Investment Management

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https://github.com/sprasadhpy/IIT_Kharagpur_Course-code-BM51001

Topic to be covered

1) Module - 1 : Portfolio optimization

- Estimate annual returns, volatility, Sharpe ratio, and maximum drawdown
- Markowitz optimization problem
- Maximum Sharpe Ratio (MSR) portfolio
- MSR in the efficient frontier
- Buy and hold strategy with MSR

Assignment 1 (5)

2) Module -2: Factor models

- Linear regression in python - intro
- Fama French models & Fama Macbeth regression
- Predict stock returns using FF factors
- Statistical inference using linear ML models
- Evaluating FF alphas using the alpha lens
- Alpha strategy- sector rotation using FF factors

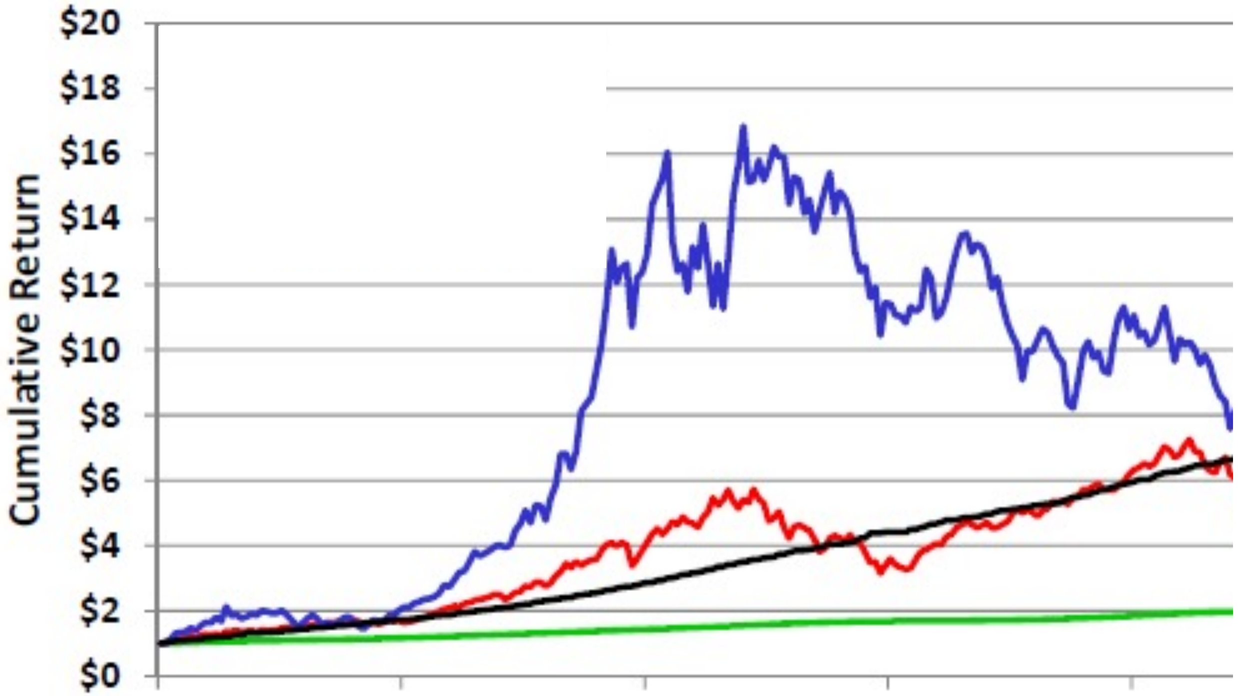
Assignment 2 (5)

Programming Assessment (10)

