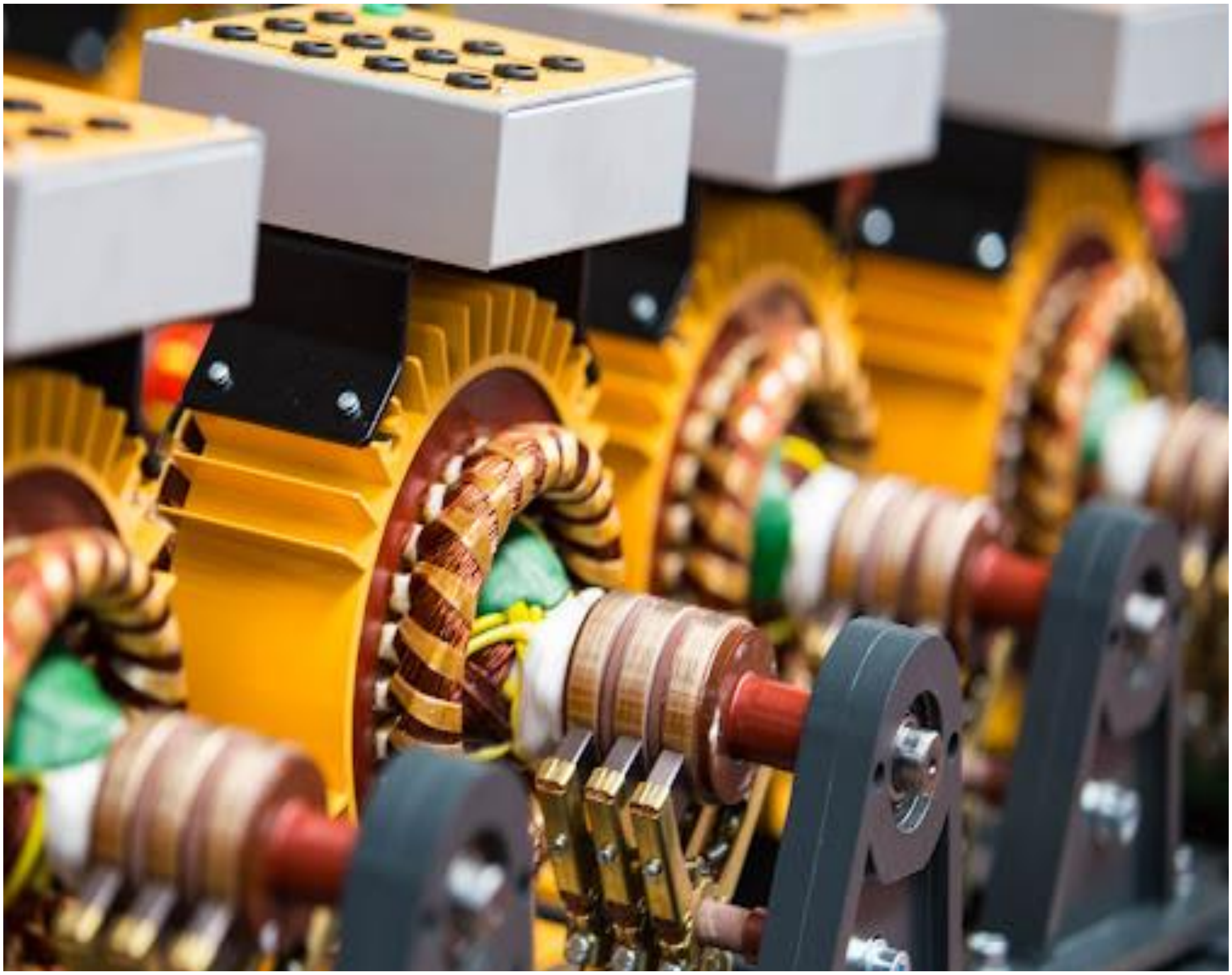
A REFERENCE MANUAL ON ADVANCED ELECTRICAL MACHINE

Bachelor in Electrical and Electronics Engineering

III/I



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UNIT 1: PRINCIPLE OF ELECTROMECHANICAL CONVERSION

An electromechanical energy conversion device is a link between an electrical and mechanical system and electromechanical energy conversion needs the presence of natural phenomenon which interrelate electric and magnetic field on one hand and mechanical force and motion on the rotor.

1.1 FORCES AND TORQUES IN MAGNETIC FIELD SYSTEMS

The *Lorentz Force Law*

$F = q(E + v \times B)$ ……….. (1)

gives the force F on a particle of charge q in the presence of electric and magnetic fields. In SI units, F is in *newtons*, q in *coulombs*, E in *volts per meter*, B in *Tesla*, and v, which is the velocity of the particle relative to the magnetic field, in *meters per second.*

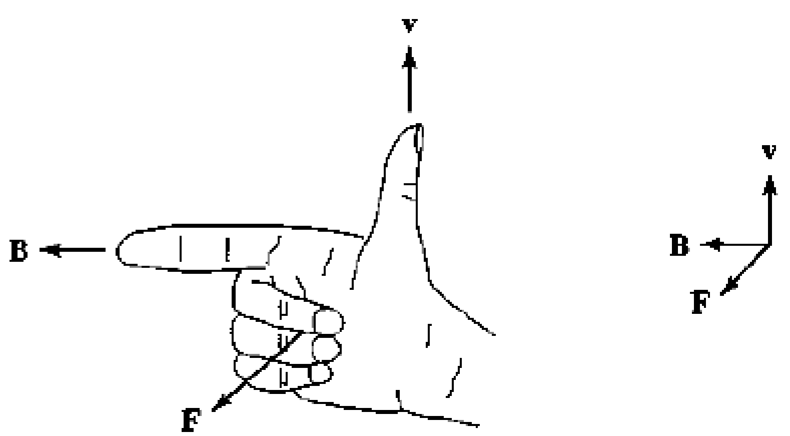
Thus, in a pure electric-field system, the force is determined simply by the charge on the particle and the electric field

$F=qE$…………..(2)

The force acts in the direction of the electric field and is independent of any particle motion. In pure magnetic-field systems, the situation is somewhat more complex. Here the force

$F = q(v \times B)$……….. (3)

is determined by the magnitude of the charge on the particle and the magnitude of the B field as well as the velocity of the particle. In fact, the direction of the force is always perpendicular to the direction of both the particle motion and that of the magnetic field. Mathematically, this is indicated by the vector cross product v x B in Eq. 3. The magnitude of this cross product is equal to the product of the magnitudes of v and B and the sine of the angle between them; its direction can be found from the right-hand rule, which states that when the thumb of the right hand points in the direction of v and the index finger points in the direction of B, the force, which is perpendicular to the directions of both B and v, points in the direction normal to the palm of the hand, as shown in Fig: 1.



For situations where large numbers of charged particles are in motion, it is convenient to rewrite Eq.1 in terms of the *charge density* ρ (measured in units of *coulombs per cubic meter)* as

$F_v = \rho\, eE + v \times B)$ ……………… (4)

where the subscript v indicates that F v is *a force density* (force per unit volume) which in SI units is measured in *newtons per cubic meter.*

The product ρ v is known as the *current density*

$J = \rho\, v$ ……………… (5)

which has the units of *amperes per square meter*. The magnetic-system force density corresponding to Eq. 3 can then be written as

Fv = J x B ……………….. (6)

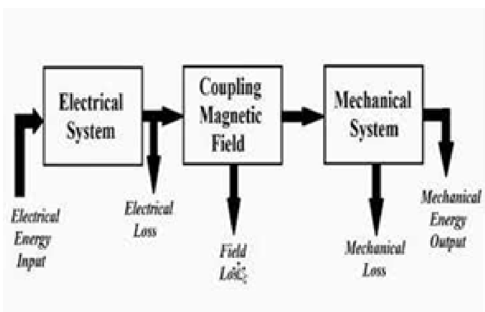
For currents flowing in conducting media, Eq. 6 can be used to find the force density acting on the material itself.

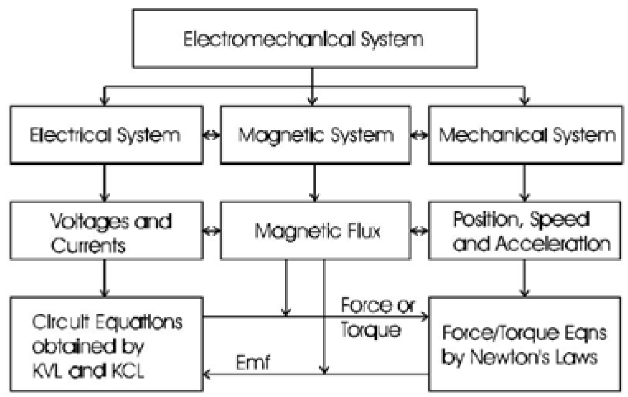
The torque T acting is given by the sum of the force-moment arm products for each wire

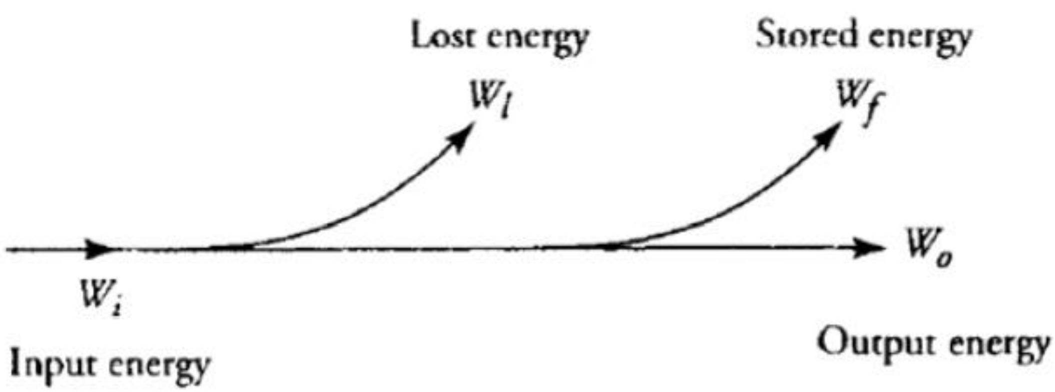
$T = F \times r$

1.2 ENERGY BALANCE

The principle of conservation of energy states that energy is neither created nor destroyed; it is merely changed in form. Electromechanical energy conversion is a reversible process except for the losses in the system. The term "reversible" implies that the energy can be transferred back and forth between the electrical and the mechanical systems.







From energy diagram we can see that principle of energy conservation is accurately followed.

i.e Input Energy = Losses + Stored Energy + Output Energy.

1.3 SINGLY EXCITED SYSTEM

Consider a singly excited linear actuator as shown below. The winding resistance is R. At a certain time instant *t*, we record that the terminal voltage applied to the excitation winding is *v*, the excitation winding current *i*, the position of the movable plunger *x*, and the force acting on the plunger **F** with the reference direction chosen in the positive direction of the *x* axis, as shown in the diagram. After a time interval *dt*, we notice that the plunger has moved for a distance *dx* under the action of the force **F**. The mechanical done by the force acting on the plunger during this time interval is thus

$dw_m = Fdx$

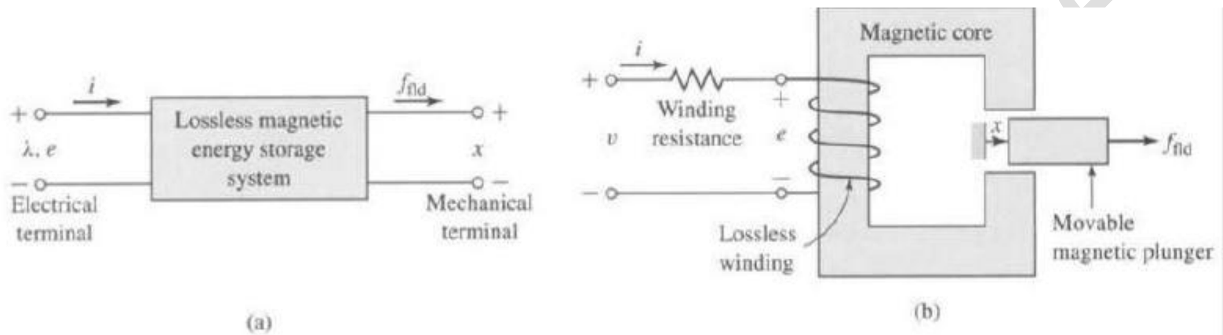


Fig: Singly Excited system energy conversion

The amount of electrical energy that has been transferred into the magnetic field and converted into the mechanical work during this time interval can be calculated by subtracting the power loss dissipated in the winding resistance from the total power fed into the excitation winding as

$$dw_e = dw_f + dw_m = vidt - Ri^2 dt$$

Since,

$$e = \frac{d\lambda}{dt} = v - Ri$$

So,

$$dw_f = dw_e - dw_m = eidt - Fdx = id\lambda - Fdx$$

we can also write,

$$e = \frac{d\lambda}{dt} = v - Ri$$

$$dw_f(\lambda, x) = \frac{dw_f(\lambda, x)}{d\lambda} d\lambda + \frac{dw_f(\lambda, x)}{dx} dx$$

the energy stored in a magnetic field can be expressed as

$$w_f(\lambda, x) = \int_0^{\lambda} i((\lambda, x)d\lambda$$

For a magnetically linear (with a constant permeability or a straight line magnetization curve such that the inductance of the coil is independent of the excitation current) system, the above expression becomes

$$W_f(\lambda, x) = \frac{1}{2}\frac{\lambda^2}{L(x)}$$

and the force acting on the plunger is then

$$F = -\frac{\partial W_f(\lambda, x)}{\partial x} = \frac{1}{2}\left[\frac{\lambda}{L(x)}\right]^2 \frac{dL(x)}{dx} = \frac{1}{2} i^2 \frac{dL(x)}{dx}$$

In the diagram below, it is shown that the magnetic energy is equivalent to the area above the magnetization or λ-i curve. Mathematically, if we define the area underneath the magnetization curve as the ***coenergy*** (which does not exist physically), i.e.

$$W_f'(i, x) = i\lambda - W_f(\lambda, x)$$

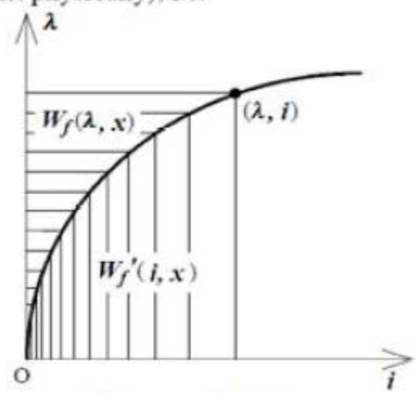
we can obtain

$$dW_f'(i, x) = \lambda di + id\lambda - dW_f(\lambda, x)$$
$$= \lambda di + Fdx$$
$$= \frac{\partial W_f'(i, x)}{\partial i} di + \frac{\partial W_f'(i, x)}{\partial x} dx$$

Therefore,

$$\lambda = \frac{\partial W_f'(i, x)}{\partial i}$$

and $$F = \frac{\partial W_f'(i, x)}{\partial x}$$



Energy and coenergy

From the above diagram, the coenergy or the area underneath the magnetization curve can be calculated by

$$W_f'(i, x) = \int_0^i \lambda(i, x) di$$

For a magnetically linear system, the above expression becomes

$$W_f'(i, x) = \frac{1}{2} i^2 L(x)$$

and the force acting on the plunger is then

$$F = \frac{\partial W_f'(i, x)}{\partial x} = \frac{1}{2} i^2 \frac{dL(x)}{dx}$$

1.4 CO-ENERGY

Co-energy has no physical significance. It is given as input electrical energy less than the stored energy in the field.

i.e Co- energy (W_t) = input electrical energy – stored energy in the field

Co-energy is expressed in the same unit as energy and is especially useful for calculation of magnetic forces and torque in rotating machines. The co-energy E' or W' is zero (0) for systems incapable of storing energy.

We have, $W_t = \int_0^{\Psi} i(\Psi)d\Psi = \int_0^{\Phi} M(\Phi)d\Phi$ …………… (1)

Assuming linearity,

$W_t = \frac{1}{2}i\psi = \frac{1}{2}M\Phi = \frac{1}{2}S\Phi^2$ ………………… (2)

Where, S= reluctance of the magnetic circuit $=\frac{mmf}{\phi} = \frac{M}{\phi}$

Since, coil inductance is given as $L=\frac{N\phi}{i} = \frac{\psi}{i} \Longrightarrow i = \frac{\Psi}{L}$

And field energy is given by

$W_t = \frac{1}{2}\frac{\Psi^2}{L}$ ……………….. (3)

In linear case, the inductance L is independent of current i but is a function of configuration x. Thus, the field energy is a special function of two independent variables Ψ and x i.e

$W_t = \frac{1}{2}\frac{\Psi^2}{L(x)}$ ……………….. (4)

The field energy is distributed throughout the space occupied by the field. Neglecting losses,

$W_{Field} = \frac{W_t}{a_t} = \int_0^{\psi} \frac{Ni}{l}\frac{d\psi}{Na} = \int_0^{B} HdB = \frac{1}{2}HB = \frac{1}{2}\frac{B^2}{\mu}$ J/m^3

Where, H = Magnetic field intensity in AT/m $=\frac{Ni}{l}$

B = magnetic field density in T

So, $W_t' = W_t = \frac{1}{2}i\psi = \frac{1}{2}M\Phi = \frac{1}{2}S\Phi^2$

Also, in terms of coil inductance,

$W_t' = \int_0^{i}\Psi \mathrm{di} = \int_0^{i}\mathrm{Lidi} = \frac{1}{2}Li^2$

In general, $W_t' = \frac{1}{2}L(x)i^2$

The expression for co-energy density is given as,

$W_t' = \int_0^{B} HdB = \frac{1}{2}HB = \frac{1}{2}\frac{B^2}{\mu}$ J/m^3

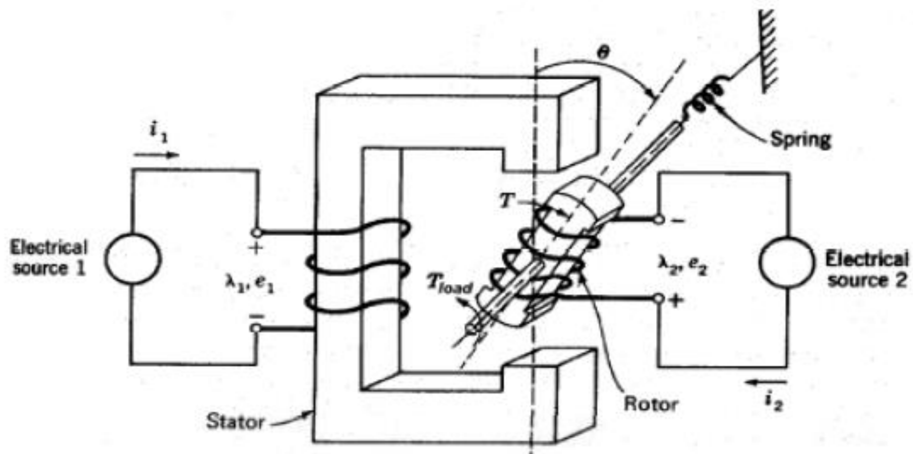
Hence, $W_t = \frac{1}{2}Li^2 = \frac{1}{2}i\psi = \frac{1}{2L}\psi^2$

1.5 DOUBLY EXCITED SYSTEM

The general principle for force and torque calculation discussed above is equally applicable to multi-excited systems. Consider a doubly excited rotating actuator shown schematically in the diagram below as an example. The differential energy and coenergy functions can be derived as following:

$$dW_f = dW_e - dW_m$$

where $$dW_e = e_1 i_1 dt + e_2 i_2 dt$$



A doubly excited actuator

$$e_1 = \frac{d\lambda_1}{dt}, \qquad e_2 = \frac{d\lambda_2}{dt}$$

and $$dW_m = Td\theta$$

Hence,

$$dW_f(\lambda_1, \lambda_2, \theta) = i_1 d\lambda_1 + i_2 d\lambda_2 - Td\theta$$

$$= \frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \lambda_1} d\lambda_1 + \frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \lambda_2} d\lambda_2 + \frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \theta} d\theta$$

and

$$
\begin{aligned}
dW_f'(i_1,i_2,\theta) &= d\left[i_1\lambda_1 + i_2\lambda_2 - W_f(\lambda_1,\lambda_2,\theta)\right] \\
&= \lambda_1 di_1 + \lambda_2 di_2 + Td\theta \\
&= \frac{\partial W_f'(i_1,i_2,\theta)}{\partial i_1}di_1 + \frac{\partial W_f'(i_1,i_2,\theta)}{\partial i_2}di_2 \\
&+ \frac{\partial W_f'(i_1,i_2,\theta)}{\partial \theta}d\theta
\end{aligned}
$$

Therefore, comparing the corresponding differential terms, we obtain

$$T = -\frac{\partial W_f(\lambda_1,\lambda_2,\theta)}{\partial \theta}$$

$$T = \frac{\partial W_f'(i_1,i_2,\theta)}{\partial \theta}$$

For magnetically linear systems, currents and flux linkages can be related by constant inductances as following

$$\begin{bmatrix}\lambda_1\\ \lambda_2\end{bmatrix} = \begin{bmatrix}L_{11} & L_{12}\\ L_{21} & L_{22}\end{bmatrix}\begin{bmatrix}i_1\\ i_2\end{bmatrix}$$

or

$$\begin{bmatrix}i_1\\ i_2\end{bmatrix} = \begin{bmatrix}\Gamma_{11} & \Gamma_{12}\\ \Gamma_{21} & \Gamma_{22}\end{bmatrix}\begin{bmatrix}\lambda_1\\ \lambda_2\end{bmatrix}$$

where $L_{12}=L_{21}$, $\Gamma_{11}=L_{22}/\Delta$, $\Gamma_{12}=\Gamma_{21}=-L_{12}/\Delta$, $\Gamma_{22}=L_{11}/\Delta$, and $\Delta=L_{11}L_{22}-L_{12}^2$. The magnetic energy and coenergy can then be expressed as

$$W_f(\lambda_1,\lambda_2,\theta) = \frac{1}{2}\Gamma_{11}\lambda_1^2 + \frac{1}{2}\Gamma_{22}\lambda_2^2 + \Gamma_{12}\lambda_1\lambda_2$$

and

$$W_f'(i_1,i_2,\theta) = \frac{1}{2}L_{11}i_1^2 + \frac{1}{2}L_{22}i_2^2 + L_{12}i_1i_2$$

respectively, and it can be shown that they are equal.

1.6 RELUCTANCE TORQUE

In rotating electrical machines, when the working flux varies with the movement of rotor, the reluctance torque is developed. In accordance with the rotational equivalent,

$T_e = -\frac{1}{2}\phi^2 \frac{dS}{d\theta_r}$ .... (1)

Where, T_e= instantaneous torque acting in direction to increase the space angle θ_r.

The direction of torque is determined by the sign of $\frac{dS}{d\theta_r}$. The torque is generated between the two fields twisting the object around the line with the magnetic field. Thus, torque is exerted on the object so that it tries to position itself to give minimum reluctance for the magnetic flux. It is also known as saliency torque. A reluctance motor depends upon the reluctance torque for its operation.

Metals like iron provides a low reluctance path and the flux lines like to pass through the least reluctance path. The least reluctance path is along the pole axis where the air gap is maximum. So, the flux line will exert force on the rotor to align with the flux lines. This torque creates additional torque to the produced torque by locking of excited poles with synchronously rotating magnetic field. This effect is absent in cylindrical rotor machine as the air gap is uniform.

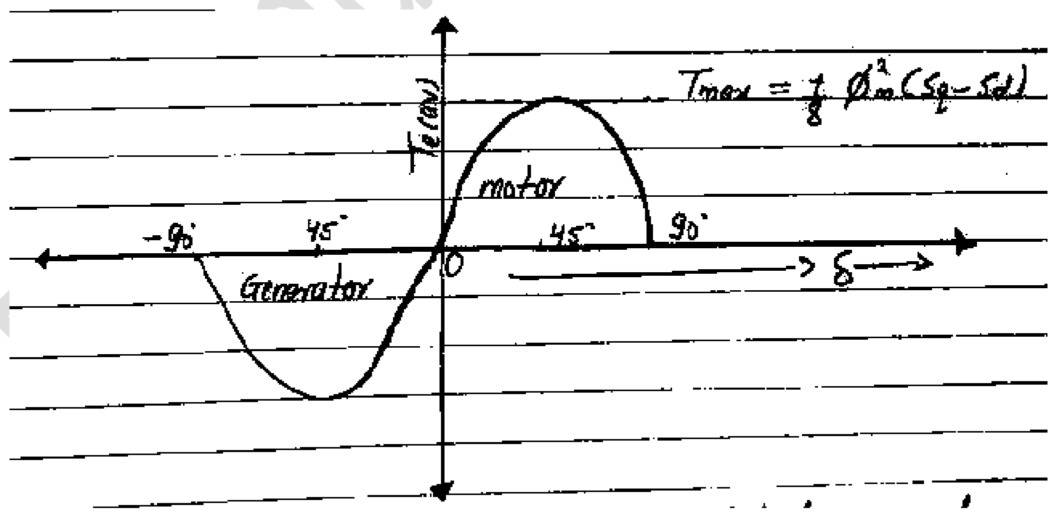
The average torque is given by,

$T_e(av) = \frac{1}{8}{\phi_m}^2(S_q - S_d)\sin 2\delta$ .... (2)

Where, $S_q = \frac{N^2}{L_q}$ and $S_d = \frac{N^2}{L_d}$

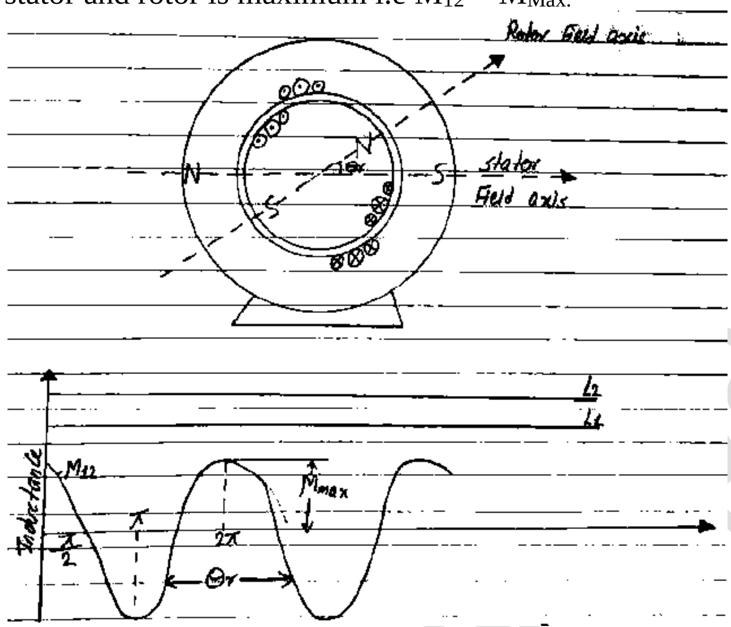
Maximum torque occurs at $\delta = 45°$

$T_e(av)Max = \frac{1}{8}{\phi_m}^2(S_q - S_d)$



1.7 TORQUE FOR SINGLE PHASE CYLINDRICAL ROTOR MACHINE

Consider a basic form of single-phase synchronous machine of smooth cylindrical rotor type. A single-phase supply is connected to the stator and a voltmeter across the rotor winding to determine the variation of stator to rotor mutual inductance, M_{12} with $\theta_r = 0$, the value of linkage flux is maximum, which is indicated by high voltmeter reading. It means that for $\theta_r = 0$, the mutual inductance between stator and rotor is maximum i.e $M_{12} = M_{Max}$.



When rotor angle is $\pi/2$, the value of flux linkage is zero, which is indicated by 0 reading on voltmeter i.e
$\theta_r = \pi/2$. The mutual inductance M_{12} =0, this is because the stator winding axis is perpendicular to the rotor winding axis. Inductance L_1 and L_2 are constant, as they are independent of movement of rotor. The mutual inductance between stator and rotor at any angle θ_r is given by
$M_{12} = M_{Max} \cos\theta_r$
Let i_1 and i_2 be the instantaneous current in the stator and rotor windings respectively, then
$W(i_1, i_2, \theta_r) = \frac{1}{2}L_1{i_1}^2 + \frac{1}{2}L_2{i_2}^2 + i_1 i_2 M_{12}$
$= \frac{1}{2}L_1{i_1}^2 + \frac{1}{2}L_2{i_2}^2 + i_1 i_2 M_{Max}\cos\theta_r$
And torque $T_e = \frac{\partial W}{\partial \theta_r}$
= 0+0- $i_1 i_2 M_{Max} \sin\theta_r$
=- $i_1 i_2 M_{Max} \sin\theta_r$ ………………… (1)
-ve sign indicates that the torque T_e acts to reduce the space angle θ_r.

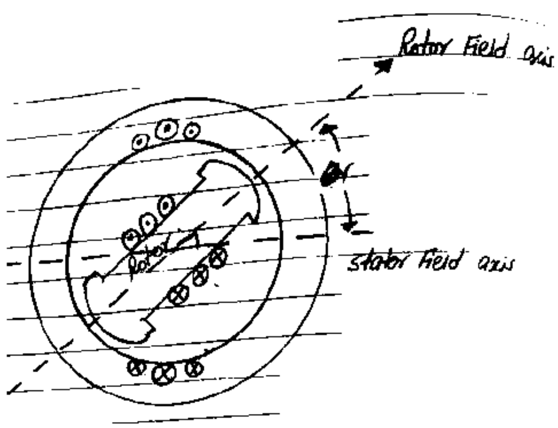
In case of synchronous machine, dc excitation is applied on the rotor winding,
$i_1 = I_{Max}\cos\omega t$ and $i_2 = I_{DC}$
$\theta_r = \omega_r t - \delta$
Where, ω = rotor angular velocity in rps
δ= rotor angular position at time t=0
$I_{Max} = \sqrt{2}I$, Maximum value of stator current.
Thus,
$T_e = -\, I_{Max}\cos\omega t.\ I_{DC}.\ M_{Max}\sin(\omega_r t - \delta)$
$= -\, I_{DC}.\ I_{Max}.\ M_{Max}.\cos\omega t.\sin(\omega_r t - \delta)$
$=-\, I_{DC}.\ I_{Max}.\ M_{Max}.\frac{1}{2}[\sin(\omega_r t - \delta + \omega t) + \sin(\omega_r t - \delta - \omega t)]$

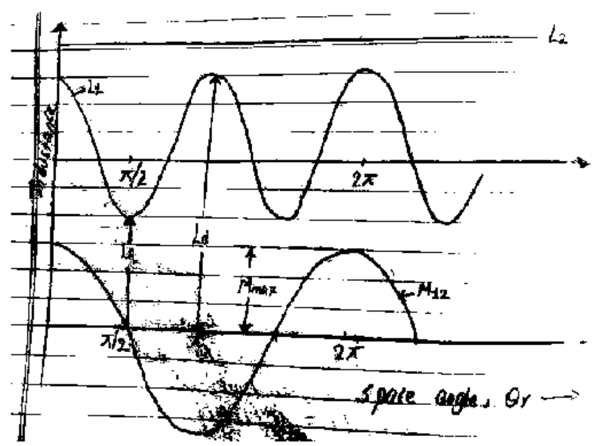
If $\omega_r \neq \omega\ then,\ T_e(\theta_r) = 0$
At synchronous speed of rotor i.e when $\omega_r = \omega$, then
$T_e = - I_{DC}.\ I_{Max}.\ M_{Max}.\ \frac{1}{2}[\sin(2\omega t - \delta) + \sin(-\delta)]$
$\therefore T_e(av) = \frac{I_{DC}.I_{Max}.M_{Max}}{2} \sin \delta$
This is the required expression for torque of a single-phase cylindrical rotor type machine.

