Electrical Supply Systems

Introduction

The supply of electric power to an electrical load is called power supply. The main function of the power supply is to convert electric current from a source to the correct voltage, current and frequency to power the load.

Typical A.C Power Supply Schemes

An electrical supply system has three main components, the generating stations, the transmission lines and distribution systems.

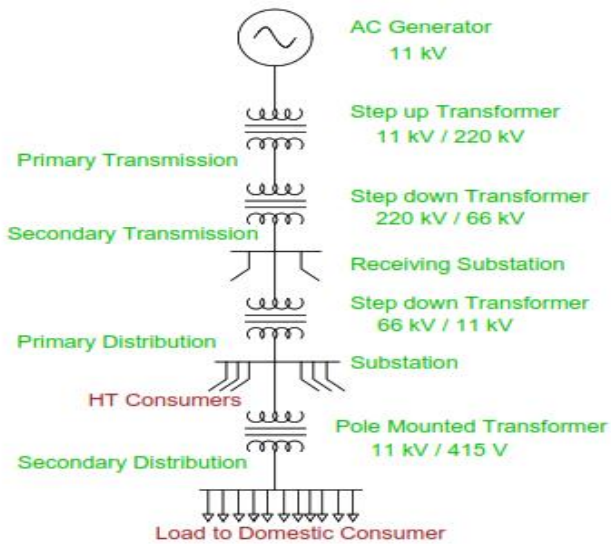


FIG : Single Line Diagram of Power Supply System

Comparison of DC and AC Transmission System

The electric power can be transmitted either by using AC transmission system or DC transmission system. Each transmission system has its own advantages and disadvantages. Therefore, to choose the right system for electric power transmission, we need to compare the two systems.

Content	AC Transmission	DC Transmission
Definition	The transmission system which transmits alternating current is called 'AC transmission'.	The transmission system which transmits direct current is called DC transmission.
Construction of AC & DC Transmission	Construction of AC transmission lines is more complicated.	Construction of DC transmission lines is less complicated.
Electric power	In this transmission, electric power can easily generate at high voltage.	In this transmission, electric power cannot easily generate at high voltage because of commutation problems.
Conductor	For the high voltage AC (HVAC) transmission, three conductors (i.e. R, Y, and B phases) are used.	For the high voltage DC (HVDC) transmission, only two conductors (positive and negative) are used.
Insulation	It requires more insulation due to more conductors.	It requires less insulation due to less conductors.
Voltage Regulation	AC transmission can easily increase & decrease the voltage level by using transformer.	DC transmission can increase & decrease voltage level by chopper & booster.

Required Factors	For the AC transmission lines, capacitance, inductance, and phase displacement need.	For the DC transmission lines, capacitance, inductance, and phase displacement do not need.
Capacitance effect	Due to the presence of capacitance in AC transmission lines, a continuous power loss occurs.	Due to the absence of capacitance in DC transmission lines, less power loss occurs.
Inductance effect	The voltage drop is more due to the presence of inductance. So, it does not provide good voltage regulation.	The voltage drop is less due to the absence of inductance. So, it provides good voltage regulation.
Skin Effect	Skin effect occurs in the AC transmission system.	Skin effect does not occur in the DC transmission system.
Corona Effect	Corona losses mostly exist in the AC system.	Corona losses rarely exist in the HVDC system.
Effect of transmission	Stability and surge effects find in AC transmission.	The DC transmission is free from the stability and surge effects.
Required parts	It does not require a rectifier and inverter.	It requires a rectifier and inverter.
Installation Cost	Under construction and maintenance, the AC transmission system is cheap.	Under construction and maintenance, the DC transmission system is expensive.
Distance over power transmission	For short-distance power transmission, the AC transmission system is mostly preferred.	For large-distance power transmission, the DC transmission system is mostly preferred.
Maintenance	Maintenance of this system is easy through connected substations.	Maintenance of this system is difficult as compared to the AC system.
Repairing	Repairing of AC transmission is very simple over DC transmission.	Repairing of DC transmission is not simple over AC transmission

