

POWER SYSTEM OPERATION & CONTROL

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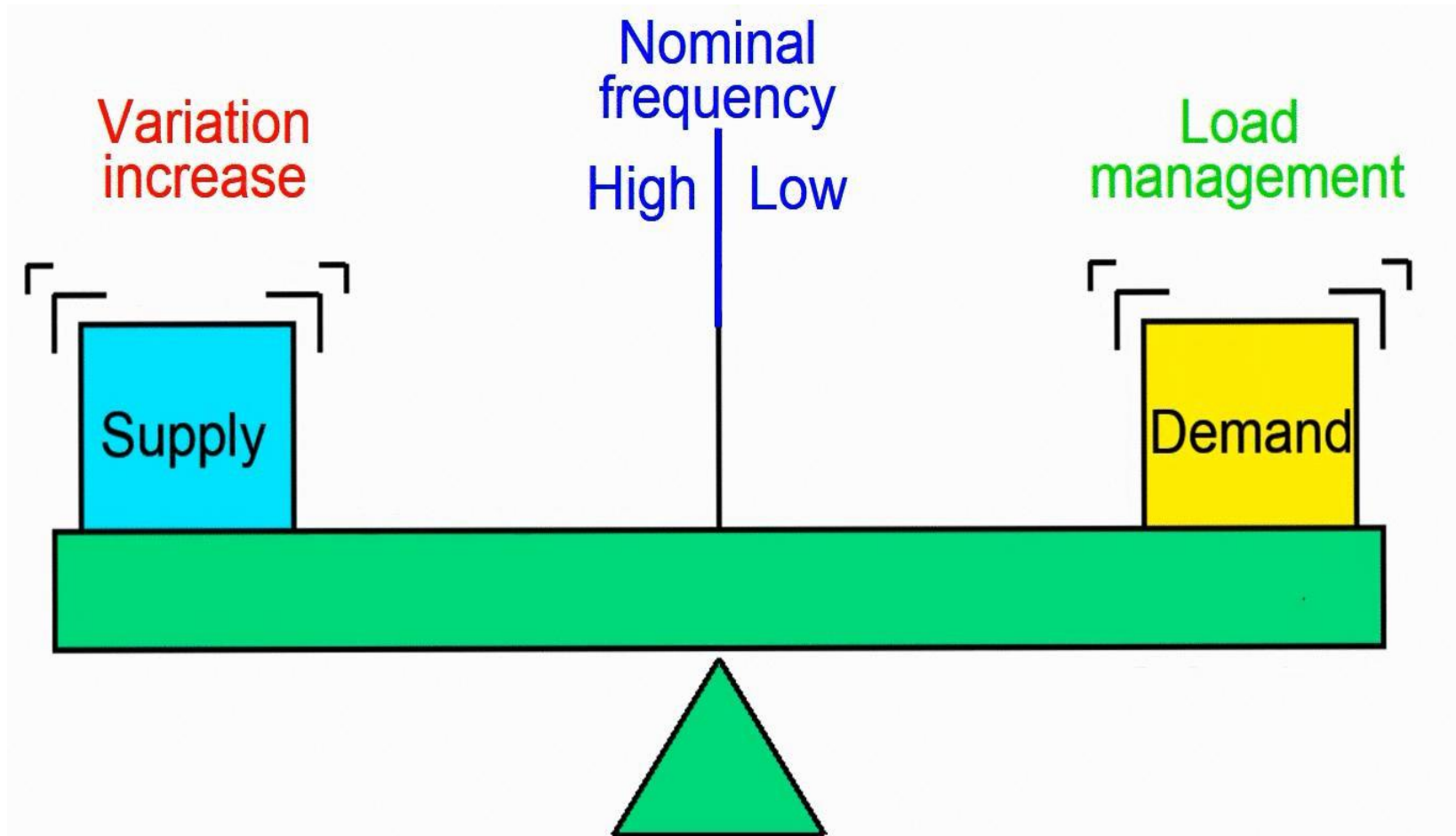
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Part 4

Active Power and Frequency Control



Active Power and Frequency Control

1- Introduction:

- The frequency of a system is dependent on active power balance
- Frequency is a common factor throughout the system, a change in active power demand at one point is reflected throughout the system frequency.
- Because there are many generators supplying power into the system, then there are (some means) must be provided to allocate change in demand to the generators as
 - 1) speed governor on each generating unit provides primary speed control function
 - 2) supplementary control originating at a central control (center allocates generation)
- In an interconnected system, with two or more independently controlled areas, the generation within each area has to be controlled so as to maintain scheduled power interchange
- The control of generation and frequency is commonly known as **load frequency control (LFC)**

REAL POWER FREQUENCY CONTROL

2- TECHNICAL TERMS

Control area: Most power systems normally control their generators in unison. The individual control loops have the same regulation parameters. The individual generator turbines tend to have the same response characteristics then it is possible to let **the control loop in the whole system which then would be referred to as a control area.**

Prime Mover: The engine, turbine, water wheel, or similar machine that drives an electric generator.

Pumped-Storage Hydroelectric Plant: A plant that usually generates electric energy during peak-load periods by using water pumped into an elevated storage reservoir. (is not applicable in Jordan)

Regulation: The governmental function of controlling or directing economic entities through the process of rule making and adjudication

Reserve Margin (Operating): The amount of unused available capability of an electric power system at peak load for a utility system as a percentage of total capability.

Scheduled Outage: The shutdown of a generating unit, transmission line, or other facility, for inspection or maintenance, in accordance with an advance schedule.

ISOCHRONOUS GOVERNOR: a governor that maintains the same speed in the mechanism controlled regardless of the load.

governor (speed limiter) is a device used to measure and regulate the speed of generators.

Speed Governor: It is an error sensing device in load frequency control. It includes all the elements that are directly responsive to speed and influence other elements of the system to initiate action.

Governor Controlled Valves: They control the input to the turbine and are actuated by the speed control mechanism.

Speed Control Mechanism: all equipment such as levers and linkages, servomotors, amplifying devices and relays that are placed between the speed governor and the governor controlled valves.

Speed Changer: It enables the speed governor system to adjust the speed of the generator unit while in operation

3- Primary Speed Controls (Isochronous speed governor)

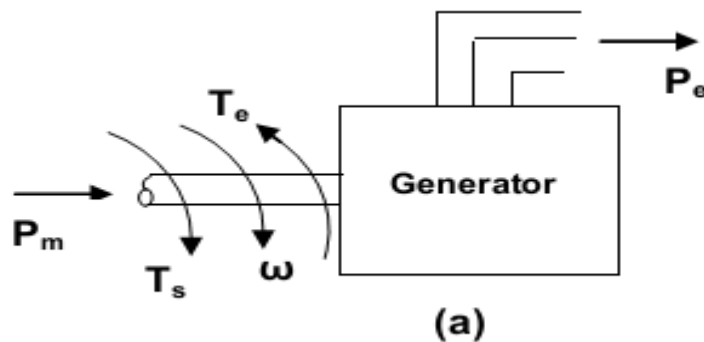
- an integral controller resulting in constant speed.
- not suitable for multimachine systems (**is isolated machine or non-coherent machine**) ; slight differences in speed settings would cause them to fight or conflict against each other (**since is not connected at same shaft**)
- can be **used** only when a generator is supplying an **isolated load** **or** when only one generator in a system is required **to respond to load changes**

4- Governor with Speed Droop

- **speed regulation or droop is provided to assure proper load sharing**
- a proportional controller with a gain of $1/R$
- The speed-load characteristic can be adjusted by changing governor settings; this is achieved in practice by operating speed-changer motor

5- Review of SWING EQUATION :

The differential equation that relates the angular momentum (M) the acceleration power (P_a) and the rotor angle (δ) is known as swing equation.



Consider the generator shown in Fig.(a). It receives **mechanical power P_m** at the **shaft torque T_m** and the **angular speed ω** via. shaft from the **prime-mover**. It delivers **electrical power P_e** to the power system network.

The generator develops **electromechanical torque T_e** in opposition to the shaft torque T_s . **At steady state, $T_m = T_e$**

accelerating torque acting on the rotor is given by

$$T_a = T_m - T_e$$

Multiplying by ω on both sides, we get power acceleration (P_a) :

$$P_a = P_m - P_e$$

or

$$P_a = T_a \cdot \omega = J \cdot \alpha \cdot \omega = M \cdot \alpha$$

$$\therefore \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \text{angular acceleration}$$

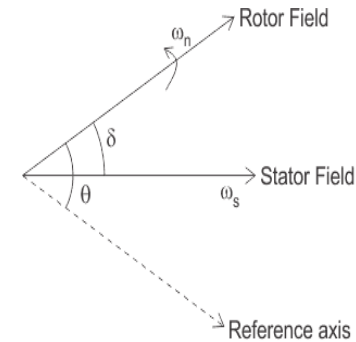
$$P_a = M \cdot \frac{d\omega}{dt} = M \cdot \frac{d^2\theta}{dt^2}, (\theta = \text{angular displacement of rotor (rad.)})$$

$$\Rightarrow \boxed{P_a = M \cdot \frac{d^2\theta}{dt^2}}, (\theta - \text{it's more convenient to measure the angular position}$$

of rotor with respect to synchronously rotation frame

$(\delta = \theta - \omega t)$ of reference $\omega t = 0 \Rightarrow (\delta = \theta)$ then;

$$\frac{d^2\theta}{dt^2} = \frac{d^2\delta}{dt^2} \Rightarrow \boxed{P_a = M \cdot \frac{d^2\delta}{dt^2}}$$



Angular position of rotor with respect to reference axis

Where $\dot{\theta} = \text{Angular displacement (rad.)}$

$$\omega = \text{Angular velocity (rad / sec)} = \frac{d\theta}{dt}$$

$$\alpha = \text{Angular acceleration (rad}^2 \text{ / sec}^2\text{)}$$
$$= \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$J = \text{Moment of inertia (kg.m}^2\text{)}$$

$$T = \text{Torque (N.m)} = J.\alpha$$

$$M = \text{Angular momentum}$$
$$(\text{kg.m}^2.\text{rad / sec}) = J \omega$$

$$K.E = \frac{1}{2} J \omega^2 = S.H \quad (\text{Joule})$$

$$H = \text{Inertia constant}$$

$$P = \text{power (watt)} = T \omega$$

$$W = \text{work} = \int P_e d\theta = \int T \frac{d\theta}{dt} = T \theta$$

$$P_a = P_m - P_e = M \cdot \frac{d^2 \delta}{dt^2} = \frac{S.H}{180^\circ \cdot f} \cdot \frac{d^2 \delta}{dt^2} \quad \text{Known as swing equation}$$

The swing equation is the differential equation that relates the angular momentum (M) the acceleration power (Pa) and the rotor angle (δ).

In case damping power (D) is to be included, then equation is modified as

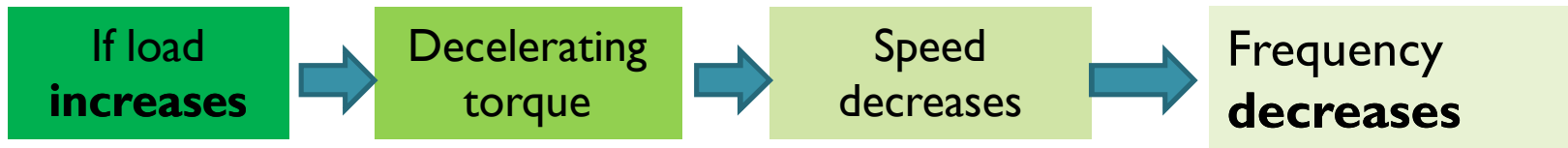
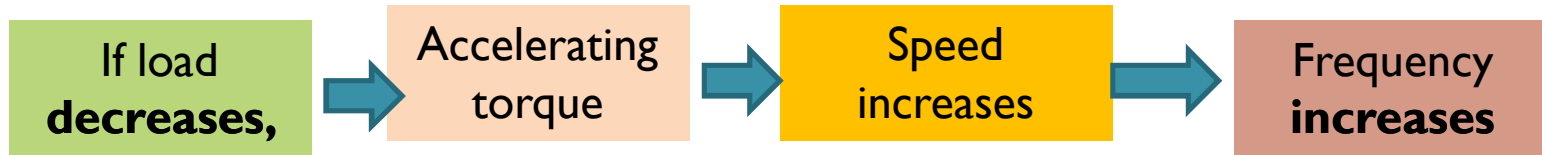
$$P_a = P_m - P_e = M \cdot \frac{d^2 \delta}{dt^2} + D \frac{d \delta}{dt} = \left(\frac{d \delta}{dt} \cdot \frac{S.H}{180^\circ \cdot f} + D \right) \frac{d \delta}{dt}$$

As result of swing equation for generator is

$$P_a = P_m - P_e = P_m - P_{\max} \cdot \sin \delta = M \cdot \frac{d^2 \delta}{dt^2}$$

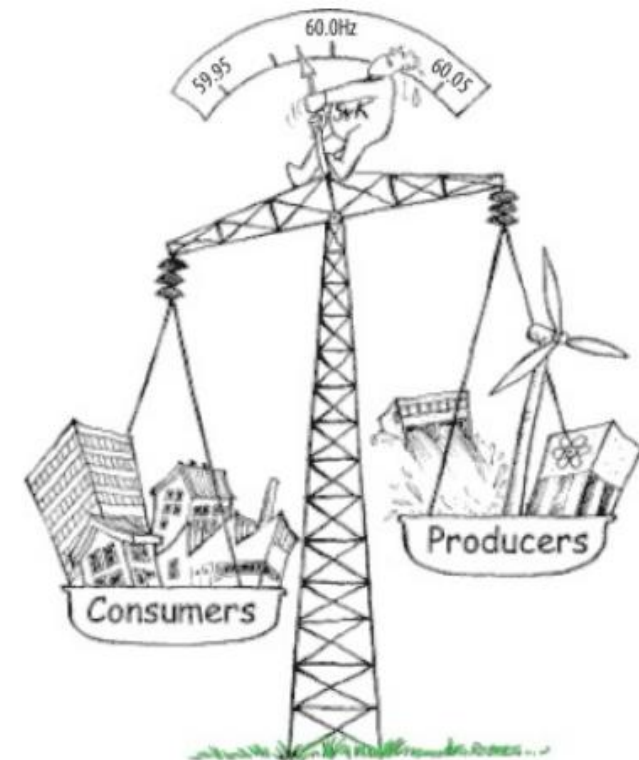
$$M \cdot \frac{d^2 \delta}{dt^2} = P_m - P_{\max} \cdot \sin \delta \quad \text{swing equation for generator}$$

$$P_e = P_{\max} \cdot \sin \delta = T_{\max} \cdot \omega_{\max} \cdot \sin \delta = T_{\max} \cdot 2\pi f \cdot \sin \delta$$



$$f = \frac{n_m P}{120} \Rightarrow n_m = \frac{120f}{P}$$

$$T_{\max} = \frac{P_{\max}}{\omega_{\max}} = \frac{P_{\max}}{2\pi \left(\frac{n_s}{60} \right)}$$



6- SPEED GIVERNING MECHANISM AND MODELLING

Governor: The power system is basically dependent upon the synchronous generator and its satisfactory performance.

