infrastructure construction (repair, rennovation, alteration)
43	Cobalt ores	121	Infrastructure demolition
44	Bauxite and other aluminium ores	122	Waste treatment: landfill (Steel)
45	Manganese ores	123	Waste treatment: landfill (Copper)
46	Cadmium ores	124	Waste treatment: landfill (Aluminium)
47	Chromium ores	125	Waste treatment: landfill (Lead)
48	Titanium ores	126	Waste treatment: landfill (Other metals)
49	Zirconium ores	127	Waste treatment: landfill (Concrete, Masonry)
50	Uranium ores	128	Waste treatment: landfill (Plasterboard)
51	All other ores	129	Waste treatment: landfill (Timber)
52	Bitumen	130	Waste treatment: landfill (Glass)
53	Float, cast and rolled glass	131	Waste treatment: landfill (All other waste)
54	Glass wool and fibre	132	Waste treatment: recycling (Steel)
55	Other glass products	133	Waste treatment: recycling (Copper)
56	Ceramics and other refractories	134	Waste treatment: recycling (Aluminium)
57	Clay bricks	135	Waste treatment: recycling (Lead)
58	Terracotta roofing tiles	136	Waste treatment: recycling (Other metals)
59	Sawn hardwood	137	Waste treatment: recycling (Concrete, masonry)
60	Sawn softwood	138	Waste treatment: recycling (Plasterboard)
61	Wood particleboard and fibreboard	139	Waste treatment: recycling (Timber)
62	Plywood	140	Waste treatment: recycling (Glass)
63	Veneer	141	Waste treatment: recycling (All other waste)
64	Pig iron production	142	Waste treatment: Energy recovery
65	Alumina production	143	Waste treatment: Exports
66	Basic steel production	144	Waste treatment: residues
67	Basic copper production	145	Waste imports
68	Basic aluminium production	146	Water
69	Basic zinc production	147	Households
70	Basic lead production	148	Mining services
71	Basic nickel production	149	RoE
72	Basic tin production	150	RoW
73	Basic gold production	151	Nature
74	Basic silver production	152	Increase/Decrease in residential building stock
75	Basic cobalt production	153	Increase/Decrease in commercial building stock
76	Basic cadmium production	154	Increase/Decrease in other non-building stock
77	Other metals production	155	Increase/Decrease in infrastructure stock
78	Cement	156	Increase/Decrease in other material stock

B Basic RAS

In the basic RAS, an initial estimate matrix $\mathbf{A}_0$ is adjusted using row and column scaling. Diagonalised row- and column-scaling matrices are found according to:

$$\hat{\mathbf{r}}_{ii} = \frac{a_i}{\sum\limits_j \mathbf{A}_{0,ij}} \tag{17}$$

and

$$\hat{\mathbf{s}}_{jj} = \frac{a_j}{\sum\limits_i \mathbf{A}_{0,ij}} \tag{18}$$

where a_i is the known (desired) row i total and a_j is the known column j total. The improved estimate of $\mathbf{A}_1$ is found by scaling the rows first and then the columns:

$$\mathbf{A}_1 = (\hat{\mathbf{r}}\mathbf{A}_0)\hat{\mathbf{s}} \tag{19}$$

or vice-versa:

$$\mathbf{A}_1 = \hat{\mathbf{r}}(\mathbf{A}_0\hat{\mathbf{s}}) \tag{20}$$

C Ratio constraints

C.1 Generalised formulation

Starting with the following ratio constraint:

$$ax_1 : bx_2 : cx_3 = k_1 : k_2 : k_3 \tag{21}$$

where x_n is the current value of the elements addressed by this constraint. a,b and c are coefficients. k_m are the relative proportions from the data source.

