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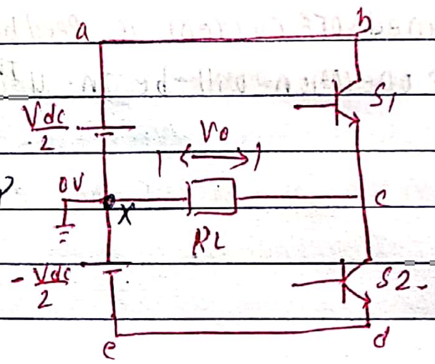
Inverter:

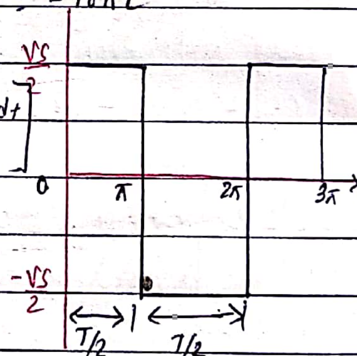
Inverter is a power electronic circuit which converts dc signal into ac signal of a specific frequency. The output of inverter could be single-phase ac or three-phase ac, but it is not possible to obtain a pure sinusoidal ac voltages from the practical inverters. The various application of inverters are battery back up, ac supply, variable frequency ac motor drives, etc.

Single phase Inverters:



let, f_0 = Required frequency o/p ac voltage.
Then, time period of ac output voltage $T = 1/f_0$
For, first $T/2$, S_1 is turned on keeping S_2 OFF. The output follows the path a-b-S_1-c-RL-x i.e. $V_o = i_o R_L$

For another $T/2$, S_2 is turned on keeping S_1 OFF. Then current will flow the path x-RL-c-S_2-d-e i.e. $V_o = -i_o R_L$



Thus, Average output voltage $V_{av} = \frac{1}{2\pi}\left[\int_0^{\pi} \frac{V_s}{2}\cdot dt - \int_{\pi}^{2\pi} V_s \cdot dt\right]$

$= 0$

RMS value of output voltage is,

$$V_{rms} = \left[\frac{1}{2\pi}\int_0^{\pi}\left(\frac{V_s}{2}\right)^2 d\omega t + \int_{\pi}^{2\pi}\left(-\frac{V_s}{2}\right)^2 d\omega t\right]^{1/2}$$

$$= \left[\frac{1}{2\pi}\left(\frac{V_s}{2}\right)^2\{(\pi - 0) + (2\pi - \pi)\}\right]^{1/2}$$

$$= \sqrt{\frac{1}{2\pi} * \left(\frac{V_s}{2}\right)^2 * 2\pi}$$

$$= \frac{V_s}{2}$$

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Fourier Analysis:

$$V_0 = \frac{A_0}{2} + \sum_{n=1}^{\infty} (A_n \cos n\omega t + B_n \sin n\omega t)$$

$$A_0 = \frac{2}{2\pi}\left[\int_0^{\pi}\left(\frac{V_s}{2}\right) d\omega t + \int_{\pi}^{2\pi}\left(-\frac{V_s}{2}\right) d\omega t\right]$$

$$= \frac{1}{\pi}\left[\left(\frac{V_s}{2}\right)(\pi - 0) + (\pi - 2\pi)\right] = \frac{V_s}{2\pi}[\pi - \pi] = 0$$

$$A_n = \frac{2}{2\pi}\left[\int_0^{\pi}\left(\frac{V_s}{2}\right)\cos n\omega t\, d\omega t + \int_{\pi}^{2\pi}\left(-\frac{V_s}{2}\right)\cos n\omega t\, d\omega t\right]$$

$$= \frac{1}{\pi}\left(\frac{V_s}{2}\right)\left[\int_0^{\pi}\cos n\omega t \cdot d\omega t - \int_{\pi}^{2\pi}\cos n\omega t \cdot d\omega t\right]$$

$$= \frac{1}{\pi}\left(\frac{V_s}{2}\right)\left[\frac{\sin n\omega t}{n}\Big|_0^{\pi} - \frac{\sin n\omega t}{n}\Big|_{\pi}^{2\pi}\right]$$

$$= 0$$

$$B_n = \frac{2}{2\pi}\left[\int_0^{\pi}\left(\frac{V_s}{2}\right)\sin n\omega t\, d\omega t + \int_{\pi}^{2\pi}\left(-\frac{V_s}{2}\right)\sin n\omega t\, d\omega t\right]$$

$$= \frac{2}{2\pi}\left(\frac{V_s}{2}\right)\left[\int_0^{\pi}\sin n\omega t\, d\omega t - \int_{\pi}^{2\pi}\sin n\omega t\, d\omega t\right]$$

$$= \frac{1}{\pi}\left(\frac{V_s}{2}\right)\left[\frac{-\cos n\omega t}{n}\Big|_0^{\pi} + \frac{\cos n\omega t}{n}\Big|_{\pi}^{2\pi}\right]$$

$$= \frac{1}{n\pi}\left(\frac{V_s}{2}\right)\left[1 - \cos n\pi + \cos 2\pi n - \cos n\pi\right]$$

$$= \frac{1}{n\pi}\left(\frac{V_s}{2}\right)\left[1 - 2\cos n\pi + \cos 2\pi n\right]$$

$$= \frac{1}{n\pi}\left(\frac{V_s}{2}\right)\left[1 - 2 \times (-1)^n + 1\right]$$

$$= \frac{1}{n\pi}\left(\frac{Vs}{2}\right)\left[2-2(-1)^n\right]$$

For n = odd

$$= \frac{Vs}{2\pi n}\, 2\left[1-(-1)^n\right]$$

$$= \frac{Vs}{\pi n}\left[1-(-1)^n\right]$$

For n = even.