1.8 TORQUE FOR SINGLE PHASE SALIENT POLE ROTOR MACHINE

Let L_1 and L_2 be the self-inductance of stator and rotor winding and M_{12} be the mutual inductance and θ_r be the space angle.
Rotor self-inductance L_2 = Constant
Stator self-inductance $L_1 = \frac{1}{2}(L_d + L_q) + \frac{1}{2}(L_d - L_q)\cos 2\theta_r$
And Mutual inductance $M_{12} = M_{Max}\cos\theta_r$
Magnetic field stored,

$$W(i_1, i_2, \theta_r) = \frac{1}{2} i_1{}^2 [\frac{1}{2}(L_d + L_q) + \frac{1}{2}(L_d - L_q)\cos 2\theta_r] + \frac{1}{2} L_2 i_2{}^2 + i_1 i_2 M_{Max}\cos\theta_r$$





The equation for instantaneous magnetic torque is

$T_e = \frac{\partial W}{\partial \theta_r}$
$= \frac{\partial}{\partial \theta_r}\left(\frac{1}{2} i_1{}^2 \left[\frac{1}{2}(L_d + L_q) + \frac{1}{2}(L_d - L_q)\cos 2\theta_r\right] + \frac{1}{2} L_2 i_2{}^2 + i_1 i_2 M_{Max}\cos\theta_r\right)$
$= - \frac{1}{2} i_1{}^2 [\frac{1}{2} \times 2(L_d - L_q)\sin 2\theta_r] - i_1 i_2 M_{Max}\sin\theta_r$
$= - \frac{1}{2} i_1{}^2 (L_d - L_q)\sin 2\theta_r - i_1 i_2 M_{Max}\sin\theta_r$
In salient pole machine, rotor winding carries direct current
$i_1 = I_{Max}\cos\omega t$ and $i_2 = I_{DC}$
$\theta_r = \omega_r t - \delta$
Thus,
$T_e = - \frac{1}{2} I_{max}{}^2 \cos^2 \omega t (L_d - L_q)\sin(2\omega_r t - 2\delta) - I_{Max}\cos\omega t\, I_{DC} M_{Max}\sin(\omega_r t - \delta)$
$\therefore T_e(av) = - \frac{1}{4} I_{max}{}^2 (L_d - L_q)[\frac{1}{2}\sin(-2\delta)] - \frac{I_{Max}.I_{DC}M_{Max}}{2}\sin(-\delta)$

$= \frac{1}{8} I_{max}{}^2 \left(L_d - L_q\right) \sin(2\delta) \; + \frac{I_{Max} \cdot I_{DC} M_{Max}}{2} \; \sin(\delta)$

If field current is reduced to zero, then

$\therefore T_e(av) = \frac{1}{8} I_{max}{}^2 \left(L_d - L_q\right) \sin(2\delta)$

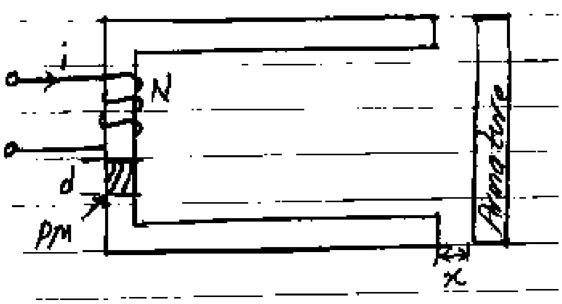
This is the required torque equation for single phase salient pole rotor machine.

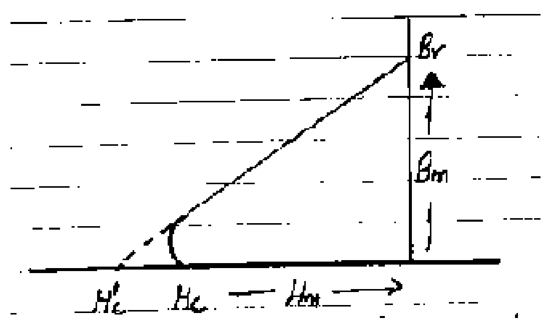
1.9 FORCE DEVELOPED ON PERMANENT MAGNET SYSTEM

Consider a permanent magnet moving armature relay. Magnetic flux density in permanent magnet
$B_m = \mu_r (H_m - H_c{}') = \mu_r H_m - B_r$

Where, μ_r= recoil permeability of the permanent magnetic materials
H_m = magnetic field intensity in the permanent magnet
B_r = residual flux density

Recoil permeability of the permanent magnetic materials, $\mu_r = \frac{B_r}{H_c{}'}$





We have,

$F_{field} = \frac{\partial W'_{field}(i,x)}{\partial x}$

Since, there is no exciting current i.e (i=0) in the system. System co-energy is independent of current and co-energy will be a function of space variable x only i.e $W'_{field}(i, x) = W'_{field}(x)$

Co-energy is given by, $W'_{field} = \int_0^i \Psi \, di$

The current is assumed to be adjusted to a value I causing the core flux to reduce to zero and the original state is then attained by imaging the current I to reduce to zero. Thus,

$W'_{field} = \int_I^0 \Psi \, di$

For any value of current, Ni = H_md + H_g 2x …………….. (1)

If A is uniform cross-sectional area,

$B_m A = B_g A$

$B_m = \mu_r H_m = B_g = \mu_0 H_g$

Substituting $H_g = \frac{B_m}{\mu_o}$ in equation (1)

Ni = H_md + $\frac{B_m}{\mu_o}$ 2x ………………. (2)

Replacing $H_m = \frac{B_m + B_r}{\mu_r}$ in equation (2)

Ni = $\left(\frac{B_m + B_r}{\mu_r}\right)$d + $\frac{B_m}{\mu_o}$ 2x

Or, Ni = $B_m \left(\frac{d}{\mu_r} + \frac{2x}{\mu_0}\right)$d + $\frac{B_r}{\mu_r}$ d

Or, Ni = $B_m \left(\frac{d}{\mu_r} + \frac{2x}{\mu_0}\right)$d + $H_c{}'$ d

$\therefore B_m = \frac{\mu_r(\text{Ni}-H_c{}'\text{ d})}{d+2\frac{\mu_r}{\mu_0}x}$ …………………. (3)

Flux linkages of the fictitious coil, $\Psi = \varphi \times \text{N} = B_m \times A \times \text{N}$

= $\frac{\mu_r(\text{Ni}-H_c{}'\text{ d})}{d+2\frac{\mu_r}{\mu_0}x} \times A \times \text{N}$ ………….. (4)

For flux linkage and flux to be zero, i = $\frac{H_c{}'\text{ d}}{N}$ ………………… (5)

Thus,

$W'_{field} = \int_i^0 \frac{\mu_r AN(\text{Ni}-H_c{}'\text{ d})}{d+2\frac{\mu_r}{\mu_0}x}\, di$

= $\frac{\mu_r A(H_c{}')^2\, d^2)}{2(d+2\frac{\mu_r}{\mu_0}x)}$

The force on armature is given as

$F_{field} = \frac{\mu_r{}^2(H_c{}')^2\, d^2 A}{\mu_0(d+2\frac{\mu_r}{\mu_0}x)^2}$

Also, $\mu_r H_c{}' = B_r$

$\therefore F = \frac{AB_r{}^2}{\mu_0[1+2\frac{\mu_r}{\mu_0}\left(\frac{x}{d}\right)]^2}$

Numerical

Determine the approximate maximum torque that a motor can develop when it is connected to a 230 V, 50 Hz supply. It is given that the exciting winding has 1500 turns, the inductance of the winding in direct axis position is 0.8 H and when the rotor is in quadrature axis position: inductance is 0.4 H.

Sol^n

f = 50 Hz

V = 230 V

N = 1500

When the rotor is in direct-axis position, L_d = 0.8 H

" " " " quadrature " " , L_q = 0.4 H

$$\phi_m = \frac{V}{4.44 \times f \times N}$$

$$= \frac{230}{4.44 \times 50 \times 1500}$$

$$\phi = 0.00069 \text{ wb}$$

$$S_q = \frac{N^2}{L_q}$$

$$= \frac{(1500)^2}{0.4}$$

$$\therefore S_q = 5.625 \times 10^6 \text{ AT/wb}$$

Since, $L = \dfrac{N^2}{\text{reluctance } (S)}$

and,

$$S_d = \frac{N^2}{L_d}$$

$$= \frac{(1500)^2}{0.8}$$

$$\therefore S_d = 2.812 \times 10^6 \text{ AT/wb}$$

The value of maximum torque that the motor can develop is,

$$T_e = \frac{1}{8} \phi_m^2 (S_q - S_d)$$

$$= \frac{1}{8} \times (0.00069)^2 \times 10^6 \times (5.625 - 2.8125)$$

$$\therefore T_e = 0.1677 \text{ Nm}$$

Hence, maximum torque developed is 0.1677 Nm.

Determine the magnitude of force in the system with permanent magnet for air gaps of (i) $x = 0$ and (ii) $x = 6$ mm. The other data are as: $B_r = 0.95T$, $H_c' = 7.2 \times 10^5$ AT/m

$d = 30$ mm, $A = 500$ mm^2, $\mu_0 = 4\pi \times 10^{-7}$ H/m

Sol^n.

$$\mu_r = \frac{B_r}{H_c'} = \frac{0.95}{7.2 \times 10^5} = 13.2 \times 10^{-7}$$

For $x = 0$,

$$\text{magnetic field, } F_f = -\frac{AB_r^2}{\mu_0\left[1 + 2\frac{\mu_r}{\mu_0}\left(\frac{x}{d}\right)\right]^2}$$

$$= -\frac{500 \times 10^{-6} \times (0.95)^2}{4\pi \times 10^{-7}}$$

$$= -359 \text{ N.}$$

For $x = 6$ mm,

$$F_f = -\frac{500 \times 10^{-6} \times (0.95)^2}{4\pi \times 10^{-7}\left[1 + \frac{2 \times 13.2 \times 10^{-7}}{4\pi \times 10^{-7}} \times \left(\frac{6}{30}\right)\right]^2}$$

$$= -178 \text{ N.}$$

-ve sign indicates that the force is restoring in nature, ie, it acts in a direction to reduce the air gap.

In a doubly-excited rotary machine, inductance coefficients are $L_{11} = (1.1 + 0.4\cos 2\theta)$, $L_{22} = (0.03 + 0.005\cos 2\theta)$, $L_{12} = 0.2\cos\theta$.
The exciting currents are $i_1 = 8$ A & $i_2 = 50$ A. Obtain the torque. Derive the expression.

$$L_{11} = 1.1 + 0.4\cos 2\theta$$
$$L_{22} = 0.03 + 0.005\cos 2\theta$$
$$L_{12} = 0.2\cos\theta$$

Now, $i_1 = 8A$ & $i_2 = 50A$.

$$T = \frac{1}{2} i_1^2 \frac{dL_{11}}{d\theta} + \frac{1}{2} i_2^2 \frac{dL_{22}}{d\theta} + i_1 i_2 \cdot \frac{dL_{12}}{d\theta}$$

$$= \frac{1}{2} i_1^2 \frac{d}{d\theta}(1.1 + 0.4\cos 2\theta) + \frac{1}{2} i_2^2 \frac{d}{d\theta}(0.03 + 0.005\cos 2\theta) + i_1 \cdot i_2 \cdot \frac{d}{d\theta}(0.2\cos\theta)$$

$$= \frac{1}{2} i_1^2 (0.4 \times 2 \times -\sin 2\theta) + \frac{1}{2} i_2^2 \times -(0.005 \times 2 \times \sin 2\theta) - i_1 \cdot i_2 \times 0.2\sin\theta$$

$$= -0.4 i_1^2 \sin 2\theta - i_2^2 \times 0.005 \times \sin 2\theta - i_1 \cdot i_2 \cdot 0.2\sin\theta$$

For $i_1 = 8A$
$i_2 = 50A$,

$$T = -0.4 \times 8^2 \times \sin 2\theta - 50^2 \times 0.005 \times \sin 2\theta - 0.2 \times 8 \times 50 \times \sin\theta$$

$$= -25.6\sin 2\theta - 12.5\sin 2\theta - 80\sin\theta$$

$$\therefore T = -38.1\sin 2\theta - 80\sin\theta \quad Nm$$

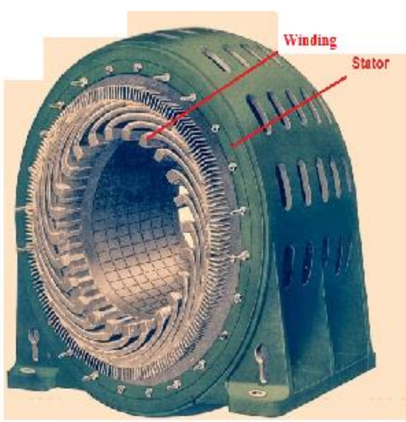
Hence, torque relation is, $-38.1\sin 2\theta - 80\sin\theta$ Nm.

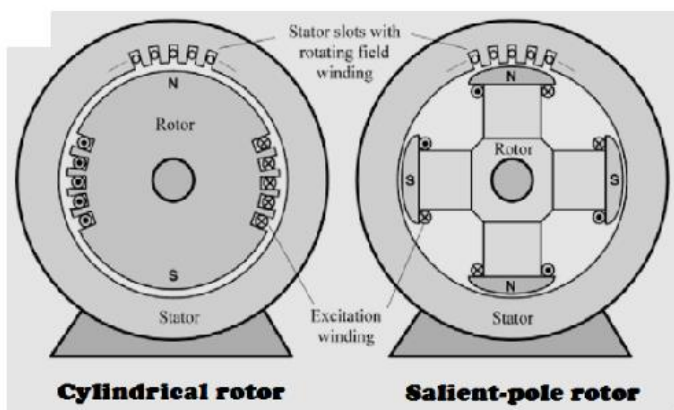
UNIT 2: SYNCHRONOUS GENERATOR

Synchronous Machines are the rotating machine that rotates at a speed fixed by supply frequency and number of poles. Thus, the speed of machines is independent of load. A Synchronous machine rotates at a constant speed called synchronous speed. Synchronous Generators (alternators) are rotating machines that rotate at synchronous speed and convert mechanical power from prime mover to ac electric power at a specific voltage and frequency.
Synchronous generators are usually three phase type because of several advantages of 3-phase generation, transmission and distribution. Most of the generating stations use Synchronous generators so they are the primary source or world's electric power system.
As we studied earlier in D.C. generators, the field poles are stationary and the armature conductors (where voltage is produced) rotates. But in case of Synchronous generators magnetic fields are rotating and stationary armature conductors produce voltage.

2.1 CONSTRUCTION DETAILS

Similar to other rotating machines, an alternator consists of two main parts, Rotor (Rotating part that produces the main field flux) and stator (stationary part where voltage is generated).
Stator: It is made up of cast iron for small size machines and of welded steel type for larger machines. In order to reduce the hysteresis and eddy current loss, the stator core is assembled with high grade silicon content. A 3-phase armature winding is put in slots on the inner periphery as shown in fig. The winding is star connected. In armature winding voltage is induced. A winding is formed is formed by connecting several coils in series. A coil is formed by connecting several turns in series. A turn consists of 2 active conductors (where emf is induced) connected end by end by inactive conductors (where emf is not induced).





Rotor: Rotors are the rotating part of a machine with numbers of magnetic poles excited by a dc source (110v to 400V) from exciter. They are of 2 types
1**.Salient-pole rotor:** Term salient means *'projecting'*. Thus, they are the rotor that consist of poles projecting out from the surface of rotor core. Since rotor is subjected to changing magnetic field, it is made up of steel laminations to reduce eddy current loss.
Construction of this type of rotor is easier and cheaper than cylindrical poles. They are used in those alternators that are driven at low speed prime mover such as water turbine, diesel engine etc. They have larger number of poles, has comparatively larger diameter and short axial length. They have concentrated winding on poles. They have non uniform air gap. Their pole faces are so shaped

that the radial air gap length increases from pole center to pole tips so that flux distribution in air gap is sinusoidal in nature.

2.Non-Salient-pole rotor (Cylindrical rotor): They are the rotor so constructed that it forms a smooth cylinder. The construction is such that no physical poles to be seen as in salient pole rotor. Cylindrical rotors are made from solid forgings of high-grade nickel-chrome-molybdenum steel.

Construction is more compact. They are generally used in generators driven by high speed prime mover like steam turbine, gas- turbine. Generally, they have 2 or 4 poles on rotor for greater mechanical strength and to permit accurate dynamic balancing. Distributed winding is preferred. They have uniform air gap so less noisy.

Exciter: Here exciter is a self-excited dc generator mounted on shaft of the alternator This will produce dc current required to magnetize magnetic poles of rotor. The dc current generated by exciter is fed to field winding of alternator through slip ring and carbon brush arrangement as shown

2.2 OPERATING PRINCIPLE

Like d.c. generator, Synchronous generators also operates on principle of electromagnetic induction. In case of an alternator field poles are rotating and armature conductor are stationary. When the shaft of the machine is driven by prime mover at synchronous speed (Ns), the exciter mounted on shaft, builds up its voltage by self-excitation and supply dc to the rotating field winding. Thus, the field winding produces rotating magnetic field so that the produced magnetic flux cuts the stationary three-phase stator winding and hence according to Electro Magnetic Induction, three-phase emf is induced in stator armature winding. In actual generating station, speed governor is used to keep speed of machine constant at any load condition so that frequency of generated emf remains constant.

Advantage of Rotating Magnetic Field and Stationary Armature Over Rotating Armature type

- A stationary armature is more easily insulated for high voltage (as 33kv) in comparison to rotating armature winding.
- Cooling of armature winding in stator is easier because the armature can be made larger to provide a number of cooling ducts.
- Since armature winding is more complex than field winding, it can be constructed more easily on stationary structure.
- The voltage level of emf generated in armature is very high, if it was rotating brushes and slip rings were required so, sparking problem would have occurred. If it is stationary sparking doesn’t arise.
- The field winding is comparatively lighter in weight and can be run in higher speed producing higher voltage as output.
- Armature windings being stationary are not subjected to vibration and centrifugal force so, no chance of loosening.
- Rotating field winding is d.c. so only 2 slip rings are required. At least 3 slip rings would be required for 3- phase rotating armature winding.

2.3 EMF EQUATION

Let,

P=Total numbers of poles.

ϕ=Useful flux per pole in webers (Wb)

Zp=Total numbers of conductors in series per phase

Tp=Total numbers of coil or turns per phase (Zp=2Tp)

N=speed of rotation in rpm

f=Frequency of generated voltage (Hz)

Since, flux per pole is ϕ, flux cut by each conductor in one revolution is Pϕ.

Flux cut by each conductor per second=$\frac{\mathrm{P\varphi N}}{60}$

Then from law of Electro Magnetic Induction, average emf induced in conductor= $\frac{\mathrm{P\varphi N}}{60}$

We know, N=120f/P $\Longrightarrow \frac{PN}{60} = 2f$

So,

Average emf induced in each conductor=2fϕ

Hence, average emf induced per phase=2fϕZp =4fϕZp

We have, form factor= $\frac{RMS\ value}{Average\ Value}$ = 1.1

RMS value of emf induced per phase=4.44fϕTp

Again, Considering effect of winding factor, RMS value of emf induced = 4.44fϕTp.Kw

Erms =4.44fϕTp.Kp.Kd

Where, Kp=pitch factor and Kd= Distribution factor

Beside f, ϕ, Tp ,there are other factors which affects emf induced i.e. pitch factor and distribution factor of stator winding.

2.4 WINDING FACTOR

a) Pitch Factor(K_p) or coil span Factor

The distance between 2 sides of a coil is known as coil span or pitch. A coil having a span equal to 180° electrical is known as Full-pitch coil as shown in fig a. A coil having a span less than 180° electrical is known as Short-pitch coil/chorded coil/fractional pitch coil as shown in fig(b).

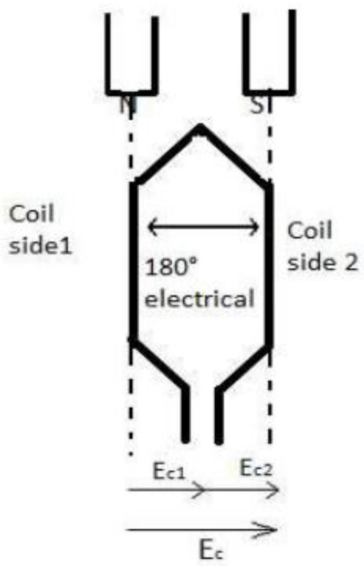
Pitch factor (Kp) is defined as ratio of emf induced when coil is short pitch to emf induced when coil is full pitched.

Let E_{c1} and E_{c2} be the voltage generated in coil sides and Ec be the resultant coil voltage. Then

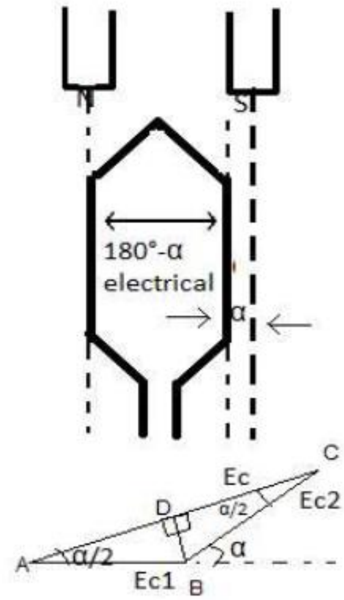
$E_c=E_{c1}+E_{c2}$ let, $|E_{c1}|=|E_{c2}|=|E|$

For full pitched coil, Ec and Ec2 are in phase, the resultant voltage Ec is equal to its arithmetic sum.

$E_c=E_{c1}+E_{c2}=2E$



Figa Full Pitched Coil



Figb Short-Pitched coil

If the coil is short-pitched generated voltage by coil sides are not in phase. Resultant voltage is equal to phasor sum of E_{c1} and E_{c2}. If the coil span is reduced by angle α electrical degree, the coil span is 180-α electrical degree. From fig b phasor diagram,

Ec=AC =2AD =2Ecos(α/2)

$$K_P = \frac{Vector\ sum\ of\ enf\ induced\ when\ the\ coil\ is\ short\ pitched}{Arithmetic\ sum\ of\ emf\ induced\ when\ the\ coil\ is\ full\ pitched} = \frac{2\text{Ecos}(\alpha/2)}{2E} = \cos(\alpha/2)$$

Note: Value of Kp is 1 for full pitch coil as α=0 and for short pitch coil Kp is always less than 1 as value of cosine never exceed 1.

Following are some advantage of short pitch coil over full pitch coil.

- Saving of the conductor material
- Reduction in distorting harmonics, and thus the waveform of the generated voltage is improved and makes wave form approaching sine wave.

Distribution Factor or Breadth Factor (Kd) :

In actual machine, the stator windings aren't concentrated in a slot, the windings are uniformly distributed over many numbers of slots to form polar group under each pole.

The distribution factor is defined as the ratio of actual voltage obtained to possible voltage if all coils of a
polar group were concentrated in a single slot.

$$K_d = \frac{Phasor\ sum\ of\ coil\ voltage\ per\ phase}{Aithmetic\ \ sum\ of\ coil\ voltage\ per\ phase}$$

Let m= slots per pole per phase = $\frac{Slots}{Pole*Phase}$

β=angular displacement between adjacent slot in degree = $\frac{180*Poles}{Phase}$

Thus, one phase of the winding consists of coil arrange in m consecutive slots. Voltages E_{c1} E_{c2}, E_{c3}…… are individual coil voltages. Each coil voltages Ec will be out of phase with next coil voltage by slot pitch β. Figure below shows the voltage polygon in the three coil of a group (m=4). Voltages Ec1,Ec2,Ec3 are represented by AB,BC,CD respectively.

Each of the phasor is a chord of a circle with center 'o' and subtend and an angle β at o. The phasor sum
AD, representing the resultant winding voltage, subtends an angle mβ at the center.

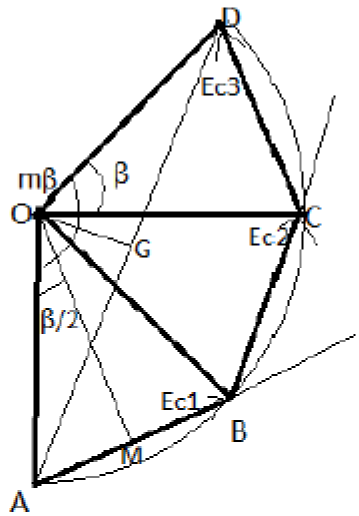


Fig. Voltage polygon

Arithmetic sum of individual coil voltages= mEc =mAB = m(2AM) = 2m.OA.sinA ÔM =2m.OA.sinβ/2

Phasor sum of individual coil voltages= AF =2AG =2OA sinA ÔG =2.OA.sin(mβ/2)

$K_d = \frac{2.OA.\sin(m\beta/2)}{2m.OA.\sin\beta/2} = \frac{\sin(m\beta/2)}{m\sin\beta/2}$

It should be noted that Kd for a given number of phases is dependent only on the number of distributed slots under given pole. It is independent of type of winding, lap or wave, or numbers of turns per coil e.t.c. As number of slots per pole increase, Kd decrease.

2.5 ARMATURE REACTION

When current flows through armature winding of an alternator, resulting mmf produces flux. The armature flux reacts with the main pole flux, causing the resulting flux to become either less than or greater than original main field flux. The effect of armature (stator) flux on the flux produced by the rotor field pole is called armature reaction.

Nature of armature reaction depends on magnitude of the current flowing through the armature winding and on power factor of load. For simplicity we shall consider the following 3 extreme condition.

At Unity Power Factor:

If load is purely resistive, there will be no phase difference between terminal voltage(V) and armature current. Since nature of flux will be in phase with armature current, the magnetic flux produced by 3-phase windings will have similar waveform as that of terminal voltage as shown in fig a and b.

$\Phi_R = \varphi_m \sin\omega t$

$\Phi_y = \varphi_m \sin(\omega t + 120°)$

$\Phi_B = \varphi_m \sin(\omega t + 240°)$

When magnet rotates 90° from its zero position

$\Phi_R = \varphi_m$

$\varphi_y = \varphi_m \sin(90 + 120°) = -0.5\varphi_m$

$\Phi_B = \varphi_m \sin(\omega t + 240°) = -0.5\varphi_m$

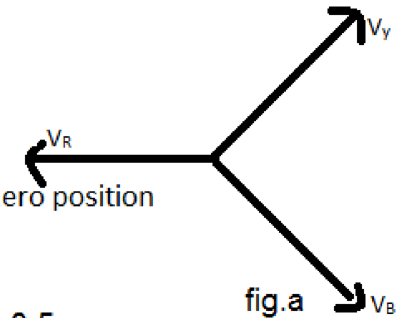


fig.a

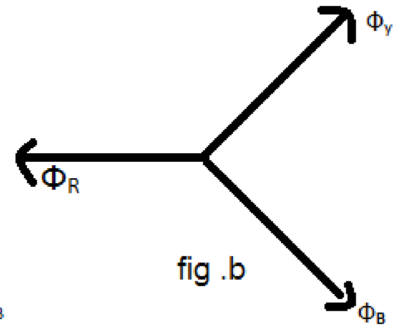
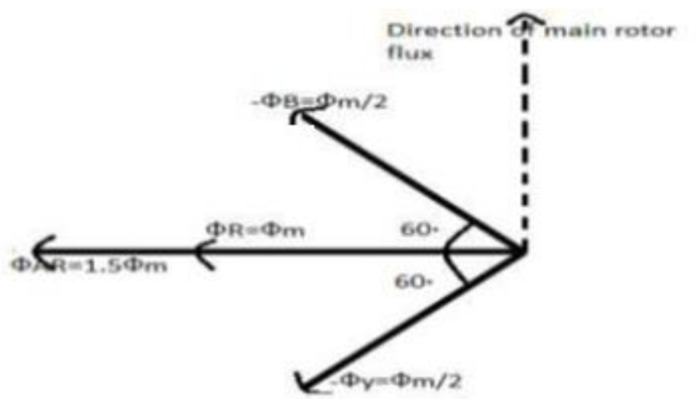


fig .b



Then net magnetic flux set by armature is given by resultant as shown in figure

$$\varphi_{AR} = \varphi_m + \sqrt{(\frac{\varphi_m}{2})^2 + (\frac{\varphi_m}{2})^2 + 2\frac{\varphi_m}{2}\frac{\varphi_m}{2}cos120} = \varphi_m + \frac{\varphi_m}{2} = 1.5\varphi_m$$

Hence, resulting magnetic flux $\phi_{AR} = 1.5\phi_m$ lags by 90° with direction of main flux. But rotates with the same speed in same direction so, ϕ_{AR} try to distort main flux.

At lagging Power Factor:

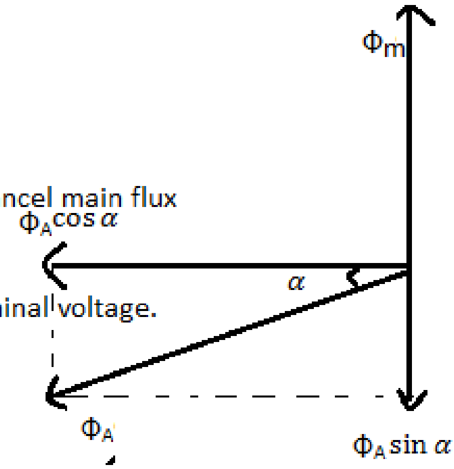
If the load is inductive (say load current lags the voltage V by an angle α) then the waveforms of armature flux will also lag by an angle of α with respect to that in case of unity power factor. Hence, the resultant armature flux (ϕ_{AR}) lags themain flux (ϕ_m) by an angle (90+α) as shown.

The armature flux has two component as:

i) $\varphi_A \cos\alpha$=cross magnetizing component

ii) $\varphi_A \sin\alpha$=In opposite direction of main flux which tries to cancel main flux and is called demagnetizing component.

As this effect causes reduction in main flux and then the terminal voltage.



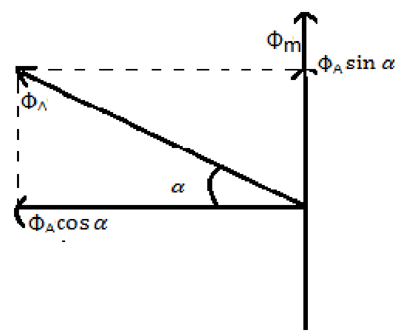
At leading Power Factor:

If the load is capactive(say load current leads the voltage V by an angle α) then the waveforms of armature flux will also lead by an angle of α with respect to that in case of unity power factor. Hence, the resultant armature flux (ϕ_{AR}) lags the main flux (ϕ_m) by an angle (90-α) as shown.

The resultant flux has two component as:

i) $\Phi_A \cos\alpha$=cross magnetizing component

ii) $\varphi_A \sin\alpha$=In direction of main flux which helps in addition of main flux and is called magnetizing component and effect is called magnetizing effect.



Due to this effect adds the flux to the main flux, greater emf gets induced in the armature and there is increase in terminal voltage.

2.6 ALTERNATOR ON LOAD AND ITS PHASOR DIAGRAM

Let V=terminal voltage across load per phase

E=emf induced per phase in stator winding

The equivalent circuit is

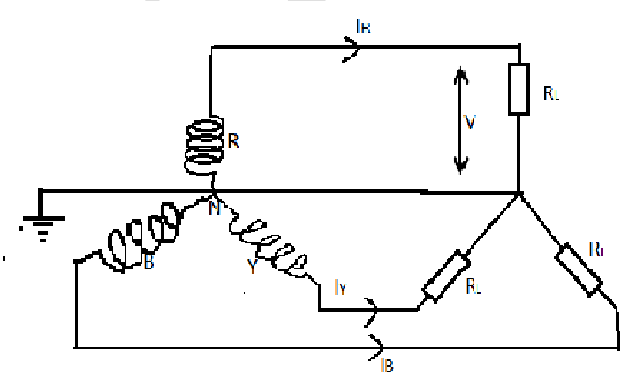


fig.3 phase alternator on 3 phase load

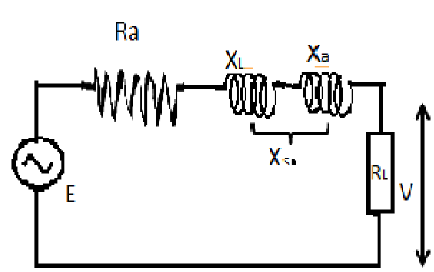


fig. equivalent circuit diagram

Where,
R_a=armature winding resistance per phase
X_L=leakage reactance of armature winding per phase
X_a=fictitious reactance
X_s=synchronous reactance
When generator is loaded, current will flow through stator winding and some voltage drop will take place in the stator winding. Therefore, the terminal voltage across the load will not be equal to emf induced in rotor winding. This is due to following reasons.
i) Voltage drop due to armature effective resistance
ii) Voltage drop due to armature leakage reactance
iii) Voltage drop due to armature reactance
The terminal voltage is given by

$$\vec{V} = \vec{E} - \overrightarrow{IaRa} - j\overrightarrow{IaX_l} - j\overrightarrow{IaXa} = \vec{E} - \overrightarrow{IaRa} - j\overrightarrow{IaX_s}$$

Phasor Diagram:

For lagging power factor: Figure below shows the phasor diagram of lagging load. Power factor cosϕ is lagging. Here terminal voltage V is taken as reference phasor along OA, Such that OA=V. I_a lags V by an angle ϕ. The voltage drop on armature resistance is I_aRa, represented by phasor AB. Voltage drop on synchronous reactance is given by BC. BC leads I_a by 90°so drawn perpendicular to I_a.

