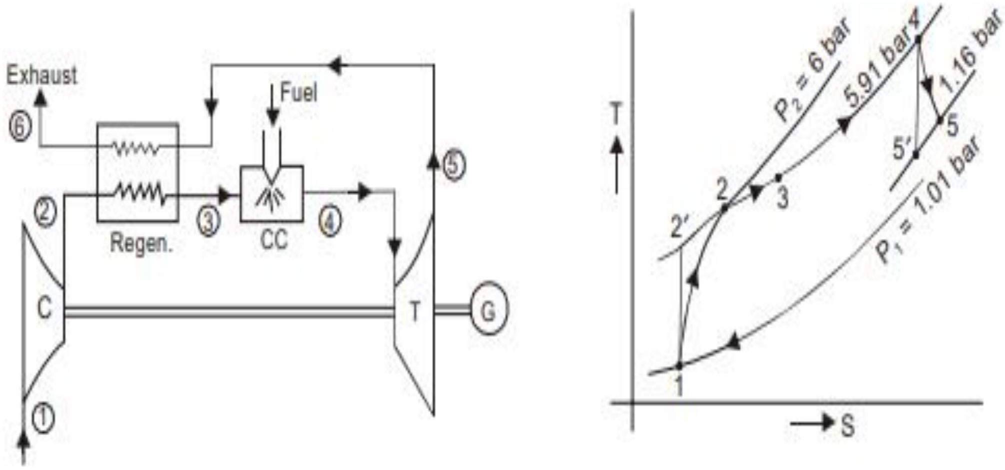
UNIT:-3 GAS TURBINE POWER PLANT NUMERICAL

SOLVED EXAMPLES

Example 1. *A gas turbine plant of 800 kW capacities takes the air at 1.01 bar and 15°C. The pressure ratio of the cycle is 6 and maximum temperature is limited to 700°C. A regenerator of 75% effectiveness is added in the plant to increase the overall efficiency of the plant. The pressure drop in the combustion chamber is 0.15 bars as well as in the regenerator is also 0.15 bars. Assuming the isentropic efficiency of the compressor 80% and of the turbine 85%, determine the plant thermal efficiency. Neglect the mass of the fuel.*

Solution. The arrangement of the components is shown in Fig. 9.30(*a*) and the processes are represented on T-s diagram as shown in Fig. 9.30(*b*).



(*a*) (*b*)

Fig. 9.30

The given data is

$$T_1 = 15 + 273 = 288 \text{ K}$$

$$p_1 = 1.01 \text{ bar}$$

$$p_2 = 1.01 \times 6 = 6.06 \text{ bar}$$

$$R_p = \frac{P_2}{p_1} = 6$$

Pressure at point 4 = 6.06 – 0.15 = 5.91 bar

Applying isentropic law to the process 1 – 2

$$T_2' = T_1(R_P)^{(\gamma-1)/\gamma} = 288(6)^{0.286} = 480 \text{ K}$$

$$\eta_c = \frac{(T_{2'} - T_1)}{(T_2 - T_1)}$$

But $$T_2 = T_1 + \eta_c(T_{2'} - T_1) = 288 + 0.8(480 - 288) = 528 \text{ K}$$

$$p_3 = 6.06 - 0.15 = 5.91 \text{ bar}$$

and $$p_4 = 1.01 + 0.15 = 1.16 \text{ bar}$$

Applying isentropic law to the process 4 – 5′

$$T_{5'} = \frac{T_4}{\left[\left(\frac{P_3}{P_4}\right)^{(\gamma-1)/\gamma}\right]} = \frac{(700+273)}{\left[\left(\frac{5.91}{1.16}\right)^{0.286}\right]} = 612 \text{ K}$$