$$= \frac{Vs}{2\pi n}\left[2-2\times 1\right]$$

$$= 0$$

Then, $B_1 = \frac{Vs}{\pi\times 1}\, 1-(-1)^1 \Rightarrow \frac{2Vs}{\pi} = \left(\frac{Vs}{2}\right)\cdot\frac{4}{\pi} = 1.2732\left(\frac{Vs}{2}\right)$

$B_3 = \frac{Vs}{3\pi}\, 1-(-1)^3 = \frac{2}{3}\frac{Vs}{\pi} = \left(\frac{Vs}{2}\right)\cdot\frac{4}{3\pi} = 0.4244\left(\frac{Vs}{2}\right)$

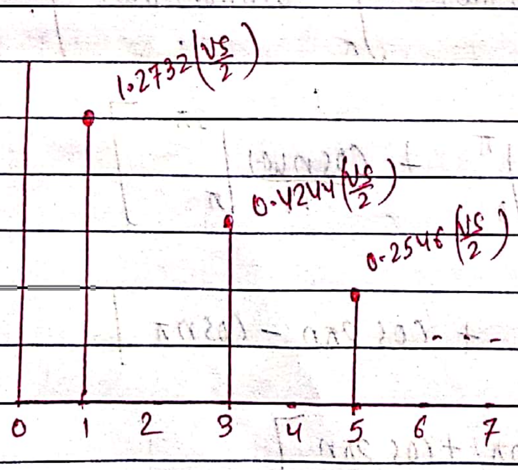
$B_5 = \frac{2Vs}{5\pi} = \left(\frac{Vs}{2}\right)\frac{4}{5\pi} = 0.2546\left(\frac{Vs}{2}\right)$

Now,

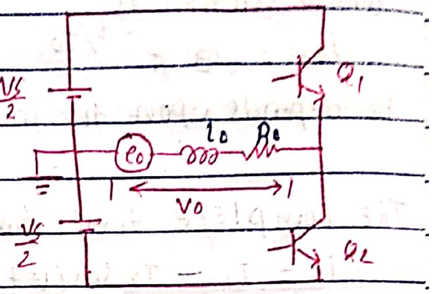
$$Vo = \frac{Ao}{2} + \sum_{n=1}^{\infty} An\cos n\omega t + Bn\sin n\omega t.$$

$$= \sum_{n=1,3,5,7,\dots}^{\infty} \frac{Vs}{\pi n}\left[1-(-1)^n\right]\sin n\omega t.$$

$$= 2\frac{Vs}{\pi}\sin\omega t + \frac{2}{3}\frac{Vs}{\pi}\sin 3\omega t + \frac{2}{5}\frac{Vs}{\pi}\sin 5\omega t + \dots$$

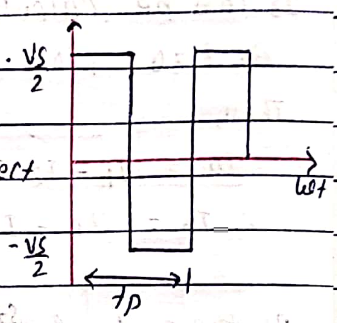


Single phase Inverter with AC Motor load.



In order to make mathematical analysis of this circuit simpler, let us make an assumption that sinusoidal back emf of ac motor (e_0) is in synchronism with inverter switching frequency and is purely sine wave without any harmonics.

let, Inverter frequency $f_0 = \frac{1}{t_p}$ Hz.



$\therefore \omega_0 = 2\pi f_0$ rad/sec.

$e_0 = E_0 \sin(\omega t + \epsilon_0)$, ϵ_0 = angle of lead with respect to o/p. voltage.

For positive half cycle.

$$\frac{V_S}{2} = E_0 \sin(\omega t + \epsilon_0) + R_0 i_0 + L_0 \frac{di_0}{dt}.$$

The solution of this equation has three parts.

i) i_{01} = A Steady State part associated with driving function $\frac{V_S}{2}$ and given by

$$i_{01} = \frac{V_S}{2R_0} = I_1$$

ii) i_{02} = A steady state part associated with driving ~~ef~~ function $E_0 \sin(\omega t + \epsilon_0)$ and is given by.

$$i_{02} = \frac{E_0}{|R_0 + j\omega_0 L_0|} \sin(\omega t + \epsilon_0 - \delta_0) \quad \text{where, } \delta_0 = \tan^{-1}\left(\frac{\omega_0 L_0}{R_0}\right)$$

$$i_{02} = \frac{E_0}{|Z_0|} \sin(\omega t + \epsilon_0 - \delta_0) \quad \text{where, } |Z_0| = |R_0 + j\omega_0 L_0|$$

$$i_{02} = I_2 \sin(\omega t + \epsilon_0 - \delta_0) \quad \text{where, } I_2 = \frac{E_0}{Z_0}$$

iii) i_{03} = Transient part is given by solution of the following equation.

$$L_0 \frac{di_{03}}{dt} + R_0 i_{03} = 0$$

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Whose solution is

$$i_{03} = I_3 e^{-t/\tau_0} \quad \text{where, } \tau_0 = L_0/R_0 = \text{time constant.}$$

I_3 depends upon the initial condition of the circuit.