The magnitude of E can be found from the phasor.

$$E^2=(V+IaRacos\phi+I_aX_ssin\phi)^2+(IaX_scos\phi-I_aRasin\phi)^2$$

$$E= \sqrt{(V + IaRacos\varphi + I_aX_ssin\varphi)2 + (IaX_scos\varphi - I_aRasin\varphi}$$

Also, here θ=ϕ+δ where δ=load angle or torque angle.

$$Tan\theta=\frac{IaXscos\varphi-IaRasin\varphi}{V+IaRacos\varphi+IaXssin\varphi}$$

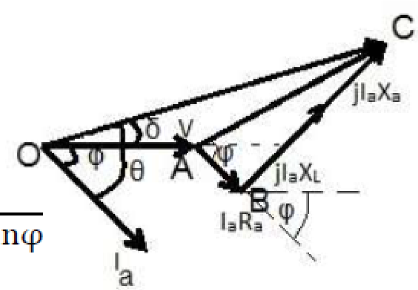


Fig. lagging power factor

For leading power factor: The phasor diagram for leading power factor shown below:

$$OC^2=E^2=(V+IaRacos\phi-I_aX_ssin\phi)^2+(IaX_scos\phi+I_aRasin\phi)^2$$

$$\therefore E = \sqrt{(V + IaRacos\varphi - IaXssin\varphi)2 + (IaXscos\varphi + IaRasin\varphi)2}$$

From phasor, θ=ϕ-δ

$$Tan\theta=\frac{(IaXscos\varphi+IaRasin\varphi)}{(V+IaRacos\varphi-IaXssin\varphi)}$$

$$\theta=\tan^{-1}(\frac{(IaXscos\varphi+IaRasin\varphi)}{(V+IaRacos\varphi-IaXssin\varphi)})$$

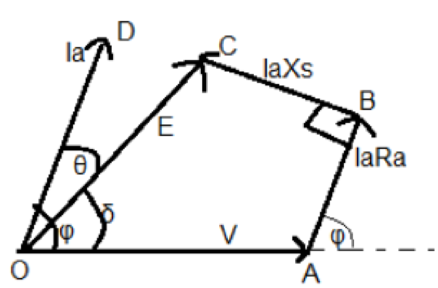


Fig. Leading power factor

For unity power factor: The phasor diagram for unity power factor is shown in fig. From, right angle triangle OBC value of E can be determined as follow:

$OC^2=OB^2+BC^2$

$E=\sqrt{(V+IaRa)2+(IaXs)2}$

$\tan\theta=\frac{IaXs}{(V+IaRa)}$

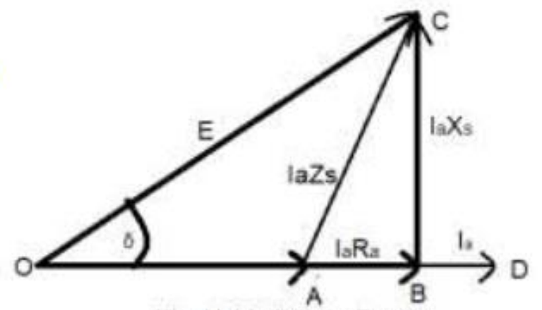


Fig. Unitiy Power Factor

The value of load angle varies with load and is a measure of air gap power developed in the machine.

Effect of variation of power factor on terminal voltage: Thus, at lagging power factor the terminal voltage falls from that on leading power factor mainly owing to decrease in generated emf. On changing the power factor rom leading to lagging one, if the load current, excitation and speed remain constant the terminal voltage falls.

- When the power factor is leading the effect of armature flux is to help the main flux, hence generates more emf
- When power factor is lagging, the effect of armature flux is to oppose the main flux, hence to generate less emf.

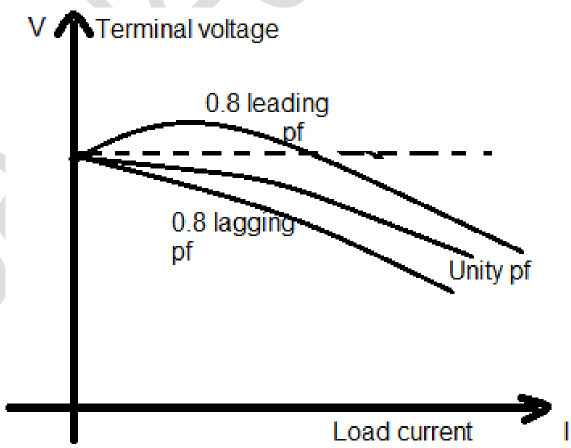


fig.Variation of terminal voltage due to pf variation

2.7 VOLTAGE REGULATION

The voltage regulation of a synchronous generator is the rise in voltage at the terminals when the load is reduced from full rated value to zero, speed and field current being constant. The terminal voltage of an alternator changes from no-load to full load due to voltage drop in internal resistance and reactance of stator winding. The magnitude and nature of voltage drop depends on not only upon current but also on power factor of load as shown.

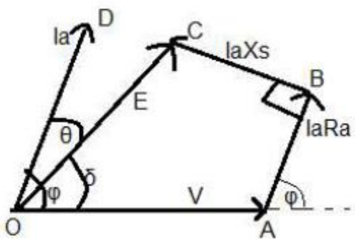


Fig. Leading power factor

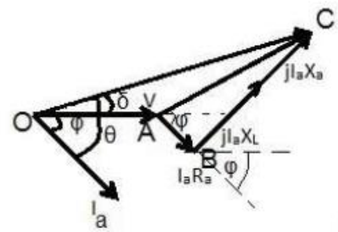


Fig. lagging power factor

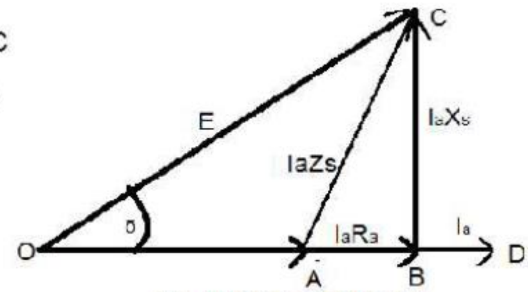


Fig. Unitiy Power Factor

$$V.R = \frac{no\ load\ voltage - full\ load\ terminal\ voltage}{full\ load\ terminal\ voltage}$$

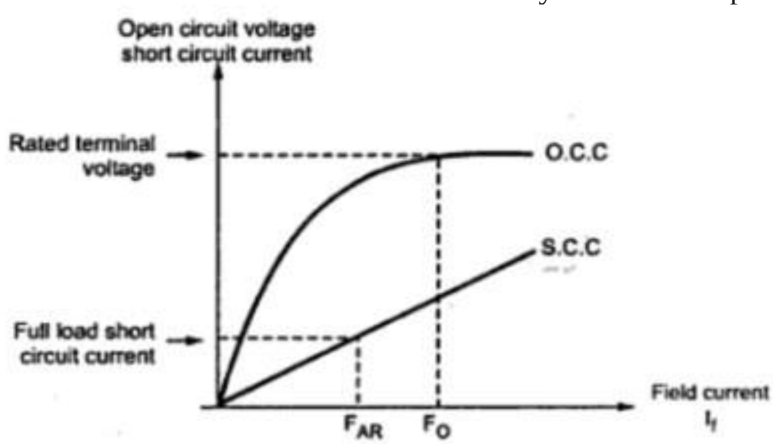
$$\therefore V.R = \frac{E-V}{V} * 100\%$$

2.7.1 Synchronous Impedance Method

The Synchronous Impedance Method or Emf Method is based on the concept of replacing the effect of armature reaction by an imaginary reactance. The method requires following data to calculate the regulation.

The open -circuit characteristic (O.C.C):

- The O.C.C is a plot of the armature terminal voltage as a function of field current with a symmetrical three phase short-circuit applied across the armature terminals with the machine running at rated speed.
- At any value of field current, if E is the open circuit voltage and Isc is the short circuit current then for this value of excitation Zs = E/Sic
- At higher values of field current, saturation increases and the synchronous impedance decreases.
- The value of Zs calculated for the unsaturated region.
- The O.C.C is called the unsaturated value of the synchronous impedance.



The short-circuit characteristic (S.C.C)

- The S.C.C is a plot of short-circuit armature current versus the field current.

- The current range of the instrument should be about 25-50 % more than the full load current of the alternator.
- Starting with zero field current, increase the field current gradually and cautiously till rated current flows in the armature.
- The speed of the set in this test also is tom be maintained at the rated speed of the alternator.

Resistance of the armature winding.

- Measure the D.C. resistance of the armature circuit of the alternator.
- The effective a.c resistance may be taken to be 1.2 times the D.C. resistance.

Regulation Calculation

- From O.C.C. and S.C.C., Z_s can be determined for any load condition.
- The armature resistance per phase can be measured by different methods.
- One of the methods is applying d.c. known voltage across the two terminals and measuring current. So, value of R_a per phase is known.

$$\mathbf{Xs = \sqrt{Zs^2 - Ra^2}}$$

So synchronous reactance per phase can be determined.

- No load induced e.m.f. per phase, E_{ph} can be determined by the mathematical expression derived earlier.

$$\text{Eph} = \sqrt{\text{I}(\text{V}\cos\varphi + \text{IRa})^2 + \text{I}(\text{V}\sin\varphi + \text{IXs})^2}$$

where V_{ph} = Phase value of rated voltage I_a = Phase value of current depending on the load condition cosΦ = p.f. of load

- Positive sign for lagging power factor while negative sign for leading power factor, R_a and X_s values are known from the various tests performed.

The regulation then can be determined by using formula,

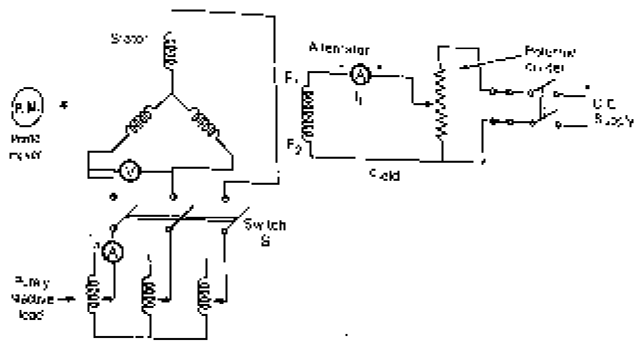
$$\text{Percentage Voltage Regulation} = \left(\frac{|E_0| - |V|}{|V|}\right)$$

2.7.2 Zero Power Factor Method

This Zero power factor (ZPF) method is used to determine the voltage regulation of synchronous generator or alternator. This method is also called Potier method. In the operation of an alternator, the armature resistance drop IRa and armature leakage reactance drop IXL are actually emf quantities while the armature reaction is basically MMF quantity. In the synchronous Impedance, all the quantities are treated as EMF quantities as against this in MMF method all are treated as MMF quantities. To determine armature leakage reactance and armature reaction MMF separately two tests are performed on the alternator. The two tests are

1. Open circuit test
2. Zero power factor test

Open circuit test: The below is the block diagram to perform open circuit test on the alternator.



Open circuit test is done step by step from the following points,

1. The switch S is opened.

2. The alternator is made to rotate using prime mover at synchronous speed and same speed is maintained constant throughout the test.

3. The excitation value is changed using a potential divider, from zero up to the rated value in a definite number of steps. The open circuit EMF is measured with the help of voltmeter. The readings are tabulated.

4. A graph of If and (Voc)ph i.e. field current and open circuit voltage per phase is plotted to some scale. This is open circuit characteristics.

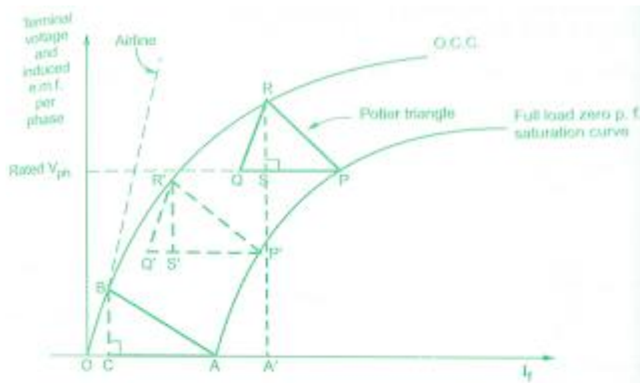
Zero power factor test: To conduct *zero power factor test*, the switch S is kept closed. Due to this, a purely inductive load gets connected to an alternator through an ammeter. A purely inductive load has a power factor of cos 90° i.e. zero lagging hence the test is called zero power factor test.

The machine speed is maintained constant at its synchronous value. The load current delivered by an alternator to purely inductive load is maintained constant at its rated full load value by varying excitation and by adjusting variable inductance of the inductive bad. Note that, due to purely inductive load, an alternator will always operate at zero power factor lagging.

The below is the graph of terminal voltage against excitation when delivering full load zero power factor current. One point for this curve is zero terminal voltage (short circuit condition) and the field current required to deliver full load short circuit armature current. While other point field current required to obtain rated terminal voltage while delivering rated full load armature current. With the help of these two points, the zero-power factor saturation curve can be obtained as

1. Plot open circuit characteristics on a graph paper as shown in the below figure.

2. Plot the excitation corresponding to zero terminal voltage i.e. short circuit full zero power factor armature current. This point is shown as A in the below figure which the x-axis. Another point is the rated voltage when the alternator is delivering full current at zero p.f. lagging. This point is P as shown in the below figure.



3. Draw the tangent to O.C.C. through origin which is line OB as shown dotted in below figure. This is called the airline.

4. Draw the horizontal line PQ parallel and equal to OA.

5. From the point, Q draw the line parallel to the airline which intersects O.C.C. at point R. Join RQ and join PR. The triangle PQR is called Potier triangle.

6. From point R, drop a perpendicular on PQ to meet at point S.

7. The zero-power factor full load saturation curve is now be constructed by moving triangle PQR so that R remains always on OCC and line PQ always remains horizontal. The dotted triangle is shown in the above figure. It must be noted that the Potier triangle once obtained is constant for a given armature current and hence can be transferred as it is.

8.Though point A, draw a line parallel to PR meeting OCC at point B. From B, draw a perpendicular on OA to meet it at point C. Triangles OAB and PQR are similar triangles.

9. The perpendicular RS gives the voltage drop due to the armature leakage reactance i.e. IXL

10. The length PS gives field current necessary to overcome the demagnetising effect of armature reaction at full load.

11. The length SQ represents field current required to induce an EMF for balancing leakage reactance drop RS. These values can be obtained from any Potier triangle such as OAB, PQR and so on.

So armature leakage reactance can be obtained as,

$$l\,(RS) = l\,(BC) = (I_{aph})_{F.L.} \times X_{L\,ph}$$

$$X_{L\,ph} = \frac{l\,(RS) \text{ or } l(BC)}{(I_{aph})\ F.L.} \quad \Omega$$

Use of Potier reactance to determine regulation of alternator:

To determine regulation using Potier reactance, draw the phasor diagram using the following procedure:

1. Draw the rated terminal voltage Vph as a reference phasor. Depending upon at which power factor (cos Φ) the regulation is to be predicted, draw the Current phasor Iph lagging or leading Vph by angle Φ.

2. Draw Iph Raph voltage drop to Vph which is in phase with Iph. While the voltage drop Iph XLph is to be drawn perpendicular to Iph Raph, vector but leading Iph Raph at the extremity of Vph.

3. The Rph is to be measured separately by passing a d.c current and measuring the voltage across armature winding. While XLph is *Potier reactance* obtained by Potier method.

Phasor sum of Vph rated, Iph Raph and Iph XLph gives the e.m.f. which is say E1ph.

E1ph = Vph +Iph Raph +Iph XLph

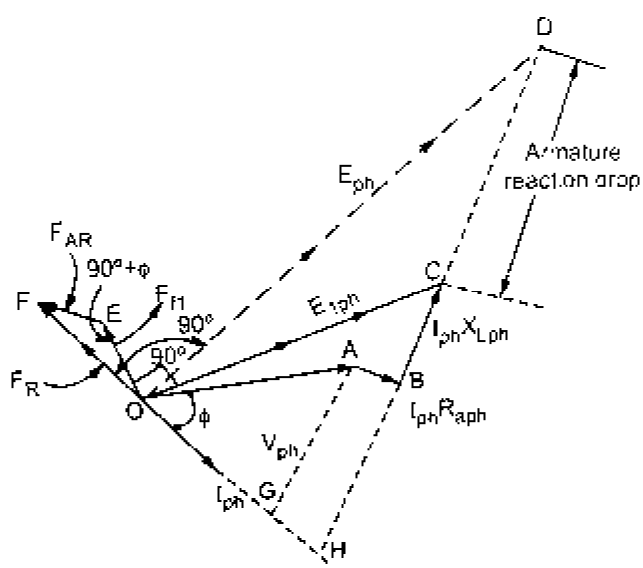
4. Obtain the excitation corresponding to E1ph from OCC which is drawn. Let this excitation be Ff1. This is excitation required for inducing EMF which does not consider the effect of armature reaction.

5. The field current required to balance armature reaction can be obtained from Potier triangle method, which is say FAR. FAR = I (PS) = I (AC)

6. The total excitation required is the vector sum of the Ff1 and FAR. This can be obtained exactly similar to the procedure used in MMF method.

7. Draw vector Ff1 to some scale, leading E1ph by 90°. Add FAR to Ff1 by drawing vector FAR in phase opposition to Iph. The total excitation to be supplied by field is given by FR.

The complete phasor diagram is shown in the below figure:



Once the total excitation is known which is FR, the corresponding induced emf Eph can be obtained from OCC. This Eph lags FR by 90°. The length CD drops due to the armature reaction. Drawing perpendicular from A and B on current phasor meeting at points G and H respectively, we get triangle OHC as right-angle triangle. Hence E1ph can be determined, analytically also. Once Eph is known, the regulation of an alternator can be predicted as,

$$\% \ R = \frac{E_{ph} - V_{ph}}{V_{ph}} \times 100$$

ZPF method takes into consideration the armature resistance and leakage reactance voltage drops as EMF quantities and the effect of armature reaction as MMF quantity. This is the reality hence the results obtained by this method are nearer to the reality the those obtained by synchronous impedance method and ampere-turns method. The only drawback of the ZPF method is that the separate curve for every load condition is necessary to plot if Potier triangles for various load conditions are required.

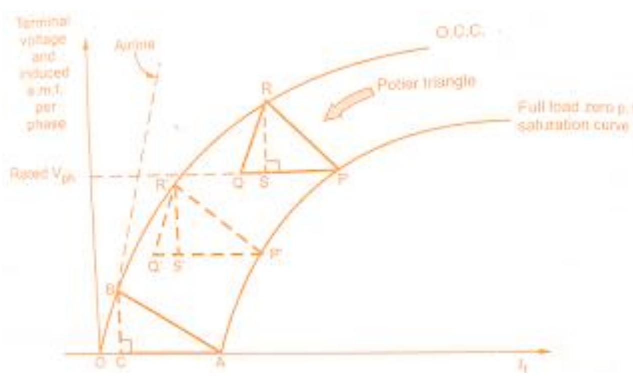
Assumptions Made in the Potier Method:

Some assumptions are made in the *Potier method* which is listed below:

1.ln the entire calculation procedure of Potier method, the armature resistance is neglected. But practically armature resistance is very small and hence this assumption does not cause significant error in the accuracy.

2. In Potier method, a zero-power factor test is required to be done. But practically when inductors are used, a perfect *zero power factor* cannot be achieved.

3. Consider the graphical interpretation of Potier method shown in the below figure.



In this graph, the distances RS, R' S' and BC are assumed equal. This represents the voltage drop across the leakage reactance which is (Iaph)FL * XLph.This indicates that the point P in the zero-power factor method and point A in the short circuit test represent the same leakage reactance of the machine. But this is not true as the excitation under short circuit condition is OA while that for point P is OA as shown.

Now the excitation OA' is much higher than OC and hence point P corresponding to saturated conditions represents larger leakage flux which in the method assumed unchanged. Hence practically the leakage reactance corresponding to saturated conditions is higher than that assumed in the method. This introduces the error in the calculations.

2.8 POWER ANGLE CHARACTERISTICS OF CYLINDRICAL ROTOR MACHINE

Fig (a) shows the circuit diagram and phasor diagram of synchronous machine in generating mode and fig (b) shows the circuit diagram and phasor diagram of synchronous machine in motoring mode. The machines are assumed to be connected to finite bus bar having voltage V_t and the resistance of stator winding is neglected and only the reactance of the machine has been considered.

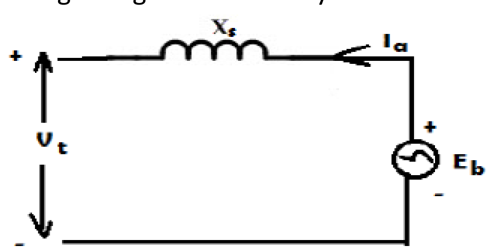


fig no (a) circuit diagram for generating mode

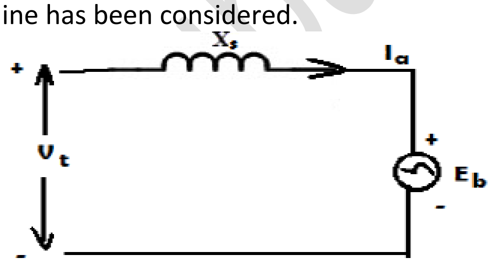


fig no (b) circuit diagram fo motoring mode

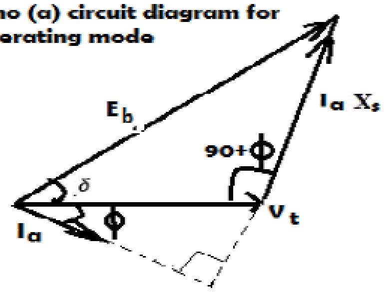


fig no (a) phasor for generating mode

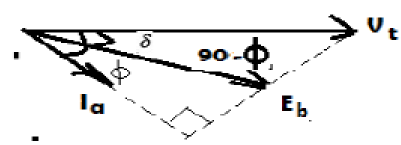


fig no (b) phasor for motoring mode

For generating mode:

$$\overrightarrow{E_b} = \overrightarrow{V_t} + j\, I_a \overrightarrow{X_s}$$

$$\frac{E_b}{\sin(90+\emptyset)} = \frac{I_a X_s}{\sin\delta}$$

for motoring mode:

$$\overrightarrow{E_b} = \overrightarrow{V_t} - j\, I_a \overrightarrow{X_s}$$

$$\frac{E_b}{\sin(90+\emptyset)} = \frac{I_a X_s}{\sin\delta}$$

In general:

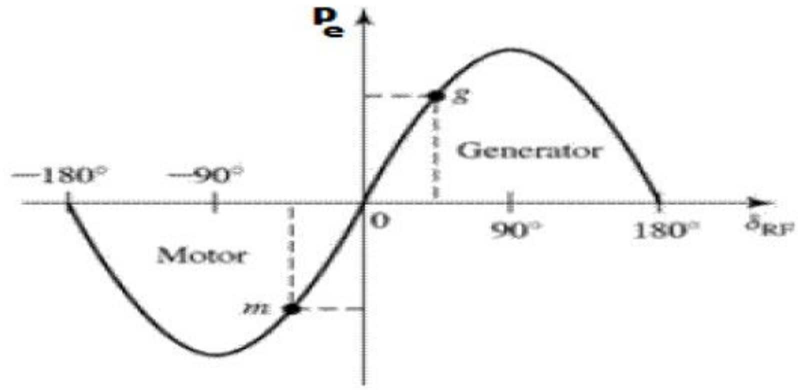
$$\frac{E_b}{\sin(90\pm\emptyset)} = \frac{I_a X_s}{\sin\delta} \quad \text{or,} \quad \frac{E_b}{\cos(\emptyset)} = \frac{I_a X_s}{\sin\delta} \quad \text{or,} \quad I_a \cos(\emptyset) = \frac{E_b}{X_s}\sin\delta$$

$$\text{or,} \quad I_a\ V_t \cos(\emptyset) = \frac{V_t E_b}{X_s}\sin\delta$$

i.e. Electrical power exchanged between infinite bus and machine,

$$P_e = \frac{V_t E_b}{X_s}\sin\delta \text{...(1)}$$

From above equation it is clear that the electrical power of machine (P_e) is proportional to $\sin\delta$. Where, δ is the power angle between V_t and E_b . This equation can be represented by a curve as shown and is known as power angle characteristics of cylindrical rotor synchronous machine.



Maximum power occurs at δ =90. At no load the machine operates δ =0. As the load increases, will increase and more electrical power will be exchanged. If the machine is overloaded so that the δ >90, then the machine will lose the synchronism, the machine will not able to exchange the more power and machine will lose stability limit. The machine is normally operated at δ much less than 90. This is to prevent the machine from going into an unstable region during transient power swing.

2.9 TWO REACTION MODEL OF SALIENT POLE MACHINE

All the above analysis of the synchronous machine is based on the assumption that the rotor is cylindrical where; the reluctance to the air gap flux is uniform at any position of the rotor. But in case of salient pole rotor, this is not true. The reluctance to air gap flux is not uniform at any position of the rotor due to asymmetrical construction of the rotor. The armature flux and field flux can't vary sinusoidally in the air gap. Hence, the armature and field mmf can't be treated in a simple way as they can be in non-salient pole alternators.

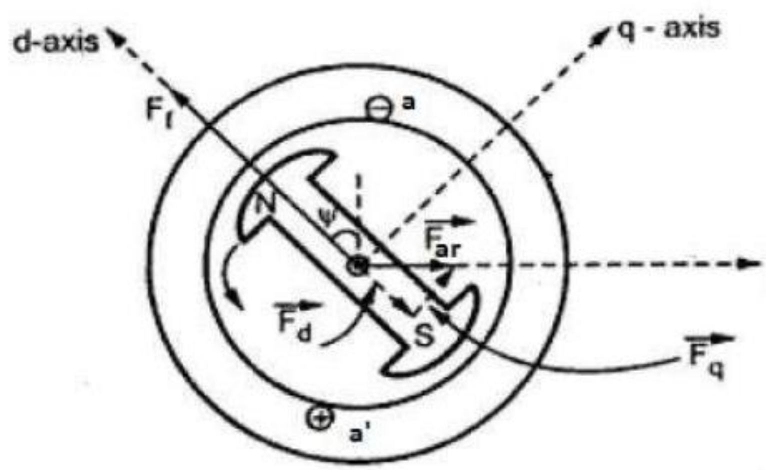


fig: Two reaction model of salient pole machine

The theory which gives the method of analysis of the distributing effects caused by salient pole Construction is called two-reaction theory. According to this theory, the salient pole motor has two axes:
1) Along the field pole, axis is direct axis (d-axis).
2) Along the right angle to the field pole axis is called quadrature -axis (q-axis).
The armature current of motor can also be resolved along d-axis and q-axis called I_d and I_q respectively.
Let us assume that $I_a$lags E_b by an angle, when the rotor position is as shown in figure. Although the field winding in a salient pole is of concentrated type, the magnetic flux wave is nearly sinusoidal because of

the shaping of pole shoe. Due to physical shape of the rotor, the reluctance to the air gap flux is minimum along the axis of field (d-axis) and maximum along the axis perpendicular to the field (q-axis).

As the φ varies, the reluctance offered to F_{ar} (armature reaction flux) will also vary. Therefore the voltage drop due to armature reaction can't be simply represented X_a as in case of cylindrical rotor machine. This problem can be overcome by dividing F_{ar} into two components:

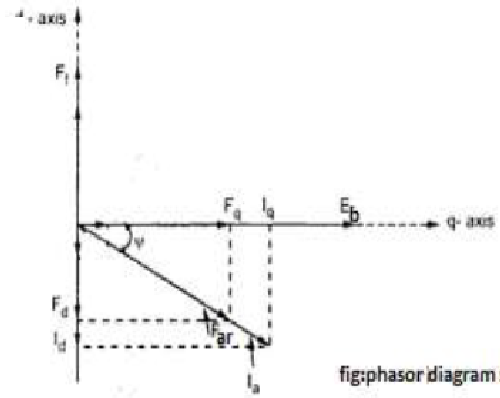


fig:phasor diagram showing the two reaction model

F_d=F_{ar}sinφ(demagnetizing component)

F_q-along the q-axis

F_q=F_{ar}cosφ(cross-magnetizing component)

Therefore, two types of synchronous reactance can be realized for salient pole machines:

X_d=X_{ar}^d+X_l=d-axis synchronous reactance.

X_q=X_{ar}^q+X_l=q-axis synchronous reactance.

Where, X_{ar}^d=reactance equivalent to d-axis component of armature reaction.

X_{ar}^q=reactance equivalent to q-axis component of armature reaction.

From following phasor we have:

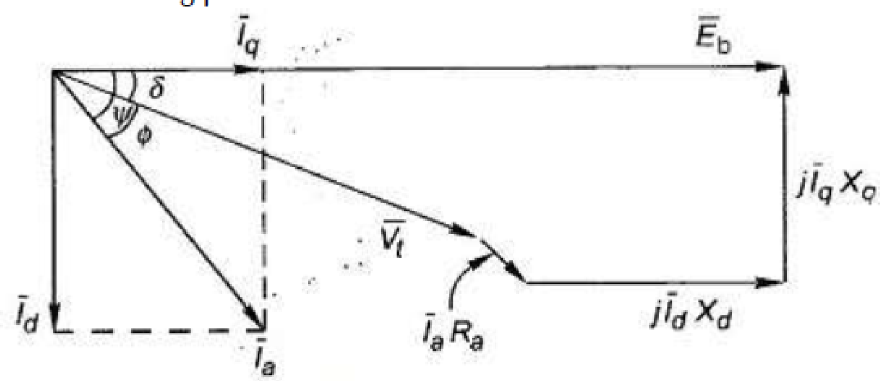


fig. (2) phasor diagram

$$\overrightarrow{E_b} = \vec{V} + \overrightarrow{I_a}R_a + j\overrightarrow{I_d}\,X_d + j\overrightarrow{I_q}\,X_q$$

2.10 POWER ANGLE CHARACTERISTICS OF SALIENT POLE MACHINE

The single line diagram of a salient pole synchronous machine connected to infinite bus bar of voltage V_b through a line series reactance X_{ext} per phase. The total d-axis and q-axis reactance are:

X_d =X_d+X_{ext}

X_q =X_q+X_{ext}

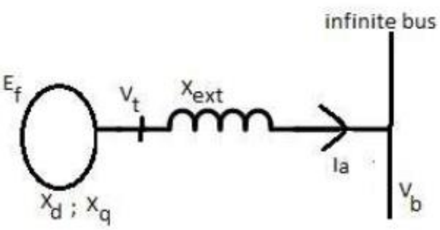


fig: single line diagram m/c connected to infinite bus.

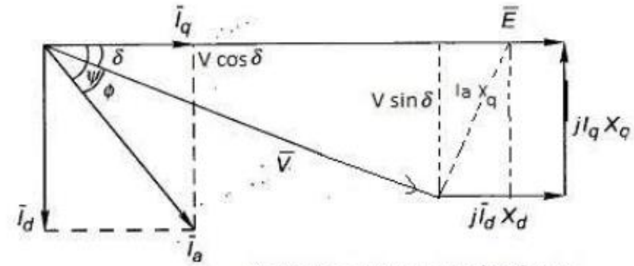


fig. (2) phasor diagram neglectting Ra

From phasor diagram (R_a is neglected

for generating mode: $\delta = \varphi$ - $\emptyset$ and for motoring mode $\delta = \emptyset$ - φ

We also have from phasor: E_o=Vcos δ+ $I_d X_d$...................... (For generating mode) ....(A)

E_o=Vcos δ- $I_d X_d$...................... (For motoring mode) ....(A)

Considering generating mode:

I_q=I_a cos φ=I_a cos(δ + $\emptyset$)...............(1)

I_d=I_a sin φ=I_a sin(δ + $\emptyset$)............(2)

We can also see that:

Vsin(δ)=$I_q X_q$=I_a cos(δ + $\emptyset$) X_q.................(3) (from equation no 1)

Vsin(δ) =$I_a X_q$cos$\emptyset$ cos δ-$I_a X_q$sin$\emptyset$ sin δ

V=$I_a X_q$cos$\emptyset$ cotδ-$I_a X_q$sin$\emptyset$

V+$I_a X_q$sin$\emptyset$ =$I_a X_q$cos$\emptyset$ cotδ

tanδ=$\frac{I_a X_q \cos\emptyset}{V+I_a X_q \sin\emptyset}$.................................(4)

If R_a and copper loss is neglected then power delivered =power output,

P_{out}=VI_a cos$\emptyset$... (B)

From equation no (A)

I_d= $\frac{E_o - V\cos\delta}{X_d}$ .... (5)

from equation no(3)

I_q=$\frac{V\sin\delta}{X_q}$........................(6)

again from equation no (1 and 2)

I_q=$I_a$cos$\emptyset$ cos δ-$I_a$sin$\emptyset$ sinδ ....(7)

I_d=$I_a$sin$\emptyset$ cos δ+$I_a$cos$\emptyset$ sinδ....................(8)

Multiplying by cosδ and sinδ respectively on eqn no 7 and 8.

$I_q$cos δ=$I_a cos\emptyset$ cos$^2\delta$-$I_a$sin$\emptyset$ sinδ cos δ ....(9)

$I_d$sinδ=$I_a sin\emptyset$ cos δ $sin\delta$+$I_a$cos$\emptyset$ sin$^2\delta$..............(10)

Adding 9 and 10

$I_d \sin\delta + I_q \cos\delta = I_a \cos\emptyset$......................(11)

putting the value of $I_a\cos\emptyset$ in eqn no (B)

P_{out}=V$I_q$cos δ+ V$I_d$sin

from eqn no (5and 6)

$$P_{out} = \frac{V*V\sin\delta\cos\delta}{X_q} + \frac{V(E_o - V\cos\delta)\sin\delta}{X_d} = \frac{V^2\sin\delta\cos\delta}{X_q} + \frac{VE_o\sin\delta}{X_d} - \frac{V^2\cos\delta\sin\delta}{X_d}$$

$$= \frac{VE_o\sin\delta}{X_d} + V^2\sin\delta\cos\delta\left(\frac{1}{X_q} - \frac{1}{X_d}\right)$$

$$P_{out} = \frac{VE_o\sin\delta}{X_d} + \frac{V^2}{2}\sin 2\delta\left(\frac{1}{X_q} - \frac{1}{X_d}\right) \text{.............................}(12)$$

Equation no 12is the required output power equation for single phase salient pole machine. For three phase

$$P_{out} = 3\frac{VE_o\sin\delta}{X_d} + 3\frac{V^2}{2}\sin 2\delta\left(\frac{1}{X_q} - \frac{1}{X_d}\right) \text{............................}(13)$$

Where excitation power is given by $3\frac{VE_o\sin\delta}{X_d}$ and reluctance power is given by $3\frac{V^2}{2}\sin 2\delta(\frac{1}{X_q} - \frac{1}{X_d})$.