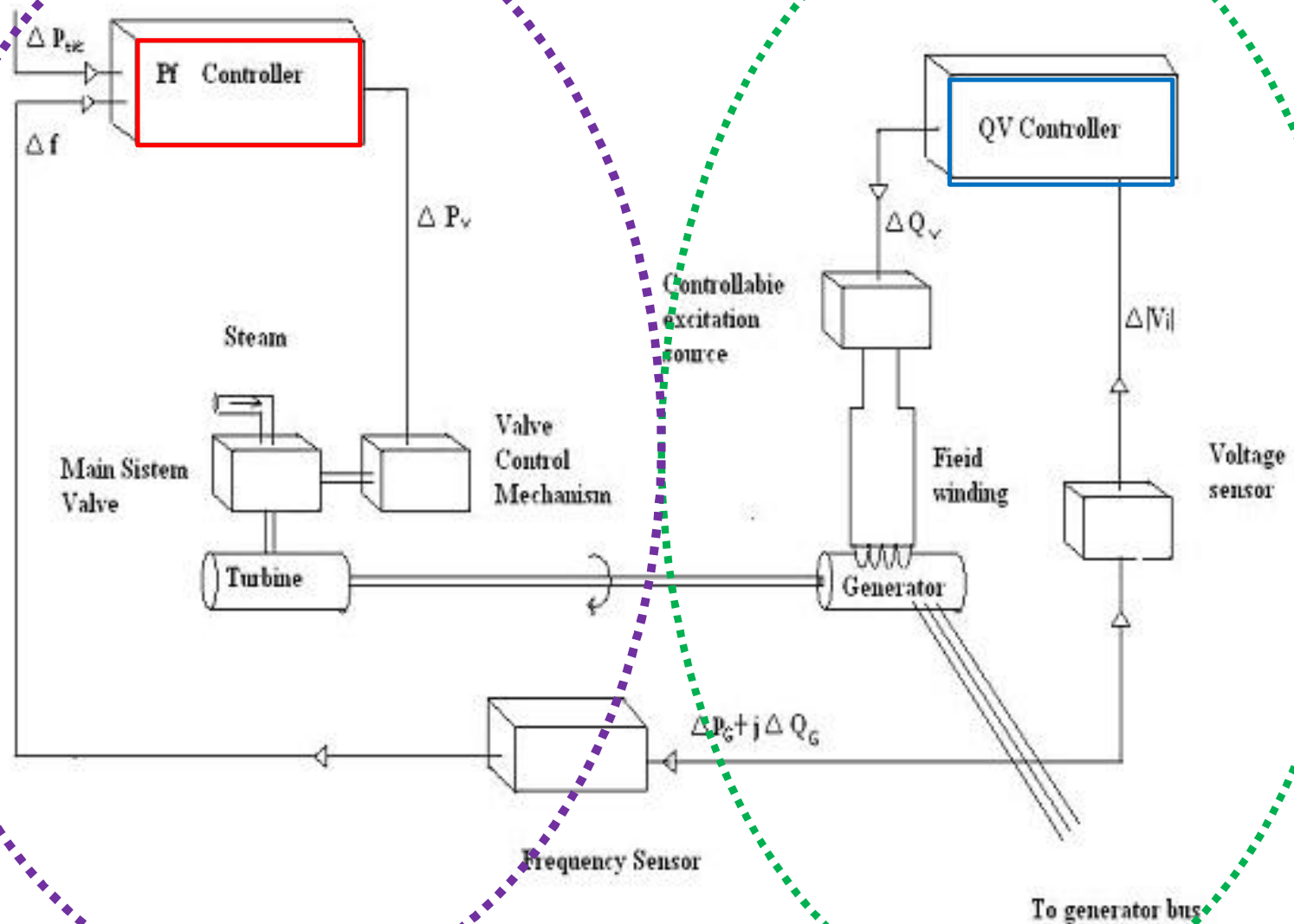
The important control loops in the system are:

- (i) Frequency control ,and
- (ii) Automatic voltage control

Frequency control is achieved through generator control mechanism (governing systems) . The governing systems for thermal and hydro generating plants are different in nature (since the inertia of water that flows into the turbine presents additional constrains which are not present with steam flow in a thermal plant). However, the basic principle is still the same, **the speed of the shaft is sensed and compared with a reference, and the feedback signal is utilized to increase or decrease the power generated by controlling the valve to turbine of steam or water.**

P and Freq. control

Q and V control



Basic Generator Control loops

LOAD FREQUENCY CONTROL

For successful operation of the system The following basic requirements are to be meet :

1. The **generation** must be adequate to **for all the load demand**.
2. The **system frequency** must be maintained **within narrow limits**.
3. The **system voltage** profile must be maintained **within reasonable limits** and In case of **interconnected** operation, the **tie line** power flows must be **maintained at the specified values**.
4. When real power balance between generation and demand is achieved the frequency specification is automatically satisfied.
5. **An independent aim of the automatic generation control (AGC) is to reschedule the generation changes to preselected machines in the system after the governors have accommodated the load change in a random manner.**

Remembering that under **steady state conditions**, the total real power generation in the system equals **the total MW demand plus real power losses**. Any difference is immediately indicated by a change in speed or frequency .

Automatic load frequency control (ALFC)

The ALFC is to control the frequency deviation by maintaining the real power balance in the system. The main functions of the ALFC are :

- 1) To maintain the steady frequency;
- 2) Control the tie-line flows; and
- 3) Distribute the load among the participating generating units.

The control (input) signals are the tie-line deviation ΔP (measured from the tie-line flows), and the frequency deviation Δf (obtained by measuring the angle deviation $\Delta \delta$).

These error signals Δf and ΔP tie are amplified, mixed and transformed to a real power signal, which then controls the valve position.

Depending on the valve position, the turbine (prime mover) changes its output power to establish the real power balance. The complete control schematic as shown in Fig.1

Block schematic for Load frequency control

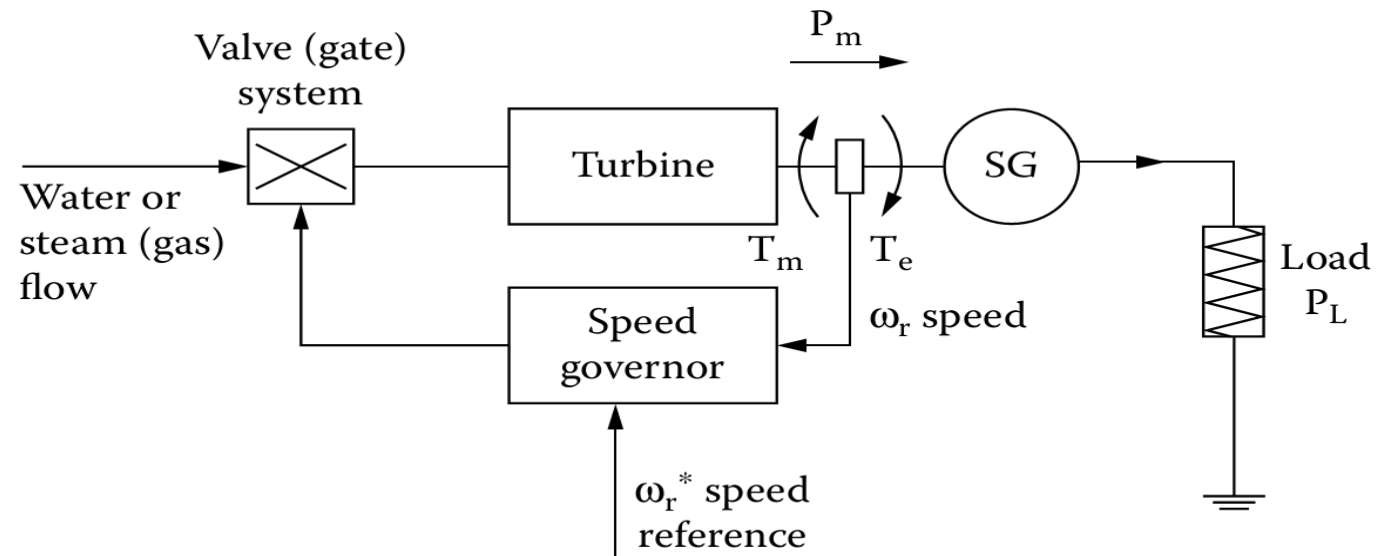


Fig 1 The Schematic representation of ALFC system

Note : For single system the sensor between turbine and generator (governor) sense ω and control the speed through the steam valve, but in case of tie line there is another sensor to match the freq.

ALFC required to analyzed, each the model of the blocks in Fig 1.

I- MATHEMATICAL MODELLING OF A **GENERATOR**:

With the use of (pu.) swing equation of a synchronous machine to small perturbation, we have

$$\Delta P_a = \Delta P_m - \Delta P_e = \frac{H}{180^\circ \cdot f} \cdot \frac{d^2 \Delta \delta}{dt^2} = \frac{H}{\frac{\omega}{2} \cdot f} \cdot \frac{d^2 \Delta \delta}{dt^2}$$

$$\omega = 2\pi = 360^\circ \text{ and } f = 1$$

$$\frac{2H}{\omega} \frac{d^2 \Delta \delta}{dt^2} = \Delta P_m - \Delta P_e$$

Or in terms of small change in speed
(See reference in the next slide)

$$\frac{d\Delta \frac{\omega}{\omega_s}}{dt} = \frac{1}{2H} (\Delta P_m - \Delta P_e)$$

Laplace Transformation gives,

$$\Delta \omega_{(s)} = \frac{1}{2H_{(s)}} \cdot [\Delta P_{m(s)} - \Delta P_{e(s)}]$$

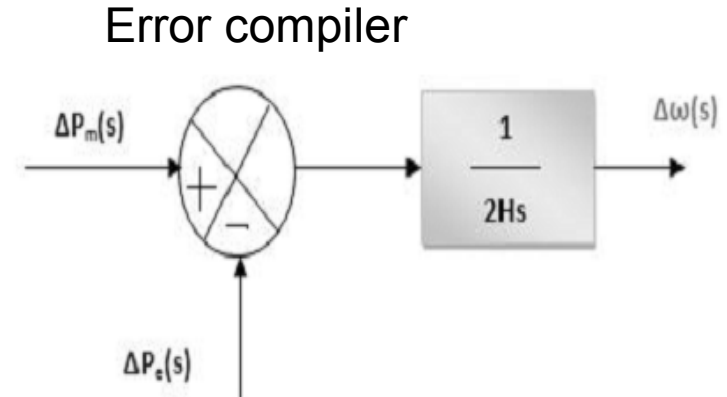


Fig. of mathematical modelling block diagram for a generator

Reference

$$\frac{2H}{\omega_{syn}} \cdot \frac{d^2 \Delta \delta}{dt^2} = \Delta P_m - \Delta P_e \quad \text{or} \quad \frac{d^2 \Delta \delta}{dt^2} = \frac{\omega_{syn} (\Delta P_m - \Delta P_e)}{2H} \quad \text{or}$$

cancelling Δ in both sides then
$$\frac{d^2 \delta}{dt^2} = \frac{\omega_{syn} (P_m - P_e)}{2H}$$

And since $\omega_e = \frac{d\delta}{dt}$ then for
$$\frac{d^2 \delta}{dt^2} = \frac{d}{dt} \cdot \frac{d\delta}{dt}$$

$$\frac{d}{dt} \omega_e = \frac{\omega_{syn} (P_m - P_e)}{2H} \Rightarrow 2H \cdot \frac{d}{dt} \omega_e = \omega_{syn} (P_m - P_e)$$

dividing both sides by ω_{syn} we get;

$$2H \frac{d}{dt} \frac{\omega_e}{\omega_{syn}} = (P_m - P_e) \quad , \text{where} \quad \frac{\omega_e}{\omega_{syn}} = \omega \quad \text{We get ;}$$

$$\boxed{\frac{d\omega}{dt} = \frac{1}{2H} P_m - P_e}$$

Where:-

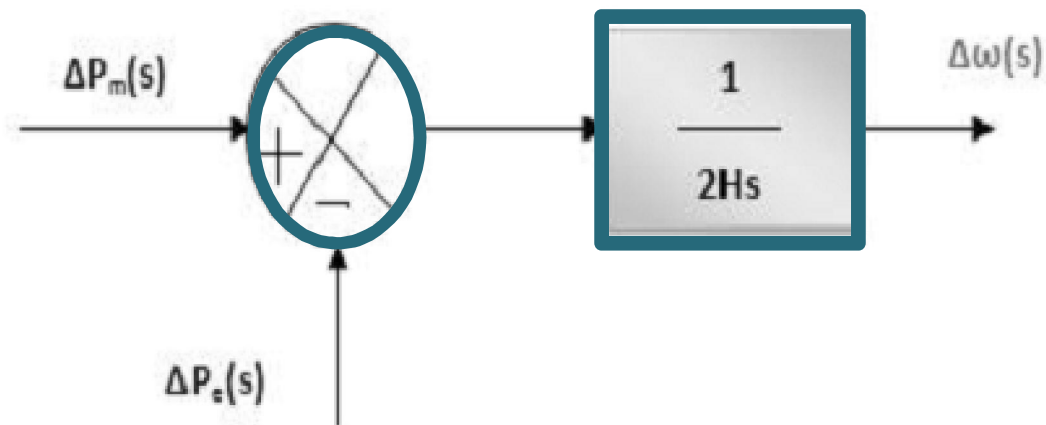
δ - is the machine “torque” (electrical) angle by which the rotor leads the synchronously rotating reference.

ω_{syn} - is the synchronous rotor speed in mechanical rad/sec.

The valve and the hydraulic amplifier represent the speed governing system using the swing equation, the generator can be using the swing equation and can be modeled also as:

$$\Delta\delta = \Delta P_m - \Delta P_e$$

$$\Delta P_m - \Delta P_e$$



Block diagram representation of The Generator

2- MATHEMATICAL MODELLING OF LOAD:

The load on a power system consists of variety of electrical drives. The **load speed characteristic of the load** is given by:

$$\Delta P_e = \Delta P_L + D \Delta \omega$$

where

ΔP_L is the non-frequency sensitive **change in load**,

$D\Delta\omega$ is the **load change that is frequency sensitive**.

