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Angle Modulation →

– In angle modulation, either phase or frequency of carrier signal is varied in accordance with the instantaneous value of message signal.

– Amplitude of the carrier kept constant and it is non-linear modulation.

Angle Modulation

Frequency Modulation	Phase Modulation
Frequency of the carrier is varied according to the Message signal	Phase angle of the carrier is varied according to the message signal.

Frequency Modulation as well as phase modulation are forms of angles modulation. Angle modulation has several advantages, over the amplitude modulation such as reduction, improved system fidelity and more efficient use of power.

But there are some disadvantages too such as increased bandwidth and use of more complex circuits.

Angle modulation is being used for the following applications:

(i) Radio broadcasting
(ii) TV sound transmission
(iii) Two way mobile radio
(iv) cellular radio
(v) Microwave and satellite communication

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The principle of angle modulation can be stated as, the standard equation of Angle modulation is given as,

$$S(t) = A_c \cos(\omega_c t + \phi).$$

$$= A_c \cos(\theta(t)) \longrightarrow \text{phase or freq. mod.}$$

where,

A_c = Carrier amplitude constant.

ω_c = angular velocity

ϕ = phase angle

$\theta(t) = \omega_c t + \phi \Rightarrow$ angle of modulation.

Phase Modulation →

– In phase modulation, phase of carrier signal changes with respect to modulating signal m(t).

If we have carrier signal c(t) ∴ we have modulating signal,

$$c(t) = A_c \cos(\omega_c t + \phi) \qquad m(t)$$

$$= A_c \cos(2\pi f_c t + \phi)$$

$$= A_c \cos(\theta(t))$$

$\theta(t)$ changes as $\phi(t)$ changes

→ For phase modulation,

$$\theta(t) = 2\pi f_c t + K_p m(t).$$

∴ PM wave is given as,

$$S(t)_{PM} = A_c \cos(2\pi f_c t + K_p m(t))$$

K_p = phase sensitivity (rad/volt).

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Frequency Modulation →

In frequency modulation, frequency of carrier signal $c(t)$ changes with respect to modulating signal $m(t)$.

- If we have carrier signal,

$c(t) = A_c \cos(\omega_c t + \phi)$.

$= A_c \cos(2\pi f_c t + \phi)$.

$= A_c \cos \theta(t)$.

- We have modulating signal $m(t)$.

- In frequency modulation,

$\theta(t) = 2\pi f_c t + 2\pi K_f \int_0^t m(t)\, dt$, $\theta(t)$ changes as f_c changes

- FM wave is given as,

$s_{FM}(t) = A_c \cos(2\pi f_c t + 2\pi K_f \int_0^t m(t) dt$.

$K_f \Rightarrow$ frequency sensitivity (Hz/volt).

Also,

$f_i(t) = f_c + K_f\, m(t)$. $f_i(t) \rightarrow$ instantaneous frequency.

On Integrating, both sides we get,

$\int f_i(t) \delta t = \int f_c \delta t + \int K_f\, m(t) \delta t$

$\int 2\pi f(t) \delta t = \int f_c 2\pi \delta t + \int K_f \cdot 2\pi \cdot m(t) \cdot \delta t$

$\int \omega \delta(t) = 2\pi f_c t + 2\pi K_f \int m(t) \delta t$.

$\theta(t) = 2\pi f_c t + 2\pi K_f \int m(t) \cdot \delta t$.

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Relationship between FM and PM wave :-

- PM wave $\Rightarrow$ $A_c \cos\left[2\pi f_c t + K_p m(t)\right]$

FM wave $\Rightarrow$ $A_c \cos\left[2\pi f_c t + \underbrace{2\pi K_f \int m(t)\, dt}_{\text{deviation of frequency}}\right]$

Generation of FM using Phase Modulator

The schematic diagram for generation of FM using PM Modulator (Block diagram) • is shown below →

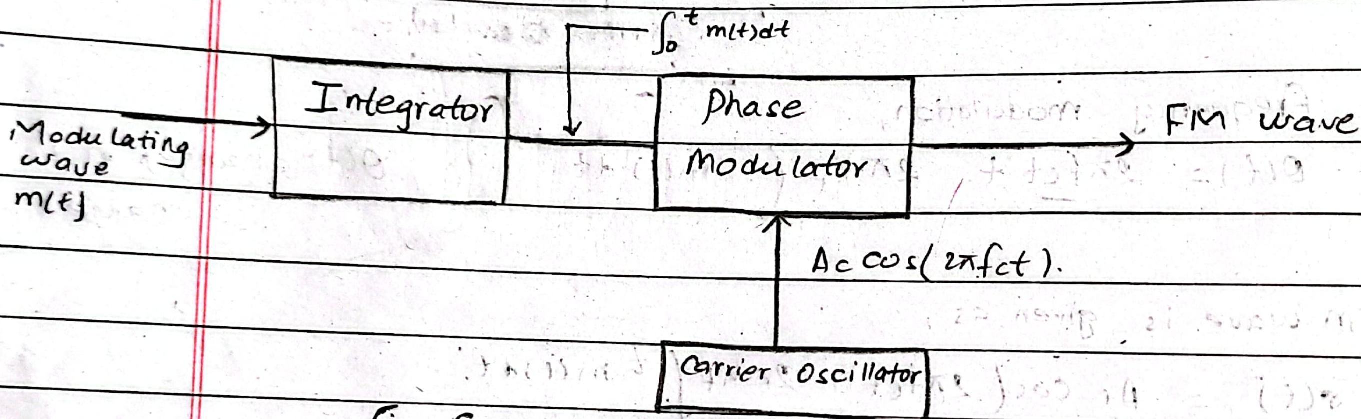


Fig: Generation of FM from phase modulator

Comparing these expressions, we can conclude that an FM wave is actually a PM wave having a modulating signal $\int_0^t m(t)\, dt$ instead of $m(t)$.

This means that we can generate FM wave by applying the integrated version of ~~e(t)~~ $m(t)$ to a phase modulator as shown above.

Generation of PM using Frequency Modulator

The block diagram for generation of PM using FM is shown in figure below →

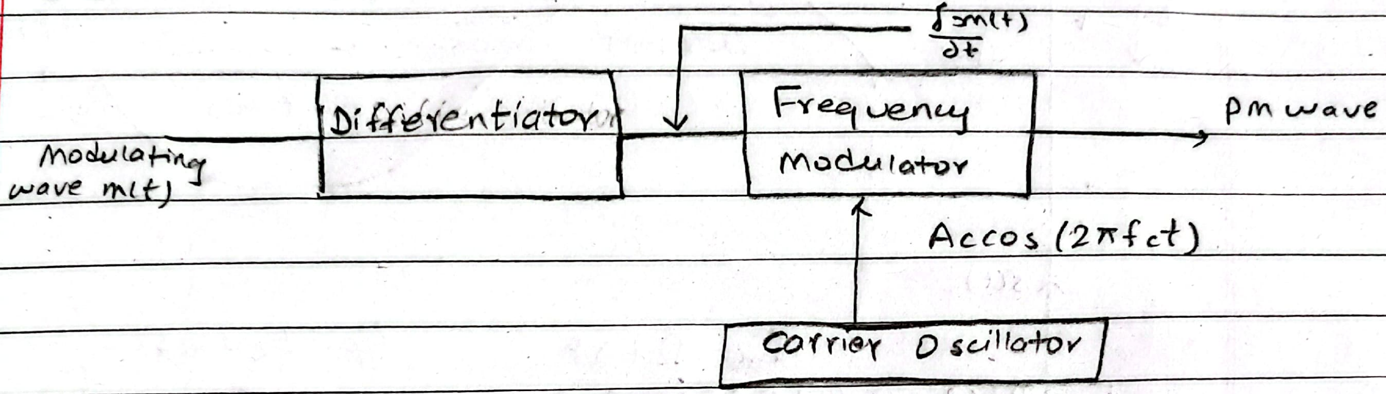


Fig : Generation of PM from frequency Modulator

It is also possible to generate a PM wave using a frequency modulator as shown in figure. The modulating signal is first passed through a differentiator and then applied to a frequency modulator.

Hence, output of FM will be,

$$s(t) = A_c \cos\left\{2\pi f_c t + 2\pi K_f \int_0^t \left(\frac{dm(t)}{dt}\right)\right\}$$

$$= A_c \cos[2\pi f_c t + 2\pi K_f\, m(t)]$$

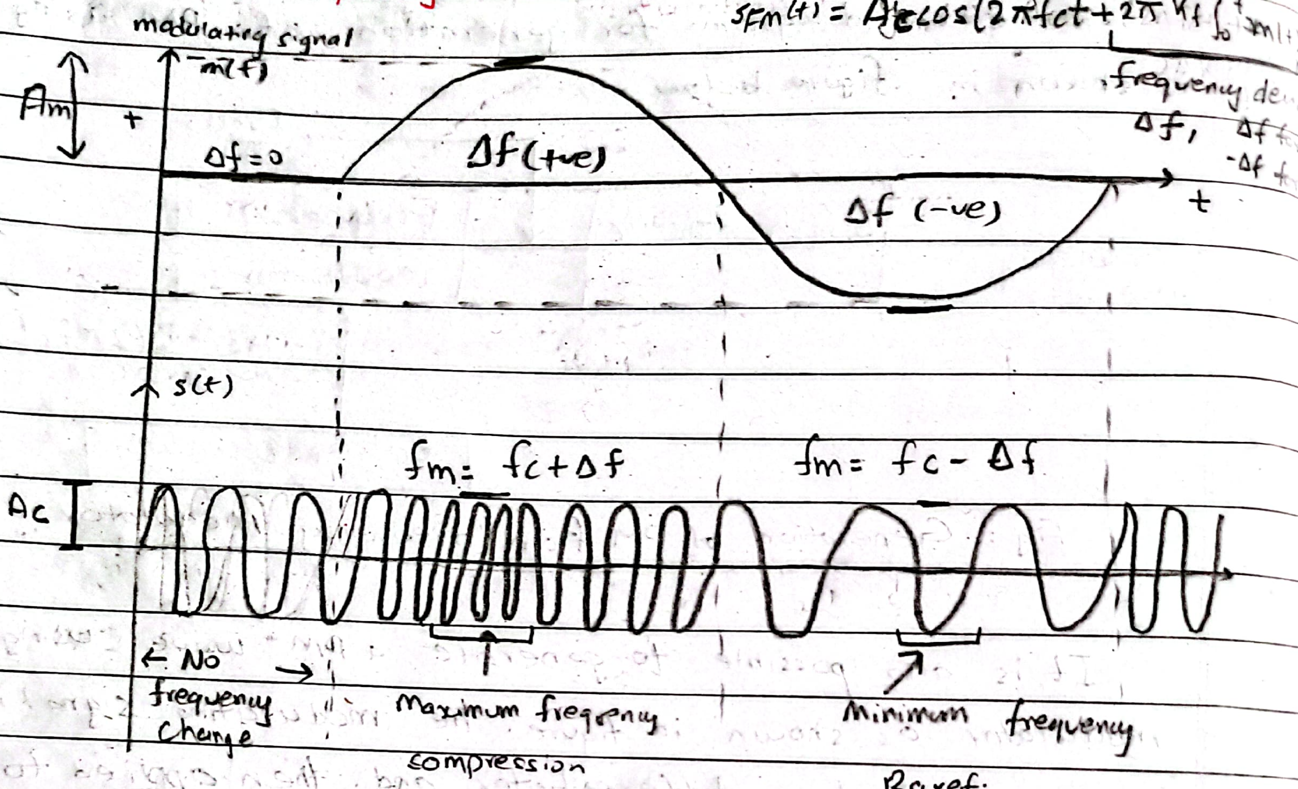
Substituting $2\pi K_f = K_p$, we get, (on comparing above eqn to PM signal we get) $K_p = 2\pi K_f$.