The reluctance power varies as $\sin 2\delta$with maximum value at δ=45°. It is to be noted that this term is independent of field excitation and would be present even if the field is unexcited. A synchronous motor with salient poles but without field winding is known as the reluctance motor. It is used for low power constant speed application. Power angle characteristic is shown in given figure.

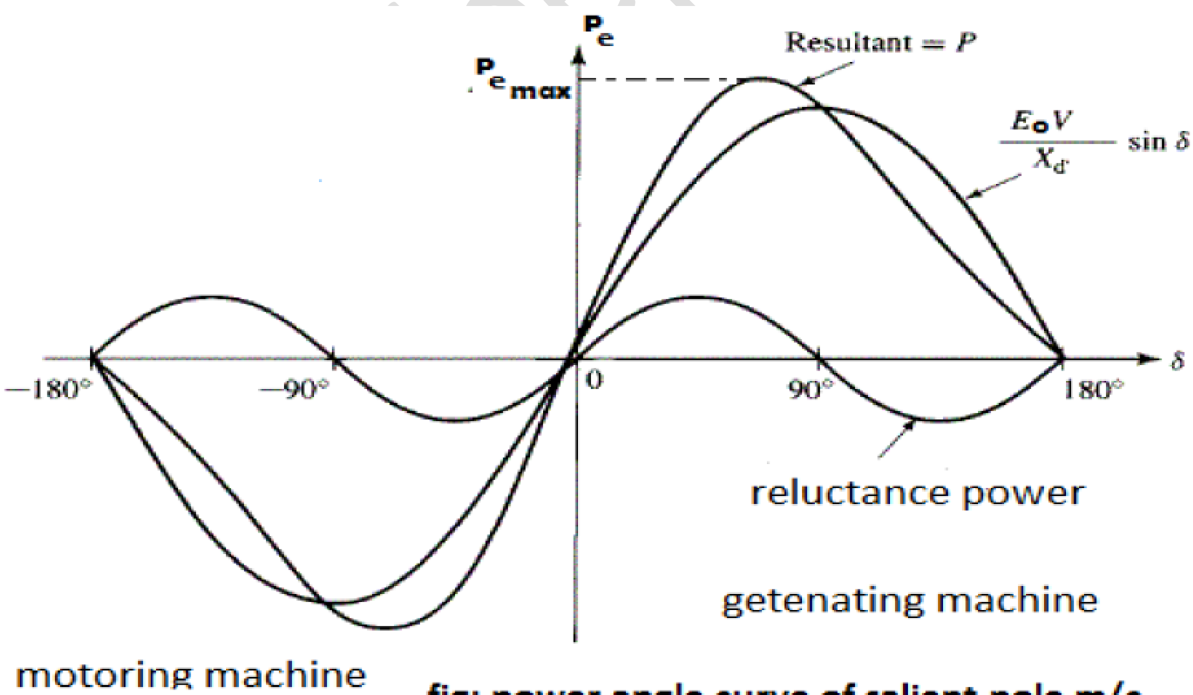


fig: power angle curve of salient pole m/c

To calculate torque we have: $T=\frac{P}{\omega_s}=\frac{P}{2\pi n_s}$

For maximum power we have: $\frac{\partial P}{\partial \delta}=\frac{VE_o cos\delta}{X_d}+V^2\cos2\delta\left(\frac{1}{X_q}-\frac{1}{X_d}\right)=0$

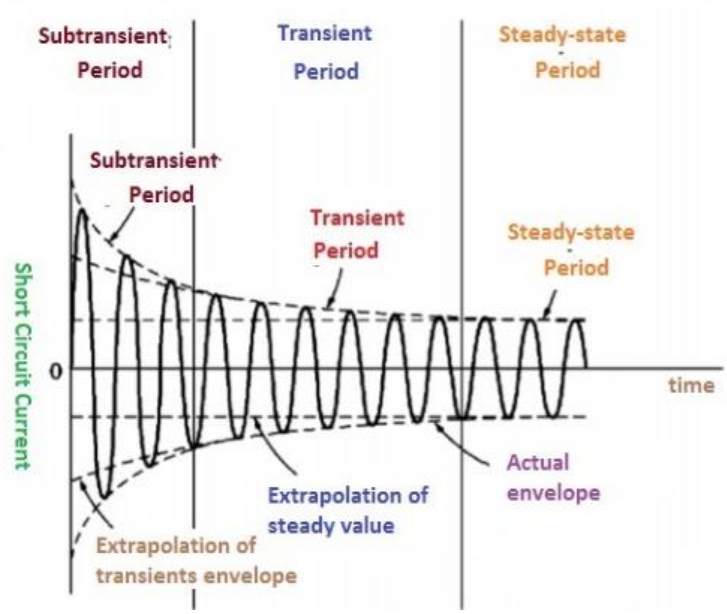
$$\cos\delta_{max}=-\frac{E_o}{4V(\frac{X_d-X_q}{X_q})}+\sqrt{\frac{1}{2}+\left(\frac{E_oX_q}{4V(X_d-X_q)}\right)^2}$$

2.11 TRANSIENT CONDITIONS OF ALTERNATOR

The synchronous generator is also known alternator that transforms mechanical energy that is provided by its prime mover at shaft into electrical energy at specific voltage and frequency. Its rotation speed is known as synchronous speed because its rotation speed equals the speed of rotation of field at stator that known as synchronous speed.

Due to this, it used in such an application where constant speed is essential. It mostly used in our generation structure for the creation of energy. In today's post, we will have a look at its transients and their effect on the working of the synchronous generator. So, let's get started with *Synchronous Generator Transients.*

- When the torque provided to the shaft of the synchronous generator by the prime mover or the load connected with the generator varies abruptly, then transients produce in the generator that will remain until the generator again gets the stability.
- For instance, when the generator is connected in parallel with the already working power system, as we discussed in Parallel Operation of Synchronous Generator that the incoming generator frequency should be higher than the running system, so its frequency is larger than the system.
- When a generator is paralleled, there are transient's intervals before it works steadily and starts to operate at the frequency equal to the system.



- The alternating current components are divided into the 3 main time periods. after the short circuit (fault) occurrence you can see that the alternating current amplitude is very high, and its decays very rapidly. This interval is known as the sub transients.
- After the sub transients' period the large amplitude of the current continues to decay slowly, finally it get steady state.
- The time interval in which the amplitude of the current decay slowly is known as a transient period and interval then it gets steady-state condition is called steady state period.
- The steady-state current during a short circuit is represented as I_{ss}.
- It is given almost by the fundamental frequency constituent of the E_A and the synchronous reactance Xs.

$I_{ss} = E_A / X_s$

- The root means square amplitude of the alternating fault current in generator changes continuously as function the T.
- If I'' is the sub transient constituent of current at the time of the short circuit, I' is the transient constituent of current at the time of the short circuit and I_{ss} is the steady-state short (fault) current then the root mean square magnitude of the current at any time after a short circuit (fault) at the output terminals of the generator is given as.

$I(t) = (I'' - I')e^{-t/T''} + (I' - 1_{ss})^{e-t/T'} + I_{ss}$

- It is usual to describe sub transient and transient reactance's for a synchronous generator as an appropriate way to define the sub transient and transient elements of short circuit (fault) current.
- The sub transient's reactance of synchronous generator can be described as it is the ratio of the E_A to the sub transient element of current at the start of short circuit current.

$X'' = E_A/I''$ sub transient

- Correspondingly, the transient's reactance of generator can be defined as it the ratio of the E_A to the transient's element of the current I' at the start of the short circuit (fault.)

$X' = E_A/I'$ transients

Numericals:

1) The stator of a 3-phase, 8 pole,750 RPM alternator has 72slots, each of which contains 10 conductors. Calculate the RMS value of the emf per phase if the flux per pole is 0.1Wb sinusoidally distributed. Assume full pitch coils and a winding distribution factor of 0.96
   - Solution

m=3 =number of phase

P=8 S=72 ϕ=0.1wb $Z = 72 * 10 / m = 240$

N=750 rpm=120f/p

∴f=750*8/120 =50 also Tp=Z/2 =240/2 =120

∴ *rms value* E=4.44fϕTp*Kd=4.44*50*0.1*120*0.96 =2557.44V

2) A 3-phase, 50 Hz, 2-pole, star-connected turbo alternator has 54slotswith 4 conductors per slot. The pitch of the coil is 2 slots less than the pole pitch. If the machine gives 3300v between lines on open circuit with sinusoidal flux distribution, determine the useful flux per pole.

➢ Solution

m=$\frac{slots}{poles * phase} = \frac{54}{2*3} = 9$

$\beta = 180° * \frac{poles}{slots} = \frac{20°}{3}$degree

Coil span=25* slot angle= 25*β = 25*$\frac{20°}{3}$degree

α=2β=$\frac{40°}{3}$degree Kp=cos(α/2)=.9932

$Kd = \frac{\sin\left(\frac{m\beta}{2}\right)}{m\sin\left(\frac{\beta}{2}\right)}$ = 0.95547

Total no. of conductors per phase(Z)=54*4/3=72

Tp=72/2=36

Erms=4.44fϕTp*Kd*kp

$\frac{300}{\sqrt{3}} = 4.44 * .9932 * 0.95547 * 50 * \varphi * 36$

$\therefore \varphi = 0.2512wb$

3) A 3-phase, star-connected alternator is rated at 1500 kVA, 12KV. The armature effective resistance and synchronous reactance are 2Ω and 25Ωrespctively per phase. Calculate the emf generated, percentage voltage regulation for a load 1250kw for the power factor of a. 0.75lagging pf b. 0.75 leading pf [2072 chaitra]

➢ Solution

Cosϕ=0.75 $\therefore Vp = \frac{12}{\sqrt{3}}$=6.928kv

P=1250 =$\sqrt{3}V_l I_l \cos\varphi$ =$\sqrt{3}$*12*I_l*0.75

$\therefore I_l = I_s$ = 80.188A,

for lagging power factor

E=$\sqrt{(V + IaRacos\varphi + I_aX_s\sin\varphi)2 + (IaX_s\cos\varphi - I_aRasin\varphi)2}$

$$E = \sqrt{\begin{matrix}(6928 + 80.19 * 2 * 0.75 + 80.19 * 25 * 0.66)^2 + \\ (80.19 * 25 * 0.75 - 80.19 * 2 * 0.66)^2\end{matrix}}$$

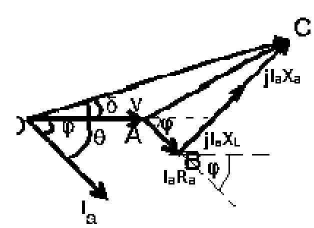


Fig. lagging power factor

E=8487.3v

Line to line voltage=8487.3*√3=14700.44V

$VR = \frac{E-V}{V} * 100\% \;=\; \frac{14700.44-12000}{12000} * 100\% = 22.5\%$

Or students can solve easily as follow to get power angle directly from phasor , $\vec{E} = \vec{V} + \overrightarrow{Ia}Ra + j\overrightarrow{Ia}X_s$

E=6928+80.19$\angle -41.41$*2+j80.19$\angle -41.41$*25 =8490.11$\angle$9.47°

$$\therefore VR = 22.5\%$$

For leading power factor,

From phasor,,$\vec{E} = \vec{V} + \overrightarrow{Ia}Ra + j\overrightarrow{Ia}X_s$

E=6928+80.19$\angle$41.41*2+j80.19$\angle$41.41*25 =8490.11$\angle$9.47°=5944.3$\angle$15.71°

Line to line voltage=5944.3*$\sqrt{3}$ =10295.83V

$$VR = \frac{E-V}{V} * 100\% \;=\; \frac{10295.83-12000}{12000} * 100\% = 14.2\%$$

4. An alternator has a direct axis synchronous reactance of 0.8p.u and a quadrature-axis synchronous reactance of 0.5 p.u. Determine the p.u open circuit voltage for full load at a lagging power factor of 0.8.Neglect saturation.

➢ Solution

Let, Vp be the reference phasor Vp=1$\angle 0 = 1 + j0\ p.u$

Ia=1 pu at 0.8 pf lagging. $=1\angle -\cos^{-1}0.8 = 1\angle -36.87$

Xd=0.8pu

Xq=0.5p.u

$$\therefore E = Vp + IaXq + (Xd - Xq)I_d \quad where, I_d = Iasin\Psi, \quad \Psi = \phi + \delta$$

Also,$tan\delta = \frac{(IaXq\cos(\phi))}{(V+IaXq\sin(\phi))}$ $\therefore \delta = 17.1 \quad hence, \Psi = 36.87 + 17.1 = 53.97$

Id=1*sin53.97=0.809 pu

$$\therefore E = 1 + 1 * 0.5 + (0.8 - 0.5) * 0.809 = 1.593\ p.u$$

5. A 3-phase,11000v, star-connected turbo-alternator, having synchronous reactance of 6Ω per phase and negligible resistance has an armature current of 200A at unity power factor when operating on constant frequency constant voltage bus-bar. If the steam admission remains the same and the emf is raised by 25%, determine the new values of current and power factor.

➢ Solution

Phase voltage=Vp= 11000/$\sqrt{3}$=6350.85

Ia_1=200A Xs=6Ω

At unity power factor, $Ea_1^2=V1^2+(Ia_1Xs)^2$=6350.85+(200*6)^2=6463.23v

When emf is increased by 25% Ea_2=1.25*6463.23=8079.03v

Let, the new armature Ia_2 and the new power factor be $\cos\phi_2$.since Ea_2>Vp, the pf $\cos\phi_2$ is lagging,

$\therefore Ea_2=\sqrt{(Va_2 + Ia_2Ra_2\cos\varphi 2 + Ia2Xs\sin\varphi)2 + (Ia_2Xs\cos\varphi - Ia2Ra_2\sin\varphi)2}$

At infinite bus Vp2=Vp1=6350.85V

Since, the steam supply remains the same, the power output will not change

$Vp_2Ia_2\cos\phi_2=Vp_1Ia_1\cos\phi_1$ $\qquad Ia_2\cos\phi_2$=200*1

At lagging pf $\cos\phi_2$ with Ra=0,

$Ea_2=\sqrt{(Vp + Ia2Xs\sin\varphi)2 + (Ia_2Xs\cos\varphi)2}$

$8079.03^2=(6350.85+6*Ia_2*\sin\phi_{2})^2+(200*6)^2$

$$\therefore Ia2 * \sin\varphi 2 = 273.09$$

Also, **Ia_2**$\cos\phi_2$=200

Squaring and adding we get, $\therefore Ia_2$=338.494

New power factor=$\cos\phi_2$=200/338.494=0.59(lagging)

6. The effective resistance of a 1200-Kva,3.3Kv 50Hz, 3 phase, Y-connected alternator is 0.25Ω/phase. A field current of 35A produces a current of 200A on short-circuit and 1.1Kv(line to line) on open circuit. Calculate the power angle and p.u change in magnitude of the terminal voltage when the full load of 1200Kva at 0.8 pf(lag) is thrown off.

- Solution

Zs=$\frac{o.c\ voltage}{s.c\ voltage} = \frac{1.1*10^3/\sqrt{3}}{200} = 3.175\Omega$

Xs=$\sqrt{3.175^2 - 0.25^2} = 0.86$

V=$3.3*10^3/\sqrt{3}$=1905V

Tanθ=Xs/Ra=3.165/0.25θ=85.48°

∴Zs=3.175∠85.48°

Rated Ia=$1200*10^3/\sqrt{3}$ $*3.3*10^3$ =210

Let, V=1905∠0°, $Ia = 210\angle - 36.87°$

From the phasor diagram, E=V+IaZs =1905+20$\angle - 36.87°$*3.175∠85.48° =2400∠12°

Phasor angle δ=12°

Per unit change in terminal voltage is (2400-1905)/1905=0.26

7. An alternator delivers 100A at 0.8 lagging pf to a 3-phase, 11kv infinity bus bar system. The synchronous reactance of alternator is 4Ω/phase. Find the open circuit emf and load angle. The steam supplied is increased and load angle is increased by 10°, find the new load current and pf. The excitation is now changed till pf becomes 0.8 lagging. Find new value of load current.

- Solution

We have

E=$\sqrt{(\mathrm{V} + \mathrm{IaXssin\varphi})2 + (\mathrm{IaXscos\varphi})2}$

=$\sqrt{(6350.85 + 100 * 4 * 0.6)^2 + (100 * 4 * 0.8)^2} = 6600v$

Open circuit phase emf (E)=$6600v$ power angle(δ)=$\tan^{-1}\frac{\mathrm{IaXscos\varphi}}{\mathrm{V+IaXssin\varphi}}$=2.27°

New power angle is δ_2=10+2.27=12.7° then

tanδ=$\frac{\mathrm{IaXscos\varphi}}{\mathrm{V+IaXssin\varphi}} \Rightarrow tan12.27 = \frac{4\mathrm{Iacos\varphi}}{6350.85+4\mathrm{Iasin\varphi}}$

$\therefore\ \mathrm{Iacos\varphi} = 0.055(6350.85 + 4\mathrm{Iasin\varphi}) \ldots\ldots\ldots$ 1

$\therefore\ \mathrm{Iasin\varphi} = 0.25(18.18\mathrm{Iacos\varphi} - 6350.85) \ldots\ldots$ 2

E_2=6600V ⇒6600^2=(6350+IaXssinφ)2 + (IaXscosφ)2……………3

From equation 2 and 3 Iacosϕ=364.9684

From equation 1 and 3 Iasinϕ=23.75

Solving above equation new armature current Ia=365.7A

Power factor cosϕ=cos(tan^{-1}(23.75/364.96)=0.997(lag)

For again change in excitation Iacosϕ=365.97

Or, Ia=365.97/0.8=456.21

∴New current=(Ia)=456.21A

UNIT 3: PARALLEL OPERATION OF ALTERNATORS

In an actual power system, according to consumer demand thee is a requirement of numbers of alternator to be connected to the system. The process of connecting two alternators in parallel is known as "synchronization". In an interconnected system many numbers of alternators are connected in parallel though bus at station and transmission line. In such case an alternator will be synchronized to an infinite bus bar on which many numbers of alternator had been connected. For parallel operation the following conditions are to be satisfied.

- The terminal voltage of two alternators should be equal.
- Frequency must be same.
- The waveform of emf generated by both alternators should be is same phase.
- The percentage impedance of both alternators should be same.
- PHASE SEQUENCE MUST BE SAME.

3.1 PARALLEL OPERATION

Before an alternator is synchronized with other generators for the first time, its phase sequence must be checked to determine that it has same phase sequence as that of the other alternators. The phase sequence can be checked by a phase sequence indicator, a small 3-phase induction motor that rotates in one direction for one phase and in opposite direction for other phase sequence. If the motor rotates in the same direction with both voltages of G1 and G2 when connected separately, then it is cleared that the both alternators have same phase sequence.

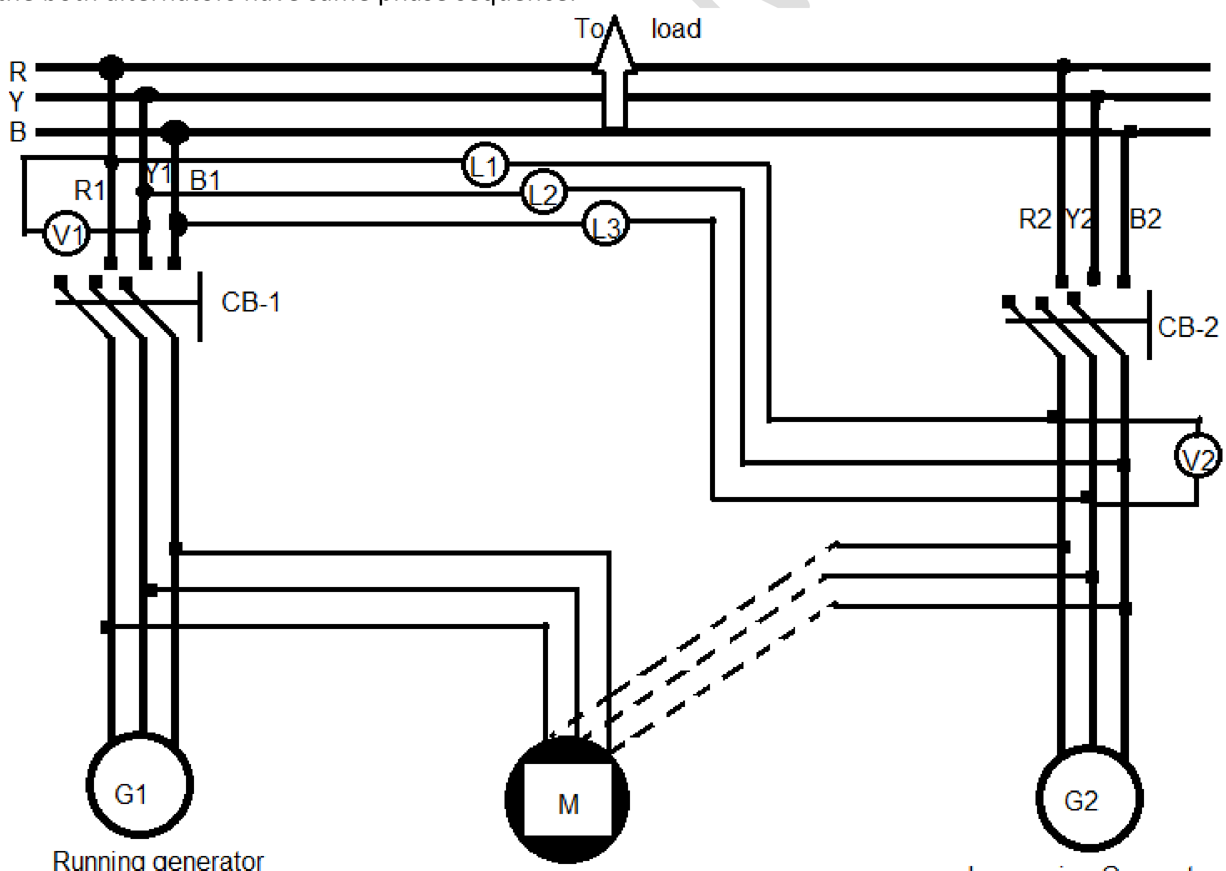


Fig.connection diagram of synchronization of 2 alterntors

Above figure shows the connection diagram for the synchronization of 2 alternatives G_1 and G_2.G_1 is running already and supplying current to the load. G_2 is to be connected in parallel with G_1, voltmeter V_1 measures the main bus bar. Voltage and voltmeter V_2 measure the output voltage of generator G_2. L_1, L_2 and L_3 are 3 lamps placed physically in triangular form and are known as synchronizing lamps. L_1 is connected across R_1-R_2, L_2 across Y_1-B_2& L_3 across B_1-Y_2.

The incoming operator G_2 is rotated by its prime mover approximately up to its synchronous speed keeping that CB-2 open. The excitation of G_2 is adjusted so that the voltage generated by incoming generator as measured by V_2 is to match main bus bar voltage, as measured by V_2 is match the main bus bar voltage as measured by V_1. The synchronization lamps are used to make sure that the voltage generated by G_2 is in phase with bus bar voltage & frequency of incoming generator is same as that of bus bar frequency.

Figure a shows phasor diagram of two machines super imposed on each other. If the frequency of both voltage state is equal, each vector rotates with same speed & difference between R_1&R_2, Y_1& B_2, B_1&Y_2remains constant. Hence, at this condition L_1 remains dark, L_2& L_3 will glow with equal brightness. Then in such case the CB-2 of incoming generator can be closed so that both generators operate in parallel.

If frequency of G_2 is greater than that of G_1, then R_2-Y_2-B_2 vector rotates faster than R_1-Y_1-B_1 vector as shown in figure b .In such situation voltage across L_1 goes on increasing, L_2 goes on decreasing & L_3 goes on increasing .After some time , the vectors B_2&Y_1 will get coincide resulting in L_2 dark. Hence light gets dark one after another in anticlockwise direction. n such a situation the speed of incoming generator G_2 has to be reduced until situation is in figure a, then CB-2 can be closed so that both G_1& G_2 are operated in parallel.

If the frequency of G_2 is less than that of G_1 then R_2-Y_2-B_2 vector rotates slower than the R_1-Y_1-B_1 vector as shown in figure c .In such a situation voltage across L_1 goes on increasing ,L_2 goes on increasing & L_3 goes on decreasing .After some time , the vectors B_1&Y_2 will get coincide resulting L_3 dark. Hence, the light gets dark one after another in clockwise direction. In such a situation the speed of incoming generator G_2 has to be increased until the situation is as fig c. Then CB-2 can be closed for parallel operation.

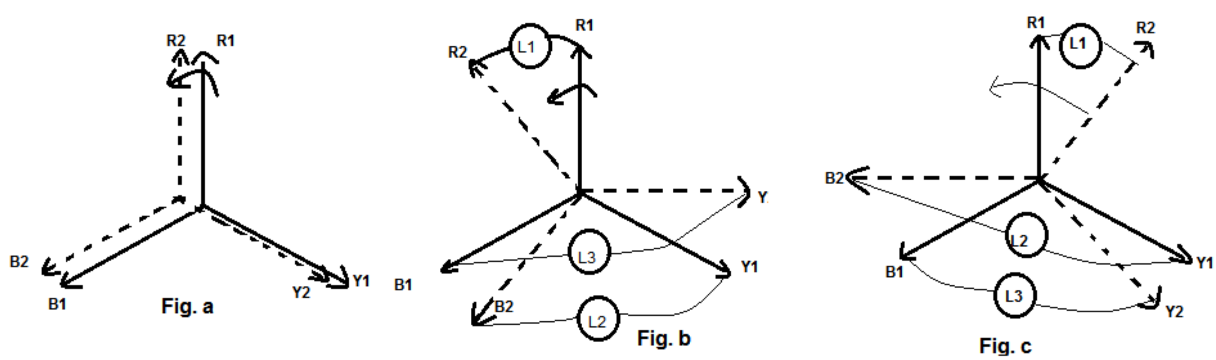
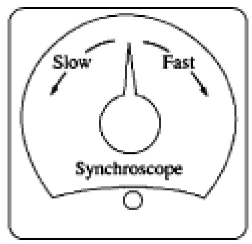


Fig. a Fig. b Fig. c

Advantage of dark lamp method:
- Cheap
- Easy determination of proper phase sequence

Disadvantage of dark lamp method:
- Flicker of lamp doesn't indicate which machine has higher frequency.
- Lamp filament might burn out.
- May cause a high circulating current which may damage the machine.

When the three light bulbs all go out, the voltage difference across them is zero and the systems are in phase. This simple scheme works, but it is not very accurate. A better approach is to employ a synchroscope. A synchroscope is a meter that measures the difference in phase angle between the phases of the two systems. The face of a synchroscope is shown in Fig. The dial shows the phase difference between the two a phase, with 0 (meaning in phase) at the top and 1800 at the bottom. Since the frequencies of the two systems are slightly different, the phase angle on the meter changes slowly. If the oncoming generator or system is faster than the running system, then the phase angle advances and the synchroscope needle rotates clockwise. If the oncoming machine is slower, the needle rotates counterclockwise. When the synchroscope needle is in the vertical position, the voltages are in phase, and the switch can be shut to connect the systems.



Notice, though, that a synchroscope checks the relationships on only one phase. It gives no information about phase sequence. In large generators belonging to power systems, this whole process of paralleling a new generator to the line is automated, and a computer does this job. For smaller generators, though, the operator manually goes through the paralleling steps as described earlier.

3.2 OPERATION ON INFINITE BUS

An infinite bus is a power system so large that its voltage and frequency remains constant regardless of how much real and reactive power is drawn from or supplied to it. Connection or disconnection of a single small machine or a load on such system would not affect the magnitude and phase of the voltage and frequency. Thus, the system behaves like a large generator having virtually zero internal impedance and infinite rotational inertia.

Characteristics of an infinite bus:

Terminal voltage remains constant because the incoming machine is too small to increase or decrease it. Frequency remains constant, because rotational inertia is large to enable the incoming machine to alter the system. Synchronous impedance is very small since the system has large number of alternators in parallel. The behavior of machines on infinite bus is quite different from its isolation operation. In isolated operation the change of excitation changes its terminal voltage, the power factors depend upon the load only. While working with infinite bus its excitation change, pf of machine change but change of excitation does not change terminal voltage which is held constant for the system.

Characteristics of alternator connected on infinite bus:

The terminal voltage and frequency of the generator are controlled by the system to which it is connected. The governor sets the points of alternator control the real power supplied by the alternator to infinite bus. The field current (excitation) in the alternator controls the reactive power supplied by the alternator to the infinite bus. Increasing the field current in alternator operating in parallel with an infinite bus increase the reactive power output of the alternator. Consider n generators G1, G2, G3……, Gn connected to an infinite bus as shown in figure.

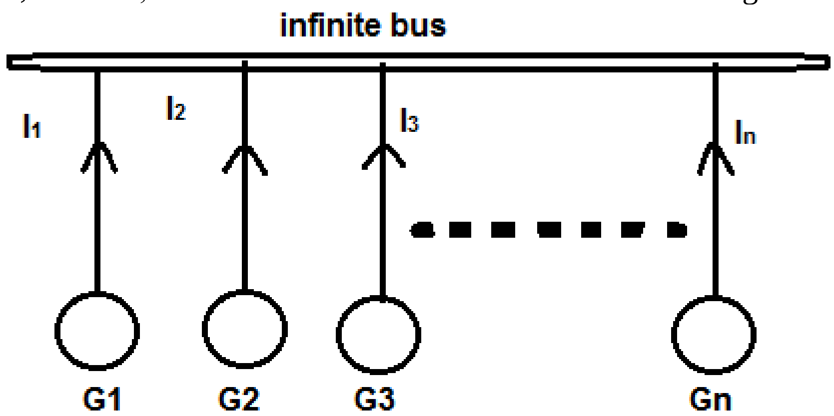


fig. n generators connected to an infinite bus

i)Proof of voltage remains constant:

Let V=terminal voltage

E=induced emf of each generator

Zs=synchronous impedance of each generator

$\therefore V = E - IZseq$

$Zseq = \frac{Zs}{n}$for large value of n, Zseq→0 so value of IZseq→0

$\therefore V = E$(constant) (If the number of alternators operating in parallel is infinite)

i)Proof of frequency remains constant

Let, J=total moment of inertia of each alternator

$$\therefore Total\ moment\ of\ inertia\ of\ all\ alternator = nJ$$

$$\therefore Acceleration\ of\ alternator(a) = \frac{accelerating\ torque}{moment\ of inertia} = \frac{\tau a}{nJ}$$

As n→ ∞, $a \to 0$

Acceleration is zero hence speed is constant, consequently frequency is constant.

3.3 SYNCHRONIZING POWER

A synchronous machine after properly synchronizing to infinite bus bar has an inherent tendency to remain in synchronism. For a cylindrical rotor machine, the power output per phase is

$P = \frac{V}{Z_s}[E\cos(\theta - \delta) - V\cos\theta]$ …………………… (1)

And synchronizing power coefficient is $P_{syn} = \frac{dP}{d\delta} = \frac{EV}{Z_s}\sin(\theta - \delta)$ ……………………….. (2)

And synchronizing torque coefficient is $\tau_{syn} = \frac{d\tau}{d\delta} = \frac{1}{2\pi n_s}\frac{dP}{d\delta} = \frac{EV}{2\pi n_s Z_s}\sin(\theta - \delta)$ ……………………….. (3)

Since, effective resistance is generally negligible, synchronous impedance can be taken equal to synchronous reactance and internal angle $\theta = 90°$. So,

$P_{syn} = \frac{EV}{X_s}\cos\delta$ W/electrical-radian ………………… (4)

$\tau_{syn} = \frac{EV}{2\pi n_s X_s}\cos\delta$ Nm/electrical-radian ……………………… (5)

We know 1 electrical radian = $\frac{180}{\pi}$ electrical degree

$P_{syn} = \frac{\pi}{180}\frac{EV}{X_s}\cos\delta$ W/electrical-degree

Since, 1 electrical degree in space = $\frac{2}{P}$ Mechanical degree

Where, P = total number of poles and m = number of phases

$P_{syn} = \frac{\pi P}{360}\frac{EV}{X_s}\cos\delta$ W/Mechanical-degree

$\tau_{syn} = \frac{\pi P}{360} m \frac{1}{2\pi n_s}\frac{dP}{d\delta}$ Nm/Mechanical-degree

For a salient Pole Machine

$$P = \frac{EV}{X_s}\sin\delta + \frac{V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta$$

$$P_{syn} = \frac{dP}{d\delta} = \frac{EV}{X_s}\cos\delta + V^2\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\cos 2\delta$$

3.4 REAL AND REACTIVE POWER SHARING

All generators are driven by a prime mover, which is the generator's source of mechanical power. The most common type of prime mover is a steam turbine, but other types include diesel engines, gas turbines, water turbines, and even wind turbines. Regardless of the original power source, all prime movers tend to behave in a similar fashion as the power drawn from them increases, i.e., the speed at which they run tends to decrease. The decrease in speed is generally nonlinear. Therefore, different forms of governor mechanism are usually included to make the decrease in speed linear with the increase in power demand.