$$\eta_t = \frac{(T_4 - T_5)}{(T_4 - T_{5'})}$$

or, $$T_5 = T_4 - \eta_t(T_4 - T_{5'})$$

$$= 973 - 0.85(973 - 612) = 666 \text{ K}$$

The effectiveness of the regenerator is given by

$$\varepsilon = \frac{(T_4 - T_5)}{(T_4 - T_5)}$$

$$T_3 = T_2 + 0.75\,(T_5 - T_2) = 528 + 0.75(666 - 528) = 631.5 \text{ kW}$$

$$W_c = C_p(T_2 - T_1) = 1 \times (528 - 288) = 240 \text{ kJ/kg}$$

$$W_t = C_p(T_4 - T_5) = 1 \times (973 - 666) = 307 \text{ kJ/kg}$$

$$W_n = W_t - W_c = 307 - 240 = 67 \text{ kJ/kg}$$

$$Q_S = C_p(T_4 - T_3) = 1 \times (973 - 631.5) = 341.5 \text{ kJ/kg}$$

$$\eta_{th} = \frac{W_n}{Q_s} = \frac{67}{341.5} = 0.196 = 19.6\%.$$

Example 2. *In a constant pressure open cycle gas turbine air enters at 1 bar and 20°C and leaves the compressor at 5 bar. Using the following data; Temperature of gases entering the turbine = 680°C, pressure loss in the combustion chamber = 0.1 bar, $\eta_{compressor}$ = 85%, $\eta_{turbine}$ = 80%, $\eta_{combustion}$ = 85%, γ = 1.4 and c_p = 1.024 kJ/kgK for air and gas, find:*

(1) The quantity of air circulation if the plant develops 1065 kW.

(2) Heat supplied per hg of air circulation.

(3) The thermal efficiency of the cycle. Mass of the fuel may be neglected.

Solution. $P_1 = 1$ bar

$P_2 = 5$ bar

$P_3 = 5 - 0.1 = 4.9$ bar

$P_4 = 1$ bar

$T_1 = 20 + 273 = 293$ K

$T_3 = 680 + 273 = 953$ K

$\eta_{compressor} = 85\%$

$\eta_{turbine} = 80\%$

$\eta_{combustion} = 85\%$

For air and gases: $c_{p'} = 1.024$ kJ/kgK

$y = 1.4$

Power developed by the plant,

P = 1065 kW

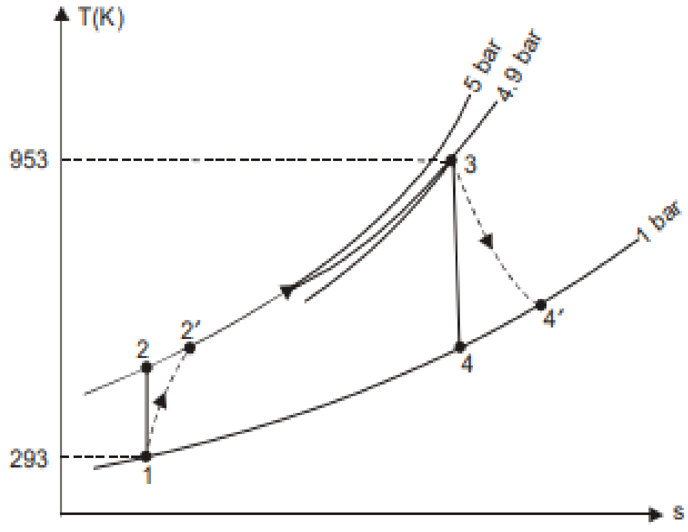


Fig. 9.31

(1) The quantity of air circulation, m_a = ?

For isentropic compression 1 – 2,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} = \left(\frac{5}{1}\right)^{(1.4-1)/1.4} = 1.584$$

$$T_2 = 293 \times 1.584 = 464 \text{ K}$$

Now,

$$\eta_{compressor} = \frac{(T_2 - T_1)}{(T_{2'} - T_1)} = 0.85$$

$$0.85 = \frac{(464 - 293)}{(T_{2'} - 293)}$$

$$T_{2'} = 494 \text{ K}$$

For isentropic expansion process 3 – 4,

$$\frac{T_4}{T_3} = \left(\left(\frac{P_4}{P_3}\right)^{(\gamma-1)/\gamma}\right) = \left(\left(\frac{1}{4.9}\right)^{(1.4-1)/1.4}\right) = 0.635$$

$$T_4 = 953 \times 0.635 = 605 \text{ K}$$

Now, $$\eta_{turbine} = \frac{(T_3 - T_{4'})}{(T_3 - T_4)} = 0.80$$

$$\frac{0.85}{0.80} = \frac{(953 - T_{4'})}{(953 - 605)}$$

$$T_{4'} = 674.6 \text{ K}$$

$$W_{compressor} = C_p(T_{2'} - T_1) = 1.024(494 - 293) = 205.8 \text{ kJ/kg}$$

$$W_{turbine} = C_p(T_3 - T_{4'}) = 1.024(953 - 674.6) = 285.1 \text{ kJ/kg}$$

$$W_{net} = W_{turbine} - W_{compressor}$$

$$= 285.1 - 205.8 = 79.3 \text{ kJ/kg of air}$$

If the mass of air flowing is m_a kg/s,

the power developed by the plant is given by $P = m_a \times W_{net}$ kW

$$1065 = m_a \times 79.3$$

$$m_a = \frac{1065}{13.43} \text{ kg}$$

i.e., Quantity of air circulation = **13.43 kg.**

(2) Heat supplied per kg of air circulation = ?

