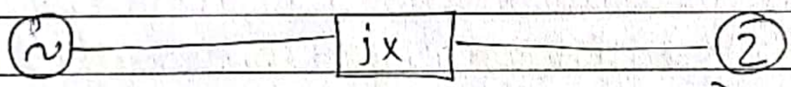
Power flow analysis:

Power flow analysis also known as load flow analysis, is a widely used technique in electrical power engineering to study and analyze the steady state behaviour of an electric power system. It involves the calculation of voltages, current and power flows in a network of interconnected power system components, such as generators, transformers, transmission lines and loads under steady state conditions.

Power flow in transmission line.



$\vec{V_S} = V_S \angle \partial$ $\qquad$ $\vec{V_R} = V_R \angle 0°$

where V_S and V_R are the sending and receiving ends voltages.
∂ is the phase angle between the sending and the receiving end voltage.
V_S lead V_R by ∂.

Generalized line constants are

$A = |A| \angle \alpha$
$B = |B| \angle \beta$
$C = |C| \angle \gamma$
$D = |D| \angle \Delta$

Complex power at receiving end is

$S_R = P_R + jQ_R$
$= V_R I_R^*$

Date: ....
Page: ....

I_R^* is the conjugate of Receiving end Current I_R.

We know that

$$V_S = AV_R + BI_R$$

So, $$I_R = \frac{V_S - AV_R}{B}$$

$$= \frac{|V_S|\angle\partial - |A|\angle\alpha\,|V_R|\angle 0^\circ}{|B|\angle\beta}$$

$$= \frac{|V_S|}{|B|}\angle(\partial-\beta) - \frac{|A|\,|V_R|}{|B|}\angle\alpha-\beta$$

$$I_R^* = \frac{|V_S|}{|B|}\angle(\beta-\partial) - \frac{|A|\,|V_R|}{|B|}\angle\beta-\alpha$$

So,

$$S_R = P_R + jQ_R$$

$$= V_R I_R^*$$

$$= \frac{|V_S|\,|V_R|}{|B|}\angle\beta-\partial - \frac{|A|\,|V_R^2|}{|B|}\angle\beta-\alpha$$

Now, we separate the real and imaginary parts, then we get the values of P_R and Q_R

So the receiving end active power

$$P_R = \frac{|V_S|\,|V_R|}{|B|}\cos(\beta-\partial) - \frac{|A|\,|V_R^2|}{|B|}\cos(\beta-\alpha)$$

Also, reactive power is

$$Q_R = \frac{|V_S|\,|V_R|}{|B|}\sin(\beta-\partial) - \frac{|A|\,|V_R^2|}{|B|}\sin(\beta-\alpha)$$

Date: ....
Page: ....

Importance of power flow analysis:

- Power flow analysis is essential in planning and designing power systems, optimizing system configurations, and evaluating system performance under different scenario.
- Power flow analysis is critical for real-time operation and control of power systems.
- It is carried out to study short circuit conditions for any interconnected system.
- For economic system operation and system loss minimization.
- It helps to determine the best location as well as optimal Capacity of the proposed generating stations, and substations

Informations obtained from Load flow studies:

- Magnitude of ~~load~~ Bus voltage
- Phase angle of ~~load~~ Bus voltage.
- Real and reactive power flow.
- Power system losses.

Classification of buses:

1. Swing Bus/slack Bus/reference bus:

This is the reference bus in a power system and is usually connected to a large power source, such as generator or an infinite bus. The voltage magnitude and phase angle at the slack bus are specified and kept constant during load flow analysis. The slack bus is used to maintain the overall balance between generation and ~~comp~~ consumption in the power system.

Narayani

2. Generator bus or voltage controlled bus or PV bus.
Here $|V_i|$ and and active power (P) is specified. and reactive power (Q) is allowed to vary. PV buses represent the buses where generators are connected and the real power output is controlled.

3. Load bus or PQ bus
This typeo of bus represents the load buses in the power system where both the real power (P) and reactive power (Q) are specified as load demands. PQ buses represents the loads in the power system

	Bus-type	specified variable	unknown
(1)	Slack/swing bus	$\|V_i\|$ $\|\delta_i\|$	P_i, Q_i
2.	Generator bus	P_i $\|V_i\|$	Q_i, δ_i
3.	Load/PQ bus	P_i, Q_i	$\|V_i\|$ δ_i

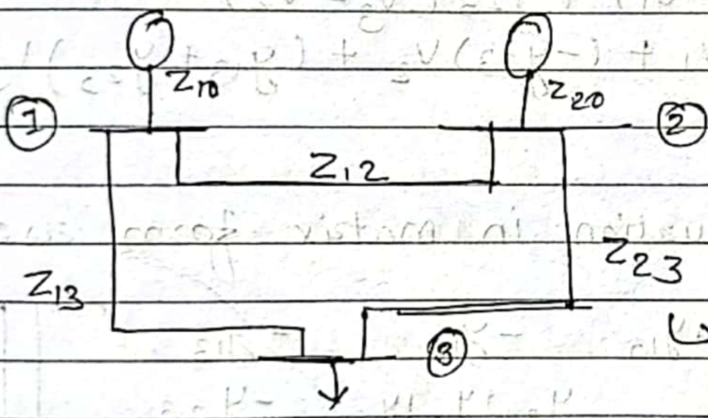
Narayani

Date: ....
Page: ....

The bus admittance matrix.

Junction formed by 2 or more elements are called node. Major nodes are those where more than two elements are connected. For power network, ground is the reference node. The remaining major nodes are called buses.

