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191318

OLD QUESTION SOLUTIONS

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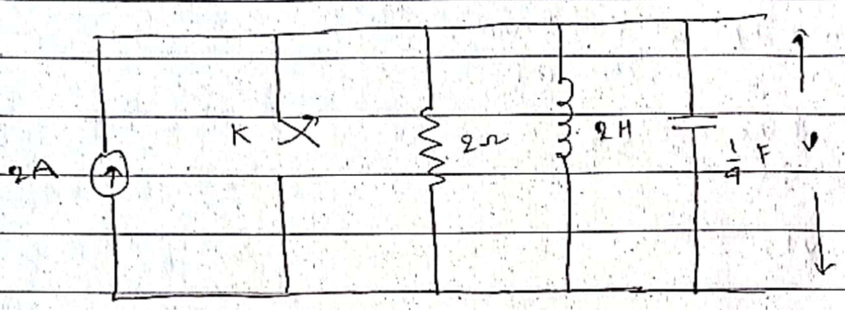
2018 Fall

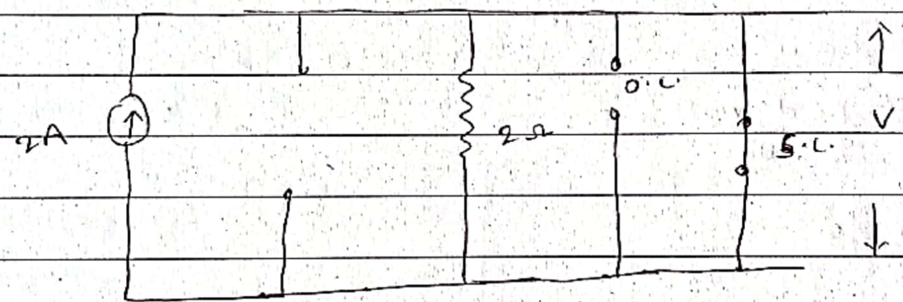
Q.1. (a) In the given RLC parallel circuit, the switch K is opened at $t = 0$. Determine

$v(0^+)$, $\frac{dv(0^+)}{dt}$, $\frac{d^2 v(0^+)}{dt^2}$



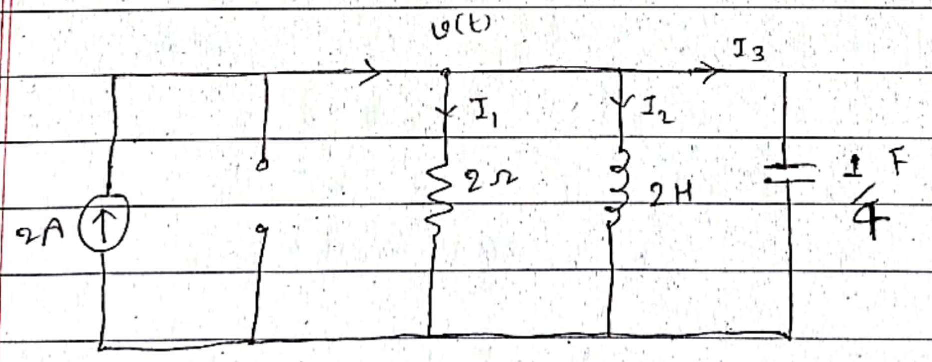
Soln

At $t = 0^+$, equivalent circuit is



$v(0^+) = 0V$ due to short circuit.

To find derivatives, consider original circuit



~~KCL @~~

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KCL @ node,

$$I_1 + I_2 + I_3 = 2$$

$$\frac{v(t)}{2} + \frac{1}{2}\int v(t)dt + \frac{1}{4}\frac{d\,v(t)}{dt} = 3$$

At $t = 0^+$

$$\frac{v(0^+)}{2} + \frac{1}{2}\int v(0^+)\,dt + \frac{1}{4}\frac{d\,v(0^+)}{dt} = 3 \quad \text{——— (1)}$$

or, $$0 + 0 + \frac{1}{4}\frac{d\,v(0^+)}{dt} = 3$$

$$\frac{d\,v(0^+)}{dt} = 12 \quad V/s$$

Diff. (1)

$$\frac{1}{2}\frac{d\,v(0^+)}{dt} + \frac{1}{2}v(0^+) + \frac{1}{4}\frac{d^2\,v(0^+)}{dt} = 3$$

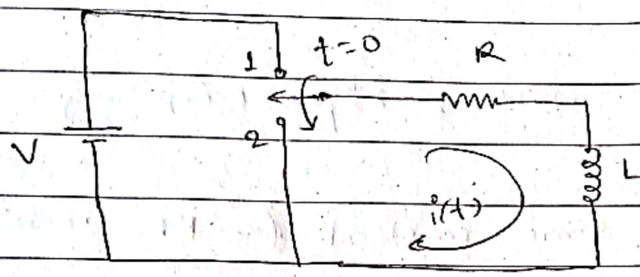
or, $$\frac{1}{\cancel{2}}\times \overset{6}{\cancel{12}} + 0 + \frac{1}{4}\frac{d^2\,v(0^+)}{dt} = 3$$

$$\frac{1}{4}\frac{d^2\,v(0^+)}{dt} = -3$$

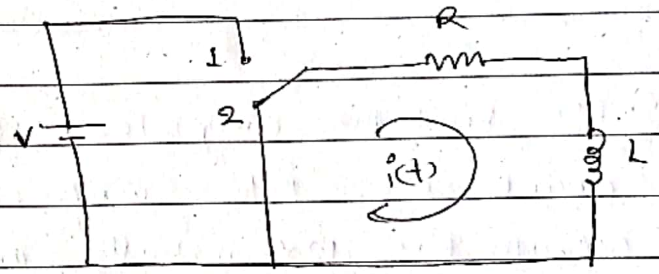
$$\frac{d^2\,v(0^+)}{dt} = -12 \quad V/s^2$$

(b) Find the response of R-L circuit connected to DC voltage source when switch is closed at $t=0$ and define the time constant for it.

=>



At $t=0$, final ckt is



Applying mesh analysis

$$-R\,i(t) - L\frac{d\,i(t)}{dt} = 0$$

$$L\frac{d\,i(t)}{dt} + R\,i(t) = 0$$

it is first order homogeneous differential equation

$$\frac{d\,i(t)}{dt} + \frac{R}{L}\,i(t) = 0$$

$$\frac{d\,i(t)}{dt} = -\frac{R}{L}\,i(t)$$

$$\frac{d\,i(t)}{i(t)} = -\frac{R}{L}\,dt$$

$$i(t) = K e^{-\frac{R}{L}t}$$ it is general solution.

To find K, we use initial position circuit.

$i(0^-) = \frac{V}{R}$ (due to no reactance)

From general eq^n

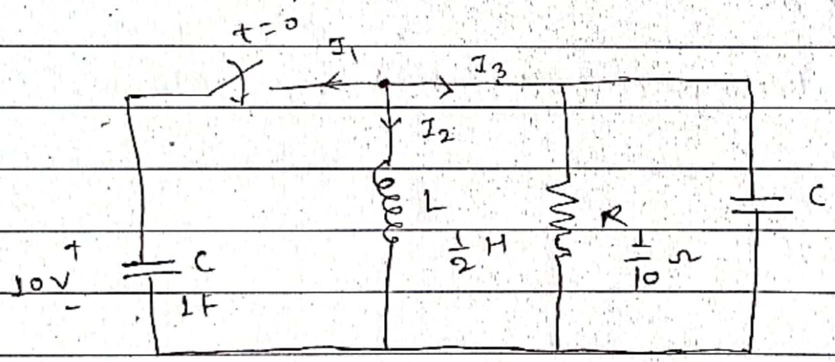
$$\frac{V}{R} = k e^{-R/L * 0}$$

$$k = \frac{V}{R}$$

Then, $i(t) = \frac{V}{R} e^{-\frac{R}{L} t}$ which is required expression.

Where $L/R = \tau$ is called time constant for RL circuit.

Q.9. (a) Find the complete response v(t) for the parallel RLC circuit shown below with R = 1/10 ohm, L = 1/2 H and C = 1 F. Capacitor has 10V initially and switch is closed at t = 0. Use classical approach.



At t = 0 apply KCL

$$I_1 + I_2 + I_3 = 0$$

$$1 \cdot \frac{d\,v(t)}{dt} + 2\int v(t)\,dt + 10\,v(t) = 0 \quad \text{——— (1)}$$

Diff w. r. to t

$$\frac{d^2 v(t)}{dt^2} + 2\,v(t) + 10\,\frac{d\,v(t)}{dt} = 0$$

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Its characteristics eqn is

$$m^2 + 2m + 10 = 0$$

$$m = \frac{-2 \pm \sqrt{4 - 40}}{2} = \frac{-2 \pm \sqrt{-36}}{2}$$

$$= \frac{-2 \pm j6}{2}$$

$$= -1 \pm j3$$

$$v(t) = e^{-t}(A \sin 3t + B \cos 3t) \quad \text{---} \; (2)$$

Initially $V(0^-) = 10V$ Then

$$10 = e^{-0}(A \sin 0 + B \cos 0)$$

$$B = 10$$

Diff (2) w.r. to t.

$$\frac{d\,v(t)}{dt} = -e^{-t}(A \sin 3t + B \cos 3t) + e^{-t}(3A \cos 3t - 3B \sin 3t) \quad \text{---} \; (3)$$

initially from (1)

$$\frac{d\,v(t)}{dt} + 0 + 10 \times 10 = 0$$

$$\frac{d\,v(t)}{dt} = -20$$

from (3) $\quad -20 = -1(B) + 1(3A)$

$$-20 = -10 + 3A$$

$$-10 = 3A \Rightarrow A = -10/3$$

Hence from (2)

$$v(t) = e^{-t}\left(-\frac{10}{3} \sin 3t + 10 \cos 3t\right)$$ As.

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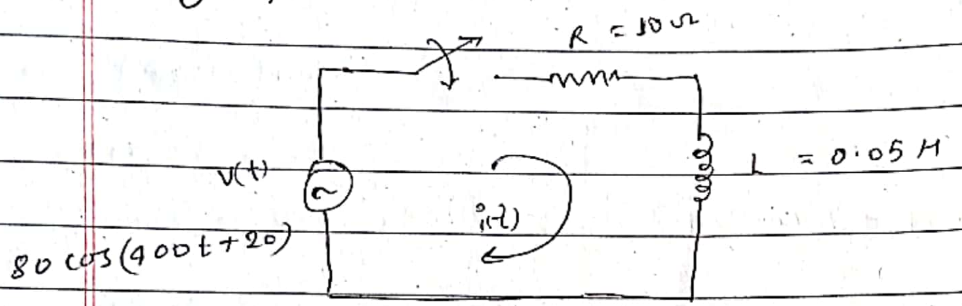
(b) An R-L series circuit is connected to an AC voltage $v(t) = 80\cos(400t + 20)$, $R = 10\,\Omega$ & $L = 0.05H$. At $t=0$, the switch K is closed. Find the equation for the current using laplace method.



Sol^n At $t=0$ applying KVL

$$10\,i(t) + 0.05\frac{d\,i(t)}{dt} = 80\cos(400t+20)$$

Using laplace transform.

$$10\,I(s) + 0.05[s\,I(s) - i(0+)] = 80\left(\frac{s\cos 20 - 400\sin 20}{s^2+400^2}\right)$$

or $$10\,I(s) + 0.05s\,I(s) = 80\,\frac{s\cos 20 - 400\sin 20}{s^2+400^2}$$

$$I(s) = \frac{80\,(s\cos 20 - 400\sin 20)}{(s^2+400^2)(10+0.05s)}$$

now,

$$\frac{80s\cos 20 - 32000\sin 20}{(s^2+400^2)(10+0.05s)} = \frac{A}{10+0.05s} + \frac{Bs+C}{s^2+400^2}$$

or $$75.17s - 10944.69 = As^2 + A.400^2 + 10Bs + 0.05Bs^2 + 10C + 0.05sC$$

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equating like terms

$A + 0.05B = 0$ ——— (1)

$10B + 0.05C = 75.17$ ——— (2)

$160000A + 10C = -10944.64$ ——— (3)

solving (1) (2) & (3)

$A = -0.13$

$B = 2.6$

$C = 983.8$

now

$$I(s) = \frac{-0.13}{10 + 0.05s} + \frac{2.6s + 983.8}{s^2 + 400^2}$$

$$I(s) = \frac{-2.6}{s + 200} + \frac{2.6s}{s^2 + 400^2} + \frac{983.8}{400} \cdot \frac{400}{s^2 + 400^2}$$

using inverse laplace transform

$$i(t) = -2.6e^{-200t} + 2.6\cos 400t + 2.46\sin 400t$$

Ans.