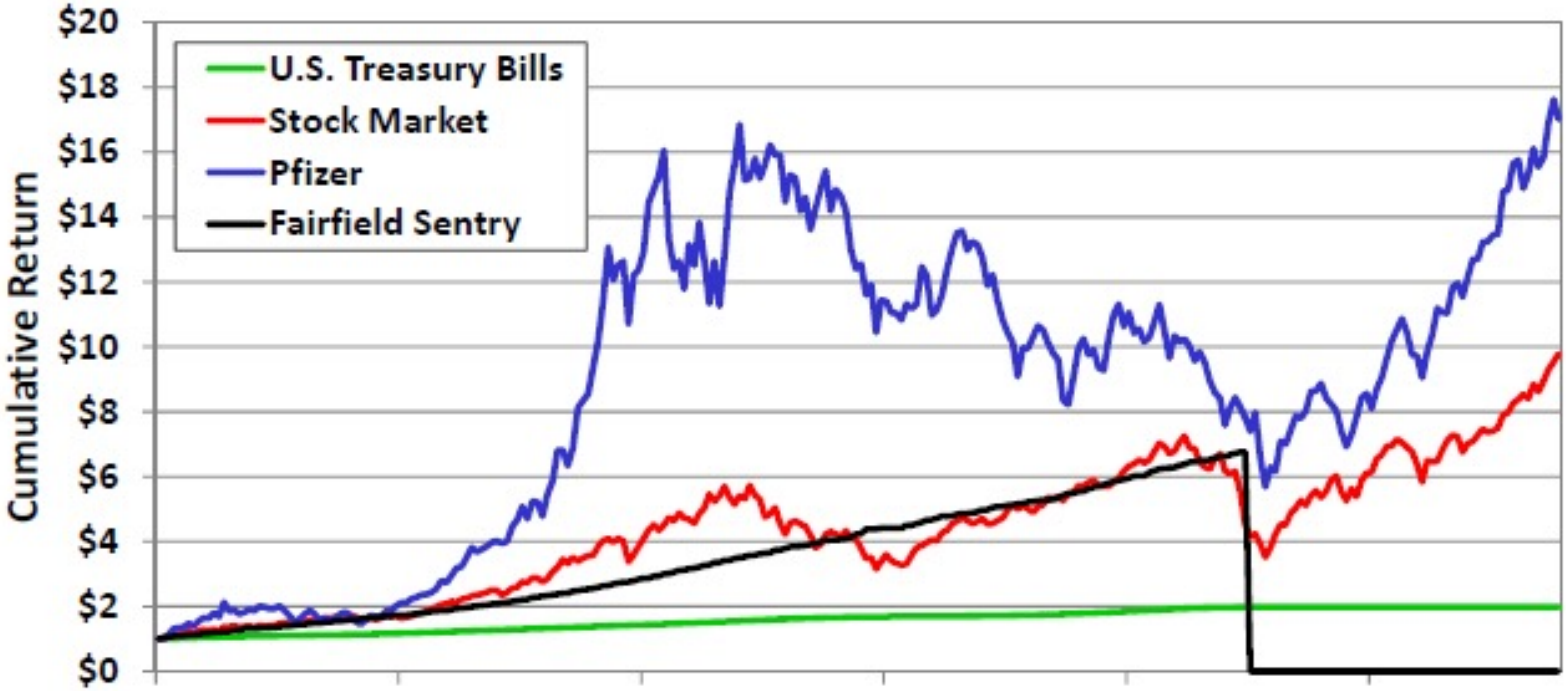
Introduction to Portfolio Management

Module 1

Myopia of Investors



Myopia of Investors



Questions to be addressed

- What is meant by ***risk*** and what are some of the alternative measures of risk used in investments?
- What do we mean by ***risk aversion*** and what evidence indicates that investors are generally risk averse?

Questions to be addressed

- How do you compute the **expected rate of return** for an individual risky asset or a portfolio of assets?
- How do you compute the **standard deviation** of rates of return for an individual risky asset?
- What is meant by the **covariance between rates of return** and how do you compute covariance?

Questions to be addressed

- What is the relationship between **covariance and correlation**?
- What is the formula for the **standard deviation for a portfolio of risky assets** and how does it differ from the standard deviation of an individual risky asset?
- Given the formula for the standard deviation of a portfolio, how and why do you diversify a portfolio?

Questions to be addressed

- What happens to the **standard deviation of a portfolio** when you change the **correlation between the assets** in the portfolio?
- What is the **risk-return efficient frontier**?
- Is it reasonable for **alternative investors** to select different portfolios from the portfolios on the efficient frontier?
- What determines which **portfolio on the efficient frontier** is selected by an individual investor?

Asset pricing models - Assumptions

- As an investor you want to maximize the returns for a given level of risk.
- Your portfolio includes all of your assets and liabilities
- The relationship between the returns for assets in the portfolio is important.
- A good portfolio is not simply a collection of individually good investments.

Risk Aversion

- Given a choice between two assets with equal rates of return, most investors will select the asset with the lower level of risk.

Evidence That Investors are Risk Averse

- Many investors purchase insurance for: Life, Automobile, Health, and Disability Income. The purchaser trades known costs for unknown risk of loss
- Yield on bonds increases with risk classifications from AAA to AA to A….