Advantages Of High Voltage Transmission

i) Power Loss Reduces

$P_{loss} = I^2R$

But, $P = V\, I \cos\phi$ = Constant

$I = \frac{P}{V \cos\phi}$

Thus, $P_{loss} = I^2R = \left(\frac{P}{V \cos\phi}\right)^2 R$

Since, P, R and $\cos\phi$ remains constant

Hence, $P_{loss} = \left(\frac{1}{V}\right)^2$

Thus, power loss reduces as voltage increases.

ii) Power transfer capacity of the line increases

$P = \frac{Vs\, VR}{X} \sin\boldsymbol{\delta} = \frac{V^2}{X} \sin\boldsymbol{\delta}$

From above relation, we say that power transfer capacity of line increases.

iii) Size of conductor decreases

$P_{loss} = I^2R$

But, $P = V\, I \cos\phi$ = Constant

$I = \frac{P}{V \cos\phi}$

Thus, $P_{loss} = I^2R = \left(\frac{P}{V \cos\phi}\right)^2 R = \left(\frac{P}{V \cos\phi}\right)^2 \boldsymbol{\rho} \frac{\boldsymbol{l}}{\boldsymbol{A}}$

For constant Ploss, P, ρ, l, $\cos\phi$ remain constant

$A \ \alpha \ \frac{1}{V^2}$

iv) Voltage drop in line decreases

Vdrop = I (R + jX)

As for high voltage current is less so voltage drop reduces.

Disadvantage of High Transmission Voltage

i. Switch Gear Cost increase
ii. Transmission tower height increase
iii. Corona loss in our transmission line also starts increasing
iv. Insulator size increase
v. For the higher transmission installation, huge space is essential.

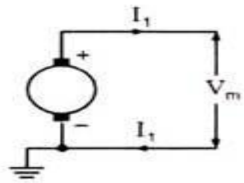
Various system of power Transmission

For the transmission of electrical power from the generating stations to the substations for distribution, there are various types of power transmission systems are adopted. However, for the transmission of electric power, three-phase three wire transmission system is universally adopted.

The different possible systems for electric power transmission are discussed below.

DC Systems:

(i) DC 2-Wire System with One Conductor Earthed



Maximum voltage between conductors = V_m volts

Power to be transmitted = P watts

Load current, $I_1 = P/V_m$

$$\text{Line losses, } W = 2 I_1^2 R_1 = 2\left(\frac{P}{V_m}\right)^2 \rho \frac{l}{a_1}$$

$$\text{Since } R_1 = \rho \frac{l}{a_1}$$

$$\text{or Area of x} - \text{section of conductor, } a_1 = \frac{2\rho P^2 l}{W V_m^2}$$

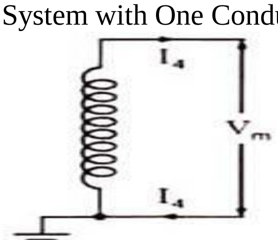
$$\text{Volume of conductor material required} = 2\, a_1 l = \frac{4\rho P^2 l^2}{W V_m^2} \ldots (2.3)$$

This system is usually formed the basis, for comparison with other systems. Thus the volume of conductor material required in this system shall be taken as the basic quantity i.e.

$$\frac{4\rho P^2 l^2}{W V_m^2} = K \text{ (say)}$$

(ii) Single Phase AC Systems:

Consider an AC Single Phase Two-Wire System with One Conductor Earthed



Peak value of voltage between conductors = V_m volts

RMS value of voltage between conductors = $V_m/\sqrt{2}$ volts

$$\text{Load current}, I_4 = \frac{P}{\frac{V_m}{\sqrt{2}} \cos \phi} = \frac{\sqrt{2} P}{V_m \cos \phi}$$

where cos ϕ is the power factor of the load

$$\text{Line losses}, W = 2 I_4^2 R_4 = 2 \left(\frac{\sqrt{2} P}{V_m \cos \phi} \right)^2 \frac{\rho l}{a_4} = \frac{4 P^2 \rho l}{a_4 V_m^2 \cos^2 \phi}$$