Actual heat supplied per kg of air circulation

$$= \frac{c_p\,(T_3 - T_{2'})}{\eta_{combustion}} = \frac{1.024(953 - 494)}{0.85} = \mathbf{552.9\ kJ/kg.}$$

(3) Thermal efficiency of the cycle, $\eta_{thermal}$ = ?

$$\eta_{thermal} = \frac{\text{work output}}{\text{heat supplied}}$$

$$= \frac{79.3}{552.9} = 0.1434 \quad \text{or} \quad \mathbf{14.34\%.}$$

Example 3. *In an open cycle regenerative gas turbine plant, the air enters the compressor at 1 bar abs 32°C and leaves at 6.9 bar abs. The temperature at the end of combustion chamber is 816°C. The isentropic efficiencies of compressor and turbine are respectively 0.84 and 0.85. Combustion efficiency is 90% and the regenerator effectiveness is 60 percent, determine:*

(a) Thermal efficiency, (b) Air rate, (c) Work ratio.

Solution. $P_1 = 1.0$ bar,

$T_1 = 273 + 32 = 305$ K

$P_2 = P_{2a} = 6.9$ bar

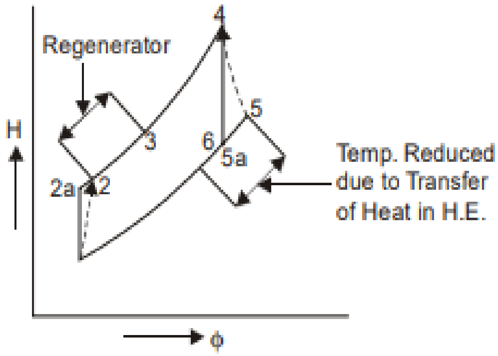
$T_4 = 816 + 273 = 1089$ K

$$\frac{T_{2a}}{T_1} = \left(\left(\frac{P_{2a}}{P_1}\right)^{(\gamma-1)/\gamma}\right)$$

$$= \left(\left(\frac{6.9}{1.0}\right)^{(1.4-1)/1.4}\right) = 1.736$$

$$T_{2a} = 1.736 \times 305 = 529.4 \text{ K}$$

Now, $\eta_{compressor} = \dfrac{(T_{2a} - T_1)}{(T_2 - T_1)} = 0.84$



Regeneration cycle

Fig. 9.32

$$0.84 = \frac{(529.4 - 305)}{(T_2 - 305)}$$

$$T_2 = 572.2\text{ K}$$

Again $\dfrac{T_4}{T_{5a}} = 1.736$

$$T_{5a} = \frac{1089}{1.736} = 627.3\text{ K}$$

Now, $\eta_{turbine} = \dfrac{(T_4 - T_5)}{(T_4 - T_{5a})} = 0.85$

$$T_4 - T_5 = 0.85(1089 - 627.3) = 392.4$$

$$T_5 = 1089 - 392.4 = 696.6\text{ K}$$

$$0.84 = \frac{(529.4 - 305)}{(T_2 - 305)}$$

$$T_2 = 572.2\text{ K}$$

Again $\dfrac{T_4}{T_{5a}} = 1.736$

$$T_{5a} = \frac{1089}{1.736} = 627.3\text{ K}$$

Now,

Regenerator efficiency $\eta_{rg} = \dfrac{(T_3 - T_2)}{(T_5 - T_2)}$

$$T_3 - T_2 = 0.6 \times (696.6 - 572.2) = 74.65$$

$$T_3 = 572.2 + 74.65 = 646.85\text{ K}$$

(*a*) Thermal efficiency

$$\eta_t = \frac{\text{Useful workdone}}{\text{Heat supplied}} = \frac{[C_p\,(T_4 - T_5) - C_p(T_2 - T_1)]}{\left[\dfrac{C_p(T_4 - T_3)}{\eta_c}\right]}$$

$$\eta_t = \frac{(392.4 - 267.2)}{\left[\dfrac{(1089 - 646.85)}{0.90}\right]} = 25.48\ \%$$