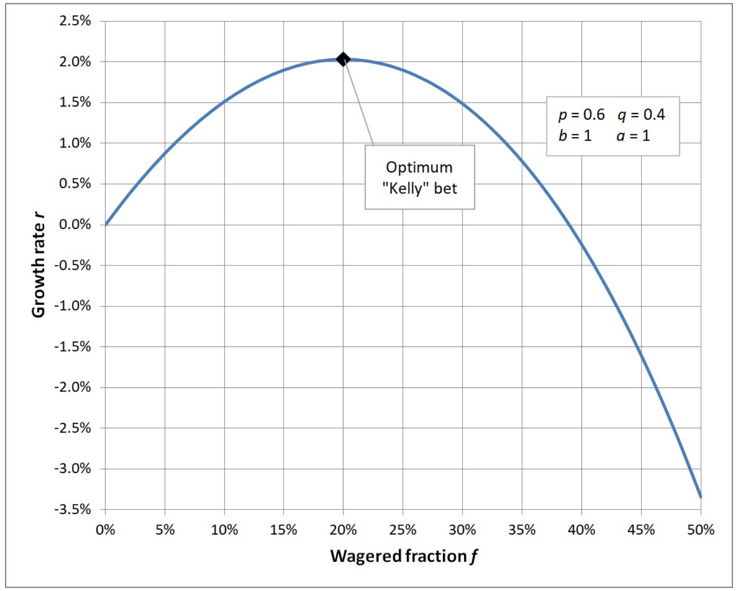
Not all investors are risk averse

Risk preference may have to do with amount of money involved - risking small amounts, but insuring large losses

Optimal theoretical size for a bet. It is valid when the expected returns are known

$$f^* = \frac{bp - q}{b}$$

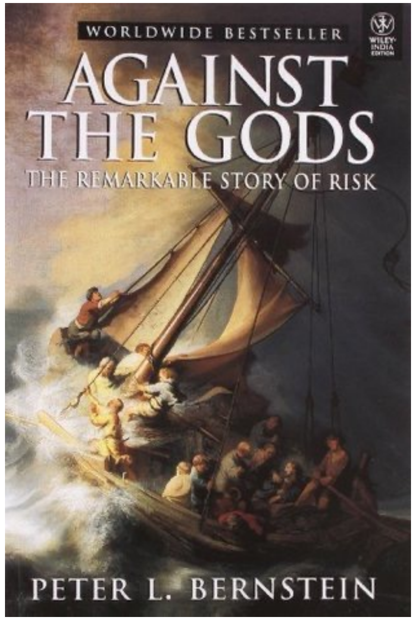
- f = the fraction of the bankroll to bet
- b = the decimal odds – 1
- p = the probability of winning
- q = the probability of losing, which is 1 – p



b : If betting \$10 on a 2-to-1 odds bet, (upon win you are returned \$30, winning you \$20).b =20 /10 = \$2.0

if a gamble has a 60% chance of winning p=0.6; q=0.4, and the gambler receives 1-to-1 odds on a winning bet then the gambler should bet 20% of the bankroll at each opportunity in order to maximize the long-run growth rate of the bankroll.

Definition of Risk



1. Uncertainty of future outcomes

or

2. Probability of an adverse outcome

- Risk is the possibility that the actual return on an investment will be different from its expected return
- Knight’s definition, risk is often defined as quantifiable uncertainty about gains and losses
- Knightian uncertainty : https://archive.org/details/riskuncertaintyp00knigrich

Markowitz Portfolio Theory

- Objective : Quantifies risk
- Derives the expected rate of return for a portfolio of assets and an expected risk measure
- Shows that the variance of the rate of return is a meaningful measure of **portfolio risk**
- Derives the formula for computing the variance of a portfolio, showing how to effectively diversify a portfolio

Assumptions of Markowitz Portfolio Theory

1. Investors consider each investment alternative as being presented by a probability distribution of expected returns over some holding period.
2. Investors maximize one-period expected utility, and their utility curves demonstrate diminishing marginal utility of wealth.
3. Investors estimate the risk of the portfolio on the basis of the variability of expected returns.
4. Investors base decisions solely on expected return and risk, so their utility curves are a function of expected return and the expected variance (or standard deviation) of returns only.
5. For a given risk level, investors prefer higher returns to lower returns. Similarly, for a given level of expected returns, investors prefer less risk to more risk.

Single-Period Utility Analysis

- U(candy bar) < U(apple).