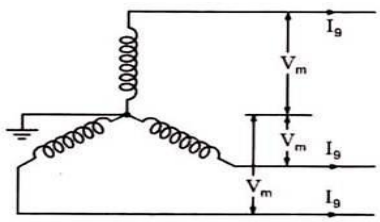
$$\text{or } a_4 = \frac{4 P^2 \rho l}{W V_m^2 \cos^2 \phi}$$

$$\text{Volume of conductor material required} = 2 a_4 l = \frac{8 P^2 \rho l^2}{W V_m^2 \cos^2 \phi} = \frac{2}{\cos^2 \phi} K \dots (2.6)$$

Hence volume of conductor material required in this system is $2/\cos^2 \phi$ times of that required in 2-wire dc system with one conductor earthed.

c) <u>3-Phase AC Systems</u>:

(i) AC 3-Phase 3-Wire System



$$\text{RMS value of voltage per phase} = \frac{V_m}{\sqrt{2}}$$

$$\text{Load current per phase}, I_9 = \frac{P/3}{\frac{V_m}{\sqrt{2}} \cos \phi} = \frac{\sqrt{2} P}{3 V_m \cos \phi}$$

$$\text{Line losses}, W = 3 I_9^2 R_9$$

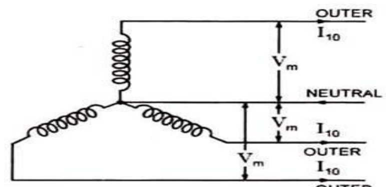
$$= 3 \left(\frac{\sqrt{2} P}{3 V_m \cos \phi} \right)^2 \frac{\rho l}{a_9} = \frac{2 P^2 \rho l}{3 V_3^2 \cos^2 \phi \, a_9}$$

$$\text{or } a_9 = \frac{2 P^2 \rho l}{3 V_m^2 \cos^2 \phi \, W}$$

$$\text{Volume of conductor material required} = 3 a_9 l = \frac{2 P^2 \rho l^2}{V_m^2 \cos^2 \phi \, W} = \frac{0.5 K}{\cos^2 \phi} \dots (2.11)$$

Hence, volume of conductor material required in this system is $0.5/\cos^2 \phi$ times of that required in two-wire dc system with one conductor earthed.

(ii) AC 3-Phase 4-Wire System



Assuming balanced load, there will be no current in neutral wire and copper losses will be same as in 3-phase 3-wire system,

$$\text{i.e. } W = \frac{2P^2 \rho l}{3\, V_m^2 \cos^2 \phi\, a_{10}}$$

$$\text{or } a_{10} = \frac{2\, P^2 \rho l}{3\, V_m^2 \cos^2 \phi\, W}$$

Taking x-section of neutral wire as half of either outer,

$$\text{Volume of conductor material required} = 3.5\, a_{10}\, l = \frac{7\rho\, P^2\, l^2}{3 \cos^2 \phi\, V_m^2\, W}$$

$$= \frac{0.583}{\cos^2 \phi} K \ldots (2.12)$$

Hence volume of conductor material required in this case is $0.583/\cos^2 \phi$ times of that required in case of two-wire dc system with one conductor earthed.

Q) For same power through same distance with same efficiency, volume of copper required in three phase wire system is 75% of single phase system. Justify it.

Solution

from above equation, For single phase

$$\text{Volume of conductor material required} = 2\, a_4 l = \frac{8\, P^2\, \rho l^2}{W\, V_m^2 \cos^2 \phi} = \frac{2}{\cos^2 \phi} K \ldots (2.6)$$

Similarly,

For three phase

$$\text{Volume of conductor material required} = 3\, a_9 l = \frac{2\, P^2\, \rho l^2}{V_m^2 \cos^2 \phi\, W} = \frac{0.5\, K}{\cos^2 \phi} \ldots (2.11)$$

Saving = volume of conductor in three phase – volume of conductor in single phase

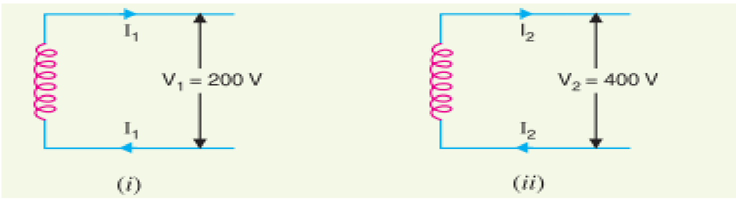
Saving = $\frac{1.5K}{cos^2\phi}$

So, % saving = $\frac{\frac{1.5K}{cos^2\phi}}{\frac{2\,K}{cos^2\phi}}$ * 100% = 75%

Q) What is the percentage saving in feeder copper if the line voltage in a 2-wire d.c. system is raised from 200 volts to 400 volts for the same power transmitted over the same distance and having the same power loss?