$$s(t) = A_c \cos(2\pi f_c t + K_p m(t)].$$

which is the expression of PM wave.

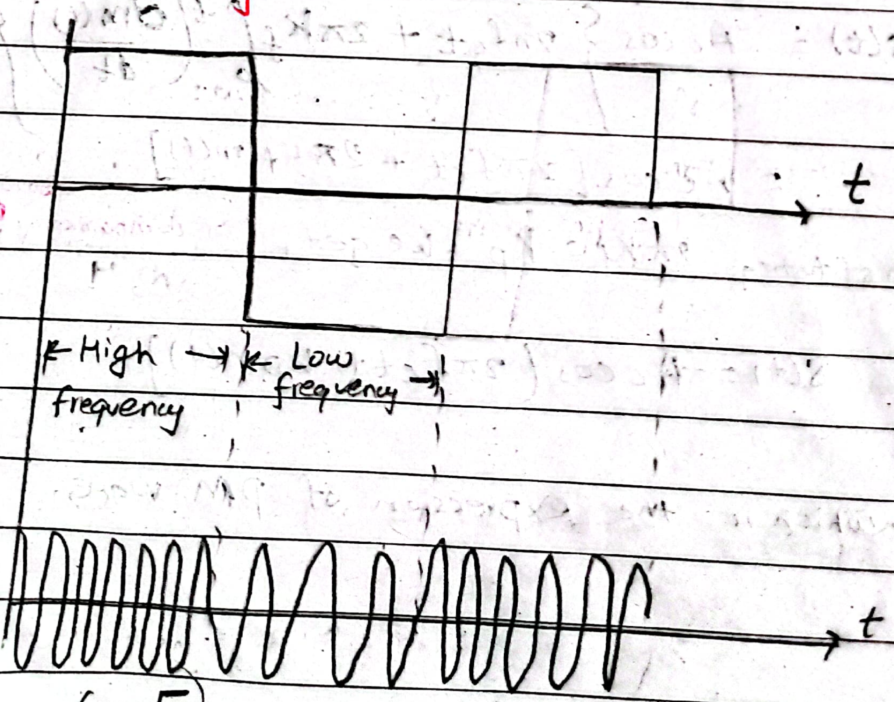
Sinusoidal Frequency Modulation →

$s_{FM}(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int_0^t m(t)$



(a) Sinusoidal Frequency Modulation

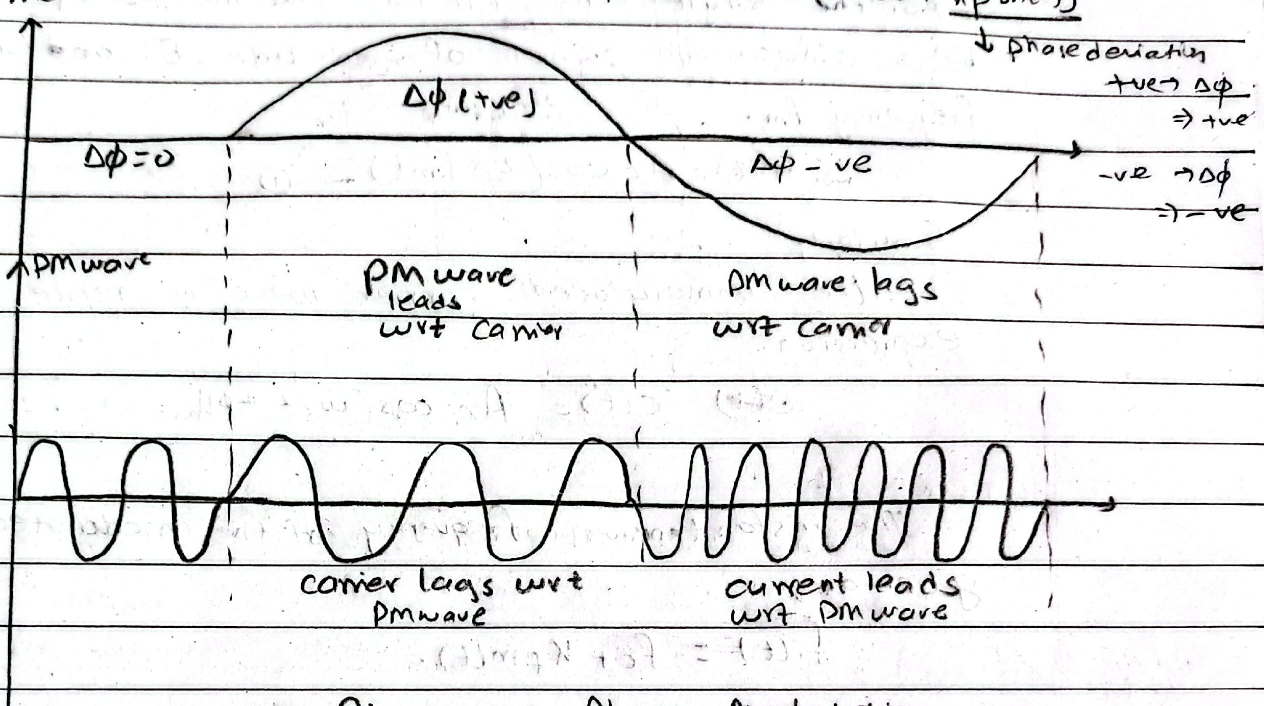
Squared Frequency Modulation →



(b) Frequency Modulated wave

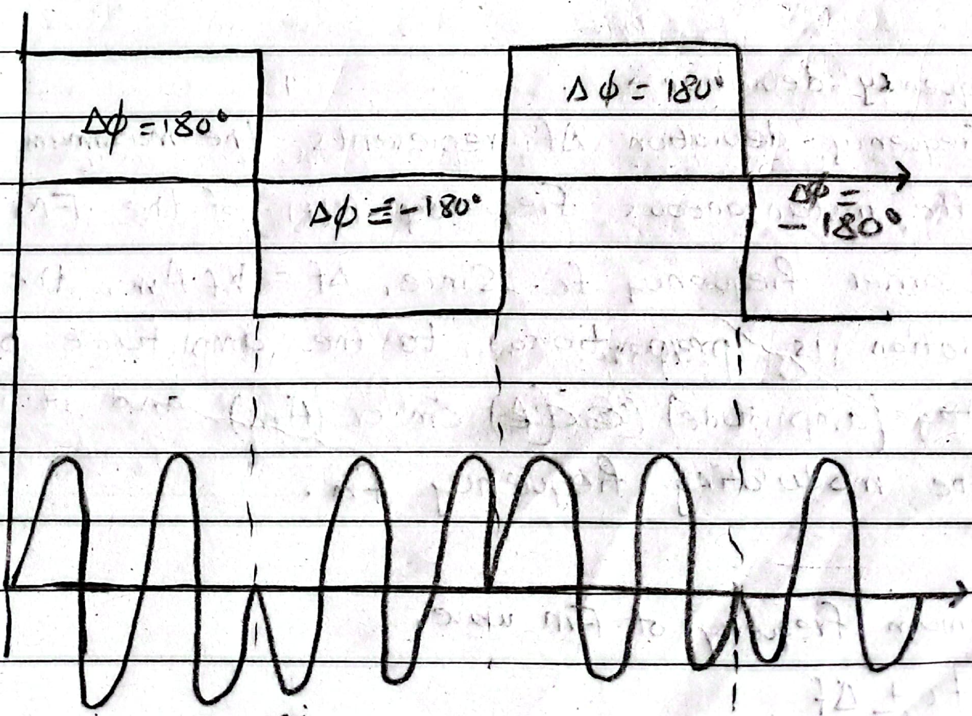
Sinusoidal Phase Modulation →

$S_{pm}(t) = A_c \cos(2\pi f_c t + \underline{K_p m(t)})$



(a) Sinusoidal Phase Modulation

Squared Phase Modulation →



(b) Phase modulated wave

Single Tone Frequency Modulation→

For the single tone FM i.e. the modulating signal m(t) be a sinusoidal signal of amplitude Em ~~and~~ (or Am) and frequency fm.

$$\therefore m(t) = A_c \cos(2\pi f_m t) \quad —(1)$$

Similarly,

the unmodulated carrier wave is represented by the expression:

$$\cancel{c(t)} \quad c(t) = A_c \cos(\omega_c t + \phi)$$

The instantaneous, frequency of the modulated wave is given as,

$$f_i(t) = f_c + K_f m(t).$$

$$f_i(t) = f_c + K_f A_m \cos(2\pi f_m t).$$

$$f_i(t) = f_c + \Delta f \cos 2\pi f_m t \quad —(2).$$

where, $\Delta f = K_f A_m$ = frequency deviation.