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Q.3 (a) What is Routh-Hurwitz criteria? Form Routh array for the following characteristics equation and state whether the system is stable or not.

$$\phi(s) = 5s^5 + 3s^4 + 2s^3 + 2s^2 + s + 1$$

⇒ Routh-Hurwitz criteria :-

let system polynomial be

$$F(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n s^0$$

Routh Table

s^n	a_0	a_2	a_4	a_6	----
s^{n-1}	a_1	a_3	a_5	a_7	----
s^{n-2}	b_1 (let)	b_2	b_3		
s^{n-3}	c_1	c_2	c_3		
⋮	⋮	⋮	⋮		
s^0	a_n				

where, $b_1 = \dfrac{a_1 a_2 - a_0 a_3}{a_1}$

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}$$

If all the elements of first column are positive then the system is stable otherwise system is unstable.

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Here $q(s) = 5s^5 + 3s^4 + 2s^3 + 2s^2 + s + 1$

s^5	5	2	1
s^4	3	2	1
s^3	$-4/3$	$-2/3$	0
s^2	$-1/2$	1	0
s^1	$1/2$	0	
s^0	1		

$= -\frac{4}{3} \times 2 + 3 + 2/3$

$- \frac{8}{3} + 2$

$-\frac{2}{3} / -4/3$

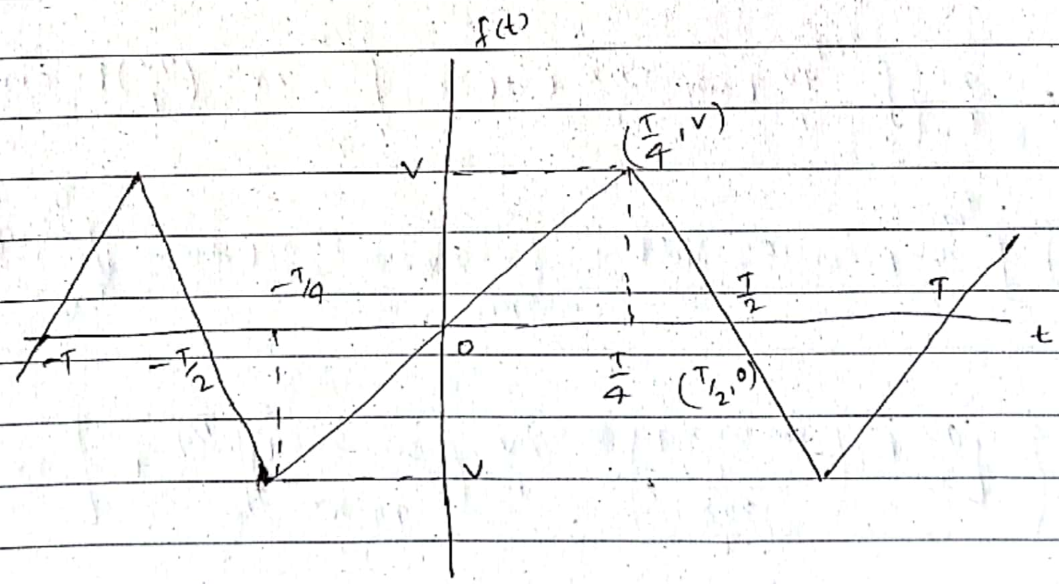
$\frac{1}{2} \times \frac{-2}{3} + 4/3$
$\overline{\quad 1/2 \quad}$

$\Rightarrow = \frac{-1/3 + 4/3}{1/2}$

$\Rightarrow$

Here all the elements in 1st row are not positive so the system is not stable.

(b) Find the following waveform; find the trigonometric fourier series expansion.



Time period $(T) = T$

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$f(t) = \frac{V}{T/4} t = \frac{4V}{T} t$ for $0 \le t \le T/4$

$y - 0 = \frac{V - 0}{\frac{T}{4} - \frac{T}{2}} (t - T/2)$

$f(t) = 2V - \left(\frac{4V}{T}\right) t$ for $\frac{T}{4} \le t \le \frac{T}{2}$

$= \frac{V}{\frac{T - 2T}{4}} \left(\frac{2t - T}{2}\right)$

$= \frac{4V}{T - 2T} \cdot \frac{2t - T}{2}$

$= \frac{2V(2t - T)}{-T}$

$= \frac{-4Vt}{T} + \frac{2VT}{T}$

$= 2V - \left(\frac{4V}{T}\right) t$

Given function is odd Hence, $a_0 = 0$, $a_n = 0$

and $b_n = \frac{4}{T} \int_0^{T/2} f(t) \sin n\omega_0 t \, dt$

$= \frac{4}{T} \left\{ \int_0^{T/4} \frac{4V}{T} t \sin \frac{2n\pi}{T} t \, dt + \int_{T/4}^{T/2} 2V - \left(\frac{4V}{T}\right) t \sin \frac{2n\pi}{T} t \, dt \right\}$

$= \frac{4}{T} \left\{ \int_0^{T/4} \frac{4V}{T} t \cdot \sin\left(\frac{2n\pi}{T}\right) t \, dt + \int_{T/4}^{T/2} 2V \sin \frac{2n\pi}{T} t \, dt - \int_{T/4}^{T/2} \frac{4V}{T} t \sin \frac{2n\pi}{T} t \, dt \right\}$

$= \frac{4}{T} \left\{ \left[\frac{4V}{T} t \left(\frac{-\cos \frac{2n\pi}{T} t}{\frac{2n\pi}{T}} \right) - \frac{4V}{T} \left(\frac{-\sin \frac{2n\pi}{T} t}{\frac{4n^2\pi^2}{T^2}} \right) \right]_0^{T/4} + \left[2V \left(\frac{-\cos \frac{2n\pi}{T} t}{\frac{2n\pi}{T}} \right) \right]_{T/4}^{T/2} \right.$

$\left. - \left[\frac{4V}{T} t \left(\frac{-\cos \frac{2n\pi}{T} t}{\frac{2n\pi}{T}} \right) - \frac{4V}{T} \left(\frac{-\sin \frac{2n\pi}{T} t}{\frac{4n^2\pi^2}{T^2}} \right) \right]_{T/4}^{T/2} \right\}$

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$$= \frac{4}{T}\left\{ \frac{4V}{T}\times\frac{T}{4}\cdot\frac{-\cos\frac{2n\pi}{T}\times\frac{T}{4}}{\frac{2n\pi}{T}} + \frac{4V}{T}\,\frac{\sin\frac{2n\pi}{T}\times\frac{T}{2}}{\frac{4n^2\pi^2}{T^2}} + 0 - 0 + \right.$$

$$\frac{2V\cdot -\cos\frac{2n\pi}{T}\times\frac{T}{2}}{\frac{2n\pi}{T}} - \frac{2V\cdot -\cos\frac{2n\pi}{T}\times\frac{T}{4}}{\frac{2n\pi}{T}} - \frac{4V}{T}\times\frac{T}{2}\cdot\frac{-\cos\frac{2n\pi}{T}\times\frac{T}{2}}{\frac{2n\pi}{T}}$$

$$\left. + \frac{4V}{T}\cdot\frac{-\sin\frac{2n\pi}{T}\times\frac{T}{2}}{\frac{4\pi^2n^2}{T^2}} + \frac{4V}{T}\times\frac{T}{4}\cdot\frac{-\cos\frac{2n\pi}{T}\times\frac{T}{4}}{\frac{2n\pi}{T}} - \frac{4V}{T}\cdot\frac{-\sin\frac{2n\pi}{T}\times\frac{T}{4}}{\frac{4n^2\pi^2}{T^2}} \right\}$$

$$= \frac{4}{T}\left\{ -\frac{2V\cos\left(\frac{n\pi}{2}\right)}{\frac{2n\pi}{T}} + \frac{4V}{T}\,\frac{\sin\left(\frac{n\pi}{2}\right)}{\frac{4n^2\pi^2}{T^2}} - \frac{2V\cos(n\pi)}{\frac{2n\pi}{T}} + \frac{2V\cos\left(\frac{n\pi}{2}\right)}{\frac{2n\pi}{T}} + \right.$$

$$\left. 2V\,\frac{\cos(n\pi)}{\frac{2n\pi}{T}} + \frac{4V}{T}\,\frac{\sin\left(\frac{n\pi}{2}\right)}{\frac{4n^2\pi^2}{T^2}} \right\}$$

$$= \frac{4}{T}\cdot 2\cdot\frac{4V}{T}\cdot\frac{\sin\left(\frac{n\pi}{2}\right)}{4n^2\pi^2}\times T^2$$

$$= \frac{8\sin\left(\frac{n\pi}{2}\right)}{n^2\pi^2}$$

Hence fourier series is

$$f(n) = \sum_{n=1}^{\infty} \frac{8\sin\left(\frac{n\pi}{2}\right)}{n^2\pi^2}\sin n\omega_0 t$$

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Q.4 (a) plot the straight line asymptotic magnitude response for

$$G(s) = \frac{40(s+10)}{s(s+1)(s^2+7s+16)}$$

⇒ step-1. convert into $\left(1 + \frac{s}{\text{something}}\right)$ form

$$= \frac{40 \times 10\left(1 + \frac{s}{10}\right)}{s\left(1+\frac{s}{1}\right) 16\left(1 + \frac{7}{16}s + \frac{s^2}{16}\right)}$$

$$= \frac{25\left(1+\frac{s}{10}\right)}{s\left(1+\frac{s}{1}\right)\left(1+\frac{7}{16}s + \frac{1}{16}s^2\right)}$$

<u>step-2</u> put $s = j\omega$

$$G(s) = \frac{25\left(1+\frac{j\omega}{10}\right)}{j\omega\left(1+\frac{j\omega}{1}\right)\left(1+\frac{7j\omega}{16}+\left(\frac{j\omega}{4}\right)^2\right)}$$

<u>step-3</u> convert into db.

$$= 20\log_{10}\left(\frac{25}{j\omega}\right) + 20\log_{10}\left(1+\frac{j\omega}{10}\right) - 20\log_{10}\left(1+\frac{j\omega}{1}\right) -$$

$$40\log_{10}\left(1+\frac{7j\omega}{16}+\left(\frac{j\omega}{4}\right)^2\right)$$

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step - 4

Table

S.N.	Component	corner frequency	slope	overall slope
~~1.~~	~~$20\log_{10}\left(\frac{25}{j\omega}\right)$~~	~~None~~	-20 db/dec.	-20 db/decade.
2.	$-20\log_{10}\left(1+\frac{j\omega}{1}\right)$	1	-20 db/dec.	-40
~~3.~~	~~$+20\log_{10}\left(1+\frac{j\omega}{10}\right)$~~	~~10~~	20	~~-20~~
3.	$-40\log_{10}\left[1+\frac{7j\omega}{16}+\left(\frac{j\omega}{4}\right)^2\right]$	4	-40	-80
4.	$20\log_{10}\left(1+\frac{j\omega}{10}\right)$	10	20	-70 db/decade

starting point. $20\log_{10}\left(\frac{25}{j\omega}\right)$

$= 20\log_{10}\left(\frac{25}{0.1}\right)$

$= 47.96$ db.

For phase angle plot:

$$\phi = -90 - \tan^{-1}\left(\frac{\omega}{1}\right) - \tan^{-1}\left(\frac{7\omega}{16-\omega^2}\right) - \tan^{-1}\left(\frac{\omega}{10}\right)$$

ω	0.1	0.2	0.4	0.8	1	2	4	6
ϕ	-98.79	-107.96	-127.11	-153.26	-165.72	-219.14		

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(b) Write three necessary and sufficient conditions of PRF.
Check given function is PRF or not

$$\frac{(s+2)(s+4)}{(s+1)(s+3)}$$

$\Rightarrow$ Necessary and sufficient conditions.