Solution:

Fig. (i) Shows 200 volts system, whereas Fig. (ii) Shows 400 volts system. Let P be the power delivered and W be power loss in both cases. Let v_1 and a_1 be the volume and area of X-section for 200 V system and v_2 and a_2 for that of 400 V systems



Now,	$P = V_1 I_1 = 200\, I_1$		...(*i*)
And	$P = V_2 I_2 = 400\, I_2$		...(*ii*)

As same power is delivered in both cases,

$$\therefore \quad 200 I_1 = 400\, I_2 \text{ or } I_2 = (200/400)\, I_1 = 0{\cdot}5 I_1$$

Power loss in 200 V system, $W_1 = 2 I_1^{\,2} R_1$

Power loss in 400 V system, $W_2 = 2 I_2^{\,2} R_2 = 2(0{\cdot}5\, I_1)^2 R_2 = 0{\cdot}5 I_1^{\,2} R_2$

As power loss in the two cases is the same,

$$\therefore \quad W_1 = W_2$$

$$\text{or} \quad 2 I_1^2 R_1 = 0{\cdot}5\, I_1^2 R_2$$

$$\text{or} \quad R_2/R_1 = 2/0{\cdot}5 = 4$$

$$\text{or} \quad a_1/a_2 = 4$$

$$\text{or} \quad v_1/v_2 = 4$$

$$\text{or} \quad v_2/v_1 = 1/4 = 0{\cdot}25$$

$$\therefore \quad \text{\% age saving in feeder copper} = \frac{v_1 - v_2}{v_1} \times 100 = \left(\frac{v_1}{v_1} - \frac{v_2}{v_1}\right) \times 100$$

$$= (1 - 0{\cdot}25) \times 100 = \mathbf{75\%}$$

Q. A d.c. 2-wire system is to be converted into a.c. 3-phase, 3-wire system by the addition of a third conductor of the same cross-section as the two existing conductors. Calculate the percentage additional load which can now be supplied if the voltage between wires and the percentage loss in the line remain unchanged. Assume a balanced load of unity power factor.

Solution:

Fig. (i) Shows the 2-wire d.c. system, whereas Fig. (ii) Shows the 3-phase, 3- wire system. Suppose V is the voltage between conductors for the two cases. Let R be the resistance per conductor in each case.

Two-wire d.c. system. Referring to Fig. 7.25 (*i*),

$$\text{Power supplied, } P_1 = V I_1$$

$$\text{Power loss, } W_1 = 2\, I_1^2 R$$

$$\text{Percentage power loss} = \frac{2\, I_1^2\, R}{V\, I_1} \times 100 \qquad ...(i)$$

3-phase, 3-wire a.c. system. Referring to Fig. 7.25 (*ii*),

$$\text{Power supplied, } P_2 = \sqrt{3}\, V I_2$$

$$\text{Power loss, } W_2 = 3\, I_2^2 R$$

$$\text{Percentage power loss} = \frac{3\, I_2^2\, R}{\sqrt{3}\, V\, I_2} \times 100 \qquad ...(ii)$$

As the percentage power loss in the two cases is the same,

$$\therefore \quad \text{exp. } (i) = \text{exp. } (ii)$$

$$\text{or} \quad \frac{2\, I_1^2\, R}{V\, I_1} \times 100 = \frac{3\, I_2^2\, R}{\sqrt{3}\, V\, I_2} \times 100$$

$$\text{or} \quad 2\, I_1 = \sqrt{3}\, I_2$$

$$\text{or} \quad I_2 = \frac{2}{\sqrt{3}}\, I_1$$

$$\text{Now,} \quad \frac{P_2}{P_1} = \frac{\sqrt{3}\, V\, I_2}{V\, I_1} = \frac{\sqrt{3}\, V \times \frac{2}{\sqrt{3}}\, I_1}{V\, I_1} = 2$$

$$\therefore \quad P_2 = 2\, P_1$$

i.e. additional power which can be supplied at unity p.f. by 3-phase, 3-wire a.c. system = **100%**.

