

Modelling the Long-Term Dynamics of Yield Curves

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September 28, 2024

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Motivation

Motivation

- Linear regression and factor models analyze equity returns based on specific factors, requiring explicit selection and testing for statistical significance.
- Dimensionality reduction techniques manage high-dimensional data by capturing significant variation while reducing variables.
- Unsupervised learning methods are valuable for analyzing unlabeled data, balancing interpretability and dimensionality.
- Residuals provide additional insights into relative value opportunities in fixed income markets.
- Understanding how yield curve dynamics change across regimes and the role of regime-dependent factors in forecasting yield curve movements is crucial.

Objectives

Primary Objective:

To identify and model the distributions of both raw yield and yield returns across different market regimes.

Secondary Objectives:

① Regime-Switching PCA Implementation:

- Integrate regime-switching Principal Component Analysis (PCA) / Probabilistic PCA (PPCA) with a Hidden Markov Model (HMM) framework.

② Identification of Market Regimes Using HMM Variations:

- Standard HMM
- Restricted HMM
- Input-Output HMM (IOHMM)
- Autoregressive dynamics PCA (AR-PCA)
- VAE based PCA

③ Impact of Bank of England (BoE) announcement Days on Yield Curve Dynamics

④ Analysis of Residuals

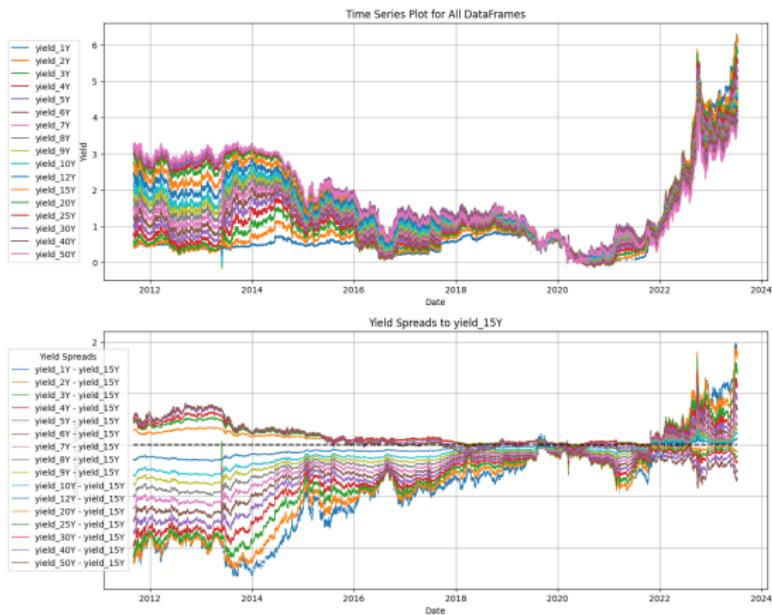
Data

SONIA Benchmark Data Details

- **Source:** Bank of England (BoE)
- **Sample Period:** September 2011 to November 2023
- **Number of Observations:** 26,839
- **Tenors:** 17 different tenors
- **Data Frequency:** 10-minute intervals
- **Processing:** Later undersampled One-Hour to have consistent ticks
- **Exogenous Information:** S&P Futures
- **Analysis Focus:**
 - Raw Yield
 - Hourly Yield returns
 - Weekly Yield returns to analyse long-term behaviour

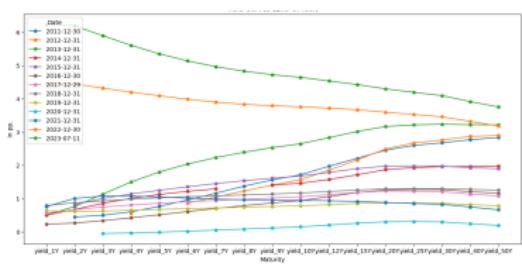
Experiments & Results

Results

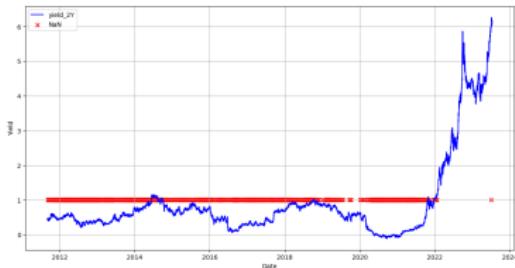


(a,b) Time Series Plot of SONIA (GBP) Yields

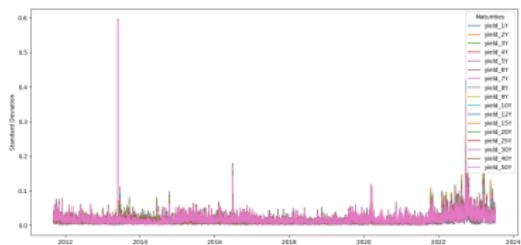
Results



Year-End Yield Curves (2011-23)

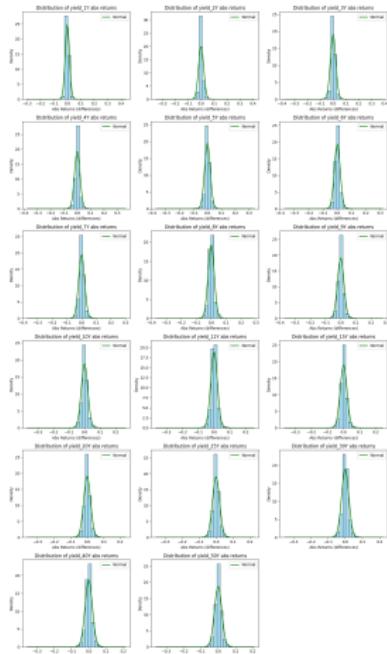


2y Tenor NaN values marked



Volatility of Tenors

Results



Absolute returns of yields are closer to a Normal distribution but with fat tails