1. All the poles and zeros must lie on the left half of s-plane.
2. The poles and zeros may lie on the $j\omega$ but the residue must be positive.
3. Real $M_1 M_2 - N_1 N_2 \geq 0$ for all ω

M_1 = even part of numerator
N_1 = odd term of numerator
M_2 = even term of denomenator
N_2 = odd term of denomenator.

Here,
$$H(s) = \frac{(s+2)(s+4)}{(s+1)(s+3)} = \frac{s^2+6s+8}{s^2+4s+3}$$

(a) All the coefficient are positive.

(b) $s+1=0 \Rightarrow s=-1$
$s+3=0 \Rightarrow s=-3$

All the poles lie on $-$ve real axis
No residue or Hurwitz test required.

(c) $M_1 M_2 - N_1 N_2$

$= (s^2+8)(s^2+3) - 6s \times 4s$
$= s^4 + 11s^2 + 24 - 24s^2$
$= s^4 - 13s^2 + 24$

put $s = j\omega$

$= (j\omega)^4 - 13(j\omega)^2 + 24$

$= \omega^4 + 13\omega^2 + 24$

> 0

for all ω

(d) power difference < 1

Hence given function is positive real function.

Q.5. (a) The driving point impedance of a LC network is given by

$$Z(s) = \frac{10(s^2+4)(s^2+10)}{s(s^2+9)}$$

obtain the Foster I form of realization.

⇒ Foster I form

$$Z(s) = \frac{10(s^2+4)(s^2+10)}{s(s^2+9)}$$

$$= \frac{10(s^4+14s^2+40)}{s^3+9s}$$

$$= \frac{10s^4+140s^2+400}{s^3+9s}$$

~~Here power of~~ numerator > denominator so we use ~~actual division~~ method.

$$s^3+9s \,\big)\; 10s^4 + 140s^2 + 400 \;\big(\; 10s$$
$$\underline{-\;10s^4 + 90s^2}$$
$$50s^2 + 400$$

$$Z(s) = 10s + \frac{50s^2+400}{s^3+9s}$$

now,
$$\frac{50s^2+400}{s(s^2+9)} = \frac{A}{s} + \frac{Bs+C}{s^2+9}$$
$$= As^2 + 9A + Bs^2 + Cs$$

equating coefficients.

$A+B = 50$

$C = 0$

$9A = 400$

$A = \frac{400}{9}$

$B = 50 - \frac{400}{9} = \frac{50}{9}$

$$Z(s) = 10s + \frac{400/9}{s} + \frac{50/9\, s}{s^2+9}$$

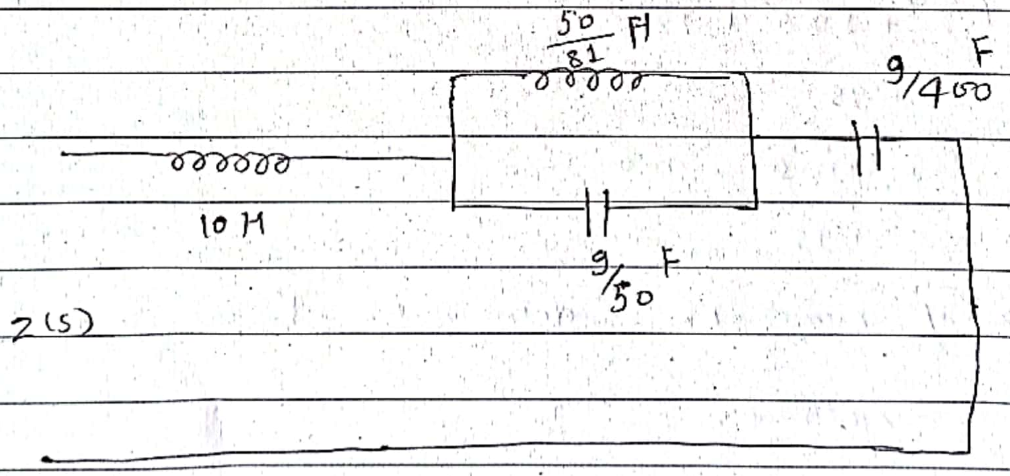

fig. foster I form.

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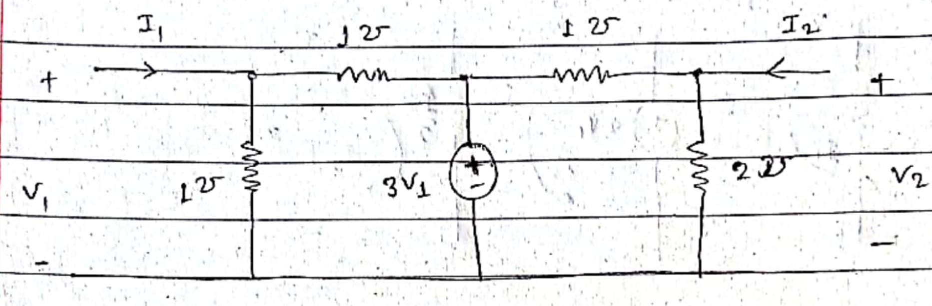
(b) Determine admittance parameters for the circuit given below:

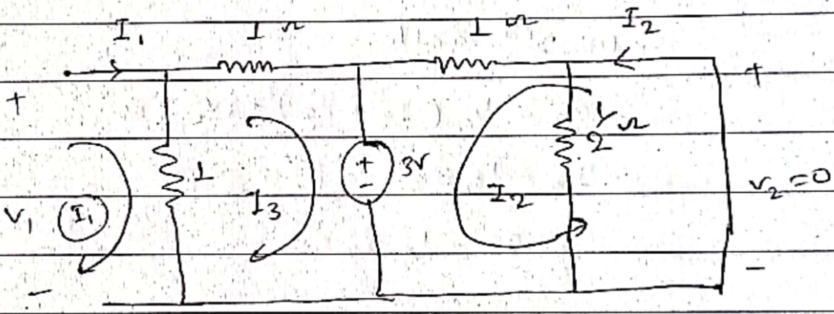


Admittance, $I_1 = y_{11}V_1 + y_{12}V_2$

$I_2 = y_{21}V_1 + y_{22}V_2$

let, $V_2 = 0$

i.e. short circuiting o/p port



In 1st mesh

$-(I_1 - I_3) + V_1 = 0$

$V_1 = I_1 - I_3$

$I_3 = I_1 - V_1$ —— (1)

In 2nd mesh

$-I_3 - 3V_1 - (I_3 - I_1) = 0$

$3V_1 = -2I_3 + I_1$ —— (2)

$3V_1 = -2(I_1 - V_1) + I_1$

or, $3V_1 = -2I_1 + 2V_1 + I_1$

or, $3V_1 = -I_1 + 2V_1$

$V_1 = -I_1$

$\frac{I_1}{V_1} = -1$

$\Rightarrow y_{11} = -1℧$

In 3rd mesh

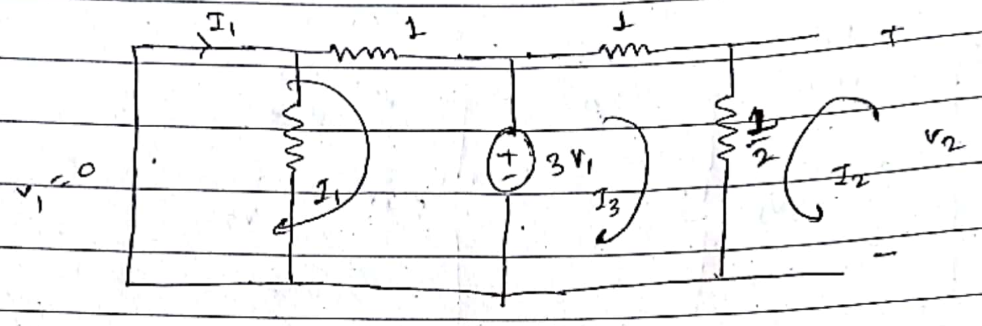
$-I_2 - 3V_1 = 0$

$-I_2 = 3V_1$

$\frac{I_2}{V_1} = -3$

$\Rightarrow y_{21} = -3℧$

let $V_1 = 0$ i.e. short circuited I/p port



In 1st mesh

$-I_1 - 3V_1 = 0$

$I_1 = 0$

$\Rightarrow y_{12} = \frac{I_1}{V_1} = 0$

In 2nd mesh

$-I_3 - \frac{1}{2}(I_3 + I_2) + 3V_1 = 0$

$-I_3 - \frac{1}{2} I_3 - \frac{1}{2} I_2 = 0$

$-\frac{3}{2} I_3 - \frac{1}{2} I_2 = 0$

$3I_3 = -I_2$

In 3rd mesh

$\frac{1}{2}(I_2 + I_3) - V_2 = 0$

$\frac{1}{2} I_2 + \frac{1}{2} I_3 - V_2 = 0$

$\frac{1}{2} I_2 - \frac{1}{2} \cdot \frac{I_2}{3} - V_2 = 0$

$\frac{2I_2}{2 \cdot 3} - V_2 = 0$

$\frac{I_2}{3} = V_2$

$\frac{I_2}{V_2} = 3$

$\Rightarrow y_{22} = 3\,\mho$

$\therefore$ $y_{11} = -1\,\mho$

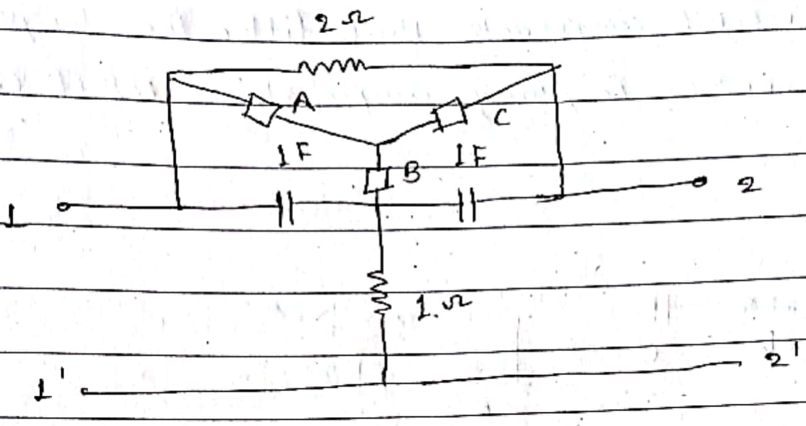
$y_{12} = 0\,\mho$

$y_{21} = -3\,\mho$

$y_{22} = 3\,\mho$

Ans

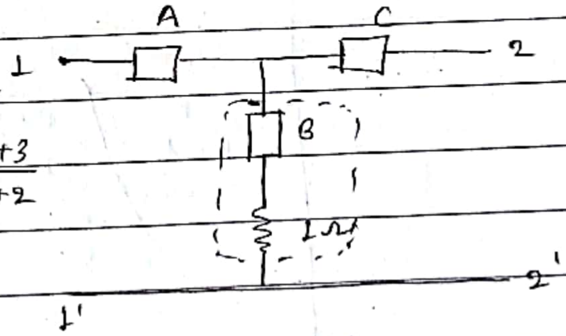
Q.6. (a) Find the equivalent T network for the given π network from the given network.



$$A = \frac{2 \times 1/s}{2 + 1/s + 1/s} \Rightarrow \frac{2/s}{\frac{2s+2}{s}} = \frac{2}{2s+2} = \frac{1}{s+1}$$

$$B = \frac{1/s \times 1/s}{2 + 1/s + 1/s} = \frac{1/s\cancel{s}}{\frac{2s+2}{\cancel{s}}} = \frac{1}{2s+2}$$

$$C = \frac{2 \times 1/s}{2 + 1/s + 1/s} = \frac{2/s}{\frac{2s+2}{s}} = \frac{1}{s+1}$$

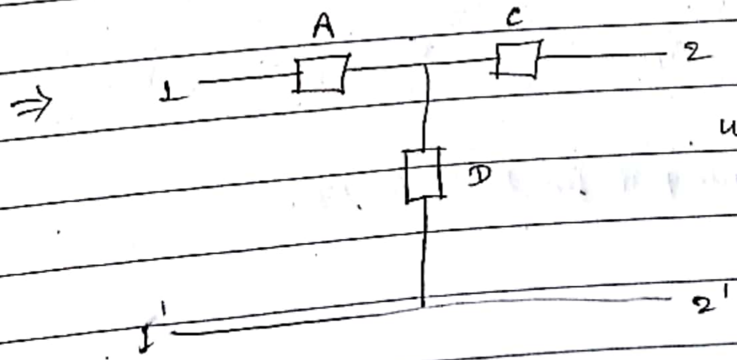


B and 1Ω are in series

gives,

$$D = 1 + \frac{1}{2s+2} \Rightarrow \frac{2s+2+1}{2s+2} \Rightarrow \frac{2s+3}{2s+2}$$

$\Rightarrow$



where, $A = \frac{1}{s+1}$

$B = \frac{1}{s+1}$

& $D = \frac{2s+3}{2s+2}$

which completes conversion.