Q. A d.c. 3-wire system is to be converted into a 3-phase, 4-wire system by adding a fourth wire equal in X-section to each outer of the d.c. system. If the percentage power loss and voltage at the consumer's terminals are to be the same in the two cases, find the extra power at unity power factor that can be supplied by the a.c. system. Assume loads to be balanced.

Solution

Fig. (i) Shows the 3-wire d.c. system, whereas Fig. (ii) Shows 3-phase, 4- wire system. Suppose that V is consumer's terminal voltage (i.e., between conductor and neutral) in the two cases. Let R be the resistance per conductor in each case

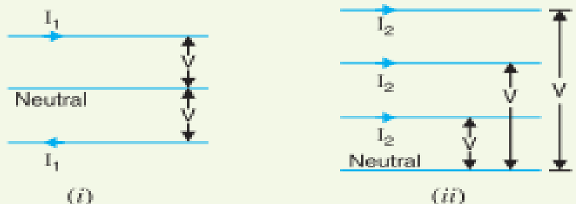


Fig. 7.26

3-wire d.c. system. Refer to Fig. 7.26 (*i*). As the loads are balanced, therefore, neutral wire carries no current. Consequently, there is no power loss in the neutral wire.

$$\text{Power supplied, } P_1 = 2\,V I_1$$

$$\text{Power loss, } W_1 = 2\,I_1^2 R$$

$$\text{Percentage power loss} = \frac{2\,I_1^2\,R}{2\,V\,I_1} \times 100 \qquad \text{...(i)}$$

3-phase, 4-wire a.c. system. Refer to Fig. 7.26 (*ii*). Since the loads are balanced, the neutral wire carries no current and hence there is no power loss in it.

$$\text{Power supplied, } P_2 = {}^{*}3\,V I_2 \qquad [\because \cos\phi = 1]$$

$$\text{Power loss, } W_2 = 3\,I_2^2 R$$

$$\text{Percentage power loss} = \frac{3\,I_2^2\,R}{3\,V\,I_2} \times 100 \qquad \text{...(ii)}$$

As the percentage power loss in the two cases is the same, therefore, exp. (*i*) is equal to exp. (*ii*) *i.e.*,

$$\frac{2\,I_1^2\,R}{2\,V\,I_1} \times 100 = \frac{3\,I_2^2\,R}{3\,V\,I_2} \times 100$$

or $$I_1 = I_2$$

Now $$\frac{P_2}{P_1} = \frac{3\,V\,I_2}{2\,V\,I_1} = \frac{3\,V\,I_1}{2\,V\,I_1} = 1{\cdot}5$$

$\therefore$ $$P_2 = 1{\cdot}5\,P_1$$

i.e., extra power that can be supplied at unity power factor by 3-phase, 4-wire a.c. system = **50%.**

Q. A single phase a.c. system supplies a load of 200 kW and if this system is converted to 3-phase, 3-wire a.c. system by running a third similar conductor, calculate the 3-phase load that can now be supplied if the voltage between the conductors is the same. Assume the power factor and transmission efficiency to be the same in the two cases.

Solution

Fig. (i) Shows the single phase 2-wire a.c. system, whereas Fig. (ii) Shows 3- phase, 3-wire system. Suppose that V is the voltage between the conductors in the two cases. Let R be the resistance per conductor and cos φ the power factor in each case.

Single phase 2-wire system. Referring to Fig. 7.27 (*i*),

$$\text{Power supplied, } P_1 = V I_1 \cos\phi$$

$$\text{Power loss, } W_1 = 2 I_1^2 R$$

$$\text{\% age power loss} = \frac{2 I_1^2 R}{V I_1 \cos\phi} \times 100 \qquad \text{...}(i)$$

3-phase, 3-wire a.c. system. Referring to Fig. 7.27 (*ii*),

$$\text{Power supplied, } P_2 = \sqrt{3} V I_2 \cos\phi$$

$$\text{Power loss, } W_2 = 3 I_2^2 R$$

$$\text{\% age power loss} = \frac{3 I_2^2 R}{\sqrt{3} V I_2 \cos\phi} \times 100 \qquad \text{...}(ii)$$