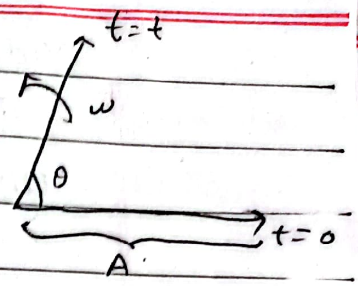
Frequency deviation→

Frequency deviation Δf represents the maximum departure of the instantaneous frequency $f_i(t)$ of the FM wave from the carrier frequency f_c. Since, $\Delta f = K_f A_m$, the frequency deviation is proportional to the amplitude of modulating voltage (amplitude) ~~(Em or A)~~ Em or (Am) and it is independent of the modulating frequency fm.

Maximum frequency of FM wave,

$$f_{max} = f_c \pm \Delta f$$

Mathematical expression for FM



We know that the FM wave is sinewave having a constant amplitude and a variable instantaneous frequency.

As the instantaneous frequency is changing continuously, the angular velocity ω of an FM wave is the function of ω_c and ω_m.

Therefore, FM wave is represented by,

$$s(t) = A_c \cos \theta t$$

$$s(t) = A_c \cos [f(\omega_c, \omega_m)]$$

$$\theta(t) \Rightarrow f(\omega_c, \omega_m).$$

Now, the argument angle $\theta(t)$ of FM is given as,

$$\theta(t) = 2\pi \int_0^t f_i(t)\, dt$$

$$\Delta f = K_f \cdot A_m.$$

$$= 2\pi \int_0^t (f_c + \Delta f \cos 2\pi f_m t)\, dt$$

$$= 2\pi f_c t + \frac{2\pi \Delta f}{2\pi f_m} \cdot \sin 2\pi f_m t$$

$$= \underbrace{2\pi f_c t}_{\omega_c t} + m_f \cdot \sin \underbrace{2\pi f_m t}_{\omega_m t}$$

where, m_f = modulation index of FM.

$$\boxed{s(t) = A_c \cos [\omega_c t + m_f \sin \omega_m t]}$$

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Modulation Index of FM (m_f) :

The modulation ~~ki~~ index of an FM wave is defined as under:

$$m_f = \frac{\text{frequency deviation}}{\text{Modulating frequency}}$$

$$= \frac{\Delta f}{f_m}$$

The modulation index (m_f) is very important in FM because it decides the bandwidth of the FM wave. The modulation index also decides the number of sidebands having significant amplitudes.

In AM, the maximum value of the modulation index m is 1. But for FM, the modulation index can be greater than 1.

Deviation Ratio →

In FM broadcasting, the maximum value of deviation is limited to 75 KHz. The maximum modulating frequency is also limited to 15 KHz. The modulation index corresponding to the maximum deviation and maximum modulating frequency is called as the deviation ratio.

$$\text{Deviation Ratio} = \frac{\text{Maximum deviation}}{\text{Maximum modulating frequency}}$$

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Percentage modulation of FM wave →

The percent modulation is defined as the ratio of the actual frequency deviation produced by the modulating signal to the maximum allowable frequency deviation.

Thus, $\% \text{ modulation} = \dfrac{\text{Actual frequency deviation}}{\text{Maximum allowed deviation}}$.

Narrow Band Frequency Modulation and Wide band Frequency Modulation-

[A] Narrow band Frequency Modulation →

A narrow band FM is the FM wave with a small bandwidth. The modulation index m_f of narrowband FM is small, as ~~compared~~ compared to one radian. Hence the spectrum of narrow band FM consists of the carrier and upper side-band and a lower sideband.

The carrier wave $c(t)$ is given as,

$c(t) = A_c \cos(\omega_c t + \phi)$, $m(t) = A_c \cos(2\pi f_m t)$.

$= A_c \cos(2\pi f_c t + \phi)$

$f_i(t) = f_c + k_f m(t)$

$f_i(t) = f_c + K_f * A_m \cos(2\pi f_m t)$

$f_i(t) = f_c + \Delta f \cos 2\pi f_m t$

$s(t) = A_c \cos \theta(t)$.

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on integrating $f_i(t)$ we get $\theta(t)$.

$$\theta(t) = \int f_i(t)dt = \int f_c dt + \int \Delta f \cos 2\pi f_m t$$

$$\Rightarrow \int 2\pi f_i(t)dt = \int_0^t 2\pi f_c t \, dt + \int_0^t 2\pi \Delta f \cos 2\pi f_m t \, dt$$

$$\theta(t) \Rightarrow 2\pi f_c t + \frac{2\pi \Delta f}{2\pi f_m} \sin 2\pi f_m t$$

$$= 2\pi f_c t + \frac{\Delta f}{f_m} \sin 2\pi f_m t$$

$$\therefore s(t) = A_c \cos\left(\underbrace{\omega_c t}_{A} + \underbrace{m_f \sin\omega_m t}_{B}\right)$$

As compare, $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$s(t) = A_c[\cos\omega_c t \cdot \cos(m_f \sin\omega_m t) - \sin\omega_c t \cdot \sin(m_f \sin\omega_m t)]$$

If modulation index m_f is very small then,

$\{\cos(m_f \sin\omega_m t) \approx 1\}$

$\{\sin(m_f \sin\omega_m t) \approx m_f \sin\omega_m t$
$\sin(\alpha) \Rightarrow \alpha\}$

$$\therefore s(t) = A_c \cos\omega_c t - A_c \sin\omega_c t \cdot m_f \cdot \sin\omega_m t$$

$$= A_c \cos\omega_c t - A_c m_f \sin\omega_c t \cdot \sin\omega_m t$$

$$= A_c \cos\omega_c t + \frac{A_c m_f}{2} \cos(\omega_c + \omega_m)t - \frac{A_c m_f}{2} \cos(\omega_c - \omega_m)t$$

$$\boxed{\therefore s(t) = \underbrace{A_c \cos\omega_c t}_{\text{carrier}} + \underbrace{\frac{A_c m_f}{2} \cos(\omega_c + \omega_m)t}_{\text{upper side band}} - \underbrace{\frac{A_m m_f}{2} \cos(\omega_c - \omega_m)t}_{\text{lower side band}}}$$

A_c $\quad\quad$ $\frac{A_c m_f}{2}$ $\quad\quad$ $\frac{A_m m_f}{2}$

$\therefore$ This is the required expression of NBFM.

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on integrating $f_i(t)$ we get, $\theta(t)$.

$$\theta(t) = \int f_i(t)dt = \int f_c dt + \int \Delta f \cos 2\pi f_m t$$

$$\Rightarrow \int 2\pi f_i(t)dt = \int_0^t 2\pi f_c t\, dt + \int_0^t 2\pi \Delta f \cos 2\pi f_m t\, dt$$

$$\theta(t) \Rightarrow 2\pi f_c t + \frac{2\pi \Delta f}{2\pi f_m} \sin 2\pi f_m t$$

$$= 2\pi f_c t + \frac{\Delta f}{f_m} \sin 2\pi f_m t$$

$$\therefore s(t) = A_c \cos\left(\underset{A}{\underline{\omega_c t}} + \underset{B}{\underline{m_f \sin\omega_m t}}\right)$$

As compare, $\cos(A+B) = \cos A \cos B - \sin A \sin B$

1) $s(t) = A_c[\cos\omega_c t \cdot \cos(m_f \sin\omega_m t) - \sin\omega_c t \cdot \sin(m_f \sin\omega_m t)]$

If modulation index m_f is very small then,

$\{\cos(m_f \sin\omega_m t) \approx 1\}$

$\left\{\begin{array}{l}\sin(m_f \sin\omega_m t) \approx m_f \sin\omega_m t \\ \sin(\alpha) \Rightarrow \alpha\end{array}\right\}$

$$\therefore s(t) = A_c \cos\omega_c t - A_c \sin\omega_c t \cdot m_f \cdot \sin\omega_m t$$

$$= A_c \cos\omega_c t - A_c m_f \sin\omega_c t \cdot \sin\omega_m t$$

$$= A_c \cos\omega_c t + \frac{A_c m_f}{2} \cos(\omega_c + \omega_m)t - \frac{A_c m_f}{2} \cos(\omega_c - \omega_m)t$$

$$\boxed{\therefore s(t) = \underbrace{A_c \cos\omega_c t}_{\text{carrier}} + \underbrace{\frac{A_c m_f}{2} \cos(\omega_c + \omega_m)t}_{\text{upper side band}} - \underbrace{\frac{A_m m_f}{2} \cos(\omega_c - \omega_m)t}_{\text{lower side band}}}$$

A_c $\qquad \frac{A_c m_f}{2}$ $\qquad \frac{A_m m_f}{2}$

$\therefore$ This is the required expression of NBFM.

Eular relation;
$(e^{\pm j\theta} = \cos \pm j\sin\theta)$

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[B] Wide Band frequency Modulation →

- For large value of modulation index m_f, the FM wave ideally contains the carrier and an infinite number of sidebands located symmetrically around the carrier. Such a FM wave has infinite bandwidth and hence called as wide band FM.
- The modulation index of wideband FM is higher than 1. The maximum permissible deviation is 75 kHz and it is used in the entertainment broadcasting applications such as FM radio, TV, etc.

- frequency spectrum of WBFM, (Bessels coefficient) →

The expression for the FM wave is not simple. It is complex since it is sine of sine function.

