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The Bloomberg Corporate Default Risk Model (DRSK) for Public Firms

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Abstract

The DRSK public model estimates forward-looking real-world default probabilities for publicly traded firms. The model also assigns credit grades based on the estimated default probabilities. The product covers firms in all regions and sectors of operation for which the necessary data is available.

The DRSK public model was last updated in 2015. This year we are releasing an updated model which improves on the previous model's performance in a variety of ways. The new model's accuracy ratio is above 92%, adjusted pseudo R-squareds have improved, and performance is more in line with observed historical default rates. We describe the new model, analyze its performance in various ways and compare it to the previous model.

Keywords. Merton, Black-Cox, distance to default, real-world default probability, logistic regression, public firms, credit risk.

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1 Introduction

Corporate credit risk is the risk that a firm fails to satisfy its financial obligations. This typically first occurs when a firm fails to make a scheduled bond or loan payment, or is unable to meet a margin call. Such defaults often lead to bankruptcy proceedings where the firm is restructured, acquired or liquidated, and creditors incur losses. As such, it is important for investors to measure and manage corporate credit risk.

There are a variety of approaches to quantifying corporate credit risk. One approach is that taken by nationally recognized statistical rating organizations (NRSROs) such as Moody's, S&P and Fitch. A firm engages an NRSRO to issue a rating for the company. The NRSRO analyzes the firm and assigns a credit rating based on the agency's internal proprietary analysis of credit risk [Moo; Snp; JF19]. The agency will update the firm's rating when the agency observes a sufficient change in the credit quality of the firm. The firm can cancel the contract with the agency, after which the agency will no longer produce ratings for the firm.

The rating produced by NRSROs is essentially an estimated stack ranking of firms by credit risk. Translating this to default probabilities (DPs) is a statistical exercise of analyzing the historical default rates for each rating. The end result is a coarse-grained mapping of each firm to the average, through the cycle default rate for the firm's credit grade. As a result, default probabilities calculated in this fashion do not take into account current market conditions, and are insensitive to each firm's idiosyncratic risks.

In 1968, Altman [Alt68] introduced the Z-score. This was one of the first approaches to formalizing estimation of an individual firm's credit risk on a more fine-grained basis. The Altman Z-score is a linear combination of five key financial ratios. The model is statistical in nature. It is calibrated to a set of historical observations of each firm's accounting ratios and default status. The coefficients are chosen by multiple discriminant analysis (MDA) (also known as linear discriminant analysis) so as to optimally differentiate between the defaulted firms and the solvent firms.

The Altman Z-score gives a fine-grained measure of the credit risk of each firm. A Z-score less than 1.8 is considered an indicator of distress and a score greater than 3.0 is considered an indicator of a healthy firm. But additional work would have to be done to estimate default probabilities based on Z-scores.

The first approach to directly model the relationship between a firm's capital structure and its default was the Merton model [Mer74]. Merton assumed that the firm had one liability (a zero coupon bond) that must be paid at maturity. Default occurs if the firm has insufficient assets at the bond's maturity to pay the creditors. The equity is then a call option on the firm's assets with strike being the firm's debt, so the model can be calibrated to the firm's debt level and its market cap or bond prices.

Merton's approach was expanded upon in numerous ways. Black and Cox [BC76] introduced a default barrier to model the fact that firms can default at any time. Longstaff and Schwartz [LS95] incorporated interest rate risk. Avellaneda and Zhu [AZ01] added a time dependent barrier. All of these variants retain the general contingent claim pricing structure of the original Merton model.

Reduced-form models are an alternative to structural models. Instead of modeling the capital structure of the firm, reduced-form models directly model the default time. Reduced-form models started appearing in the 1990s in papers such as Artzner and Delbaen [AD95], Jarrow and Turnbull [JT94], and Duffie and Singleton [DS99], although such models are closely related to the long standing industry practice of pricing defaultable debt at a spread above the risk-free rate. These models are essentially survival analysis models from statistics applied to default times and are typically calibrated directly to an individual firm's bond prices or CDS spreads.

Structural models and reduced-form models are commonly used for pricing debt subject to credit risk. They cannot be directly used for measuring credit risk because the default probabilities they produce are risk neutral, not real world. Risk neutral default probabilities are subject to the willingness of investors to trade credit risk. Risk neutral default probabilities can deviate substantially from real world default probabilities [Hey+16] and using risk neutral probabilities is unsuitable for risk analysis [Ste16]. The probabilities produced by such models would have to be converted to real world before using them for credit risk analysis.

For the Bloomberg DRSK public firm model, we take a hybrid approach, in that we combine a statistical approach with a structural model. Like the Altman Z-score, our model is a statistical factor model fit to default events. However, instead of fitting the model using MDA, we use a logistic regression. The advantage of the latter is that instead of producing a Z-score, it directly produces default probabilities.

What makes this approach a hybrid approach is that one of our key factors is the distance to default that we derive from a structural model. This is similar to the model in Duffie, Saita, and Wang [DSW07], and ensures that our default probability estimates take into account the latest information available about the solvency of the firm, as expressed in the firm's market cap and stock price volatility, and their relationship to the firm's debt level. We also take into account differences between different regions and sectors by using different models for financial firms and for firms outside of North America.

This hybrid approach enables us to produce a term structure of real world default probabilities for over 65,000 public firms. The default probabilities express each individual firm's default risk and are point in time, rather than through the cycle. The default probabilities are recalculated on a daily basis so that we always produce estimates as up to date with current market conditions as possible.

The model outperforms the previous model. Both adjusted pseudo R-squareds and accuracy ratios have improved. The performance is more in line with observed historical default rates. The model is responsive to market conditions; default probabilities drop during economic expansions and rise during economic contractions. Our regression analysis exhibits an adjusted pseudo R-squared between 34% and 41% across different sub-models, which is good for default probability estimation. The model discriminates well between defaulted firms and non defaulted firms, with accuracy ratios from 92% to 97%. And the model's predictive power has improved, with average default rates rising substantially as default events are approached. For North American financial firms, it rises from less than 5% to over 20%.

The remainder of this document is organized as follows. Section 2 reviews the process of default and bankruptcy and specifies the types of credit events that the model is based on. Section 3 describes the data sets the model is based on and tested with. Section 4 describes the structure of model in detail. The behavior of the individual factors in the model is presented in Section 5. Section 6 addresses the overall performance of the model. We look at the model's goodness of fit and predictive power as well as the behavior of the model's underlying risk factors. Section 7 compares the behavior of the model to the previous model. Section 8 provides a concluding summary.

2 Default definition

When modeling default probabilities, it is important to define exactly what the events are whose probabilities are being estimated. Since, for credit risk, we are interested in defaults as well as bankruptcies, we will refer to the set of these events as "credit events". So, we will need to define default and bankruptcy and detail which types of such events are counted as credit events.

Default potentially occurs after a firm breaches the terms of a debt contract, such as failing to make a coupon payment on time or to satisfy a debt covenant. If this can lead to default, the event is referred to as a "technical default".

For example, a firm is in technical default when it fails to make a bond coupon payment on time. Bond prospectuses often grant the issuer a grace period for honoring its payment obligations (typically 30 days in the United States). If the firm makes the payment within the grace period, it is no longer in technical default. Otherwise, the firm has defaulted. Such default events are recorded in Bloomberg's corporate action database (CACS).

In some cases, a firm can avoid default by exchanging the distressed bond for a package of new debt, equity and cash. If the package is of lesser value, then this is referred to as a "distressed exchange". Similarly, government interventions (such as bail-outs) can occur. Such events are often not formally recorded.

If these or other mitigating steps are not taken, a court filing typically takes place. This is a bankruptcy filing. There are various ways that bankruptcies happen, depending on the bankruptcy laws of the firm's country of domicile and how the bankruptcy is filed. Some bankruptcy filings begin the liquidation process (chapter 7 in the US). Other bankruptcy filings mark the start of bankruptcy protection (chapter 11 in the US), enabling a more nuanced work-out of the bankruptcy. In some countries, the creditors file to petition the court to declare the debtor insolvent. Firms will often breach a number of debt obligations before entering into bankruptcy. Firms can also file for bankruptcy before having defaulted.

Once a firm has entered into bankruptcy, a variety of things can happen. It can be liquidated, or restructured and emerge from bankruptcy, or merge with or be bought by another company. A court can also dismiss the bankruptcy filing (e.g., for insufficient financial disclosures), or the firm can cancel the filing for any reason, including reaching an agreement with debt holders.

There is a difference of opinion as to which of the above events to capture within a credit risk model. Some NRSROs include technical defaults, while Basel regulations, international accounting standards, and ISDA do not. Basel, ISDA, and international accounting standards include unlikeliness to pay as a default event, but they use different criteria to define this.

The set of credit events for the DRSK calibration includes all defaults and bankruptcies recorded in the Bloomberg CACS database, except for canceled and dismissed filings. We supplement this with distressed exchanges, and government interventions that we have manually identified and recorded. Our definition is largely in alignment with the Basel regulations, except that we do not include unlikeliness to pay because it tends to be subjective.