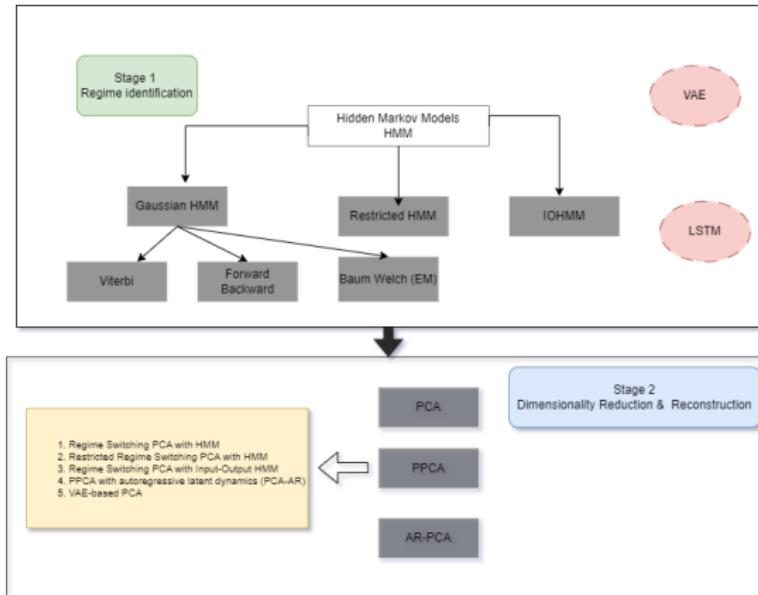
Correlation of Yield Returns across Tenors

	yield_1Y	yield_2Y	yield_3Y	yield_4Y	yield_5Y	yield_10Y	yield_12Y	yield_15Y	yield_20Y	yield_30Y	yield_40Y	yield_50Y																		
yield_1Y	1	0.95	0.9	0.87	0.81	0.77	0.73	0.69	0.65	0.62	0.6	0.58	0.56	0.51	0.53	0.52	0.51													
yield_2Y		1	0.98	0.96	0.93	0.87	0.83	0.79	0.76	0.72	0.7	0.68	0.66	0.64	0.62	0.62	0.61													
yield_3Y			1	0.98	0.94	0.92	0.88	0.85	0.82	0.78	0.77	0.75	0.72	0.7	0.69	0.68	0.67													
yield_4Y				1	0.98	0.97	0.94	0.92	0.89	0.86	0.85	0.83	0.8	0.79	0.77	0.75	0.75													
yield_5Y					1	0.91	0.94	0.99	1	0.99	0.98	0.96	0.94	0.92	0.9	0.89	0.86	0.83	0.82	0.81										
yield_10Y						1	0.87	0.92	0.97	0.9	1	0.99	0.98	0.97	0.95	0.94	0.93	0.91	0.89	0.87	0.86	0.85								
yield_12Y							1	0.83	0.88	0.94	0.99	1	1	0.99	0.98	0.97	0.96	0.94	0.92	0.9	0.89	0.89								
yield_15Y								1	0.85	0.83	0.92	0.96	0.98	1	1	0.99	0.98	0.97	0.96	0.94	0.92	0.91	0.91							
yield_20Y									1	0.86	0.76	0.82	0.89	0.97	0.99	1	1	0.99	0.98	0.97	0.95	0.94	0.93	0.92	0.92					
yield_30Y										1	0.62	0.64	0.65	0.63	0.66	0.68	0.69	1	1	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.93			
yield_40Y											1	0.66	0.66	0.72	0.68	0.69	0.72	0.74	0.76	1	1	0.99	0.99	0.98	0.97	0.96	0.95	0.94	0.94	
yield_50Y												1	0.61	0.61	0.67	0.75	0.61	0.65	0.68	0.76	0.79	1	1	1	1	1	1	1	1	1

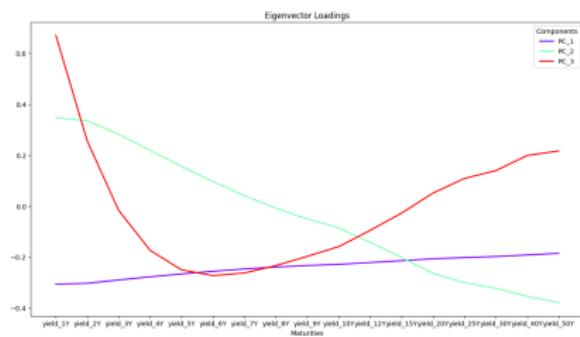
Correlation Heatmap of Yield Returns across Tenors



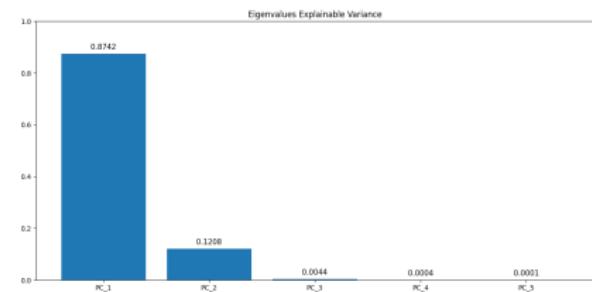
Model Zoo



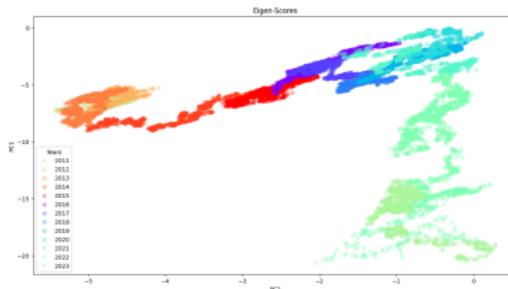
Results: PCA Analysis (Yield)



Principal Components (Yield)

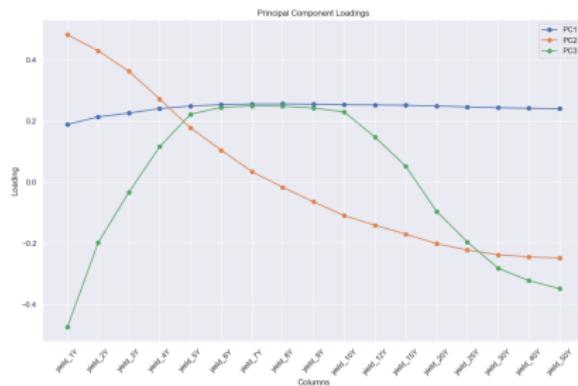


Explained Variance for Each Principal Component (Yield)

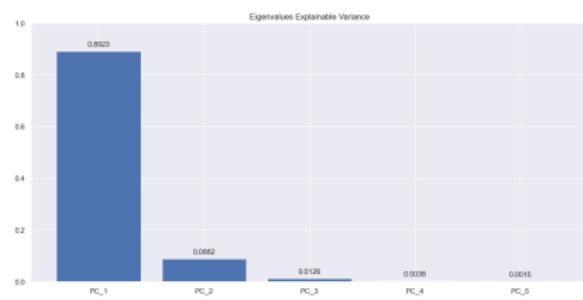


Eigen Scores (Yield)

Results: PCA Analysis (Yield Level)



Principal Components (Yield Level)