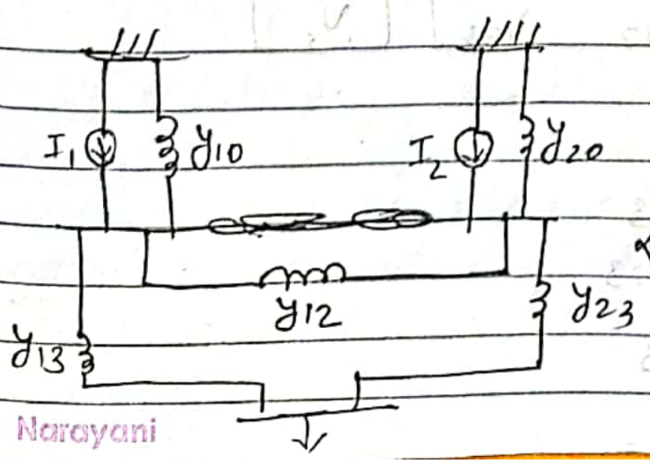
Consider a single-line diagram of the following power network.



↳ Impedance diagram.

Fig: A bus, 2 generator power system.

Now drawing the admittance diagram.



↖ admittance diagram.

$V = IR$

$I = \frac{V}{R}$

$= VY \quad \left[Y = \frac{1}{R} \right]$

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Applying KCL at nodes:

$$I_1 = y_{10}V_1 + y_{12}(V_1 - V_2) + y_{13}(V_1 - V_3)$$
$$= (y_{10} + y_{12} + y_{13})V_1 + (-y_{12})V_2 + (-y_{13})V_3.$$

$$I_2 = y_{20}V_2 + y_{12}(V_2 - V_1) + y_{23}(V_2 - V_3)$$
$$= (-y_{12})V_1 + (y_{20} + y_{12} + y_{23})V_2 + (-y_{23})V_3.$$

$$0 = y_{13}(V_3 - V_1) + y_{23}(V_3 - V_2)$$
$$= (-y_{13})V_1 + (-y_{23})V_2 + (y_{13} + y_{23})V_3$$

We can write equation in matrix form as:

$$\begin{bmatrix} I_1 \\ I_2 \\ 0 \end{bmatrix} = \begin{bmatrix} y_{10}+y_{12}+y_{13} & -y_{12} & -y_{13} \\ -y_{12} & y_{20}+y_{12}+y_{23} & -y_{23} \\ -y_{13} & -y_{23} & y_{13}+y_{23} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

Also,

$$\begin{bmatrix} I_1 \\ I_2 \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

where,

$$Y_{11} = y_{10} + y_{12} + y_{13}$$
$$Y_{22} = y_{20} + y_{12} + y_{23}$$
$$Y_{33} = y_{13} + y_{23}$$

Date:
Page:

The diagonal element of each node is the sum of the admittance connected to it. It is known as self admittance.

$$Y_{12} = Y_{21} = -y_{12}$$
$$Y_{13} = Y_{31} = -y_{13}$$
$$Y_{23} = Y_{32} = -y_{23}$$

non - diagonal element.

The non-diagonal element is equal to the -ve of the admittance between the nodes. It is also known as the mutual impedance.

Also,

$$I_{Bus} = Y_{Bus} V_{Bus}$$

Y_{Bus} is Symmetric

Extending the above relation to an n bus system, the node voltage equation in matrix form is

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \vdots & & & \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}$$

$$V_{Bus} = Y_{Bus}^{-1} I_{Bus}$$
$$= Z_{Bus} I_{Bus}.$$

Z_{Bus} is the bus impedance matrix and is inverse of Y_{Bus}.

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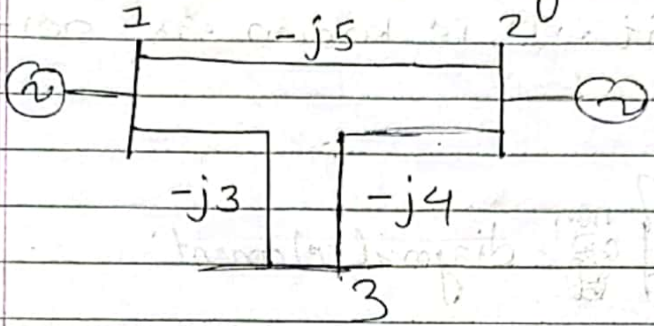
$(a+bj)$ form is usually impedance.
$(a-bj)$ form is usually admittance.

Calculate the element of Y-bus matrix.



Solution,
Since 3 buses are present so Y matrix will be

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

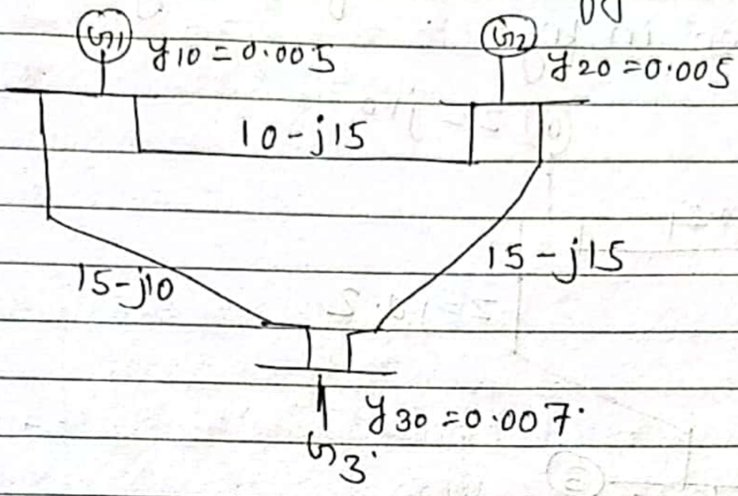
So, $Y_{11} = y_{12} + y_{13} = -j5 - j3 = -j8$
$Y_{22} = y_{21} + y_{23} = -j5 - j4 = -j9$
$Y_{33} = y_{31} + y_{32} = -j3 - j4 = -j7$

$Y_{12} = Y_{21} = -y_{12} = j5$
$Y_{13} = Y_{31} = -y_{13} = j3$
$Y_{23} = Y_{32} = -y_{23} = j4$.