D is expressed as % change in load divided by % change in frequency

$$D = \frac{\text{pu change in load}}{\text{pu change in frequency}}$$

$$\text{pu change in load} = D\Delta\omega$$

From the swing equation slide 9 we have

$$\Delta P_a = M \cdot \frac{d\Delta\omega}{dt} = \Delta P_m - \Delta P_e$$

Introducing D we obtain

$$M \frac{d\Delta\omega(t)}{dt} = \Delta P_m - \Delta P_L - D\Delta\omega(t)$$

Taking the Laplace transform we get:

- $P_m = P_{m0} + \Delta P_m$

- $P_e = P_{e0} + \Delta P_e$

$$P_a = M \cdot \frac{d\omega}{dt} = M \cdot \frac{d^2\delta}{dt^2} \Rightarrow \left[\frac{d^2\delta}{dt^2} = \frac{d\omega}{dt} \right]$$

- $\omega(t) = \frac{\omega_{e0}}{\omega_{e0}} + \Delta\omega(t) = 1 + \Delta\omega(t)$

Substitution of the above into the stability equation $M \frac{d\omega}{dt} = P_m - P_e$ we get

$$M \frac{d(1 + \Delta\omega(t))}{dt} = P_{m0} + \Delta P_m - P_{e0} - \Delta P_e$$

Simplifying, and noting that at equilibrium the $P_{m0} = P_{e0}$, then ;

$$M \frac{d\Delta\omega(t)}{dt} = \Delta P_m - \Delta P_e$$

Because we want to derive block diagrams for our control systems, we will transform all time-domain expressions into the Laplace (**s**) domain.

$$Ms\Delta\omega(s) = \Delta P_m - \Delta P_e$$

where we have assumed $\Delta\omega(t=0)=0$.

time-domain expressions into the Laplace (**s**) domain is:

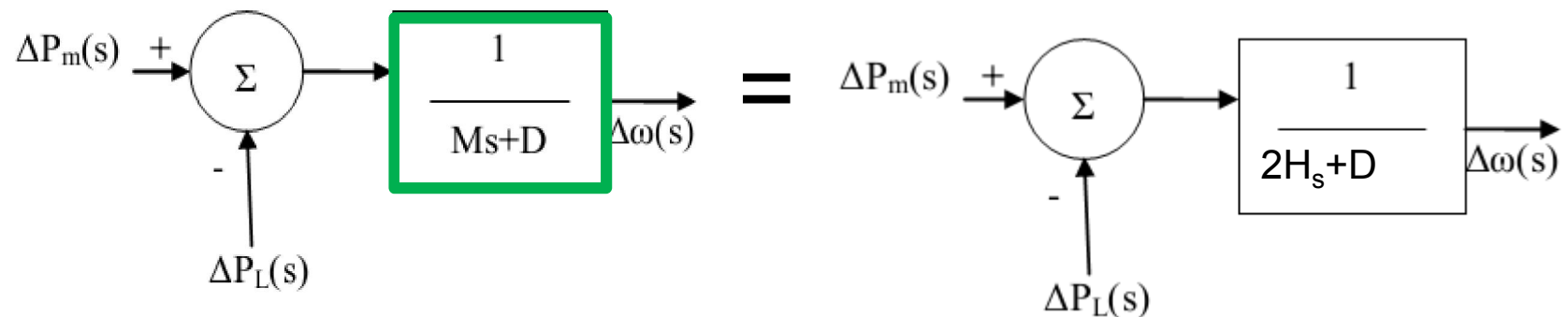
$$Ms\Delta\omega(s) = \Delta P_m(s) - \Delta P_L(s) - D\Delta\omega(s)$$

Solving for $\Delta\omega(s)$, we obtain

$$Ms\Delta\omega(s) + D\Delta\omega(s) = \Delta P_m - \Delta P_L(s)$$

$$\Delta\omega(s)(Ms + D) = \Delta P_m - \Delta P_L(s)$$

$$\Delta\omega(s) = \underbrace{(\Delta P_m - \Delta P_L(s))}_{\text{Input}} \underbrace{\frac{1}{Ms + D}}_{\text{Transfer Function}}$$



Block diagram representation of the
generator and load

M = Angular momentum

The relationship between the change in load due to the change in frequency is given by

$$\Delta P_{L(\text{freq})} = D \Delta \omega \quad \text{or} \quad D = \frac{\Delta P_{L(\text{freq})}}{\Delta \omega}$$

where D is expressed as percent change in load divided by percent change in frequency.

For example:

If load changed by 1.5% for a 1% change in frequency, then D would equal 1.5. However, the value of D used in solving for system dynamic response must be changed if the system base MVA is different from the nominal value of load.

Suppose the D above is referred to a net connected load of 1200 MVA and the entire dynamics problem were to be set up for a 1000-MVA system base.

(Note that D = 1.5 tells us that the load would change by 1.5 pu for 1 pu change in frequency.) That is, the load would change by 1.5 x 1200 MVA (actual) = 1800 MVA for a 1 pu change in frequency.

When expressed on a 1000-MVA base, D becomes

$$D_{1000\text{-MVA base}} = 1.5 \times \left(\frac{1200}{1000} \right) = 1.8 \quad \text{or} \quad D_{(1000\text{MVA base})} = \frac{1800}{1000} = 1.8$$

EXAMPLE

An isolated power system with a 600-MVA generating unit having an M of 7.6 pu MW/pu frequency/sec on a machine base. The unit is supplying a load of 400 MVA. **The load changes by 2% for a 1% change in frequency.**

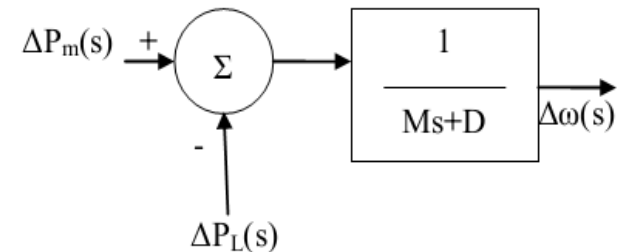
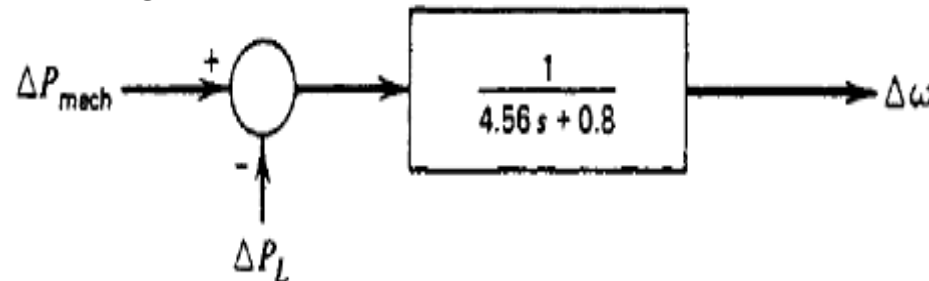
Solution : $D = \frac{\text{change in the load}}{\text{change in freq.}} = \frac{2}{1} = 2$

First, we will set up the block diagram of the equivalent generator load system. Everything will be referenced to **a 1000 MVA base**

$$M_{(\text{on base of } 1000\text{MVA})} = 7.6 \times \frac{600}{1000} = 4.56 \text{ pu.}$$

The change in the load is $\frac{400}{1000} \times 2 = 0.8$

Then the block diagram



Suppose the load suddenly increases by 10 MVA (or 0.01 pu); that is

Then

$$\Delta\omega(s) = \underbrace{(\Delta P_m - \Delta P_L(s))}_{\text{Input}} \underbrace{\frac{1}{Ms + D}}_{\text{Transfer Function}} = -\frac{0.01}{s} \left(\frac{1}{4.56s + 0.8} \right)$$

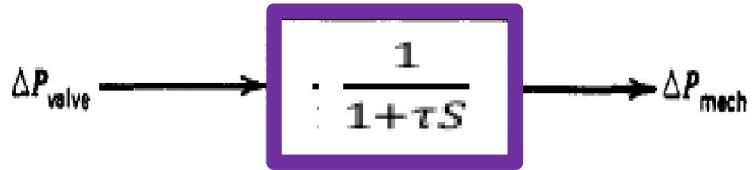
or taking the inverse Laplace transform,

$$\begin{aligned}\Delta\omega(t) &= (0.01/0.8)e^{-(0.8/4.56)t} - (0.01/0.8) \\ &= 0.0125e^{-0.175t} - 0.0125\end{aligned}$$

The final value of $\Delta\omega$ is - 0.0125 pu, which means is a **drop** of 0.75 Hz on a 60-Hz system. or drop of 0.625HZ on 50-HZ system

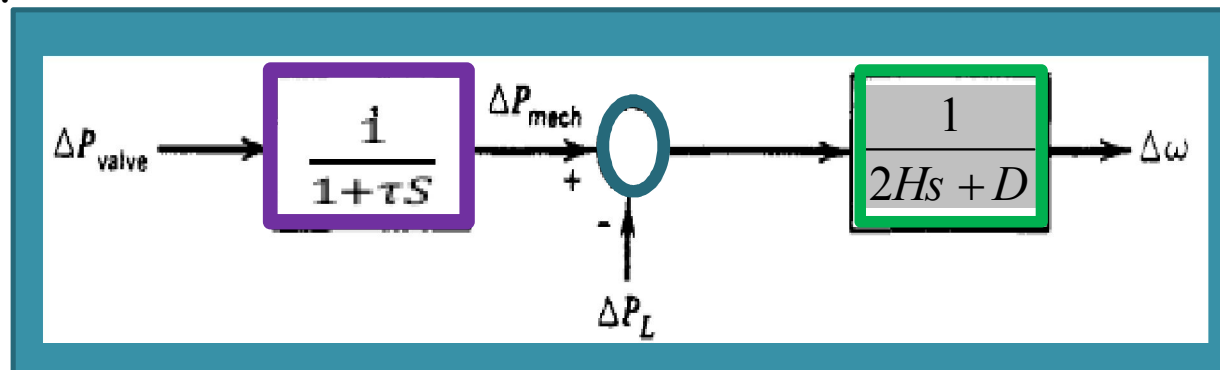
3- MATHEMATICAL MODELLING FOR **PRIME MOVER**:

The prime mover driving a generator unit may be a **steam turbine** or a **hydro turbine**. The models for the prime mover must take account of **the steam supply and boiler control system characteristics** the **mechanical power provided to the generator**, briefly discussed in next slide(not our inters) we can denote it as ΔP_m . We will denote the mechanical power control as ΔP_v .. (**valve** position) . The model of turbine relates the **changes in mechanical power output** ΔP_m and **the changes in the steam**

$$G_T = \frac{\Delta P_m(s)}{\Delta P_v(s)} = \frac{1}{1 + \tau S}$$


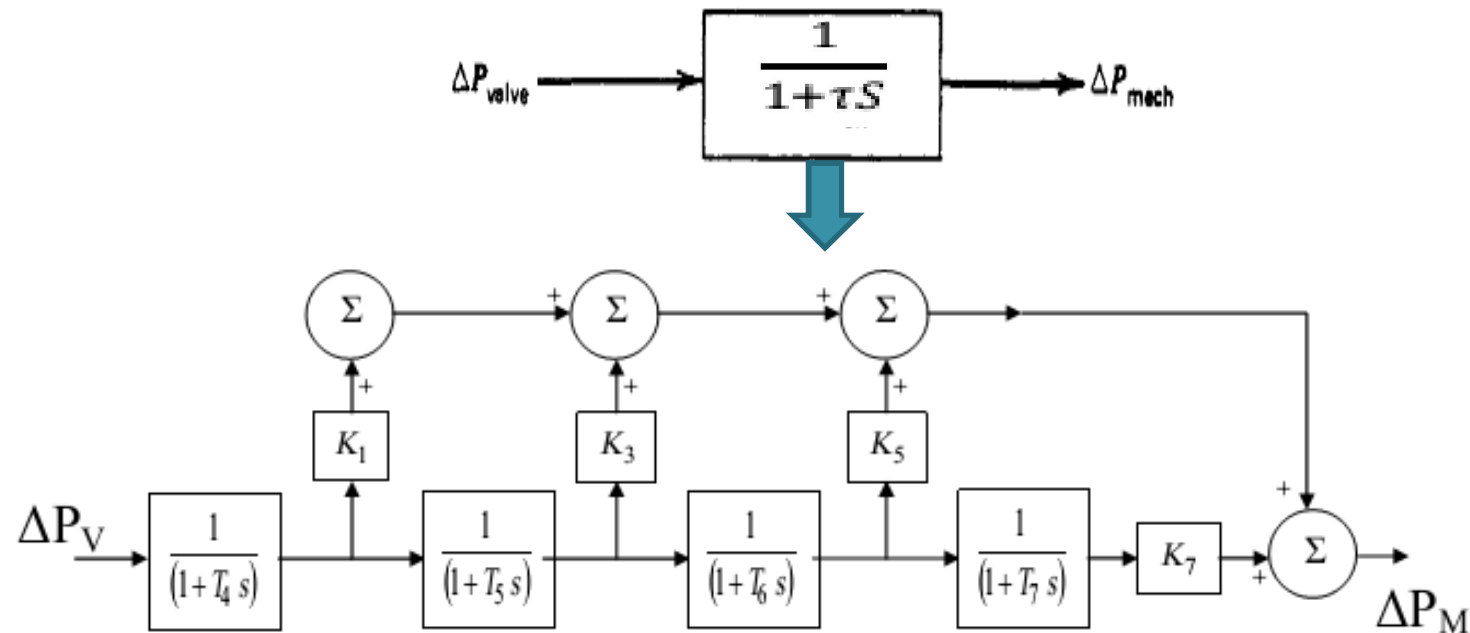
mathematical modelling for prime mover

ζ = "charging time" time constant where the turbine constant is in the range of 0.2 -2.0s.



**Combined prime mover, generator and load model
for a single generating unit**

a brief discussion about turbine models and indicates that a general turbine model is as shown in Fig.



- The time constants ζ_5 , ζ_6 , and ζ_7 are associated with time delays of piping systems for reheaters and cross-over mechanisms, **T_4 represents the first stage, often called the steam chest.**
- The coefficients K_1 , K_3 , K_5 , and K_7 represent fractions of total mechanical power outputs associated with very high, high, intermediate, and low pressure components, respectively.”