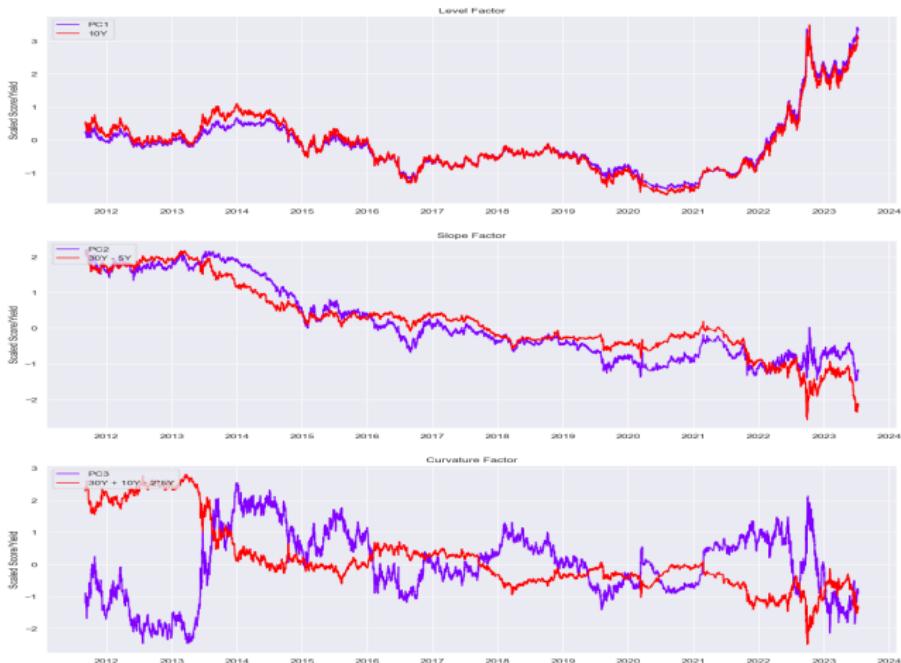


Explained Variance for Each Principal Component (Yield Level)



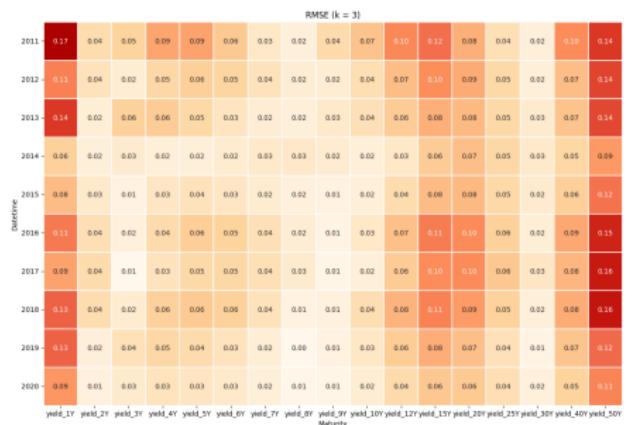
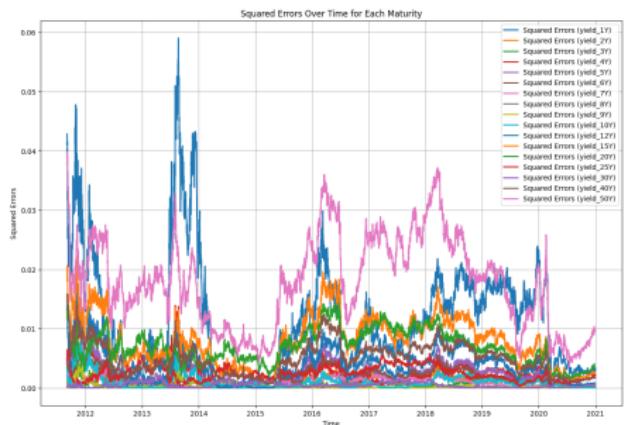
Eigen Scores (Yield Level)

Results



Time series of PCA projections vs. Tradeable counterparts (Level: Outright Trade, Slope: Flattener Trade, Curvature: Butterfly Trade)

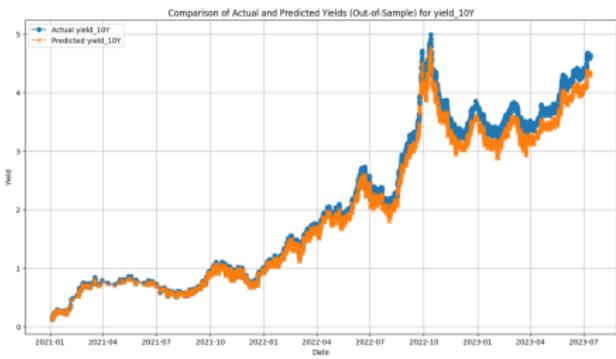
Results



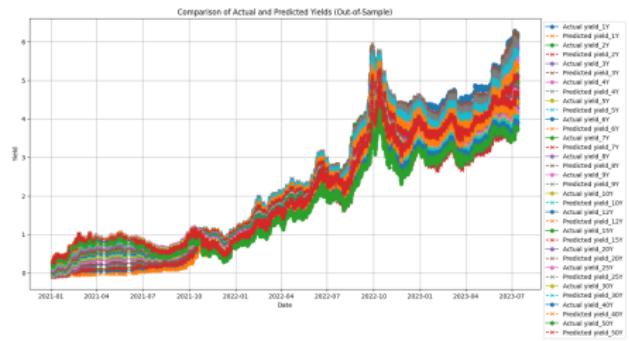
Squared Errors Over Time for Each Tenor

RMSE Heatmap Across Maturities for PCA Model (k=3)

Results

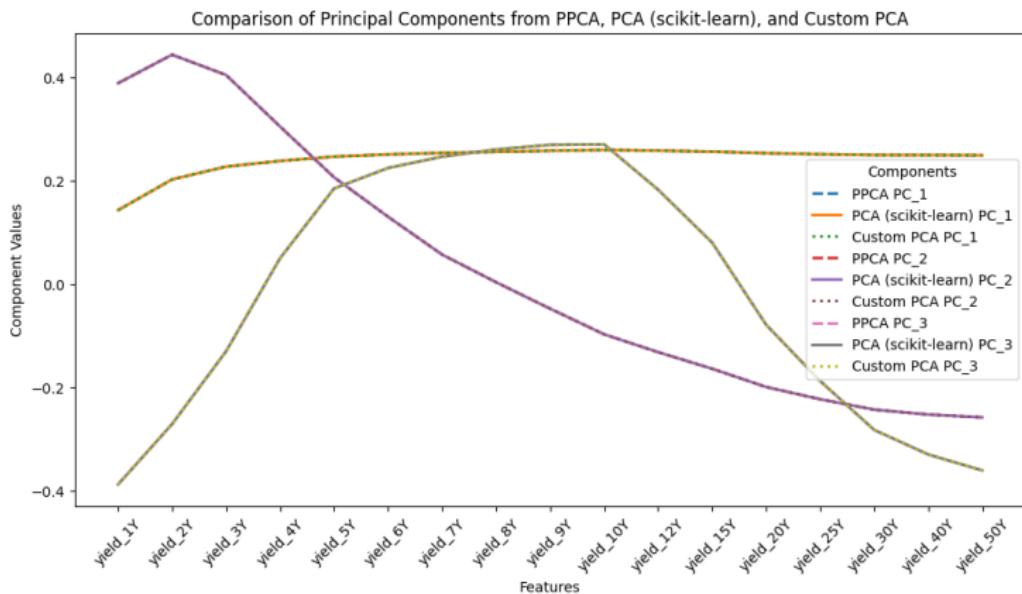


Out-of-Sample Comparison of Actual vs. Predicted (10-Year Yield)