So, $$Y_{bus} = \begin{bmatrix} -j8 & j5 & j3 \\ j5 & -j9 & j4 \\ j3 & j4 & -j7 \end{bmatrix}$$

Date: ....
Page: ....

For the network shown in the figure find Y-bus.



$Y_{11} = y_{10} + y_{12} + y_{13}$
$= 0.005 + (10 - j15) + (15 - j10)$
$= 25.005 - j25$

$Y_{22} = y_{20} + y_{21} + y_{23}$
$= 0.005 + (10 - j15) + (15 - j15)$
$= 25.005 - j30$

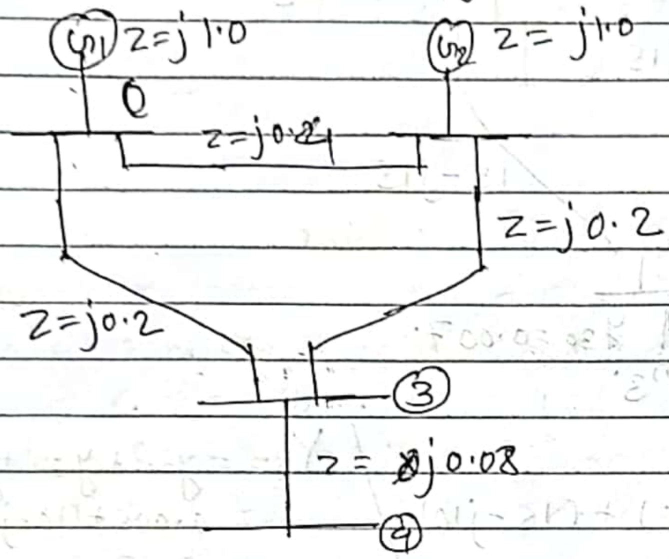
$Y_{33} = y_{30} + y_{31} + y_{32}$
$= 0.007 + (15 - j10) + (15 - j15)$
$= 30.007 + j25.$

$Y_{12} = Y_{21} = -(10 - j15) = -10 + j15.$
$Y_{13} = Y_{31} = -(15 - j10) = -15 + j10.$
$Y_{32} = Y_{23} = -(15 - j15) = -15 + j15.$

So, the bus admittance Matix is

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} = \begin{bmatrix} 25.005 - j25 & -10 + j15 & -15 + j10 \\ -10 + j15 & 25.005 - j30 & -15 + j15 \\ -15 + j10 & -15 + j15 & 30.007 + j25 \end{bmatrix}$$

Determine the [Ybus] matrix of the given system network shown in fig:



Given is the impedance so we need to convert it to the admittance.

$$Y_{10} = \frac{1}{Z_{10}} = \frac{1}{j1.0} = -j1.0$$

$$Y_{20} = \frac{1}{Z_{20}} = \frac{1}{j1.0} = -j1.0$$

$$Y_{12} = \frac{1}{Z_{12}} = \frac{1}{j0.4} = -j2.5$$

$$Y_{13} = \frac{1}{Z_{13}} = \frac{1}{j0.2} = -j5$$

$$Y_{23} = \frac{1}{Z_{23}} = \frac{1}{j0.2} = -j5.$$

$$Y_{34} = \frac{1}{Z_{34}} = \frac{1}{j0.08} = -j12.5$$

Since there is 4 buses so bus admittance matrix will be in the form

$$[Y_{bus}] = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}$$

Date: ....
Page: ....