$$\frac{ax_1}{ax_1 + bx_2 + cx_2} = \frac{k_1}{k_1 + k_2 + k_3} \tag{22}$$

$$\frac{bx_2}{ax_1 + bx_2 + cx_3} = \frac{k_2}{k_1 + k_2 + k_3} \tag{23}$$

$$\frac{cx_3}{ax_1 + bx_2 + cx_3} = \frac{k_3}{k_1 + k_2 + k_3} \tag{24}$$

Equations 22-24 can be generalised. For example, if:

$$c_1x_1 : c_2x_2 : c_3x_3 = k_1 : k_2 : k_3 \tag{25}$$

then

$$\frac{c_ix_i}{\sum\limits_j c_jx_j} = \frac{k_i}{\sum\limits_j k_j} \quad \forall i \tag{26}$$

$$c_ix_i \sum_j k_j - k_i \sum_j c_jx_j = 0 \tag{27}$$

Simplifying:

$$c_ix_i \sum_{j \neq i} k_j - k_i \sum_{j \neq i} c_jx_j = 0 \tag{28}$$

The following method describes how the table elements are updated given the ratio constraints. Beginning with the following generalised ratio constraint:

$$ax_1 - bx_2 - cx_3 = \rho \tag{29}$$

where ρ is the realisation or current value of the elements x_1,x_2,x_3. An adjustment α is introduced:

$$ax_1(1-\alpha) - bx_2(1+\alpha) - cx_3(1+\alpha) = 0 \tag{30}$$

Expanding:

$$ax_1 - ax_1\alpha - bx_2 - bx_2\alpha - cx_3 - cx_3\alpha = 0 \tag{31}$$

Substituting 29 into 31:

$$\rho - \alpha(ax_1 + bx_2 + cx_3) = 0 \tag{32}$$

$$\alpha = \frac{\rho}{ax_1 + bx_2 + cx_3} \tag{33}$$

The table element x_1 is then scaled by a factor of $(1-\alpha)$ and the elements x_2,x_3 scaled by factors of $(1+\alpha)$.

C.2 Worked example

For example, if

$$ax_1 : bx_2 : cx_3 = 2 : 3 : 4 \tag{34}$$

then

$$\frac{ax_1}{ax_1 + bx_2 + cx_2} = \frac{2}{9} \tag{35}$$

$$7ax_1 - 2bx_2 - 2cx_3 = 0 \tag{36}$$

Similarly (as per equations 23 and 24)