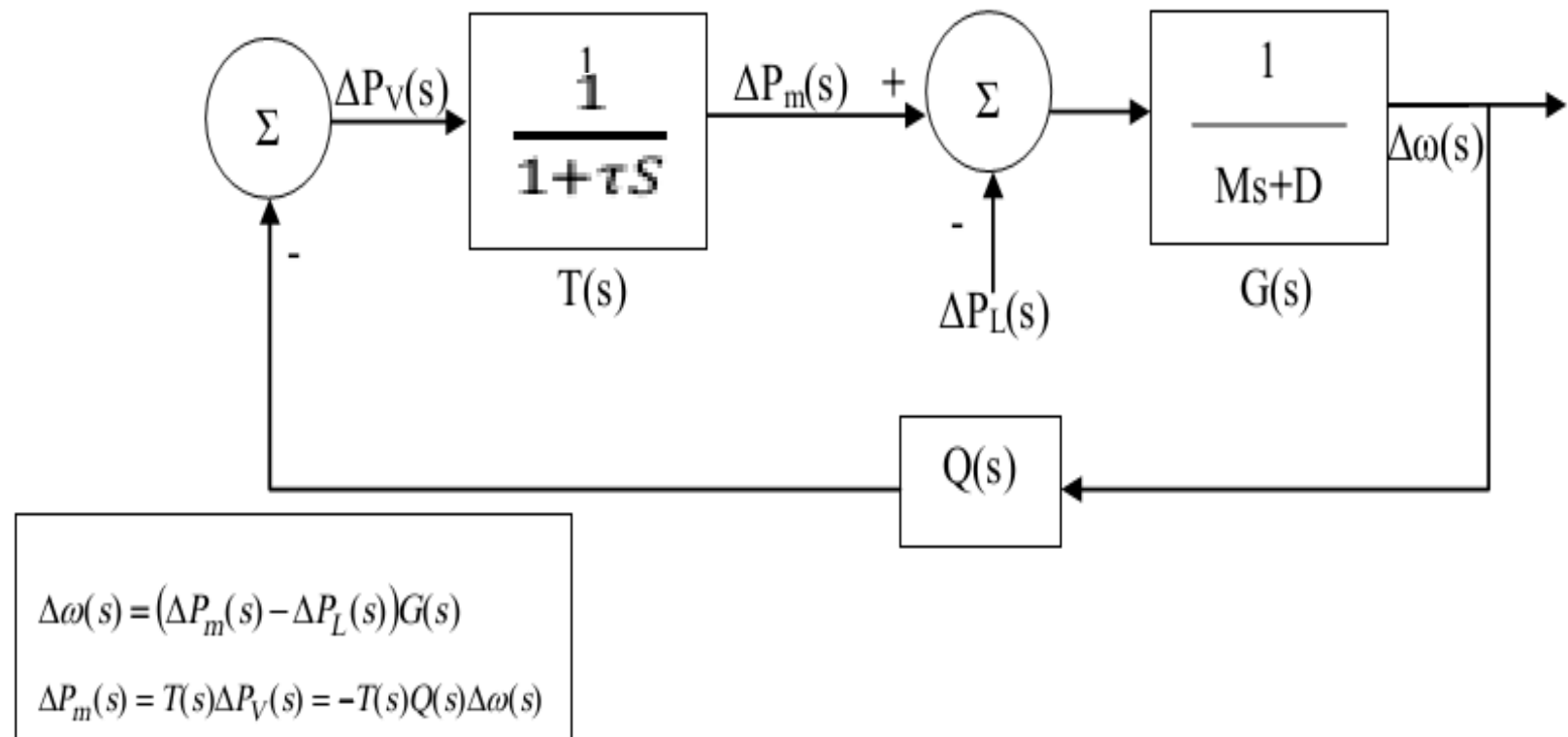
$$\Delta P_m(s) = \frac{1}{1 + T_4 s} \Delta P_V(s) = \Delta P_m(s) = \frac{1}{1 + \tau S} \Delta P_V(s)$$

transfer functions providing frequency deviation as a function of:

- change in steam valve setting and
- change in connected load.

The block diagram for this appears as in Figure.

Also we have a mechanism (speed governor) through which secondary speed control can act. This requires having feedback from $\Delta\omega$ to ΔPV . We denote this feedback as $Q(s)$,



4- MATHEMATICAL MODELLING FOR GOVERNOR :

Governor : adjusts the input valve (to change the mechanical power output to compensate for electrical load changes and to restore frequency to nominal value) .

When the electrical load is increased suddenly then the electrical power exceeds the input mechanical power ($P_e > P_m$). Due to this reason the energy that is stored in the machine is decreased and the governor sends signal for supplying more volumes of steam , gas or water, to increase the speed of the prime mover.

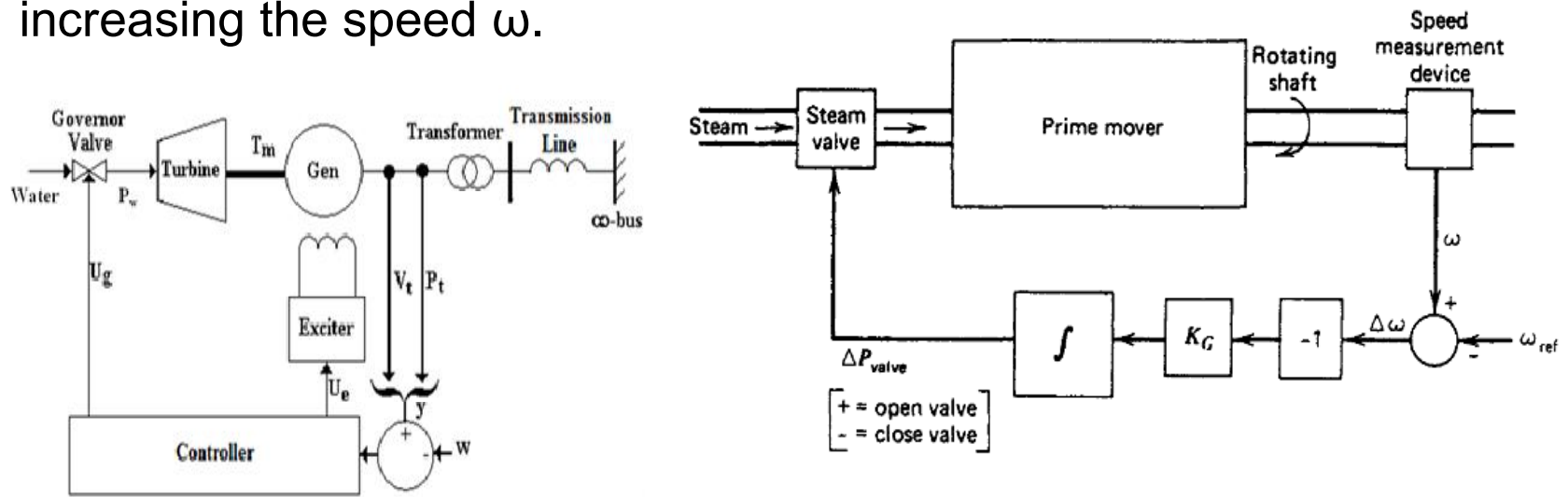
Modern governors use combination of electronic, mechanical, and hydraulic to effect the required valve position changes. The simplest governor, called the **isochronous governor**,.

$$\Delta P_g = \Delta P_{ref} - \frac{1}{R} \Delta f$$

The command ΔP_g is transformed through amplifier to the steam valve position command ΔP_v . We assume here a linear relationship and considering simple time constant we get this s-domain relation

$$\Delta P_v = \frac{1}{1 + \tau_g s} \Delta P_g(s)$$

We will illustrate such a speed-governing mechanism with the diagram shown in Figure. The speed-measurement device's output, ω , is compared with a reference, ω_{ref} , to produce an **error signal**, $\Delta\omega$. The error, $\Delta\omega$, is negated and then **amplified by a gain K_G** and **integrated to produce a control signal, ΔP_{valve}** which causes the main steam supply valve to open (ΔP_{valve} position) when $\Delta\omega$ is negative. If, for example, **the machine is running at reference speed and the electrical load increases**, ω will fall below ω_{ref} and $\Delta\omega$ will be negative then the action of the gain and integrator will be to open the steam valve, and the turbine to increase its mechanical output, thereby increasing the electrical output of the generator and increasing the speed ω .



Isochronous governor

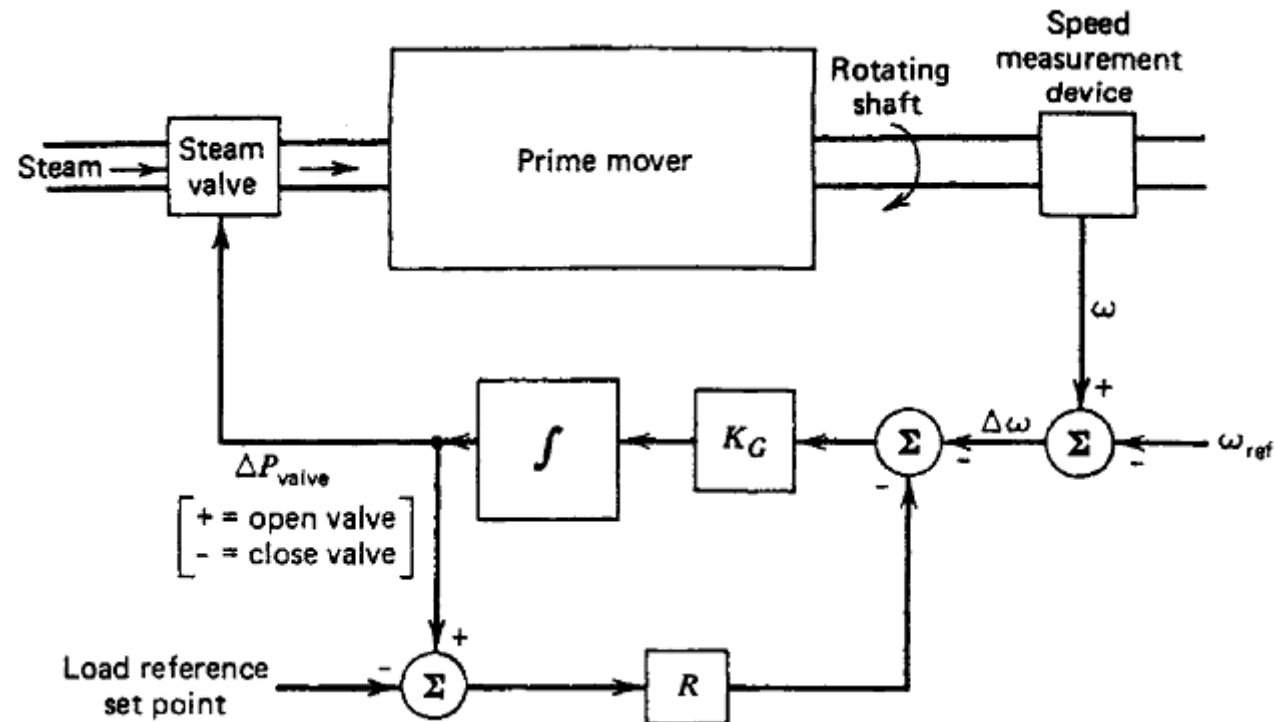
When ω exactly equals ω_{ref} , the steam valve stays at the new position (further opened) to allow the turbine generator to meet the increased electrical load .

Note that the isochronous governor of Figure cannot be used if two or more generators are electrically connected to the same system since each generator would have precisely the same speed setting , each trying to pull the system's speed (or frequency) to its own setting.

To be able **to run two or more generating units in parallel** on a generating system, the governors are provided with a feedback signal that causes the speed error to go to zero at different values of generator output

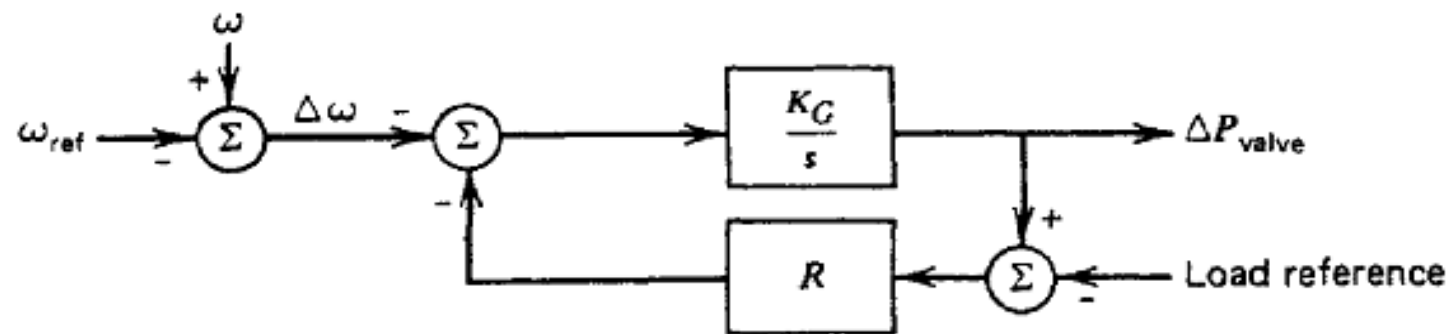
This can be accomplished by adding a feedback loop around the integrator as shown in next figure .

(Note that we have also inserted a new input, called the load reference, that we will discuss shortly. The block diagram for this governor is shown in Figure of slide 40 , where the governor now has a net gain of $1/R$ and a **time constant TG**

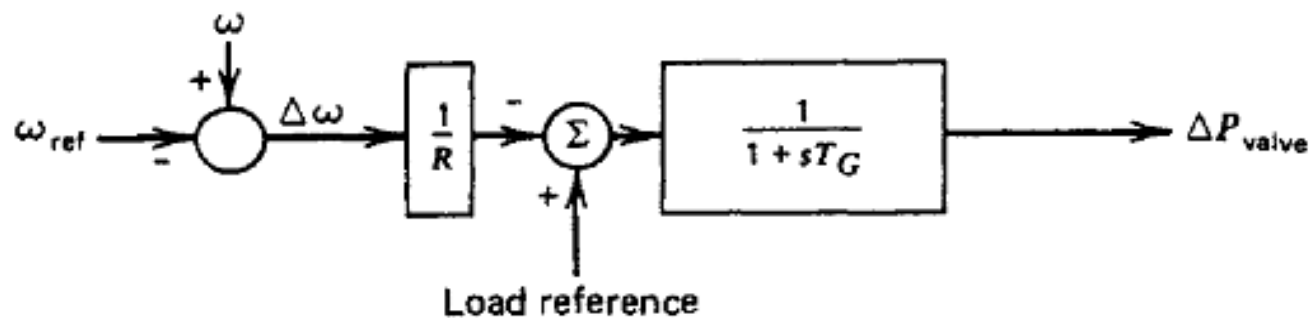
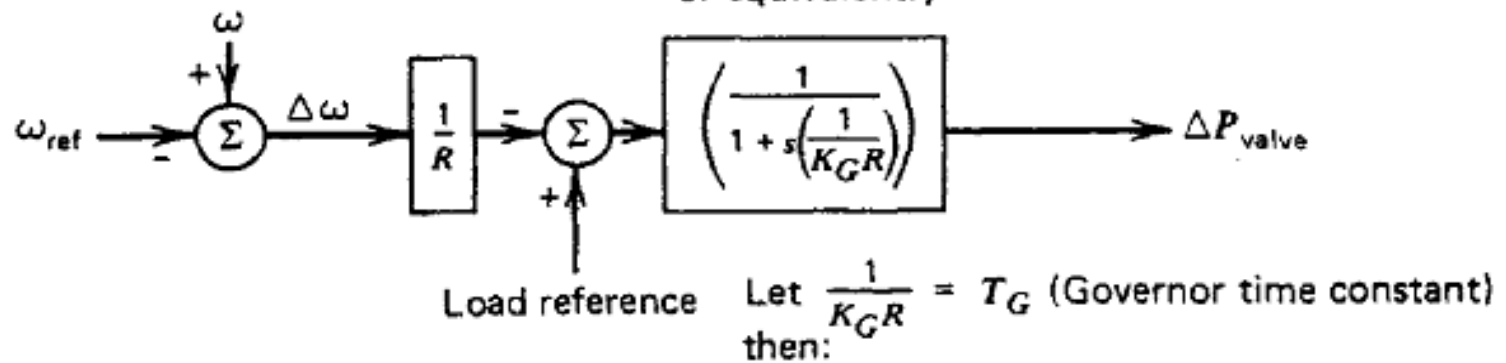


For two or more generating units in parallel Governor with speed-droop feedback loop added at each machine

Figure shows an input labeled "load reference set point." By changing the load reference, the generator's governor characteristic can be set to give reference frequency at any desired unit output.