$Y_{11} = y_{10} + y_{12} + y_{13}$
$= -j1.0 + (-j2.5) + (-j5)$
$= -j8.5$

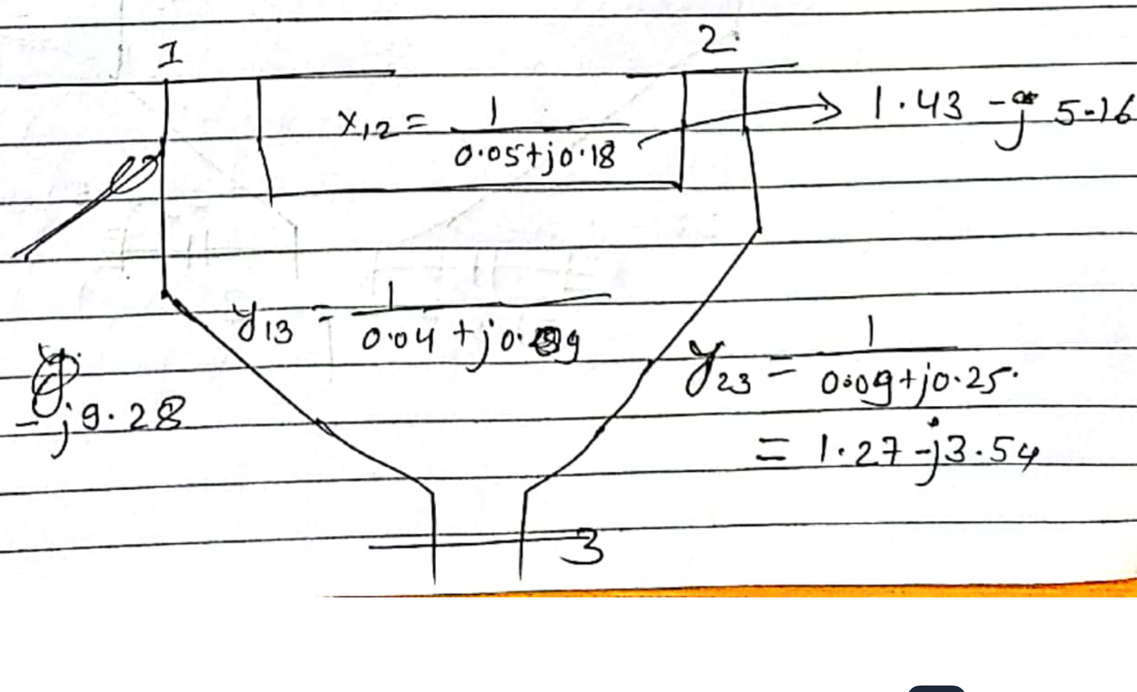
Similarly,

$$\begin{bmatrix} -j8.5 & j2.5 & j5 & 0 \\ j2.5 & -j8.5 & j5 & 0 \\ j5 & j5 & -j22.5 & j12.5 \\ 0 & 0 & j12.5 & -j12.5 \end{bmatrix}$$

Determine Ybus for the 3 bus system, The line impedance are as follows:

Line (bus to bus)	Impedance (PU)
1-2	0.05 + j0.18
1-3	0.04 + j0.09
2-3	0.09 + j0.25

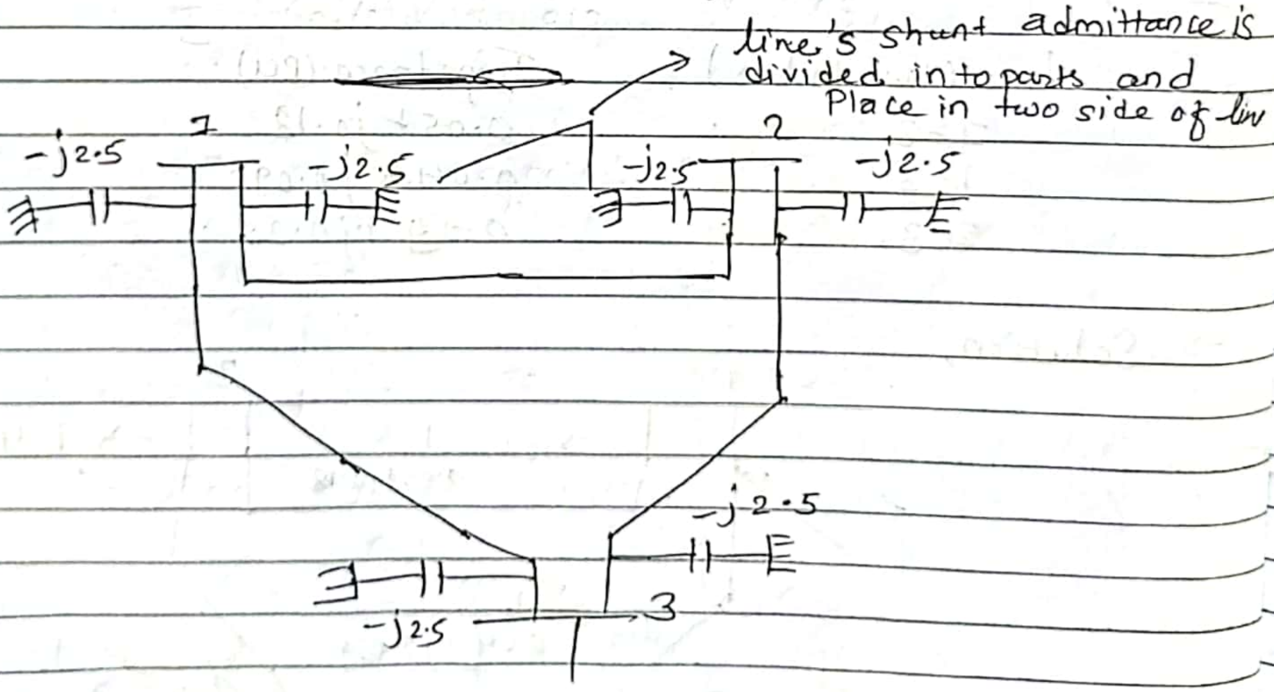
→ Solution,



$$Y_{bus} = \begin{bmatrix} 5.55-j14.44 & -1.43+j5.16 & -4.12+j9.28 \\ -1.43+j5.16 & 2.70-j8.70 & -1.27+j3.54 \\ -4.12+j9.28 & -1.27+j3.54 & 5.39-j12.82 \end{bmatrix}$$

Determine Y_{Bus} for the 3-bus system shown in fig. The line series impedances are as follows:
Each line has a total shunt admittance of $-j5.0$ p.u.
Determine the Y_{Bus}.

line (Bus to Bus)	Impedance p.u
1-2	$0.05+j0.18$
1-3	$0.04+j0.09$
2-3	$0.09+j0.25$.



$y_{12} = \frac{1}{0.05+j0.18} = 1.43 - j5.16$

$y_{13} = \frac{1}{0.044+j0.09} = 4.12 - j9.28$

$y_{23} = \frac{1}{0.09+j0.25} = 1.27 - j3.54$

$Y_{11} = y_{12} + y_{13} - j2.5 + j2.5$

$= 5.55 - j19.44$

$Y_{22} = y_{21} + y_{23} - j2.5 - j2.5$

$= 1.43 - j5.16 + 4.12 - j9.28 - j0.5$

$= 2.70 - j13.70$

Similarly, $Y_{33} = y_{31} + y_{32} - j2.5 - j2.5$

$= 5.39 - j17.82$

So, the Y_{Bus} is $\begin{bmatrix} 5.55 - j19.44 & -1.43 + j5.16 & -4.12 + j9.28 \\ -1.43 + j5.16 & 2.70 - j13.70 & -1.27 + j3.54 \\ -1.27 + j3.54 & -1.27 + j3.54 & 5.39 - j17.82 \end{bmatrix}$

Date: ....
Page: ....

