# CS5560 Knowledge Discovery and Management

**Problem Set 6**

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References

https://www.analyticsvidhya.com/blog/2015/09/naive-bayes-explained/

https://nlp.stanford.edu/IR-book/html/htmledition/text-classification-and-naive-bayes-1.html

http://www.nltk.org/book/ch06.html

I. Consider the problem of classifying the origination point of passenger travel itineraries. Suppose we have the following training set of travel itineraries:

| Itinerary | Document | Class |
|---|---|---|
| 1 | "smith: new york - chicago - san francisco - new york" | JFK |
| 2 | "chen: san francisco - london - paris - san francisco" | SFO |
| 3 | "chen: san francisco - tokyo - singapore- san francisco" | SFO |
| 4 | "o'brien: chicago - buenos aires - new york - chicago" | ORD |

a) Assume that we use a Bernoulli (i.e., binary) Naive Bayes model. Compute the following feature probabilities:
- P(Xfrancisco=true | Class=SFO)
- P(Xlondon=true | Class=SFO)
- P(Xfrancisco=true | Class=JFK)

b) Assume that we use a multinomial NB model instead. Compute the following probabilities:
- P(X=francisco | Class=SFO)
- P(X=london | Class=SFO)
- P(X=francisco | Class=JFK)

c) Consider a standard Naive Bayes classifier trained on the training set and applied to a similar test set. How accurate is this classifier for:
(i) the Bernoulli model, and
(ii) the multinomial model?

d) Construct a non-standard feature representation that is 100% accurate for either model.

II. This problem concerns smoothing Naïve Bayes classifiers. Consider the following formula for Laplace (add-1) smoothing for Naïve Bayes

$$\hat{P}(w_i \mid c) = \frac{count(w_i,c)+1}{\sum_{w \in V} (count(w,c)+1)}$$

$$= \frac{count(w_i,c)+1}{\left(\sum_{w \in V} count(w,c)\right) + |V|}$$

a) Suppose we build a Naive Bayes classifier (multinomial or Bernoulli) with no smoothing of the respective P(word | class) probabilities. If a word was unseen in a class, it will thus have a probability of 0. Describe in words the decision procedure of this classifier (emphasizing the effect of the lack of smoothing, and how its decisions will differ from a smoothed Naive Bayes classifier).

b) Suppose we take a smoothed multinomial classifier and double the amount of smoothing (e.g., for a variant of "add 1 smoothing", add 2 to each count, and add to the denominator 2k, where k is the number of samples). What qualitative effect will this have on decisions of the classifier?

III. An IR system returns 3 relevant documents, and 2 irrelevant documents. There are a total of 8 relevant documents in the collection.

a) What is the precision of the system on this search, and what is its recall?

b) Instead of using recall/precision for evaluating IR systems, we could use accuracy of classification. Consider a classifier that classifies documents as being either relevant or non-relevant. The accuracy of a classifier that makes c correct decisions and i incorrect decisions is defined as: c/(c+i).

(i) Why do the recall and precision measures reflect the utility (i.e., quality or usefulness) of an IR system better than accuracy does?

(ii) Suppose that we have a collection of 10 documents, and two different boolean retrieval systems A and B. Give an example of two result sets, Aq and Bq, assumed to have been returned by the system in response to a query q, constructed such that Aq has clearly higher utility and a better score for precision than Bq, but such that Aq and Bq have the same scores on accuracy.

1 a) **Bernoul Model** :- It is equivalent to the binary independance model, which generates an indicator for each term in the vocabulary either "1" indicating presence of the term in the document or "0" indicating absence.