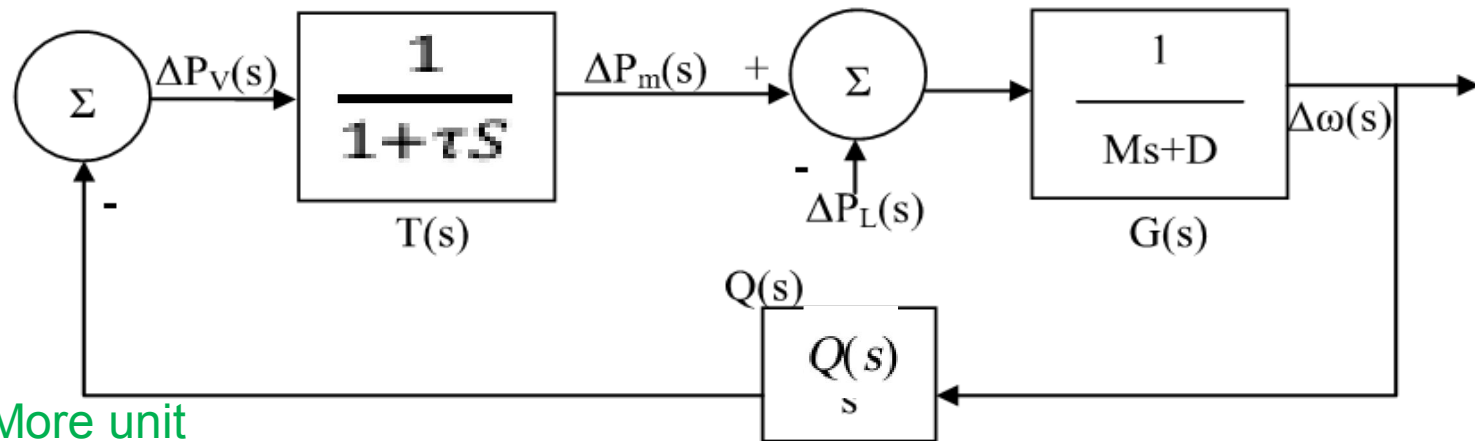
or equivalently



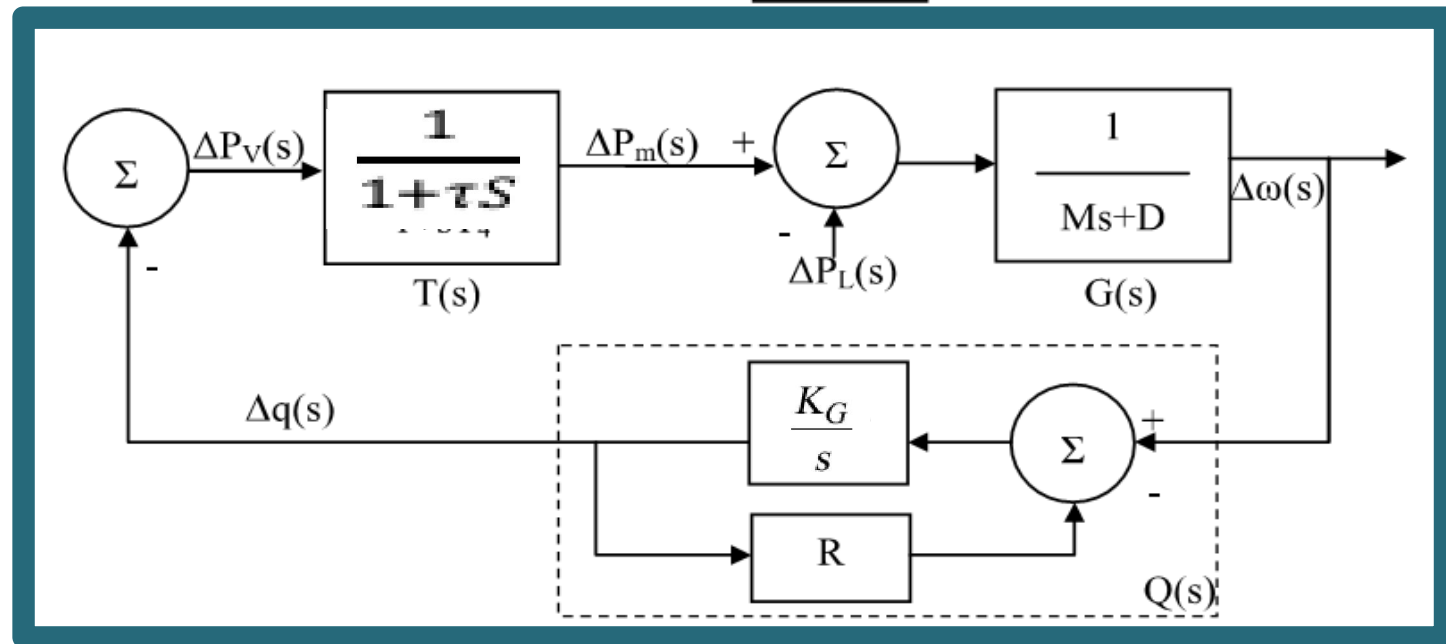
Block diagram of governor

Block diagram of frequency control in power system

a) Single unit



b) More unit

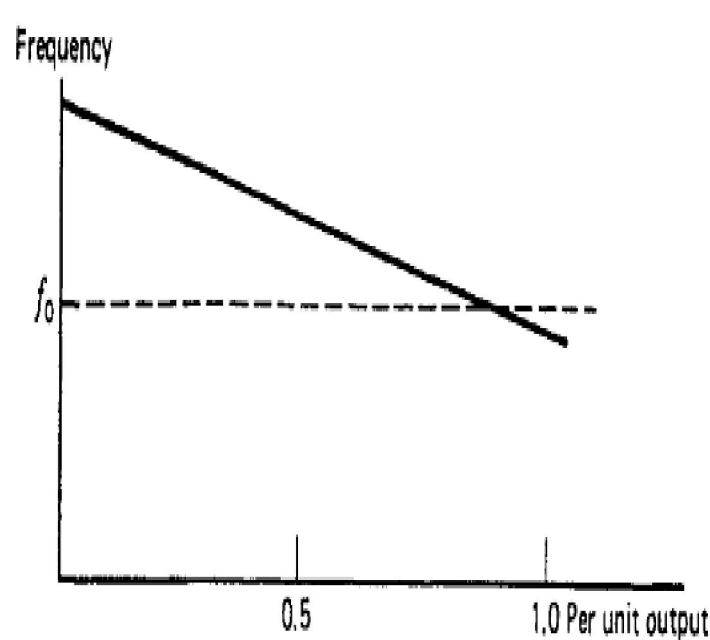


Block diagram of governor+ Primary mover + generator +load

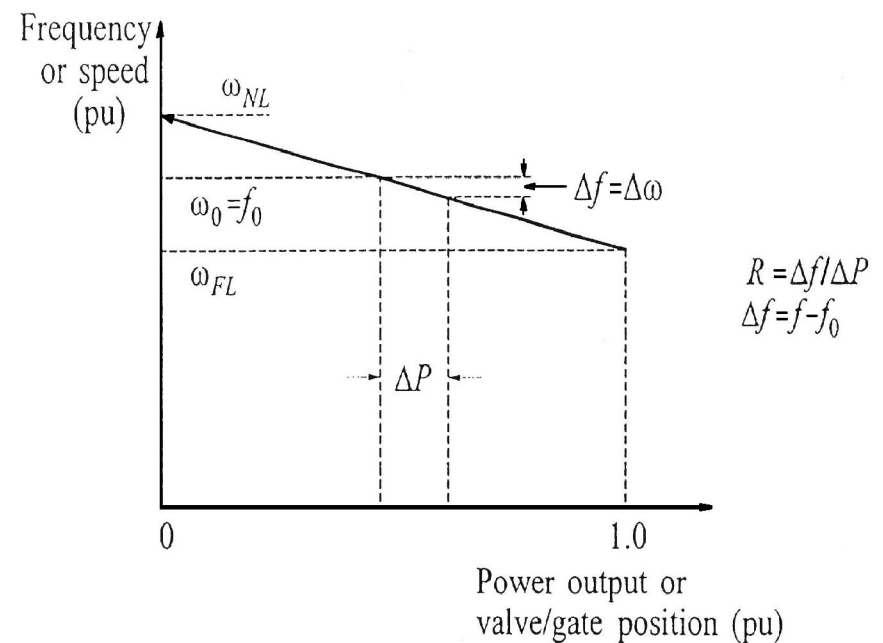
Speed-droop characteristic (R)

The result of adding the feedback loop with gain R is a governor characteristic as shown in Fig. The value of R (speed slope) determines the slope of the characteristic. Where R determines the change on the unit's output for a given change in frequency.

Common practice is to set R on each generating unit, so that a change from 0 to 100% (rated) output will result in the same frequency change for each unit. Also, a change in electrical load on a system will be compensated by generator unit output changes proportional to each unit's rated output



Speed-droop characteristic



R_u is the speed regulation per unit which is define as:

$$R_u = \frac{f_{(pu)}}{P_{(pu)}} = \frac{\frac{\text{Actual change in frequency}}{\text{base of frequency}}}{\frac{\text{Actual change in load}}{\text{base of power}}}$$

$$R_u = \frac{\left(\frac{\Delta f}{f_R} \right)}{\left(\frac{\Delta P_{gR}}{S_R} \right)} = \frac{f_2 - f_1}{f_R} \cdot \frac{S_R}{\Delta P_{gR}} = \frac{S_R}{f_R} \cdot \frac{f_2 - f_1}{\Delta P_{gR}} \quad \text{or}$$

$$R_u = \frac{S_R}{f_R} \cdot \frac{\Delta f}{\Delta P_{gR}} \quad \text{multiply both side by } \frac{f_R}{S_R} \text{ then}$$

$$\boxed{R_u \cdot \frac{f_R}{S_R} = \frac{\Delta f}{\Delta P_{gR}}} \quad (1)$$

Where:

f_2 = Frequency at no load ,Hz

f_1 = Frequency at rated MW output(P_{gR}),Hz

f_R =rated frequency of the unit, Hz

S_R =MW base

R =the magnitude of the slop of speed drop (Hz/MW)

If the load is increased by ΔP then the generators correspond for this load is $P_g = P_{g_0} + \Delta P_g$ and the changing in load MW obtain from relation (1) is:

$$\boxed{\Delta P_{gR} = \frac{\Delta f \cdot S_R}{R_u \cdot f_R}} \text{ or generally } \boxed{\Delta P_{g \text{ (total) } R} = \left[\sum_i^n \frac{S_{R(i)}}{R_{(i)u}} \right] \cdot \frac{\Delta f}{f_R}} \quad (2)$$

For example if we have **two units** then for first unit:

$$\Delta P_{g(1)R} = \left(\frac{S_{R1}}{R_{1u}} \right) \cdot \frac{\Delta f}{f_R} \quad , (\text{MW}) \quad (3)$$

Note that the changing in load is distributed between both units. From relation (2) for two unit we get

$$\frac{\Delta f}{f_R} = \frac{\Delta P_{g \text{ (total) } R}}{\left(\frac{S_{R1}}{R_{1u}} + \frac{S_{R2}}{R_{2u}} \right)} \quad , (pu) \quad (4)$$

Substitute (3) into (4) we get;

$$\Delta P_{g(1)R} = \Delta P_{g(total)R} \cdot \left(\frac{S_{R1}}{R_{1u}} \right) \cdot \left(\frac{S_{R1}}{R_{1u}} + \frac{S_{R2}}{R_{2u}} \right)^{-1}, MW$$

And

$$\Delta P_{g(2)R} = \Delta P_{g(total)R} \cdot \left(\frac{S_{R2}}{R_{2u}} \right) \cdot \left(\frac{S_{R1}}{R_{1u}} + \frac{S_{R2}}{R_{2u}} \right)^{-1}, MW$$

Example:

If two units operated in parallel at 60Hz to supply a load of 700MW, the rated of the units are given in table

	Output rated(MW)	Supply load(MW)	Speed drop characteristic
Unit 1	600	400	4%
Unit 2	500	300	5%
Total	1100	700	

If the load increased to 800MW,determine the new loading of each unit and common frequency change before supplementary control (AGC) take action (neglected losses).

Solution;

The change in load (increased) is 100MW then the change in frequency is

$$\frac{\Delta f}{f_R} = \frac{\Delta P_{g(total)R}}{\left(\frac{S_{R1}}{R_{1u}} + \frac{S_{R2}}{R_{2u}} \right)} = \frac{100}{\left(\frac{600}{0.04} + \frac{500}{0.05} \right)} = 0.004$$

But since the load is increased the frequency is decreased then

$$\frac{\Delta f}{f_R} = -0.004$$

And the change in frequency is

$$\Delta f = f_R \times (-0.004) = 60 \times (-0.004) = -59.76 \text{ Hz}$$

Now for the change of the units load we have for unit (1)

$$\Delta P_{g(1)R} = \Delta P_{g(total)R} \cdot \left(\frac{S_{R1}}{R_{1u}} \right) \cdot \left(\frac{S_{R1}}{R_{1u}} + \frac{S_{R2}}{R_{2u}} \right)^{-1}$$

$$\Delta P_{g(1)R} = 100 \times \left(\frac{600}{0.04} \right) \times \left(\frac{600}{0.04} + \frac{500}{0.05} \right)^{-1} = 60 \text{ MW}$$

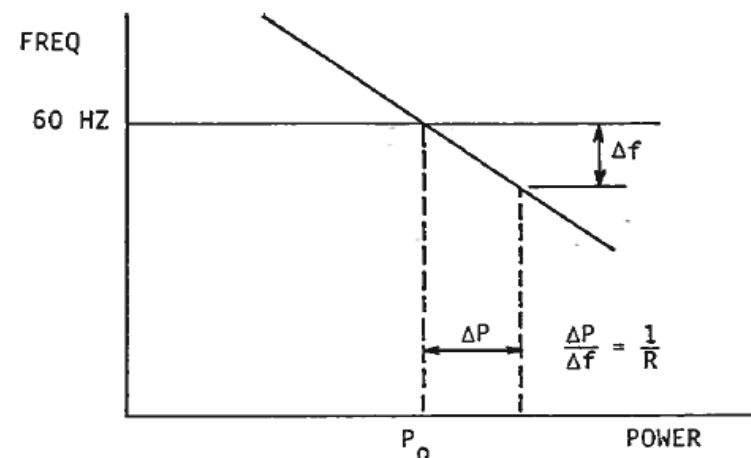
$$\Delta P_{g(2)R} = 100 \times \left(\frac{500}{0.04} \right) \times \left(\frac{600}{0.04} + \frac{500}{0.05} \right)^{-1} = 40 \text{ MW}$$