The complete time domain solution is.

$$i_0 = I_1 - I_2 \sin(\omega t + \epsilon_0 - \delta_0) + I_3 e^{-t/\tau_0}$$

I_3 can be calculated by setting initial condition as,

At $t = 0$, $i_0 = I_a$ (may be zero also)

Then,

$$I_a = I_1 - I_2 \sin(\epsilon_0 - \delta_0) + I_3$$

$$\therefore I_3 = I_a - I_1 + I_2 \sin(\epsilon_0 - \delta_0)$$

At the end of 1st half cycle, At $\theta = \pi$, let.

$\omega t = \pi$, $i_0 = I_b$, then.

$$I_b = I_1 - I_2 \sin(\pi + \epsilon_0 - \delta_0) + I_3 e^{-\pi \tan\delta_0}$$

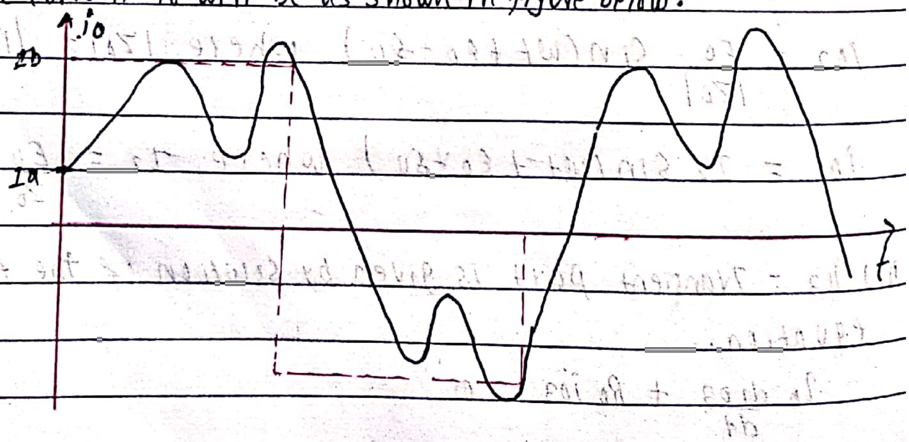
The solution of –ve half cycle is given by

$$i_0 = -I_1 - I_2 \sin(\theta + \epsilon_0 - \delta_0) + I_4 e^{-\theta \tan\delta_0}.$$

To calculate I_4 at $\theta = \pi$, $i_0 = I_b$.

$$I_4 = \frac{I_b + I_1 + I_2 \sin(\pi + \epsilon_0 - \delta)}{e^{-\pi \tan\delta_0}}$$

Following this procedure we can step through successive half cycle and determine two cycle circuit response to any initial condition I_a. The waveform of load current i_0 will be as shown in figure below.



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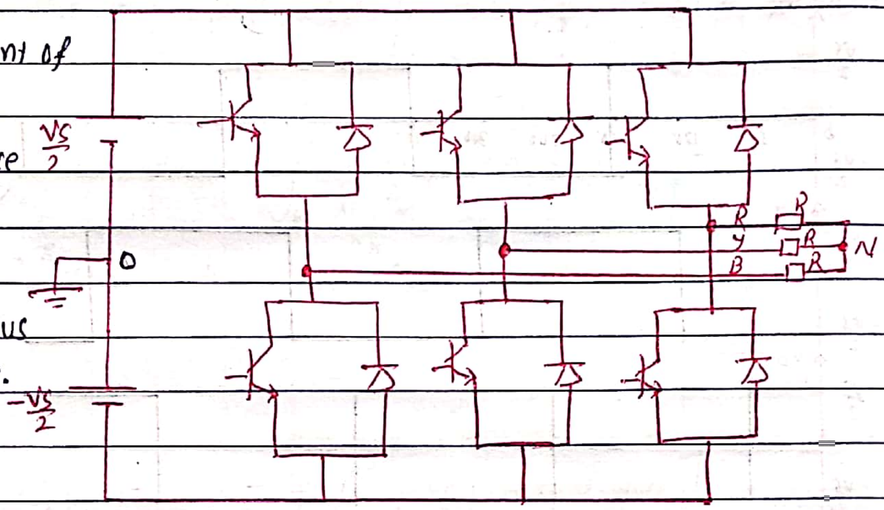
Three phase Inverter:

Basically a three phase inverter is the combination of three sets of single phase inverters supplied from a common dc source and each set conducted with a phase difference of 120° to each other.

Let, 'O' is the mid-point of dc supply which is considered as reference potential point for evaluating the potential of various points of the circuit.



Let, V_R = Potential of R phase terminal w.r.t 'O'

V_Y = Potential of Y phase terminal w.r.t 'O'.

V_B = potential of B phase terminal w.r.t 'O'.

The voltage between o/p terminals are given by

$V_{RY} = V_R - V_Y$

$V_{YB} = V_Y - V_B$

$V_{BR} = V_B - V_R$

Considering 3-phase balanced load.

$V_{RN} + V_{YN} + V_{BN} = 0$

And, $V_{RN} = V_R - V_N$. $\Rightarrow V_R = V_{RN} + V_N$.

$V_{YN} = V_Y - V_N$ $\Rightarrow V_Y = V_{YN} + V_N$.

$V_{BN} = V_B - V_N$ $\Rightarrow V_B = V_{BN} + V_N$.