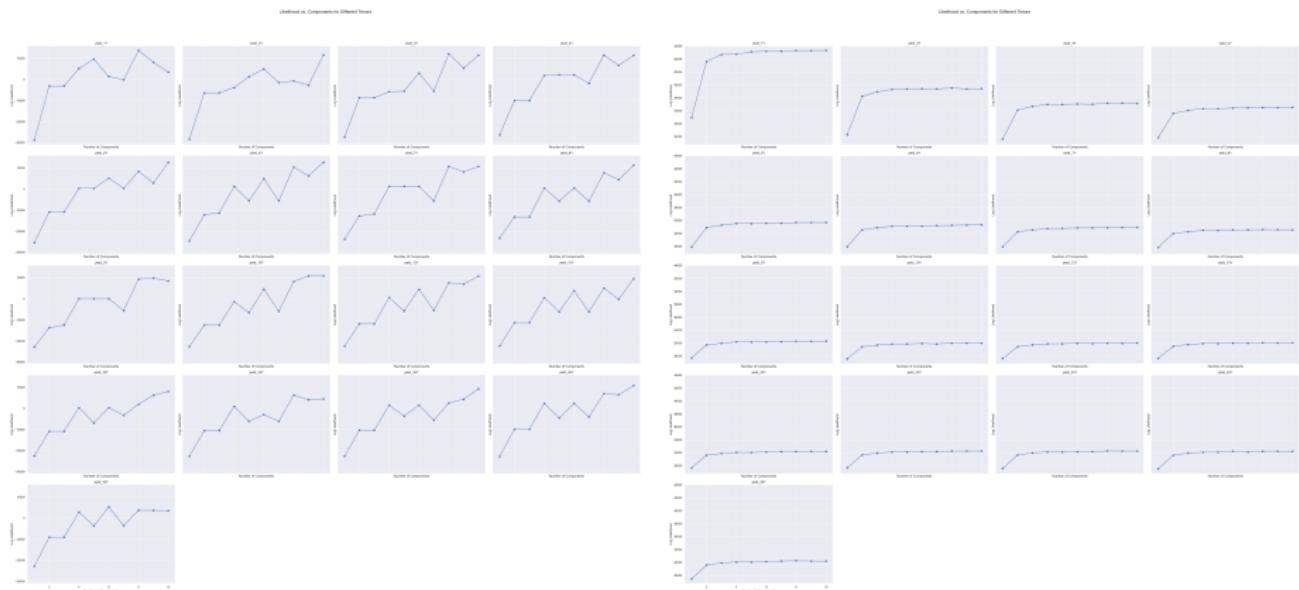
Out-of-Sample Comparison of Actual vs. Predicted (All Tenors)

Results



Comparison of the loadings between PCA and PPCA

Results : Regime Identification



Hyper-parameter selection on number of states from HMM (Yield)

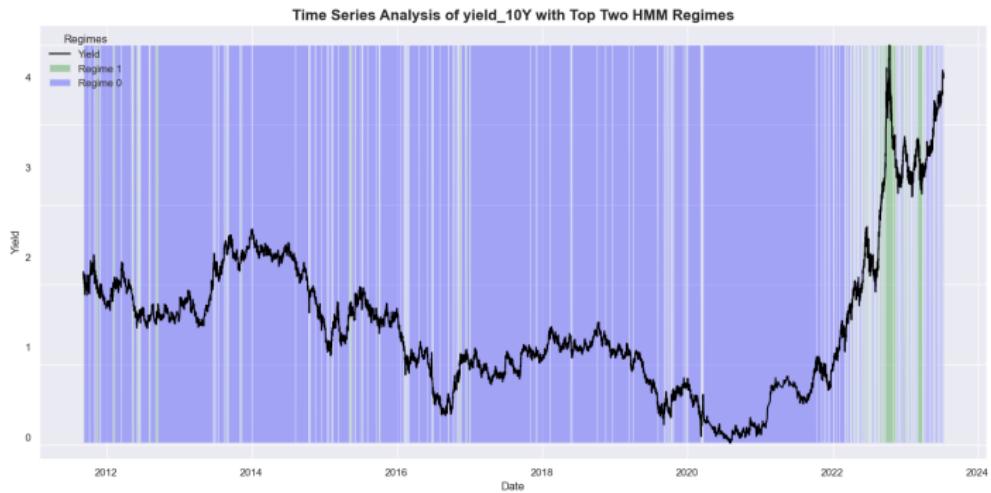
Hyper-parameter selection on number of states from HMM (Yield Level)

Results: Sufficiency of a Two-State Model for Optimal Interpretability and Robustness

Tenor	Unique Regimes Identified	Most Frequent States	Counts of Most Frequent States
yield_1Y	[0 1 2 3 4 5 6 7 8 9]	[6 9]	[1446, 7658]
yield_2Y	[0 1 2 3 4 5 6 7 8 9]	[6 8]	[3755, 4142]
yield_3Y	[0 1 2 3 4 5 6 7 8 9]	[8 3]	[3074, 3886]
yield_4Y	[0 1 2 3 4 5 6 7 8 9]	[2 7]	[1531, 6537]
yield_5Y	[0 1 3 4 5 6 7 8 9]	[7 9]	[4871, 5673]
yield_6Y	[0 1 2 3 4 5 6 7 8 9]	[2 7]	[1659, 6301]
yield_7Y	[0 1 2 3 4 5 6 7 8 9]	[2 9]	[2201, 6214]
yield_8Y	[0 1 2 3 4 5 6 7 8 9]	[2 3]	[1350, 7211]
yield_9Y	[0 1 2 3 4 5 6 7 8 9]	[0 9]	[1773, 7912]
yield_10Y	[0 1 2 3 4 5 6 7 8 9]	[2 9]	[2875, 5116]
yield_12Y	[0 1 2 3 4 5 6 7 8 9]	[8 3]	[1548, 8122]
yield_15Y	[0 1 2 3 4 5 6 7 8 9]	[0 9]	[2593, 6764]
yield_20Y	[0 1 2 3 4 5 6 7 8 9]	[2 9]	[2126, 6553]
yield_25Y	[0 1 2 3 4 5 6 7 8 9]	[2 9]	[1529, 6889]
yield_30Y	[0 1 2 3 4 5 6 7 8 9]	[8 9]	[2635, 3964]
yield_40Y	[0 1 2 3 4 5 6 7 8 9]	[2 3]	[2559, 4507]
yield_50Y	[0 1 2 3 4 5 6 7 8 9]	[2 7]	[1991, 4966]