The FM wave for sinusoidal signal tone modulation is given as,

$$s(t)_{FM} = A_c \cos(2\pi f_c t + m_f \sin\omega_m t). \quad -(1)$$

Let, $m_f \Rightarrow$ modulation index of FM wave,
$\Rightarrow \beta$.

$$s(t)_{FM} = A_c \cos(2\pi f_c t + \beta \sin\omega_m t) — (1).$$

This equation can be expressed as trigonometric relation,

$$S_{FM}(t) = A_c \cos(2\pi f_c t) \cdot \cos(\beta \sin 2\pi f_m t) - A_c \sin(2\pi f_c t) \cdot \sin(\beta \sin 2\pi f_m t) \quad -(2)$$

From this expanded form we can see the Inphase and Quadrature components of FM wave $s(t)$ is given as,

$$S_I(t) = A_c \cos[\beta \sin(2\pi f_m t)] — (3)$$

$$S_Q(t) = A_c \cos[\beta \sin(2\pi f_m t)] — (4)$$

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Hence, the complex envelope of FM wave is given as,

$$\tilde{s}(t) = s_I(t) + j s_Q(t)$$
$$= A_c \cos[\beta \sin(2\pi f_m t)] + j A_c \sin(\beta \sin(2\pi f_m t)].$$
$$= A_c e^{j\beta \sin(2\pi f_m t)} \quad \text{(Euler's relation)}$$
$$= A_c \exp(j\beta \sin(2\pi f_m t)) \quad \text{—(5)}$$

The complex envelope $\tilde{s}(t)$ retains all the information related to modulated process indeed we may radily express FM wave $s(t)$ interms of complex envelope $\tilde{s}(t)$.

$$s(t) = \text{Re}[A_c \exp(j2\pi f_c t + j\beta \sin(2\pi f_m t))].$$
$$= \text{Re}[\tilde{s}(t) \cdot \exp(j 2\pi f_m t)] \quad \text{—(4)}$$

The expression of $\tilde{s}(t)$ in complex fourier series as,

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} C_n \exp(j 2\pi f_m t) \quad \text{—(6)}$$

where,

$$C_n = \frac{1}{T_m} \int_{-T_m/2}^{T_m/2} A_c \exp(-j2\pi f_m t) \cdot \underset{\tilde{s}(t)}{\exp(+j\beta \sin(2\pi f_m t))}.dt$$
$$= A_c f_m \int_{-1/2f_m}^{+1/2f_m} \exp[(-j2\pi f_m t) + j\beta \sin(2\pi f_m t)].dt \quad \text{—(7)}$$

Let us assume,

$$x = 2\pi f_m t$$
$$\frac{dx}{dt} = 2\pi f_m \cdot \frac{d(t)}{dt}$$
$$dx = 2\pi f_m \, dt.$$
$$\therefore dt = dx/2\pi f_m. \quad \text{—(8)}$$

Note, properties of Bessel function,

1. $J_{-n}(\beta) = \begin{cases} J_n(\beta); & n = \text{even} \\ -J_n(\beta); & n = \text{odd} \end{cases}$

2. $J_{n-1}(\beta) + J_{n-1}(\beta) = \frac{2n}{\beta} J_n(\beta)$.

3. $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$ for all β.

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when, $t = -\frac{1}{2} f_m$ then $x = -\pi$

when, $t = +\frac{1}{2} f_m$ then $x = +\pi$

[$\because x$ ranges from $-\pi$ to $+\pi$]

Substuting the value of dt in eqn (7) we get,

$$C_n = \frac{A_c}{2\pi} \int_{-\pi}^{\pi} \exp(j\beta \sin x - jnx)\, dx$$

$$= \frac{1}{2\pi} A_c \int_{-\pi}^{\pi} \exp(j\beta \sin x - jnx)\, dx$$

$$= A_c \left[\frac{1}{2\pi} \int_{-\pi}^{\pi} \exp(j(\beta \sin x - nx)) \right] dx \quad \text{--- (9)}$$

(Bessel's function)

the Integral on right hand side is recognized as the, nth order Bessel function of the first kind and argument β. and it is denoted by symbol $J_n(\beta)$.

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp(j(\beta \sin x - nx))\, dx$$

$$C_n = A_c \cdot J_n(\beta) \quad \text{--- (10)}$$

Hence, this is the required expression of C_n in Bessel's form.

Also we have,

$$e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta) \exp(j\, 2\pi f_m t) \quad \text{--- (11)}$$

From eqn (4) and (11) we get,

$$s(t)_{FM} = Re \left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \exp(j \cdot 2\pi f_m t) \right]$$

$$= A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(2\pi(f_c + n f_m)t].$$

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Performance comparison of wide band and Narrow band FM

S.N.	Parameter/Characteristics	Wideband FM	Narrowband FM
1.	Modulation Index	Greater than 1	Less than or slightly greater than 1
2.	Maximum deviation	75KHz	5KHz
3.	Range of modulating frequency.	30Hz to 15KHz	30Hz to 3KHz
4.	Maximum modulation index	5 to 2500	Slightly greater than 1.
5.	Bandwidth	Large about 15 times higher than BW of narrowband FM.	Small. Approximately same as that of AM
6.	Applications	Entertainment broad-casting (can be used for high quality music transmission)	FM mobile communication like police wireless, ambulance etc. (This is used for speech transmission).
7.	Pre-Emphasis and De-emphasis	Needed	Needed

Sidebands and Modulation Index :- (Relation)

We know that any modulation process produces sidebands. We have seen that in AM, two sidebands are produced with frequencies equal to $f_c + f_m$ and $f_c - f_m$.

In FM and PM, also, sum and difference sideband frequencies are produced.

Note: the number of side bands with significant amplitude increase with increase in m_f.

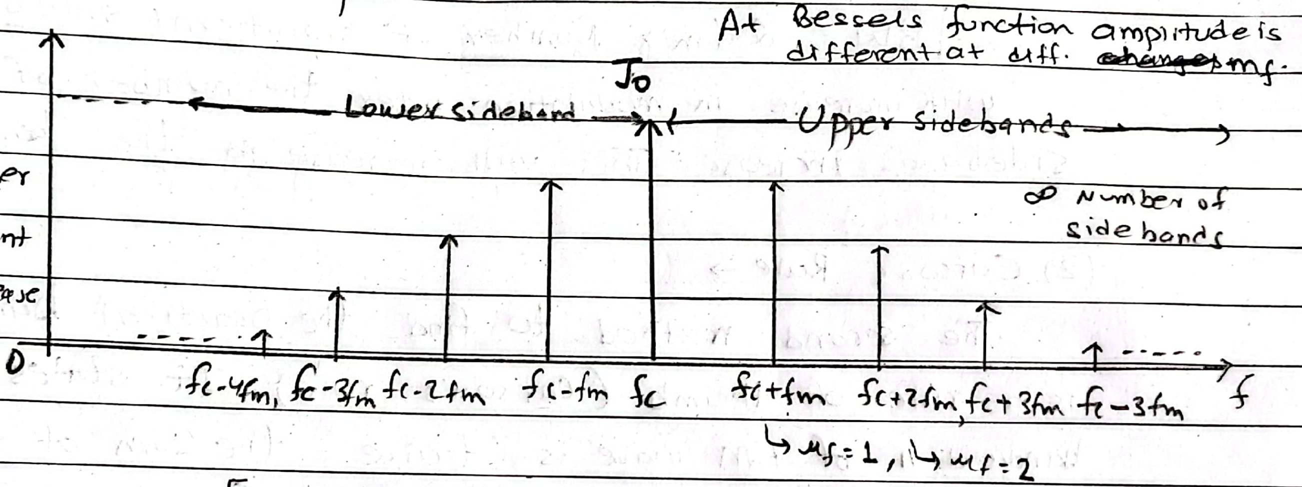

Figure: Ideal frequency spectrum of FM;

In addition theoretically infinite number of pairs of upper and lower sidebands are also generated. Hence, the spectrum of FM or PM signal is generally wider than the spectrum of AM.

Effect of Modulation Index →

As the amplitude of modulating signal varies, the frequency deviation will change, the number of sidebands produced, their amplitudes will change. ~~[illegible]~~ above figure, illustrates the effects of modulation index on the frequency spectrum of FM. Theoretically, the bandwidth of FM is infinite.

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Transmission bandwidth of frequency modulation

(1) Practical Bandwidth →

Theoretically, the bandwidth of the FM wave is infinit[e]. But, practically, it is calculated based on how many sidebands have significant amplitude. The simplest method to calculate the bandwidth is as under:

$$\boxed{BW = 2 \cdot f_m * \text{Number of significant sidebands.}}$$

with increase in modulation index, the number of signific[ant] sidebands increase. This will increase the bandwidth.

(2) Carson's Rule →

The second method to find the practical bandwidth is a rule of thumb (Carson's rule). It states that the bandwidth of FM wave is twice the sum of the devia-tion and the highest modulating frequency.

Thus,

$$BW = 2[\Delta f + f_{m(max)}]$$

The Carson's Rule gives correct results if the modula[tion] index is greater than 6.

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Constant Average Power of frequency modulation →

we know that the envelope of FM wave always has a constant magnitude. Therefore, the average power of such a wave dissipated in $1\,\Omega$ resistance will always be constant.

The transmitted power in FM is given as,

$$P_t = \frac{(E_c/\sqrt{2})^2}{R_L} \qquad [\text{Since } E_c = A_c]$$

$(E_c/\sqrt{2}) \Rightarrow$ RMS value of FM wave,

Hence, $R_L = 1$,

$$P_t = \frac{1}{2} E_c^2$$

Since, E_c is constant, P_t also will be constant. It is possible to express the transmitted power in the form of series expansion as under,

$$P_t = \frac{1}{2} E_c^2 \sum_{n=-\infty}^{\infty} J_n^2(m_f).$$

But, $\sum_{n=-\infty}^{\infty} J_n^2(m_f) = 1$. $\left.\begin{matrix} \text{for } m_f = 1, 2, --- \infty, \\ P_t = 1, 2, -- \infty \end{matrix}\right\}$.

$$\therefore P_t = \frac{1}{2} E_c^2 = \frac{1}{2} A_c^2.$$

Transmission power in FM wave →

$$P_T = A_c^2/2$$

practically 98 % power is transmitted.

$$\text{So, } P_T = 0.98 * \frac{A_c^2}{2}.$$

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Multiple-frequency Modulation -

We have already derived the expression for FM wave with a single modulating input signal. Let us now extend this concept to obtain the expression of FM for more than one modulating signals.