Whatever governor mechanism is present on a prime mover, it will always be adjusted to provide a slight drooping characteristic with increasing load. The speed droop (SD) of a prime mover is defined by the equation

$$SD = \frac{n_{nl} - n_{fl}}{n_{fl}} \times 100$$

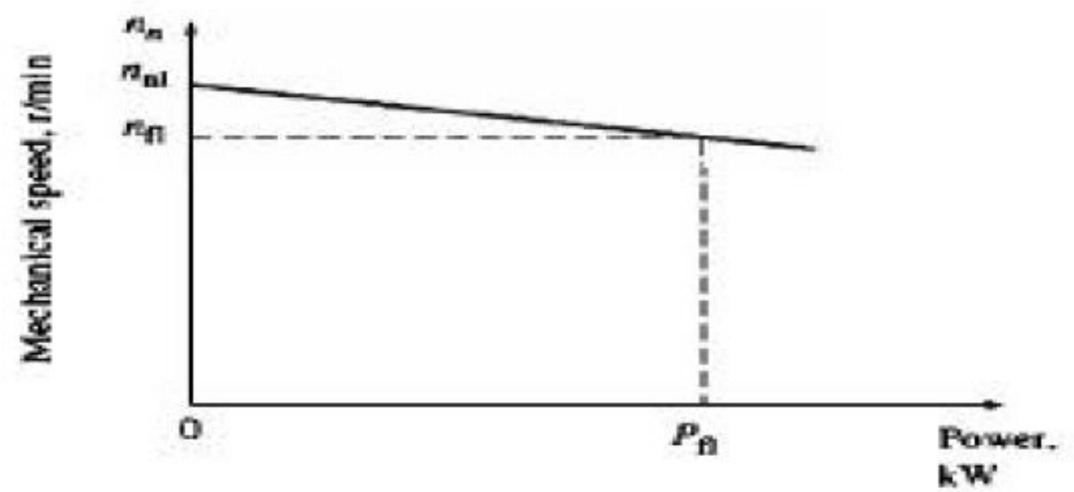
Where n_{nl} is the no- load prime-mover speed and n_{fl} is the full-load prime-mover speed. Most generator prime movers have a speed droop-of 2 to 4 percent. It has to be noted that, most governors have some type of set point adjustment to allow the no-load speed of the turbine to be varied. A typical speed-vs-power plot is shown in Fig. Since the shaft speed is related to the resulting electrical frequency by Equation given below.

$$f_e = \frac{n_{nl}120}{P}$$

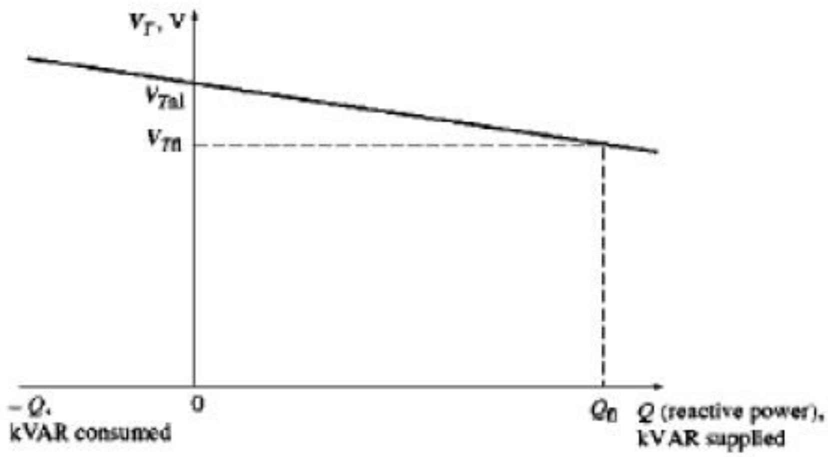
The electrical power output of a synchronous generator is related to its frequency. A typical plot of frequency versus power is shown in Fig. Frequency-power characteristics of this plot play an essential role in the parallel operation of synchronous generator. The relationship between frequency and power can be described quantitatively by the equation

$$P = S_p\left(f_{nl} - f_{syn}\right)$$

Where P = power output of the generator in kW
f_{nl}= no- load frequency of the generator in Hz
f_{fl}= full- load frequency of the generator in Hz
f_{syn}= operating frequency of system in Hz
s_p= slope of curve, in kW/Hz



A similar relationship can be derived for the reactive power *Q* and terminal voltage *VT*. As previously seen, when a lagging load is added to a synchronous generator, its terminal voltage drops. Similarly, when a leading load is added to asynchronous generator, its terminal voltage increases. Therefore, it is possible to make a plot of terminal voltage versus reactive power, such type of plot has a drooping characteristic like the one shown in Fig. This characteristic is not intrinsically linear, but voltage regulators of many generators include a feature to make it so. The Q-V characteristic curve can be moved up and down by changing the no-load terminal voltage set point on the voltage regulator. As like as the frequency-power characteristic, this curve plays an important role in the parallel operation of synchronous generators.



The relationship between the terminal voltage and reactive power of a generator can be expressed by an equation similar to the frequency-power relationship, if the voltage regulator produces an output that is linear with changes in reactive power. It is important to realize that, when a single generator is operating individually, the real power P and reactive power Q supplied by the generator will be the amount demanded by the load attached to the generator. The P and Q supplied cannot be controlled by the generator's controls. Therefore, for any given real power, the governor set point controls the generator's operating frequency f_e and for any given reactive power, the field current controls the generator's terminal voltage V_t .

Unit 4: Synchronous Motor

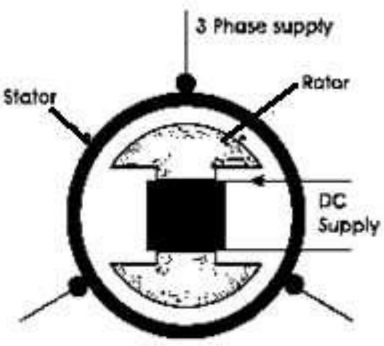
Like most of rotating machine, synchronous motor also operates as both generator and motor. It is a machine that converts ac electric power to mechanical power at constant speed (i.e. synchronous speed). It is *doubly-excited* machine. Rotor poles are excited by(dc) and stator winding is connected to ac supply. Hence, air gap flux is the resultant of flux due to rotor and stator current.

Important features:

1) It runs either at synchronous speed or not at all i.e. while running it maintains constant speed.
2) It can draw either lagging or leading reactive current from ac supply.
3) it is not self-starting.

4.1 OPERATING PRINCIPLE

Synchronous motor works on the principle of *magnetic locking*. When two unlike poles are brought near to each other, if the magnets are strong, there exits tremendous force of attraction between these two poles. In such condition the two magnets are said to be magnetically locked. If now one of the two magnets are rotated, the other also rotates in the same direction i.e. due to magnetic locking condition.



Synchronous Motor

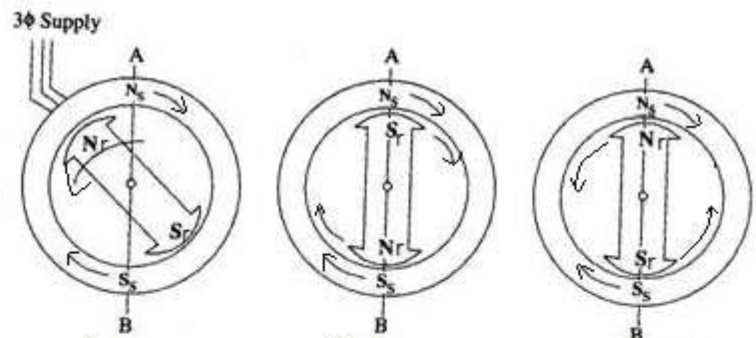


fig.a fig.b fig. c

Synchronous motors are not self-starting. When stator windings are supplied with thee phase voltage, rotating magnetic field is produced. At the same time rotor windings is excited by the dc motor so, rotor poles also get magnetized. But this interaction is not able to produce the continuous rotation.

At starting, let relative position of stator and rotor are as shown in fig (a). Here like poles get repealed. After some instant N_s and S_r come face to face so, the opposite poles tend to attract and rotor tends to rotate clockwise. But due to the heavy mass rotor can't respond such high alternation of reaction. At the starting, if relative position is as shown in fig (b). Here unlike poles attract, rotor tends to rotate in clockwise direction but heavy mass of rotor can't pick up the synchronous speed. So, after some time N_s and N_r comes face to face and repeal, so rotor tendency alters to anti clock direction but heavy mass rotor doesn't response to a quick reversal of direction of rotation. Similarly, it is not possible as if it starts as in fig(c). So, remains at rest. Hence, at any position, motor is not self-starting. If rotor is rotated up to or near the synchronous speed (before supply voltage to stator) by some other methods without exciting rotor windings, the rotor poles will get magnetically locked with stator are excited by respective supply. Then rotor rotates continuously even other method is removed.

4.2 STARTING METHODS

Since synchronous motor is not self-starting it must be accelerated up to synchronous speed by some auxiliary means. Some of methods are described below.

a) Dc motor coupled to the shaft of synchronous motor:

Here, unexcited rotor is rotated by the means of a dc motor coupled to the shaft of synchronous motor. Speed of dc motor is adjusted by field regulator. As speed reach near synchronous speed, field windings of synchronous motor excited by dc and dc motor is switched off and motor rotates continuously.

b) Using field exciter of synchronous motor as dc motor:

It is similar to the above method except exciter of synchronous motor (i.e. dc shunt generator) is operated as dc motor for the time being and as the speed reaches to synchronous speed the dc machine is again used as exciter.

c) A small induction motor of at least one pair less than synchronous motor:

Using an auxiliary induction motor at least one pair pole less and involves the same synchronizing process as that of first method.

d) Using the damper windings as a squirrel cage induction motor:

A damper winding consists of heavy copper bar inserted in slots of pole faces of rotor. The bars are short circuited by end rings at both ends of rotor i.e. squirrel cage winding. When three phase supply is connected to stator, the synchronous motor with damper winding will starts as three phase induction motor. The exciter moves along the rotor. When the rotor attains 95% of synchronous speed, dc is supplied to rotor winding and rotor get magnetically locked by rotating field of stator and motor runs at synchronous speed.

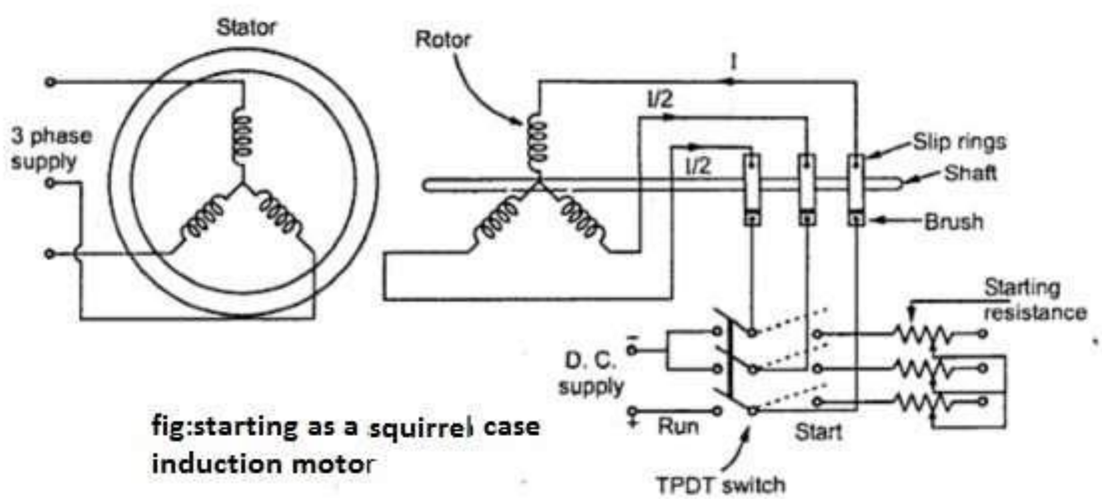


fig:starting as a squirrel case induction motor

Advantages:

When motor is overloaded, it doesn't stop, runs continuously as squirrel cage induction motor.

Disadvantages:

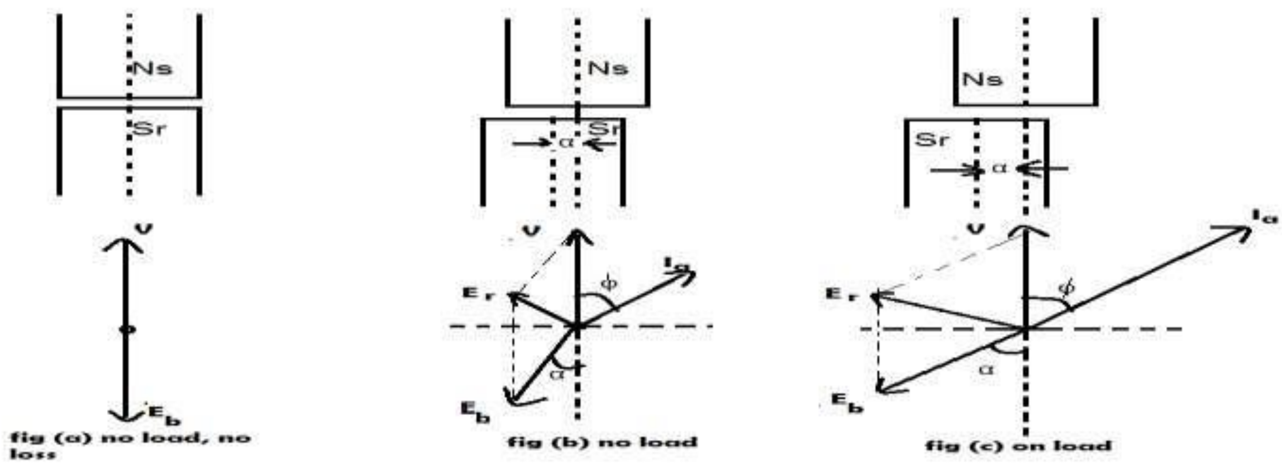
Since, damper winding resistance is low, high current is withdrawn from the supply. So, to avoid high current, synchronous motor is started is usually as slip-ring induction motor. Rotor winding is connected in series with triplex rheostat at time of starting by throw over switch. As motor attains synchronous speed switch is put on side of exciter terminal and dc supply passes through rotor winding.

4.3 NO-LOAD AND LOAD OPERATION

The synchronous motor is not self-starting. It had to be run up to its synchronous speed by some auxiliary means and supply to the dc winding of the rotor had to be switched on, then the rotor pole will get magnetically locked up with stator poles. However, engagement between the stator and rotor poles is not absolutely rigid one. As the load on the motor increases, the rotor continuously tends to fall in phase (not in speed) by some angle, but the motor still continues to run with the synchronous speed. At no load, if there is no power loss in the motor, the stator poles and rotor poles will be along the same axis and phase difference between the applied voltage and back emf E_b will be exactly $180°$ as shown in fig.(a). But this is not in practice because some power loss takes place due to iron loss and friction loss. Hence, the rotor pole will be lags by some angle α with the stator pole and the phasor diagram will be shown in fig (b). The current drawn by armature at no load is

$I_a = \frac{\vec{V} - \vec{E_b}}{Z_s} = \frac{\vec{E_R}}{Z_s}$, where $\overrightarrow{E_R}$ = resultant voltage and Z_s=synchronous impedance per phase.

When the load on the motor increases (speed remains constant) the rotor pole lags the stator pole by large angle α and the phase angle between E_b and V will increase whereas magnitude E_b of remains constant. So that the net voltage E_R will increase and armature current I_a also increases.



4.4 EFFECT OF EXCITATION AND POWER FACTOR CONTROL

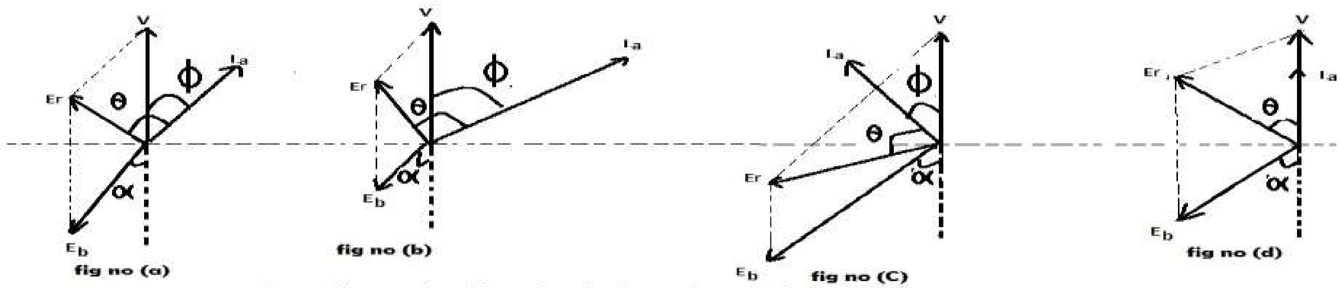
The dc current supply to rotor field winding is known as excitation in synchronous motor. As the speed and magnitude of back emf remains constant incase of synchronous motor, provided that flux per pole produced by rotor doesn't change. So, the magnitude of back emf can be changed by field excitation. If the excitation is changed at constant load, the magnitude of armature current and power factor will be change by changing the excitation, the motor can be operated at both leading and lagging p.f. Consider a synchronous motor loaded with constant mechanical load and operating at a constant supply voltage.

When the excitation is 100% i.e. when E_b=V, armature current lags I_a lags behind the resultant voltage E_R by a fixed angle θ where $\theta = \tan^{-1}\frac{X_s}{R_e}$ and lags behind the applied voltage v by an angle of $\emptyset$, if angle as shown in fig (a) since X_s and R_e are constant angle θ *also* remains constant.

When the excitation is reduced (under excitation) the excitation voltage E_b decreases in magnitude to value shown in fig (b) which shifts the resultant voltage E_R in clockwise direction and since the phase angle θ between E_R and I_a being equal to $\tan^{-1}\frac{X_s}{R_e}$ is fixed so, armature current phasor is also shifted in clockwise direction. Thus power angle Ø increases and hence power factor decreases but active component $I_a \cos \emptyset$ remains constant. So that output power remains constant.

When excitation is increased (over excitation). The excitation voltage E_b increases in magnitude to the value shown in fig(c) which shifts the resultant voltage E_R in counter clockwise direction. θ also remains constant so that the value of I_a also increases to keep the $I_a \cos \emptyset$(output power) constant as shown in fig(c). The phase of E_R changes so that I_a become leading with respect to V in over excitation condition. So power factor of the motor becomes leading in nature and overexcited synchronous motor works on leading power factor.

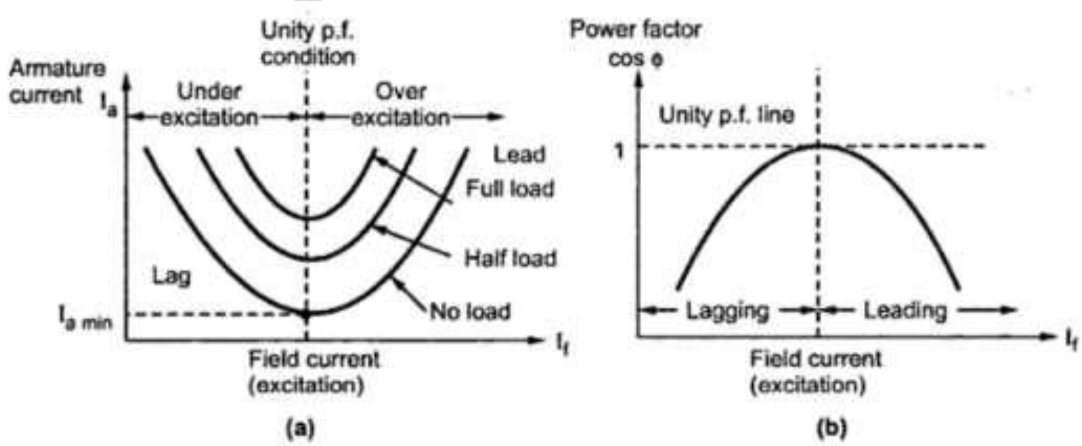
When the excitation is changed, the power factor also changes. The excitation for which the power factor of motor is unity (cos Ø=1) this is called critical excitation. For a certain excitation power angle Ø between applied voltage V and current I_a is zero. At this instant the value of current drawn from the main supply will be minimum as shown if fig(d), which occurs only when power factor is unity.



phasor diagram for effect of excitation and power factor control

V-curves and inverted V-curves:

From above, it is clear that if excitation is varied from very low (under excitation) to very high (over excitation) value, then value of armature current also varies. The current has larger value at both over and under excitation. In between, it has minimum value corresponding to a certain excitation for which power factor is unity. If the graph of armature current drawn by the motor (I_a) against field current (I_f) is plotted, then its shape looks like an English alphabet V. If such graphs are obtained at various load condition, we get family of curves, all looking like V. Such curves are called V-curves shown below. For the same input armature current varies between a wide ranges and power factor also vary accordingly with excitation, when over excited motor runs with leading power factor and motor runs with lagging power factor when under excited. If power factor is plotted against, then the shape of graph looks like an inverted V as shown below.



4.5 HUNTING

It is seen that, when synchronous motor is on no load, the stator and the rotor pole axes almost coincide with each other. When the synchronous motor is loaded, the rotor axis falls back with respect to stator. The angle by which rotor falls back called load angle and denoted by δ. If the load is suddenly through off, the rotor tries to retard to take exact position to the forward field but due to inertia of rotor the rotor poles travels to far. They are then pulled back again and so on. Thus, an oscillation is set up about the equilibrium position, due to sudden application or removal of load is called hunting.

Due to such hunting, the load angle changes its value about its final value δ. As changes for same excitation i.e. the current drawn by the motor also changes. Hence, during hunting there are changes in the current drawn by the motor which may cause the problem to the other appliances connected to the same line. The change in armature current due to hunting is shown below. If such variations, synchronies with neutral period of oscillation of the rotor, the amplitude of swing may become great that the motor may come out of synchronism. At this instant mechanical stress on the rotor is sever and current drawn by the motor is also large. So, the motor gets subjected to large electrical and mechanical stress.

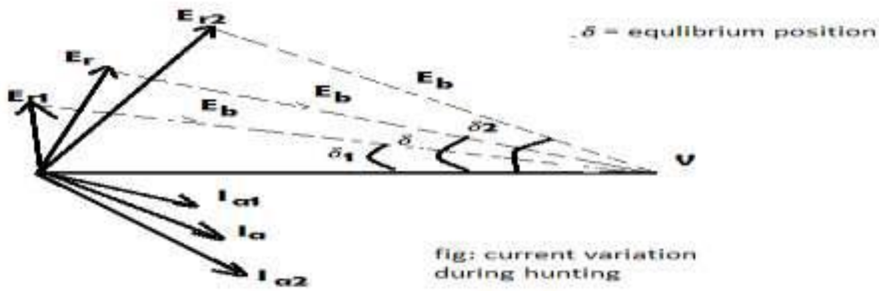
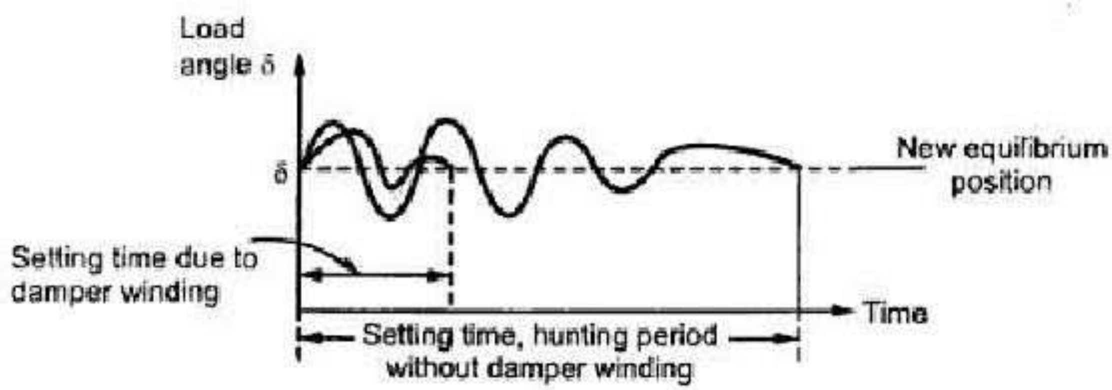


fig: current variation during hunting

Prevention:

In the slots provided in the pole faces, a short-circuited winding is placed. This is called damper winding. When the rotor starts rotating i.e. when hunting starts, a relative motion between damper winding and rotating magnetic field is created. Due to this relative motion emf gets induced in damper winding. According to Lenz's law, the direction of induced emf is always oppose the case producing it. The cause in this case is hunting. So, such induced emf oppose the hunting. The induced emf tries to damp the oscillation as quickly as possible. Thus, hunting is minimized due to damper winding. The time required by rotor to take its equilibrium position after hunting is called settling time.

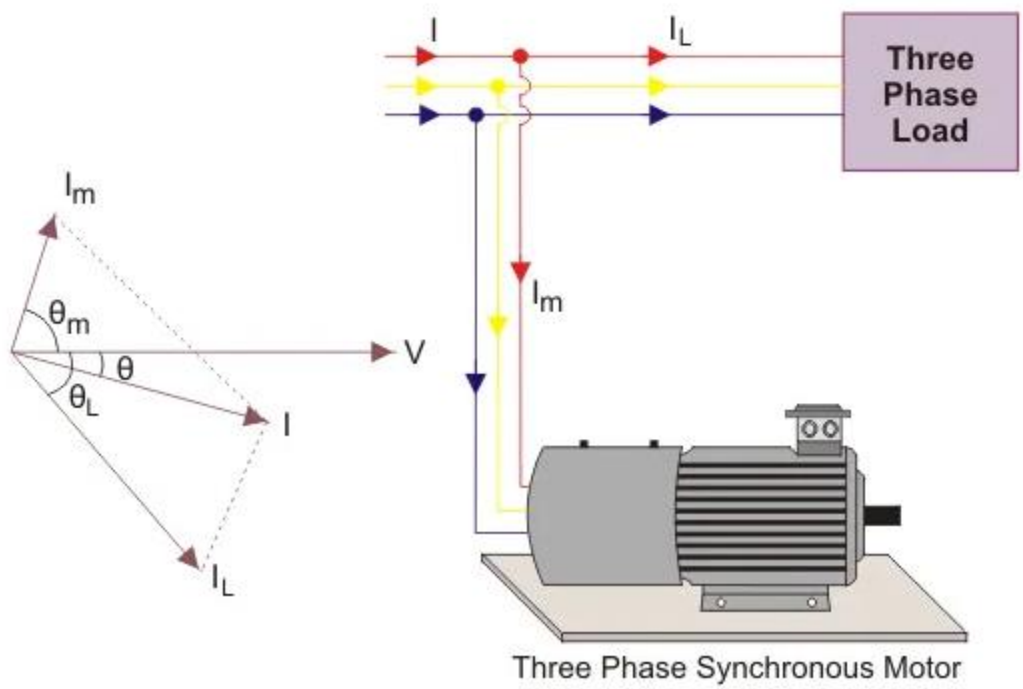


. fig: effect of damper winding on hunting

Er. Sandeep Neupane

4.6 SYNCHRONOUS CONDENSER

Like capacitor bank, we can use an overexcited synchronous motor to improve the poor power factor of a power system. The main advantage of using synchronous motor is that the improvement of power factor is smooth. When a synchronous motor runs with over-excitation, it draws leading current from the source. We use this property of a synchronous motor for the purpose. Here, in a three-phase system, we connect one three-phase synchronous motor and run it at no load.
Suppose due to a reactive load of the power system the system draws a current I_L from the source at a lagging angle θ_L in respect of voltage. Now the motor draws a I_M from the same source at a leading angle θ_M. Now the total current drawn from the source is the vector sum of the load current I_L and motor current I_M. The resultant current I drawn from the source has an angle θ in respect of voltage. The angle θ is less than angle θ_L. Hence power factor of the system cosθ is now more than the power factor cosθL of the system before we connect the **synchronous condenser** to the system.



The synchronous condenser is the more advanced technique of improving power factor than a static capacitor bank, but power factor improvement by synchronous condenser below 500 kVAR is not economical than that by a static capacitor bank. For major power network we use synchronous condensers for the purpose, but for comparatively lower rated systems we usually employ capacitor bank.
The advantages of a synchronous condenser are that we can control the power factor of system smoothly without stepping as per requirement. In case of a static capacitor bank, this fine adjustments of power factor cannot be possible rather a capacitor bank improves the power factor stepwise. The short circuit withstand-limit of the armature winding of a synchronous motor is high. Although, synchronous condenser system has some disadvantages. The system is not silent since the synchronous motor has to rotate continuously. An ideal load less synchronous motor draws leading current at 90^o(electrical).

NUMERICALS:

1) A 75 KW,3 phase , Y-connected, 50 Hz, 440-V cylindrical rotor synchronous motor operates at rated condition with 0.8 p.f. leading. The motor efficiency excluding field and stator losses is 95% and X_s=2.5Ω. calculate i) mechanical power developed ii) armature current iii) back emf iv) power angle v) maximum or pull out torque of motor.

- Solution

i) Mechanical power developed (P_m) = P_{in}=$\frac{P_{out}}{\eta}$=$\frac{75*10^3}{.95}$=78,950W

ii) ii) we know when input power is known: P_{in}= √3 I_a $V_l$cos(∅) =>78,950=√3 *440*0.8 I_a

∴ I_a =129A

iii) Applied voltage per phase=440/√3 =254V

let V=254∠0V and I_a =129∠ $\cos^{-1}$ 0.8 =129∠36.86 (here power factor is positive so that angle of current is taken positive).

we know in case of motor $\overrightarrow{E_b}$ = $\overrightarrow{V_t}$ - j I_a $\overrightarrow{X_s}$

=254∠0V – (129∠36.86 *2.5∠90) => 516.52∠-30 V

∴Required back emf is 516.52∠-30 V

iv) From above we can write power angle =-30^0

v) maximum torque occurs when the value of δ= 90^0

we know that P_e =3$\frac{V_t E_b}{X_s}$ $\sin\delta$ =3$\frac{256*516.2}{2.5}$ =>158576.64 Watt

we also know that n_s=$\frac{2*f}{P}$=$\frac{2*50}{4}$ =25 rps ∴ maximum torque= T=$\frac{P}{\omega_s}$=$\frac{P}{2\pi n_s}$=$\frac{158576.64}{2*3.14*25}$ =1010 N-m.

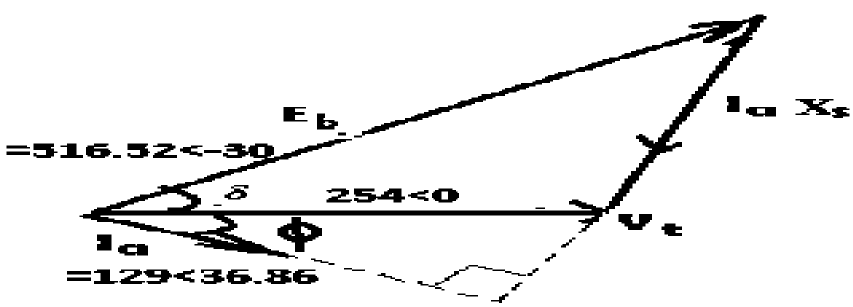


fig :phasor diagram

2) A 1000 KVA, 11 KV, 3 phase , Y connected synchronous motor has an armature resistance and reactance per phase of 3.5Ω and 40Ω respectively. Determine the induced e.m.f per phase(suggestion if question has not declared per phase back emf then calculate back line emf) and angular retardation of the rotor when fully loaded at i) unity p.f ii) 0.8 p.f. lagging iii) 0.8 p.f leading.

- Solution

Here, full load current (I_a) = $\frac{KVA}{\sqrt{3}*KV}$ = $\frac{1,000*1000}{\sqrt{3}*11000}$ = 52.5 A

voltage per phase (V) = 11,000/√3 =6351 V

Armature resistance drop per phase= I_a*R_a=184v

Reactance drop per phase= I_a*X_s =2,100V impedance drop per phase = I_a*z_s= $\sqrt{184^2+2100^2}$ =2100 V(approx)

Z_s=$\sqrt{3.5^2+40^2}$ = 40.15Ω

∴θ=$\tan^{-1}\frac{Z_s}{R_a}$=$\tan^{-1}\frac{40.15}{3.5}$ =85^0

i) At unity p.f. I_a=52.5 A

we know for motor

$\overrightarrow{E_b} = \vec{V}$ - $\overrightarrow{I_a}R_a$ - j$\overrightarrow{I_a}\,X_s$ = 6351∠0 - 52.5∠0*3.5 -52.5∠0*40∠90 =6515∠ − 18

∴Back emf is=6515V per phase and angular retardation is α = 18^0.

ii) At 0.8 lagging power factor

I_a=52.5∠-$\cos^{-1}$ 0.8=52.5∠-36.86

Here

$\overrightarrow{E_b} = \vec{V}$ - $\overrightarrow{I_a}R_a$ - j$\overrightarrow{I_a}\,X_s$ = 6351∠0 - 52.5∠ − 36.86*3.5 -52.5∠ − 36.86*40∠90

= 5187.55∠-17^0V

∴Back emf is=5187.55V per phase and angular retardation is α = 17^0.

iii) At 0.8 leading p.f

I_a=52.5∠ $\cos^{-1}$ 0.8=52.5∠36.86

Here

$\overrightarrow{E_b} = \vec{V}$ - $\overrightarrow{I_a}R_a$ - j$\overrightarrow{I_a}\,X_s$ = 6351∠0 - 52.5∠36.86*3.5 -52.5∠36.86*40∠90 =7675.43∠-13.48.

∴Back emf is=7675.43V per phase and angular retardation is α = 13.48^0.