Static load flow equation / Load flow equation / Bus loading equation.

Consider a 'n' bus power system network and let $V_1, V_2,$ --- V_n be the voltage at bus 1, 2, --- n respectively

Here, $I_{bus} = [Y_{bus}][V_{bus}]$

So the current entering the i^{th} bus of an n-bus system is given as:

$$I_i = Y_{i1}V_1 + Y_{i2}V_2 + --- + Y_{ii}V_i + ---- Y_{in}V_n \quad --- (1)$$

$$I_i = \sum_{p=1}^{n} Y_{ip} \cdot V_p \quad --- (2)$$

$$= \sum_{p=1}^{n} |Y_{ip}||V_p| \angle(\theta_p + \theta_{ip}) \quad --- (3)$$

The complex power injected at i^{th} Bus is.

$$S_i = V_i * I_i^* \quad --- (4)$$
$$= P_i + jQ_i$$
$$S_i^* = P_i - jQ_i = V_i^* I_i \quad --- (5)$$

So from eq^n (2) and (5)

$$S_i^* = V_i^* \sum_{p=1}^{n} Y_{ip} \cdot V_p \quad --- (6)$$

$a\angle\theta = a(\cos\theta + i\sin\theta)$

Also,

$V_i = |V_i| \angle \partial_i$

$V_i^* = |V_i| \angle -\partial_i$

$$P_i - jQ_i = |V_i| \sum_{p=1}^{n} |Y_{ip}||V_p| \angle(\partial_p + \partial_{ip} - \partial_i)$$

Seprating real and imaginary part:

$$P_i = |V_i| \sum_{p=1}^{n} |Y_{ip}||V_p| \cos(\partial_p + \partial_{ip} - \partial_i) \quad \text{---} \quad (7)$$

$$Q_i = -|V_i| \sum_{p=1}^{n} |Y_{ip}||V_p| \sin(\partial_p + \partial_{ip} - \partial_i) \quad \text{---} \quad (8)$$

eq^n (7) and (8) are known as load flow equation. These eq^n are non-linear and hence only a numerical solution is possible.

Gauss Seidel Method:

Gauss seidel method is one of the common methods ~~Advantages~~ used in solving power flow equations.

Advantages:

- Simplicity in technique
- Small computer memory requirement.
- Less computational time per iteration.

Disadvantages:

- Slow rate of convergence hence large number of iteration
- Increase no. of iteration directly with increasing in the number of buses.
- Effect of convergence due to the choice of slack bus.
- Need to program using complex number.

~~Tends to diverge~~

The Gauss seidel method - Procedure ~~Algorithm~~

Case I: When PV buses is absent.

Initially assume all buses to be PQ type buses, except the slackbus. This means that (n-1) complex bus voltages have to be determined. The slack bus is generally Numbered as bus 1.

Consider the expression for the current injected at bus-i,

Date: ....
Page: ....

$$I_i = \sum_{k=1}^{n} Y_{ik} V_k$$

$$I_i = Y_{ii} V_i + \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k$$

$$V_i = \frac{1}{Y_{ii}} \left[I_i - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k \right] \quad \text{---} (1)$$

The complex power at bus-i is

$$S_i = V_i * I_i^* = P_i + j Q_i \quad \text{---} (2)$$

$$I_i = \frac{P_i - j Q_i}{V_i^*}$$

From eq^n (1) and (2)

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - j Q_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k \right] \quad \text{---} (3)$$

Equation (3) is the implicit eq^n since the unknown variable, appears on both sides of the equation. Hence it needs to be solved by an iterative technique.

The value of the updated voltage are used in the computation of Subsequent voltages in the same iteration, thus speeding up convergence.
Iteration are carried out till the magnitude of all bus voltages do not change by more than the tolerance level.

Date: ....
Page: ....

Algorithm for Gauss-Seidel. (when PV bus is absent)

1. Formulation of the bus admittance Matrix Y_{BUS}.

2. Assume the initial voltage of all load buses as $1\angle 0°$ $(1+j0)$ also assume slack bus as bus 1.

3. Update the voltages value by iteration method, the voltages are given by,

$$V_i^{N+1} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k\neq i}}^{n} Y_{ik}V_k\right]$$

Update the new value and perform the operation for the remaining buses.

4. Continue the iteration until $|V_i^{N+1} - V_i^N|$ is equal or less than the tolerance value.

5. Compute the slack bus power after voltage have converged using:

$$S_i^* = P_i - jQ_i = V_i^* \sum_{k=1}^{n} Y_{ik}V_k$$

When PV bus is present:

At PV buses, the magnitude of voltage and active power is known and hence reactive power and voltage angle (∂) is needed to be calculated

So,

$$Q_i = -\text{imag}\left[\left(Y_{ii} V_i + \sum_{j=1}^{n} Y_{ij} V_j\right) V_i^*\right], \; j \neq i \quad \text{--- (i)}$$

Now, voltage is calculated as:

$$V_i = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_i\right] \quad \text{--- (ii)}$$

V_i is corrected on each iteration.

V_i corrected $= |V_i|$specified $\angle \partial_i$, it is because in PV buses V is known so its value does not changes.

where, ∂_i is the calculated phase angle on each iteration.

For load buses (PQ buses) V_i is calculated using ~~formula~~ eq^n (ii)

Continue the iteration until $|V_i^{N+1} - V_i^{N}|$ is equal or less than the tolerance.