→ P (x fransisco = true | class SFO)

- Total no. of class of SFO = 2
- Presence of term "fransico" in no. of doc = 2

⟹ 2/2 = 1

→ P (x london = true | class = SFO)

- Total SFO classes = 2
- Presence of "london" in doc = 1

⟹ 1/2 = 0.5

→ P (x fransico = true | class JFK)

- Total JFK classes = 1
- Presence of "fransico" in doc = 1

⟹ 1/1 = 0.

b) **Multinomial Model** :- It generates one term from the vocabulary in each position of the document, where we assume a generative model

| | Multinomial Model | Bernoulli model |
|---|---|---|
| Event model | Generation of token | Generation of document |
| Random Variable | $X=t$ iff $t$ occurs at given pos | $U_t=1$ iff $t$ occurs in doc |
| document representation | $d=\{t_1,\dots,t_k,\dots,t_{n_d}\}, t_k \in V$ | $d=\{e_1,\dots e_i,\dots e_M\}, e_i \in \{0,1\}$ |
| Parameter estimation | $\hat{P}(X=t\|c)$ | $\hat{P}(U_i=e\|c)$ |
| decision rule: maximize | $\hat{P}(c)\prod_{1\le k\le n_d}\hat{P}(X=t_k\|c)$ | $\hat{P}(c)\prod_{t\in V}\hat{P}(U_i=e_i\|c)$ |
| Multiple Occurences | taken into account | ignored |
| Length of docs | can handle longer docs | Work best for short docs |
| # feature | can handle more | Works best with fewer |
| estimate for term the | $\hat{P}(X=\text{the}\|c)\approx 0.5$ | $\hat{P}(U_{\text{the}}=1\|c)\approx 1.0$ |