- In a world of certainty, utility theory says a person should assign a numerical value to each alternative and then choose the alternative with the largest numerical value.

- The investor's utility function is used to determine the numerical values for each alternative investment.

- https://web.stanford.edu/~wfsharpe/art/euaarevised.pdf

Markowitz Portfolio Theory

Using these five assumptions, a single asset or portfolio of assets is considered to be efficient if no other asset or portfolio of assets offers higher expected return with the same (or lower) risk, or lower risk with the same (or higher) expected return.

Alternative Measures of Risk

- Variance or standard deviation of expected return. - **Why ?**
- Range of returns
- Returns below expectations
    - Semivariance – a measure that only considers deviations below the mean
    - These measures of risk implicitly assume that investors want to minimize the damage from returns less than some target rate

Expected Rates of Return

- For an individual asset - sum of the potential returns multiplied with the corresponding probability of the returns
- For a portfolio of assets - weighted average of the expected rates of return for the individual investments in the portfolio

Computation of Expected Return for an Individual Risky Investment

Probability	Possible Rate of Return (Percent)		Expected Return (Percent)
0.25	0.08		0.0200
0.25	0.10		0.0250
0.25	0.12		0.0300
0.25	0.14		0.0350
		E(R) =	0.1100

Computation of the Expected Return for a Portfolio of Risky Assets

Weight (W_i) (Percent of Portfolio)	Expected Security Return (R_i)	Expected Portfolio Return (W_i X R_i)
0.20	0.10	0.0200
0.30	0.11	0.0330
0.30	0.12	0.0360
0.20	0.13	0.0260
		$E(R_{por\ i}) =$ 0.1150

$$E(R_{\text{por i}}) = \sum_{i=1}^{n} W_i R_i$$

where :

W_i = the percent of the portfolio in asset i

$E(R_i)$ = the expected rate of return for asset i

Variance (Standard Deviation) of Returns for an Individual Investment

- Standard deviation is the square root of the variance
- Variance is a measure of the variation of possible rates of return R_i, from the expected rate of return $[E(R_i)]$

Variance (Standard Deviation) of Returns for an Individual Investment

$$\text{Variance}\,(\sigma^2) = \sum_{i=1}^{n} [\mathrm{R}_i - \mathrm{E}(\mathrm{R}_i)]^2\,\mathrm{P}_i$$

where P_i is the probability of the possible rate of return, R_i

Variance (Standard Deviation) of Returns for an Individual Investment

$$(\sigma) = \sqrt{\sum_{i=1}^{n} [\mathrm{R}_{\mathrm{i}} - \mathrm{E}(\mathrm{R}_{\mathrm{i}})]^2 \mathrm{P}_{\mathrm{i}}}$$

Variance (Standard Deviation) of Returns for an Individual Investment

Possible Rate of Return (R_i)	Expected Return $E(R_i)$	$R_i - E(R_i)$	$[R_i - E(R_i)]^2$	P_i	$[R_i - E(R_i)]^2 P_i$
0.08	0.11	0.03	0.0009	0.25	0.000225
0.10	0.11	0.01	0.0001	0.25	0.000025
0.12	0.11	0.01	0.0001	0.25	0.000025
0.14	0.11	0.03	0.0009	0.25	0.000225
					0.000500