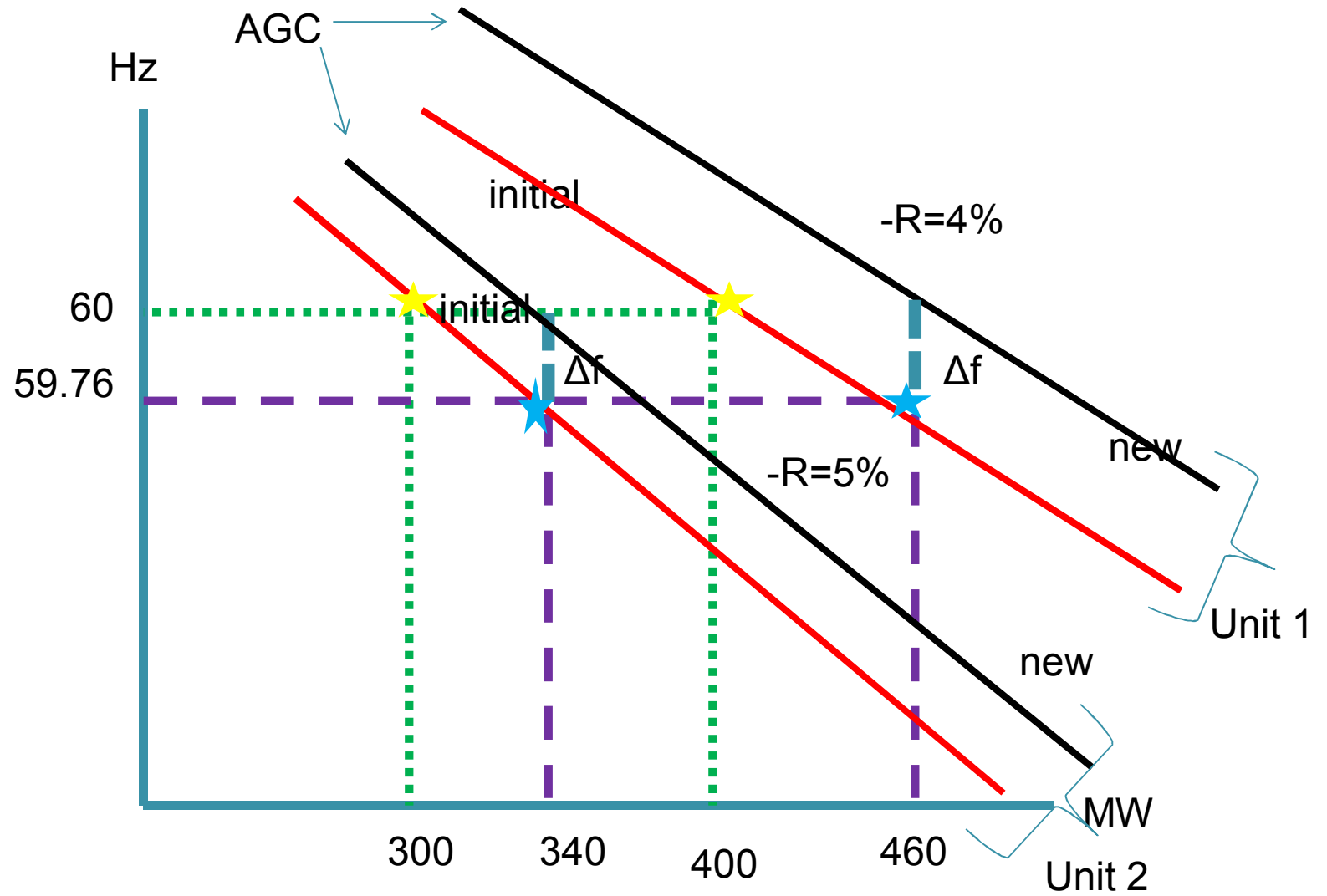
The new load change are

	Output rated(MW)	Supply load(MW)
Unit 1	600	460
Unit 2	500	340
Total	1100	800

The droop of the load frequency curve is thus mainly determined by $-R$ as shown in next figure.

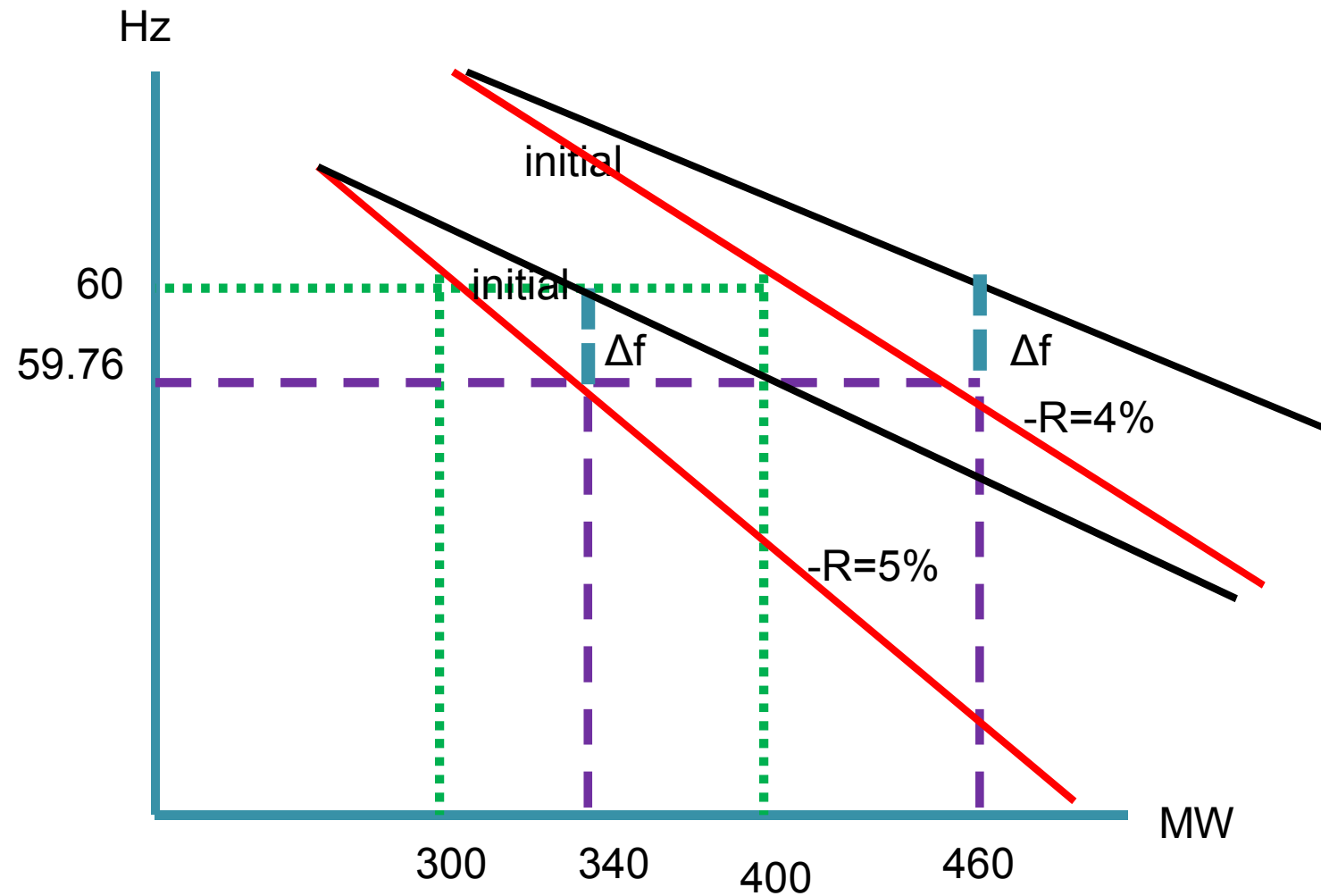
$$\frac{1}{R} = \frac{\Delta f}{\Delta P_{gR}} = -R$$





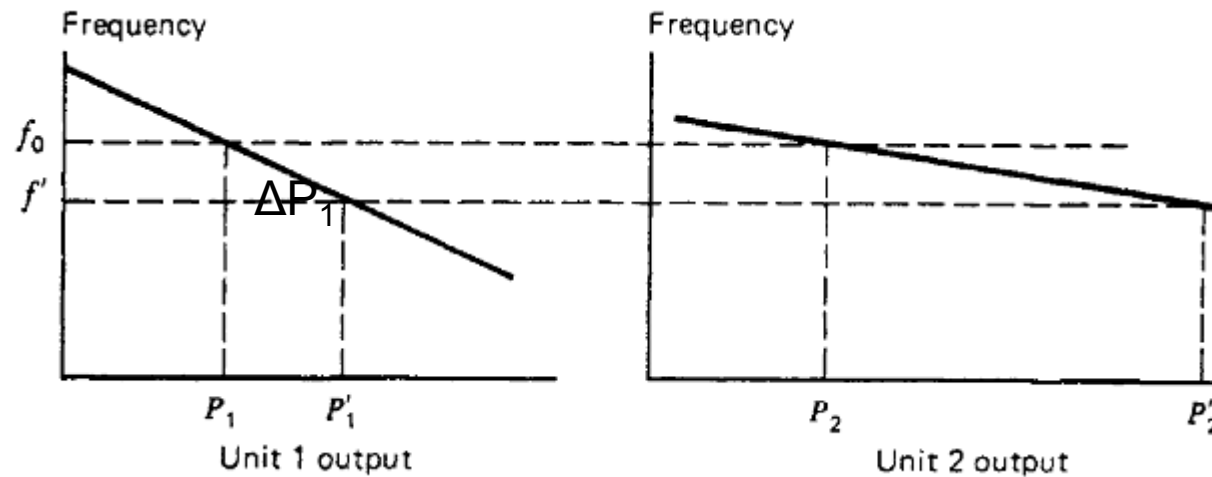
Speed drop characteristics

The question is can you modify the slop value to be common for both units



Then if **two generators** with drooping governor characteristics are connected to a power system, there will always be a unique frequency, at which they will share a load change between them.

This is illustrated in Figure , showing two units with drooping characteristics connected to a common load as shown in example before



Allocation of unit outputs with governor droop

Figure shown, the governors increase output until the units seek a new, common operating frequency, f' . The amount of load pickup on each unit is proportional to the slope of its droop characteristic.

Unit 1 increases its output from P_1 to P'_1 and unit 2 increases its output from P_2 to P'_2 such that the net generation increase.

$$\Delta P_L = (P'_1 - P_1) + (P'_2 - P_2)$$

Parallel operation of two units with different capacity and regulation

The case when two units of different frequency and regulation characteristics are operated in parallel is as shown below. The regulation characteristics are $R_1 = \Delta f(\text{pu}) / \Delta P_1(\text{pu})$, $R_2 = \Delta f(\text{pu}) / \Delta P_2(\text{pu})$

$$\Delta P_{1(\text{pu})} = \frac{\Delta f_{(\text{pu})}}{R_1} \Rightarrow \Delta f_{(\text{pu})} = \Delta P_{1(\text{pu})} \cdot R_1 \text{ and } \Delta P_{2(\text{pu})} = \frac{\Delta f_{(\text{pu})}}{R_2} \text{ then ;}$$

$$\Delta P_{2(\text{pu})} = \Delta P_{1(\text{pu})} \cdot \frac{R_1}{R_2} \text{ divided both side by } \Delta P_{1(\text{pu})} \Rightarrow \frac{\Delta P_{2(\text{pu})}}{\Delta P_{1(\text{pu})}} = \frac{R_1}{R_2} \text{ or}$$

$$\frac{\Delta P_{1(\text{pu})}}{\Delta P_{2(\text{pu})}} = \frac{R_2}{R_1} \text{ (in pu) or } \Rightarrow$$

$$\frac{\Delta P_{1(\text{actual})}}{\Delta P_{2(\text{actual})}} = \boxed{\frac{\Delta P_{1(\text{pu})}}{\Delta P_{2(\text{pu})}} \cdot \frac{P_{1(\text{rated})}}{P_{2(\text{rated})}} = \frac{R_2}{R_1}}$$

Initial Loads - P1 and P2, change in load

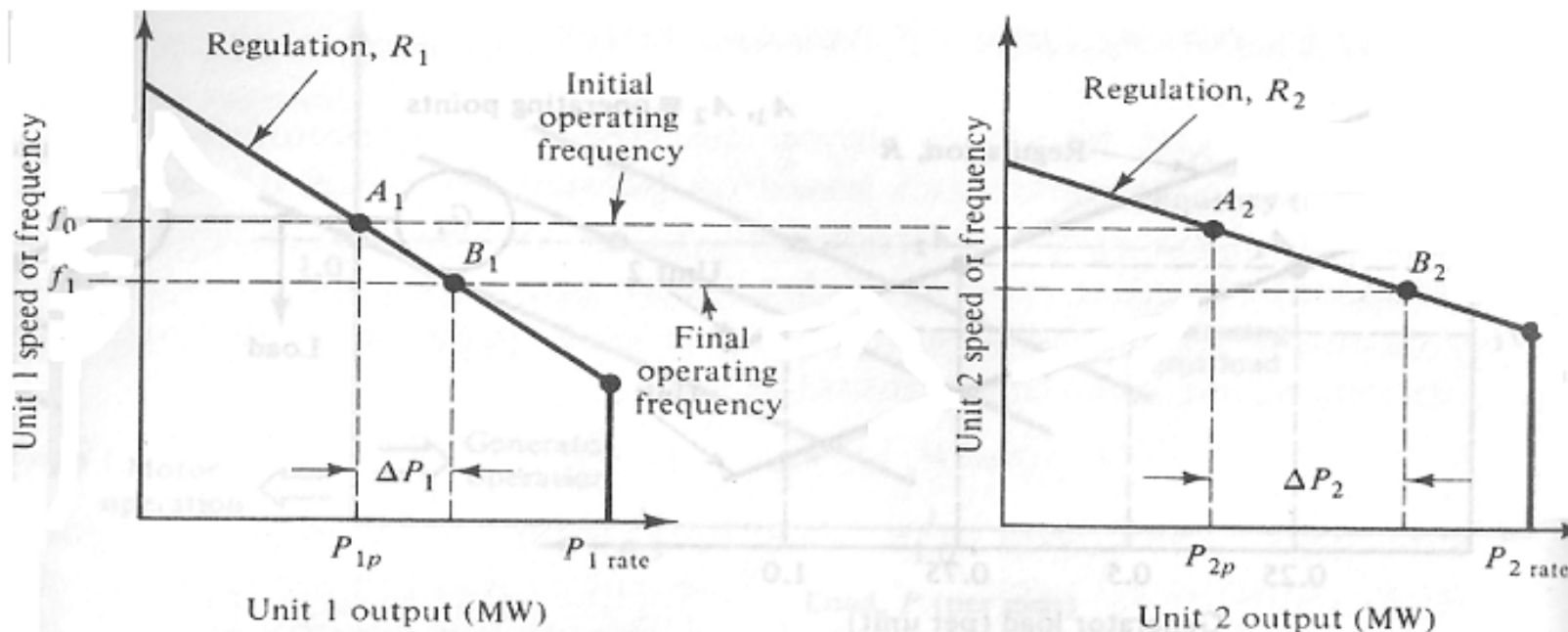
$$\Delta P_{(\text{pu})} = \frac{\Delta f}{R} = \frac{\Delta P_{(\text{actual})}}{\Delta P_{(\text{rated})}} \Rightarrow \boxed{\Delta P_{(\text{actual})} = \frac{\Delta f \cdot P_{(\text{rated})}}{R}}$$

$$\Delta L = \Delta P_{1(\text{actual})} + \Delta P_{2(\text{actual})} = \frac{\Delta f \cdot P_{1(\text{rated})}}{R_1} + \frac{\Delta f \cdot P_{2(\text{rated})}}{R_2}$$

Equivalent System Regulation

$$\Delta L = \Delta f \left(\frac{P_{1(rated)}}{R_1} + \frac{P_{2(rated)}}{R_2} \right) \text{ or } \frac{\Delta L}{\Delta f} = \left(\frac{P_{1(rated)}}{R_1} + \frac{P_{2(rated)}}{R_2} \right) \text{ or } r$$

$$\frac{\Delta f}{\Delta L} = \left(\frac{P_{1(rated)}}{R_1} + \frac{P_{2(rated)}}{R_2} \right)^{-1} = \frac{\Delta f}{\Delta P_g} = -R \quad (\text{neglecting losses})$$



Example :

Two parallel operating generators, 60Hz. Unit1 = 337 MW with 0.03 pu droop, Unit 2=420MW with 0.05 pu droop.