It is often unclear whether a subsequent credit event is part of a firm's transition to bankruptcy or is a credit event occurring after a firm has resolved its previous insolvency. Similarly, a firm can enter into bankruptcy before having defaulted, and then emerge and subsequently default. To avoid over-counting credit events, we do not include credit events occurring after a firm's first credit event, and curtail the calibration data set accordingly.

As a result, the model's default probabilities are estimates of the probability of such defaults, bankruptcies, government interventions, and distressed exchanges occurring within a particular time frame.

3 Data set

The public model data set consists of monthly observations of firms from May 1998 to November 2018. Tables 1 and 2 show the breakdown of this data set by region and sector, respectively. Our data set contains 65,520 public firms, with the bulk of the firms domiciled in Asia (45%), North America (29%), and Western Europe (16%). The data set contains 2,826 first firm defaults, which are mostly recorded in North America (66%) and Asia (17%).

Firms in our sample fall into ten different sectors[1], with consumer discretionary having the largest representation (18%), and utilities the smallest (2%). The remaining sectors range from 5% to 17%. The financials sector accounts for 17% of the firms followed by industrials, with 13%.

3.1 Default rates

As can be seen in Table 1, regional default rates vary substantially. Differences in bankruptcy laws play a large role in this. In the United States, the Bankruptcy Reform Act of 1978 created Chapter 11. This enabled more orderly reorganization and/or liquidation of bankrupt companies and made bankruptcies a matter of public record. It was only in 2015 that European countries

[1] The sectors referred to here are the top level sectors in the Bloomberg Industry Classification System (BICS).

Table 1: Public model data set by region. We give the number and percentage of firms in each region, the number and percentage of defaults in each region, and each region's default rate.

	Count		Percentage		
Region	Firms	Defaults	Firms	Defaults	Default rate
Africa	1,290	6	2.0	0.2	0.5
Asia	29,435	480	44.9	17.0	1.6
Eastern Europe	2,622	25	4.0	0.9	1.0
Latin America	1,115	53	1.7	1.9	4.8
Middle East	1,809	20	2.8	0.7	1.1
North America	18,932	1,858	28.9	65.7	9.8
Western Europe	10,317	384	15.7	13.6	3.7
Total	65,520	2,826	100.0	100.0	4.3

Table 2: Public model data set by sector. We give the number and percentage of firms in each sector, the number and percentage of defaults in each sector, and each sector's default rate.

	Count		Percentage		
Sector	Firms	Defaults	Firms	Defaults	Default rate
Communications	3,526	283	5.4	10.0	8.0
Consumer Discretionary	11,598	661	17.7	23.4	5.7
Consumer Staples	4,082	131	6.2	4.6	3.2
Energy	3,589	273	5.5	9.7	7.6
Financials	11,192	326	17.1	11.5	2.9
Health Care	5,233	236	8.0	8.4	4.5
Industrials	8,778	342	13.4	12.1	3.9
Materials	7,860	210	12.0	7.4	2.7
Technology	8,459	335	12.9	11.9	4.0
Utilities	1,203	29	1.8	1.0	2.4
Total	65,520	2,826	100.0	100.0	4.3

started enacting similar laws [Man16], with convergence towards Chapter 11 continuing [Zol19; Jac+20]. Other regions lag further behind.

In countries with bankruptcy laws only allowing for liquidation, it is often the case that defaults and bankruptcies are managed behind closed doors, so records of such events are much more rare. And some countries are more inclined to prop up failing firms than others, often to the detriment of the creditors.

Calibrating a default model given this lack of default records is challenging. We address our approach to this problem in section 3.2.

3.2 Calibration data set

Credit modeling tries to predict the probability of future defaults and bankruptcies based on observations of the model factors. As noted above, defaults are complicated and come under different rules and regulations in different countries. Their resolution often depends on the nature of the loan agreement and the business opportunity between the lender and the borrower. In particular, the lender might make arrangements directly with the borrower to avoid a default event. As a result, when a firm fails to pay a loan, or breaches a covenant, that breach and its resolution might not be a matter of public record. This is corroborated by the large differences seen in default rates across regions (section 3.1).

Similarly, the model factors themselves also need to be accurately recorded. Since the factors depend on financial reports, this requires the financial reports themselves to be detailed and reliable.

To accurately model the probability of default, the issue of missing default events and inaccurate data needs to be addressed. One way of doing this is to calibrate to a more accurate subset of the data. Since missing a payment on a publicly traded bond is a mater of public record, we restrict our calibration set to bond issuers. And since small firms tend to have less reliable financial reports, we only include firms in the calibration set that are sufficiently large (e.g., in the US, we only include firms with over \$200 million in asset value over enough of their history).

When data quality varies from region to region, region size itself can also skew model results. We control for this by adjusting the asset value cut off to deemphasize regions with lower quality data. The final calibration set includes about 14,000 bond issuers.

4 DRSK model structure

DRSK is a hybrid model, in that it combines a statistical approach with a structural model.

We use a logistic regression to estimate the probability of default events based on factors that best capture credit risk. But the factors are not purely relevant accounting ratios – one of the key factors is the distance to default (DD) derived from a hybrid Merton-Black-Cox structural model.

In that the primary model factor is the DD, we first provide the details of our hybrid Merton-Black-Cox model and the calculation of the DD factor. Then, we detail the other model factors and the logistic regression used to estimate real world default rates as a function of these factors.

4.1 The structural model of default

Under the basic Black-Cox model (strong covenant) [BC76], the default time is the first passage time of the firm's asset value hitting a barrier before a horizon time T. The interest rate r is assumed to be constant, and the firm's asset value process (A) is assumed to follow a geometric Brownian motion. Thus, we have:

$$dA = \mu A dt + \sigma_A A dW \tag{4.1}$$

$$dB = rBdt \tag{4.2}$$

where B is the money market account. Default occurs at the first time that A crosses a barrier K.

Under the risk neutral measure with respect to B, the assets have a drift of r, so the risk neutral asset value process (before default) is given by:

$$A_t = A_0 e^{(r-\frac{\sigma^2}{2})t+\sigma W_t} \tag{4.3}$$

DRSK uses the Black-Cox model with r being the 5-year swap rate, and the barrier K set to a linear combination of the short term debt and the long term debt with various accounting adjustments applied (see section 4.2).

The equity (market cap) is then a down and out call on the assets with maturity T years and strike equal to the barrier level. Let

$$M_t = \min_{0\le s\le t} A_s \tag{4.4}$$

Then the payoff of the equity is:

$$S_T = (A_T - K)^+ 1_{M_T>K} \tag{4.5}$$

$$= (A_T - K) 1_{M_T>K}, \tag{4.6}$$

where the latter equality holds because $A_T - K < 0$ implies $1_{M_T>K} = 0$.

Choosing an equivalent pricing measure Q with respect to a numéraire N, we get the value of the equity at time t as:

$$S_t = N_t E^Q[(A_T - K)^+ 1_{M_T>K}/N_T|\mathcal{F}_t] \tag{4.7}$$

$$= C(A_t, T-t, K) - \left(\frac{A_t}{K}\right)^{1-2r/\sigma_A^2} C(K^2/A_t, T-t, K) \tag{4.8}$$

where $C(A, t, K)$ is today's value of a call on A with strike K maturing at time t:

$$C(A, t, K) = A\Phi(d_1) - Ke^{-rt}\Phi(d_2) \tag{4.9}$$

$$d_1 = \frac{(r + \sigma_A^2/2)t + \log(A/K)}{\sigma_A\sqrt{t}} \tag{4.10}$$

$$d_2 = \frac{(r - \sigma_A^2/2)t + \log(A/K)}{\sigma_A\sqrt{t}} \tag{4.11}$$

$$\tag{4.12}$$

Once the model is calibrated, the distance to default is

$$DD = d_2 \tag{4.13}$$

As a result, the DD for a given horizon can be viewed as the number of standard deviations by which the forward asset level exceeds the default barrier at the horizon.

4.2 Debt level estimation

Within the Black-Cox framework, the default threshold is the value of the firm's total debt K, which cannot be readily estimated from the complex debt structure. In practice, the firm is willing to pay short-term debt first, before paying long-term debt. Therefore, a common practical estimation of the debt threshold is to set it somewhere between the face value of the short-term debt and long-term debt. Specifically, K is defined as:

$$K = STD + \frac{1}{2} \times LTD \tag{4.14}$$

where K, STD, and LTD denote total debt, short-term debt, and long-term debt, respectively. Total debt changes at announcement dates when new financial reports are released. The final calculation of total debt is subject to additional debt adjustments as follows:

- Lease adjustment: We add a lease adjustment if firms haven't adopted the new International Financial Reporting Standards (IFRS) 16, and Accounting Standard Codification (ASC) 842, and haven't provided the lease data disclosure described therein (see for instance, *IFRS 16 Leases* [Int16]).
- Core deposits: Another debt adjustment is made for banks to account for the regulatory proportion of their core deposits used for lending. Core deposits are defined as the segment of bank non-maturity deposits (also known as NMDs) that are partially insured by the Federal Deposit Insurance Corporation and benefit from a low-cost and stable source of funding. The Basel Committee on Banking Supervision standards on interest rate risk in the banking book provides some guidelines on how to estimate levels and caps of regulatory core deposits (see for instance, *Standards, Interest Rate Risk in the Banking Book* [Bas16]).