Variance = .0050

Standard Deviation = .02236

Variance (Standard Deviation) of Returns for a Portfolio

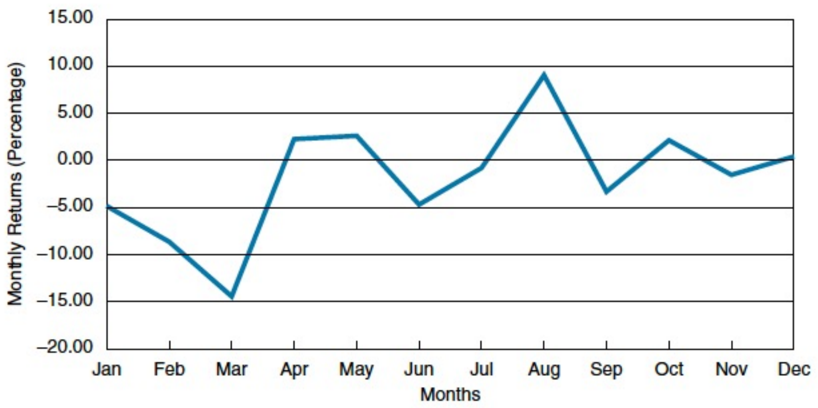
Computation of Monthly Rates of Return

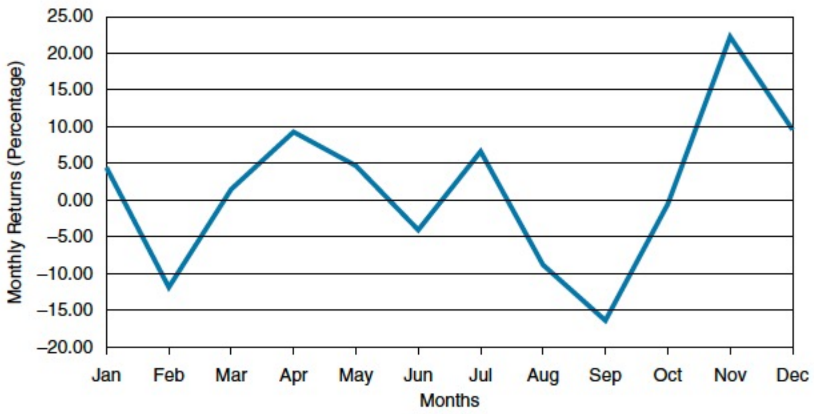
Date	Closing Price	Dividend	Return (%)	Closing Price	Dividend	Return (%)
Dec.00	60.938			45.688		
Jan.01	58.000		-4.82%	48.200		5.50%
Feb.01	53.030		-8.57%	42.500		-11.83%
Mar.01	45.160	0.18	-14.50%	43.100	0.04	1.51%
Apr.01	46.190		2.28%	47.100		9.28%
May.01	47.400		2.62%	49.290		4.65%
Jun.01	45.000	0.18	-4.68%	47.240	0.04	-4.08%
Jul.01	44.600		-0.89%	50.370		6.63%
Aug.01	48.670		9.13%	45.950	0.04	-8.70%
Sep.01	46.850	0.18	-3.37%	38.370		-16.50%
Oct.01	47.880		2.20%	38.230		-0.36%
Nov.01	46.960	0.18	-1.55%	46.650	0.05	22.16%
Dec.01	47.150		0.40%	51.010		9.35%
		E(RCoca-Cola)=	-1.81%		E(Rhome Depot)==	1.47%

Variance (Standard Deviation) of Returns for a Portfolio

Time series plots

Coke vs. Home depot.





Covariance of Returns

- A measure of the degree to which two variables “move together” relative to their individual mean values over time
- Positive covariance : the rates of return for two investments tend to move in the same direction relative to their individual means during the same time period
- Negative covariance : the rates of return for two investments tend to move in different directions relative to their means during specified time intervals over time.
- The magnitude of the covariance - variances of the individual return series, relationship between the series.

Covariance of Returns

For two assets, i and j, the covariance of rates of return is defined as:

$$Cov_{ij} = E\{[R_i - E(R_i)][R_j - E(R_j)]\}$$

If the rates of return for one stock are above (below) its mean rate of return during a given period and the returns for the other stock are likewise above (below) its mean rate of return during this same period, then the product of these deviations from the mean is positive.

If this happens consistently, the covariance of returns between these two stocks will be some positive

Covariance of Returns

$$\frac{1}{12}\sum_{i=1}^{n}\left[R_i - E(R_i)\right]\left[R_j - E(R_j)\right]$$

	Monthly Return						
Date	Coca-Cola (R_i)	Home Depot (R_j)	Coca-Cola $R_i - E(R_i)$	Home Depot $R_j - E(R_j)$	Coca-Cola $[R_i - E(R_i)]$	$\times$	Home Depot $[R_j - E(R_j)]$
Jan-01	–4.82	5.50	–3.01	4.03			–12.13
Feb-01	–8.57	–11.83	–6.76	–13.29			89.81
Mar-01	–14.50	1.51	–12.69	0.04			–0.49
Apr-01	2.28	9.28	4.09	7.81			31.98
May-01	2.62	4.65	4.43	3.18			14.11
Jun-01	–4.68	–4.08	–2.87	–5.54			15.92
Jul-01	–0.89	6.63	0.92	5.16			4.76
Aug-01	9.13	–8.70	10.94	–10.16			–111.16
Sep-01	–3.37	–16.50	–1.56	–17.96			27.97
Oct-01	2.20	–0.36	4.01	–1.83			–7.35
Nov-01	–1.55	22.16	0.27	20.69			5.52
Dec-01	0.40	9.35	2.22	7.88			17.47
	$E(R_i)$ = –1.81	$E(R_j)$ = 1.47			Sum =		76.42

Cov_{ij} = 76.42/12 = 6.37

Covariance and Correlation

- Covariance and Correlation Covariance is affected by the variability of the two individual return series.