(*b*) Air rate AR = $\dfrac{3600}{\text{Useful work in kW/kg}}$

$$= \frac{3600}{(1.005 \times 125.4)} = 28.56 \text{ kg/kW-hr}$$

(*c*) Work ratio = $\dfrac{\text{Useful work}}{\text{Turbine work}} = \dfrac{(1.005 \times 125.2)}{(1.005 \times 392.4)}$ = **0.32.**

Example 4. *A gas turbine power plant is operated between 1 bar and 9 bar pressures and minimum and maximum cycle temperatures are 25°C and 1250°C. Compression is carried out in two stages with perfect intercooling. The gases coming out from HP. turbine are heated to 1250°C before entering into L.P. turbine. The expansions in both turbines are arranged in such a way that each stage develops same power. Assuming compressors and turbines isentropic efficiencies as 83%,*

(1) determine the cycle efficiency assuming ideal regenerator. Neglect the mass of fuel.

(2) Find the power developed by the cycle in kW if the airflow through the power plant is 16.5 kg/sec.

Solution. The arrangement of the components and the processes are shown in Fig. 9.33(*a* and *b*). The given data is

$T_1 = 25 + 273 = 298 \text{ K} = T_3$ (as it is perfect intercooling),

$p_1 = 1$ bar and $p_3 = 9$ bar

$p_2 = \sqrt{p_1 p_3} = \sqrt{(1 \times 9)} = 3$ bar

$R_{P1} = R_{p2} = 3$

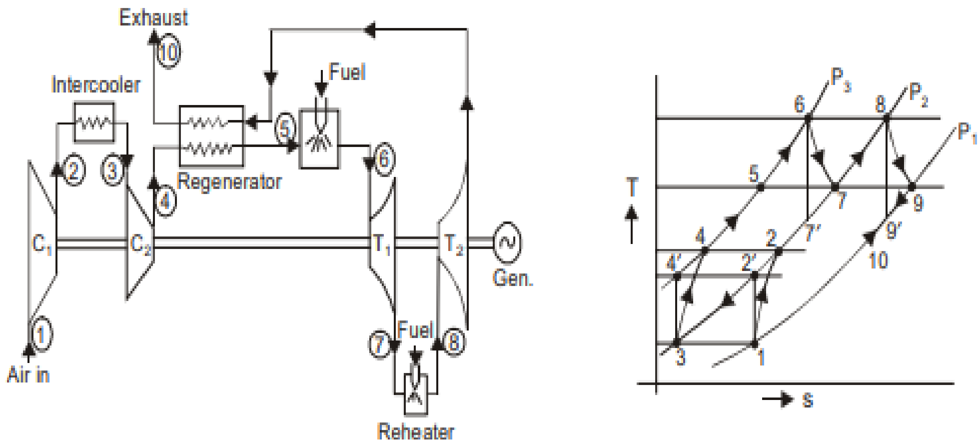
$\eta_{c1} = \eta_{c2} = \eta_{t1} = \eta_{t2} = 0.83,$

$T_6 = T_8 = 1250 + 273 = 1523 \text{ K}$

$T_{10} = T_5$ (as perfect regenerator is given)

Applying isentropic law to the process 1 – 2′

$$T_{2'} = T_1 \left(\frac{P_2}{P_1} \right)^{(\gamma-1)/\gamma} = 298(3)^{0.286} = 408 \text{ K}$$



(*a*) (*b*)

Fig. 9.33

$$\eta_{c1} = \frac{(T_{2'} - T_1)}{(T_2 - T_1)}$$

$$T_2 = \frac{T_1 + (T_{2'} - T_1)}{\eta_{c1}}$$

$$= \frac{298 + (408 - 298)}{0.83} = 430.5 \text{ K}$$

$$T_4 = T_2 = 430.5 \text{ K}$$

Applying isentropic law to the process 6 – 7′

$$\frac{T_6}{T_{7'}} = \left(\frac{P_3}{P_2}\right)^{(\gamma-1)/\gamma} = (3)^{0.286} = 1.37 \text{ K}$$

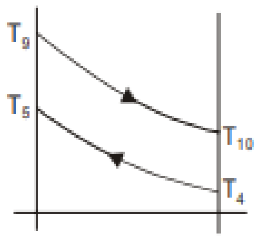


Fig. 9.34

$$T_{7'} = \frac{1523}{1.37} = 1111 \text{ K}$$

$$\eta_{t1} = \frac{(T_6 - T_7)}{(T_6 - T_{7'})}$$

$$T_7 = T_6 - \eta_{t1}(T_6 - T_{7'})$$
$$= 1523 - 0.83(1523 - 1111) = 1181 \text{ K}$$