Fig. 7.27

As the transmission efficiency in the two cases is the same, therefore, percentage power loss will also be the same *i.e.*,

$$\text{exp. } (i) = \text{exp. } (ii)$$

or
$$\frac{2 I_1^2 R}{V I_1 \cos\phi} \times 100 = \frac{3 I_2^2 R}{\sqrt{3} V I_2 \cos\phi} \times 100$$

or
$$2 I_1 = \sqrt{3} I_2$$

or
$$I_2 = \frac{2}{\sqrt{3}} I_1$$

Now,
$$\frac{P_2}{P_1} = \frac{\sqrt{3} V I_2 \cos\phi}{V I_1 \cos\phi} = \frac{\sqrt{3} V \frac{2}{\sqrt{3}} I_1 \cos\phi}{V I_1 \cos\phi} = 2$$

As the power supplied by single phase, 2-wire (*i.e.*, P_1) is 200 kW,

∴ Power supplied by 3-phase, 3-wire a.c. system is

$$P_2 = 2P_1 = 2 \times 200 = \mathbf{400\ kW}$$

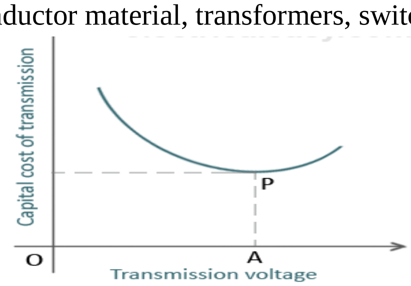
It may be seen that 3-phase, 3-wire system can supply 100% additional load.

Choice of Voltage; Conductor Size

While designing any transmission line, economy is one of the most important factors the engineer must consider. An electrical power transmission line must be designed in such a way that the maximum economy is achieved.

Economic Choice Of Transmission Voltage

Increase in transmission voltage, cost of conductor material can be reduced & efficiency can be increased. But the cost of transformers, insulators, switchgear etc. is increased at the same time. Thus, for overall economy, there is an optimum transmission voltage. Economical transmission voltage is one for which sum of cost of conductor material, transformers, switchgear, insulators & other equipment is minimum.



A graph is drawn for the total cost of transmission with respect to various transmission voltages as shown in the figure at right. The lowest point on the curve gives the optimum transmission voltage. As here in the graph, point P is the lowest and the corresponding voltage OA is the optimum transmission voltage.

The **empirical formula** is used in the modern power system networks to find the economical transmission voltage. According to American practice, the economic voltage between lines in a three-phase AC system is defined as the following equation.

$$V = 5{\cdot}5\sqrt{0{\cdot}62\,l + \frac{3P}{150}}$$

where
V = line voltage in kV
P = maximum kW per phase to be delivered to single circuit
l = distance of transmission line in km

From the above equation, it is noted that the power to be transmitted and the distance of the transmission line is considered in the equation.

Economic Choice of Conductor Size

The cost of conductor material required for designing a transmission line is a very considerable part of the total cost of a transmission line. Therefore, the determination of proper size of the conductor for the transmission line is very important.

Kelvin's law states that the most economical area of conductor is that for which the total annual cost of transmission line is minimum.

The total annual cost of the transmission line can be divided into two parts viz. −

- Annual charges on capital cost
- Annual cost of energy wasted in conductor.

(I) Annual charge on capital outlay. This is on account of interest and depreciation on the capital cost of complete installation of transmission line. In case of overhead system, it will he the annual interest and depreciation on the capital cost of conductors, supports and insulators and the cost of their erection. Now, for an overhead line, insulator cost is constant, the conductor cost is proportional to the area of X-section and the cost of supports and their erection is partly constant and partly proportional to area of X-section of the Conductor Size. Therefore, annual charge on an overhead' transmission line can be expressed as :

$$\text{Annual charge} = P_1 + P_2\,a \qquad \ldots(i)$$

where P_1 and P_2 are constants and a is the area of X-section of the conductor.