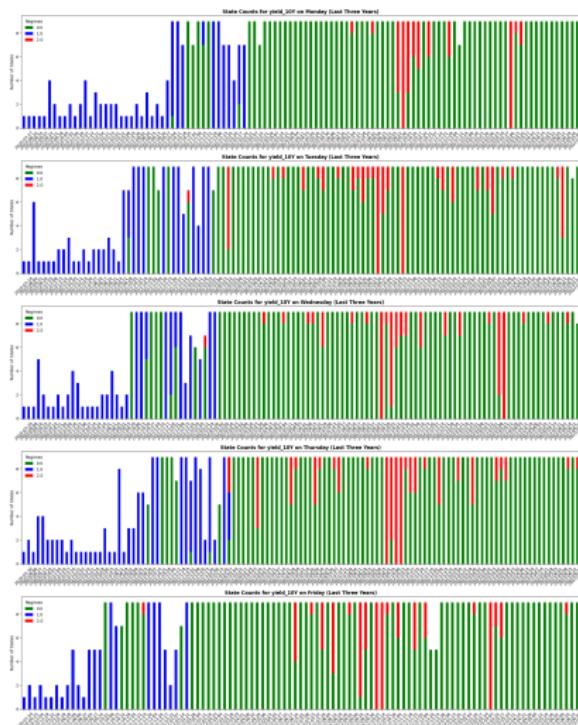
Experiment 1 : Most Frequent States and Counts for Each Tenor

Results: Sufficiency of a Two-State Model for Optimal Interpretability and Robustness



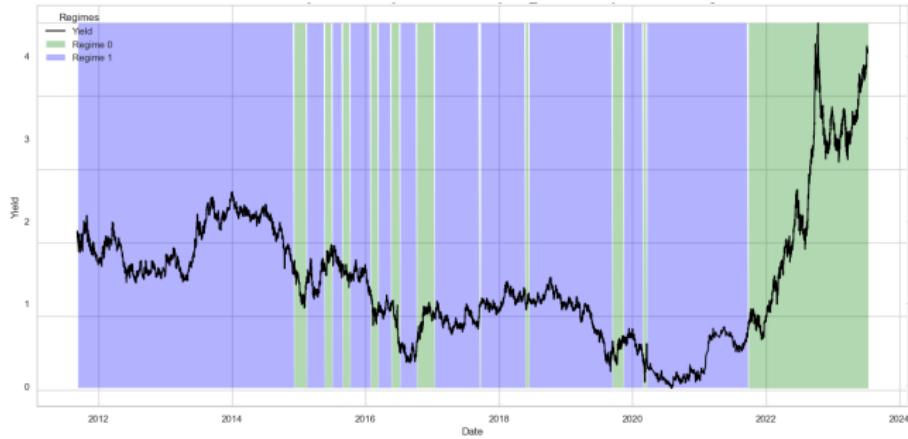
Experiment 2 : Regime identified using standard HMM on hourly returns of a 10Y Tenor

Results: Sufficiency of a Two-State Model for Optimal Interpretability and Robustness



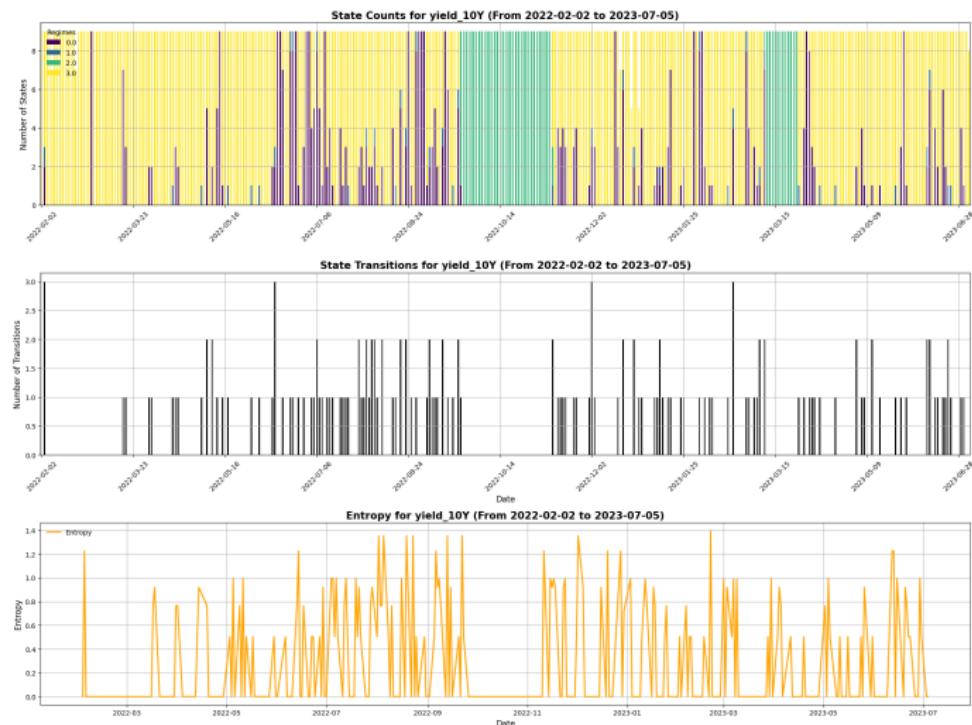
State Counts of 10Y Tenor on different Weekdays

Results: Sufficiency of a Two-State Model for Optimal Interpretability and Robustness



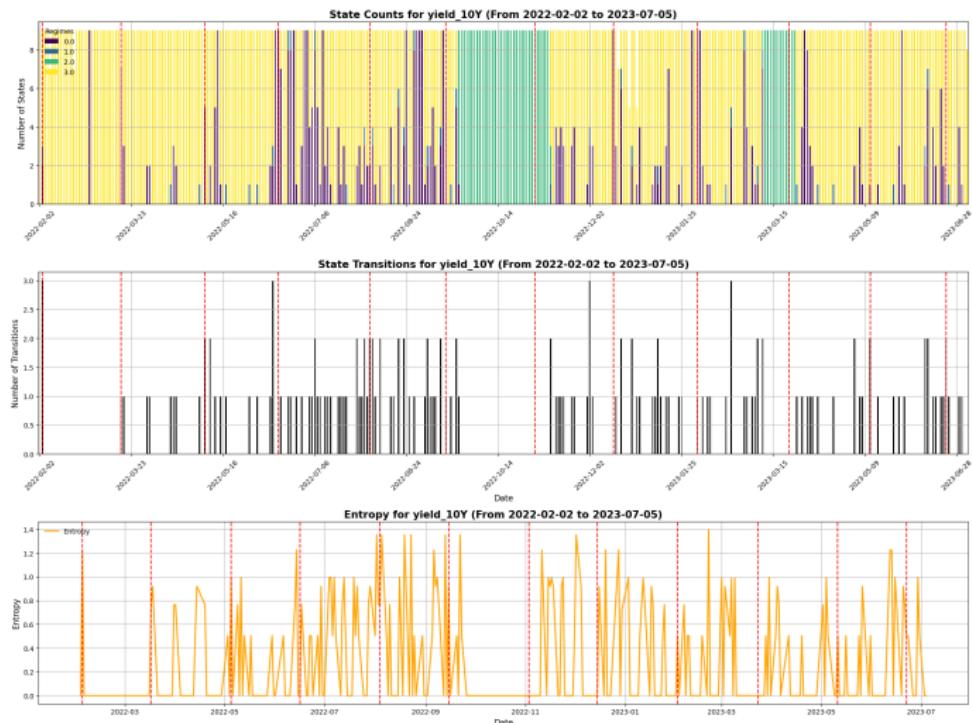
Regime identified using standard HMM on weekly abs returns of a 10Y Tenor