Let $m_1(t)$ and $m_2(t)$ be two modulating signals and let,

$$m(t) = m_1(t) + m_2(t)$$

Let, $m_1(t) = A_{m1} \cos \omega_{m1} t \quad = E_{m1} \cos \omega_{m1} t$

$m_2(t) = A_{m2} \cos \omega_{m2} t \quad = E_{m2} \cos \omega_{m2} t$

Therefore,

$$m(t) = A_{m1} \cos \omega_{m1} t + A_{m2} \cos \omega_{m2} t$$

The instantaneous frequency is given by,

$$\omega_i = \omega_c + K_f m(t)$$

$$= \omega_c + K_f [E_{m1} \cos \omega_{m1} t + E_{m2} \cos \omega_{m2} t]$$

The maximum frequency deviation is given by,

$$\Delta\omega = (E_{m1} + E_{m2}) K_f$$

$$\theta(t) = \int \omega_i \, dt = \omega_c + \frac{E_{m1} K_f}{\omega_{m1}} \sin \omega_{m1} t + \frac{E_{m2} K_f}{\omega_{m2}} \sin \omega_{m2} t$$

Let, $\frac{E_{m1} K_f}{\omega_{m1}} = m_{f1}$ and $\frac{E_{m2} K_f}{\omega_{m2}} = m_{f2}$.

$$\therefore \theta(t) = \omega_c + m_{f1} \sin \omega_{m1} + m_{f2} \sin \omega_{m2} t$$

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Hence, the expression for FM wave is given by,

$$s(t) = E_c \sin \theta(t)$$

$$s(t) = E_c \sin[\omega_c t + m_{f1} \sin \omega_{m1} t + m_{f2} \sin \omega_{m2} t]$$

Spectrum of this FM wave contains sidebands of frequencies $(f_c \pm n f_{m1})$ and $(f_c \pm n f_{m2})$.

In addition to that, there will be cross modulation terms such as $(\omega_c \pm n\omega_{m1} \pm k\omega_{m2})$.

In AM, each new frequency added gives rise to its own pair of side bands only. There are no cross modulation terms. Hence, AM is called as linear modulation while FM is called as a non-linear modulation.

Difference/ comparison of FM and PM ~~signals~~ systems →

FM	PM
1. $s(t) = V_c \sin\{w_c t + m_f \sin w_m t\}$	1. $s(t) = V_c \sin[w_c t + m_p \sin w_m t]$
2. Frequency deviation is proportional to modulating voltage.	2. Phase deviation is proportional to the modulating voltage.
3. Associated with the change in f_c, there is some phase change.	3. Associated with the changes in phase, there is some change in f_c.
4. m_f is proportional to the modulating voltage as well as the modulating frequency f_m.	4. m_p is proportional only to the modulating voltage.
5. It is possible to receive FM on a PM receiver.	5. It is possible to receive PM on FM receiver.
6. Noise immunity is better than AM and PM.	6. Noise immunity is better than AM but worse than FM.
7. Amplitude of the FM wave is constant.	7. Amplitude of the PM wave is constant.
8. Signal to noise ratio is better than that of PM.	8. Signal to noise ratio is inferior to that of FM.
9. FM is widely used.	9. PM is used in some mobile systems.
10. In FM, the frequency deviation is proportional to the modulating voltage only.	10. In PM, the frequency deviation is proportional to both the modulating voltage and modulating frequency.

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Performance Comparison of FM and AM System →

S.N:	FM	AM
1.	Amplitude of FM wave is constant. It is independent of the modulation index.	Amplitude of AM wave will change with the modulating voltage.
2.	Hence, transmitted power remains constant. It is independent of mf.	Transmitted power is dependent on the modulation index.
3.	All the transmitted power is useful.	Carrier power and one sideband power are useless.
4.	FM receivers are immune to noise.	AM receivers are not immune to noise.
5.	It is possible to decrease noise further by increasing deviation.	This feature is absent in AM.
6.	Bandwidth = $2[\Delta f + f_m]$. The BW depends on modulation index.	$BW = 2f_m$. It is not dependent on the modulation inde
7.	BW is large. Hence, wide channel is required.	BW is much less than F
8.	Space wave is used for propagation. So, radius of transmission is limited to line of sight.	Ground wave and sky propagation is used. Theref larger area is covered tha
9.	Hence, it is possible to operate several transmitters on same frequency.	Not possible to opera more channels on sam frequency.

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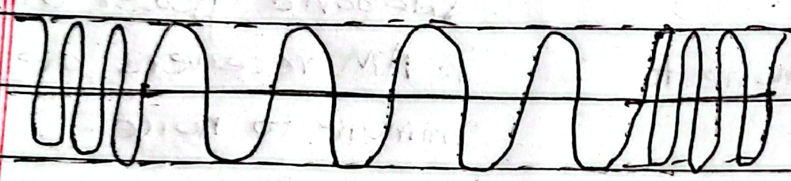
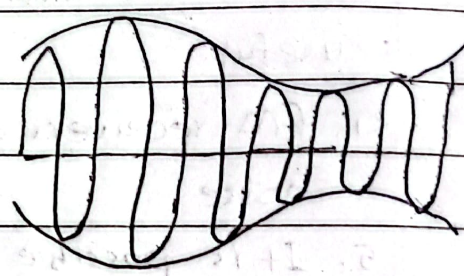
6.	Bandwidth $= 2[\Delta f + f_m]$. The BW depends on modulation index.	$BW = 2f_m$. It is not dependent on the modulation index.
7.	BW is large. Hence, wide channel is required.	BW is much less than FM.
8.	Space wave is used for propagation. So, radius of transmission is limited to line of sight.	Ground wave and sky wave propagation is used. Therefore, larger area is covered than FM.
9.	Hence, it is possible to operate several transmitters on same frequency.	Not possible to operate more channels on same frequency.

FM	AM
10. FM transmission and reception equipment are more complex.	10. AM equivalents are less complex.
11. The number of sidebands having significant amplitudes depends on modulation index m_f.	11. Number of sidebands in AM will be constant & equal to 2.
12. The information is contained in the frequency variation of the carrier.	12. The information is contained in the amplit variation of the carrier.
13. FM wave: 	13. AM wave. 
14. Applications: Radio, TV broadcasting, police wireless, point to point communications.	14. Applications: Radio and TV broadcasting

CHAPTER-2

PULSE MODULATION

* **Introduction →**

- A type of modulation in which pulses are varied in some respect, such as amplitude, width, position to represent the signal. i.e. the transmission of analog data or speech which is in continuous form is known as pulse modulation.

- The transmission of the voice signal by a carrier is represented as a voltage signal that varies continuously with time, In amplitude modulation and frequency modulation, the carrier signal is varied continuously in analog manner; this continuous transmission of information in an analog manner is used in the Frequency Division multiplex system, while in the Time division Multiplex system it is not necessary. The Time Division Multiplex system use pulse modulation. In the Pulse modulation, the continuous signal is converted into a series of pulses, each proportional to the amplitude of the signal and corresponding in time to it. Thus in pulse modulation, a series of pulses carries the information instead of continuous modulated signal.

- Classification of Pulse Modulation →

Pulse Modulation
- Analog Pulse Modulation
  - PAM
  - PWM
  - PPM
- Digital Pulse Modulation
  - PCM
  - DM (Delta Modulation)

RZ   NRZ

* Analog Pulse Modulation →

Analog Pulse Modulation is the technique to sampled analog information signal. In PM system the carrier is a pulse train instead of sinusoidal carrier as in Analog Modulation. Some Characterest Parameter of a carrier signal (amplitude, Width, position) is changed wrt instantaneous value of message signal. PM system is that it permits the simultaneous transmission of several signals on time sharing basis i.e. ~~TDM~~, PWM, PPM, ~~PM~~ and PAM are the types of analog PM.

[A] Pulse Amplitude Modulation →

- The modulation in which the amplitudes of regularly spaced rectangular pulses vary according to instantaneous value of modul or message signal is called PAM. The pulses in PAM signal may be flat-top, or natural or ideal type. Flat top is most popul and widely used because it has high immunity to noise.

- the schematic diagram of flat-top sampling is shown in fig below →

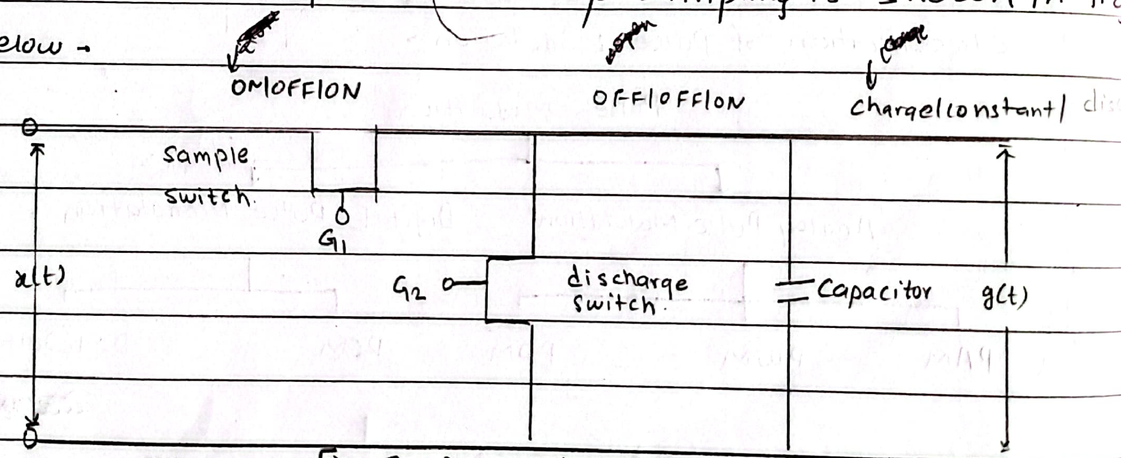
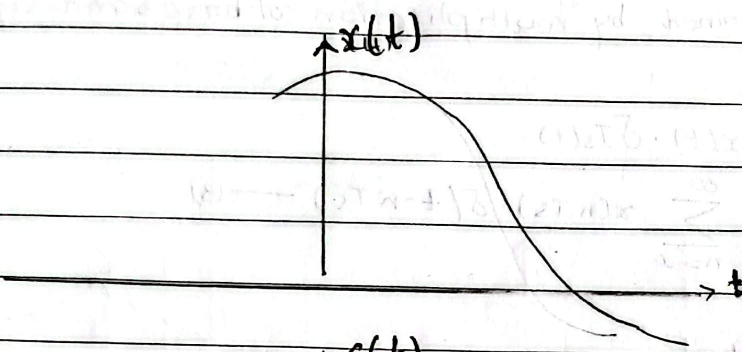


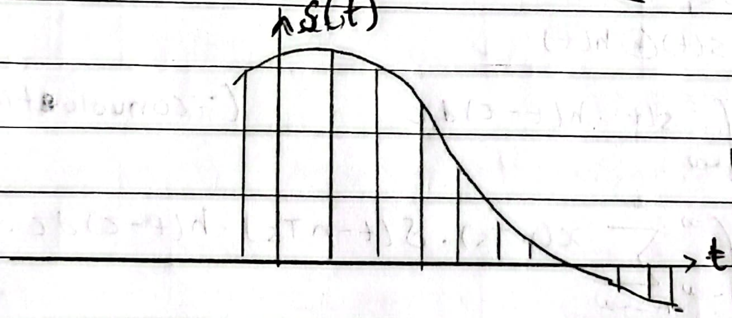
Fig: sampe and hold circuit

• Above figure shows the sample and hold circuit. It consists of tw FET switches and a capacitor. The sampling switch (G1) is closed for short duration by a short pulse applied to the Gate (G2) of transistor. During this period the capacitor 'C' is quickly Charged upto a voltage equal to the Instantaneous sample value of the in signal x(t). Now sampling switch (G_2) is opened and the capacitor holds the charge. The discharge switch is then closed by a pulse applied to Gate(G_2), of the other ~~tranist~~ transistor. The capacitor is open and thus capacitor has no voltage. Therefore, Flat top samples is produced in the output of sample and Hold circuit.