- Pension liabilities: Another regulatory debt adjustment includes liabilities due to funding requirements for pensions and other post retirement benefits (see for instance, *Pension Protection Act of 2006* [10906]).

4.3 Structural model calibration

Structural models are difficult to calibrate because there are two unknowns – the asset level A_t, and the implied volatility σ_A. Given the debt barrier and the market cap, different implied volatilities can be chosen, yielding different asset levels.

One approach to calibrating the model is to use the historical equity volatility to infer the asset volatility. This is done using the pure Merton model (default only at time T), where Equation 4.8 would instead be:

$$S_t = C(A_t, T - t, K) \tag{4.15}$$

Applying Ito's formula and comparing volatility terms, we then have that the asset volatility differs from the equity volatility by a factor of the delta (dS/dA):

$$\frac{dS}{dA} A_t \sigma_A = \sigma_S S_t \tag{4.16}$$

This approach tends to produce unstable asset values, so for our hybrid Merton-Black-Cox model, we instead solve for the implied volatility σ_A which matches the historical asset volatility over the past 12 months. This yields two equations and two unknowns which can be solved for σ_A numerically through a fixed-point method. We find that this approach yields more realistic behavior.

4.4 Real-world default probability: DP

DRSK estimates real-world default probabilities (DPs) using a logistic regression of historical realized defaults against the structural model DD and additional risk factors such as profitability and insolvency. Specifically, the real-world DP for a firm, F, at tenor, t, is modeled as:

$$DP(F,t) = p(B(F,t)) = \frac{1}{1 + \exp(-f(B(F,t)))} \tag{4.17}$$

$$f(B(F,t)) = \beta_0^t + \beta_1^t DD(F,t) + \sum_i \beta_i^t B_i(F) \tag{4.18}$$

where $i \in \{BANK\_FLAG, ROA, NPL\}$ denotes the list of factors in the financials model and $i \in \{ICR\}$ the list of factors in the non-financials model (Table 3). The parameters β_i^t are the calibrated regression coefficients for each tenor $t \in \{0.25, 0.5, 0.75, 1, 2, 3, 4, 5\}$ in years.

The definition of each factor, the rationale for including it, and its statistical general behavior is detailed in Section 5.

Table 3: DRSK public model predictors

Sector	Model predictors	Notation
Financials	Distance to default Bank flag Return-on-assets Non-performing loans	DD $BANK\_FLAG$ ROA NPL
Non-financials	Distance to default Interest coverage	DD ICR

4.5 Logistic calibration

The model is calibrated by logistic regression of the model factors against the indicator of default over the month end calibration set. So, for example, for estimating the occurrence of a credit event within 3 months, we regress against the indicators of defaults or bankruptcy of each firm occurring within 3 months.

As noted in section 3.2, to correct for missing default events in the full data set, the calibration set only includes bond issuers. To further correct for unrecorded credit events outside of North America, we also over-sample default events in these regions.

4.6 DRSK term structure

Using marginal DPs from the logistic regression, one can obtain cumulative DPs for each tenor t∈ {0.25, 0.5, 0.75, 1, 2, 3, 4, 5} in years as follows:

$$DP_i^{cumulative} = 1 - \prod_{j=3m}^{i} \left(1 - DP_j^{marginal}\right) \tag{4.19}$$

Equivalently, annualized DPs can be obtained as follows:

$$DP_i^{annualized} = 1 - \left(1 - DP_i^{cumulative}\right)^{\frac{1}{t}} \tag{4.20}$$

Both cumulative and annualized DPs are used to evaluate the default probability of a firm over different horizons. They are useful for comparing and validating stylized facts between samples within different grades of credit quality.

4.7 DRSK mapping to credit grades

To facilitate comparing the default risk of different firms, we group firms into credit risk buckets, which we call credit grades. The range of each credit grade's bucket is given in Table 4. The ranges were obtained so that the default rate in each range is broadly consistent with the 1- year probability of transition to default across major NRSRO ratings of a comparable level. Except for transition smoothing (see below), a firm is assigned the credit grade whose range contains the firm's one year default probability.

Because the distance to default depends on the market cap, and the market cap fluctuates from day to day with equity price changes, it is possible for a firm's default probability to fluctuate around the boundary between credit grades. If we mapped the default probability directly to a credit grade, this would cause the credit grade to fluctuate back and forth between two neighboring credit grades. To avoid this, we employ a credit grade transition smoothing technique.

When the default probability for a firm breaches one of the barriers defining its current credit grade, the credit grade is only changed if the default probability surpasses this boundary by at least 10% or stays in the new range for at least 90 days.

Table 4: DRSK credit grade map. The figure shows the mapping between alphanumeric credit grades and corresponding ranges of default probabilities expressed in percentage.

Credit level	Credit grade	DP range (Min, Max)
Investment grade (IG)	IG1	0.0000 - 0.0020
	IG2	0.0020 - 0.0040
	IG3	0.0040 - 0.0080
	IG4	0.0080 - 0.0152
	IG5	0.0152 - 0.0286
	IG6	0.0286 - 0.0529
	IG7	0.0529 - 0.0960
	IG8	0.0960 - 0.1715
	IG9	0.1715 - 0.3000
	IG10	0.3000 - 0.5200
High yield (HY)	HY1	0.5200 - 0.8800
	HY2	0.8800 - 1.5000
	HY3	1.5000 - 2.4000
	HY4	2.4000 - 4.0000
	HY5	4.0000 - 6.0000
	HY6	6.0000 - 10.0000
Distressed (DS)	DS1	10.0000 - 15.0000
	DS2	15.0000 - 22.0000
	DS3	22.0000 - 30.0000
	DS4	30.0000 - 50.0000
	DS5	50.0000 - 100.0000

5 Factor analysis

Here we detail the rationale for including each model factor, and its ability to predict credit events. To analyze the importance of each factor, we split the calibration set into two subsets. The defaulting subset consists of the set of observations occurring within one year of a credit event. The non-defaulting subset is its complement. We then compare the behavior of the factor on the defaulting subset to the non-defaulting subset to analyze the extent to which the factor distinguishes between defaulting firms and non-defaulting firms. Our analysis covers three sets of data with respect to regions and sectors. Throughout the document, we refer to financials in North America as Fin_NA, financials outside of North America as Fin_GLB, and non-financials as NonFin_GLB.

We use box plots to compare the distribution of factor values on the defaulting subset to the non-defaulting subset, The box plots illustrate the extent to which the median and extremes of the factor differs on the two subsets.

We also plot the rolling average default rate (the proportion of observations in the defaulting subset) as a function of the factor. This visually indicates how the empirical realized default rate varies as a function of the underlying factor.

As is illustrated in the following sections, while all of the factors have some explanatory power, the DD is the strongest predictor of default.

5.1 Distance to default, DD

In a structural framework, DD is a market-based measure of default risk. Although it is derived from a simple capital structure, it is shown to be a good predictor of default. Specifically, the smaller the distance, the more likely a particular firm is to default within the specified horizon. Figure 1 shows that DDs are inversely related to the observed default rates. It also shows that the DD distribution is different between defaulted and non-defaulted firms. As can be seen in the box plots, firms that default within a year have a median DD close to 1, whereas the remaining firms have a median DD close to 5, and a much higher percentage of large DDs. The default rate curves show that default rates are substantially higher on average for low DDs and rapidly fall off as DDs rise, with almost no defaults occurring for firms with DDs greater than 8.

5.2 Bank, $BANK\_FLAG$

The empirical study of systemic banking crises by Hoelscher and Quintyn [HQ03] and updated by Laeven and Valencia [LV18] provides insights regarding the effect of bank defaults on the real economy. Specifically, a systemic banking crisis creates disruptions of the payments system and credit flows, and leads to the destruction of asset values. For some economies, this translates into a fiscal cost ranging from about 4% (Sweden) up to 57% (Indonesia) of a country's gross domestic