Then, $V_R + V_Y + V_B = 3V_N + (V_{RN} + V_{YN} + V_{BN})$

$\therefore V_N = \frac{1}{3}(V_R + V_Y + V_B)$

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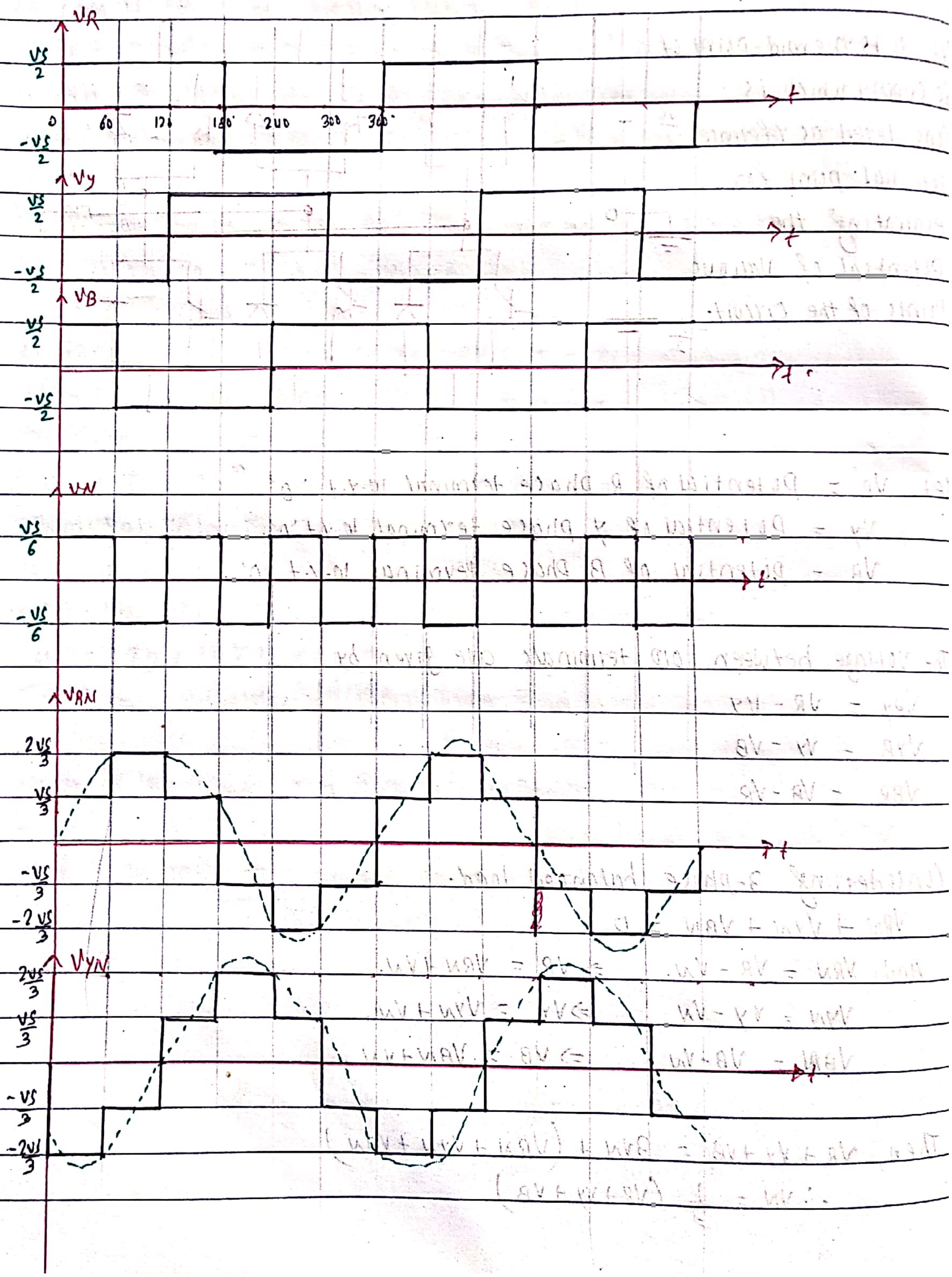
Now, the phase voltage is given by $V_{RN} = V_R - V_N$.
Similarly the V_{YN} and V_{BN} will be similar to that of V_{RN}, but with $120°$ out of phase with each other. It is clear from the waveform that frequency of V_N is 3 times greater than that of frequency of V_R.



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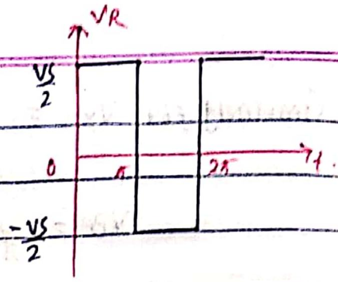

Fourier analysis of three phase inverters.

The wave form of VR is as shown in figure below.

$$V_0(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} (A_n \cos n\omega t + B_n \sin n\omega t)$$

$$A_0 = \frac{2}{2\pi}\left[\int_0^{\pi}\left(\frac{V_S}{2}\right) \cdot d\omega t + \int_{\pi}^{2\pi}\left(-\frac{V_S}{2}\right) d\omega t\right] = \frac{1}{\pi}\left(\frac{V_S}{2}\right)\left[(\pi - 0) - (2\pi - \pi)\right]$$

$$= 0$$

$$A_n = \frac{2}{2\pi}\left[\int_0^{\pi}\left(\frac{V_S}{2}\right)\cos n\omega t \cdot d\omega t + \int_{\pi}^{2\pi}\left(-\frac{V_S}{2}\right)\cos n\omega t \cdot d\omega t\right]$$

$$= \frac{2}{2\pi}\cdot\left(\frac{V_S}{2}\right)\left[\int_0^{\pi}\cos n\omega t \cdot d\omega t - \int_{\pi}^{2\pi}\cos n\omega t \cdot d\omega t\right]$$