- The correlation coefficient is obtained by standardizing (dividing) the covariance by the product of the individual standard deviations

Covariance and Correlation

Correlation coefficient varies from -1 to +1

$$r_{ij} = \frac{\text{Cov}_{ij}}{\sigma_i \sigma_j}$$

where :

r_{ij} = the correlation coefficient of returns

σ_i = the standard deviation of R_{it}

σ_j = the standard deviation of R_{jt}

Correlation Coefficient

- It can vary only in the range +1 to -1. A value of +1 would indicate perfect positive correlation. This means that returns for the two assets move together in a completely linear manner. A value of –1 would indicate perfect correlation. This means that the returns for two assets have the same percentage movement, but in opposite directions

Computation of the expected return for a portfolio of risky assets

Weight (W_i) (Percent of Portfolio)	Expected Security Return $E(R_i)$	Expected Portfolio Return $[W_i \times E(R_i)]$
.20	.10	.0200
.30	.11	.0330
.30	.12	.0360
.20	.13	.0260
		$E(R_{port}) = .1150$

Portfolio Standard Deviation Formula

$$\sigma_{port} = \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{i=1}^{n} w_i w_j Cov_{ij}}$$

where :

σ_{port} = the standard deviation of the portfolio

W_i = the weights of the individual assets in the portfolio, where weights are determined by the proportion of value in the portfolio

σ_i^2 = the variance of rates of return for asset i

Cov_{ij} = the covariance between the rates of return for assets i and j,

where $Cov_{ij} = r_{ij} \sigma_i \sigma_j$

Portfolio with a large number of securities, this formula reduces to the sum of the weighted covariances

Portfolio Standard Deviation Calculation

- Any asset of a portfolio may be described by two characteristics:
    - The expected rate of return
    - The expected standard deviations of returns
- The correlation, measured by covariance, affects the portfolio standard deviation
- Low correlation reduces portfolio risk while not affecting the expected return

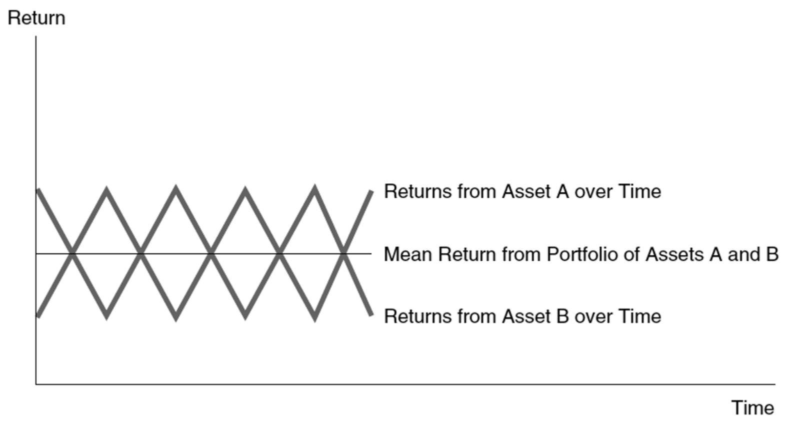
Combining Stocks with same Returns and Risk

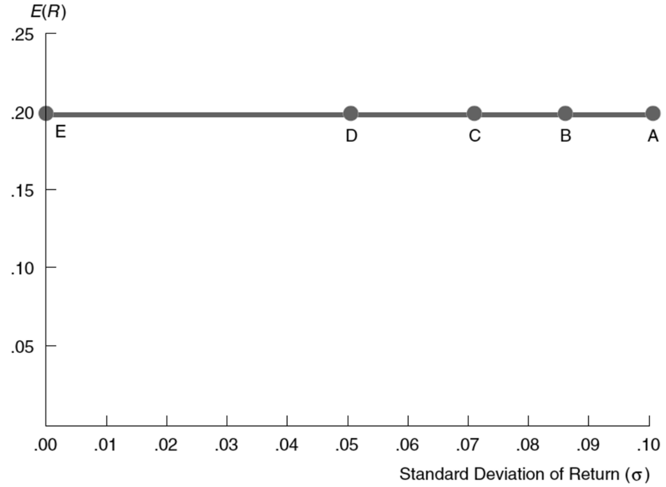
Asset	$E(R_i)$	W_i	$\sigma^2{}_i$	σ_i
1	.20	.50	.0100	.10
2	.20	.50	.0100	.10