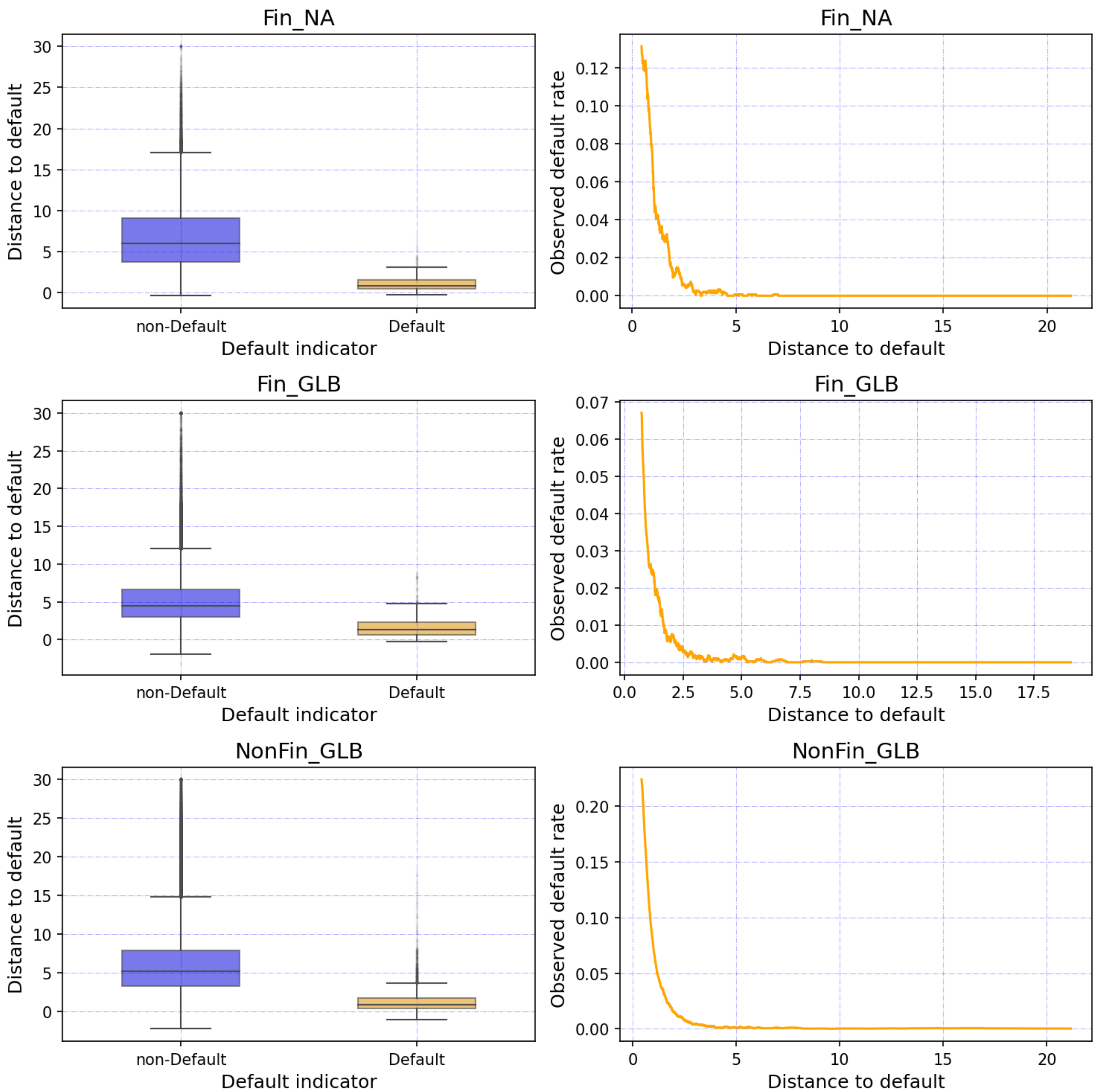

Figure 1: DD distribution. The figure shows the distribution of DD against the observed default rate (right panel), and DD box plot by default indicator (left panel) for each sub-model. The graphs show a strong relationship between DD and default rates. Default rates decrease sharply as DDs increase.

product.

Because of such failure costs, regulators have strong incentives to intervene to prevent bank defaults. As a result, we would expect banks to have lower default rates than other financial firms. This is borne out by the data. We observe that in North America, banks have a default rate of 0.56%, whereas the non-bank default rate is 0.71%. For financials outside of North America, the realized default rate is 0.31% for banks and 0.39% for non-banks.

Structural models do not capture the regulatory and supervisory complexities associated with bank interventions and bank closures [CLS07], so these differences do not show up in the model's DD factor. To account for this, we add an indicator factor ($BANK\_FLAG$) to the model.

5.3 Profitability, ROA

The return on assets (ROA) is the ratio of the firm's net income to its total assets. This accounting ratio is a useful indicator of the firm's profitability. While the firm's DD captures the relationship between the assets and debt of the firm, the ROA helps to capture the relationship between cash flows and assets. A negative ROA indicates the firm is operating at a loss and is in a state of distress.

Figure 2 compares the ROA to realized default rates. Firms with positive ROAs rarely default, and default rates increase as ROAs become more negative. We also see that the relationship between ROAs and default rates differs between banks and other institutions, justifying splitting the factor between these two subsets.

5.4 Non-performing loans, NPL

Financial institutions have a more complex liability structure than non-financial institutions do. That extra complexity, especially for banks, is not generally captured by structural models. For instance, the bank failures experienced in the aftermath of the 2000-2002 dot-com bubble were associated with banks that were holding a large quantity of non-performing loans on their balance sheets [CLS07]. The effect was more pronounced for several large Japanese banks that ended up being severely under-capitalized. For other regions, the growth in troubled assets reduced capital ratios to the point where regulatory interventions were called for.

To capture the impact of non-performing loans on financial institutions, we include the non-performing loan ratio (NPL) as a factor. The NPL is the ratio of the notional of non-performing loans minus the loan loss reserves to the current market cap. Firms with high NPLs are more likely to default.

The box plots in Figure 3 show that the NPL is substantially higher for defaulting firms. In North America, 75% of defaulting firms have an NPL between 2 and 5, whereas non-defaulting firm NPLs tend to be close to zero. The default rate plots show that default rates rise quickly

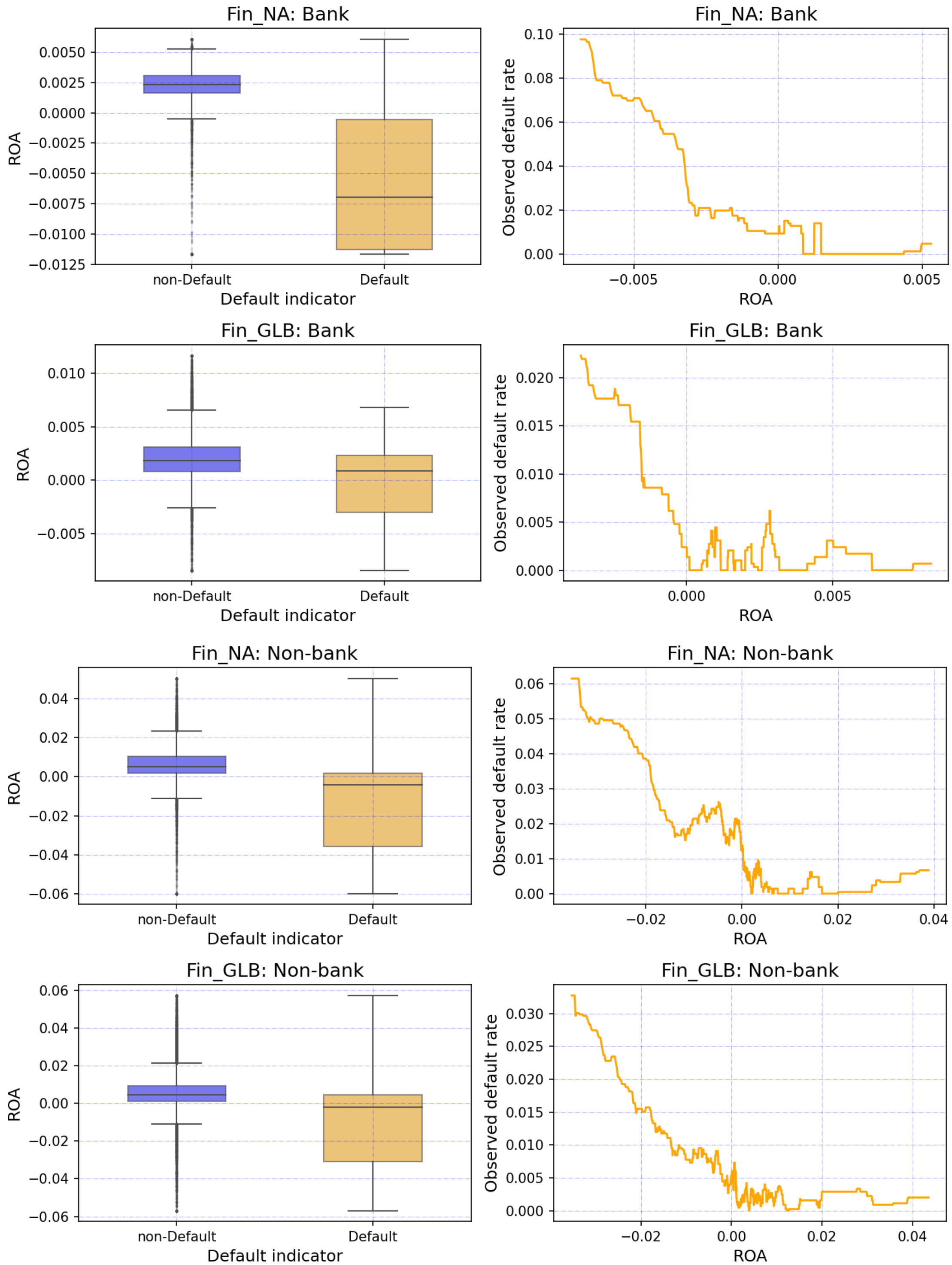

Figure 2: *ROA* distribution. The figure shows the distribution of the average of realized default rates as a functions of the average of *ROA* (right panel), and *ROA* box plot by default indicator (left panel). The figure shows that default rates are very high for negative *ROA*s and decrease as *ROA*s become positive.

with increasing NPLs once the NPLs exceed a threshold of about 0.1 in North America and 0.5 outside of North America.