$$= 0$$

$$B_n = \frac{2}{2\pi}\left[\int_0^{\pi}\left(\frac{V_S}{2}\right)\sin n\omega t \cdot d\omega t + \int_{\pi}^{2\pi}\left(-\frac{V_S}{2}\right)\sin n\omega t \cdot d\omega t\right]$$

$$= \frac{2}{2\pi}\left(\frac{V_S}{2}\right)\left[-\frac{\cos n\omega t}{n}\Big|_0^{\pi} + \frac{\cos n\omega t}{n}\Big|_{\pi}^{2\pi}\right]$$

$$= \frac{1}{n\pi}\left(\frac{V_S}{2}\right)\left[1 - \cos n\pi + \cos 2\pi n - \cos n\pi\right]$$

$$= \frac{1}{n\pi}\left(\frac{V_S}{2}\right)\left[1 + 1 - 2\cdot(-1)^n\right]$$

$$= \frac{1}{n\pi}\left(\frac{V_S}{2}\right)\cdot 2\left[1 - (-1)^n\right]$$

For, n = even $\qquad$ For n = odd.

$B_n = 0$ $\qquad$ $B_n = -\frac{1}{n\pi}\left(\frac{V_S}{2}\right)\cdot 2 * 2 \Rightarrow \frac{4}{n\pi}\left(\frac{V_S}{2}\right)$

$$\text{Then, } V_0 = \sum_{n=1,3,5,\dots}^{\infty} \frac{4}{n\pi}\left(\frac{V_S}{2}\right)\sin n\omega t.$$

$$= \frac{4}{\pi}\left(\frac{V_S}{2}\right)\sum_{n=1,3,5}^{\infty}\frac{1}{n}\sin n\omega t.$$

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Similarly for $v_Y = \frac{4}{\pi}\left(\frac{V_S}{2}\right) \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n} \sin\left[n\left(\omega t - \frac{2\pi}{3}\right)\right]$

$$v_B = \frac{4}{\pi}\left(\frac{V_S}{2}\right) \sum_{n=1,3,5}^{\infty} \frac{1}{n} \sin\left[n\left(\omega t - \frac{4\pi}{3}\right)\right]$$

We can break these series in three sub series as follows.

$n = 1, 7, 13, \ldots \quad 3, 9, 15, \ldots \quad 5, 11, 17, \ldots$

So that,

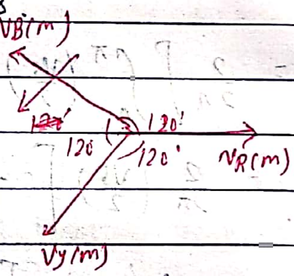
In 1st sub series we get 1st, 7th, 13th, ... harmonics and so on.

In 2nd sub-series we get 3rd, 9th, 15th, ---- harmonics and so on.

In 3rd sub-series we get 5th, 11th, 17th, ---- harmonics and so on.

For the 1st sub series, $n = 1, 7, 13, \ldots$

$n = (1+6K)$, $K = 0$ to ∞



$$V_R = \frac{4}{\pi}\left(\frac{V_S}{2}\right) \sum_{n=1,7,13}^{\infty} \frac{1}{n} \sin(n\omega t)$$

$$V_Y = \frac{4}{\pi}\left(\frac{V_S}{2}\right) \sum_{n=1,7,13}^{\infty} \frac{1}{n} \sin\left[n\left(\omega t - \frac{2\pi}{3}\right)\right]$$

$$V_B = \frac{4}{\pi}\left(\frac{V_S}{2}\right) \sum_{n=5,11,17}^{\infty} \frac{1}{n} \sin\left[n\left(\omega t - \frac{4\pi}{3}\right)\right]$$

The first sub-series represents positive sequence system voltage.

For the 2nd sub series, $n = 3, 9, 15 = 3+6K$ where, $K = 0$ to ∞.

$$V_Y = \frac{4}{\pi}\left(\frac{V_S}{2}\right) \sum_{n=3,9,15,\ldots}^{\infty} \frac{1}{n} \sin\left[n\left(\omega t - \frac{2\pi}{3}\right)\right]$$

$$V_R = \frac{4}{\pi}\left(\frac{V_S}{2}\right) \sum_{n=3,9,15}^{\infty} \frac{1}{n} \sin(n\omega t)$$

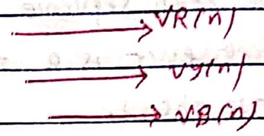
$$V_B = \frac{4}{\pi}\left(\frac{V_S}{2}\right) \sum_{n=3,9,15}^{\infty} \frac{1}{n} \sin\left[n\left(\omega t - \frac{4\pi}{3}\right)\right]$$

Now, $\sin\left[n\left(\omega t-\frac{2\pi}{3}\right)\right] = \sin\left[n\omega t-(3+6K)\frac{2\pi}{3}\right]$

$= \sin\left(n\omega t - 2\pi - 2\pi \cdot 2K\right)$

$= \sin(n\omega t)$ for all values of K.

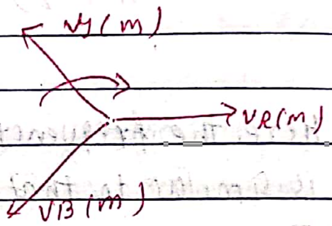
Similarly $\sin\left[n\left(\omega t-\frac{4\pi}{3}\right)\right] = \sin(n\omega t)$



The 2nd sub-series represents zero sequence system voltage and can be represented by the phasor diagram as shown in figure.