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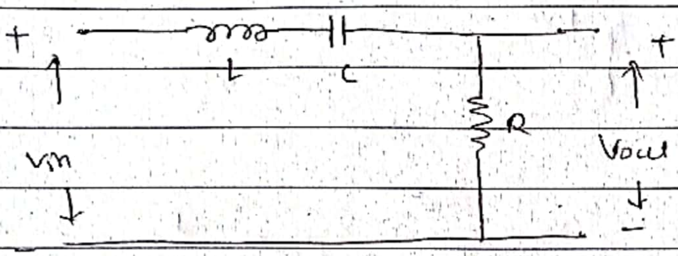
(b) Define filter. Explain the concept of band filter with magnitude plot.

⇒ Filter are the circuit components that filter the frequency components. ie. they allow certain frequency components or reject certain components.



RLC band pass filter

Band filter used to filter certain band. In band pass it allows certain band and rejects low and high band.

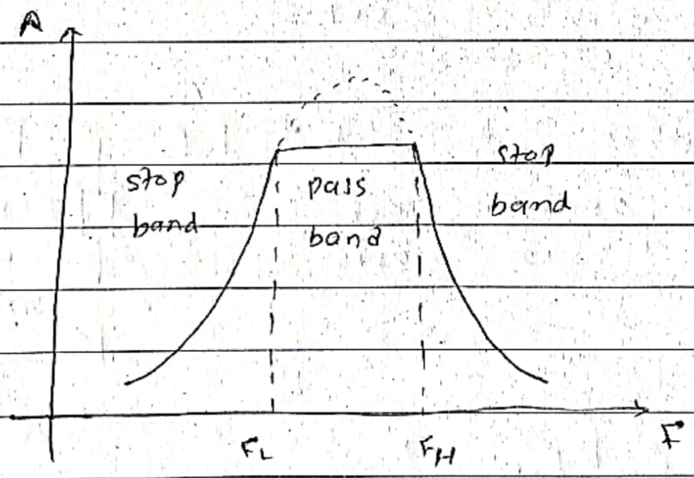


fig. magnitude plot for band filter.

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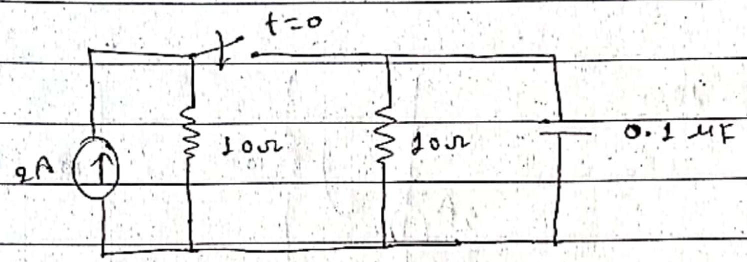
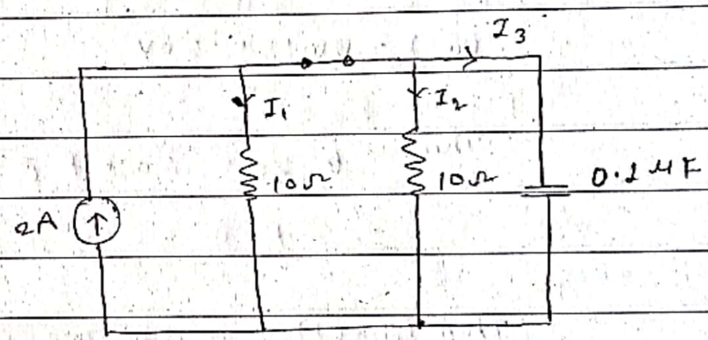
2018 spring.

1.(a) 2018 Fall. 1.(a)

(b) Find the total response $v_c(t)$ for the voltage across the capacitor in circuit given below using classical method. The switch S is closed at $t=0$.



⇒ switch S is closed at $t=0$



Applying KCL

$I_1 + I_2 + I_3 = 2$

$$\frac{v(t)}{10} + \frac{v(t)}{10} + 0.1\times10^{-3}\frac{d\,v(t)}{dt} = 2$$

$$\frac{1}{5}v(t) + 0.1\times10^{-3}\frac{d\,v(t)}{dt} = 2$$

$$10^{-4}\frac{d\,v(t)}{dt} + \frac{1}{5}v(t) = 2$$

$$\frac{d\,v(t)}{dt} + 2000\,v(t) = 20000$$

which is non homogeneous first order differential eq^n

when, $p = 2000$, $q = 20000$

$$v(t) = e^{-pt}\int q\,e^{pt}dt + ke^{-pt}$$

$$= e^{-2000t}\int 20000\cdot e^{2000t}dt + ke^{-2000t}$$

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$$v(t) = \cancel{e^{-2000t}} \cdot 20000 \cdot \frac{\cancel{e^{2000t}}}{2000} + k e^{-2000t}$$

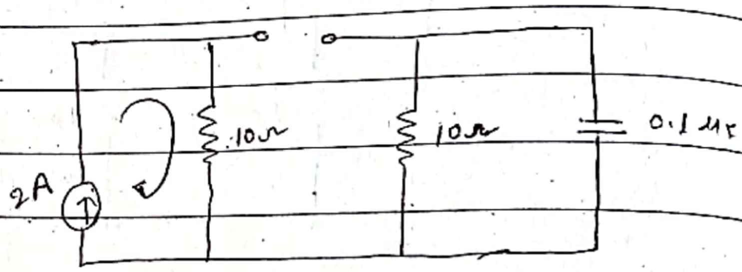
$$v(t) = 20 + k e^{-2000t} \quad \text{---} \; (1)$$

At initially, switch is open.

voltage drop across 10Ω

$= 10 \times 2$

$= 20V$



at $t = 0^-$

$v(0^-) = v(0^+) = 20V$

Then, $20 = 20 + k \cdot e^{-2000 \times 0}$

$k = 0$

Then from (1) $v(t) = 20 + 0 \times e^{-2000t}$

$v_c(t) = 20 \, V$. Ans.

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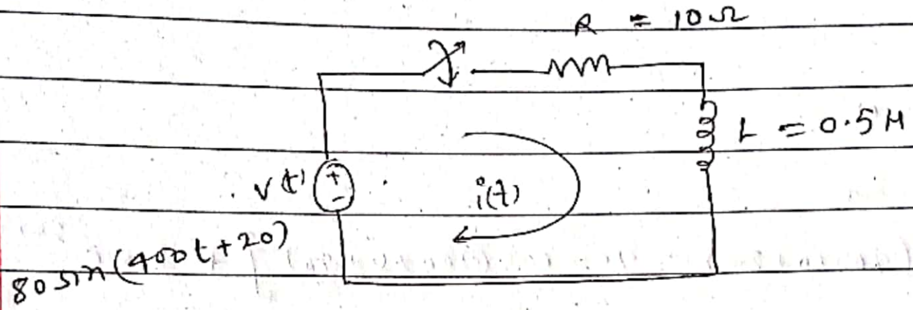
Q.2. (a) An RL circuit is connected to an AC voltage $v(t) = 80\sin(400t + 20)$ volts at $t = 0$, $R = 10\Omega$ & $L = 0.5H$. Find the equation for the current using classical method.



At $t = 0$, switch is closed.

Applying KVL,

$$10\, i(t) + 0.5 \frac{d\, i(t)}{dt} = 80\sin(400t + 20)$$

or $$\frac{d\, i(t)}{dt} + 20\, i(t) = 80\sin(400t + 20)$$

which is non homogeneous diff. eqn

where $p = 20$, $Q = 80\sin(400t + 20)$

Then

$$i(t) = e^{-20t}\int 80\sin(400t+20)\, e^{20t}\, dt + K e^{-20t}$$

$$= 80e^{-20t}\int e^{20t}\sin(400t+20)\, dt + K e^{-20t}$$

$$= 80\cancel{e^{-20t}}\, \frac{\cancel{e^{20t}}}{20^2 + 400^2}\left[20\sin(400t+20) - 400\cos(400t+20)\right]$$

$$+ K e^{-20t}$$

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$$i(t) = \frac{80}{20^2+400^2}\left[20\sin(400t+20) - 400\cos(400t+20)\right] + Ke^{-20t}$$

$$i(t) = \frac{1}{2005}\left[20\sin(400t+20) - 400\cos(400t+20)\right] + Ke^{-20t} \quad \text{--- (1)}$$

Initially, $i(0^-) = 0$

Then (1) becomes

$$\frac{1}{2005}\left[20\sin(400\times0+20) - 400\cos(400\times0+20)\right] + K\times e^{-20\times0} = 0$$

or, $\frac{1}{2005}\times -369.036 + Ke^{0}$

$K = 0.184$

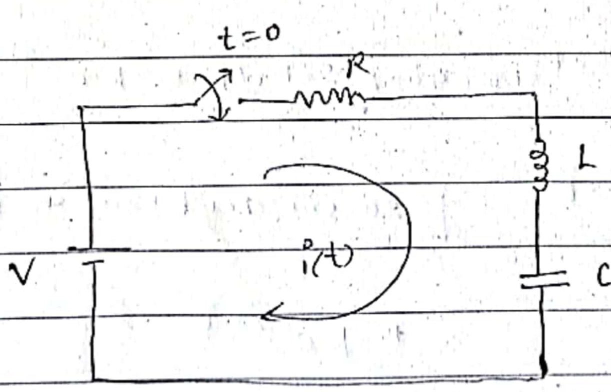
Then from (1)

$$i(t) = \frac{1}{2005}\left[20\sin(400t+20) - 400\cos(400t+20)\right] + 0.184\,e^{-20t}$$

Ans.

(b) obtain the sinusoidal response of RLC series circuit with necessary assumption.

→ consider switch is closed at $t=0$ and we have find current response.



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At $t=0$ switch is closed; applying KVL

$$-R\,i(t) - L\frac{di(t)}{dt} - \frac{1}{C}\int i(t)\,dt + V = 0$$

$$L\frac{di(t)}{dt} + R\,i(t) + \frac{1}{C}\int i(t)\,dt = V \quad \text{——— (1)}$$

Differentiating above eqn

$$L\frac{d^2 i(t)}{dt^2} + R\frac{di(t)}{dt} + \frac{1}{C}\,i(t) = 0$$

or, $$\frac{d^2 i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{RC}\,i(t) = 0$$

which is homogeneous eqn and its characteristics eqn is

$$m^2 + \frac{R}{L}m + \frac{1}{RC} = 0$$

Let m_1 and m_2 be solution of given eqn min

$$m_1, m_2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$m_1, m_2 = \frac{-\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - \frac{4}{LC}}}{2}$$

Case I when $\frac{4}{LC} < \frac{R^2}{L^2}$; we will get real and distinct root.

say m_1 and m_2

$$\Rightarrow i(t) = K_1 e^{m_1 t} + K_2 e^{m_2 t}$$

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case II when $\frac{4}{LC} = \frac{R^2}{L^2}$

we will get real and equal root

$m_1 = m_2 = m$

then

$\Rightarrow i(t) = (K_1 + K_2)\, e^{mt}$

case III when $\frac{4}{LC} > \frac{R^2}{L^2}$

we will get imaginary roots say $m_1 \pm j m_2$

$\Rightarrow i(t) = e^{m_1 t}(K_1 \cos m_2 t + K_2 \sin m_2 t)$

which is required ~~sinusoidal~~ response.