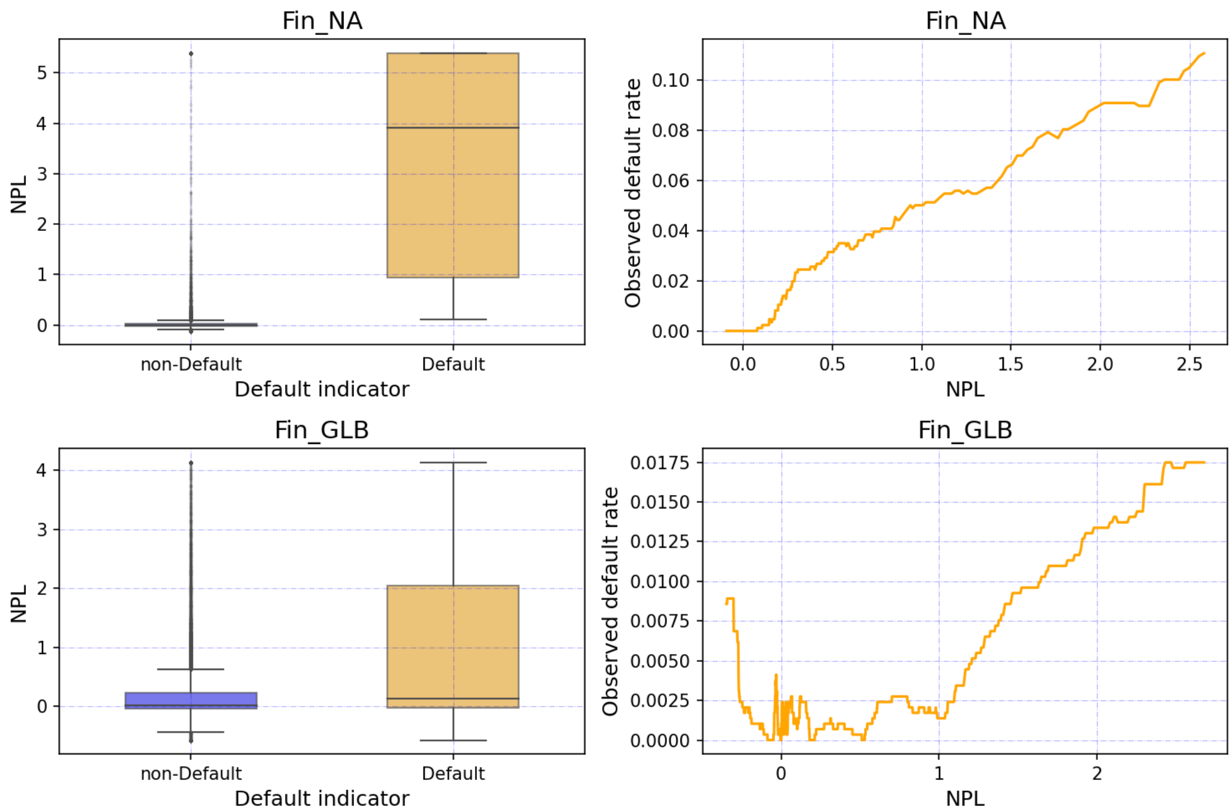

Figure 3: NPL distribution. The figure shows the distribution of NPL against the observed default rate (right panel), and NPL box plot by default indicator (left panel). The figure shows that default rates increase as NPLs increase.

5.5 Interest coverage, ICR

The ICR is defined as the ratio between total operation cash-flows and interest expenses. It measures the ability of a firm to service its debt using internal cash-flows. A high ICR indicates that the firm has sufficient liquidity and a stronger financial position. In a recent publication by the Federal Reserve, Palomino et al. [Pal+19] suggest that critical levels of ICR are indicative of financial distress for publicly traded non-financial firms. They also suggest that the combination of debt structure and ICR provide good information about corporate vulnerability.

The box plot in Figure 4 shows that the ICR distribution is substantially lower for defaulting firms. The median is about 1.9 for non-defaulting firms, versus about 0.6 for defaulting firms. Similarly, ICRs over 0.8 are highly correlated with default rates. While realized default rates are not monotonic as a function of ICR, it's still the case that low ICRs have much higher default rates than high ICRs.

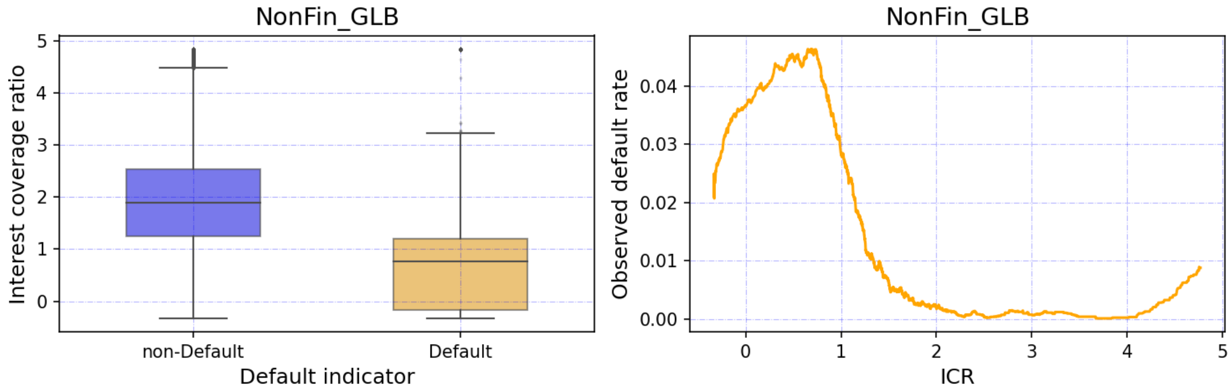

Figure 4: ICR distribution. The figure shows the distribution of ICR against the observed default rate (right panel), and ICR box plot by default indicator (left panel). The figure shows that default rates decrease as ICRs increase.

6 Model performance

We use a variety of metrics to analyze model performance:

1. The overall model fit
2. The DP distribution
3. The relation between model DP and realized default rates
4. The DP predictive power
5. The model coefficient behavior

The overall model fit is analyzed through the model's adjusted pseudo R-squared, receiver operating characteristic (ROC) curves and the accuracy ratios (ARs). The ROC curves also illustrate the model's ability to distinguish between defaulting firms and the remaining firms. This analysis is in Section 6.1.

To confirm that the model DPs follow expectations, we look at the distribution of DPs over time, across the 21 DRSK credit grades. This analysis is in Section 6.2.

Another check is to compare realized and model estimated default rates. Given that the model is regression based, overall averages will match on the calibration set. So, instead of overall averages, we perform two other tests. We compare the realized default rate over time for the Bloomberg Corporate High Yield Index (hereafter, HY index) to the corresponding average model estimated default rate. Then, on a rolling average basis, we compare each realization of aggregate default rates to our model predictions within the same rolling window. This analysis is in Section 6.3.

The model's predictive power is analyzed by exploring the average model default rate as a function of time to default. For each sub-model, we run this test using the full set of data and the the calibration subset. This analysis is in Section 6.4.

Section 6.5 analyzes the behavior and stability of the model coefficients. By comparing the model coefficients for different marginal DP regressions, we can confirm that the factors are meaningful and exhibit appropriate behavior throughout the term structure. Lastly, using a time based k-fold cross validation test, we confirm that the model is stable and does not over-fit to the data.

6.1 Overall model fit

Figure 5 plots the adjusted pseudo R-squared for eight tenors and for different regions and sectors. Over the shorter horizon of three months, the logistic regression accounts for 34% (Fin_GLB and NonFin_GLB) to 41% (Fin_NA) of the total variation. As the horizon increases, the uncertainty about the factors increases and the model fit decreases to a level below 7% for the 5-year horizon.

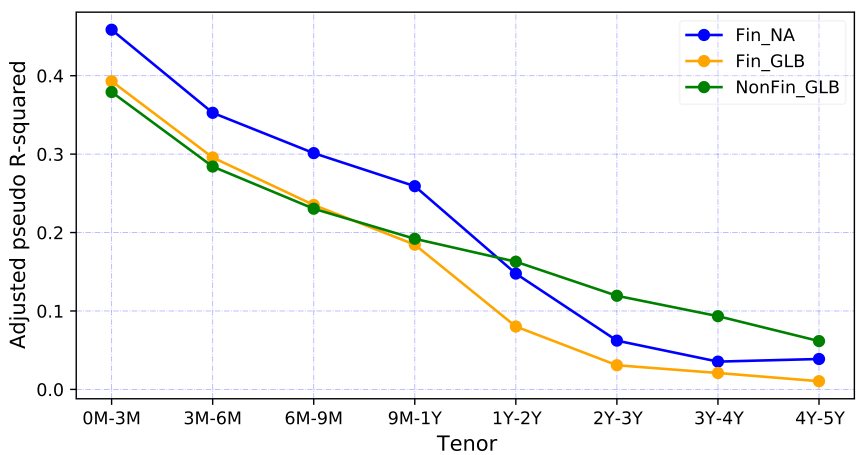

Figure 5: Adjusted pseudo R-squared. We show the adjusted pseudo R-squared for the three sub-models, obtained for different time horizons. The figure shows that adjusted pseudo R-squared are above 38% for the short horizon (three months), and decrease as the horizon increases, reaching near zero for the 4-5 year horizon (Fin_GLB).