$T_9 = T_7 = 1181$ K (as equal work is developed by each turbine)

$$W_c = 2C_{Pa}(T_2 - T_1) = 2 \times 1(430.5 - 298) = 266 \text{ kJ/kg}$$

$$W_t = 2C_{Pa}(T_6 - T_7) = 2 \times 1(1523 - 1181) = 687.5 \text{ kJ/kg}$$

$$W_n = W_t - W_c = 687.5 - 266 = 421.5 \text{ kJ/kg}$$

When the ideal regeneration is given, then

$$\varepsilon = 1 \text{ therefore } T_5 = T_9 = 1181 \text{ K} = T_7$$

$$Q_S \text{ (heat supplied)} = 2C_{pa}(T_6 - T_5)$$
$$= 2 \times 1(1523 - 1181) = 684 \text{ kJ/kg}$$

(1) Thermal $\eta = \dfrac{W_n}{Q_s} = \dfrac{421.5}{684} = 0.615 = 61.5\%$

(2) Power developed by the plant = $W_n \times m = 421.5 \times 16.5 = 6954.75$ kW.

Example 5. *A gas-turbine power plant generates 25 MW of electric power. Air enters the compressor at 10°C and 0.981 bar and leaves at 4.2 bar and gas enters the turbine at 850°C. If the turbine and compressor efficiencies are each 80%, determine*

(1) The temperatures at each point in the cycle

(2) The specific work of the cycle

(3) The specific work of the turbine and the compressor

(4) The thermal efficiencies of the actual and ideal cycle

(5) The required airflow rate.

Solution.

$$T_1 = 273 + 20 = 293 \text{ K}$$

$$T_3 = 273 + 850 = 1123 \text{ K}$$

$$T_{2a} = T_1\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} = 293.(4.28)^{0.2857} = 443.9 \text{ K}$$

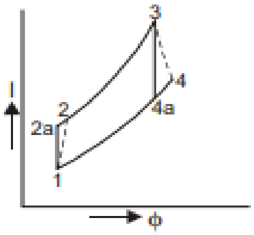


Fig. 9.35

Similarly

$$T_{4a} = \frac{1123}{(4.28)^{0.2857}} = 741.25 \text{ K}$$

Now

$$\eta_{\text{compressor}} = \frac{(T_{2a1} - T_1)}{(T_2 - T_1)}$$

$$\eta_{\text{turbine}} = \frac{(T_3 - T_4)}{(T_3 - T_{4a})}$$

$$T_2 = \frac{T_1 + (T_{2a} - T_1)}{\eta_{\text{compressor}}} = \frac{293 + (443.9 - 293)}{0.8} = 481.6 \text{ K}$$

$$T_4 = T_3 - \eta_{\text{turbine}}(T_3 - T_{4a})$$

$$= 1123 - 0.8(1123 - 741.25) = 817.6 \text{ K}$$

(2) and (3) specific work of compressor $= C_p (T_2 - T_1)$

$$= 1.005(481.6 - 293) = 189.54 \text{ kJ/kg}$$

Specific work of turbine $= 1.005\ (T_3 - T_4)$

$$= 1.005(1123 - 817.6) = 306.93 \text{ kJ/kg}$$

Net work = 306.93 – 189.54 = 117.4 kJ/kg

(4) Thermal efficiency (η_t) of ideal cycle,

$$\eta_t = \frac{1 - 1}{\left(\frac{P_2}{P_1}\right)^{(\gamma-1)\gamma}} = 1 - 0.66 = 34\%$$

Thermal efficiency of actual cycle,

$$\eta_t = \frac{\text{(Heat supplied-Heat rejected)}}{\text{Heat supplied}}$$

$$= \frac{\{C_p(T_3 - T_2) - C_p(T_4 - T_1)\}}{\{C_p(T_3 - T_2)\}} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

$$= 1 - \frac{(817.6 - 293)}{(1123 - 481.6)} = 1 - 0.818 = 18.20\%$$

(5) Air flow rate $= \dfrac{3600}{\text{net work output in kJ/kg}}$ kg/kW-hr.

$$= \left(\frac{3600}{117.4}\right) \times 25{,}000 \text{ kg/hr}$$

$$= \frac{(3600 \times 25000)}{(117.4 \times 3600)} \text{ kg/s} = 212.95 \text{ kg/s}$$