Q(3)(a) From pole zero plot, obtain the time domain response for i(t) using pole zero plot method.

$$I(s) = \frac{3s}{(s+2)(s+3)}$$

$\Rightarrow$ zeros: $s = 0$ (origin).

poles: $s+2 = 0$

$\Rightarrow s = -2$

$s+3 = 0$

$\Rightarrow s = -3$

scaling factor = 3

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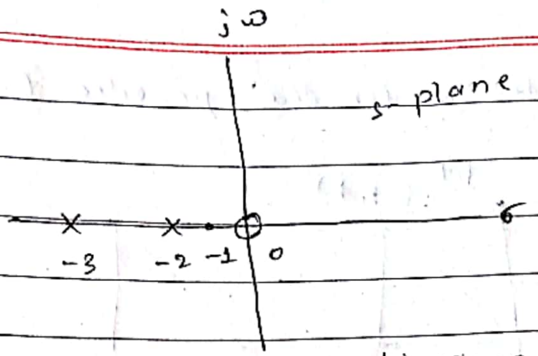

pole-zero diagram

$$I(s) = \frac{A}{s+2} + \frac{B}{s+3}$$

$$A = 3 \times \frac{2\angle 0}{1\angle 180^\circ} = -6$$

$$B = 3 \times \frac{3\angle 0}{1\angle 0} = 9$$

$$I(s) = \frac{-6}{s+2} + \frac{9}{s+3}$$

Taking inverse Laplace transform

$$i(t) = -6e^{-2t} + 9e^{-3t}$$ Ans.

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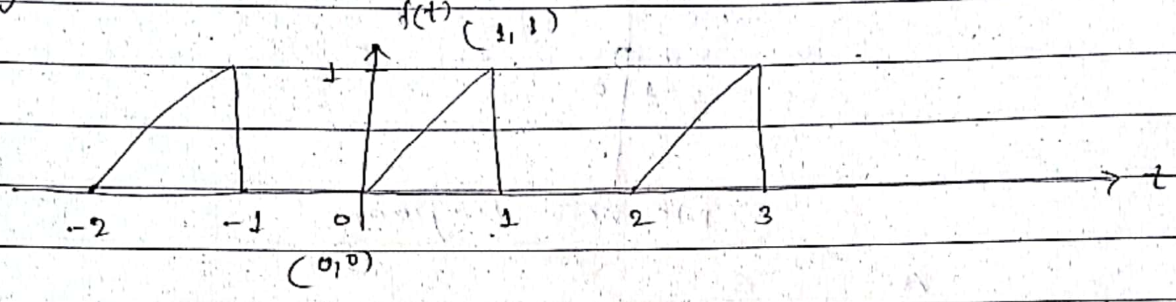
(b) Obtain the Fourier series for the periodic function shown in the figure.



Time period $(T) = 2$

$$y - 0 = \frac{1-0}{1-0}(\cancel{x}t - 0)$$

$$y = \cancel{x}t$$

$$\cancel{y}\ f(t) = t \quad \text{for} \quad 0 \leq t \leq 1$$

$$f(t) = 0 \quad \text{for} \quad 1 \leq t \leq 2$$

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt$$

$$= \frac{1}{2}\left(\int_0^1 t\,dt + \int_1^2 0\,dt\right)$$

$$= \frac{1}{2}\left[\frac{t^2}{2}\right]_0^1$$

$$= \frac{1}{2}\cdot\frac{1}{2}$$

$$= \frac{1}{4}$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_0 t\,dt$$

$$= \frac{2}{2}\int_0^1 t\cos n\omega_0 t\,dt$$

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$$a_n = 1 \left[t \, \frac{\sin n w_0 t}{n w_0} - \left(\frac{-\cos n w_0 t}{n^2 w_0^2} \right) \right]_0^1$$

$$= \left[\frac{\cancel{\sin n \frac{2\pi}{T}}^{\,0}}{n \cdot \frac{2\pi}{T}} + \frac{\cos n \frac{2\pi}{T}}{n^2 \frac{4\pi^2}{T^2}} - \frac{1}{n^2 \frac{4\pi^2}{T^2}} \right]$$

$$= \frac{T^2}{4 n^2 \pi^2} \left(\cos \frac{2n\pi}{T} - 1 \right)$$

$$= \frac{\cancel{4}}{\cancel{4} n^2 \pi^2} (\cos n\pi - 1)$$

$$= \frac{1}{n^2 \pi^2} (\cos n\pi - 1)$$

$$b_n = \frac{2}{2} \int_0^1 t \sin n w_0 t \, dt$$

$$= \left[t \cdot \frac{-\cos n w_0 t}{n w_0} - \left(\frac{-\sin n w_0 t}{n^2 w_0^2} \right) \right]_0^1$$

$$= \left(\frac{-\cos n \cdot \frac{2\pi}{2}}{n \cdot \frac{2\pi}{2}} + \frac{\cancel{\sin n \cdot \frac{2\pi}{2}}^{\,0}}{n^2 w_0^2} + 0 - 0 \right)$$

$$= \frac{-\cos n\pi}{n\pi}$$

Then Fourier series is

$$f(n) = \frac{1}{4} + \sum_{n=1}^{\infty} \frac{1}{n^2 \pi^2} (\cos n\pi - 1) \cos n w_0 t - \frac{1}{n\pi} \cos n\pi \sin n w_0 t$$

Q.4 (a) Plot the straight line asymptotic magnitude response for

$$G(s) = \frac{40(s+100)}{s(s+10)(s^2+70s+1000)}$$

⇒ Step-1 converting into $\left(1+\frac{s}{\dots}\right)$ form

$$G(s) = \frac{\cancel{40}^{4} \times \cancel{100}\left(1+\frac{s}{100}\right)}{s\times 10\left(1+\frac{s}{10}\right)\times \cancel{1000}\left[1+\frac{70}{1000}s+\left(\frac{s}{10\sqrt{10}}\right)^2\right]}$$

$$= \frac{0.4\left(1+\frac{s}{100}\right)}{s\left(1+\frac{s}{10}\right)\left(1+\frac{70}{1000}s+\left(\frac{s}{10\sqrt{10}}\right)^2\right)}$$

Step-2 put $s = j\omega$

$$G(s) = \frac{0.4\left(1+\frac{j\omega}{100}\right)}{j\omega\left(1+\frac{j\omega}{10}\right)\left(1+\frac{70}{1000}j\omega+\left(\frac{j\omega}{10\sqrt{10}}\right)^2\right)}$$

Step-3 converting into db.

$$= 20\log_{10}\left(\frac{0.4}{j\omega}\right) - 20\log_{10}\left(1+\frac{j\omega}{10}\right) - 40\log_{10}\left(1+\frac{70}{1000}j\omega+\left(\frac{j\omega}{10\sqrt{10}}\right)^2\right) + 20\log_{10}\left(1+\frac{j\omega}{1000}\right)$$

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Step-4 Table

S.N.	Component	Corner freq.	Slope	Overall slope
1.	$20\log_{10}\left(\frac{0.4}{j\omega}\right)$	None	-20	-20
2.	$20\log_{10}\left(1+\frac{j\omega}{10}\right)$	10	-20	-40
3.	$40\log_{10}\left[1+\frac{70}{1000}j\omega+\left(\frac{j\omega}{10\sqrt{10}}\right)^2\right]$	$10\sqrt{10}$	-40	-80
4.	$20\log_{10}\left(1+\frac{j\omega}{100}\right)$	100	20	-60

starting point $\Rightarrow 20\log_{10}\left(\frac{0.4}{0.1}\right)$

$\Rightarrow 12.041$ db.

Step 5. phase angle

$$\phi = -90 - \tan^{-1}\left(\frac{\omega}{10}\right) - \tan^{-1}\left(\frac{70\omega}{1000-\omega^2}\right) + \tan^{-1}\left(\frac{\omega}{100}\right)$$

ω	0.1	0.2	0.4	0.8	1	2	4
ϕ	-90.92	-91.83	-93.66	-97.32	-99.14	-108.16	-125.39

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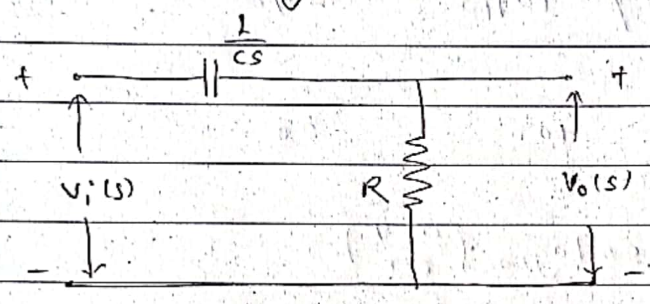
(b) Explain the concept of high pass, low pass and band pass filter with neat sketch.

$\Rightarrow$

(i) High pass filter

It only passes high frequency components. ie it rejects all low frequency components.

s-domain circuit diagram is shown below.



It consists two passive elements capacitor and resistor which are connected in series.

Transfer function is:-

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{R}{R + \frac{1}{CS}}$$

$$H(s) = \frac{CSR}{1 + CSR}$$

$$H(j\omega) = \frac{j\omega CR}{1 + j\omega CR}$$

$$\text{magnitude}, |H(j\omega)| = \frac{\omega CR}{\sqrt{1 + (\omega CR)^2}}$$

At $\omega = 0$, magnitude $= 0$

At $\omega = \frac{1}{CR}$ magnitude $= 0.707$

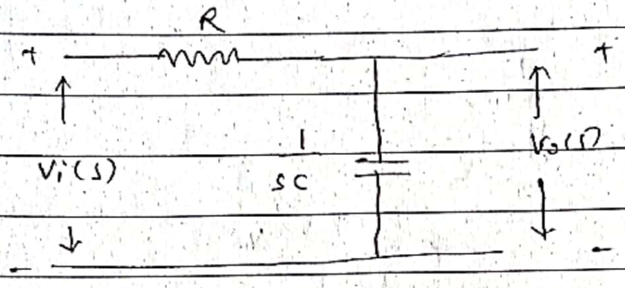
At $\omega \to \infty$, magnitude $= 1$

Therefore, magnitude of transfer function of High pass filter will vary from 0 to 1 as ω varies from 0 to ∞.

(ii) Low pass filter

It allows only low frequency components & rejects all other high frequency components.

circuit diagram:



Resistor and capacitor are connected in series. I/p voltage is applied across the entire combination and O/p is considered as the voltage across capacitor.

Transfer function:

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{1 + sCR}$$

$$H(j\omega) = \frac{1}{1 + j\omega CR}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega CR)^2}}$$

At $\omega = 0$, $|H(j\omega)| = 1$

At $\omega = \frac{1}{CR}$ $|H(j\omega)| = 0.707$

At $\omega = \infty$ $|H(j\omega)| = 0$

$|H(j\omega)|$ varies from 1 to 0 as ω varies from 0 to ∞.

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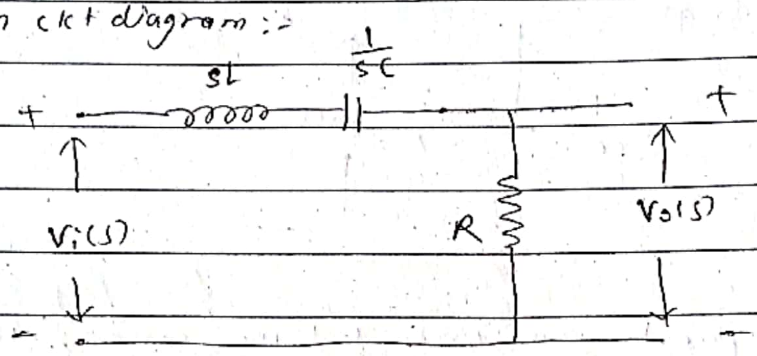
(iii) Band pass filter

It allows only one band of frequencies. This frequency band lies between low frequency range and high frequency range. i.e. this filter rejects both high and low frequency components.

s-domain ckt diagram :-



Here inductor, capacitor and resister are connected in series. I/P voltage is applied across entire combination and the O/P is considered as the voltage across resistor.