For the 3rd sub-series, $n = 5, 11, 17, \ldots = 5+6K$, where $K = 0$ to ∞

$$V_R = \frac{4}{\pi}\left(\frac{V_S}{2}\right)\sum_{n=5,11,17,\ldots}^{\infty}\frac{1}{n}\sin(n\omega t)$$



$$V_Y = \frac{4}{\pi}\cdot\left(\frac{V_S}{2}\right)\sum_{n=5,11,17}^{\infty}\frac{1}{n}\sin\left[n\left(\omega t-\frac{2\pi}{3}\right)\right]$$

$$V_B = \frac{4}{\pi}\left(\frac{V_S}{2}\right)\sum_{n=5,11,17}^{\infty}\frac{1}{n}\sin\left[n\left(\omega t-\frac{4\pi}{3}\right)\right]$$

Now, $\sin\left[m\left(\omega t-\frac{2\pi}{3}\right)\right] = \sin\left[n\omega t-(5+6K)\frac{2\pi}{3}\right]$

$= \sin\left[n\omega t - \frac{10\pi}{3} - 2K\cdot 2\pi\right]$

$= \sin\left[n\omega t - \frac{4\pi}{3}\right]$ for all values of K.

$= V_B$

$\sin\left[n\left(\omega t-\frac{4\pi}{3}\right)\right] = \sin\left[n\omega t-(5+6K)\frac{4\pi}{3}\right]$

$= V_Y$

The 3rd sub-series represents negative sequence system voltage and can be represented by the phasor as shown in figure.

$$\text{Now, } V_R = \frac{4}{\pi}\left(\frac{V_S}{2}\right)\left[\sum_{n=1,7,13}^{\infty}\frac{1}{n}\sin(n\omega t) + \sum_{n=3,9,15}^{\infty}\frac{1}{n}\sin(n\omega t) + \sum_{n=5,11,17}^{\infty}\frac{1}{n}\sin(n\omega t)\right]$$

i.e, positive sequence + zero sequence + negative sequence.

Line to line voltage:

Fourier components of line to line voltage can be now calculated as follows:

$V_{RY} = V_R - V_Y$

The zero sequence system cancel so that the fourier components whose frequency is a multiple of three times the inverter frequency (Triplines) are absent.

$$\therefore V_{RY} = \frac{4}{\pi}\left(\frac{V_S}{2}\right)\sum_{n=1,7,13}^{\infty}\left[\frac{1}{n}\sin(n\omega t) - \sin\left(n\omega t - \frac{2\pi}{3}\right)\right]$$

$$+\frac{4}{\pi}\left(\frac{V_S}{2}\right)\sum_{n=5,11,\dots}^{\infty}\left[\frac{1}{n}\sin(n\omega t) - \sin\left(n\omega t - \frac{2\pi}{3}\right)\right]$$

Here, the frequency spectrum of the line to line voltage waveform is similar to that of V_R but

* There are no triplines.
* The amplitude of the remaining components are $\sqrt{3}$ times greater.
* The negative sequence components lags by $60°$ $(\pi/3)$ with respect to positive sequence components.

Load neutral voltage:

The voltage of the load neutral (V_N) with respect to reference point is a rectangular wave having frequency equal to 3 times that of V_R and amplitude $1/3$ that of V_R. The fourier components of V_N are given by

$$V_N = \frac{4}{\pi}\left(\frac{V_S}{6}\right)\sum_{n=1,3,\dots}^{\infty}\frac{1}{n}\sin(3n\omega t)$$

Load phase voltage:

Fourier components of load voltage (phase) can be calculated as follows:

$V_{RN} = V_R - V_N$

$$= \frac{4}{\pi}\left(\frac{V_S}{2}\right)\sum_{n=1,7,13}^{\infty}\frac{1}{n}\sin(n\omega t) - \frac{4}{\pi}\left(\frac{V_S}{2}\right)\sum_{n=5,11,\dots}^{\infty}\frac{1}{n}\sin(n\omega t)$$

Thus, the frequency spectrum of the load phase voltage is similar to that of V_R except that it contains no triplines.

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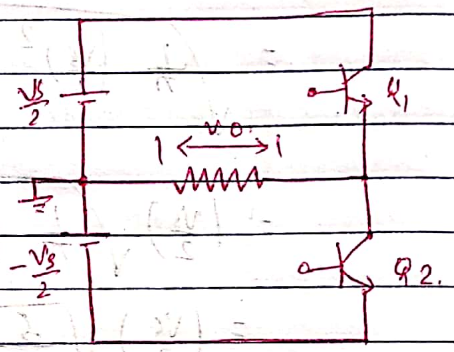
Pulse width modulated inverter:-

PWM inverter is required.

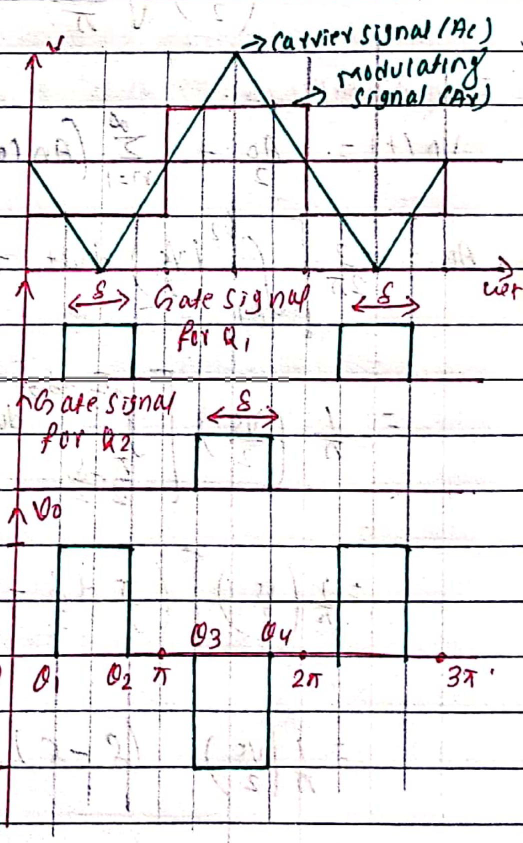
* to cope with variation with dc input voltage.
* for voltage regulation.
* for constant voltage/frequency control.