Results



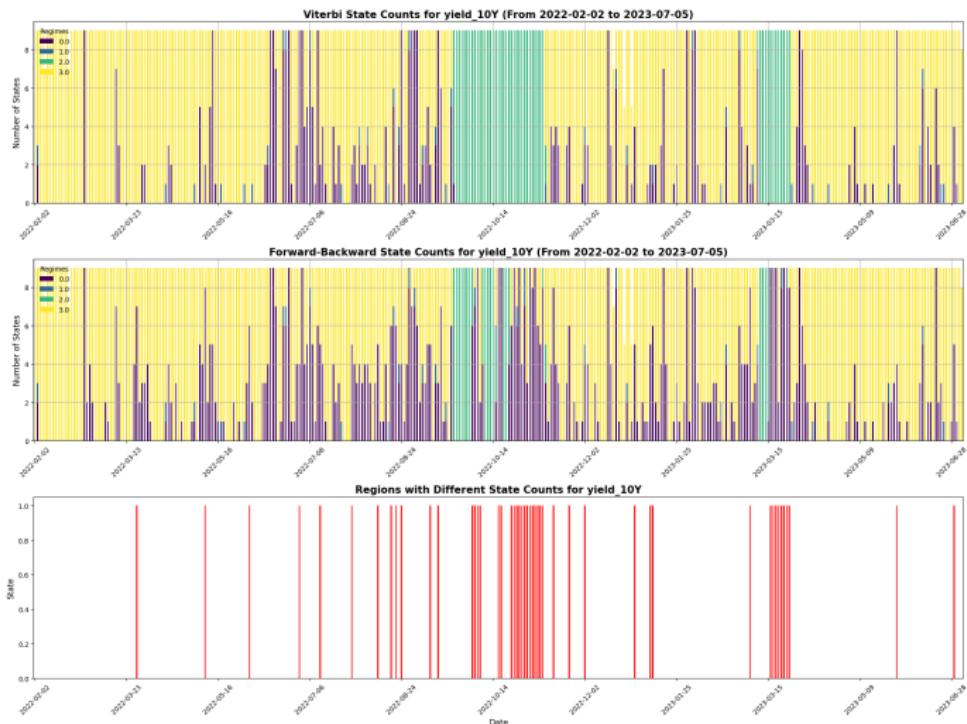
State Counts vs. Transitions for Yield 10Y on Thursdays Over the Last Three Years

Results : BoE MPC Announcements



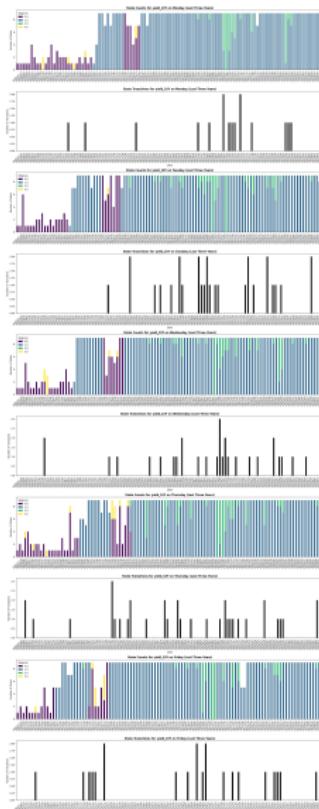
State Counts, Transitions, and Entropy for Yield 10Y Tenor with BoE MPC Announcements Marked