Figure 6 plots the ROC curves, which evaluate the models' predictive ability over a horizon of one year. Specifically, the ROC indicates how the DRSK prediction of default, over the specified horizon, can distinguish between true positives and negatives. The y-axis in Figure 6 illustrates the probability of predicting defaulted firms will default (true positive rate) and the x-axis illustrates the probability of making the wrong prediction of classifying non-defaulted firms as defaulted firms (false positive rate). The AR measure indicates how much the model is capable of distinguishing between default type and non-default type firms. The higher the AR, the better the model at predicting defaults.

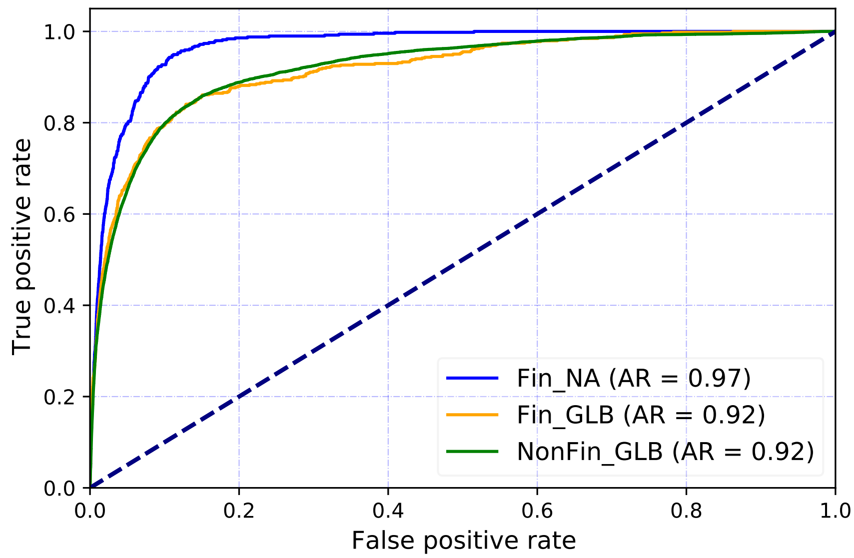

Figure 6: Receiver operating characteristic (ROC). The figure shows ROC curves for three DRSK sub-models. The figure indicates that all sub-models have very high accuracy.

As reported in Figure 6, the AR values range between 92% for Fin_GLB, and 97% for Fin_NA, suggesting that DRSK predictive ability, over the one year horizon, remains very high across different regions and sectors.

We also perform a walk-forward validation test to assess the model performance in-sample and out-of-sample. In practice, the walk-forward test consists of retraining the model as new data becomes available. Here, we first calibrate the model on the training subset ranging from 1999 to 2009. We predict DPs of 2010 using the calibration sample (in-sample, blue line) and predict DPs of 2010 using the inclusive sample data for 2010 (out-of-sample, yellow line). We then compute and store the resulting in-sample AR and out-of-sample AR for the year 2010. Next, we expand the subset with an additional year of data, now including the period from 1999 to 2010. We calibrate the new model, predict DPs for 2011, compute and store the in-sample and the out-of-sample AR for the year 2011. This process is repeated until the data is exhausted.

Figure 7 graphs the AR results of the walk-forward validation test. Overall, the in-sample AR range remains very high, greater than 92% for all three sub-models. The out-of-sample AR exhibits more volatility, yet it ranges between high levels of 60% and 97%. This validates the performance of the model in predicting the one-year DP.

6.2 DP distribution

Figure 8 gives the stacked distribution of DPs over time for the 21 DRSK credit grades across the three sub-models. The shifts of the credit grade distribution around 2000 and 2008 show that the

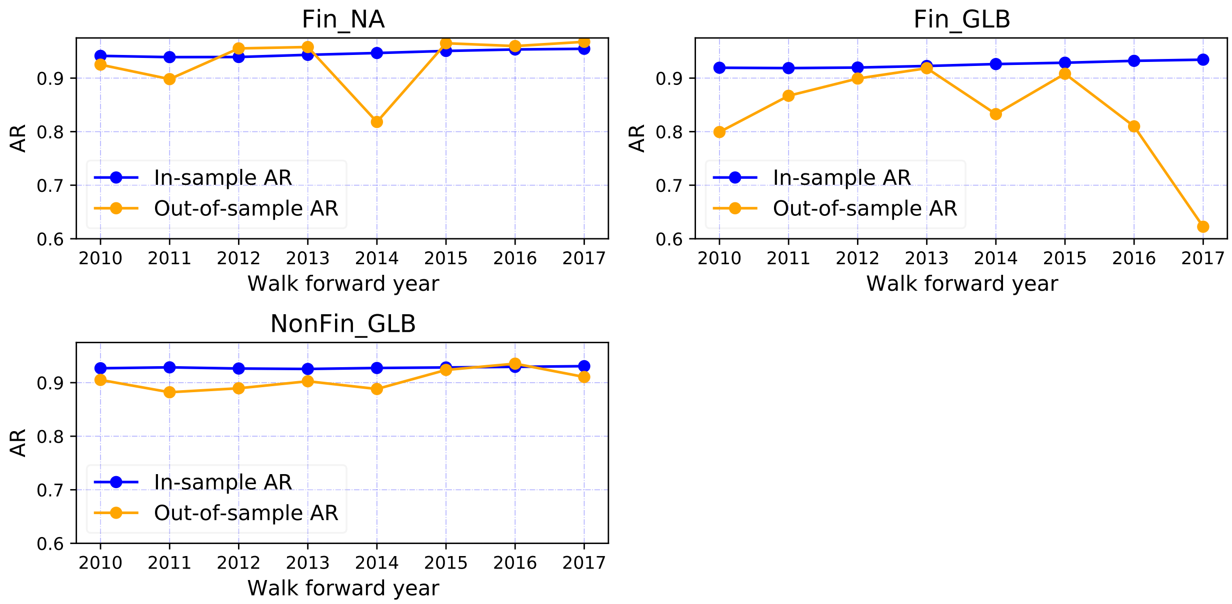

Figure 7: Walk-forward AR. The figure shows the in-sample and out-of-sample AR output of the walk-forward test for three sub-models. The figure shows that the model predictive ability, over the one year horizon, remains very high across different regions and sectors.

model captures the increases in credit risk that occur during times of economic and financial stress.

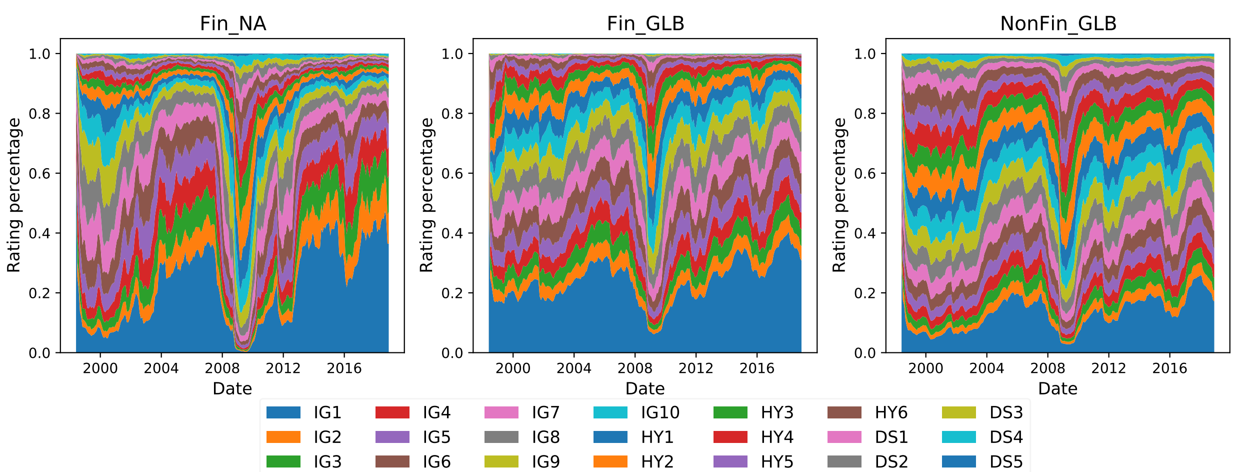

Figure 8: Credit grade distribution - stack plot. We show the distribution of DRSK risk bands over time using a stack plot. The figure shows that DRSK average credit grades are sensitive to the economic and financial crises.

6.3 Realized default rate comparisons

In this section, we test the DP model fit and performance. We look at the model's performance over time and compare estimated default rates to realized default rates.

Figure 9 shows that the model accurately captures the changes in default rates over time. Predicted default rates are close to realized default rates throughout the history for each sub-model. The non-financial global model appears to be overestimating default rates, but this is intentional, in that default events are over-sampled to adjust for missing default events (Section 4.5).

We also compare the model's average default rate over time to the realized default rate on the Bloomberg high yield index (HY index). The HY index members are bond issuers, and hence they are in our calibration set. However, it is still a good test, in that it confirms the extent to which the model's average default rates match the realized default rates on a subset of firms that are independently classified as high yield. As can be seen in Figure 9, the model does a good job of capturing the index's default rate over time.