Compute the slack bus power after voltage have converged using.

$$S_i^* = P_i - jQ_i = V_i^* \sum_{k=1}^{n} Y_{ik} V_k.$$

[Develop algorithm yourself)

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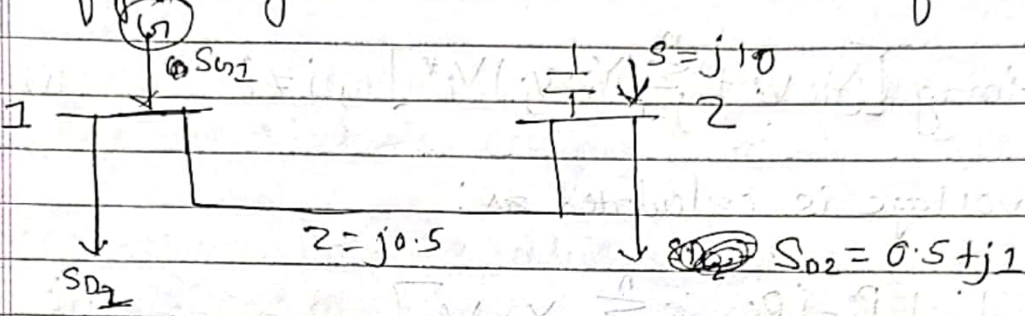
Power (P) = Power generated − Power consumed

$= P_G - P_L$

$Q = Q_G - Q_L$

Example 1:

Obtain the voltage at bus 2 for the simple system shown in fig 2, using the Gauss-Seidel method, if $V_1 = 1\angle 0°$ pu



~~Solution~~

Here bus 1 is the swing bus and bus 2 is load bus.

For bus 1:

$V_1 = 1\angle 0°$ $\quad P_1 = ?$ $\quad Q_1 = ?$

[In bus 2 capacitor has inject a reactive power]

For bus 2

$V_2 = ?$ $\quad \delta_2 = ?$ $\quad P_L = 0.5$ pu, $Q_L = 1$ pu $\quad Q_G = 1.0$ pu

So, $P_2 = P_G - P_L$

$= 0 - 0.5 = -0.5$ pu

$Q_2 = Q_G - Q_L$

$= 1 - 1 = 0$ pu

$$Y_{bus} = \begin{bmatrix} -j2 & j2 \\ j2 & -j2 \end{bmatrix}$$

Let $V_2 = 1\angle 0°$

we have,

$$V_2 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^*} - Y_{21}V_1\right]$$

Put calculator in degree

$$V_2^1 = \frac{1}{-j2}\left[\frac{-0.5}{1\angle 0^\circ} - j2 \times 1\angle 0^\circ\right]$$

$= 1 - 0.25i$

$= 1.03078\angle -14.03624^\circ$

$$V_2^2 = \frac{1}{-j2}\left[\frac{-0.5}{1.03078\angle 14.036} - j2 \times 1\angle 0^\circ\right]$$

$= 0.94118 - 0.23529i$

$= 0.97014\angle -14.036^\circ$

Do similarly.

$V_2^3 = 0.97026\angle -14.931$

$V_2^4 = 0.96623\angle -14.951^\circ$

[Do up to the steps as mentioned in the question or compare the errors]

So,

$P_1 - jQ_1 = V_1^* (V_1 Y_{11} + V_2 Y_{12})$

$= 1\angle 0^\circ (1\angle 0^\circ * (-j2) + 0.96623\angle -14.931 * j2]$

$= 0.4979 - 0.13279i$

So,

Flow in the line $S_{12} = V_1 I_{12}^*$ — y is small not capital one

$= V_1 [(V_1 - V_2) y_{12}]^*$

$= 1\angle 0^\circ (1\angle 0^\circ - 0.96623\angle -14.93)(-j2)]^*$

$= 0.4978 + 0.1327i$

Flow in the line $S_{21} = V_2 I_{21}^*$

$= V_2 [(V_2 - V_1) y_{21}]^*$

$= 0.96623\angle -14.93^\circ [(0.96623\angle -14.93 - 1\angle 0^\circ)(-j2)]^*$

$= -0.4978 + 0.001$

[acceleration factor varies from 1 to 1.6 but usually taken as 1.4]

So the loss in line is $S_{12} + S_{21} = j0.1327$

Since there is no resistance so no real power loss.

~~Using G.S. method,~~

Note:

If slack bus voltage is not specified it is taken as $1\angle 0°$.

Date: ....
Page: ....

The Newton-Raphson Method:

Known: $V_1 = V_1 \angle 0°$ [Swing bus]

V_p, P_p Gen-bus. ($p = 2, 3 --- G$)

P_p, Q_p Load bus. ($p = G+1, G+2 --- n$)

unknown:

P_1, Q_1 Swing bus:

Q_p, δ_p $p = 2, 3 --- G$.

V_p, δ_p $p = G+1, G+2 --- n$.

Algorithm for N-R method:

1. Form the Ybus.
2. Assume the initial values of voltages:

$\delta_p = 0$. $p = 2, 3 --- G$.

$|V_p| = 1$ $\delta_p = 0$, $p = G+1, G+2 --- N$.

3. Calculate P_p, Q_p using the equations,

$$P_p = \sum_{q=1}^{N} |V_p V_q Y_{pq}| \cos(\theta_{pq} + \delta_q - \delta_p)$$

$$Q_p = -\sum_{q=1}^{N} |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_q - \delta_p)$$

4. Calculate the change in P and Q as,

$\Delta P_p = P_{p,s} - P_p$

$\Delta Q_p = Q_{ps} - Q_p$

where s stands for specified ~~and~~ value, P_p and Q_p are the calculated values.