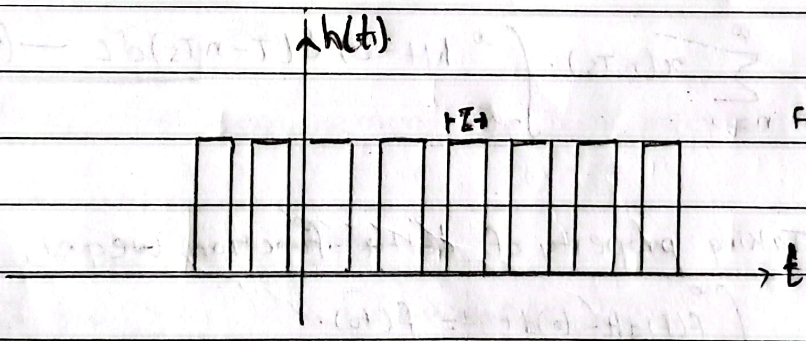
- the schematic diagram of PAM waveform using flat top sampling is shown in figure below.



Fig(a): continuous time signal x(t).



Fig(b): Instantaneous sample signal s(t).



Fig(c): Sampling function wave form (periodic pulse train).

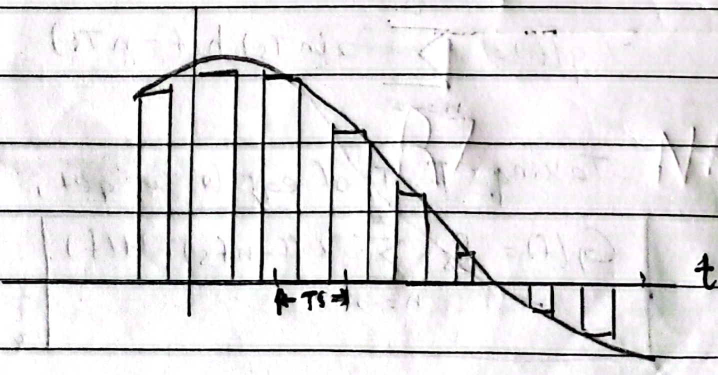


Fig: Flat top sampled signal

let the duration or width of each sample is τ and sampling rate $f_s = \frac{1}{T_s}$ $|f x(t)|$ is a baseband signal whose Instantaneous sample signal is $s(t)$ and $h(t)$ is the constant width function. Therefore, flat-top sampled signal $q(t)$ is obtained as,

$$q(t) = s(t) \circledast h(t) \quad \text{—(1)}$$

As the train of Impulses is represented as,

$$\delta_{T_s}(t) = \sum_{n=-\infty}^{\infty} \delta(T - nT_s) \quad \text{—(2)}$$

The $s(t)$ is obtained by multiplication of baseband signal $x(t)$ and $\delta_{T_s}(t)$. i.e.

$$\because s(t) = x(t) \cdot \delta_{T_s}(t).$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \delta(t - nT_s) \quad \text{—(3)}$$

Also we have,

$$g(t) = s(t) \circledast h(t)$$

$$= \int_{-\infty}^{\infty} s(t) \cdot h(t-\tau)\,d\tau \qquad (\because \text{convolution theorem}).$$

$$= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \delta(t - nT_s) \cdot h(t-\tau)\,d\tau.$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \int_{-\infty}^{\infty} h(t-\tau)\,\delta(\tau - nT_s)\,d\tau \quad \text{—(4)}$$

Taking property of delta function we get,

$$\int_{-\infty}^{\infty} f(t)\,\delta(t - t_0)\,dt = f(t_0).$$

$$\therefore q(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\,h(t - nT_s)$$

Taking F. T. of eq^n (2) we get,

$$\boxed{G(f) = f_s \cdot \sum_{n=-\infty}^{\infty} X(f - nf_s) \cdot H(f)}$$

→ Transmission Bandwidth of PAM →

In PAM signal the pulse width or duration or length 'τ' is very small in comparision to time period (sampling period) 'T_s' between two samples.

i.e. $\tau << T_s$ —— (1)

If the maximum frequency of the modulating or message signal $x(t)$, is 'f_m'. According to sampling theorem, the sampling frequency 'f_s' is given as,

$$f_s \geq 2f_m \quad \text{—— (2)}$$

$$\frac{1}{T_s} \geq 2f_m$$

$$\therefore T_s \leq \frac{1}{2f_m} \quad \text{—— (3)}$$

From eq^n (1) and (3) we get,

$$\tau << T_s \leq \frac{1}{2} f_m \quad \text{—— (4)}$$

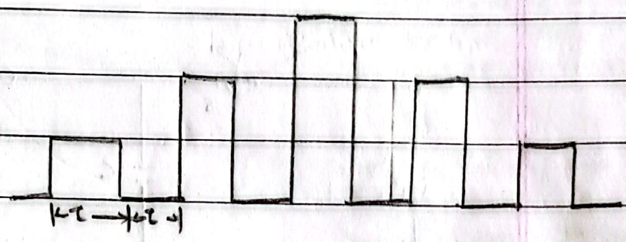


Fig: PAM Signal Illustration

For both ON and OFF time of PAM signal. The maximum frequency of PAM signal is,

$$f_{max} = \frac{1}{\tau+\tau} = \frac{1}{2\tau} \quad \text{—— (5)}$$

Therefore, BW requirement for transmission of PAM signal is given as,

$$BW \geq f_{max} \quad \text{—— (6)}$$

$$\therefore BW \geq \frac{1}{2\tau} \quad [\text{from eq}^n (5)].$$

$$\therefore BW \geq \frac{1}{2\tau} >> f_m \quad [\text{from eq}^n (4)]$$

$$\boxed{BW >> f_m}$$

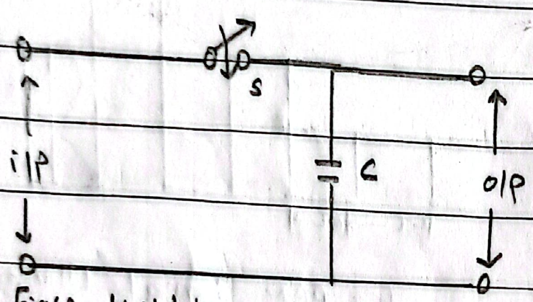
∵ Hence, this is the required expression of transmission BW of PAM.

Demodulation (Reconstruction) of PAM signal:

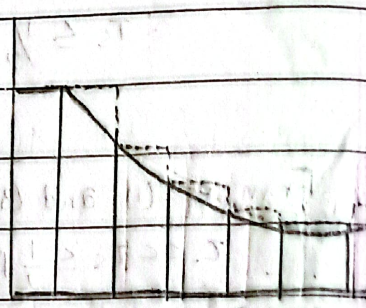
- Recovering of the modulating signal from the modulated signal is called demodulation. For PAM signal the demodulation is done by Holding circuit.

- The block diagram of PAM demodulation is shown in figure below.

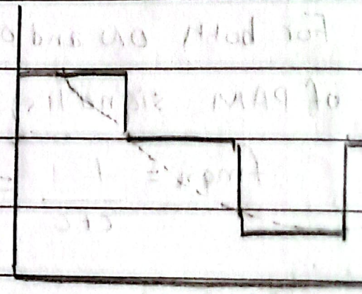
Fig: Block diagram of PAM demodulator.



Fig(a): Holding circuit (zero order holding ckt).



Fig(b): O/p of Hold ckt



Fig(c): o/p of LPF

Above figure(a), shows the simple holding circuit (zero-order holding circuit because it considers previous sample to decide the value between the two pulses). Here, the switch 's' is closed after the arrival of pulse and opened at the end of pulse. The capacitor gets charged to the pulse amplitude value and holds for the interval between two pulses. This is then passed through the LPF to reconstructed modulating signal (message signal) which is clearly illustrated by fig(b) and fig(c).

D/L Method,

[B] Pulse width/length/duration Modulation:-

- In PWM the pulse width is changed in proportion to the amplitude of the modulating signal. There are three(3) types of variation in PWM/PDM/PLM. It is fewly important in communication system.