Single pulse width modulation:-



In single phase pulse width modulation control, there is only one pulse per half cycle and the width of the pulse is varied to control the inverter output voltage.

* The gate signal for transistor Q_1 & Q_2 are controlled by comparing a rectangular modulating reference signal with a triangular carrier wave.



The control strategy is as follow:-

* If modulating signal (Ar) > carrier signal (Ac) then $V_0 = Vs/2$.
* If modulating signal (Ar) < carrier signal (Ac) then $V_0 = -Vs/2$.

Modulation index is given by

$$M_i = \frac{\text{peak value of modulating signal.}}{\text{peak value of carrier signal.}}$$

$$= \frac{Ar}{Ac}$$

Now, $\theta_1 = \frac{\pi}{2} - \frac{\delta}{2}$ $\quad \theta_2 = \frac{\pi}{2} + \frac{\delta}{2}$

$\theta_3 = \frac{3\pi}{2} - \frac{\delta}{2}$ $\quad \theta_4 = \frac{3\pi}{2} + \frac{\delta}{2}$

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$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_{\theta_1}^{\theta_2}\left(\frac{Vs}{2}\right)^2 dt + \int_{\theta_3}^{\theta_4}\left(\frac{-Vs}{2}\right)^2 \cdot dt}$$

$$= \sqrt{\frac{2}{2\pi}\int_{\theta_1}^{\theta_2}\left(\frac{Vs}{2}\right)^2 dt}$$

$$= \sqrt{\frac{1}{\pi}\left(\frac{Vs}{2}\right)^2 \int_{\frac{\pi}{2}-\frac{\delta}{2}}^{\frac{\pi}{2}+\delta/2} dt}$$

$$= \left(\frac{Vs}{2}\right)\sqrt{\frac{1}{\pi}\left(\frac{\pi}{2}+\frac{\delta}{2}-\frac{\pi}{2}+\frac{\delta}{2}\right)}$$

$$= \left(\frac{Vs}{2}\right)\sqrt{\frac{\delta}{\pi}}$$

Fourier Analysis:

$$V_0(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty}\left(A_n\cos n\omega t + B_n \sin n\omega t\right)$$

$$A_0 = \frac{2}{2\pi}\left[\int_{\theta_1}^{\theta_2}\left(\frac{Vs}{2}\right)d\omega t + \int_{\theta_3}^{\theta_4}\left(-\frac{Vs}{2}\right)\cdot d\omega t\cdot\right]$$

$$= \frac{1}{\pi}\left(\frac{Vs}{2}\right)\left[\int_{\frac{\pi}{2}-\frac{\delta}{2}}^{\frac{\pi}{2}+\delta/2} d\omega t - \int_{\frac{3\pi}{2}-\frac{\delta}{2}}^{\frac{3\pi}{2}+\frac{\delta}{2}} \cdot d\omega t\right]$$

$$= \frac{1}{\pi}\left(\frac{Vs}{2}\right)\left[\left(\frac{\pi}{2}+\frac{\delta}{2}-\frac{\pi}{2}+\frac{\delta}{2}\right) - \left(\frac{3\pi}{2}+\frac{\delta}{2}-\frac{3\pi}{2}+\frac{\delta}{2}\right)\right]$$

$$= \frac{1}{\pi}\left(\frac{Vs}{2}\right)(\delta-\delta) = 0$$

$$A_n = \frac{2}{2\pi}\left[\int_{\theta_1}^{\theta_2}\left(\frac{Vs}{2}\right)\cos n\omega t\cdot d\omega t + \int_{\theta_3}^{\theta_3}\left(-\frac{Vs}{2}\right)\cos n\omega t \cdot d\omega t\right]$$

$$= \frac{1}{\pi}\left(\frac{Vs}{2}\right)\left[\frac{\sin n\omega t}{n}\Big|_{\frac{\pi}{2}+\frac{\delta}{2}}^{\frac{\pi}{2}+\frac{\delta}{2}} + \frac{\sin n\omega t}{n}\Big|_{\frac{3\pi}{2}-\frac{\delta}{2}}^{\frac{3\pi}{2}+\frac{\delta}{2}}\right]$$

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$= \frac{1}{\pi n}\left(\frac{Vs}{2}\right)\left[-\sin n\left(\frac{\pi}{2}-\frac{\delta}{2}\right) + \sin n\left(\frac{\pi}{2}+\frac{\delta}{2}\right) - \sin n\left(\frac{3\pi}{2}+\frac{\delta}{2}\right) + \sin n\left(\frac{3\pi}{2}-\frac{\delta}{2}\right)\right]$

$= \frac{1}{\pi n}\left(\frac{Vs}{2}\right)\left[-\cos\frac{n\delta}{2} + \cos\frac{n\delta}{2} + \cos\frac{n\delta}{2} - \cos\frac{n\delta}{2}\right]$