Transfer function:

$$H(s) = \frac{R}{R + \frac{1}{sC} + sL} = \frac{sCR}{1 + s^2LC + sCR}$$

$$H(j\omega) = \frac{j\omega CR}{1 - \omega^2 LC + j\omega CR}$$

$$|H(j\omega)| = \frac{\omega CR}{\sqrt{(1 - \omega^2 LC)^2 + (\omega CR)^2}}$$

At $\omega = 0$, magnitude $= 0$

At $\omega = \frac{1}{\sqrt{LC}}$, magnitude $= 1$

At $\omega = \infty$, magnitude $= 0$

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Q.5. (a) Write the necessary and sufficient condition for a function to be positive real function (prf). Check the given function for prf.

$$M(s) = \frac{s^2+6s+8}{s^2+4s+3}$$

(a) All the coefficients are positive.

(b) $$M(s) = \frac{(s+4)(s+2)}{(s+3)(s+1)}$$

$s+3=0 \Rightarrow s=-3$

$s+1=0 \Rightarrow s=-1$

All poles lies on -ve real axis. No residue or Hurwitz test required.

(c) $M_1M_2 - N_1N_2$

$= (s^2+8)(s^2+3) - 6s \times 4s$

$= s^4+11s^2+24 - 24s^2$

$= s^4 - 13s^2 + 24$

put $s = j\omega$,

$= (j\omega)^4 - 13(j\omega)^2 + 24$

$= \omega^4 + 13\omega^2 + 24$

> 0

for all ω.

(d) power difference < 1

Hence given function is prf.

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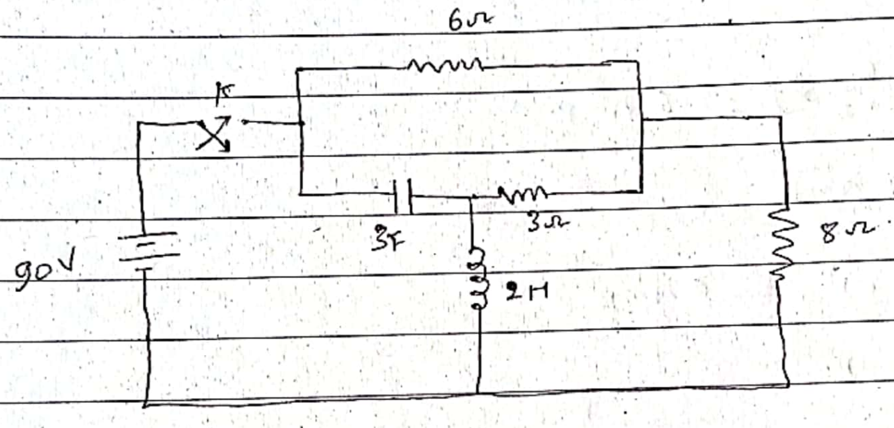
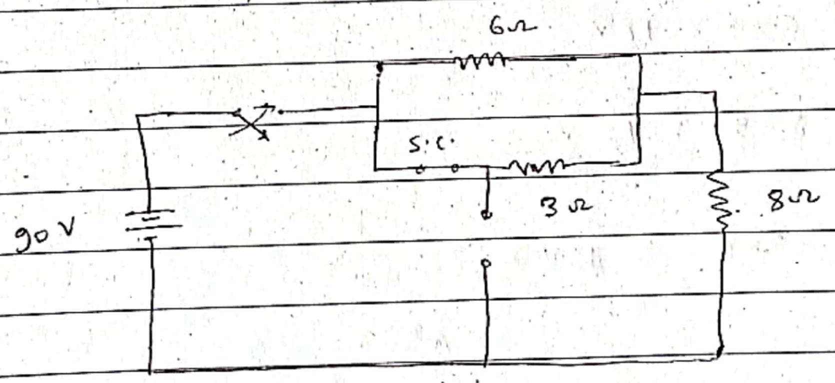
Assesment set:

1.(a) In the given circuit switch K is closed at $t=0$, find $i(0^+)$, $i_{6\Omega}(0^+)$, $i_{3\Omega}(0^+)$, $i_{8\Omega}(0^+)$, $i_L(0^+)$, $i_C(0^+)$, $V_L(0^+)$, $V_{6\Omega}(0^+)$, $V_{8\Omega}(0^+)$ and $V_{3\Omega}(0^+)$.



Initially switch is open. at $t=0^-$ so inducter acts as open circuit and capacitor acts as short circuit.



equivalent resistance/impedance

$$R_{eq} = 8 + 6 \parallel 3$$

$$= 8 + \frac{6\times3}{6+3}$$

$$= 10\,\Omega$$

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Then after just switching.

$$i(0^+) = \frac{V}{R} = \frac{90}{10} = 9\,Amp.$$

$$i_{6\Omega}(0^+) = \frac{3}{6+3} \times 9 = 3\ Amp.$$

$$i_{3\Omega}(0^+) = \frac{6}{6+3} \times 9 = 6\ Amp.$$

$$i_{8\Omega}(0^+) = 9\ Amp$$

$$i_c(0^+) = i_{3\Omega}(0^+) = 6\ Amp.$$

$$i_L(0^+) = 0\ Amp.$$

Now,

$$V_L(0^+) = 0\ V$$

$$V_{6\Omega}(0^+) = 6 \times 3 = 18\ V$$

$$V_{8\Omega}(0^+) = 8 \times 9 = 72\ V$$

$$V_{3\Omega}(0^+) = 3 \times 6 = 18\ V$$

Ans.

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(b) Determine the solution of current differential equation $i''(t) + 3i'(t) + 2i(t) = 4e^t$, initial conditions $i(0^+) = 1$, $i'(0^+) = -1$. Use classical method.

Sol

Here,

$$\frac{d^2 i(t)}{dt^2} + 3\frac{di(t)}{dt} + 2i(t) = 4e^t \quad \text{——— (1)}$$

Its solution is

$$i(t) = i_c(t) + i_p(t) \quad \text{——— (2)}$$

(i) Complementary solution.

homogeneous eqⁿ is

$$\frac{d^2 i(t)}{dt^2} + 3\frac{di(t)}{dt} + 2i(t) = 0$$

its auxilary ~~solution~~ eqⁿ

$$m^2 + 3m + 2 = 0$$

$$m^2 + m + 2m + 2 = 0$$

$$m(m+1) + 2(m+1) = 0$$

$$(m+1)(m+2) = 0$$

$$m = -1, -2$$

$$i_c(t) = Ae^{-t} + Be^{-2t} \quad \text{——— (3)}$$

(ii) For particular solution

$$Q = 4e^t$$

$$i_p = ke^t$$

$$i_p' = ke^t$$

$$i_p'' = ke^t$$

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Then from eqⁿ ①

$$ke^t + 3ke^t + 2ke^t = 9e^t$$

$$6ke^t = 9e^t$$

$$k = 9/6 = 2/3$$

$i_p = \frac{2}{3}e^t$ —— ④

Then from ②

$i = i_c + i_p$

$i = Ae^{-t} + Be^{-2t} + \frac{2}{3}e^t$ which is general solⁿ —— ⑤

$i' = -Ae^{-t} - 2Be^{-2t} + 2/3\, e^t$

$i(0^+) = 1$

$1 = Ae^{-0} + Be^0 + 2/3\, e^0$

$1 = A + B + 2/3$

$A + B = 1 - 2/3 = 1/3$

$i'(0^+) = -1$

$-1 = -A - 2B + 2/3$

$A + 2B = 2/3 + 1 = 5/2$

$B = 5/2 - 1/3 = 13/6$

$A = 1/3 - 13/6$

$= \frac{2-13}{6}$

$= -11/6$

Then from ⑤

$i(t) = \frac{-11}{6}e^{-t} + \frac{13}{6}e^{-2t} + \frac{2}{3}e^t$

Ans

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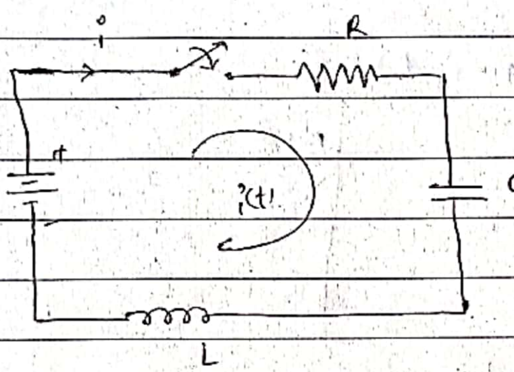
Q.2.(a) Derive an expression for current i(t) for second order networks consisting of resistance R, inductance L Henry and capacitance C Farad. When a step voltage V volts is applied at time t=0. Determine the natural frequency and damping coefficient of the network. Also sketch it.



Consider RLC n/w with step voltage V.

Applying KVL,

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt = V$$

differentiating we have,

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0$$

This is second order homogeneous diff. eqn

Its auxiliary eqn is

$$Lm^2 + Rm + \frac{1}{C} = 0$$

$$m_1 = -\frac{R}{2L} + \frac{\sqrt{(R^2 - 4L/C)}}{2L} = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$m_2 = -\frac{R}{2L} - \frac{\sqrt{(R^2 - 4L/C)}}{2L} = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

Here,

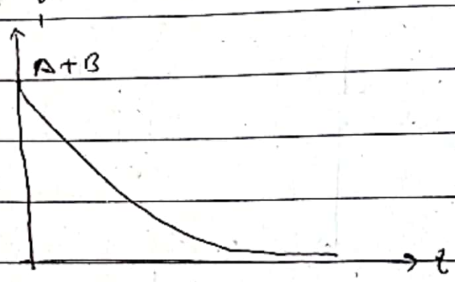
$\alpha = \frac{R}{2L}$ is called damping coefficient

$w_0 = \sqrt{\frac{1}{LC}}$ is rasmont frequency

m_1 and m_2 are called natural frequencies.

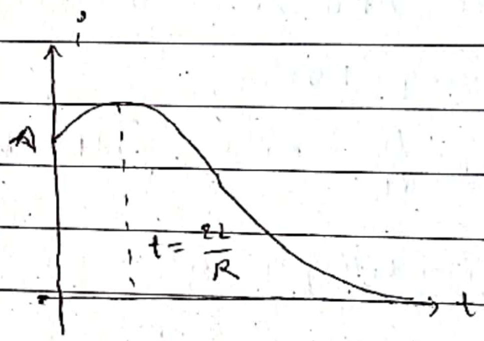
Case-1 $R^2 > \frac{4L}{C}$ (overdamped)

$$i(t) = A e^{m_1 t} + B e^{m_2 t}$$



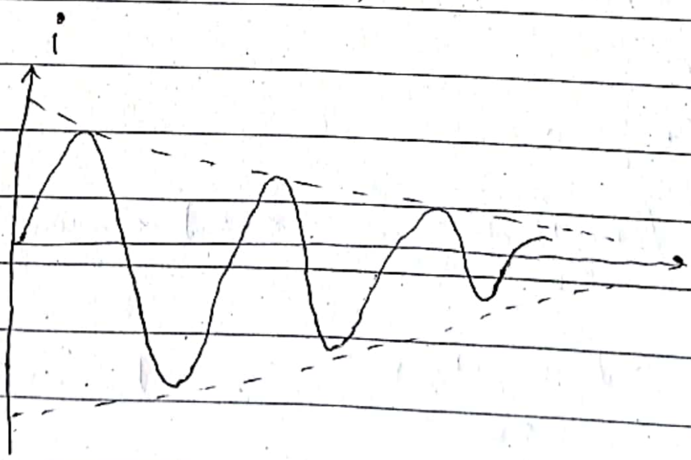
Case-2 $R^2 = \frac{4L}{C}$ (critically damped)

$$i(t) = (A + Bt)\, e^{-Rt/2L}$$



Case-3 $R^2 < \frac{4L}{C}$ (Underdamped)

$$i(t) = e^{-\alpha t}(A \cos \omega t + B \sin \omega t)$$



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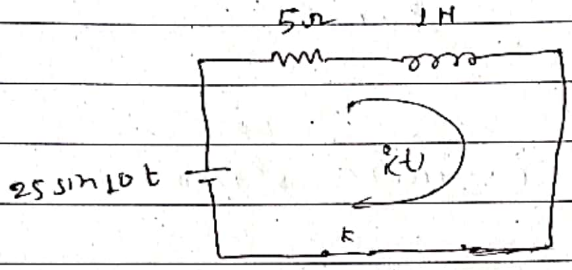
(b) A sinusoidal voltage 25 sin 10t is applied at time t=0 to a series RL circuit comparising R = 5Ω and L = 1 H. By the method of Laplace transformation, find current i(t), Assume zero current through inductor before application of voltage.