The respective phasor diagrams are:

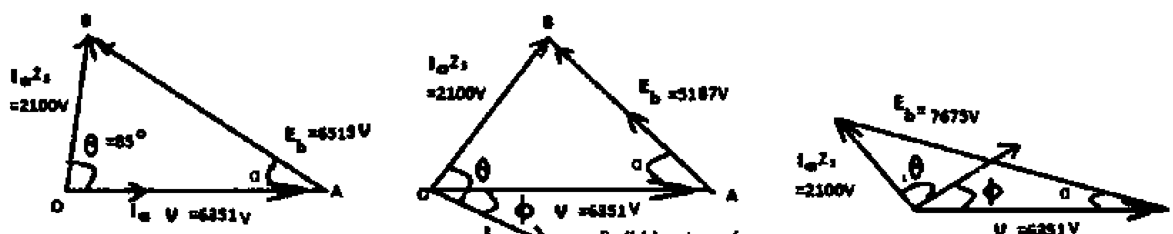


fig (a) unity p.f

fig (b) lagging p.f

fog(c) leading p.f

4) A 20 MVA, 3 phase star connected 11 KV, 12 pole, 50 Hz salient pole synchronous motor has a direct axis reactance of 50 ohm and quadrature axis reactance of 3 ohm per phase. The armature resistance being negligible. At rated load, unity p.f and rated voltage determine i) Excitation voltage b) the maximum value of power angle and corresponding power.(BE/2071 chitra)

➢ Solution

Full load rated current (I_a)=$\frac{kva}{\sqrt{3}*KV}$=$\frac{20*10^6}{\sqrt{3}*11*10^3}$=1049.72A

voltage per phase= 11/$\sqrt{3}$ =6.350V

we also have cosΦ=1 ∴Φ=0^o

we know δ=$\tan^{-1}[\frac{I_aX_q\cos\emptyset}{V-I_aX_q\sin\emptyset}]$ = $\tan^{-1}[\frac{1049.72*3*1}{6350}]$ = 26.37^o

Also; I_a=$I_a$sin($\emptyset$-δ) = 1049.72*sin(-26.37)= -466.33 A

Now; i) Excitation voltage is given by(E_0)=Vcos δ- I_dX_d =6350*cos(-26.37)-(-466.33)*50 =29.005KV

ii) For maximum value of power angle w know direct formula:

$$\cos\delta_{max}=-\frac{E_o}{4V(\frac{X_d-X_q}{X_q})}+\sqrt{\frac{1}{2}+(\frac{E_oX_q}{4V(X_d-X_q)})^2} \quad = -\frac{29005}{4*6350(\frac{50-3}{3})}+\sqrt{\frac{1}{2}+(\frac{29005*3}{4*6350(50-3)})^2} \quad =0.64$$

∴ δ_{max}=50.14^0.

iii) for corresponding maximum power we have; $P_{out\,max}$=3 $\frac{VE_o sin\delta}{X_d}$ +3 $\frac{V^2}{2}$ sin2δ($\frac{1}{X_q}$ - $\frac{1}{X_d}$)

=3 * $\frac{6350*29005*sin50.14^0}{50}$ +3 $\frac{6350^2}{2}$ sin2 * 50.14^0($\frac{1}{3}$ - $\frac{1}{50}$) = 6.41 MW.

4)A 3 phase, 10MVA, 2300V, 60HZ synchronous motor has X_s=0.9 p.u and negligible stator resistance. The motor is connected to infinite bus. If terminal voltage V_t=2300<0 Vand excitation emf E_o= 3450<120V. Determine power transfer and power factor of machine.(BE/2071 back)

- Solution

Assuming delta connected motor and base voltage 2300V

terminal voltage in p.u. (V_t) =2300/2300 =1p.u.

and excitation voltage in p.u is= $\frac{3450<120}{2300}$ =1.5<120 p.u.

we have for motor, $\overrightarrow{E_b} = \overrightarrow{V_t}$ - j $\overrightarrow{I_a}$ $\overrightarrow{X_s}$

$\overrightarrow{I_a}$=$\frac{\overrightarrow{V_t} - \overrightarrow{E_b}}{j\overrightarrow{X_s}}$ =$\frac{1<0-1.5<120}{0.90<90}$ =2.42<−126.58^{0}p.u.

∴Required p.f. of machine is cos(126.58) =0.5959 which is lagging in nature.

Now power transfer is given by $P_e = \frac{V_t E_b}{X_s} \sin\delta = \frac{1*1.5}{0.9} \sin 120$ =1.44 p.u.

5) A three phase, 5 kVA, 208V, 4 pole, 50 Hz star connected synchronous motor has negligible armature winding resistance and resistance and synchronous reactance of 8 ohm per phase. It is operated from the three phase the, 208, 50HZ supply and field excitation is adjusted to that the power factor is unity and the motor draw a power of 3 KW from the supply.

i) find the back emf voltage and power angle.

ii)keeping the excitation voltage constant, the power angle is increased by 20% due to increase in load on the shaft. Calculate the new armature current and power factor.

iii) if the excitation is held constant and the shaft load is slowly increased, determine the maximum torque. (BE/2072 chaitra)

- Solution

load power is P_l =3 kw.

Armature current is given by=$\frac{KW}{\sqrt{3}* V_l*cos\emptyset}$=$\frac{3000}{\sqrt{3}*208*1}$ =8.32A

voltage per phase is (V_t) =208/$\sqrt{3}$ =120.08V (angle of armature current is 0 as p.f is unity)

For motor we have; $\overrightarrow{E_b} = \overrightarrow{V_t}$ - j $\overrightarrow{I_{a1}}$ $\overrightarrow{X_s}$ =120.08<0 – 8.32<0*8<90 =137.30<-28.99

i) ∴Required back emf is E_{b1}=137.30 V and power angle is δ_1= −28.99^0

II) according to question we have back emf is constant E_{b2}=137.30V and power angle is increased by 20% so, we have new power angle as δ_2=δ_1+0.2δ_1=−34.788^0.

Now for motor we have ; $\overrightarrow{E_{b2}} = \overrightarrow{V_t}$ - j $\overrightarrow{I_{a2}}$ $\overrightarrow{X_s}$ or, $\overrightarrow{I_{a2}}$=$\frac{\overrightarrow{V_t} - \overrightarrow{E_b}}{j\overrightarrow{X_s}}$ = $\frac{120.08<0-137.30<-34.788}{j\,8}$

= 9.83<-5.338 A

∴ required armature current is 9.83A and power factor is cos(5.33) =0.99 lagging .

ii) Here back emf is also constant so E_{b3} = 137.30V and for maximum torque we know δ_{max}=90 at this condition power is given by: $P_{e\,max}$ =3 * $\frac{V_t E_{b3}}{X_s}$ sin δmax = $\frac{3*120.08*137.30*1}{8}$

=6182.619 watt.

$n_s = \frac{2*f}{P}$ =30 .

For maximum torque we have; T_{max} =$\frac{P_{e\,max}}{\omega_s}$=$\frac{P_{e\,max}}{2\pi n_s}$= 33 N-m.

UNIT 5: INDUCTION TYPE MACHINE

5.1 THREE PHASE INDUCTION MOTOR

5.1.1 Constructional Details

The three-phase induction motor is very simple in construction compared to dc motor or synchronous motor. The essential features of a three phase induction motor are: a laminated stator core carrying three phase winding; a laminated rotor core carrying either a cage or three phase winding, the later with the shaft mounted slip rings; a stiff shaft to preserve the very short air gap; a frame to form the stator housing and carry the end covers, bearings, and terminal box. Non-salient pole construction is used for all polyphase induction motor.
It consists of three main parts. They are;

Stator:

It is the stationary part of the motor which is like cylinder having hollow at center. It is made up of numbers of circular laminated stampings. It contains the number of alternate slots and tooth in the inner circumference on which the stator winding is placed. Generally, three phase windings are provided on these slots which are uniformly distributed and each phase windings are spaced 120^0 electrically apart. These windings when supplied by three phase voltage, creates the definite number of magnetic poles on stator core.

Rotor:

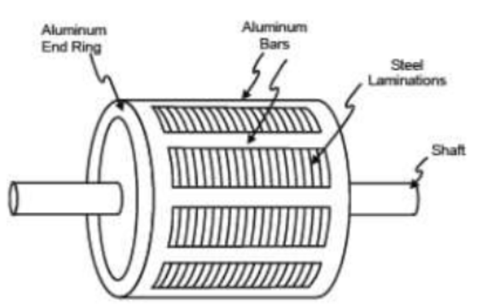
It is the central rotating part of the motor. It is cylindrical in shape with central shaft. The shaft is supported by bearings at both ends so that rotor rotates freely keeping the air gap of 1-4 mm between the rotor and stator. It is made up of laminated silicon steel. Rotor are of two types. They are;

a) Squirrel Cage Rotor:

This type of rotor is made of cylindrical laminated core with parallel slots near the outer circumference. These parallel slots carry the rotor conductors and the end of these conductors are short circuited by copper ring called end ring. Almost 90% of induction motor are provided with squirrel cage rotor because of its very simple, robust and almost instructible construction.

b) Phase Wound Rotor/ Slip Ring:

This type of rotor is made of cylindrical laminated core but it has open slots along the outer circumference on which the three phase windings are provided with same number of poles as that in the stator winding. The three end of rotor windings are connected to the separate slip rings and the slip rings are short circuited by the carbon brushes with or without external resistance. The slip ring is electrically insulated from shaft.



Squirrel cage induction motor rotor

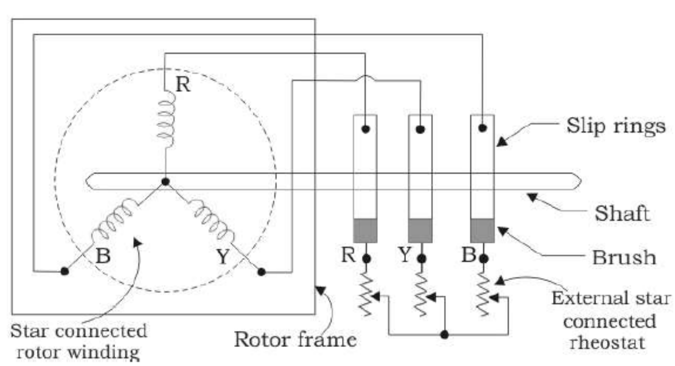


Fig : Slip ring Rotor

5.1.2 Operating Principle

When the stator or primary winding of a 3-phase induction motor is connected to a 3-phase ac supply, a rotating magnetic field is established which rotates at synchronous speed. As the rotating magnetic field produced by primary currents sweeps across the rotor conductors, an emf is induced in the rotor winding. Since the rotor winding are either directly shorted or closed through some external resistance, current starts to flow in rotor winding. Now the current carrying conductor under rotating magnetic field experience the force and Torque is setup to rotate the rotor.

Let 3-phase current flows through stator winding, each phase winding will produce their own magnetic flux whose nature will be as 3-phase current. The three fluxes ϕ_R, ϕ_Y, ϕ_B are the alternating in nature and are 120° out of phase with each other as

$\phi_R = \phi_m \sin\omega t$

$\phi_Y = \phi_m \sin(\omega t - 120)$

$\phi_B = \phi_m \sin(\omega t + 120)$

When $\omega t = 0$

$\phi_R = 0$

$\phi_Y = \phi_m \sin(0-120) = -\frac{\sqrt{3}}{2}\phi_m$

$\phi_B = \phi_m \sin(0+120) = \frac{\sqrt{3}}{2}\phi_m$

$\phi_T = \sqrt{\phi_Y^2 + \phi_B^2 + 2\phi_Y\phi_B\cos\theta} = 1.5\,\phi_m$

similarly when $\omega t = 60^0$

$\phi_R = \phi_m \sin\omega t = \phi_m \sin 60 = \frac{\sqrt{3}}{2}\phi_m$

$\phi_Y = \phi_m \sin(60-120) = -\frac{\sqrt{3}}{2}\phi_m$

$\phi_B = \phi_m \sin(60+120) = 0$

$\phi_T = \sqrt{\phi_R^2 + \phi_Y^2 + 2\phi_R\phi_Y\cos\theta} = 1.5\,\phi_m$

Again when $\omega t = 120^0$

$\phi_R = \phi_m \sin\omega t = \phi_m \sin 120 = -\frac{\sqrt{3}}{2}\phi_m$

$\phi_Y = \phi_m \sin(60-120) = 0$

$\phi_B = \phi_m \sin(60+120) = \frac{\sqrt{3}}{2}\phi_m$

$\phi_T = 1.5\,\phi_m$

Hence the stator winding produces a rotating magnetic field.

The speed of rotating magnetic field is

$N_s = \frac{120f}{P}$ rpm

Where f = frequency of applied voltage to the stator.

P = No. of the magnetic poles.

This speed is known as Synchronous speed.

The rotating magnetic field produced by the stator cuts the rotor conductor which are at rest initially, hence emf is induced on conductor due to electromagnetic induction. As the rotor conductors are shorted, current will circulate within the conductors. Now, those current carrying conductor lies under the magnetic field of stator. Hence force will develop on the rotor conductors. So, the rotor starts rotating under the action of force.

The direction of rotation can be determined by using Lenz's law. The direction of force will be in such a way that it opposes the cause by which the emf was induced in the rotor conductor. The main cause of rotor emf is relative speed between the rotating magnetic field and rotor. Therefore, in order to reduce this relative speed, the rotor will rotate in same direction of rotating magnetic field. The rotor will try to catch up the speed of rotating magnetic field (N_s) but it never success to do so and always run at a speed less than the synchronous speed.

If the rotor catches up the speed N_s the relative speed between rotating field and rotor will be zero and hence the no current will flow in the rotor conductor and no force will develop in rotor. Hence rotor rotates always with the speed less than synchronous speed. So, the induction motor is called asynchronous speed.

Slip : Let N be the speed of rotor in rpm. Then $\frac{N_s-N}{N_s}$ is a factor indicating the fraction by which the speed of rotor is less than synchronous speed N_s. This factor is known as slip(s).

Fraction slip, s = $\frac{Synchronous\ speed-rotor\ speed}{synchronous\ speed} = \frac{N_s-N}{N_s}$

And Percentage slip = $\frac{N_s-N}{N_s} * 100\%$

The slip of an induction motor change w.r.to load on the motor.

Standstill condition:

Standstill condition is the condition at the instant of starting at which the rotor is at rest. At this time speed of rotor is zero, so the relative speed N_s- N is maximum and slip is maximum (s = $\frac{N_s-N}{N_s}$ =1). So the maximum emf will induce in rotor circuit without changing the frequency(f = $\frac{N_sP}{120}$ Hz).

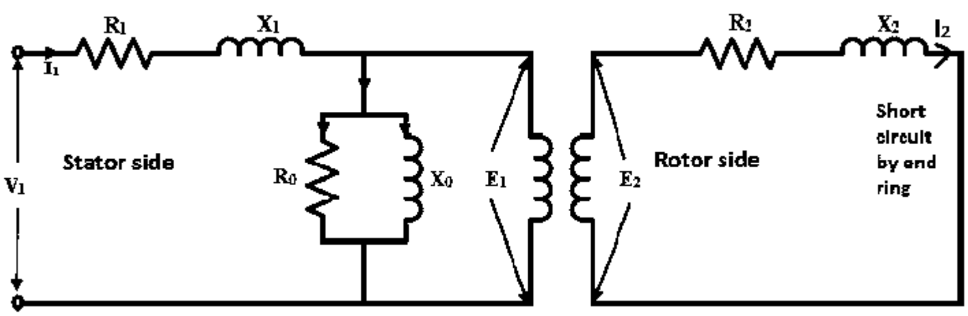


Fig: Equivalent circuit of induction motor at standstill condition

Let us consider the equivalent circuit of the induction motor as shown in the figure which is similar to the transformer. The stator winding is analogous to the primary winding of transformer and rotor winding is analogous to the secondary winding of transformer.

Consider,

V_1 = Supply voltage to stator winding per phase

I_1 = Stator current per phase

I_0 = No load current per phase

E_1 = Stator emf per phase

E_2 = Rotor emf per phase at standstill condition

R_1 = Stator winding resistance per phase

X_1 = Stator leakage reactance per phase
R_2 = Rotor winding resistance per phase
X_2 = Rotor winding leakage reactance per phase
I_2 = Rotor current per phase
N_1 = No. of turns per phase in stator
N_2 = No. of turns per phase in rotor

Now,
Rotor emf per phase at standstill condition (E_2) is given by

$$E_2 = \frac{N_2}{N_1} E_1$$

Rotor current per phase at standstill condition,

$$I_2 = \frac{E_2}{\sqrt{R_2^2 + X_2^2}}$$

This current I_2 lags with E_2 by an angle of ϕ_2

$$\text{Where } \cos\phi_2 = \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

The torque developed by the rotor at standstill condition is proportional to product of stator flux per pole and active component of I_2.

$$T_2 \propto \psi I_2 \cos\phi_2$$

$$T_2 = K \psi I_2 \cos\phi_2$$

Where ψ = stator flux per pole.
Similar in Transformer ψ remains constant and independent to I_1 and I_2. It depends only on E_1.

$E_2 \propto E_1$, $\psi \propto E_2$

$$T_s = K_1 E_2 \frac{E_2}{\sqrt{R_2^2 + X_2^2}} * \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

$$T_s = \frac{K_1 E_2^2 R_2}{R_2^2 + X_2^2}$$

Where $K_1 = \frac{3}{2\pi n_s}$

Running Condition:

When the rotor rotates, the relative speed between rotating magnetic field and rotor will decrease , there by reducing the rate of cutting the flux. Therefore the magnitude of induced emf on the rotor will decrease with compare to emf at standstill.

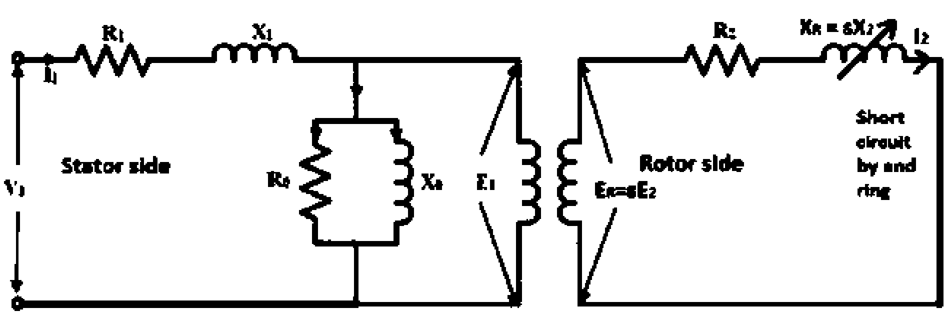


Fig: Equivalent circuit of induction motor at running condition

The magnitude of emf induced in the rotor at running condition is,

$E_R = sE_2$

As the relative speed decrease, the frequency of rotor emf will also decrease with compare to that at standstill condition.

The frequency of rotor is

$f_r = \frac{(N_s - N)P}{120}$

$\frac{f_r}{f} = \frac{(N_s - N)P}{120} * \frac{120}{N_s P} = \frac{(N_s - N)}{N_s} = s$

$$f_r = sf$$

At standstill condition s =1, but at running condition 's' is less than 1. So $f_r <$ f and the value of rotor leakage reactance is given by

$X_R = sX_2$

X_R is shown variable because it is changed w.r.to the speed of the rotor. Rotor current at running condition,

$I_R = \frac{E_R}{\sqrt{R_2^2 + X_R^2}} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$

I_R lags with E_R by an angle ϕ_R

Where $\cos\phi_R = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$

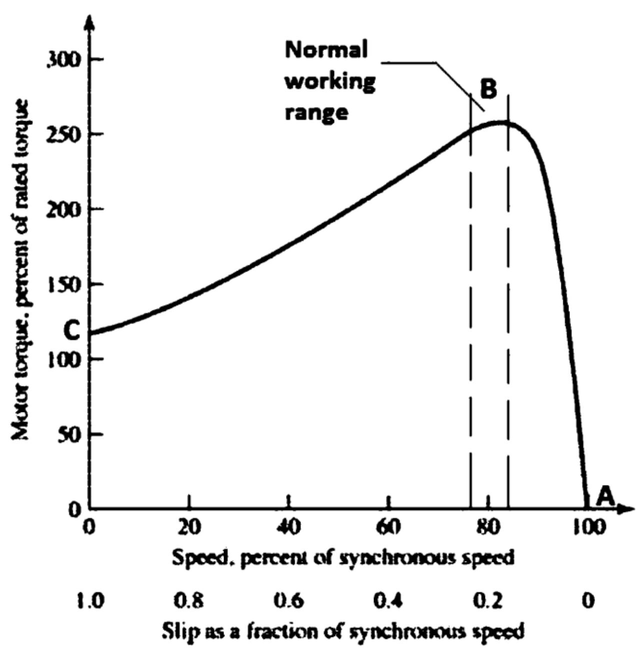
Torque developed by rotor at running condition is,

$T_R = K_1 E_2 \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} * \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$

$T_R = \frac{K_1 s E_2^2 R_2}{R_2^2 + (sX_2)^2}$

5.1.3 Torque Slip Characteristics

The curve showing the torque developed by rotor at various values of Slip or speed is termed as Torque-Slip characteristics of I/M.



The torque equation is given by;

$T_R = \frac{K_1 s E_2^2 R_2}{R_2^2 + (sX_2)^2}$

Case 1 : When rotor speed is equal to synchronous speed ($N = N_s$)

The slip is given by $s = \frac{N_s - N}{N_s} = 0$

So, Torque developed $T_R = 0$.

This represent point A on curve.

Case 2: When rotor speed is nearly equal to synchronous speed ($N \rightarrow N_s$)

The slip is given by $s = \frac{N_s - N}{N_s}$ will be very small

So $(sX_2)^2$ will be more small, so this component can be neglected with compare to R_2^2

Now from torque equation ;

$T_R = \frac{K_1 s E_2^2 R_2}{R_2^2}$

$T_R = \frac{K_1 E_2^2}{R_2} s$

$T_R \propto s \rightarrow$ if E_2^2 and R_2 are kept constant.

Hence AB is the straight line in the curve which means Torque increases proportionally with increase in the slip or with decrease in speed.

Case 3 : If the speed of the rotor is further decreased due to the overloading, the value of slip will be no more enough to make the $(sX_2)^2$ negligible. In this case R_2^2 is neglected.

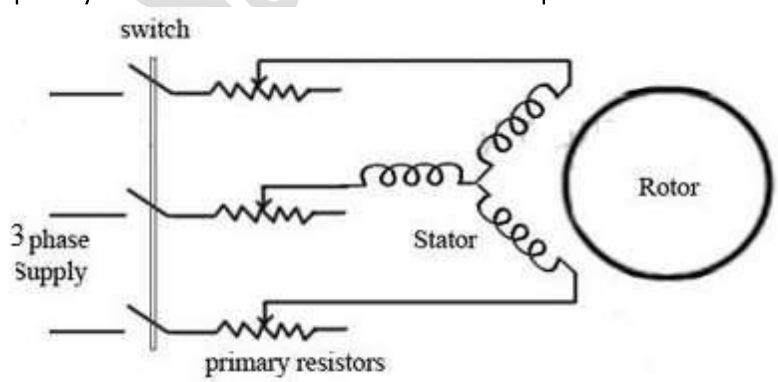
$T_R = \frac{K_1 s E_2^2 R_2}{(sX_2)^2}$

Or, $T_R = \frac{K_1 E_2^2 R_2}{s(X2)2} \rightarrow T_R \propto \frac{R_2}{s}$ i.e $T_R \propto \frac{1}{s}$ (if E_2^2 and R_2 are kept constant.)

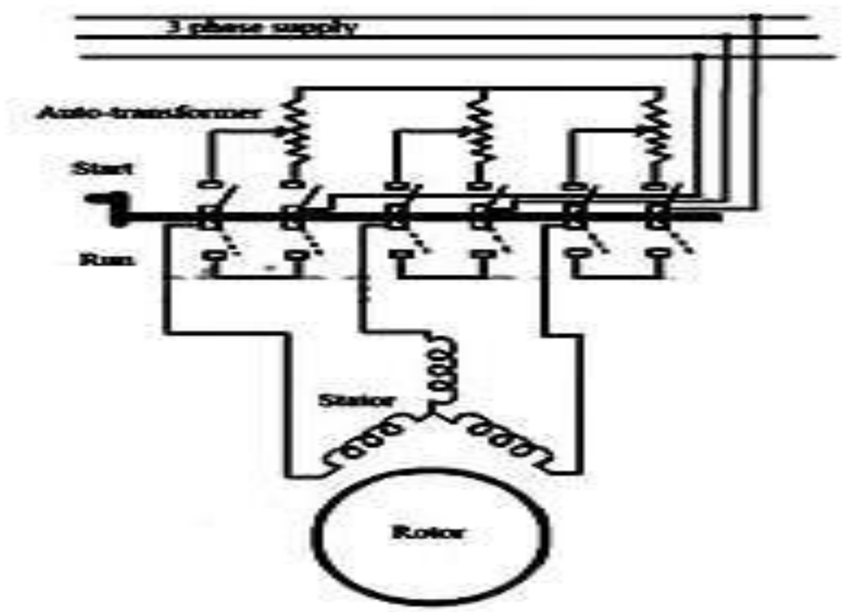
Hence the beyond the maximum torque, any further in speed results in decrease in Torque developed by the rotor which means motor slows down and stops.

5.1.4 Starting Methods

a) Primary Resistor Method: This method uses the variable resistances in series with the stator winding. The purpose of resistor is to drop some voltage and hence to reduce the applied voltage across the stator windings. At starting the whole resistances are connected in series with stator winding so that motor draws low current. As the speed picks up, the resistance is gradually cut out and finally it is completely cut out when motor runs at normal speed.



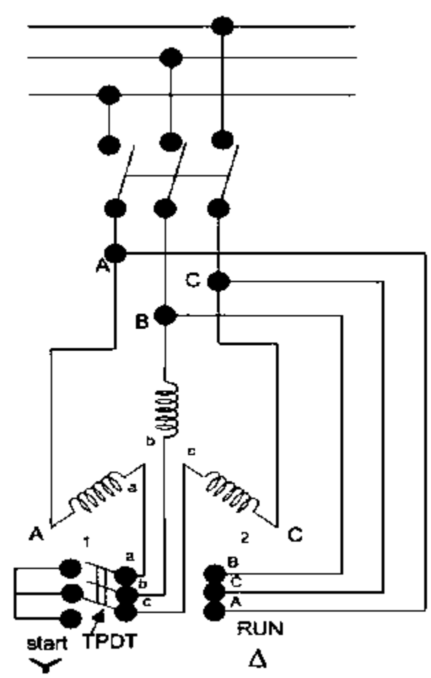
b) Auto Transformer Method : This method uses the Auto transformer to give reduced voltage at starting as shown in figure. When the switch is on start position, a reduced voltage is applied across the stator. When motor run up to 80% of its normal speed, the connections are changed so that the full supply voltage appears across the stator winding without Auto transformer.



c) Star Delta Starter: This method is generally used in case of motor designed to run normally with delta connected stator winding. This method uses a two-way switch which connects the stator winding in star connection for starting period and in delta connection for normal running condition.

If motor is started with delta connection; $I_d = \frac{\sqrt{3}\,V}{Z}$
and if the motor is started with star connection; $I_s = \frac{V/\sqrt{3}}{Z}$.

So, $\frac{I_S}{I_d} = \frac{\sqrt{3}V/Z}{V/2\sqrt{3}}$ or, $I_s = \frac{1}{3} I_d$



5.1.5 Speed Control Methods

The three-phase induction machine are practically constant speed machine more or less like a dc shunt motor. Speed can be controlled in following ways;

a) Changing Applied Voltage: The speed of the induction motor can be increased or decreased by increasing or decreasing the magnitude of the applied voltage. This method is cheaper but rarely used because large change in voltage is required for small change in speed and can create the serious disturbance in magnetic condition of motor. (V α N)

b) Changing Applied Frequency : The synchronous speed of rotating magnetic field is given by $N_s = \frac{120f}{p}$. So ($N_s \alpha f$) i.e speed can be controlled by changing the frequency of applied voltage.

c) Changing Number of Stator Poles : The synchronous speed of rotating magnetic field is given by $N_s = \frac{120f}{p}$ i.e. ($N_s \propto \frac{1}{P}$). The speed is inversely proportional to the no. of magnetic poles in the stator winding. Stator winding can be designed in such a way that the can be connected as 2- poles or 4- poles or 6- poles with the help of special switch.

d) Adding rheostat in the stator circuit : In this method of speed control of three phase induction motor rheostat is added in the stator circuit due to this voltage gets dropped .In case of three phase induction motor torque produced is given by $T \propto sV_2{}^2$. If we decrease supply voltage torque will also decrease. But for supplying the same load, the torque must remains the same and it is only possible if we increase the slip and if the slip increase motor will run reduced speed.

e) Cascade Connection Method: In this method, two motors having different number of poles are mounted on a common shaft. The stator winding of motor A is supplied by main supply voltage of frequency 'f'. The second motor B is supplied by the voltage induced in the rotor of first motor A through slip rings. So the frequency of applied voltage applied to stator of second motor will be different from the main supply frequency. Now the two motor try to move with two different speed corresponding to two different frequency. As the two motor are coupled to same shaft, the system will run at new speed.

Combined slip $s = \frac{P_1}{P_1+P_2}$

$N_s'' = \frac{120f}{P_1+P_2}$

If $N_s' = \frac{120f}{P_1}$

$N_1 = \frac{120f(1-s_1)}{P_1}$

$N_s'' = \frac{120f}{P_2}$

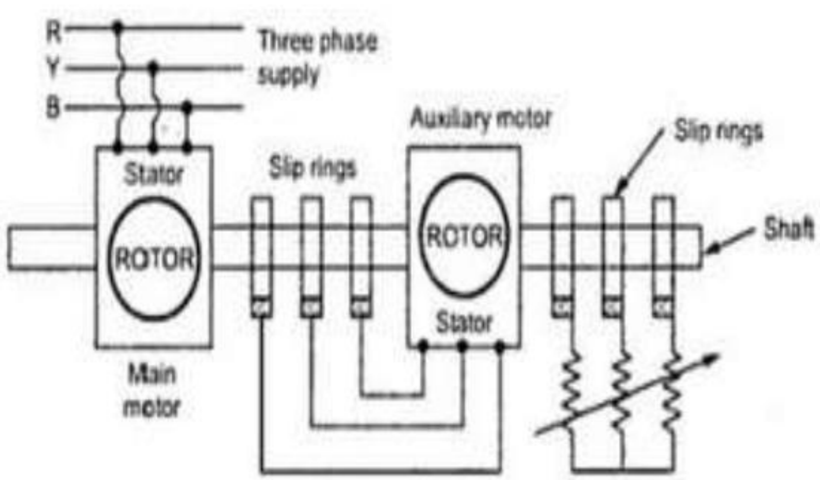


Fig : Cascade Connection.

5.1.6 Double Cage Induction Motor

The main advantages of a squirrel cage motor are poor starting torque because of its low rotor resistance. The starting torque can be increased by having cage of high resistance, but then the motor will have poor efficiency under normal running condition (because there will be more rotor copper loss). In order to have high starting torque without sacrificing its electrical efficiency, under normal condition two independent cages on the same rotor one inside the other is used. The outer cage consists of bars of a high resistance metal whereas the inner cage has low resistance copper bars.
Hence outer cage has high resistance and low ratio of reactance to resistance, whereas the inner cage has low resistance but being situated deep in the rotor has large ratio of reactance to resistance. Hence the outer cage develops maximum torque at starting, while the inner cage does so at about 15% slip.