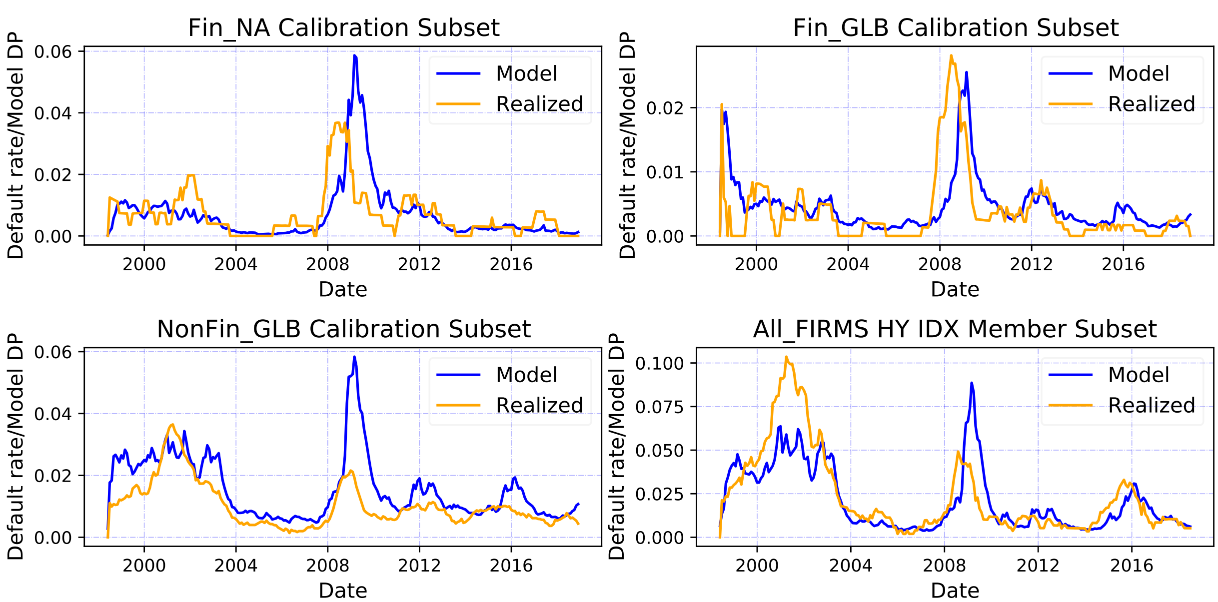

Figure 9: Model DP vs realized default rate. The figure shows the time-series of realized default rates versus estimated default rates across regions and sectors and for the HY index. The figure shows that realized default rates and estimated default rates are in alignment, over time.

Figure 10 show the realized default rates as a function of the estimated default rates. Overall, the realized default rates and the estimated default rates are in alignment for all three sectors, with a stronger relation for financials (top panel). For the high yield index (bottom right), the alignment is much better up to 15% of model DPs. After that, it seems like the model is underestimating defaults. However, given the sample size, this is more the result of a sample bias as there are fewer firms at the higher default rates.

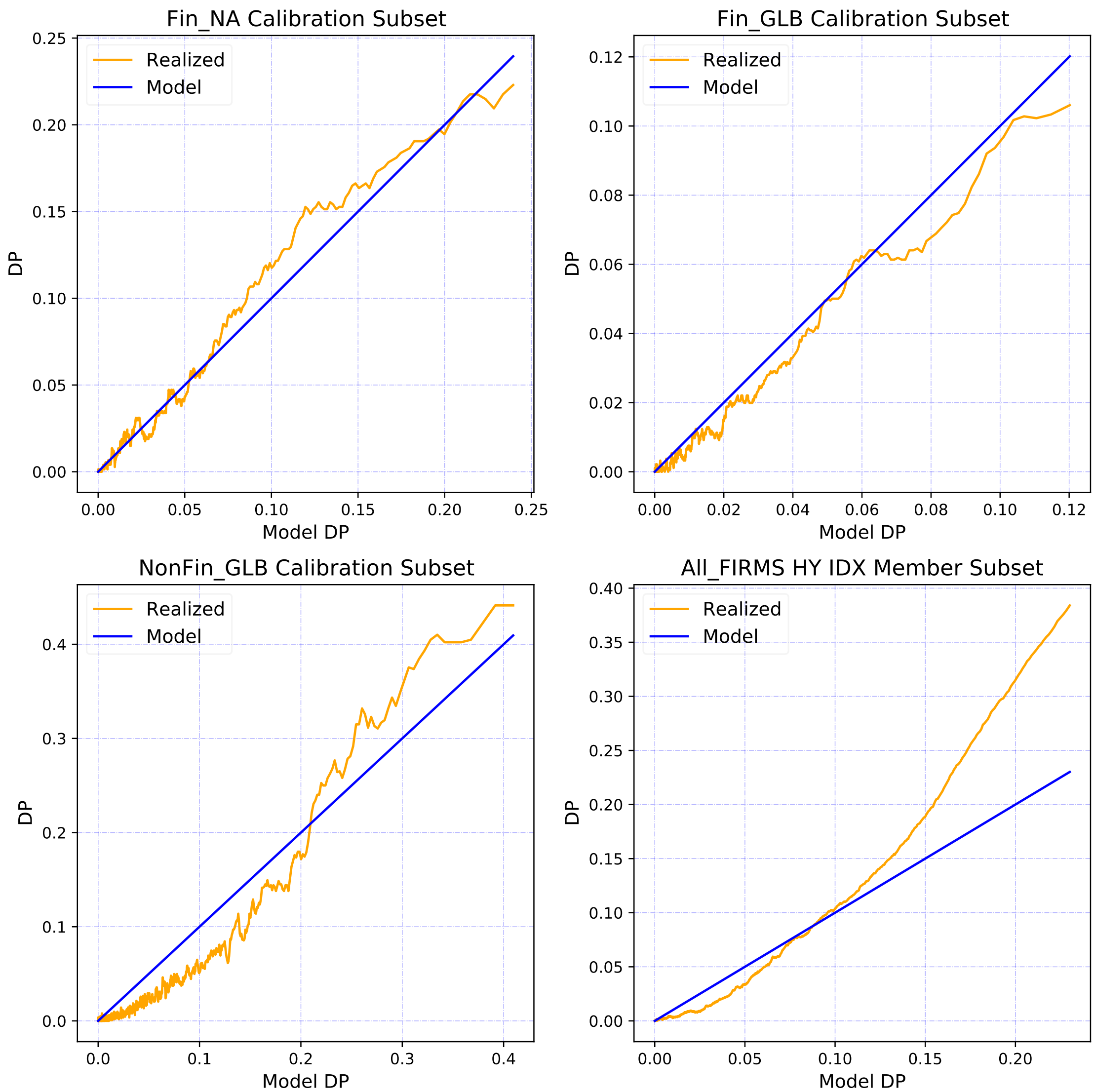

Figure 10: Model DP vs realized default rate. The figure shows the distribution of the average of the observed default rates against the average of model DPs. Each panel includes a specific region and sector on the calibration subset, except the bottom right panel, which includes members of the HY index. The figure shows that, in terms of averages, realized default rates and estimated default rates are mostly in alignment.

6.4 Predictive power

We test the model's ability to predict defaults by looking at the model's average default rate as a function of the number of months to default. A model that has good predictive power should show an increasing average default rate as the time to default decreases.

Figure 11 shows that all of the sub-models show a substantial increase in average default rates as the time to default decreases. For the Fin_NA sub-model, default rates start rising about 2-3 years before default, from about 2% to about 30%. The same holds for the NonFin_GLB sub-model. The Fin_GLB sub-model exhibits similar behavior, although the rise in default rates is less marked, rising to about 12% prior to default.

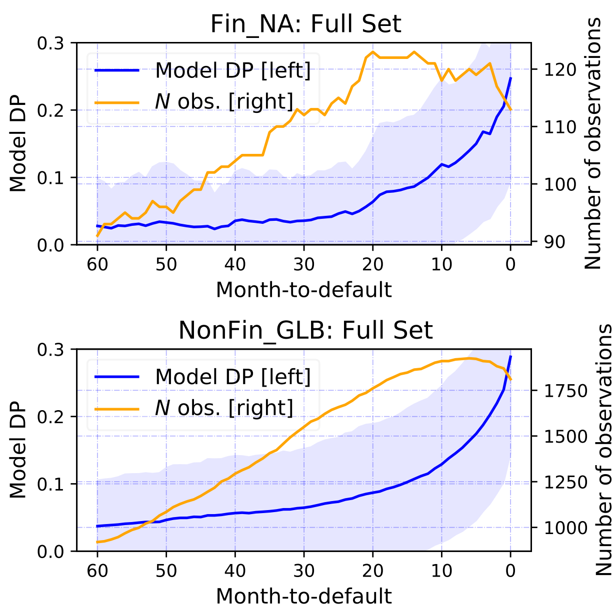

Figure 11: DP versus month-to-default. The figure shows the distribution of model DPs as a function of time to default using the full data set. The shaded area represents +/- the standard deviation of DP. Across three sub-models, DP increases with time to default.