Sn At t=0 circuit is.



applying KVL,

$$5\, i(t) + 1\, \frac{di(t)}{dt} = 25 \sin 10t$$

Taking laplace transform

$$5\, I(s) + s\, I(s) - \cancel{i(0^+)}^{0} = 25 \left(\frac{10}{s^2+10^2}\right)$$

$$I(s)\,(s+5) = 25\, \frac{10}{s^2+100}$$

$$I(s) = \frac{250}{(s+5)(s^2+100)} \quad \text{——— (1)}$$

$$\frac{250}{(s+5)(s^2+100)} = \frac{A}{s+5} + \frac{Bs+C}{s^2+100}$$

$$250 = As^2 + 100A + Bs^2 + Cs + 5Bs + 5C$$

equating like terms

$A + B = 0$

$100 + 5B + C = 0$

$5C = 250$

$C = 50$

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$100 + 5B + 50 = 0$

$5B = -150$

$B = -30$

$A = 30$

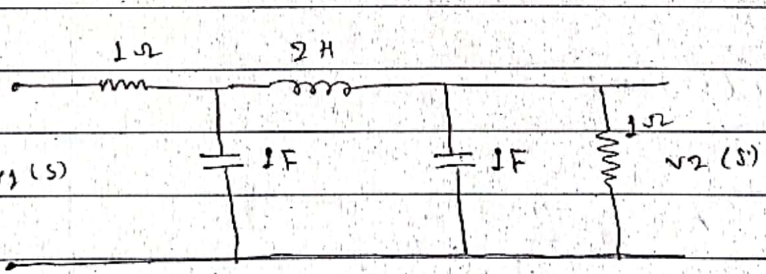
from ① $\quad I(s) = \dfrac{30}{(s+5)} + \dfrac{-30s + 50}{s^2+100}$

$$I(s) = \frac{30}{(s+5)} - \frac{30s}{(s^2+100)} + \frac{50}{s^2+100}$$

Taking inverse laplace transform.

$$i(t) = 30\,e^{-5t} - 30\cos 10t + 5\sin 10t$$

Q.3: (a) What are the advantages of laplace transform over classical method? For N/W given below obtain the voltage transfer function V2(s).



Advantages of L.T.

→ The diff eqⁿ are greatly simplified by laplace transform method.

→ In laplace transform, the eqⁿ incorporates the provision of initial values.

→ No need to evaulate initial values separately.

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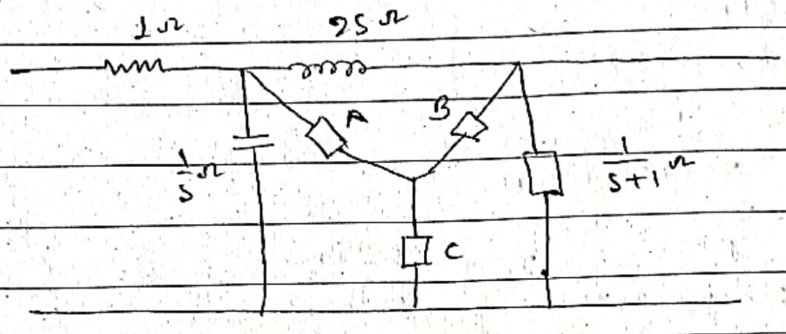
$V1(s) = z_{11} I_1 + z_{12} I_2$

$V2(s) = z_{21} I_1 + z_{22} I_2$



∇ niw to Y n/w.

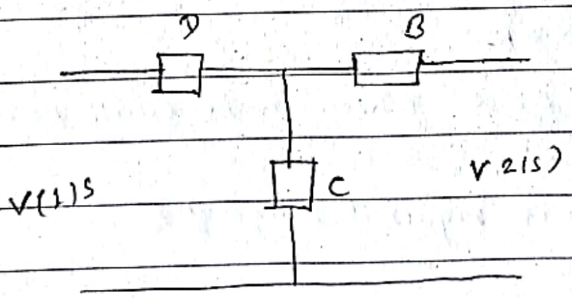
$$A = \frac{\frac{1}{s} \times 2s}{\frac{1}{s} + 2s + \frac{1}{s+1}} = \frac{2}{\frac{s+1+s+2s(s^2+s)}{s(s+1)}} = \frac{2s(s+1)}{2s^3+2s^2+2s+1}$$

$$B = \frac{2s \times \frac{1}{s+1}}{\frac{1}{s} + 2s + \frac{1}{s+1}} = \frac{2s^2}{2s^3+2s^2+2s+1}$$

$$C = \frac{\frac{1}{s} \times \frac{1}{s+1}}{\frac{1}{s} + 2s + \frac{1}{s+1}} = \frac{1}{2s^3+2s^2+2s+1}$$

$$\text{now } (A+1)\,\Omega = \frac{2s(s+1) + 2s^3+2s^2+2s+1}{2s^3+2s^2+2s+1}$$

$$D = \frac{2s^3+4s^2+4s+1}{2s^3+2s^2+2s+1}$$



now,

$$Z_{11} = \frac{2s^3+4s^2+4s+2}{2s^3+2s^2+2s+1}$$

$$Z_{22} = \frac{2s^2+1}{2s^3+2s^2+2s+1}$$

$$Z_{12} = Z_{21} = \frac{1}{2s^3+2s^2+2s+1}$$

$$\frac{V_2(s)}{V_1(s)} = \frac{\frac{1}{2s^3+2s^2+2s+1} I_1 + \frac{2s^2+1}{2s^3+2s^2+2s+1} I_2}{\frac{2s^3+4s^2+4s+2}{2s^3+2s^2+2s+1} I_1 + \frac{1}{2s^3+2s^2+2s+1} I_2}$$ Ans.

(b) The denominator polynomial of a network is given as

$$s^4+s^3+s^2+s+k=0$$

Determine the range of k for which the system is stable using Routh-Hurtwz criteria.

Routh-Hurwitz table

s^4	1	1	k
s^3	1	1	0
s^2	α	k	0
s^1	$\frac{\alpha-k}{\alpha}$	0	
s^0	k		

For system to be stable first column must be positive so that

$$\frac{\alpha-k}{\alpha} > 0$$

$$\alpha - k > 0$$

$$\alpha > k$$

ie $k < \alpha$ where α is small positive number.

which is required range of k.

Q.4. (a) Plote the poles and zeros in s-plane for the network function

$$V(s) = \frac{10s}{(s+3)(s+2)}$$

Also, obtein v(t) using pole-zero diagram.

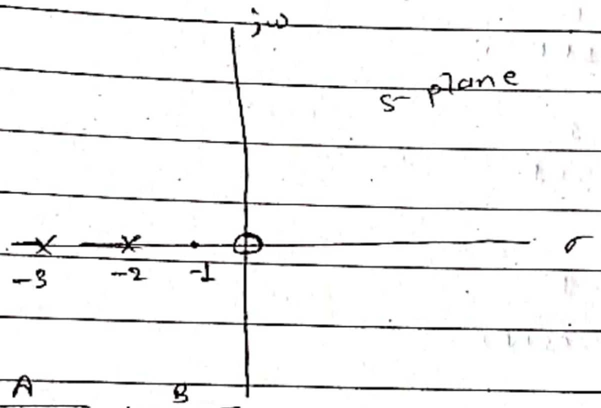
zeros:

$s = 0$

poles

$s+3 = 0 \Rightarrow s = -3$

$s+2 = 0 \Rightarrow s = -2$



$$v(s) = \frac{A}{s+3} + \frac{B}{s+2}$$

$$A = 10 \times \frac{3\angle 0}{1\angle 0} = 30$$

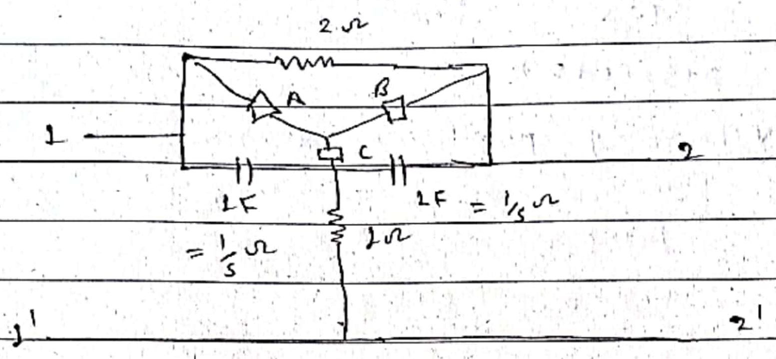
$$B = 10 \times \frac{2\angle 0}{1\angle 180} = -20$$

$$v(s) = \frac{30}{s+3} - \frac{20}{s+2}$$

Taking inverse laplace.

$$v(t) = 30e^{-3t} - 20e^{-2t} \text{ Ans.}$$

(b) Find z and y parameters of the given circuit.

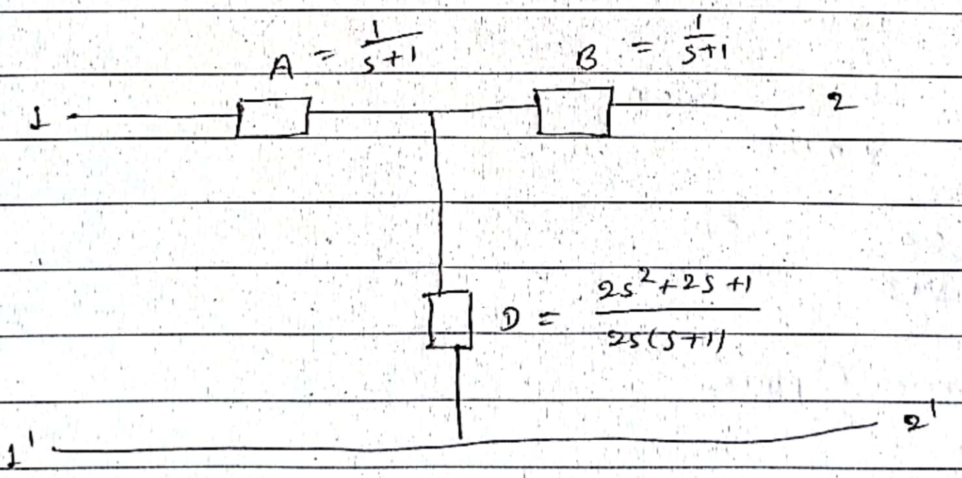


$$A = \frac{2 \times 1/s}{2 + 1/s + 1/s} = \frac{1}{s+1}$$

$$B = \frac{2 \times 1/s}{2 + 1/s + 1/s} = \frac{1}{s+1}$$

$$C = \frac{1/s \times 1/s}{2 + 1/s + 1/s} = \frac{1}{2s(s+1)}$$

$$D = C + 1 = \frac{1}{2s(s+1)} + 1 = \frac{2s^2 + 2s + 1}{2s(s+1)}$$



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$$z_{11} = A + D = \frac{2s^2+4s+1}{2s(s+1)}$$

$$z_{22} = B + D = \frac{2s^2+4s+1}{2s(s+1)}$$

$$z_{12} = z_{21} = D = \frac{2s^2+2s+1}{2s(s+1)}$$

$$Z = \begin{bmatrix} \frac{2s^2+4s+1}{2s(s+1)} & \frac{2s^2+2s+1}{2s(s+1)} \\ \frac{2s^2+2s+1}{2s(s+1)} & \frac{2s^2+4s+1}{2s(s+1)} \end{bmatrix}$$

$$Y = \frac{1}{\Delta Z}\begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}$$ where $\Delta Z = \frac{(2s^2+4s+1)^2 - (2s^2+2s+1)^2}{(2s^2+2s)^2}$

$$Y = \frac{(2s^2+2s)^2}{(2s^2+4s+1)^2-(2s^2+2s+1)^2}\begin{bmatrix} \frac{2s^2+4s+1}{2s(s+1)} & -\frac{2s^2+2s+1}{2s(s+1)} \\ -\frac{2s^2+2s+1}{2s(s+1)} & \frac{2s^2+4s+1}{2s(s+1)} \end{bmatrix}$$

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Q.5 (a) Draw the Bode diagram for the given n/w function

$$H(s) = \frac{2000(s+9)}{(s+16)(s+100)}$$

⇒

1. converting into $1 + \frac{s}{\ldots}$ form

$$H(s) = \frac{2000 \times 9 \left(1+\frac{s}{9}\right)}{16 \times \left(1+\frac{s}{16}\right) \times 100 \times \left(1+\frac{s}{100}\right)} = \frac{5\left(1+\frac{s}{9}\right)}{\left(1+\frac{s}{16}\right)\left(1+\frac{s}{100}\right)}$$

2. put $s = j\omega$

$$H(s) = \frac{5\left(1+\frac{j\omega}{9}\right)}{\left(1+\frac{j\omega}{16}\right)\left(1+\frac{j\omega}{100}\right)}$$

3. converting into dB

$$20\log_{10} H(s) = 20\log_{10}(5) + 20\log_{10}\left(1+\frac{j\omega}{9}\right) - 20\log_{10}\left(1+\frac{j\omega}{16}\right) - 20\log_{10}\left(1+\frac{j\omega}{100}\right)$$

4. magnitude frequency plot.