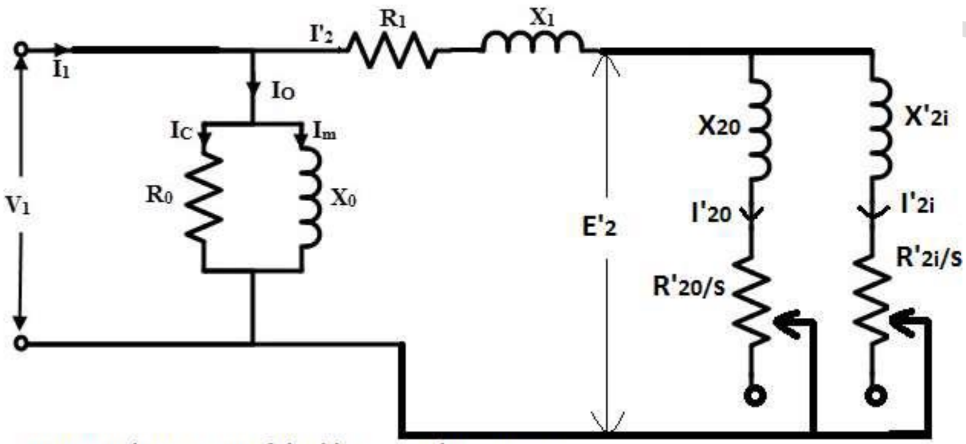
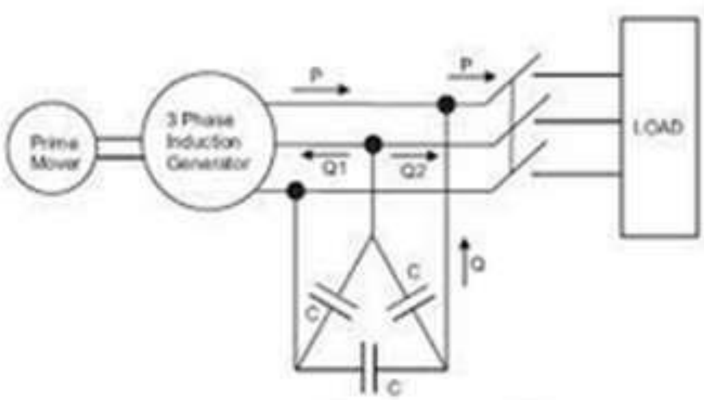
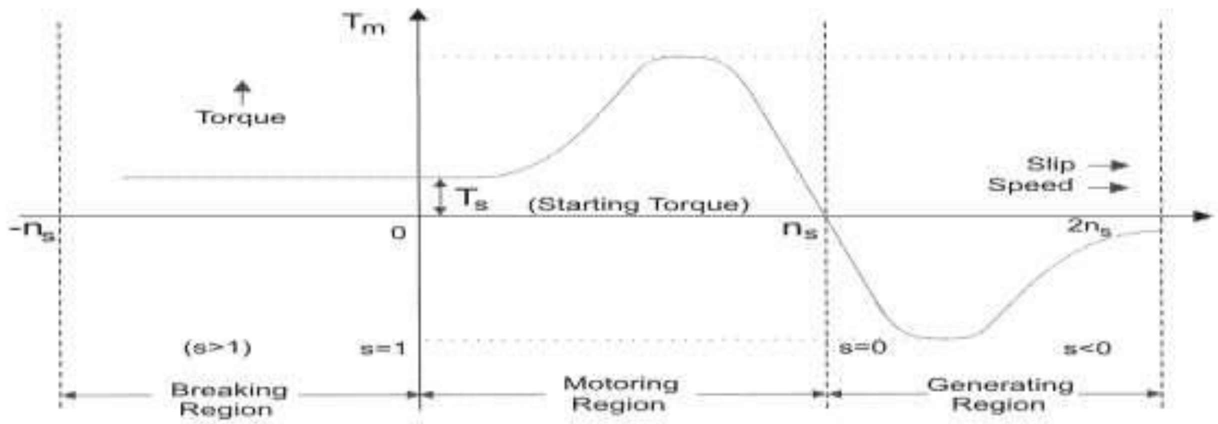


Fig: Equivalent circuit of double cage induction motor

5.2 INDUCTION GENERATOR

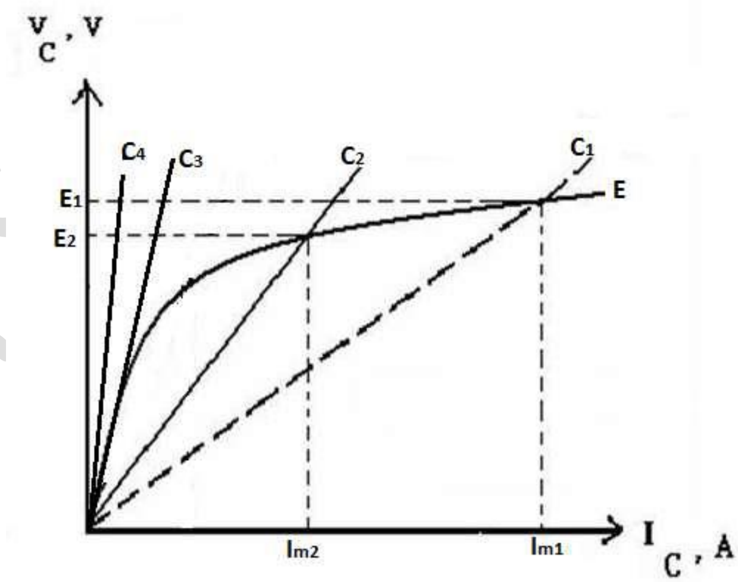
An Induction generator is the induction machine whose shaft is driven by some prime mover above the synchronous speed. If the rotor of the induction machine is driven at the speed above synchronous speed, the stator winding will generate three phase emf provided the air gap flux is maintained by supplying reactive power from external source. The reactive power required can't be obtained from the mechanical power input. So, capacitors have to be used to obtain the reactive power required for maintaining the air gap flux. These capacitor acts as excitation for machine. As the machine is driven above synchronous speed, the slip become negative which indicate that the torque is not developed by the machine but is given to machine through prime mover.





Voltage Build Up in Induction Generator

An induction generator builds up the voltage in similar manner of DC generator. The build-up process in the DC machine depends upon the residual magnetism in the field poles and the final voltage is determined by resistance of field circuit. In case of induction generator, the residual magnetism is sufficient to induce the small ac voltage in the stator. Such a voltage across a capacitor connected at its terminal causes a lagging magnetizing current to flow in stator terminals or leading current through capacitor. If the proper value of capacitor is selected, magnetizing current can be sufficient to increase the existing air gap flux. With the increased air gap flux induced voltage increases resulting in more magnetizing current flow. This process of voltage build-up continues until the induce voltage reaches limit of saturation. The slope of C_1, C_2, C_3, C_4 gives the reactance of capacitor required to produce the voltage corresponding to the point of intersection with curve 'E'. As the value of capacitance decreases, its reactance increases. The curve C_3 gives the infinite number of possible solutions. Below this capacitance the machine will not excite and hence will not operate. The capacitance corresponding to C_3 represents critical capacitance below which machine cannot able to build up the voltage.



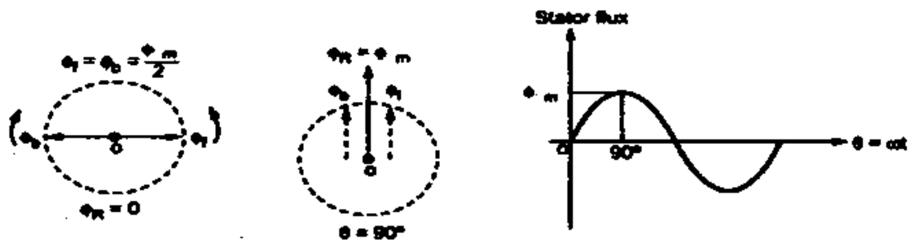
5.3 SINGLE-PHASE INDUCTION MOTOR

A single-phase induction motor is similar to a three phase Induction motor in construction except that its stator is provided with a single-phase winding instead of three phase winding. When a single-phase ac voltage is supplied to the single-phase winding, it will not produce a rotating magnetic field like in the case of three phase induction motors, rather it will produce an alternating magnetic field (Pulsating magnetic field) as shown below. Such a motor cannot inherently develop any starting torque of its own and will not start to rotate if the stator winding is connected to an ac supply. However, if the rotor is given a spin or started with the help of some auxiliary circuits, it will continue to run.

5.3.1 Double Field Revolving Theory

According to this theory, pulsating field produced in a single phase motor can be resolved into two rotating components which rotate in opposite directions with synchronous speed and each having magnitude or half of the maximum magnitude of the pulsating field.

In case of single phase induction motor, the stator winding produces an alternating magnetic field having maximum magnitude of φ_m. According to this theory, consider the two components of the stator flux be forward component (φ_f) and backward component (φ_b) each having magnitude $\frac{\phi m}{2}$ rotating in opposite direction with synchronous speed (N_s). The resultant of these two components at any instants gives the instantaneous value of stator flux at that instant. So, resultant of these two fluxes is original stator flux.



(a) (b) (c)

Fig (a) shows the stator flux and its components φ_b and φ_f . At start, resultant (φ_R) is zero (φ_R=0) because φ_f and φ_b are opposite to each other. After 90° rotation shown in fig (b) , the two components are rotated in such a way that both are pointing in the same direction and hence resultant is algebraic sum of magnitude of φ_b and φ_f. So $\varphi_R=\frac{\phi m}{2}+\frac{\phi m}{2}=\varphi_m$

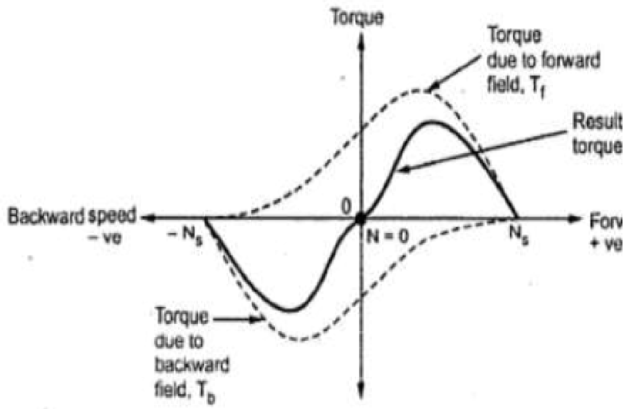
Thus, continuous rotation of φ_b and φ_f gives the original stator flux shown in fig (c).

Since, the both components are rotating and hence get cut by the motor conductors, emf gets induced in the rotor which circulates rotor current. The rotor current produces rotor flux which interacts with φ_f to produce a torque in one particular direction (say anticlockwise) and interact with φ_b produce a torque in another direction (say clockwise).

At start these two torques are equal in magnitude but in opposite direction. Each torque tries to rotate the rotor in its own direction. Thus, net torque experienced by the rotor is zero at start and hence single-phase induction motor is not self-starting.

Torque-Speed characteristics

The two oppositely directed torques and the resultant torque shown effectively with the help of torque speed characteristics below. At start N=0, resultant torque T=0. So single phase induction motors are not self-starting. However, if the rotor is given an initial rotation in any direction in which rotor initially rotates. And motor starts rotating in that direction. But in practice, it is not possible to give initial torque to rotor externally. Hence some modifications are done in the construction of single-phase induction motors to make them self-starting.



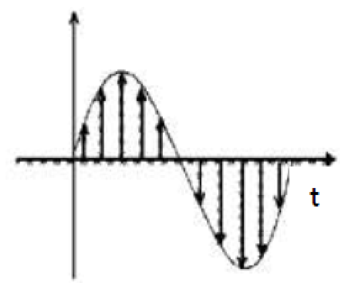


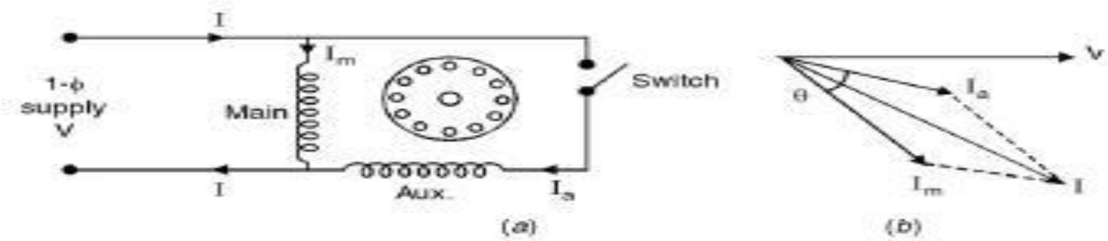
Fig: Pulsating magnetic field in the air gap

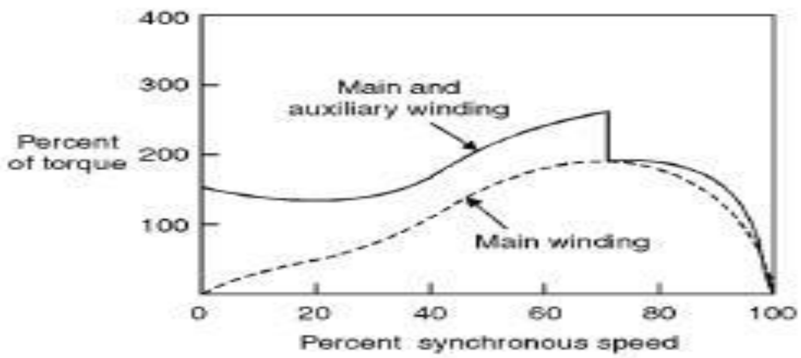
5.3.2 Starting of single-phase induction motor

Single phase induction motor with one stator winding inherently doesn't produce any starting torque. In order to make the motor start rotating some arrangement is required so that the motor produces the starting torque. In running condition, the motor will be able to produce torque with only one winding. The simplest method of starting a single-phase induction motor is to provide an auxiliary winding on the stator in addition to the main winding and start the motor as a two-phase machine. The two windings are placed in the stator with their axes displaced 90° electrical degrees. The currents in the two windings are thus phase-shifted from each other producing rotating stator field capable of producing the starting torque. However, once the motor is running, it is capable of producing the torque with only one winding (the main winding). So, as the motor speeds up the auxiliary winding can be disconnected. Based upon the various methods used to produce the phase difference between the currents in the main and auxiliary windings, the single-phase motors can be classified as follows:

(a) Split-phase induction motor.

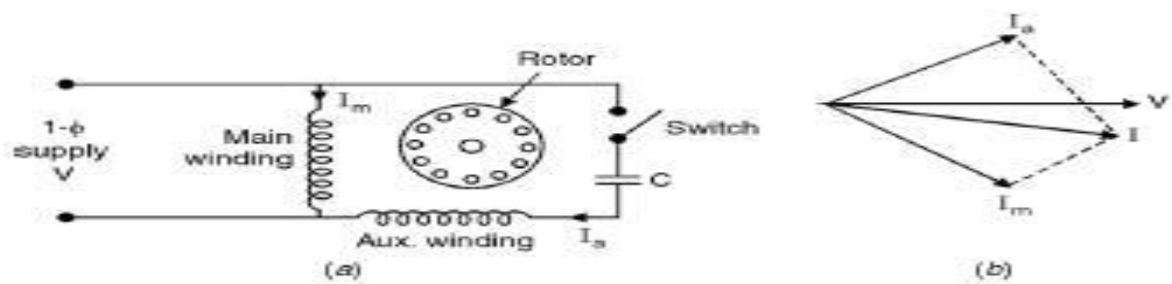
The stator of a split phase induction motor has two windings, the main winding and the auxiliary winding. These windings are displaced in space by 90° electrical as shown in Fig. The auxiliary winding is made of thin wire (super enamel copper wire) so that it has a high R/X ratio as compared to the main winding which has thick super enamel copper wire. Since the two windings are connected across the supply the current Im and Ia in the main winding and auxiliary winding lag behind the supply voltage V, Ia is leading the current Im shown in Fig. This means the current through auxiliary winding reaches maximum value first and the mmf or flux due to Ia lies along the axis of the auxiliary winding and after some time (t = θ/w) the current Im reaches maximum value and the mmf or flux due to Im lies along the main winding axis. Thus, the motor becomes a 2-phase unbalanced motor. It is unbalanced since the two currents are not exactly 90 ° apart. Because of these two fields a starting torque is developed and the motor becomes a self-starting motor. After the motor starts, the auxiliary winding is disconnected usually by means of centrifugal switch that operates at about 75 % of synchronous speed. Finally, the motor runs because of the main winding. Since this being single-phase some level of humming noise is always associated with the motor during running. A typical torque speed characteristic is shown in Fig. It is to be noted that the direction of rotation of the motor can be reversed by reversing the connection to either the main winding or the auxiliary windings.

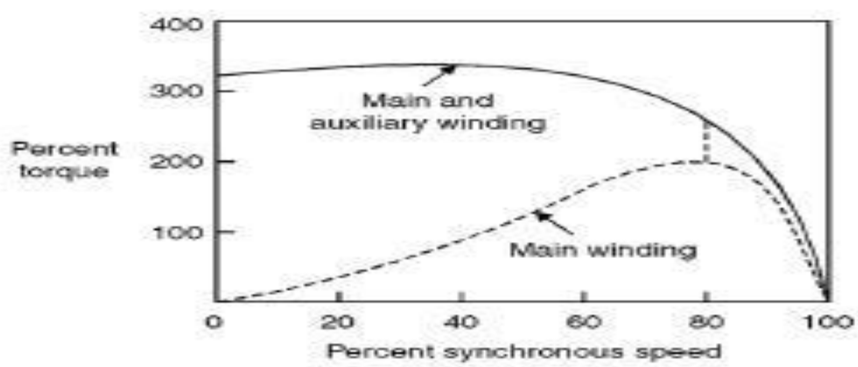




(b) Capacitor starts induction motor.

Capacitors are used to improve the starting and running performance of the single-phase induction motors. The capacitor start induction motor is also a split phase motor. The capacitor of suitable value is connected in series with the auxiliary coil through a switch such that Ia the current in the auxiliary coil leads the current Im in the main coil by 90 ° in time phase so that the starting torque is maximum for certain values of Ia and Im. This becomes a balanced 2-phase motor if the magnitude of Ia and Im are equal and are displaced in time phase by 90°. Since the two windings are displaced in space by 90 ° as shown in Fig. maximum torque is developed at start. However, the auxiliary winding and capacitor are disconnected after the motor has picked up 75% of the synchronous speed. The motor will start without any humming noise. However, after the auxiliary winding is disconnected, there will be some humming noise.

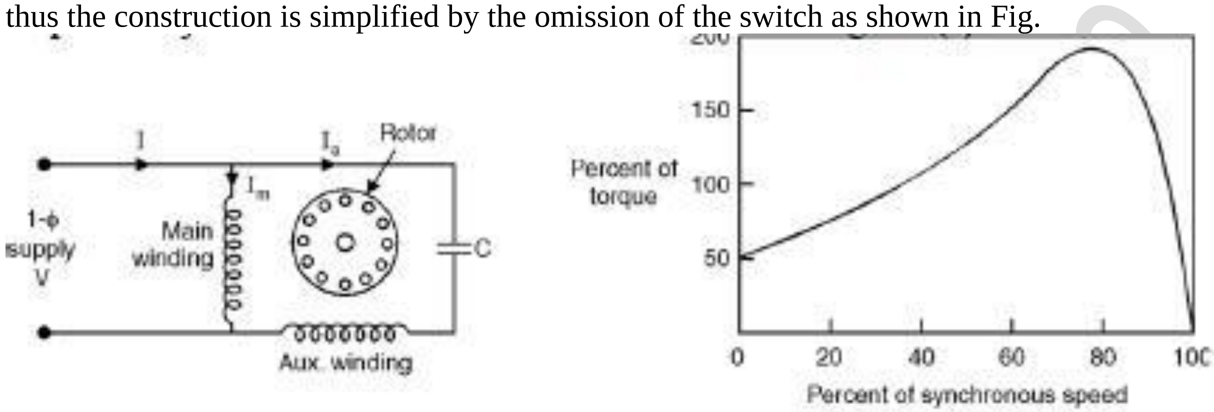




Since the auxiliary winding and capacitor are to be used intermittently, these can be designed for minimum cost. However, it is found that the best compromise among the factors of starting torque, starting current and costs results with a phase angle somewhat less than 90° between Im and Ia. A typical torque-speed characteristic is shown in Fig. high starting torque being an outstanding feature.

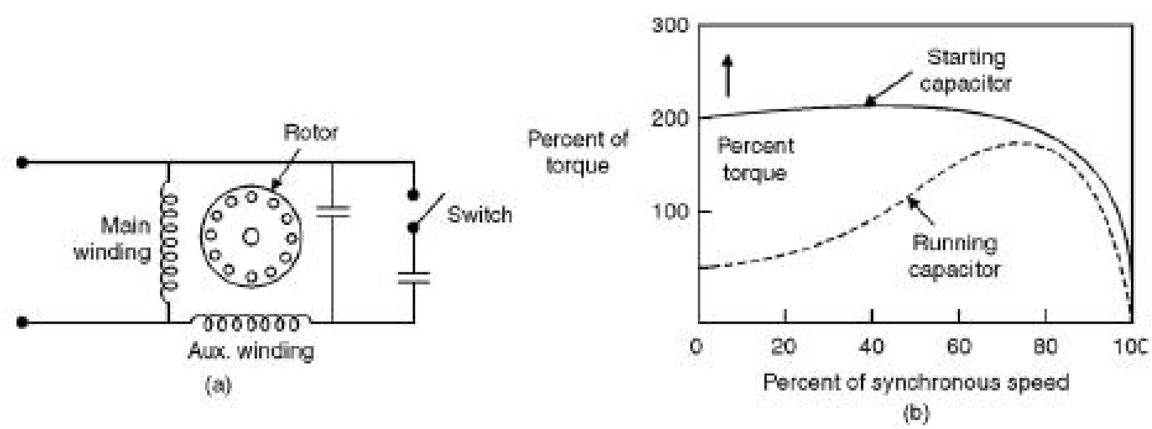
c) Permanent-split capacitor motor.

In this motor the auxiliary winding and capacitor are not disconnected from the motor after starting, thus the construction is simplified by the omission of the switch as shown in Fig.



Here the auxiliary winding and capacitor could be so designed that the motor works as a perfect 2-phase motor at anyone desired load. With this the backward rotating magnetic field would be completely eliminated. The double stator frequency torque pulsations would also be eliminated; thereby the motor starts and runs as a noise free motor. With this there is improvement in p.f. and efficiency of the motor. However, the starting torque must be sacrificed as the capacitance is necessarily a compromise between the best starting and running characteristics. The torque-speed characteristic of the motor is shown in Fig.

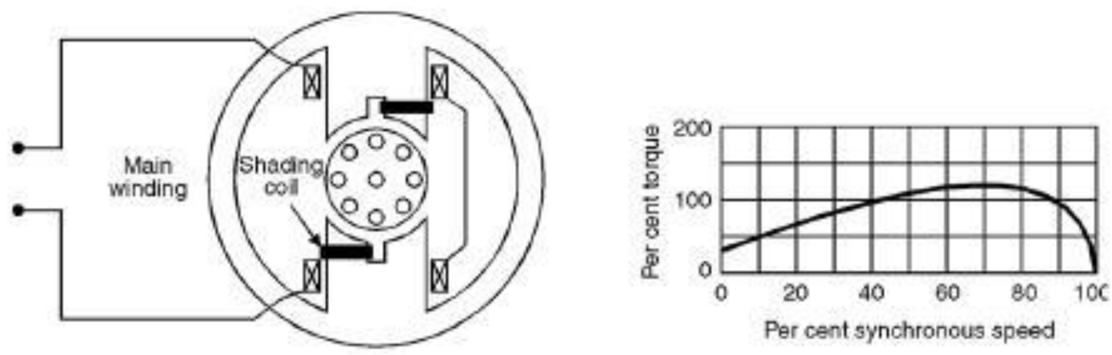
(d) Capacitor start, capacitor run motor. If two capacitors are used with the auxiliary winding as shown in Fig. one for starting and other during the start and run, theoretically optimum starting and running performance can both be achieved.



The small value capacitor required for optimum running conditions is permanently connected in series with the auxiliary winding and the much larger value required for starting is obtained by a capacitor connected in parallel with the running capacitor. The starting capacitor is disconnected after the motor starts.

(e) Shaded pole induction motor

Fig. shows schematic diagram of shaded pole induction motor. The stator has salient poles with one portion of each pole surrounded by a short-circuited turn of copper called a shading coil. Induced currents in the shading coil (acts as an inductor) cause the flux in the shaded portion of the pole to lag the flux in the other portion. Hence the flux under the unshaded pole leads the flux under the shaded pole which results in a rotating field moving in the direction from unshaded to the shaded portion of the pole and a low starting torque is produced which rotates the rotor in the direction from unshaded to the shaded pole (A typical torque speed characteristic). The efficiency is low. These motors are the least expensive type of fractional horse power motor and are built up to 1/20 hp. since the rotation of the motor is in the direction from unshaded towards the shaded part of the pole, a shaded pole motor can be reversed only by providing two sets of shading coils which may be opened and closed or it may be reversed permanently by inverting the core.



(a) (b)

Numerical

1. A 4 pole, 400 V, 50 Hz, 3- phase induction has R_2= 0.2 ohm and X_2= 1.8 ohm. It develops a maximum torque of 50 N-m. Calculate the torque developed by rotor at the speed of 1450 rpm and the starting torque.

➢ Solution:

No. of poles(P) = 4

Synchronous speed(N_s) = $\frac{120f}{P} = \frac{120*50}{4}$ = 1500 rpm.

Motor Speed (N) = 1450 rpm.

Slip(s) = $\frac{N_s-N}{N_s} = \frac{1500-1450}{1500}$ = 0.0333 and s_{max}(a)= $\frac{R_2}{X_2} = \frac{0.2}{1.8} = 0.111$

Now, $\frac{T_{max}}{T} = \frac{\frac{K_1 s_m E_2^2 R_2}{R_2^2+(s_m X_2)^2}}{\frac{K_1 s E_2^2 R_2}{R_2^2+(sX_2)^2}} = \frac{s_m(R_2^2+(sX_2)^2)}{s(R_2^2+(s_m X_2)^2)} = \frac{0.111(0.2^2+0.0333^2*1.8*1.8)}{0.0333(0.2^2+0.111^2*1.8*1.8)} = 2.25139$

So, T = 22.208 N-m.

Also,

$\frac{T_{stat}}{T_m} = \frac{2a}{a^2+1} = \frac{2*0.111}{0.111^2+1}$

So, T_{stat} = 10.9649 N-m.

2. A 4 pole, 400 V, 3-phase, 50 Hz squirrel cage induction motor runs at 1450 rpm at 0.8 pf lagging developing 11 KW. The stator losses are 1100 watt and mechanical losses are 400 watt. Determine Rotor copper loss, Rotor frequency, Line current and efficiency.

➢ Solution:

Synchronous speed (N_s) = $\frac{120f}{P} = \frac{120*50}{4}$ = 1500 rpm.

Motor speed(N) = 1450 rpm.

Slip(s) = $\frac{N_s - N}{N_s} = \frac{1500-1450}{1500}$ = 0.03

Power output(P_0) = 11 KW

Rotor Output(P_{mech}) = P_0 + Mechanical losses = 11+ 0.4 = 11.4 KW

Power input to rotor(P_2) = $\frac{P_{mech}}{1-s} = \frac{11.4}{1-0.03}$ = 11.75 KW

Power input to stator(P_1) = P_2 + Stator loss = 11.75 + 1.1 = 12.85 KW

So, Rotor copper loss = $P_2 - P_{mech}$ = 11.75 – 11.4 = 0.352 KW

Rotor Frequency f_r = sf =0.03 *50 = 1.5 Hz

Line current(I_L) = $\frac{P_L}{\sqrt{3}V_L Cos\phi} = \frac{11000}{\sqrt{3}*400*0.8}$ = 19.846 A

Efficiency(η) = $\frac{P_0}{P_1}$ * 100% = $\frac{11}{12.85}$*100% = 85.60 %.

3. The main winding and starting winding of a 50Hz capacitor start single phase induction motor have impedances as follows:

Main winding (3+3j)Ω

Starting winding (7.5+j3)Ω

Calculate the value of capacitor to be connected in series with the starting winding to produce a phase difference of 90° between main winding current and starting winding current at starting.

➢ **Solution**

Z_m=3+3j Z_a=7.5+j3

letX_c be the capacitive reactance to be connected with auxiliar

Z_a=7.5+j(3-X_c) angle (φ_a) =$\tan^{-1}\frac{3-Xc}{7.5}$

Z_m=3+3j angle (φ_m) =$\tan^{-1}\frac{3}{3}$=45°

I_a and I_m most have a phase difference of 90°. I_m will lag the vol

(90°-45°)=45°.

As leads the phase angle of I_a must be taken as negative.

i.e.$\tan -45° = \frac{3-Xc}{7.5}$

∴X_c=10.5Ω

C=$\frac{1}{2\pi fC} = \frac{1}{2\pi*50*10.5}$=3.03*10^{-4}=303.15$\mu F$

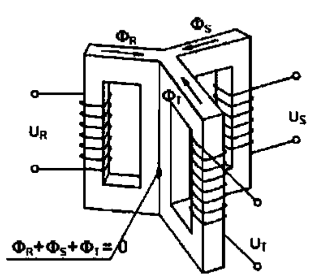
UNIT 6: THREE PHASE TRANSFORMERS

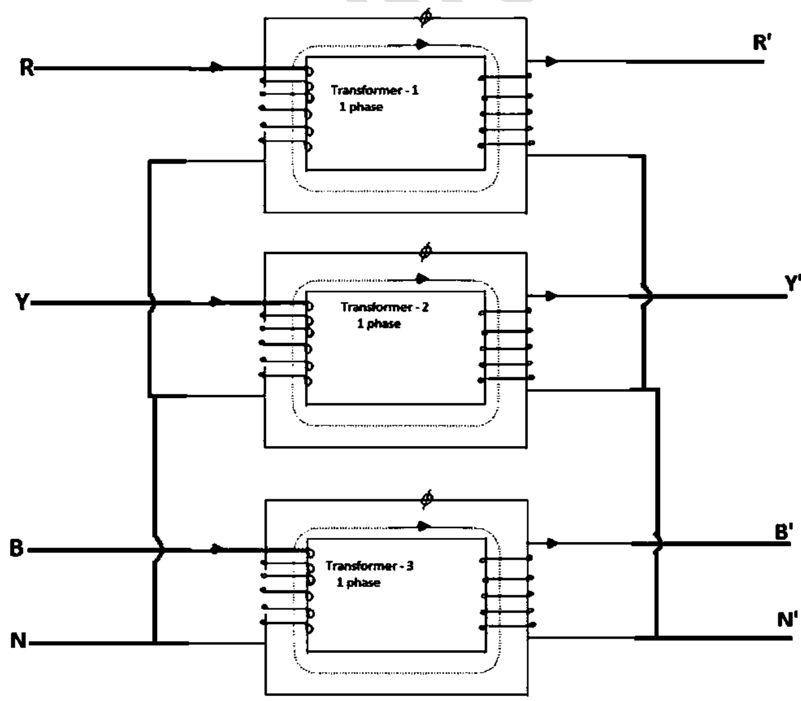
6.1 INTRODUCTION

Large scale generation of electric power is usually three-phase at 11 KV or higher voltage level. Transmission of power is generally accomplished at further higher voltage foe economy. Therefore, three phase step-up transformers are necessary at the sending end of the transmission line and three phase steps down transformers are used at load center.

Years ago, it was a common practice to use suitably interconnected three single phase transformers instead of a single three phase transformer. But these days, the latter is gaining popularity because of improvement in design and manufacture but principally because of better acquaintances of operating men with the three-phase type. As compared to a bank of single-phase transformers, the main advantages of a three-phase transformer are that it occupies less floor space for equal rating, weighs less, costs about 15% less and further that only one unit is to be handled and connected.

Like single phase transformers, three phase transformers are also of the core type or shell type. In three phase primary windings are interconnected in star and put across three phase supply. The three cores are 120° apart and their empty legs are in contact with each other. The center leg, formed by these three carries the flux produced by the three phase currents I_R, I_Y and I_B. As at any instant $I_R+I_Y+I_B = 0$, hence the sum of three fluxes is also zero. Therefore, it will make no difference if the common legs are removed. In that case any two legs will act as the return for the third just as in three phase system any two conductors act as the return for the current in the third conductor.





	Bank of 3 single phase transformers	Single unit 3 phase transformer
1	More costly due to more iron, three separate tanks, more oil and more auxiliary equipments.	Less costly as less volume of iron, one tank so less oil and less auxiliary equipments.
2	As three separate transformers it requires six h.v. bushings which increase the cost.	It requires only three bushings which reduces the cost.
3	More floor space is required which increases the capital.	Less floor space is required which decreases the capital.
4	If one phase/transformer damaged, then it can be easily replaced by a single phase unit.	If one phase is damaged, whole transformer has to be replaced, which increases the cost.
5	Only one single phase transformer is required as standby unit.	One complete three phase transformer is required as standby unit.
6	Because of more iron part, more iron loss hence less efficient.	Comparatively less iron part so more efficient.
7	Easy for transportation as each unit is small.	Comparatively to the first case more difficult for transportation.

6. 2 THREE-PHASE TRANSFORMER CONNECTIONS

A three-phase transformer can be built by suitably connecting a bank of three single phase transformers or by one three-phase transformer. The primary or secondary windings may be connected in either star (Y) or delta (Δ) arrangement. The four most common connections are (i) Y-Y (ii) Δ - Δ (iii) Y- Δ and (iv) Δ -Y. These four connections are described below. In the figure, the windings at the left are the primaries and those at the right are the secondaries. The primary and secondary voltages and currents are also shown. The notations used are given by;

V_1, V_2 Rated primary and secondary phase voltages,

I_1, I_2: Rated primary and secondary phase currents,

N_1, N_2: Primary and secondary number of turns,

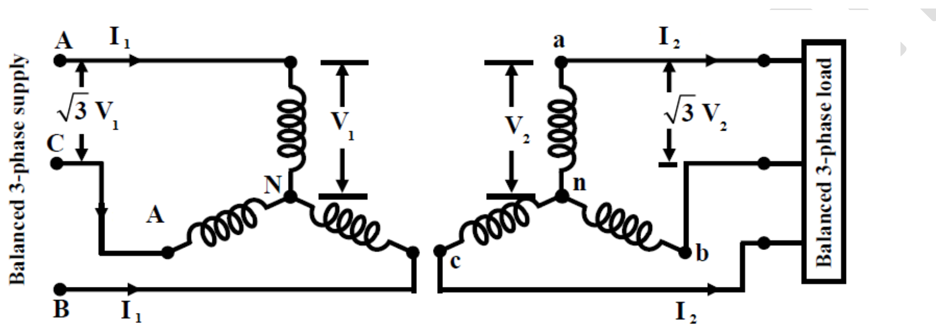
$S = V_1 I_1 = V_2 I_2$=Rated kVA

6.2.1 Star/Star

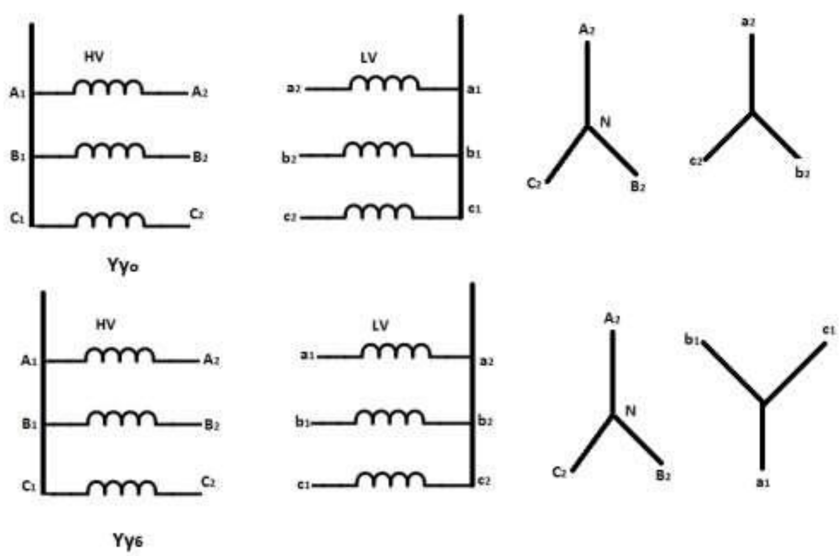
To apply a rated voltage to the primary terminals a line to line voltage of $\sqrt{3}V_1$ is supplied, so that the primary rated voltage V_1 is impressed across each of the primary coils of the individual transformer. This ensures V_2 to be induced across each of the secondary coil and the line to line voltage in the secondary will be $\sqrt{3}V_2$. Now we have to calculate how much load current or kVA can be supplied by this bank of three phase transformers without over loading any of the single phase transformers. From the individual rating of each transformer, we know maximum allowable currents of primary and secondary windings are I_1 and I_2 respectively. Since secondary side is connected in star, line current and the winding currents (phase current) are same. Therefore total kVA that can be supplied to a balanced 3-phase load is $\sqrt{3}(V_L)(I_L) = \sqrt{3}(\sqrt{3}V_2)I_2 = 3V_2I_2 = 3s$ i.e. three times the Kva rating of each single phase transformer.