Find each unit's share for 0.1pu increase in load and new frequency ?

Both generators must share an increase load of 0.1pu that means the actual increasing value are :

$$\text{gen}(1) = 0.1 \times 337 = 33.7 \text{ MW} \text{ and}$$

$$\text{gen}(2) = 0.1 \times 420 = 42 \text{ MW} \text{ as a result of both is } = 76.7 \text{ MW}$$

And the change in frequency is

$$\frac{\Delta f}{f_R} = \frac{\Delta P_{g(\text{total})R}}{\left(\frac{S_{R1}}{R_{1u}} + \frac{S_{R2}}{R_{2u}} \right)} = \frac{(P_{1(\text{rated})} + P_{2(\text{rated})})}{\frac{P_{1(\text{rated})}}{R_1} + \frac{P_{2(\text{rated})}}{R_2}} = \frac{1}{\frac{337}{0.03} + \frac{420}{0.05}} \times (337 + 420) = 0.03855 \text{ pu}$$

New system frequency due to increase in load

$$\Delta L = \Delta P = \frac{\Delta f}{R} \Rightarrow \Delta f = R_{\text{system}} \cdot \Delta L = 0.03855 \times 0.1 = 0.003855 \text{ pu}$$

And since the load is increase then the frequency is decrease by $= -0.003855$

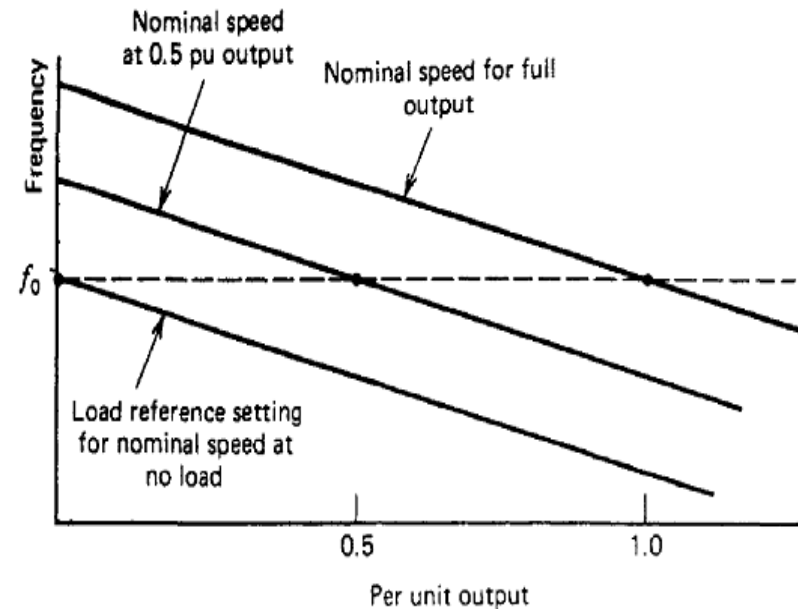
$$\text{thuse } 60(\text{rated}) - (60(\text{base}) \times 0.00386(\text{pu})) = 60 - 0.2316 = 59.769 \text{ Hz (actual)}$$

$$\Delta P_1 = \frac{\Delta f}{R_1} \cdot P_{1\text{rated}} = \frac{0.00386}{0.03} \times 337 = 43.47 \text{ MW} \text{ and}$$

$$\Delta P_2 = \frac{\Delta f}{R_2} \cdot P_{2\text{rated}} = \frac{0.00386}{0.05} \times 420 = 32.4 \text{ MW}$$

$$\text{The total increase in the load is } = 43.47 + 32.4 = 75.87 \text{ MW}$$

Note that a steady-state change in ΔP_{valve} of 1.0 pu requires a value of R pu change in frequency, $\Delta\omega$. (unit regulation referred to in percent). For instance, a 3% regulation for a unit would indicate that 3% change in frequency (for 60HZ).



Speed-changer settings

Therefore, R is equal to pu change in frequency divided by pu change in unit output. That is percentage speed regulation droop

$$R = \frac{\Delta\omega}{\Delta P} \text{ pu}$$

$$\begin{aligned} \text{Percent } R &= \frac{\text{percent speed or frequency change}}{\text{percent power output change}} \times 100 \\ &= \left(\frac{\omega_{NL} - \omega_{FL}}{\omega_0} \right) \times 100 \end{aligned}$$

where

ω_{NL} = steady-state speed at no load

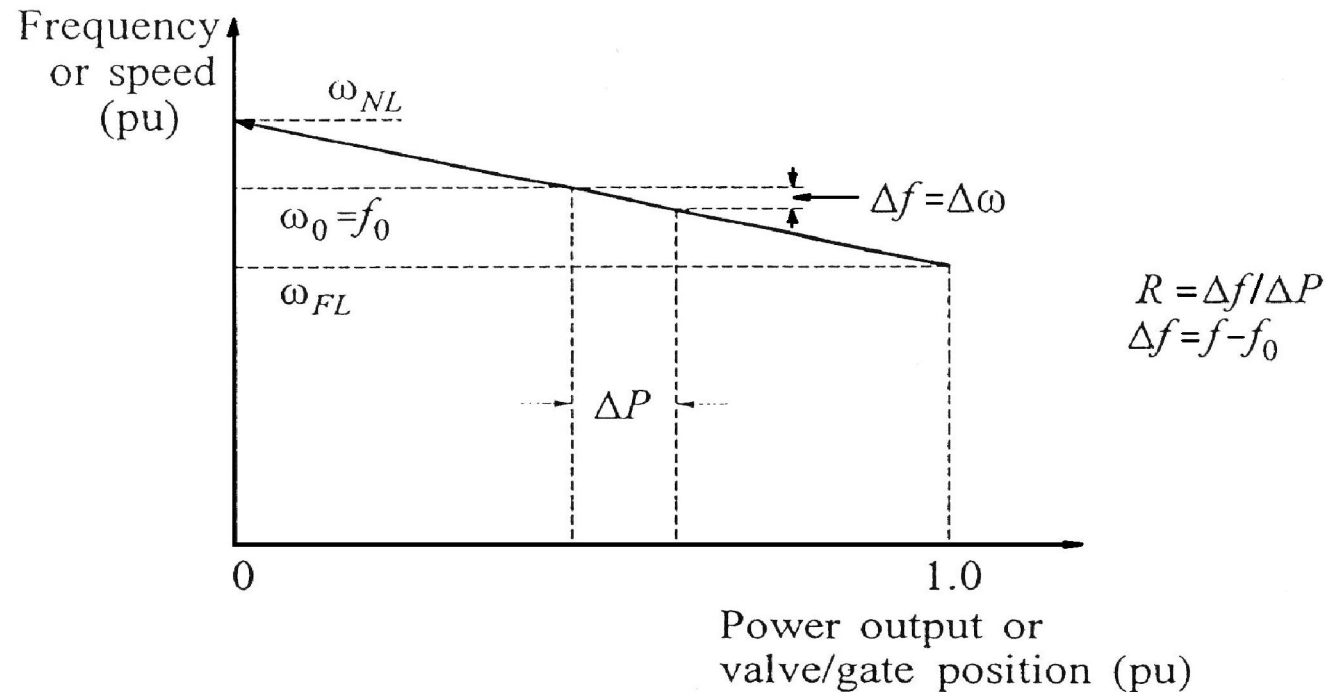
ω_{FL} = steady-state speed at full load

ω_0 = nominal or rated speed

Therefore, Unit speed regulation = $R = \Delta f(\text{pu})/\Delta P(\text{pu})$

$$= \frac{\Delta f(\text{Hz})/50(\text{Hz})}{\Delta P(\text{MW})/P_{\text{rate}}(\text{MW})}$$

For example, a 5% droop or regulation means that a 5% frequency deviation causes 100% change in valve position or power output for (for 50HZ)



Note that the transfer function relating the load change, ΔP_L , to the frequency change $\Delta \omega$, is

$$\Delta\omega(s) = \Delta P_L(s) \left[\frac{\frac{-1}{Ms + D}}{1 + \frac{1}{R} \left(\frac{1}{1 + sT_G} \right) \left(\frac{1}{1 + sT_{CH}} \right) \left(\frac{1}{Ms + D} \right)} \right]$$

The steady-state value of $\Delta\omega(s)$ may be found by

$$\Delta\omega \text{ steady state} = \lim_{s \rightarrow 0} [s \Delta\omega(s)]$$

$$= \frac{-\Delta P_L \left(\frac{1}{D} \right)}{1 + \left(\frac{1}{R} \right) \left(\frac{1}{D} \right)} = \frac{-\Delta P_L}{\frac{1}{R} + D}$$

Note that if D were zero, the change in speed would simply be as shown before

$$\Delta\omega = -R \Delta P_L$$

Example

A single area consists of two generating units with the following characteristics.

Unit	Rating	Speed regulation R (pu on unit MVA base)
1	600 MVA	6%
2	500 MVA	4%

The units are operating in parallel, sharing 900 MW at the nominal frequency. Unit 1 supplies 500 MW and unit 2 supplies 400 MW at 60 Hz. The load is increased by 90 MW.

- (a) Assume there is no frequency-dependent load, i.e., $D = 0$. Find the steady-state frequency deviation and the new generation on each unit.
- (b) The load varies 1.5 percent for every 1 percent change in frequency, i.e., $D = 1.5$. Find the steady-state frequency deviation and the new generation on each unit.

First we express the governor speed regulation of each unit to a common MVA base. Select 1000 MVA for the apparent power base, then

$$R_1 = \frac{1000}{600}(0.06) = 0.1 \text{ pu}$$

$$R_2 = \frac{1000}{500}(0.04) = 0.08 \text{ pu}$$

The per unit load change is

$$\Delta P_L = \frac{90}{1000} = 0.09 \text{ pu}$$

with $D = 0$, the per unit steady-state frequency deviation is

$$\Delta \omega_{ss} = \frac{-\Delta P_L}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{-0.09}{10 + 12.5} = -0.004 \text{ pu}$$

Thus, the steady-state frequency deviation in Hz is

$$\Delta f = (-0.004)(60) = -0.24 \text{ Hz}$$

and the new frequency is

$$f = f_0 + \Delta f = 60 - 0.24 = 59.76 \text{ Hz}$$

The change in generation for each unit is

$$\Delta P_1 = -\frac{\Delta\omega}{R_1} = -\frac{-0.004}{0.1} = 0.04 \text{ pu} \\ = 40 \text{ MW}$$

$$\Delta P_2 = -\frac{\Delta\omega}{R_2} = -\frac{-0.004}{0.08} = 0.05 \text{ pu} \\ = 50 \text{ MW}$$

Thus, unit 1 supplies 540 MW and unit 2 supplies 450 MW at the new operating frequency of 59.76 Hz.

(b) For $D = 1.5$, the per unit steady-state frequency deviation is

$$\Delta\omega_{ss} = \frac{-\Delta P_L}{\frac{1}{R_1} + \frac{1}{R_2} + D} = \frac{-0.09}{10 + 12.5 + 1.5} = -0.00375 \text{ pu}$$

Thus, the steady-state frequency deviation in Hz is

$$\Delta f = (-0.00375)(60) = -0.225 \text{ Hz}$$

and the new frequency is

$$f = f_0 + \Delta f = 60 - 0.225 = 59.775 \text{ Hz}$$

The change in generation for each unit is

$$\begin{aligned}\Delta P_1 &= -\frac{\Delta\omega}{R_1} = -\frac{-0.00375}{0.1} = 0.0375 \text{ pu} \\ &= 37.500 \text{ MW}\end{aligned}$$

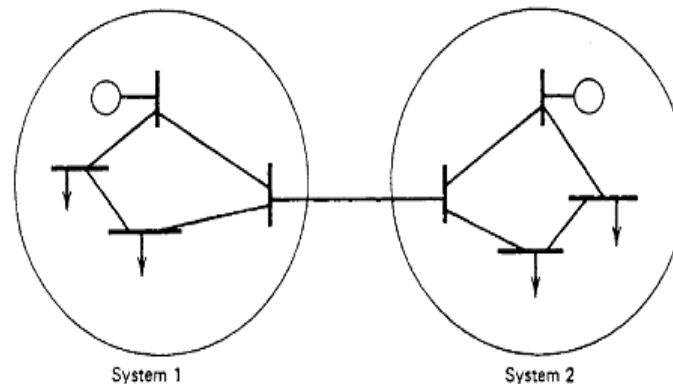
$$\begin{aligned}\Delta P_2 &= -\frac{\Delta\omega}{R_2} = -\frac{-0.00375}{0.08} = 0.046875 \text{ pu} \\ &= 46.875 \text{ MW}\end{aligned}$$

Thus, unit 1 supplies 537.5 MW and unit 2 supplies 446.875 MW at the new operating frequency of 59.775 Hz. The total change in generation is 84.375, which is 5.625 MW less than the 90 MW load change. This is because of the change in load due to frequency drop which is given by

$$\begin{aligned}\Delta\omega D &= (-0.00375)(1.5) = -0.005625 \text{ pu} \\ &= -5.625 \text{ MW}\end{aligned}$$