$$-ax_1 + 3bx_2 - cx_3 = 0 \tag{37}$$

$$-4ax_1 - 4bx_2 + 5cx_3 = 0 \tag{38}$$

E Complete qualitative prior

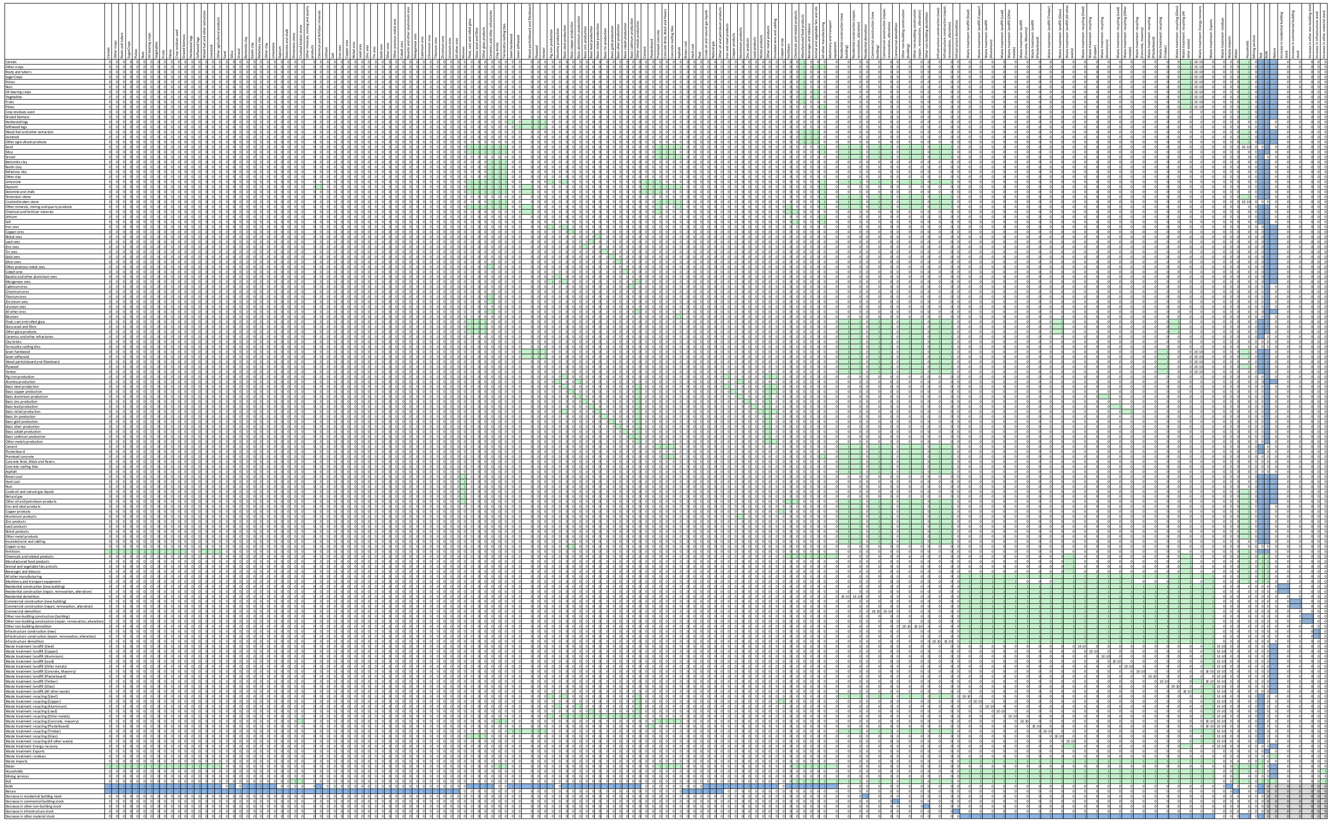

Figure 9: The qualitative prior

D Data providence heat maps

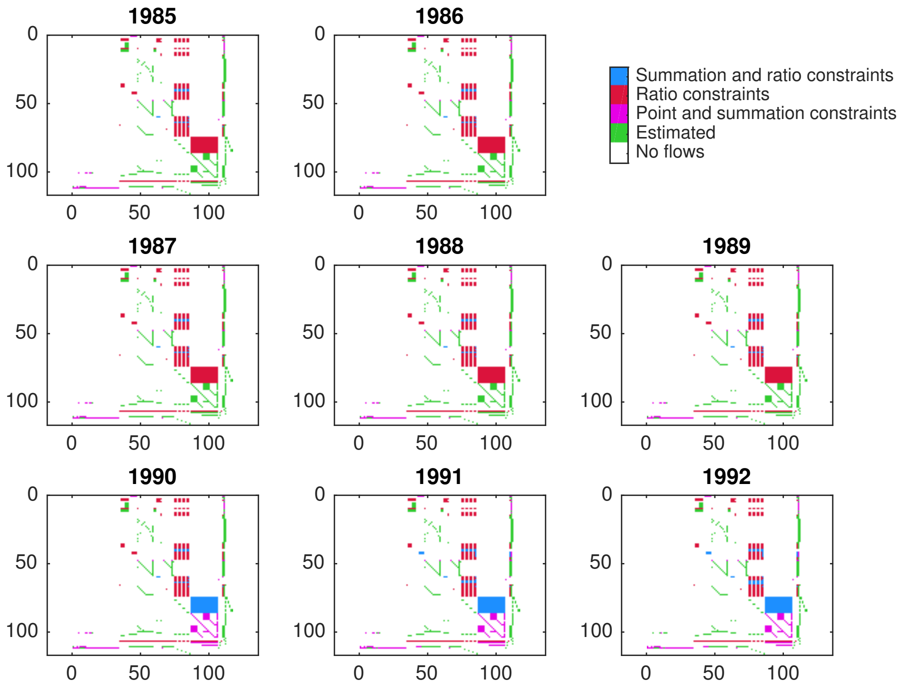

Figure 8: Data providence maps 1985–1992

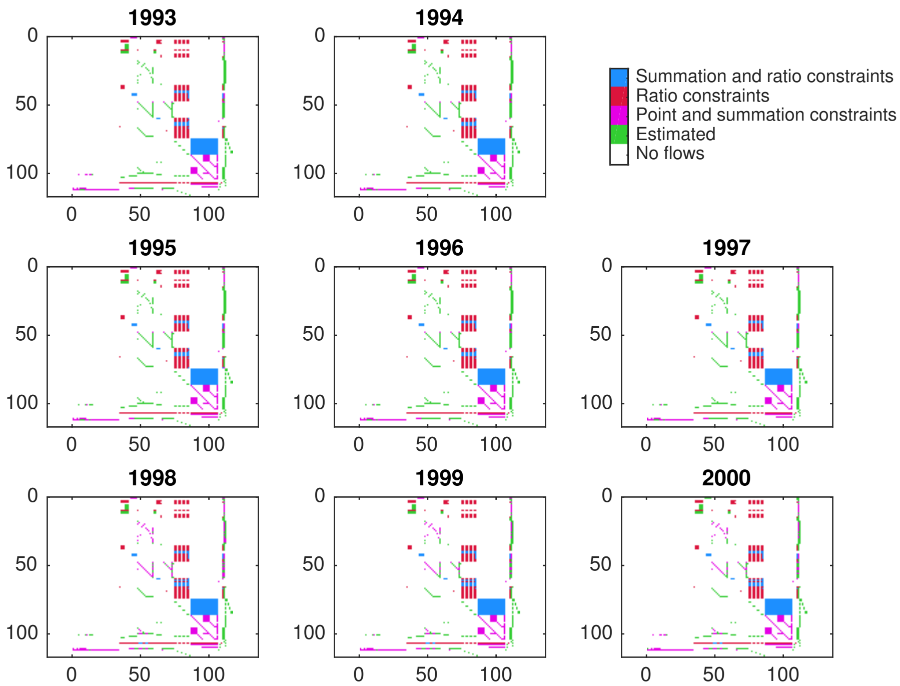

Figure 10: Data providence maps 1983–2000

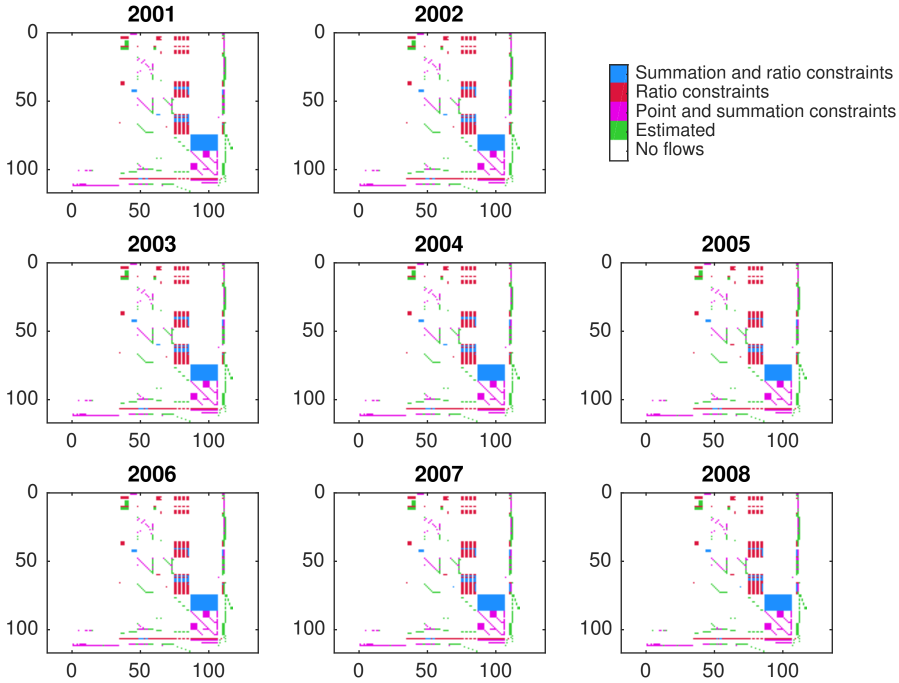

Figure 11: Data providence maps 2001–2008

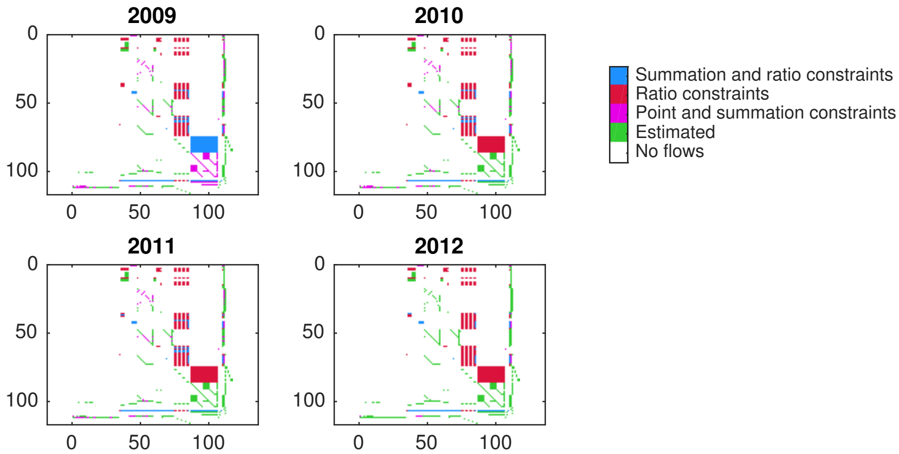