$= 0$

$Bn = \frac{2}{2\pi}\left[\int_{\theta_1}^{\theta_2}\left(\frac{Vs}{2}\right)\sin n\omega t \cdot d\omega t + \int_{\theta_3}^{\theta_4}\left(-\frac{Vs}{2}\right)\sin n\omega t \cdot d\omega t\right]$

$= \frac{1}{\pi}\left(\frac{Vs}{2}\right)\left[\left.\frac{-\cos n\omega t}{n}\right|_{\frac{\pi}{2}-\frac{\delta}{2}}^{\frac{\pi}{2}+\frac{\delta}{2}} + \left.\frac{\cos n\omega t}{n}\right|_{\frac{3\pi}{2}-\frac{\delta}{2}}^{\frac{3\pi}{2}+\frac{\delta}{2}}\right]$

$= \frac{1}{\pi n}\left(\frac{Vs}{2}\right)\left[\cos n\left(\frac{\pi}{2}-\frac{\delta}{2}\right) - \cos n\left(\frac{\pi}{2}+\frac{\delta}{2}\right) + \cos n\left(\frac{3\pi}{2}+\frac{\delta}{2}\right) - \cos n\left(\frac{3\pi}{2}-\frac{\delta}{2}\right)\right]$

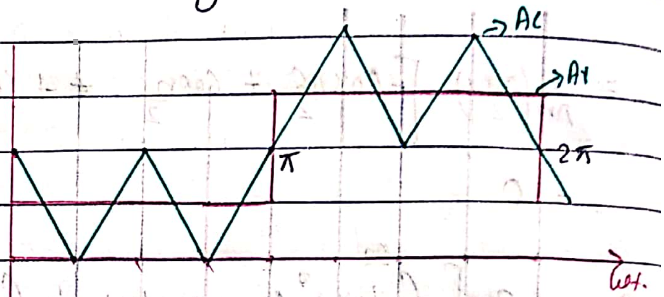
$= \frac{1}{\pi n}\left(\frac{Vs}{2}\right)\left[\sin\frac{n\delta}{2} + \sin\frac{n\delta}{2} + \sin\frac{n\delta}{2} + \sin\frac{n\delta}{2}\right]$

$= \frac{4}{\pi}\left(\frac{Vs}{2}\right)\frac{1}{n}\sin\frac{n\delta}{2}$

Then, $vo(t) = \sum_{n=1}^{\infty}\frac{1}{n}\frac{4}{\pi}\left(\frac{Vs}{2}\right)\sin\frac{n\delta}{2}\ \sin n\omega t$.

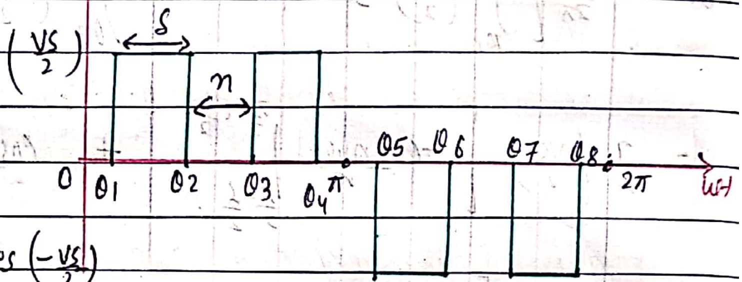
Multiple pulse width modulation:-

The harmonic content can be reduced by using several pulses in each half cycle of output voltage.



Modulation index (Mi) = $\frac{A_r}{A_c}$

Frequency modulation ratio = $\frac{\text{Frequency of carrier signal}}{\text{Frequency of reference signal}}$



$F_R = \frac{f_c}{f_r}$

let, let, $P = \frac{F_R}{2}$ = No. of pulses per half cycle.

Here,

$\pi = P\delta + Pn = P(\delta + n)$

$\therefore n = \frac{\pi}{P} - \delta$

$\theta_1 = \frac{\left(\frac{\pi}{P} - \delta\right)}{2}$ $\qquad \theta_2 = \theta_1 + \delta = \frac{\left(\frac{\pi}{P} + \delta\right)}{2}$

$$V_{rms} = \sqrt{\frac{2P}{2\pi}\left[\int_{\theta_1}^{\theta_2}\left(\frac{V_s}{2}\right)^2 . d\omega t\right.}$$

$$= \sqrt{\frac{P}{\pi}\left(\frac{V_s}{2}\right)^2 \int_{\theta_1}^{\theta_2} d\omega t}$$

$$= \left(\frac{V_s}{2}\right)\sqrt{\frac{P}{\pi}\cdot\left(\frac{\pi/P}{2} + \frac{\delta}{2} - \frac{\pi/P}{2} + \frac{\delta}{2}\right)}$$

$$= \left(\frac{V_s}{2}\right)\sqrt{\frac{P\delta}{\pi}}$$

Sinusoidal pulse width Modulation:-

In this modulation technique, the reference signal is sinusoidal wave. The control signal is a triangular wave of frequency fc. The frequency of sinusoidal reference signal controls the frequency of the output voltage of inverter. The peak value Ar of the reference signal determines the modulation index.

The carrier and reference waves are mixed in a comparator. When the magnitude of reference signal is more than that of carrier signal, the output of comparator is high and a pulse is generated. This pulse triggers the thyristor into conduction and a voltage output occurs. It is seen that the width of different output pulses is not the same. The width of the pulses depends on the angular position of the pulse. By varying the modulation index we can vary the rms value of output voltage. If δ_m is the width of the m^{th} pulse, the rms value of output voltage is

$$V_L = V\left[\sum_{m=1}^{p} \frac{\delta_m}{\pi}\right]^{0.5}$$

where, p is the number of pulses in one half cycle of output wave.