(ii) Annual cast of energy wasted. This is on account of energy lost mainlyt in the conductor due to $/^2$R losses. Assuming a constant current in the conductor throughout the year, the energy lost in the conductor is proportional to resistance. As resistance is inversely proportional to the area of X-section of the Conductor Size, therefore, the energy lost in the conductor is inversely proportional to area of X-section. Thus, the annual cost of energy wasted in an overhead transmission line can be expressed as :

$$\text{Annual cost of energy wasted} = P_3/a \qquad \ldots(ii)$$

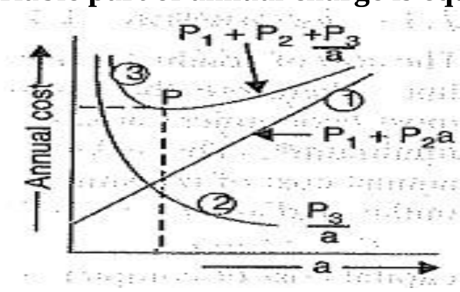
where P_3 is a constant.

$$\begin{aligned}\text{Total annual cost, } C &= \text{exp. }(i) + \text{exp. }(ii)\\ &= (P_1 + P_2\,a) + P_3/a\\ C &= P_1 + P_2\,a + P_3/a \qquad \ldots(iii)\end{aligned}$$

In exp. (iii), only area of X-section a is variable. Therefore, the total annual cost of transmission line will be minimum if differentiation of C w.r.t. a is zero i.e.

$$\begin{aligned}\frac{d}{da}(C) &= 0\\ \text{or}\quad \frac{d}{da}(P_1 + P_2\,a + P_3/a) &= 0\\ \text{or}\quad P_2 - \frac{P_3}{a^2} &= 0\\ \text{or}\quad P_2 &= P_3/a^2\\ \text{or}\quad P_2\,a &= \frac{P_3}{a}\end{aligned}$$

i.e. Variable part of animal charge = Annual cost of energy wasted
Therefore Kelvin's Law can also be stated as i.e. the most economical area of conductor is that for which the variable part of annual charge is equal to the cost of energy losses



Limitations of Kelvin's law: Although theoretically Kelvin's law holds good, there is often considerable difficulty in applying it to a proposed scheme of power transmission. In practice, the limitations of this law are:

- It is not easy to estimate the energy loss in the line without actual load curves, which are not available at the time of estimation.
- The assumption that annual cost on account of interest and depreciation on the capital outlay is in the form $P_1 + P_2a$ is strictly speaking not true. For instance, in cables neither the cost of cable dielectric and sheath nor the cost of laying vary in this manner.
- This law does not take into account several physical factors like safe current density, mechanical strength, corona loss etc.
- Conductor Size determined by this law may not always be practicable one because it may be too small for the safe carrying of necessary current.
- Interest and depreciation on the capital outlay cannot be determined accurately.

Q). A 2-conductor cable 1 km long is required to supply a constant current of 200 A throughout the year. The cost of cable including installation is Rs. (20 a + 20) per meter where 'a' is the area of X-section of the conductor in cm^2. The cost of energy is 5P per kWh and interest and depreciation charges amount to 10%. Calculate the most economical conductor size. Assume resistivity of conductor material to be 1·73 μ Ω cm.

Solution.

Resistance of one conductor $= \frac{\rho l}{a} = \frac{1\cdot 73 \times 10^{-6} \times 10^{5}}{a} = \frac{0\cdot 173}{a}\ \Omega$

Energy lost per annum $= \frac{2I^2\ Rt}{1000}$ kWh

$= \frac{2 \times (200)^2 \times 0\cdot 173 \times 8760}{1000 \times a} = \frac{1{,}21{,}238\cdot 4}{a}$ kWh

Annual cost of energy lost = Cost per kWh × Annual energy loss

$= \text{Rs}\ \frac{5}{100} \times \frac{1{,}21{,}238\cdot 4}{a}$

$= \text{Rs}\ 6062/a$ ...(*i*)

The capital cost (variable) of the cable is given to be Rs 20 *a* per metre. Therefore, for 1 km length of the cable, the capital cost (variable) is Rs. 20 a × 1000 = Rs. 20,000 a.