6.5 Coefficient stability and behavior

In that the term structure of default is generated by a set of regressions, it's important to confirm stability of the model coefficients as a function of the tenor. In general, the estimated model coefficients are fairly stable as a function of tenor.

For the coefficient walk-forward test, we compare the model coefficients produced on the 1999-2010 subset to those produced on the 1999-2011 subset, etc. The walk-forward test indicates that the average year over year change of the estimated coefficients is around 6% for the last 8 years. This implies that the model is capturing intrinsic, stable market behavior.

7 Comparison to previous model

This section highlights the differences between the previous DRSK model and the new model. Some of the differences are due to changes in the data, while others are due to changes to the methodology.

Relative to the previous model, the changes made are as follows:

- We updated the calibration data set for the model. More recent defaults have been added, and data corrections since the last calibration have been included.
- The model was updated to use version 2 of the Bloomberg Industry Classification System (BICS2). This improves the classification accuracy.
- The previous model was is calibrated to all firms meeting the size and liquidity criteria. We found that model performance in the high yield sector was improved by only including bond issuers in the calibration set (see Section 3.2).
- In the new model calibration dataset, we found some factors in the previous model were no longer significant, so they were removed.
- We made several improvements to the calculation of the DDs. We improved the convergence of the algorithm. This enabled us to lower the model's volatility floor. And we now adjust for missing equity prices when computing historical volatility.
- In the previous model, we compute the cost of funds using the weighted average of the cost of debt and the cost of equity. This required information and assumptions about each country's marginal corporate tax rate, inflation rate, government bond yield curve, and market stock indexes. We replaced this with the use of country-level 5-year swap rates. These curves have longer history and are more reliable.
- We implemented a waterfall of fall-back fields to use when primary fields are stale or missing, thus increasing the firm coverage in the new model. A recent coverage test showed that this enabled us to cover more than 2,200 additional firms.

Overall these changes improved model performance and stability:

- The adjusted pseudo R-squared of each sub-model has improved (Table 5). The Fin_GLB sub-model experienced the largest improvement. Its adjusted R-squared increased from 11% to 27%.
- The ARs for the Fin_GLB model have likewise improved, increasing from 85% to 92% (Table 5). The ARs for the other sub-models are unchanged.
- The new model more closely tracks historical default rates over time (Figure 12). This is most evident in its comparison to the default rate of the high yield index. The old model produced an average default rate significantly lower than the index's historical default rate.

The new model's average default rate is much more in alignment with history. This is largely attributed to the increase in default rates for the NonFin_GLB sub-model, which now better accounts for unreported defaults.

- The NonFin_GLB sub-model generally increases default rates on a cross-sectional basis as well. A scatter plot of the old versus new credit grades for a particular date (Figure 13, for November 2018), shows that the NonFin_GLB model generally ranks firms as riskier on that date. The impact on the other sub-models is more subtle. The new Fin_GLB sub-model is more bi-modal, with high grade credits improving, while lower grade credits tended to remain in place. The new Fin_NA sub-model sees high grade credits as slightly riskier.

Table 5: Adjusted pseudo R-squared and AR - previous vs. new. The adjusted pseudo R-squared and AR for the previous model and new model are shown for the cumulative one year DP. The table indicates that the new model outperforms the previous model.

	Adjusted pseudo R-squared		AR	
	Previous	New	Previous	New
Fin_NA	0.40	0.41	0.97	0.97
Fin_GLB	0.11	0.27	0.85	0.92
NonFin_GLB	0.31	0.34	0.92	0.92

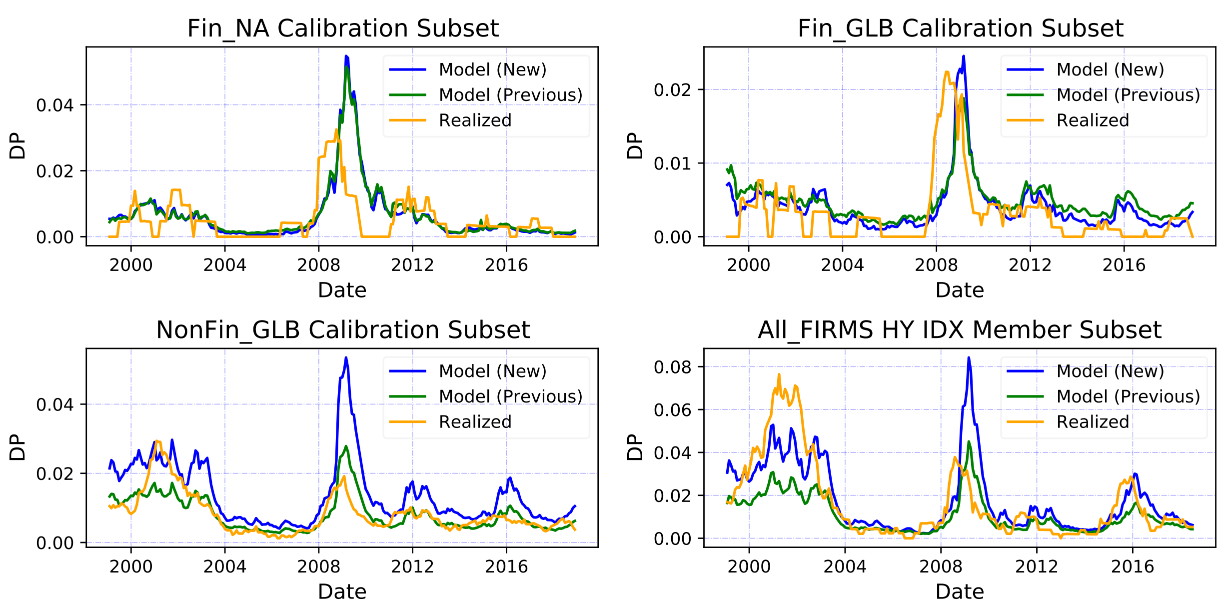

Figure 12: DP time series - previous vs. new model. We plot the old and new model DPs over time along with the realized default rates. The NonFin_GLB sub-model now better accounts for unreported defaults, which in turn brings up the overall model performance on the high yield index.

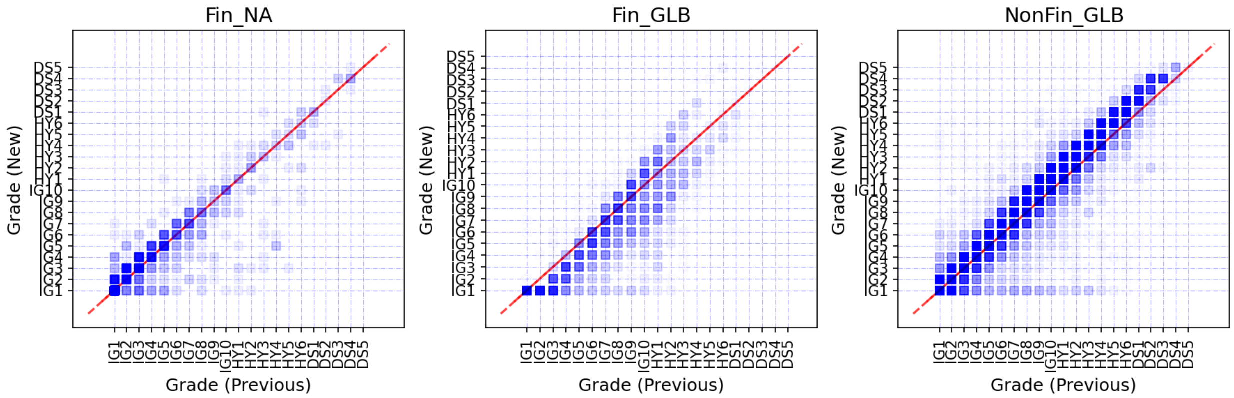

Figure 13: DP scatter plot - previous vs. new model. The scatter plots illustrate the changes in credit grades in November 2018. NonFin_GLB is significantly riskier in the new model, as are higher quality credits in Fin_GLB.

8 Summary

The public corporate DRSK model is a hybrid credit risk model that provides forward-looking, point in time estimates of default probabilities for public firms. The model is calibrated to historical financials over a 20 year period containing records for over 65,000 firms.

The model achieves high performance levels in terms of adjusted pseudo R-squared (e.g. between 34% and 47%), and AR (between 92% and 97%). The model DP is predictive of credit events up to a horizon of five years, and tracks the realized default rates closely over time. The DP distribution follows our expectations. The model is responsive to market conditions; default probabilities drop during economic expansions and rise during economic contractions. The model coefficients are robust and stable over time and in cross-section. Our results hold in the HY index subset. We showed that the model DP accurately tracks historical default rates for the HY index.

Relative to the previous model, this model release includes a number of innovations which includes changes to the data, changes to the model, and changes to the calculation methodology. These changes improved the model performance in terms of adjusted pseudo R-squared, AR, and tracking historical realized default rates. The overall impact of these changes is reflected in higher new model DPs, as compared to the previous model DPs, both in time-series and in cross-section. This pattern is more pronounced for firms in the NonFin_GLB sector, which accounts for about 82% of the population. We also increased coverage by about 2,200 additional firms, which was made possible through the use of data enhancements and fall-backs.

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