S.N.	component	corner freq.	slope	overall slope
1.	$20\log_{10}(5)$	None	0	0
2.	$20\log_{10}\left(1+\frac{j\omega}{9}\right)$	9	20	20
3.	$20\log_{10}\left(1+\frac{j\omega}{16}\right)$	16	−20	0
4.	$20\log_{10}\left(1+\frac{j\omega}{100}\right)$	100	−20	−20

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starting point = $20 \log_{10}(5)$

$= 13.979$ db.

5. Angle - frequency plot

$$\phi = \tan^{-1}\left(\frac{\omega}{9}\right) - \tan^{-1}\left(\frac{\omega}{16}\right) - \tan^{-1}\left(\frac{\omega}{100}\right)$$

ω	0	1	2	4	8	16
ϕ	0	9.88	18.29	28.63	32.19	21.87.

(b) Check for the positive realness of given function.

$$Z(s) = \frac{2s^2 + 2s + 4}{s^3 + 2s^2 + s + 2}$$

$\Rightarrow$

(i) All the coefficients are positive.

(ii) Denominator

$s^3 + 2s^2 + s + 2$

even $\rightarrow$ $2s^2 + 2$.

odd $\rightarrow$ $s^3 + s$

$$2s^2 + 2 \overline{) s^3 + s} (\tfrac{1}{2}s \quad \text{positive.}$$

$$\frac{s^3 + s}{\times \quad \times}$$

polynomial is Hurwitz.

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(iii) $M_1M_2 - N_1N_2$

$= (2s^2+4)(2s^2+1) - 2s(s^3+s)$

$= 4s^4+4s^2+8s^2+8 - 2s^4+2s^2$

$= 2s^4+14s^2+8$

put $s = j\omega$

$= 2(j\omega)^4 + 14(j\omega)^2+8$

$= 2\omega^4 - 14\omega^2+8$

$\geq 0.$

(iv) power difference ≤ 1.

Hence given function is PRF.

Q.6 (a) Synthesize the given impedance function in Foster I and Foster II forms.

$$Z(s) = \frac{8(s^2+4)(s^2+25)}{s(s^2+16)}$$

zeros

$s^2+4=0$

$s=j2$

$s^2+25=0$

$s=j5$

poles

$s=0$

$s^2+16=0$

$s=j4$

All zeros and poles lies on $j\omega$ axis. so LC n/w is synthesized.

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(i) Foster I form.

$$z(s) = \frac{8(s^2+4)(s^2+25)}{s(s^2+16)}$$

$$= \frac{8s^4 + 232s^2 + 800}{s^3 + 16s}$$

power of numerator > denomenator so actual division needed.

$$s^3+16s \overline{)\, 8s^4 + 232s^2 + 800 \,}\, (8s$$
$$\underline{8s^4 + 128s^2}$$
$$104s^2 + 800$$

$$z(s) = 8s + \frac{104s^2 + 800}{s(s^2+16)}$$

$$\frac{104s^2+800}{s(s^2+16)} = \frac{A}{s} + \frac{Bs+C}{s^2+16}$$

$$= As^2 + 16A + Bs^2 + Cs$$

$A + B = 104$ | $A = 50$
$C = 0$ | $B = 54$
$16A = 800$

$$z(s) = 8s + \frac{50}{s} + \frac{54s}{s^2+16}$$

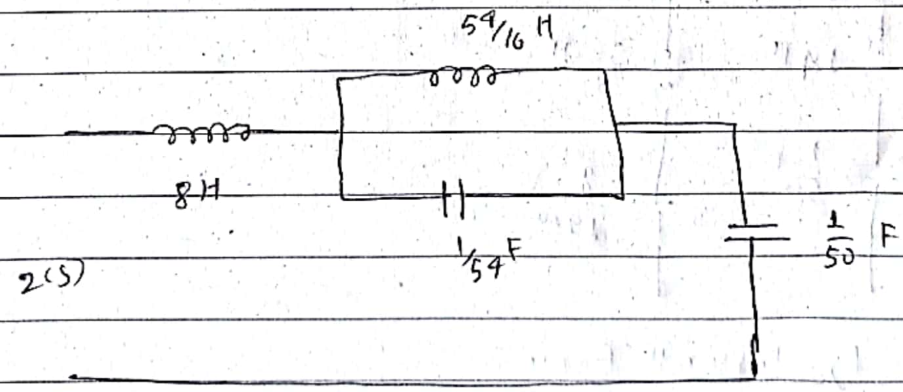


fig. foster-I form.

(ii) For foster II form

$$Y(s) = \frac{s(s^2+16)}{8(s^2+4)(s^2+25)} = \frac{0.125\ s(s^2+16)}{(s^2+4)(s^2+25)}$$

Partial fraction.

$$\frac{0.125\ s(s^2+16)}{(s^2+4)(s^2+25)} = \frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+25}$$

$$0.015s^3+2s = As^3+25As+Bs^2+25B+Cs^3+4Cs+Ds^2+4D$$

equating like terms,

$A+C=0.125$

$A=0.125-C$

$B+D=0 \Rightarrow B=-D.$

$25A+4C=2$

$25(0.125-C)+4C=2$

$-21C=-1.125$

$C=\frac{3}{56}$

$25B+4D=0$

$-25D+4D=0$

$D=0$

$A=0.125-\frac{3}{56}$

$=\frac{1}{14}$

$$Y(s) = \frac{\frac{1}{14}s}{(s^2+4)} + \frac{\frac{3}{56}s}{(s^2+25)}$$

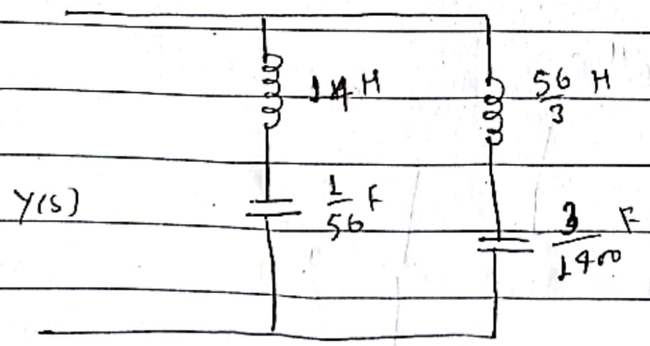
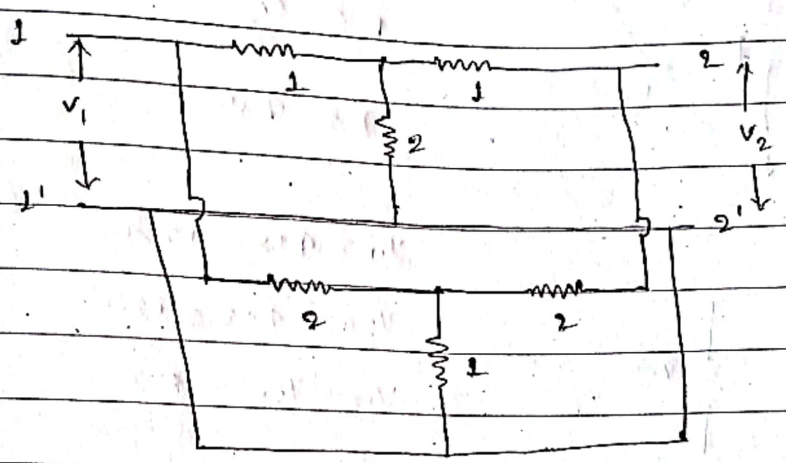


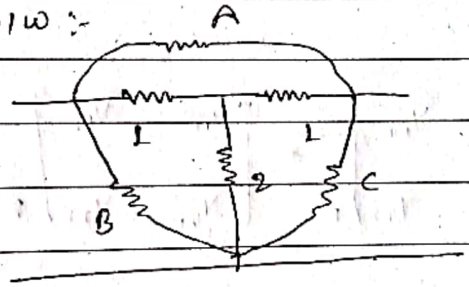
fig. Foster II form.

(b) Find the short circuit admittance parameters of the following two port n/w. All resistors are in ohm.

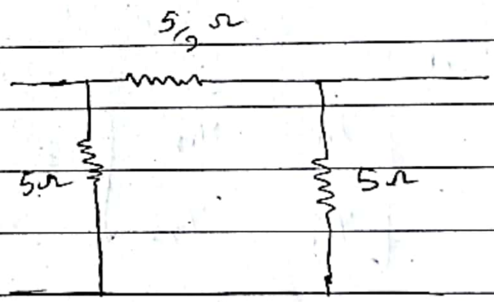


Given two port n/w are in parallel combination. From this n/w we get directly short ckt parameter or Y - parameters as:

(i) consider 1st n/w :-



$\Rightarrow$



$$A = \frac{1\times1+1\times2+2\times1}{2} = 5/2$$

$B = 5/1 = 5\,\Omega$

$C = 5/1 = 5\,\Omega$

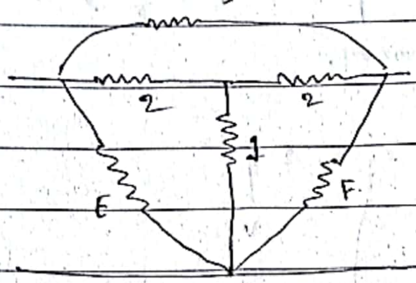
$y_{11} = 5 + 5/2 = \frac{15}{2}\,\Omega$

$y_{22} = \frac{15}{2}\,\Omega$

$y_{12} = y_{21} = 5/2\,\Omega$

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}_1 = \begin{bmatrix} 15/2 & 5/2 \\ 5/2 & 15/2 \end{bmatrix} \quad \text{—— (1)}$$

Consider 2nd n/w

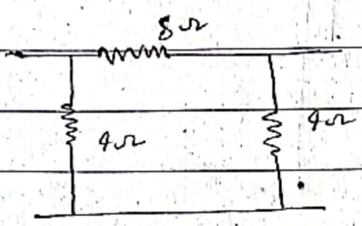


$$D = \frac{2\times2+2\times1+1\times2}{1} = 8\,\Omega$$

$$E = 8/2 = 4\,\Omega$$

$$F = 4\,\Omega$$

⇓



$$y_{11} = 4+8 = 12\,\Omega$$

$$y_{22} = 4+8 = 12\,\Omega$$

$$y_{12} = y_{21} = 8\,\Omega$$

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}_2 = \begin{bmatrix} 12 & 8 \\ 8 & 12 \end{bmatrix} \text{——} (2)$$

Now,

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}_1 + \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}_2$$

$$= \begin{bmatrix} 15/2 & 5/2 \\ 5/2 & 15/2 \end{bmatrix} + \begin{bmatrix} 12 & 8 \\ 8 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 39/2 & 21/2 \\ 21/2 & 39/2 \end{bmatrix}$$ Ans.

END