Narayani

5. Calculate the Jacobian matrices J_1, J_2, J_3 and J_4 for the estimated or given values of voltages (magnitude and angle)

6. Obtain the solution of the equation.

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

Find $\Delta \delta_p$ and $|\Delta V_p|$ by matrix inversion or solution of simultaneous equation.

7. Calculate the new bus voltages by updating δ and magnitude of V as,

$$\delta_p^1 = \delta_p^0 + \Delta \delta_p^0$$

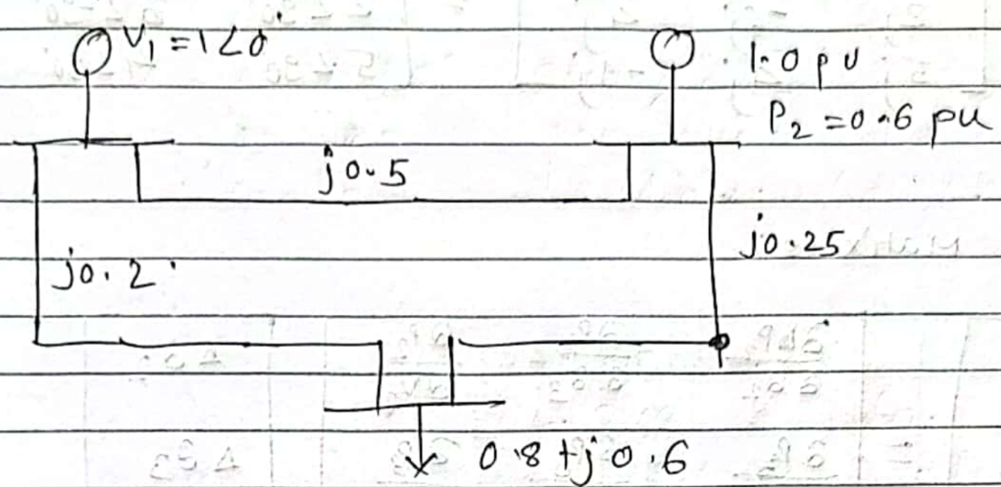
$$|V_p^1| = |V_p^0| + |\Delta V_p^0|$$

8. Check the tolerance of the solution. If within tolerance stop. Otherwise repeat steps 3 ~~to~~ to 7.

9. Calculate P_1 and Q_1, line flows and line losses at end.

Date: ....
Page: ....

Example :



Iterate till the voltage magnitude converged to 0.01.

Solution,

known quantities : V_1 , δ_1 unknown

V_2 , P_2 δ_2, Q_2

P_3 , Q_3 $|V_3|$, δ_3.

The expression for P and Q buses are

$$P_2 = |V_2 V_1 Y_{21}| \cos(\theta_{21} + \delta_1 - \delta_2) + |V_2^2 Y_{22}| \cos \theta_{22} + |V_2 V_3 Y_{23}| \cos(\theta_{23} + \delta_3 - \delta_2)$$

$$P_3 = |V_3 V_1 Y_{31}| \cos(\theta_{31} + \delta_1 - \delta_3) + |V_3 V_2 Y_{32}| \cos(\theta_{32} + \delta_2 - \delta_3) + |V_3^2 Y_{33}| \cos(\theta_{33})$$

$$Q_3 = -\left[|V_3 V_1 Y_{31}| \sin(\theta_{31} + \delta_1 - \delta_3) + |V_3 V_3 Y_{32}| \sin(\theta_{32} + \delta_2 - \delta_3) + |V_3^2 Y_{33}| \sin(\theta_{33}) \right]$$

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix} = \begin{bmatrix} & \end{bmatrix}\begin{bmatrix}\Delta\delta\\ \Delta V\end{bmatrix}$$

1-2 $\frac{1}{j0.5} = -2j$

1-3 $\frac{1}{j0.2} = -5j$

2-3 $\frac{1}{j0.25} = -4j$

$$Y_{bus} = \begin{bmatrix} -7j & 2j & 5j \\ 2j & -6j & 4j \\ 5j & 4j & -9j \end{bmatrix} = \begin{bmatrix} 7\angle -90^\circ & 2\angle 90^\circ & 5\angle 90^\circ \\ 2\angle 90^\circ & 6\angle -90^\circ & 4\angle 90^\circ \\ 5\angle 90^\circ & 4\angle 90^\circ & 9\angle -90^\circ \end{bmatrix}$$

Jacobian Matrix is:

$$\begin{bmatrix}\Delta P_2 \\ \Delta P_3 \\ \Delta Q_3\end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial V_2} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial V_2} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \frac{\partial Q_3}{\partial V_2} \end{bmatrix} \begin{bmatrix}\Delta \delta_2 \\ \Delta \delta_3 \\ \Delta V_2\end{bmatrix}$$

~~Let us calculate the cal~~

Let $V_2 = 1$ p.u and δ_2 and $\delta_3 = 0^\circ$

Calculate the calculated value of P_2, P_3 and Q_3:

$P_2 = |1 * 1 * 2| \cos 90^\circ + 1^2 * 6 * \cos(-90^\circ) + |1 \times 1 * 4| \cos 90^\circ$
$= 0 + 0 + 0 = 0$

$P_3 = |1 \times 1 * 5| \cos 90^\circ + |1 \times 1 * 4| \cos 90^\circ + |1^2 * 9| \cos(-90^\circ)$
$= 0$

$Q_3 = -[5 \sin 90^\circ + 4 \sin 90^\circ + 9 \sin(-90^\circ)]$
$= -[5 + 4 - 9]$
$= 0$

Narayani

Date: ....
Page: ....