$Cov_{ji} = r_{ij}\sigma_i\sigma_j$

$$\sigma_{port} = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2r_{1,2}\sigma_1\sigma_2}$$

Case	Correlation Coefficient	Covariance	σ_{port}
a	+1.00	.0010	0.10
b	+0.50	.0050	0.0868
c	0.00	.0000	0.0707
d	-0.50	-.0050	0.05
e	-1.00	-.0010	0



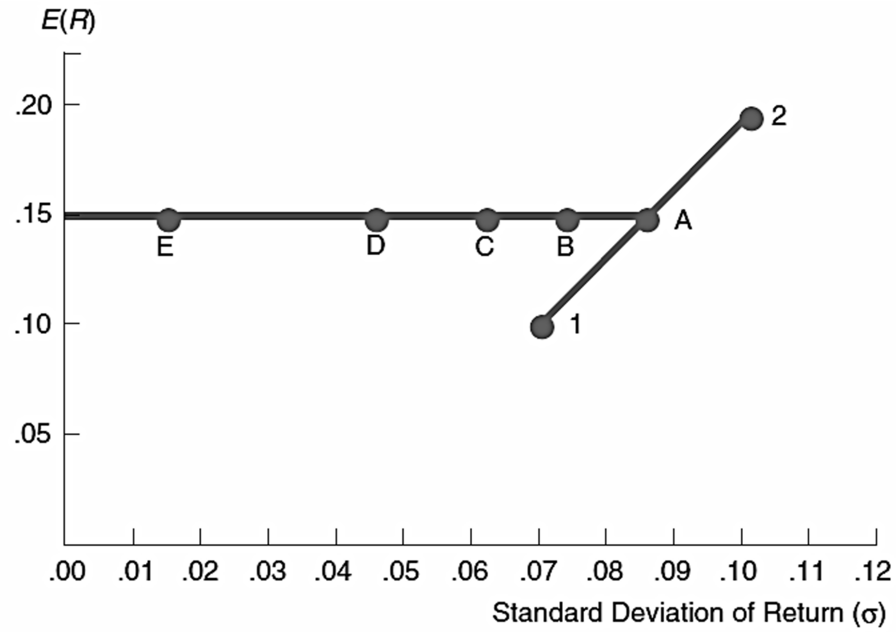


Combining Stocks with Different Returns and Risk

Asset	$E(R_i)$	W_i	$\sigma^2{}_i$	σ_i
1	.10	.50	.0049	.07
2	.20	.50	.0100	.10

$$E(R_{port}) = 0.50(0.10) + 0.50(0.20) = 0.15$$

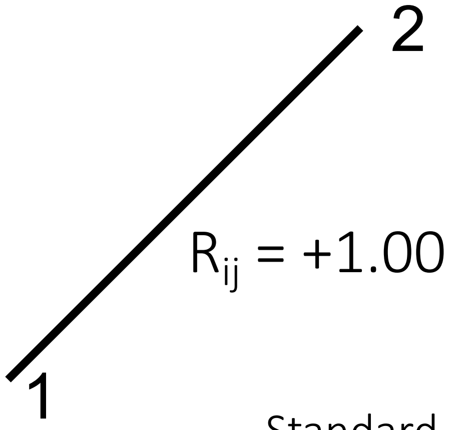
Case	Correlation Coefficient	Covariance	σ_{port}
a	+1.00	.0070	0.085
b	+0.50	.0035	0.07399
c	0.00	.0000	0.0610
d	-0.50	-.0035	0. 0444
e	-1.00	-.0070	0. 015



Portfolio Risk-Return Plots

E(R)

With two perfectly correlated assets, only possibility- to create a two asset portfolio with risk-return along a line between either single asset



Standard Deviation of Return

Combining Stocks with Different Returns and Risk

- Assets may differ in expected rates of return and individual standard deviations
- Negative correlation reduces portfolio risk
- Combining two assets with -1.0 correlation reduces the portfolio standard deviation to zero only when individual standard deviations are equal

Constant Correlation with Changing Weights

Asset	$E(R_i)$	
1	.10	$r_{ij} = 0.00$ (C)
2	.20	

Case	$\mathbf{W_1}$	$\mathbf{W^2}$	$\mathbf{E(R_i)}$
f	0.00	1.00	0.20
g	0.20	0.80	0.18
h	0.40	0.60	0.16
i	0.50	0.50	0.15
j	0.60	0.40	0.14
k	0.80	0.20	0.12
l	1.00	0.00	0.10

We would derive a set of combinations that trace an ellipse starting at Asset 2, going through the 0.50 – 0.50 point, and ending at Asset 1.