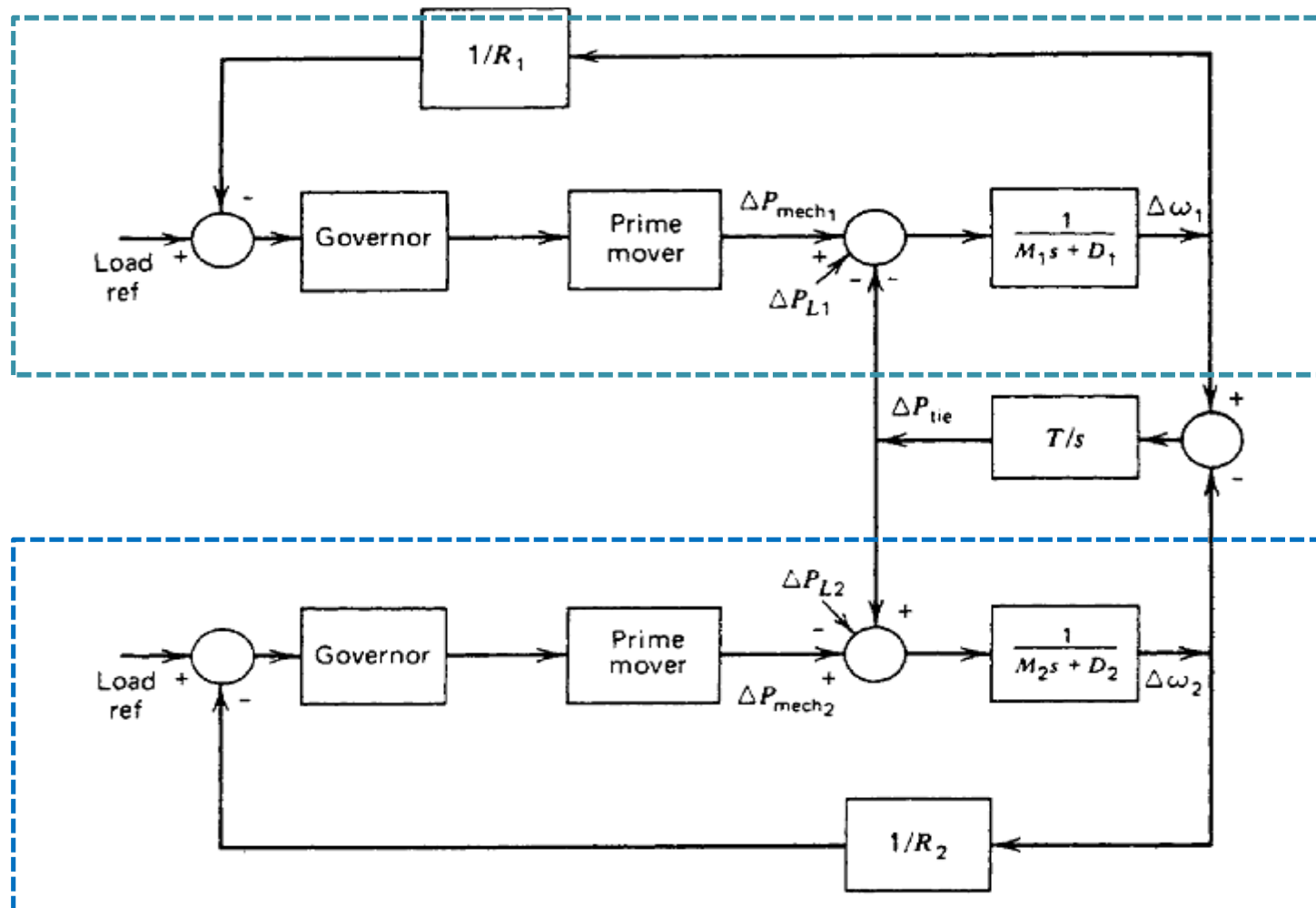
TIE-LINE MODEL(AGC)

Suppose now that we have an interconnected power system broken into two areas each having one generator. The areas are connected by a single transmission line.



The power flow over the transmission line will appear as a positive load to one area and an equal but negative load to the other, **or vice versa**, depending on the direction of flow.

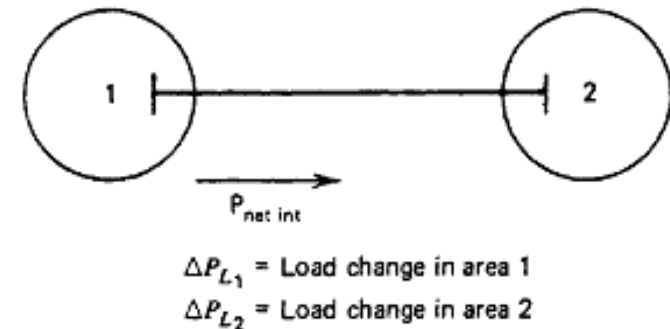
The direction of flow will be dictated by the relative phase angle between the areas, which is determined by the relative speed deviations in the areas. A block diagram representing this interconnection can be drawn as in Figure (next slide) .



Block diagram of interconnected areas.

Note that the tie power flow was defined as going from area 1 to area 2; therefore, the flow appears as a load to area 1 and a power source (negative load) to area 2. If one assumes that mechanical powers are constant, the rotating masses and tie line exhibit damped oscillatory characteristics known as synchronizing oscillations.

1.If frequency decreased and net interchange power leaving the system is increased ➡ a load increase has occurred outside the system.



2.If frequency decreased and net interchange power leaving the system is decreased ➡ a load increase has occurred inside the system

In the following table we note the increase with (+) and decrease with (-) and shown their results

$\Delta\omega$	$\Delta P_{\text{net int}}$	Load change		Resulting control action
-	-	ΔP_{L1}	+	Increase P_{gen} in system 1
		ΔP_{L2}	0	
+	+	ΔP_{L1}	-	Decrease P_{gen} in system 1
		ΔP_{L2}	0	
-	+	ΔP_{L1}	0	Increase P_{gen} in system 2
		ΔP_{L2}	+	
+	-	ΔP_{L1}	0	Decrease P_{gen} in system 2
		ΔP_{L2}	-	

It is quite important to analyze the steady-state frequency deviation, tie-flow deviation, and generator outputs for an interconnected area after a load change occurs.

Let there be a load change ΔP_L in area 1. In the steady state, after all synchronizing oscillations have damped out, the frequency will be constant and equal to the same value on both areas. Then

$\Delta \omega = \Delta \omega_1 + \Delta \omega_2$ and the change in respect with time must be zero, then

$$\frac{\partial(\Delta \omega)}{\partial t} = \frac{\partial(\Delta \omega_1)}{\partial t} = \frac{\partial(\Delta \omega_2)}{\partial t} = 0 \quad \text{and}$$

$$\Delta P_{machine 1} - \Delta P_{tie line} - \Delta P_{L1} = \Delta \omega D_1$$

$$\Delta P_{machine 2} + \Delta P_{tie line} = \Delta \omega D_2 \quad \text{also}$$

$$\Delta P_{machine 1} = \frac{-\Delta \omega}{R_1} \quad \text{and} \quad \Delta P_{machine 2} = \frac{-\Delta \omega}{R_2}$$

substitute we get for first equation $\frac{-\Delta \omega}{R_1} - \Delta P_{tie line} - \Delta P_{L1} = \Delta \omega D_1$ or

$$-\Delta P_{tie line} - \Delta P_{L1} = \frac{\Delta \omega}{R_1} + \Delta \omega D_1 = \Delta \omega \left(\frac{1}{R_1} + D_1 \right)$$

same for the second equation $\Rightarrow \quad + \Delta P_{tie line} = \Delta \omega \left(\frac{1}{R_2} + D_2 \right)$

from both equations we get the changing in $\Delta \omega$

$$-\Delta\omega\left(\frac{1}{R_2} + D_2\right) - \Delta P_{L1} = \Delta\omega\left(\frac{1}{R_1} + D_1\right) \text{ or}$$

$$-\Delta P_{L1} = \Delta\omega\left(\frac{1}{R_1} + D_1\right) + \Delta\omega\left(\frac{1}{R_2} + D_2\right) = \Delta\omega\left[\left(\frac{1}{R_1} + D_1\right) + \left(\frac{1}{R_2} + D_2\right)\right]$$

then

$$\Delta\omega = \frac{-\Delta P_{L1}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + D_2 + D_1\right)}$$

from which we can derive the change in tie flow

$$\Delta P_{tie} = \frac{-\Delta P_{L1} \left(\frac{1}{R_1} + D_2\right)}{\left(\frac{1}{R_1} + \frac{1}{R_2} + D_2 + D_1\right)}$$

EXAMPLE

You are given two system areas connected by a **tie line** with the following characteristics.

Area 1	Area 2
$R = 0.01 \text{ pu}$	$R = 0.02 \text{ pu}$
$D = 0.8 \text{ pu}$	$D = 1.0 \text{ pu}$
Base MVA = 500	Base MVA = 500

If a load change of 100 MW or (0.2 pu) occurs in area 1.(on base of 500)

What is the new steady-state frequency and what is the change in tie flow? Assume both areas were at nominal frequency (60 Hz) to begin.

Solution:

$$\Delta\omega = \frac{-\Delta P_{L_1}}{\frac{1}{R_1} + \frac{1}{R_2} + D_1 + D_2} = \frac{-0.2}{\frac{1}{0.01} + \frac{1}{0.02} + 0.8 + 1} = -0.00131752 \text{ pu}$$

$$f_{\text{new}} = 60 - 0.00132(60) = 59.92 \text{ Hz}$$

$$\begin{aligned} \Delta P_{\text{tie}} &= \Delta\omega \left(\frac{1}{R_2} + D_2 \right) = -0.00131752 \left(\frac{1}{0.02} + 1 \right) = -0.06719368 \text{ pu} \\ &= -33.6 \text{ MW} \end{aligned}$$

The change in prime-mover power would be

$$\Delta P_{\text{mech}_1} = \frac{-\Delta\omega}{R_1} = - \left(\frac{-0.00131752}{0.01} \right) = 0.13175231 \text{ pu} = 65.876 \text{ MW}$$

$$\begin{aligned} \Delta P_{\text{mech}_2} &= \frac{-\Delta\omega}{R_2} = - \left(\frac{-0.00131752}{0.02} \right) = 0.06587615 \text{ pu} = 32.938 \text{ MW} \\ &= 98.814 \text{ MW} \end{aligned}$$

The total changes in generation is (100- 98.814) MW, which is 1.186 MW short of the 100 MW load change. The change in total area load due to frequency drop would be

$$\text{For area 1} = \Delta\omega D_1 = -0.0010540 \text{ pu} = -0.527 \text{ MW}$$

$$\text{For area 2} = \Delta\omega D_2 = -0.00131752 \text{ pu} = -0.6588 \text{ MW}$$

Where $D_1=8$, $D_2=1$ and $\Delta\omega=-0.00131752$ pu and the power base is 500MW then;

Actual area 1 = $-0.0010540 \times 500 = -0.527 \text{ MW}$

And area 2 = $-0.00131752 \times 500 = -0.6588 \text{ MW}$

Therefore, the total load change is $\Delta P_L = 100 - 98.814 = 1.186 \text{ MW}$, which accounts for the difference in total generation change and total load change.





Appendix

Usefully Formula

$$1^\circ \text{ mechanical} = \left(\frac{2}{p}\right)^\circ \text{ electrical}$$

$P = \text{No. of poles}$

$$\omega(\text{mechanical}) = 2\pi n' \text{ (mech.rad / sec)}$$

$n' = \text{speed in revolution / sec. (rps) or}$

$$\omega = \frac{2\pi.n}{60}$$

$n = \text{speed in revolution / min. (rpm)}$

$\omega(\text{electrical}) = \text{Angular frequency}$

$$\omega(\text{electrical}) = 2\pi f \text{ (elect.rad / sec)}$$

$$\omega(\text{electrical}) = 360^\circ f \text{ (elect deg / sec)}$$

$$1^\circ \text{ electrical} = \left(\frac{P}{2}\right)^\circ \text{ mechanical}$$

$$\text{One revolution} = \frac{P}{2} \times 360^\circ \text{ (elect.deg.)}$$

$$1^\circ \text{ electrical} = \left(\frac{2}{P}\right)^\circ \times 360^\circ \text{ (revolution)}$$

$$1^\circ \text{ elec. / sec} = \left(\frac{2}{P}\right)^\circ \times \frac{1}{360^\circ} \text{ (rev / sec)}$$

$$1^\circ \text{ elec. / min} = \left(\frac{2}{P}\right)^\circ \times \left(\frac{1}{360^\circ} / \frac{1}{60}\right) =$$

$$= \left(\frac{2}{P}\right)^\circ \times \frac{60}{360}$$

Rotation Mechanics-vs-Linear Mechanics

Rotational mechanical is very important in stability studies. The following table show us comparison between the linear and rotational mechanical formula.

<i>Rotational mechanics</i>	<i>Linear mechanics</i>
$\theta = \text{Angular displacement (rad.)}$	$S = \text{Depalcement(distance)(m)}$
$\omega = \text{Angular velocity (rad / sec)} = \frac{d\theta}{dt}$	$V = \text{Velocity (m / sec)} = \frac{ds}{dt}$
$\alpha = \text{Angular accelaration (rad}^2 \text{ / sec}^2\text{)}$	$a = \text{accelaration (m / sec)}$
$= \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$	$m v = \frac{dv}{dt} = \frac{d^2\delta}{dt}$
$J = \text{Moment of inertia (kg.m}^2\text{)}$	$m = \text{Mass (kg)}$
$T = \text{Torque (N.m)} = J.\alpha$	$F = \text{Force} = m.a \text{ (N)}$
$M = \text{Angular momentum}$ $(\text{kg.m}^2.\text{rad / sec}) = J\omega$	$M = \text{Momentum (kg.m / sec)} = m.v$
$K.E = \frac{1}{2}J\omega^2 \text{ (Joule)}$	$K.E = \frac{1}{2}.m.v^2 \text{ (Joule)}$
$P = \text{power (watt)} = T\omega$	$P = \frac{d}{dt}(F.s) = F.v \text{ (N.m / s)}$
$W = \text{work} = \int P_e d\theta = \int T \frac{d\theta}{dt} = T\theta$	$W_{ork} = F.s$

List of Symbols

V_d, V_q : Stator voltage in d-axis and q-axis circuit

V_t : Terminal voltage

E_q' : Transient EMF in the quadratic axis of the generator

x_{ad} : Stator-rotor mutual reactance

E_{fd} : Field voltage

r_{fd} : Field resistance

X_{fd} : Self reactance of field winding

U_e : Exciter input

δ : Rotor angle

P_m : Mechanical power

P_w : Water power

H : Inertia constant

$\omega(t)$: Rotor speed of the generator

ω_0 : Angular frequency of the infinite bus bar

K_d : Mechanical damping torque coefficient

T_d : Damping torque coefficient due to damper windings

P_t : Real power output at the generator terminals

τ_e : Exciter time constant

τ_g : Governor valve time constant

τ_b : Turbine time constant

U_g : Governor input

G_v : Governor valve position

K_v : Valve constant

x_d : Total d-axis synchronous reactance between the generator and the infinite busbar

x_q : Total q-axis synchronous reactance between the generator and the infinite busbar

x_d' : Total d-axis transient reactance including the generator and the infinite busbar

T_{do}' : d-axis transient open-circuit time constant

x_T : Reactance of the transformer

x_L : Reactance of the transmission line

x_s : Reactance of the system

Laplace transform table 1

Item no.	$f(t)$	$F(s)$
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2 + \omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2 + \omega^2}$