- the schematic diagram of PWM of 3 variations is shown in figure below:-

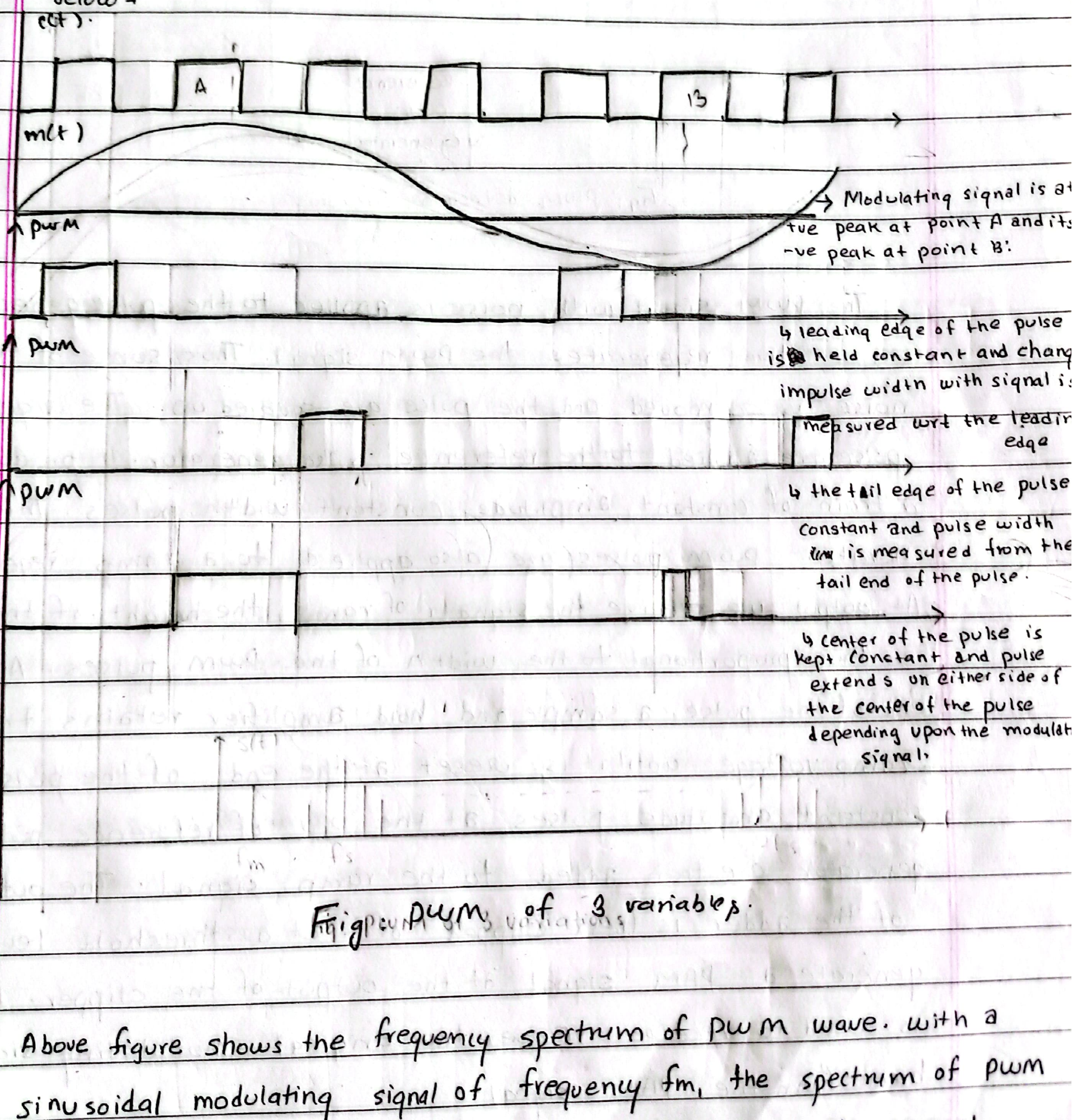


Fig: PWM of 3 variables.

Above figure shows the frequency spectrum of PWM wave. with a sinusoidal modulating signal of frequency fm, the spectrum of PWM signal consist the modulating frequency (fm) along with several-harmonics.

→ Detection of PWM signal →

The block diagram of detection of PWM signal is shown in fi... below.

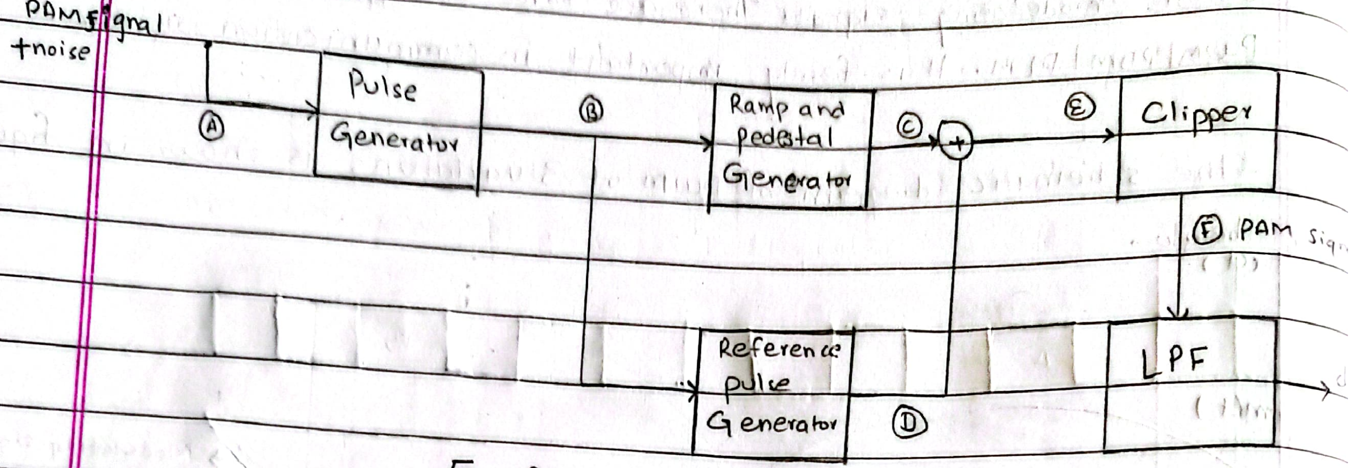


Fig: PWM detection circuit

The PWM signal with noise is applied to the pulse generato... circuit that regenerates the PWM signal. Thus, some of the noise is removed and the pulse are squared up. The regenera... pulse are applied to the reference pulse generator. It produces a train of constant amplitude, constant width pulses. The rege... nerated PWM pulses are also applied to a ramp regenerat... At output we receive the signal, of ramp, the height of the ra... is thus proportional to the width of the PWM pulses. At th... end of the pulse, a sample and hold amplifier retains the fi... ramp voltage until it is reset at the end of the pulse. constant amplitude pulses at the o/p of reference pulse generator are then added to the ramp signal. The output of the adder is then clipped off at a threshold level t... generate a PAM signal at the output of the clipper. A LP... is used to ~~recerve~~ recover the original modulating signal back from the PAM signal.

→ Generation of PWM →

the schematic diagram of PWM Generator is shown in figure below →

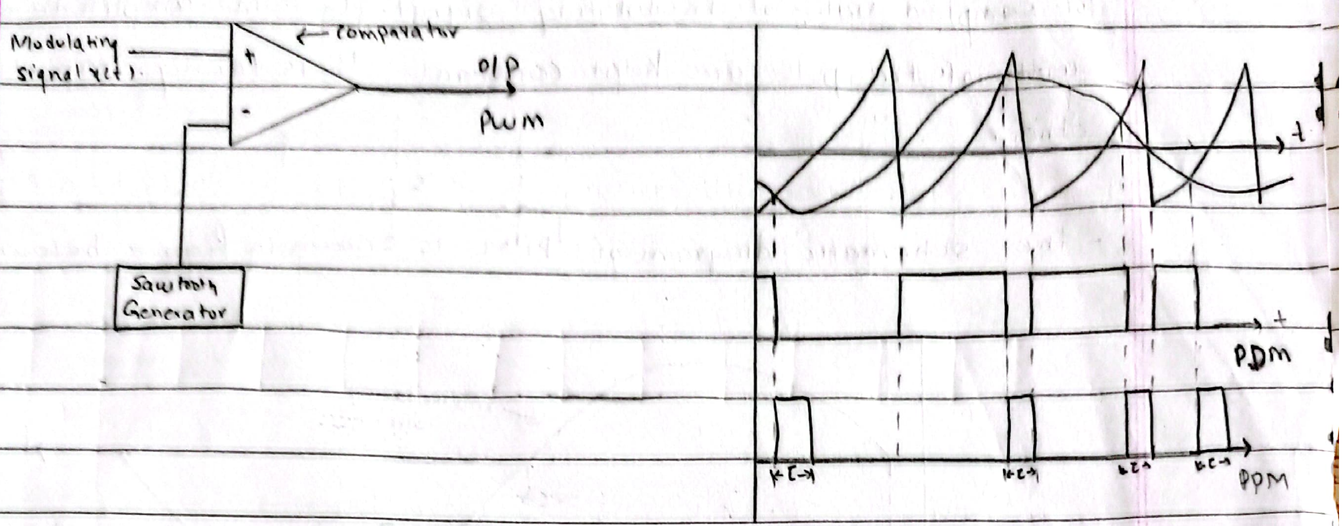


Fig: PWM Generator.

· Above figure is a PWM Generator. The sawtooth generator generates the sawtooth signal of frequency 'fs' is a sampling signal and is applied to the inverting terminal of the comparator. The message or modulating signal x(t) is applied to the Non-Inverting I/p-o/p of the comparator will be high as long as instantaneous amplitude of x(t) is higher that that of sampling signal producing PWM signal. Likewise, o/p is zero when amplitude of sampling signal is greater than the instantaneous amplitude of modulating signal x(t).

Advantages
- Better noise Immunity
- Synchronization between Tx & Rx is not required (required in PPM)
- Possible to reconstruct the signal from noise (possible in PPM).

disadvantages
- It requires large BW compared to PAM signal in order to avoid distortion
- due to variable pulse width, transmitter must be able to handle the power content of pulse width maximum width.

[c] Pulse Position Modulation →

- In PPM the position of each pulse is changed wrt the amplitude of sampled value of modulating signal. In PPM amplitude and width of the pulse are kept constant. It is fewly Important other.