→ $P(X=\text{fransisco}\,|\,\text{Class}=\text{SFO})=$

- Occurance of Fransisco in class SFO = 4
- Word Count in class SFO = 14

$\Rightarrow 4/14$

→ $P(X=\text{london}\,|\,\text{Class}=\text{SFO})$

- Occurance of Word "london" = 1
- Word Count in class SFO = 14

$\Rightarrow 1/14$

→ $P(X=\text{fransico}\,|\,\text{Class}=\text{JFK})$

- Occurance of Word fransico in class JFK = 1
- Word Count in class JFK = 8

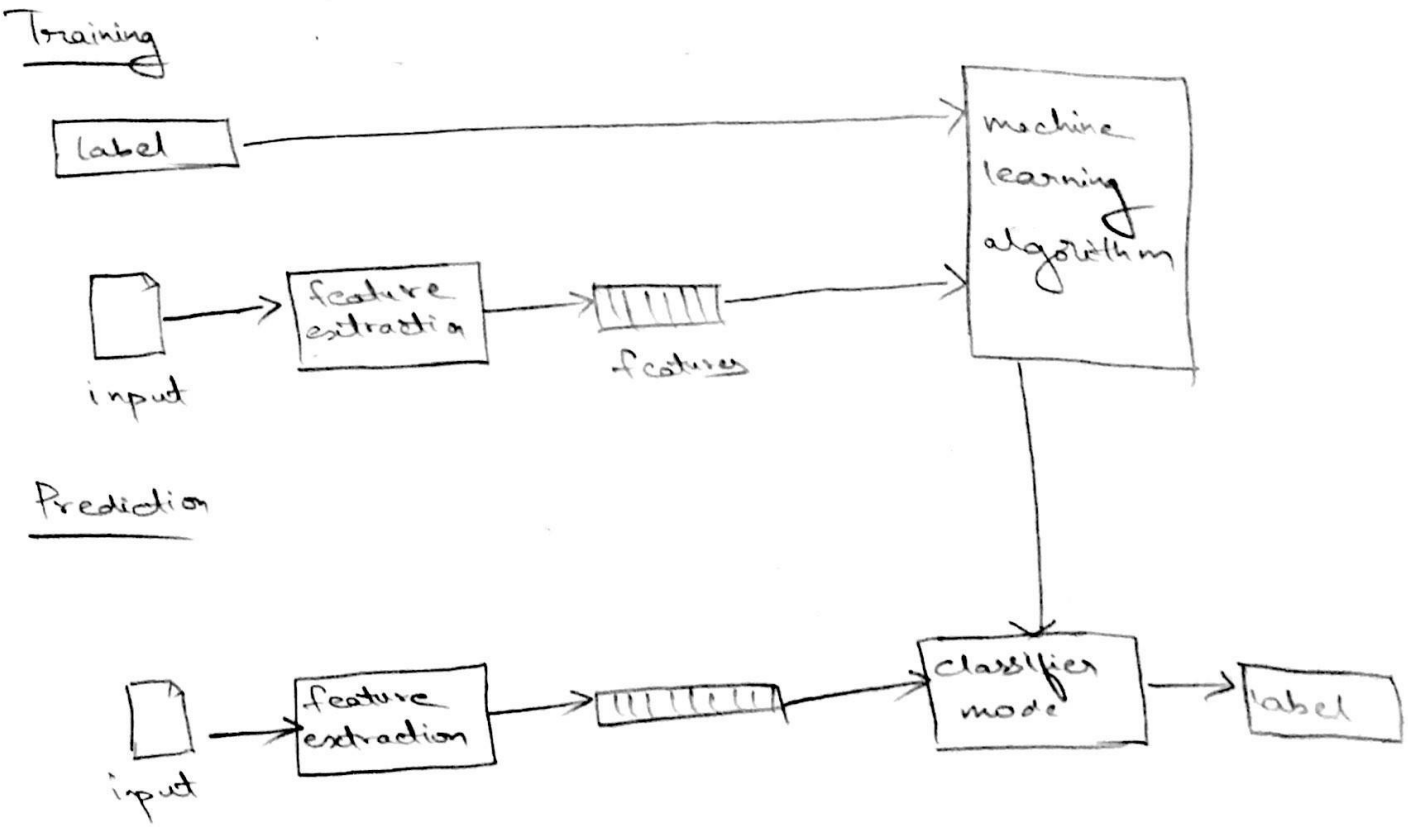
$\Rightarrow 1/8$

c) → Multinomial model takes into account multiple occurence of the word and can handle more docus and features so multinomial model is more accurate, because it uses frequency information

→ Bernoulli Model in contrast does not consider the frequent occurence of words, so it is not much accurate, as it models absence of terms explicitly.

2) Classification :-

Classification is the task of choosing the correct class label for a given input




→ A classifier is called supervised if it is built based on training corpus containing the correct label for each input.

a)-> In the naive bayes classifier when occurance of word is "0" ⇒ probability of occurance is 0

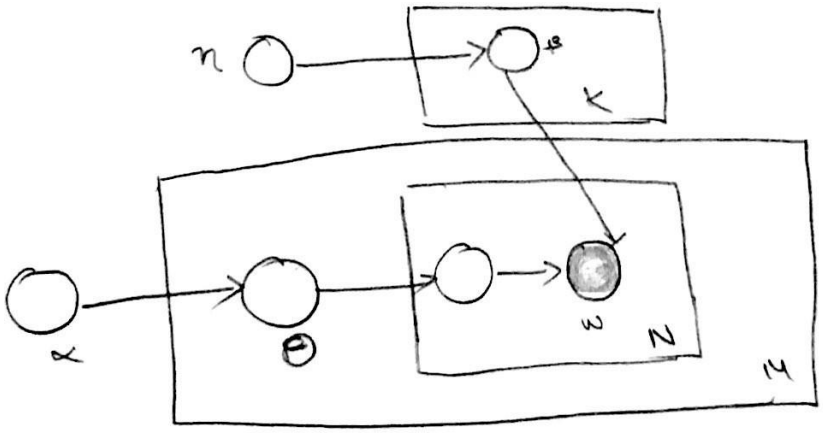
-> If $P(word|class) = 0$, then we can never choose a category.

-> Classification are based on training set, so we can rank the classes for which all words were seen similarity to the smoothed classifier.

b) Smoothing Multinomial Model :-

-> Large vocabulary size is characteristic of document corpora often problems with sparsity.

-> Maximum likelihood estimates of the multinomial parameters & assign zero probability to new words, and thus zero probability to new document.