Results :Viterbi vs. Forward-Backward Algorithm

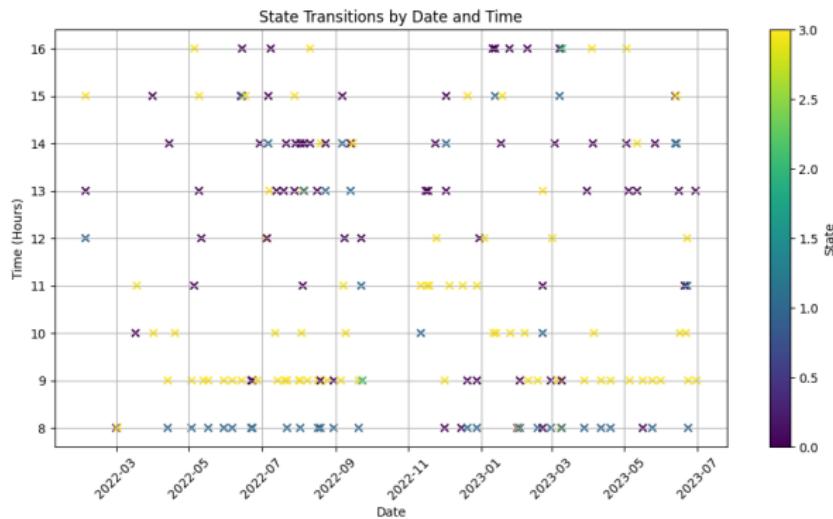


Viterbi vs. Forward-Backward Algorithm

Results : Weekday Effect

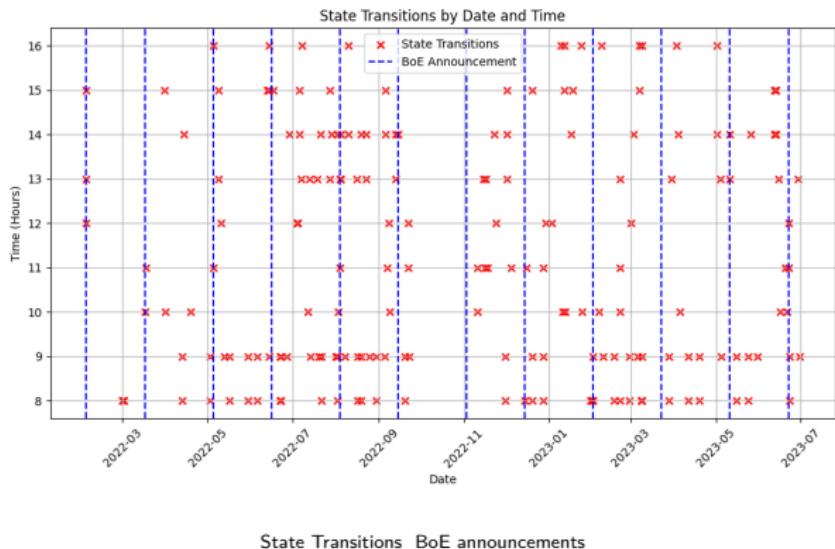


Results

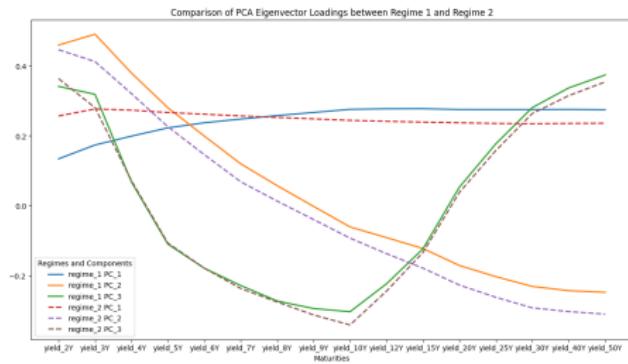
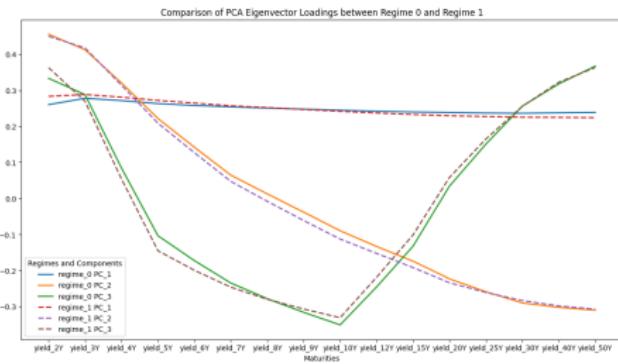