So, $\Delta P_2 = P_{specified} - P_{calculated}$
$= 0.6 - 0 = 0.6\ pu$
$\Delta P_3 = -0.8 - 0 = -0.8\ pu$
$\Delta Q_3 = -0.6 - 0 = -0.6\ pu$

$\frac{\partial P_2}{\partial \delta_2} = |V_2 V_1 Y_{21}| \sin(\theta_{21} + \delta_1 - \delta_2) + 0 + |V_2 V_3 Y_{23}| \sin(\theta_{23} + \delta_3 - \delta_2)$
$= |1 \times 1 \times 2| \sin 90° + |1 \times 1 \times 4| \sin 90°$
$= 6$

$\frac{\partial P_2}{\partial \delta_3} = -|V_2 V_3 Y_{23}| \sin(\theta_{23} + \delta_3 - \delta_2)$
$= -|1 \times 1 \times 4| \sin 90°$
$= -4$

$\frac{\partial P_2}{\partial V_2} = |V_1 Y_{21}| \cos(\theta_{21} + \delta_1 - \delta_2) + 2 V_2 Y_{22} \cos\theta_{22} + |V_3 Y_{23}| \cos(\theta_{23} + \delta_3 - \delta_2)$
$= 2\cos 90° + 2 \times 6 \cos(-90°) + 4 \cos 90°$
$= 0.$

$\frac{\partial P_3}{\partial \delta_2} = 0 - |V_3 V_3 Y_{32}| \sin 90° + 0$
$= -4$

$\frac{\partial P_3}{\partial \delta_3} = |V_3 V_1 Y_{31}| \sin(\theta_{31} + \delta_1 - \delta_3) + |V_3 V_2 Y_{32}| \sin(\theta_{32} + \delta_2 - \delta_3)$

$= 5 \sin 90° + 4 \sin 90°$
$= 9$

$\frac{\partial P_3}{\partial V_2} = |V_3 Y_{32}| \cos(\theta_{32} + \delta_2 - \delta_3)$
$= 0$

Narayani

Date: ....
Page: ....

$$\frac{\partial Q_3}{\partial \delta_2} = 0$$

$$\frac{\partial Q_3}{\partial \delta_3} = 0$$

$$\frac{\partial Q_3}{\partial V_2} = 9$$

$$\begin{bmatrix} 0.6 \\ -0.8 \\ -0.6 \end{bmatrix} = \begin{bmatrix} 6 & -4 & 0 \\ -4 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta V_3 \end{bmatrix}$$

Using Matrix inversion method;

$$\begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta V_3 \end{bmatrix} = \begin{bmatrix} 0.0578 \\ -0.063 \\ -0.066 \end{bmatrix} \rightarrow \text{Convert angle into degree} = \begin{bmatrix} 3.3171 \\ -3.618 \\ -0.066 \end{bmatrix}$$

So, The new voltages are:

$\cancel{V_2 = 1\angle 3.3171}$

$\cancel{V_3 =}$

$$\begin{bmatrix} \delta_2^1 \\ \delta_3^1 \\ V_3^1 \end{bmatrix} = \begin{bmatrix} \delta_2^0 \\ \delta_3^0 \\ V_3^0 \end{bmatrix} + \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta V_3 \end{bmatrix} = \begin{bmatrix} 3.3171^\circ \\ -3.618^\circ \\ 0.934 \text{ pu} \end{bmatrix}$$

The new voltages are:

$V_2 = 1\angle 3.3171^\circ$

$V_3 = 0.934\angle -3.618^\circ$

Date: ....
Page: ....

Here $|\Delta V_3| = 0.066$ but error margin ε is given as 0.01 so the process is needed to be repeated for next iteration

$P_2 = 1 \times 1 \times 2 \cos(90 - 3.265) + 12 \times 6 \times \cos(-90) + 1 \times 0.934 \times 4 \cos(90 - 3.609 - 3.265)$
$= 0.5665$

$P_3 = -0.74536$

$Q_3 = -0.5233$

$\Delta P_2 = 0.6 - 0.5665 = 0.0335$

$\Delta P_3 = -0.8 - (-0.7436) = -0.0546$

$\Delta Q_3 = -0.6 - (-0.5233) = -0.0767$

Calculating Jacobian matrix:

$$\begin{bmatrix} 0.0335 \\ -0.0546 \\ -0.0767 \end{bmatrix} = \begin{bmatrix} 5.702 & -3.706 & 0.483 \\ -3.706 & 8.36 & -0.777 \\ 0.4508 & -0.745 & 7.833 \end{bmatrix} \begin{bmatrix} \Delta \partial_2 \\ \Delta \partial_3 \\ \Delta |V_3| \end{bmatrix}$$

Using matrix inversion:

$$\begin{bmatrix} \Delta \partial_2 \\ \Delta \partial_3 \\ \Delta |V_3| \end{bmatrix} = \begin{bmatrix} 0.00263 \text{ rad} \\ -0.0063 \text{ rad} \\ -0.01 \text{ pu} \end{bmatrix} = \begin{bmatrix} 0.15 \text{ deg} \\ -0.36 \text{ deg} \\ -0.01 \text{ pu} \end{bmatrix}$$

$$\begin{bmatrix} \partial_2^2 \\ \partial_3^2 \\ |V_3|^2 \end{bmatrix} = \begin{bmatrix} 3.43 \text{ deg} \\ -3.95 \text{ deg} \\ 0.924 \text{ pu} \end{bmatrix}$$

The new voltages are:

$V_2 = 1 \angle 3.43^\circ$

$V_3 = 0.924 \angle -3.95^\circ$

The maximum Voltage difference ~~with~~ is 0.01.