Figure 12: Data providence maps 2009–2012

F Ratios of inputs to the construction sectors

The following figures show the estimated ratio of material outputs tothe various construction industries. These were estimated using both historical input-output tables and time-series information onf work-done by construction sector.

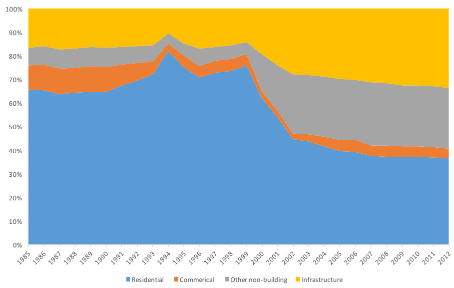

Figure 13: Ratio of bricks output to the construction sectors

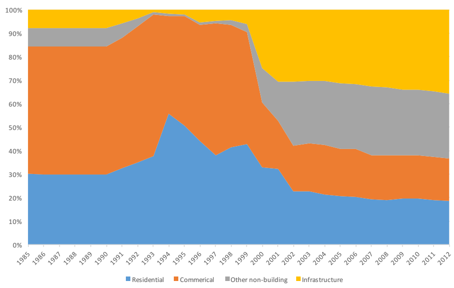

Figure 14: Ratio of pre-mixed concrete output to the construction sectors

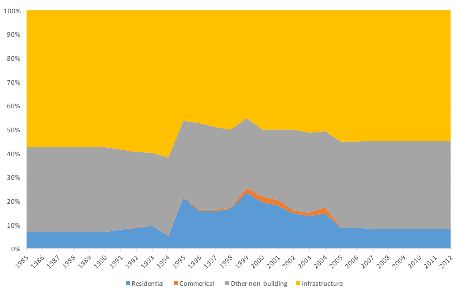

Figure 15: Ratio of gravel output to the construction sectors

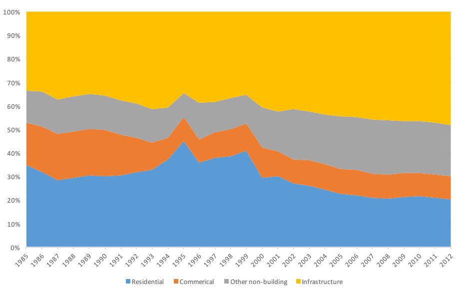

Figure 16: Ratio of iron and steel product output to the construction sectors

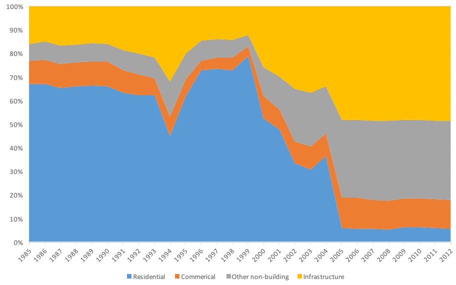

Figure 17: Ratio of sand output to the construction sectors

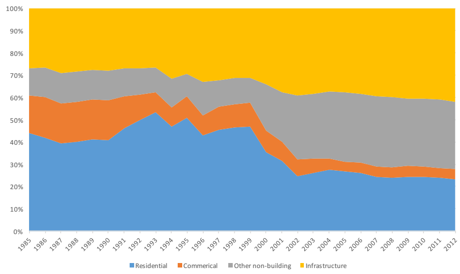

Figure 18: Ratio of sawn wood output to the construction sectors