State Transitions around Every Hour

Results



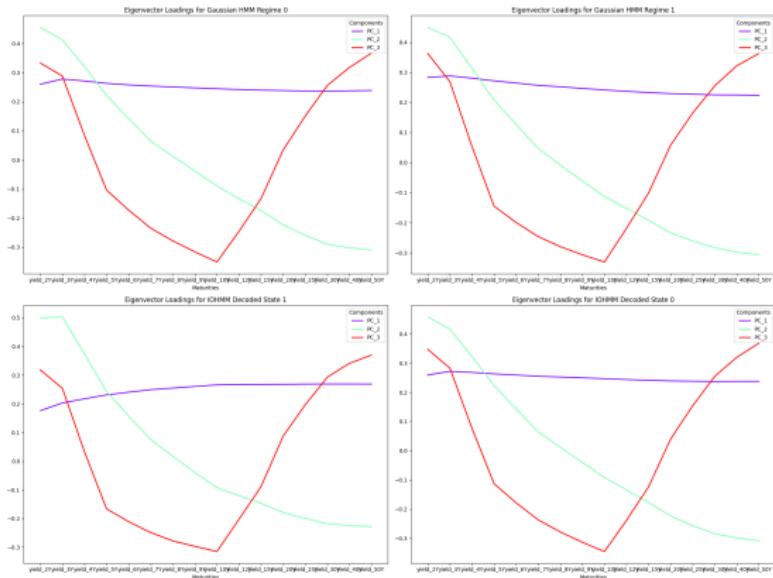
Results :Standard Vs. Restricted Regime



Standard Regime dependent PCAs

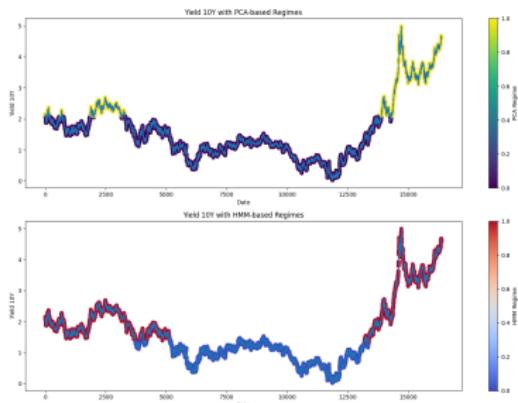
Restricted Regime dependent PCAs

Results: Gaussian HMM vs. IOHMM

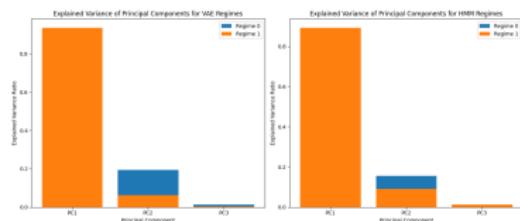


Gaussian HMM vs. IOHMM

Results: VAE Dependent PCA

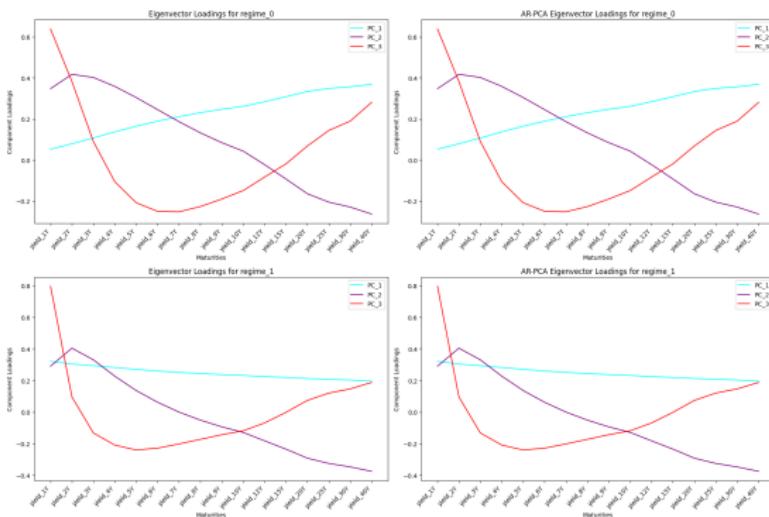


VAE Regime Detection



Explained Variance

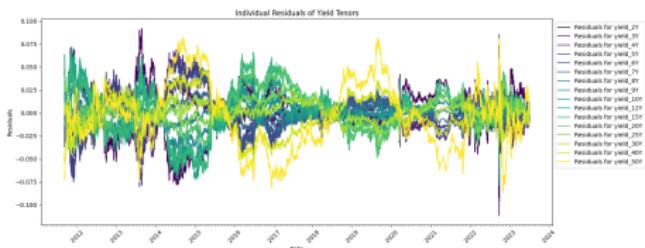
Results: AR-PCA



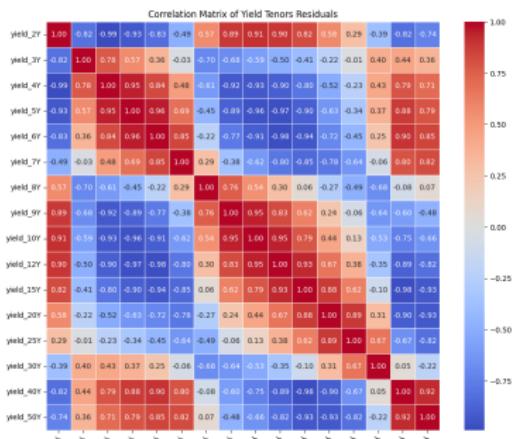
PCA with autoregressive latent dynamics (AR-PCA)

Left: Standard PCA, Right: AR-PCA

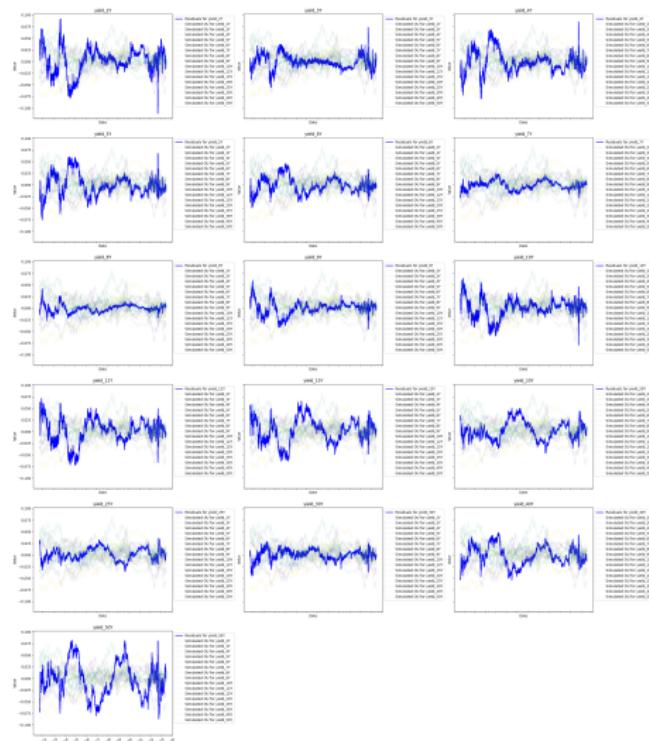
Results: Residual Analysis



Time series of Residuals (Baseline PCA)

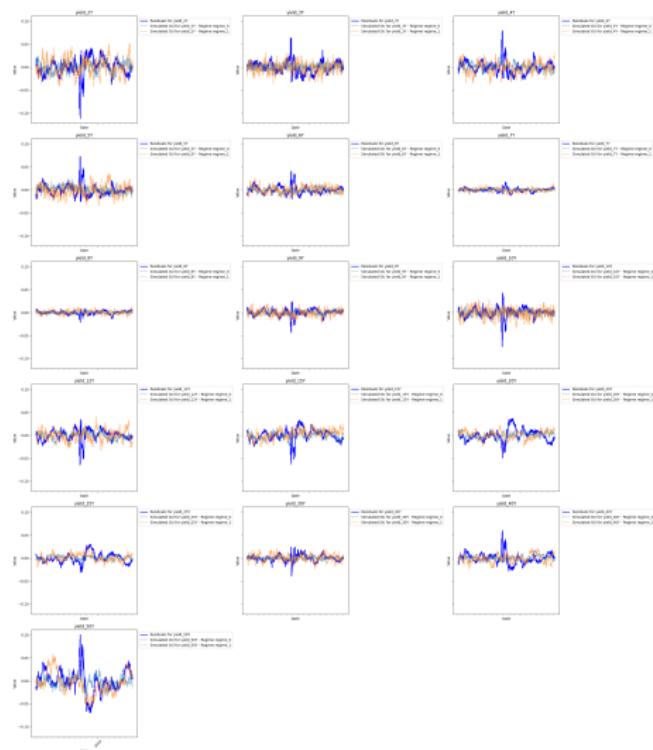


Results: Residual Analysis



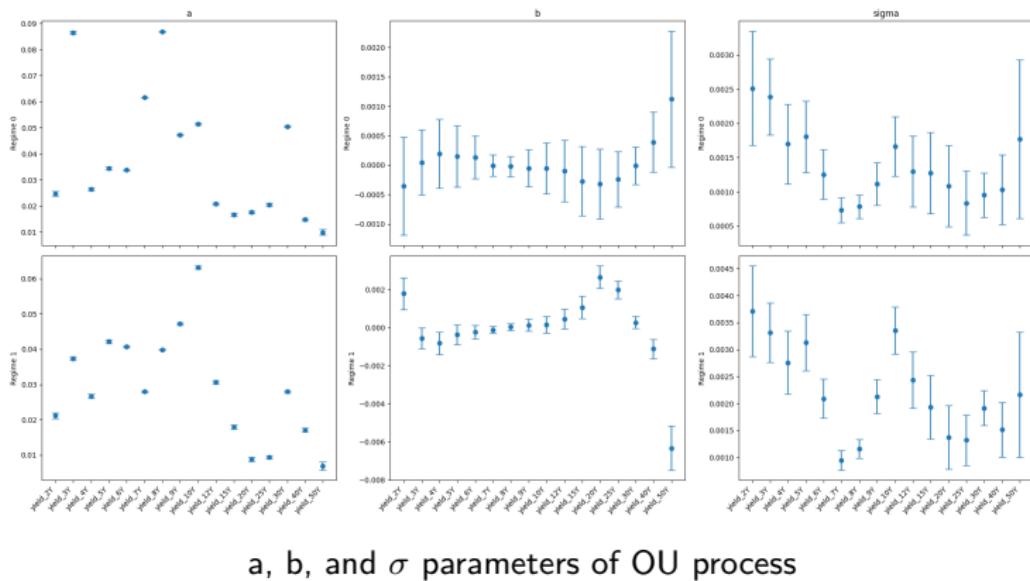
Simulated paths of a calibrated OU process

Results: Residual Analysis



Comparison of Simulated OU Process Paths and Residual Behaviors across Different Regimes

Results: Residual Analysis



The plot shows the parameter estimates for the Ornstein-Uhlenbeck (OU) process fitted to the residuals derived from a Regime Switching Hidden Markov Model (HMM) PCA model. Confidence intervals at 99% level are included to indicate the reliability of the parameter estimates.

Thank you!