Variable annual charge = Annual interest and depreciation on capital cost (variable) of cable

$= \text{Rs}\ 0\cdot 1 \times 20{,}000\ a$

$= \text{Rs}\ 2000\ a$ ...(*ii*)

According to Kelvin's law, for most economical X-section of the conductor,

Variable annual charge = Annual cost of energy lost

or $2000\ a = 6062/a$

$\therefore \quad a = \sqrt{\frac{6062}{2000}} = 1\cdot 74\ \text{cm}^2$

Q) The cost of a 3-phase overhead transmission line is Rs (25000 a + 2500) per km where 'a' is the area of X-section of each conductor in cm^2. The line is supplying a load of 5 MW at 33kV and 0·8 p.f. lagging assumed to be constant throughout the year. Energy costs 4P per kWh and interest and depreciation total 10% per annum. Find the most economical size of the conductor. Given that specific resistance of conductor material is 10^{-6} Ω cm.

Solution.

$$\text{Resistance of each conductor, } R = \frac{\rho l}{a} = \frac{10^{-6}\times 10^5}{a} = \frac{0.1}{a}\ \Omega$$

$$\text{Line current, } I = \frac{P}{\sqrt{3}V\cos\phi} = \frac{5\times 10^6}{\sqrt{3}\times 33\times 10^3\times 0.8} = 109.35\text{ A}$$

$$\text{Energy lost per annum} = \frac{3I^2Rt}{1000}\text{ kWh} = \frac{3\times(109.35)^2\times 0.1\times 8760}{1000\times a} = \frac{31{,}424}{a}\text{ kWh}$$

$$\text{Annual cost of energy lost} = \text{Rs } 0.04\times 31{,}424/a = \text{Rs } \frac{1256.96}{a}$$

The capital cost (variable) of the cable is given to be Rs 25000 a per km length of the line.

$\therefore$ Variable annual charge = 10% of capital cost (variable) of line

= Rs $0.1\times 25{,}000a$ = Rs $2{,}500\,a$

According to Kelvin's law, for most economical X-section of the conductor,

Variable annual charge = Annual cost of energy lost

$$2500\,a = \frac{1256.96}{a}$$

or $$a = \sqrt{\frac{1256.96}{2500}} = \mathbf{0.71\ cm^2}$$

Q) A 2-wire feeder carries a constant current of 250 A throughout the year. The portion of capital cost which is proportional to area of X-section is Rs 5 per kg of copper conductor. The interest and depreciation total 10% per annum and the cost of energy is 5P per kWh. Find the most economical area of X-section of the conductor. Given that the density of copper is 8·93gmcm 3 and its specific resistance is 1·73 × 10^{-8} Ω m.

Solution. Consider 1 metre length of the feeder. Let a be the most economical area of X-section of each conductor in m^2.

$$\text{Resistance of each conductor, } R = \frac{\rho l}{a} = \frac{1.73\times 10^{-8}\times 1}{a} = \frac{1.73\times 10^{-8}}{a}\ \Omega$$

$$\text{Energy lost per annum} = \frac{2I^2Rt}{1000}\text{ kWh} = \frac{2\times(250)^2\times 1.73\times 10^{-8}\times 8760}{1000\times a}$$

$$= \frac{18{,}94{,}350}{a}\times 10^{-8}\text{ kWh}$$

$$\text{Annual cost of energy lost} = \text{Rs } \frac{5}{100}\times\frac{18{,}94{,}350\times 10^{-8}}{a} = \text{Rs } \frac{94{,}717.5}{a}\times 10^{-8}$$

Mass of 1 metre feeder = 2 (Volume × density) = $2\times a\times 1\times 8.93\times 10^3$ kg

= $17.86\times 10^3 a$ kg

Capital cost (variable) = Rs $5\times 17.86\times 10^3\,a$ = Rs $89.3\times 10^3 a$

Variable Annual charge = 10% of capital cost (variable)

= $0.1\times 89.3\times 10^3 a$ = Rs 8930 a

For most economical area of X-section,

Variable annual charge = Annual cost of energy lost

or $$\text{Rs } 8930\,a = \frac{94{,}717.5}{a}\times 10^{-8}$$

$\therefore$ $$a = \sqrt{\frac{94{,}717.5\times 10^{-8}}{8930}} = 3.25\times 10^{-4}\text{m}^2 = \mathbf{3.25\ cm^2}$$