Retrieval formula using general smoothing schema

$$\log p(q|d) = \sum_{w \in V, c(w,q)>0} c(w,q) \log p(w|d)$$

$$p(w|d) = \begin{cases} p_{seen}(w|d) & \text{if } w \text{ is seen in } d \\ \alpha_d p(w|c) & \text{otherwise} \end{cases}$$

→ P(word|count)

Let initial word = w

count = c

⇒ 1/k

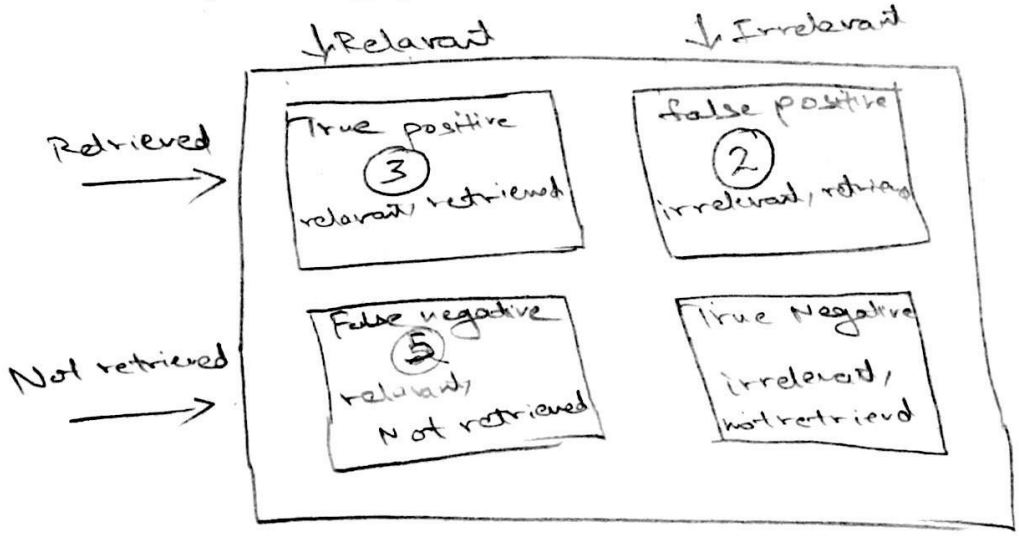
→ Increase in smoothing

⇒ 1/2k

→ It'll be more likely to choose categories for which many of the words in the document

III) Total positive (tp) = 5

Total (tp+fp) = 8




$$\text{Precision} = \frac{tp}{(tp+fp)} = \frac{3}{3+2} = \frac{3}{5}$$

$$\text{Recall} = \frac{tp}{(tp+fn)} = \frac{3}{8} = \frac{3}{8}$$

b) (i)

→ Precision, indicates how many of the items that we identified were relevant

→ Recall, indicates how many of the relevant items that we identified.

| | Relevant | Irrelevant |
|---|---|---|
| Retrieved | TP | FP |
| Not Retrieved | FN | TN |

→ For the information retrieval system, returns no results will have high accuracy for most queries, since the corpus usually contains only a few relevant documents

→ Recall & precision are two different measures that can jointly capture the tradeoff between returning more relevant results and returning fewer irrelevant results

ii) Precision (Aq) > Precision (Bq)

For Aq

| | Relevant | Irrelevant |
|---|---|---|
| Retrieved | 1 | 2 |
| Not retrieved | 1 | |

=> Precision (Aq) = 1/3

For Bq

| | Relevant | Irrelevant |
|---|---|---|
| Retrieved | 0 | 2 |
| Not Retrieved | 2 | |

=> Precision (Bq) = 0

→ In the above case both $A_q$ & $B_q$ made 2 mistakes ⟹ accuracy = 80%

→ Since $B_q$ didn't return any relevant documents, it is of no utility.

(1d) 100% accurate Naive Bayes Model

→ This can be achieved by using the term that occurs in the last position of each document.

→ Non standard feature represented with using non-standard words. The non-standard words are classified to 6 categories using SKIP ∈ Z collection to official, literature, informal, popular, educative and scientific.

$$P(X_{NewYork} = true | Class = JFK) = 1.0$$

$$P(X_{SanFrancisco} = true | Class = SFO) = 1.0$$

$$P(X_{Chicago} = true | Class = ORD) = 1.0.$$