- The schematic diagram of PPM is shown in figure below →

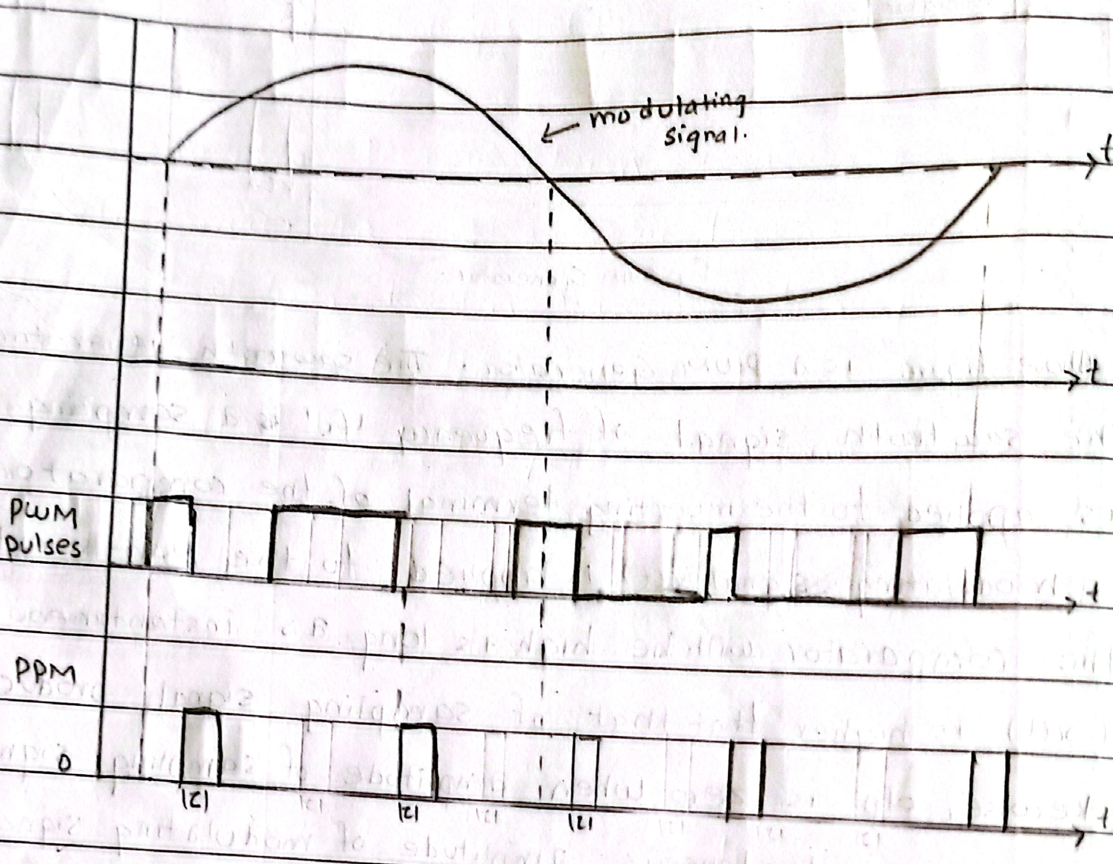


Figure : PPM

~~Demodulation of PPM~~

~~The schematic diagram of PPM demodulation is shown in figure below~~

→ Generation of PPM →

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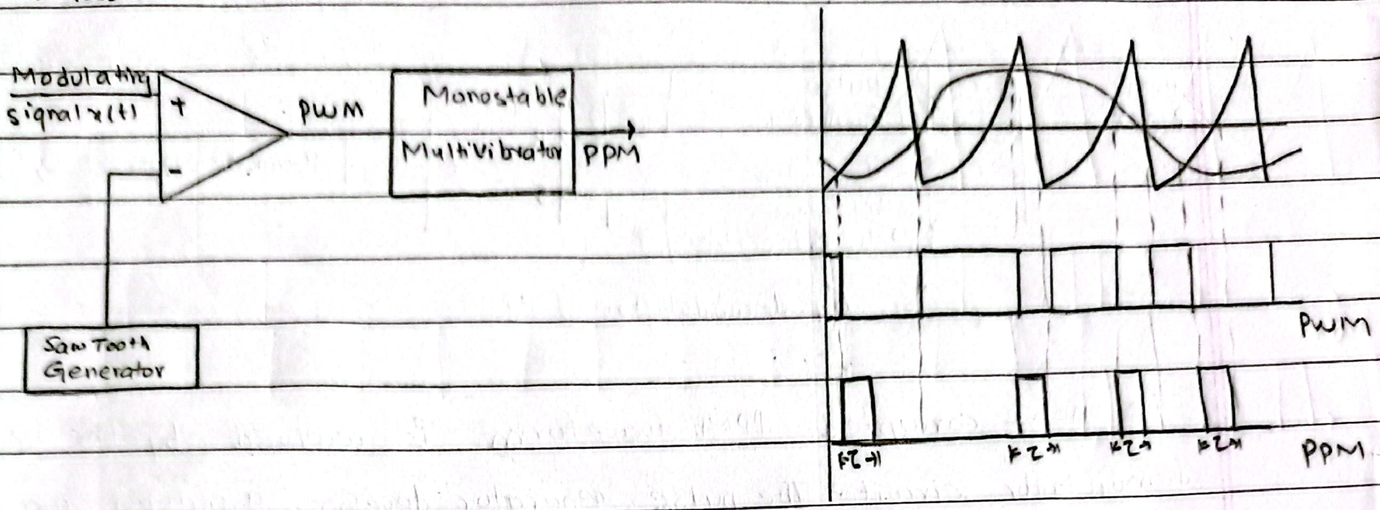

Figure: Generation of PPM

· Above figure shows the PPM Generator. The pulse width modulation signal is obtained at the o/p of the comparator which is fed to Monostable Multivibrator. The multivibrator is negative edge triggered. Triggering occurs at the trailing (falling) edge of PWM signal, which generates PPM signal with same or constant width and amplitude of modulating signal.

- advantages,

1. Like PWM, amplitude held constant in PPM results less interference of noise.
2. Signal and Noise seperation are easy.
3. Since pulse width and amplitude is constant, transmission power for each pulse is same.

→ disadvantages

1. Synchronization between Tx and Rx is required.
2. Large B/W is required compared to PAM.

- Demodulation of PPM →

The schematic diagram of PPM demodulation is shown in figure below.

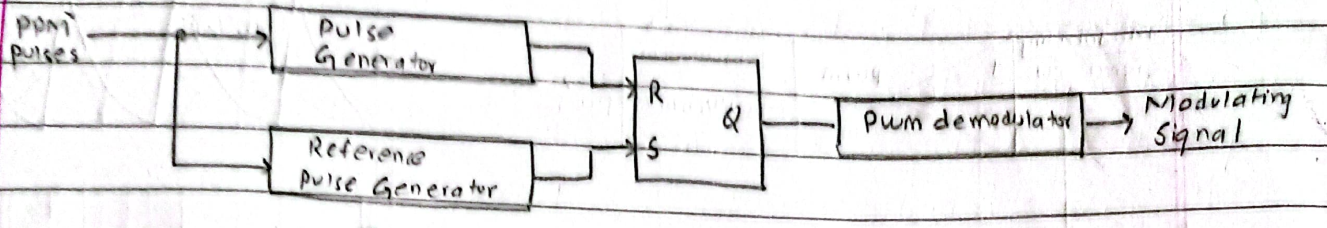

Fig: demodulation of PPM

The noise corrupted PPM waveform is received by the PPM demodulator circuit. The pulse generator develops a pulsed waveform at its output of fixed duration and applies these pulses to the ~~reset~~ reset pin (R) of a SR flip-flop. A fixed period reference pulse is generated from the incoming PPM waveform and the SR flip flop is set by the reference pulses. Due to set and reset signals applied to the flip-flop, we get a PWM signal at its output. The PWM signal can be demodulated using the PWM demodulator.

- advantages of PPM :-

(a) due to constant amplitude of PPM pulses, the information is not contained in the amplitude. Hence, the noise added to PPM signal doesnot distort the information. Thus, it has good noise immunity.

(b) It is possible to reconstruct PPM signal from the noise contaminated PPM signal. This is also possible in PWM but not possible in PAM.

(c) Due to constant amplitude of pulses, the transmitted power always remains constant. It doesnot change as it used to, in PWM.

- disadvantages of PPM :

(a) as the position of the PPM pulses is varied with respect to a reference pulse, a transmitter has to send synchronizing pulses to operate the timing circuits in the receiver. Without them, the demodulation would not be possible to acheive.

- Demodulation of PPM →

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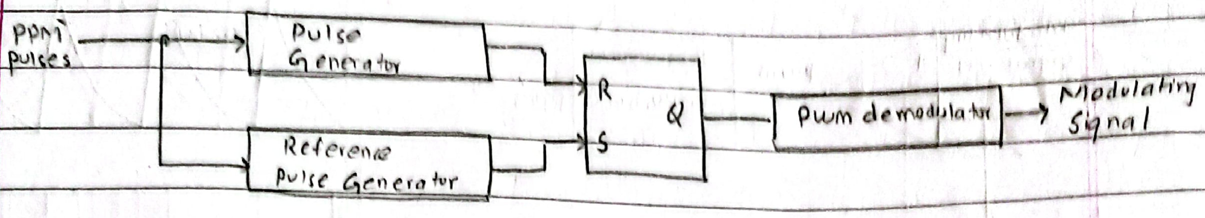

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(a) as the position of the PPM pulses is varied with respect to a reference pulse, a transmitter has to send synchronizing pulses to operate the timing circuits in the receiver. Without them, the demodulation would not be possible to acheive.

(b) Large bandwidth is required to ensure transmission of undistorted pulses.

$$\Delta P_2 = P_{2sp} - P_{2cal}$$
$$= -4 + 1.30$$
$$= -2.7$$

$$\cancel{P_{3cal} = Y_{31} V_2 V_3 \cos(\theta_{31} + \delta_2 - \delta_3) + Y_{33} V_3^2 \cos(\theta_{33} + \delta_3 - \delta_2) + Y_3}$$

$$P_3 = |Y_{31} V_1 V_3| \cos(\theta_{31} + \delta_1 - \delta_3) + |V_3 V_2 Y_{32}| \cos(\theta_{32} + \delta_2 - \delta_1) + |V_3^2 Y_{33}| \cos\theta_{33}$$

$$= 31.62 \times 1.05 \times 1.04 \cos 108.43 \cos(116.56) + 1.04^2 \times 67.23 \cos(-67.249) + 1.04 \times 1 \times 35.77$$

$$= 0.57125$$

$$\Delta P_3 = 2 - 0.57125$$
$$= \cancel{2.57125}\ 1.42875$$