Constant Correlation with Changing Weights

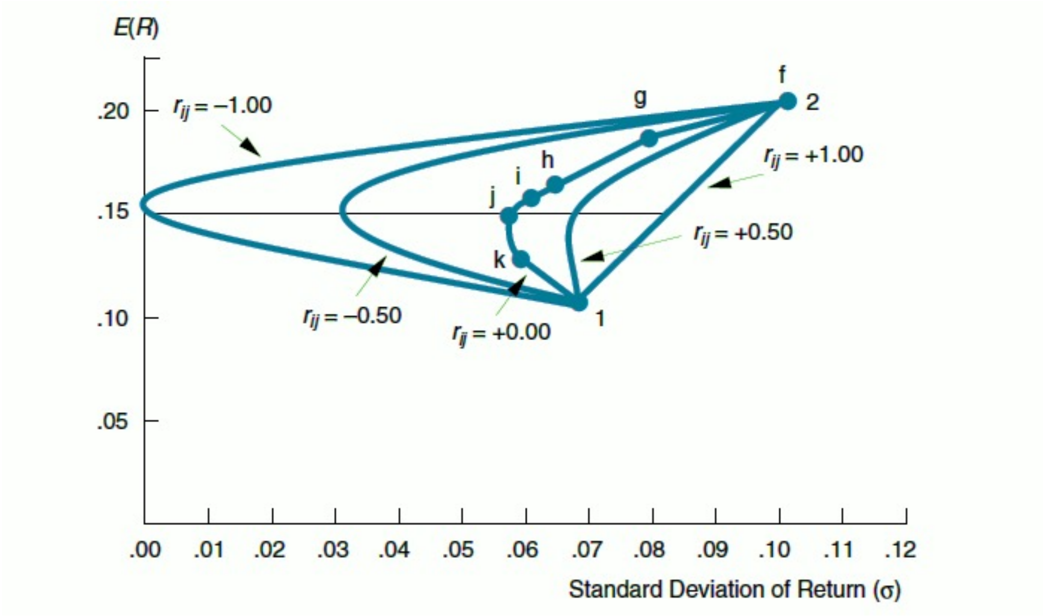
Case	W_1	W_2	$E(R_i)$	$E(R_{port})$
f	0.00	1.00	0.20	0.1000
g	0.20	0.80	0.18	0.0812
h	0.40	0.60	0.16	0.0662
i	0.50	0.50	0.15	0.0610
j	0.60	0.40	0.14	0.0580
k	0.80	0.20	0.12	0.0595
l	1.00	0.00	0.10	0.0700

$$\begin{aligned}\sigma_{port(g)} &= \sqrt{(0.20)^2(0.07)^2 + (0.80)^2(0.10)^2 + 2(0.20)(0.80)(0.00)} \\ &= \sqrt{(0.04)(0.0049) + (0.64)(0.01) + (0)} \\ &= \sqrt{(0.006596)} \\ &= 0.0812\end{aligned}$$

$$\begin{aligned}\sigma_{port(h)} &= \sqrt{(0.40)^2(0.07)^2 + (0.60)^2(0.10)^2 + 2(0.40)(0.60)(0.00)} \\ &= \sqrt{(0.004384)} \\ &= 0.0662\end{aligned}$$

$$\begin{aligned}\sigma_{port(j)} &= \sqrt{(0.60)^2(0.07)^2 + (0.40)^2(0.10)^2 + 2(0.60)(0.40)(0.00)} \\ &= \sqrt{(0.003364)} \\ &= 0.0580\end{aligned}$$

$$\begin{aligned}\sigma_{port(k)} &= \sqrt{(0.80)^2(0.07)^2 + (0.20)^2(0.10)^2 + 2(0.80)(0.20)(0.00)} \\ &= \sqrt{(0.003536)} \\ &= 0.0595\end{aligned}$$



In the case where it is assumed that the correlation was zero (0.00), the low-risk investor at Point 1 who would receive a return of 10 percent and risk of 7 percent could increase his/her return to 14 percent and experience a decline in risk to 5.8 percent by investing (diversifying) 40 percent of the portfolio in riskier Asset 2

Thank you