Phase voltage transformation ratio= $\frac{V_2}{V_1} = \frac{N_2}{N_1} = k$

Line voltage transformation ratio= $\frac{\sqrt{3}V_2}{\sqrt{3}V_1} = \frac{N_2}{N_1} = k$



This type of connection requires less insulation as the phase voltage is less than the line voltage. For which it require less number of turns/phase but more cross sectional area of conductor. It is economical for high voltage low current rating transformers. As the cross-sectional areas of conductors are more it is more mechanically strong. With both primary and secondary connected in star no closed path exists among the windings. As the tripled harmonics are always in phase, by virtue of the Y connection they get canceled in the line voltages. Non-tripled harmonics like fundamental become 3 times phase value and appear in the line voltages. Line currents remain sinusoidal except for non-tripled harmonic currents.



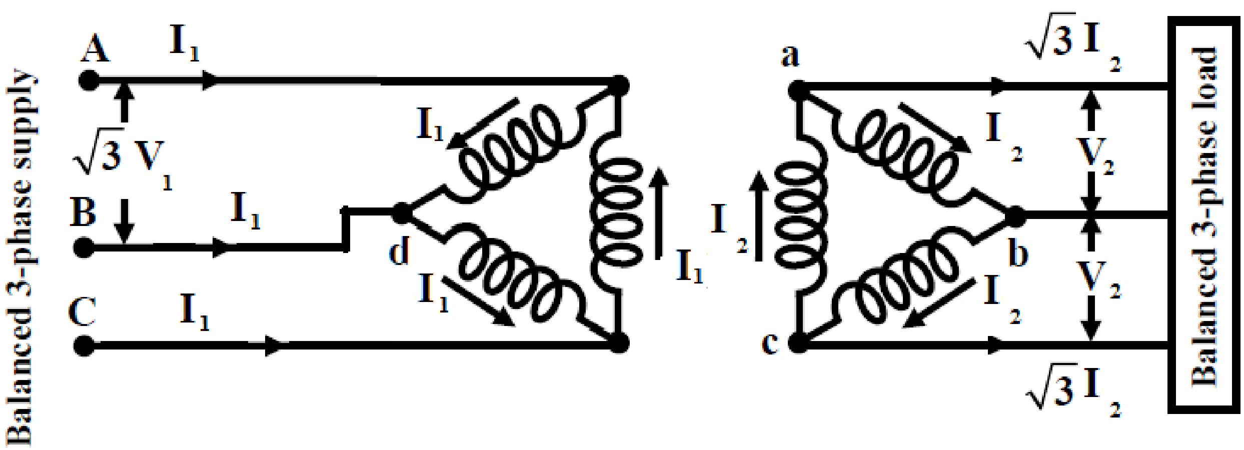
6.2.2 Delta/Delta

As discussed above, to apply a rated voltage to the primary terminals a line to line voltage of is supplied, so that the primary rated voltage V_1 is impressed across each of the primary coils of the individual transformer, as for delta connection both line and phase voltages are same.

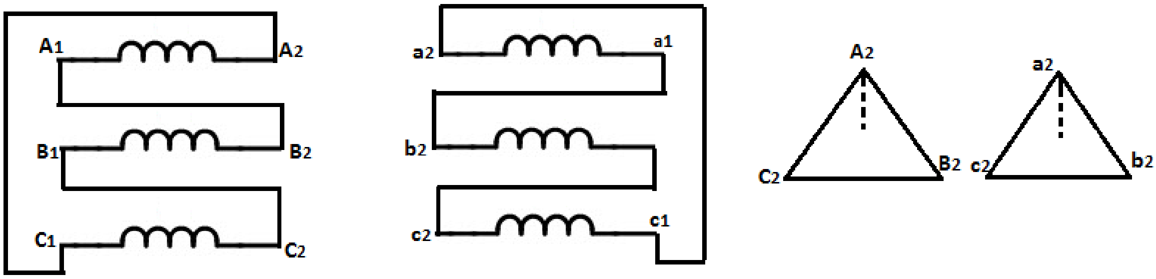
This ensures V_2 to be induced across each of the secondary coil and the line to line voltage in the secondary will also be V_2. Now we have to calculate how much load current or kVA can be supplied by this bank of three phase transformers without over loading any of the single phase transformers. Since secondary side is connected in delta, we can connect a load in such a way that the winding currents (phase current) should not exceed I_2, for which the line current becomes $\sqrt{3}I_2$. When the current in secondary winding becomes I_2, the corresponding reflected phase current in the primary becomes I_1 and the line current becomes $\sqrt{3}I_1$. Therefore total kVA that can be supplied to a balanced 3-phase load and drawn from the supply is given by $\sqrt{3}(V_L)(I_L)=\sqrt{3}V_2(\sqrt{3}I_2)=3V_2I_2=3s$ i.e. three times the Kva rating of each single phase transformer.

Phase voltage transformation ratio= Line voltage transformation ratio = $\frac{V_2}{V_1}=\frac{N_2}{N_1}=k$

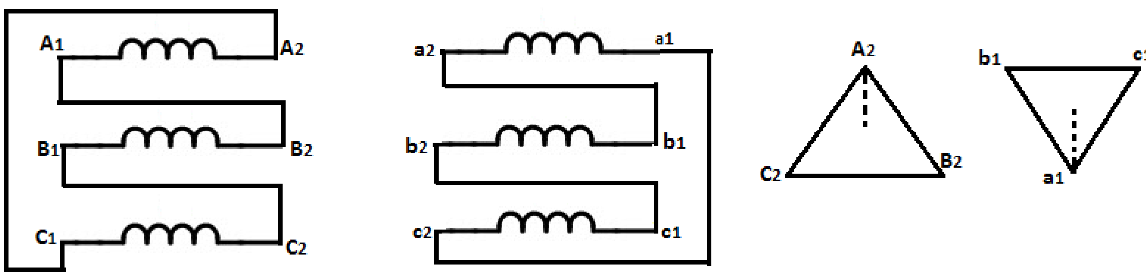
As the phase and line voltages are same it requires more insulation but more number of turns/phase, where as the cross sectional areas of conductors are less compare to Y-Y connection. For which it is more economical for low voltage and high current applications. Due to the less cross sectional areas of conductors it is mechanically weak compare to Y-Y transformers.



With mesh connection on both primary side and secondary side a closed path is available for the triplen harmonics to circulate currents. Thus, the supply current is nearly sinusoidal (but for the non-triplen harmonic currents). The triplen harmonic currents inside the closed mesh winding correct the flux density wave to be nearly sinusoidal. The secondary voltages will be nearly sinusoidal. Third harmonics currents flow both in the primary and the secondary and hence the magnitudes of these currents, so also the drops due to them will be lower.



Dd0



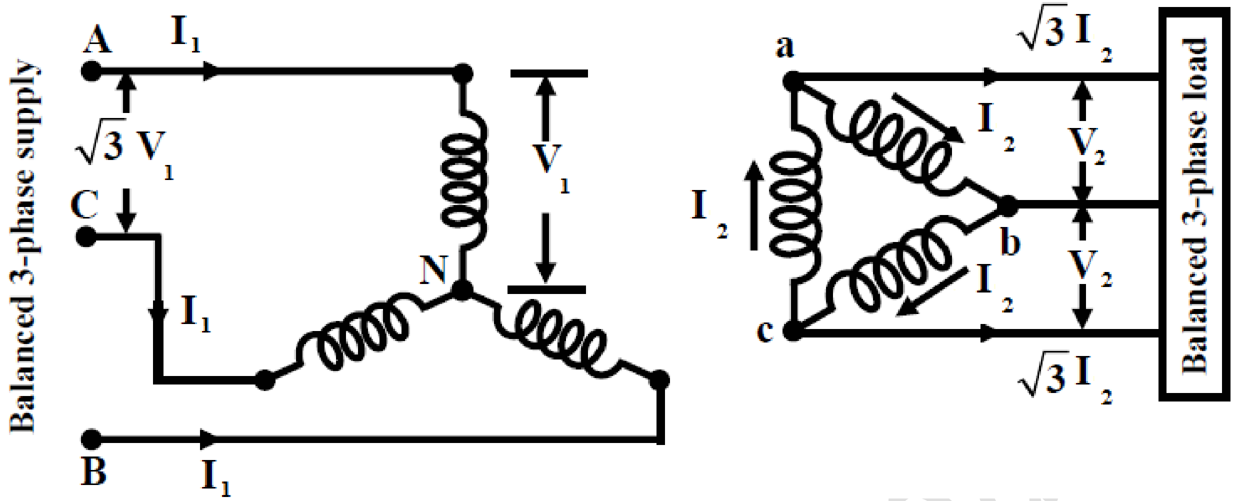
Dd6

6.2. 3 Star/Delta

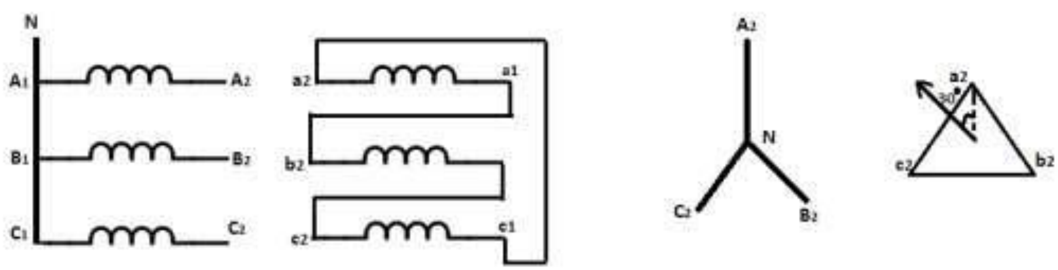
For this connection a line to line voltage of $\sqrt{3}V_1$ is supplied, so that the primary rated voltage V_1 is impressed across each of the primary coils of the individual transformer. This ensures V_2 to be induced across each of the secondary coil and the line to line voltage in the secondary will also be V_2 as it is connected in delta. Now we have to calculate how much load current or kVA can be supplied by this bank of three phase transformers without over loading any of the single phase transformers. Since secondary side is connected in delta, we can connect a load in such a way that the winding currents (phase current) should not exceed I_2, for which the line current becomes $\sqrt{3}I_2$. When the current in secondary winding becomes I_2, the corresponding reflected phase current in the primary becomes I_1 and the line currents are also I_1 as it is star connected. Therefore total kVA that can be supplied to a balanced 3-phase load and drawn from the supply is given by $\sqrt{3}(V_L)(I_L)=\sqrt{3}V_2(\sqrt{3}I_2)=3V_2I_2=3s$ i.e. three times the Kva rating of each single phase transformer.

Phase voltage transformation ratio= $\dfrac{V_2}{V_1}=\dfrac{N_2}{N_1}=k$

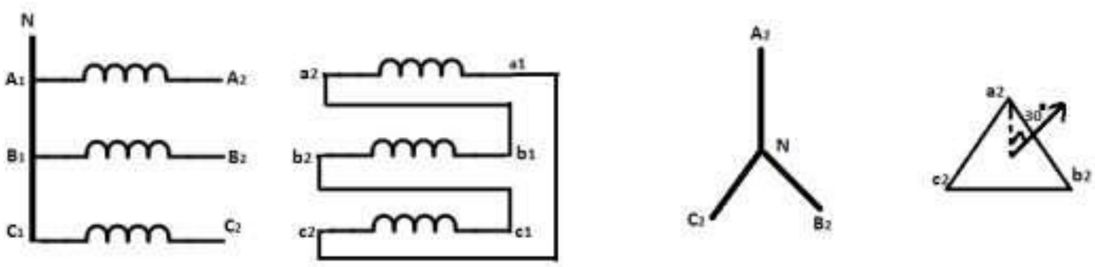
Line voltage transformation ratio= $\dfrac{V_2}{\sqrt{3}V_1}=\dfrac{N_2}{\sqrt{3}N_1}=\dfrac{k}{\sqrt{3}}$



This transformer connection gives least secondary terminal voltage among the all types of connection. Commonly used in a step-down transformer. This transformer is generally used at the end of a transmission line.wye connection on the HV side reduces insulation costs, the neutral point on the HV side can be grounded, stable with respect to unbalanced loads.



Yd1



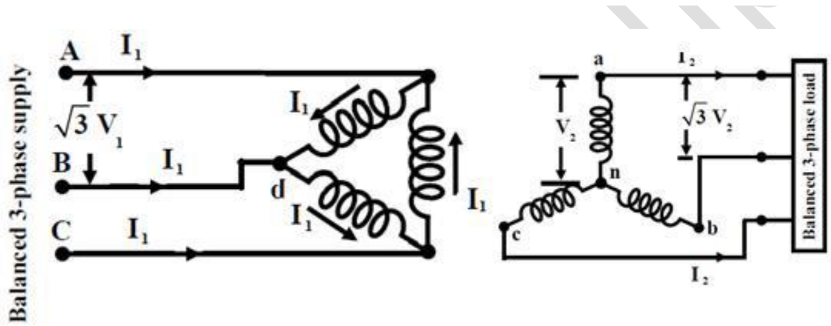
Yd11

6.2.4 Delta/Star

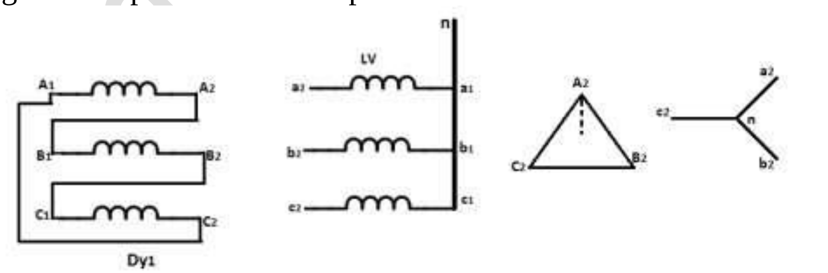
In this connection to apply a rated voltage to the primary terminals a line to line voltage of V_1 is supplied, so that the primary rated voltage V_1 is impressed across each of the primary coils of the individual transformer, as for delta connection both line and phase voltages are same. This ensures V_2 to be induced across each of the secondary coil and the line to line voltage in the secondary will also be$\sqrt{3}V_2$. Now we have to calculate how much load current or kVA can be supplied by this bank of three phase transformers without over loading any of the single phase transformers. From the individual rating of each transformer, we know maximum allowable currents of primary and secondary windings are I_1 and I_2respectively. Since secondary side is connected in star, line current and the winding currents (phase current) are same. When the current in secondary winding becomes I_2, the corresponding reflected phase current in the primary becomes I_1 and the line current becomes $\sqrt{3}I_1$. Therefore total kVA that can be supplied to a balanced 3-phase load and drawn from the supply is given by $\sqrt{3}(V_L)(I_L) = \sqrt{3}(\sqrt{3}V_2)I_2 = 3V_2I_2 = 3s$ i.e. three times the kVA rating of each single phase transformer.

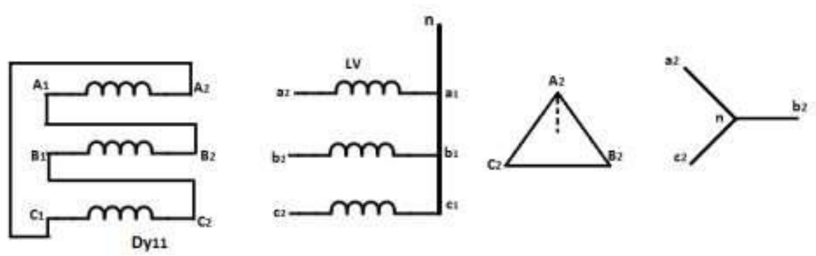
Phase voltage transformation ratio= $\frac{V_2}{V_1} = \frac{N_2}{N_1} = k$

Line voltage transformation ratio= $\frac{\sqrt{3}V_2}{V_1} = \frac{\sqrt{3}N_2}{N_1} = \sqrt{3}k$



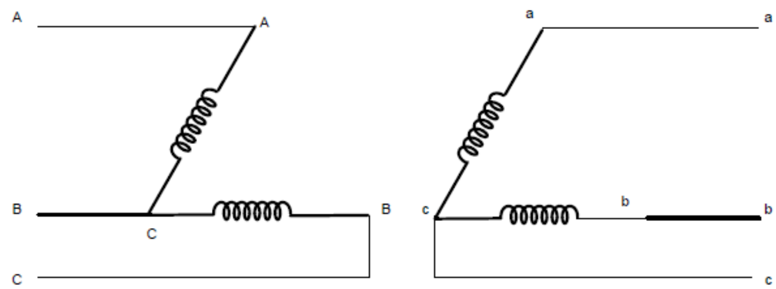
For the same turn ratio and applied voltage this transformer connection provides highest secondary terminal voltage for which it is commonly used as a step-up transformer. This transformer is generally used at the beginning end of a transmission line. Dy and Yd connection (without neutral connection) Behavior of the bank with mesh connection on one side is similar to the one discussed under Dd connection. The harmonic currents and drops and the departure of the flux density from sinusoidal are larger in the present case compared to Dd banks.



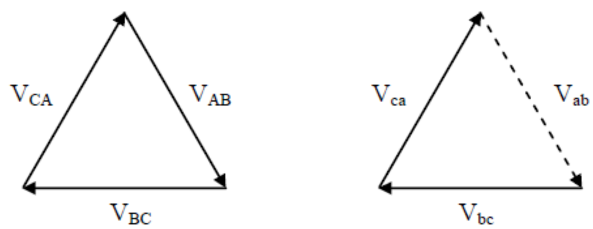


6.3 OPEN DELTA

one of the respective phase coil in primary and secondary of a delta/delta transformer is removed and three phase supply is connected to the primary as shown in the figure. Then the equal voltage will be available at the secondary terminals. This type of connection is known as open delta or V-V connection.



(i)



Let V_{ph} and I_{ph} be the rated phase voltage and current respectively of each of the transformer.

Case-I in close delta

Line voltage $V_l = V_{ph}$

and Line current $I_l = \sqrt{3}\, I_{ph}$

VA delivered by the bank of transformers in delta$=\sqrt{3}\, V_l I_l = \sqrt{3}\, V_{ph}(\sqrt{3}\, I_{ph}) = 3V_{ph} I_{ph}$

Case-II in open delta

Line voltage $V_L = V_{ph}$

and Line current $I_L = I_{ph}$

VA delivered by the bank of transformers in delta$=\sqrt{3}\, V_L I_L = \sqrt{3}\, V_{ph} I_{ph}$

It is thus seen that the VA rating of open-delta is $\sqrt{3}\, V_{ph} I_{ph}$ and not $2V_{ph}I_{ph}$.

$$\frac{kVA\ in\ open\ delta}{kVA\ in\ close\ delta} = \frac{\sqrt{3}\, V_{ph} I_{ph}}{3V_{ph} I_{ph}} = 0.577 = 57.7\%$$

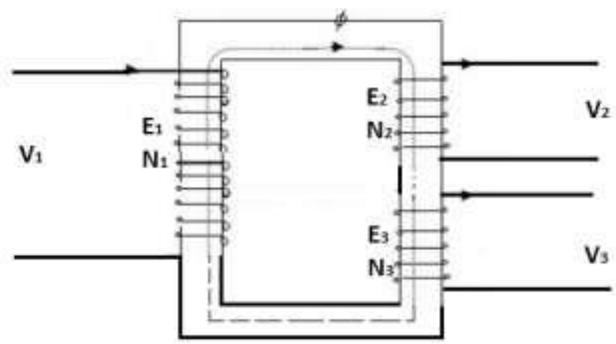
1. kVA supplied by each transformer $= \frac{\sqrt{3}\, V_{ph} I_{ph}}{2} = 0.866\, V_{ph} I_{ph}$, this implies each transformer is under loaded by a factor of 13.4%.

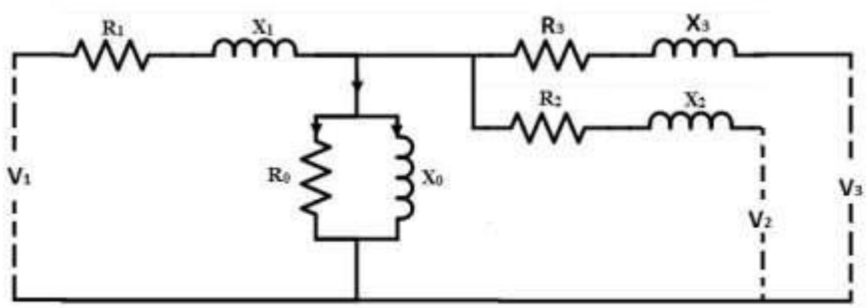
2. $utilisation\ factor\ or\ rating\ factor = \frac{Actual\ available\ kVA}{Actual\ installed\ kVA in\ open\ delta} = \frac{\sqrt{3}\, V_{ph} I_{ph}}{2\, V_{ph} I_{ph}} = 86.6\%$

6.4 THREE WINDING TRANSFORMERS

Transformers may be built with a third winding, called the tertiary, in addition to the normal primary and secondary and such transformer are called the 3 winding transformers. The tertiary winding may serve any of the following purposes:

- To supply the substation auxiliaries at a voltage different from those of the primary and secondary windings.
- To supply phase compensation devices, such as condensers operated at a voltage which is different from
- both primary and secondary voltage.
- To interconnect three supply systems operating at different voltages.
- To load large split winding generators.
- To measures voltage of an HV testing transformer.

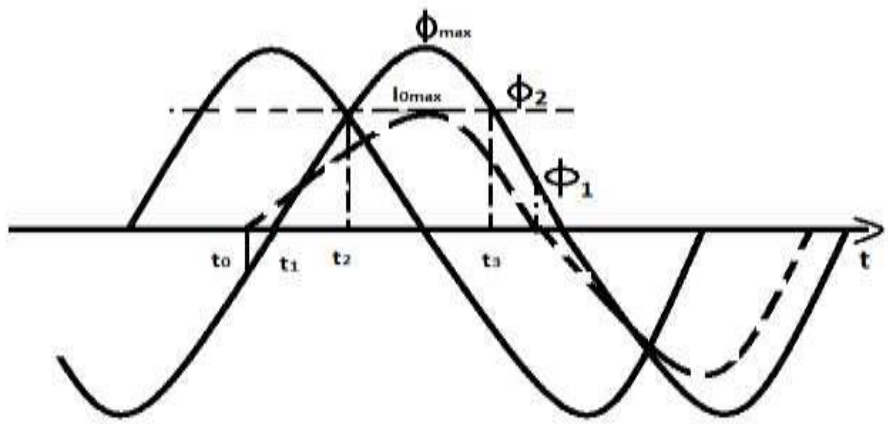


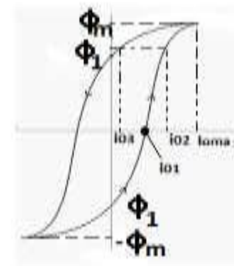


Tertiary windings are normally delta connected so that when faults and short circuits occur on the primary or secondary sides (particularly between lines and earth), the considerable unbalance produced in phase voltages may be compensated by the circulating currents flowing in the closed Δ. The reactance of the should be large enough to limit the circulating currents in order that there is no over-heating of the windings.

6.5 EXCITING CURRENT HARMONICS

It was found that the exciting current I_o is not sinusoidal due to hysteresis and saturation non-linearity of the core material. The waveform of magnetic flux in the core is lagging the applied voltage by 90°. The waveform of exciting current i_o required to set up sinusoidal flux can be obtained graphically by looking up the hysteresis loop (φ-i_o curve).

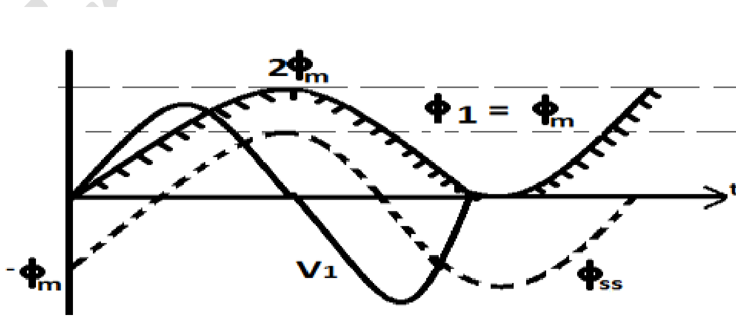


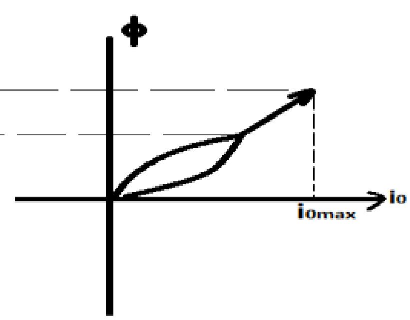


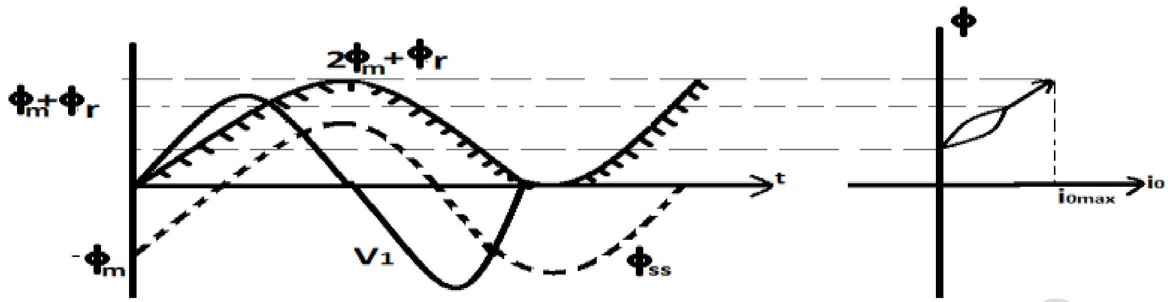
Let us assume that the steady state operation has been reached so that hysteresis loop is being repeated in successive cycles of the applied voltage. Consider the instant when the flux has a value of $-\varphi_1$, the corresponding exciting current being zero. When the flux becomes zero at t_1, the current is a small positive value of i_{o1}. when the flux has a positive value of φ_2, there are two possible values of current i_{o2} when the flux is on the increasing part of hysteresis loop and i_{o3} when the flux is on the decreasing part of hysteresis loop ($i_{o2} > i_{o3}$). The maximum flux $-\varphi_{max}$ coincides with the current maximum i_{omax}. The current becomes zero once again for flux $+\varphi_1$. So far the positive half of exciting current has been traced out, the negative half will be symmetrical (odd symmetry) to it because of inherent symmetry of hysteresis loop. From the graph, it is observed that the waveform is non-sinusoidal and peaky. The current has fundamental and odd harmonics. The highest component is found to be third harmonics which is about 40% of the fundamental component.

6.6 TRANSFORMER INRUSH CURRENT

For an ideal transformer V_1 and φ are both sinusoidal and φ lags V_1 by 90°. When the voltage V_1 is switched on to the transformer, the core flux and corresponding exciting undergo a transient before reaching steady state values. The duration of switching transient depends on the instant when the voltage wave is switched on, the worst condition being when the applied voltage has zero value at the instant of switching.

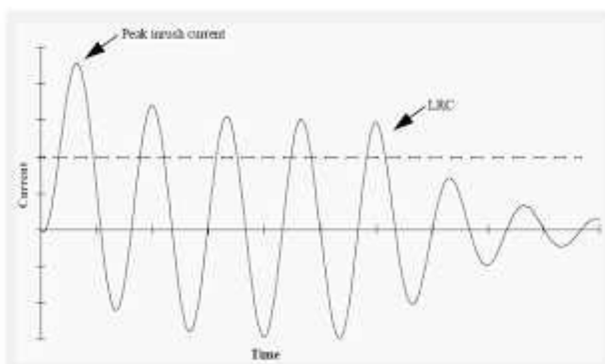






It is assumed here that the initial flux in the transformer core at instant of switching has zero value. It is seen from the figure that the steady state value of flux demanded at this instant is $-\varphi_m$ (shown by the dotted line), while the flux can only start with zero value in the inductive circuit. Ad a consequences, a transient flux component (off set of flux) $\varphi_1 = \varphi_m$ originates so that the resultant flux is $\varphi_1 + \varphi_{ss}$ which has zero value at instant of switching. The transient component φ_1 will decay according to the circuit time constant $(^L/_R)$ which is generally low in a transformer. If the circuit heat dissipation (core loss) is assumed negligible, the transient flux will go through a maximum value of 2 φ_m, a phenomenon called doubling effect. The corresponding exciting current will be very high as the core goes into deep saturation region of magnetization, it may indeed be as high as 100 times of normal exciting current and produces very high electromagnetic forces. This is why the winding of large transformer must be strongly braced. In subsequent half period φ_t gradually decays till it vanishes and the core flux reaches the steady state value. Because of low time constant of the transformer circuit, distortion effect of transient may last several seconds.

The initial core flux will not be zero as assumed above, but will have some residual value φ_r, the transient will now even more serve $\varphi_1 = \varphi_m + \varphi_r$ and the core flux will now go through a maximum value of 2 $\varphi_m + \varphi_r$. It is observed that the offset flux is unidirectional so that the transient flux and exciting current are unidirectional in the initial stage of the transient.



Numerical

1) The core of a three phase, 50 Hz, 11000/500 V delta/star, 300 KVA, core type transformer operates with a flux of 0.05 Wb. Find numbers of H.V. and L.V. turns per phase , e.m.f. per turn, full load H.V. and L.V. phase currents.

   - Solution

e.m.f. per turn (E/T) = $4.44f\varphi_m$ = 4.44*50*0.05 = 11.1 Volts

Voltage per phase on delta connected primary winding = 11000 V

Voltage per phase on star connected secondary winding = $\frac{550}{\sqrt{3}}$ V = 317.54 V

Numbers of primary turn (N_1) = $\frac{11000}{11.1}$ = 991

Numbers of secondary turn (N_2) = $\frac{317.54}{11.1}$ = 29

H.V. phase current = $\frac{300*10^3}{3*11000}$ = 9.09 A

L.V. phase current = $\frac{300*10^3}{3*317.54}$ = 314.92 A

2) A 3-phase transformer, ratio 33/6.6 KV, Δ/Y, 2 MVA has a primary resistance of 8Ω per phase and a secondary resistance of 0.08 Ω per phase. The percentage impedance is 7%. Calculate the secondary voltage with rated primary voltage and hence the regulation for full load 0.75 p.f. lagging conditions.

   - Solution

Full load secondary current = $\frac{2*10^6}{\sqrt{3}*6.6*10^3}$ = 175 A

K = $\frac{6.6}{\sqrt{3}*33}$ = 0.115

R_{02} = 0.08 + 8*0.115^2 = 0.1858 Ω per phase

Now, secondary impedance drop per phase = $\frac{7}{100} \times \frac{6600}{\sqrt{3}}$ = 266.7 V

Z_{02} = 266.7/175 = 1.523 Ω per phase

$X_{02} = \sqrt{Z^2{}_{02} - R^2{}_{02}} = \sqrt{1.523^2 - 0.1858^2}$ = 1.51 Ω per phase

Drop per phase = I_2 ($R_{02}\cos\varphi$ + $X_{02}\sin\varphi$)

= 175 (0.1867*0.75 + 1.51*0.66) = 200 V

Secondary voltage/phase = 6600/√3 = 3810 V

∴ V_2 = 3810 – 200 = 3610 V

∴ Secondary line voltage = √3*3610 = 6250 V

% regulation = 200 × $\frac{100}{3810}$ = 5.23%

UNIT 7: SPECIAL MACHINES

7.1 MAGNETIC MATERIALS USED FOR ELECTRICAL MACHINES

The magnetic properties of a magnetic material depend on the orientation of the crystals of the material and decide the size of the machine or equipment for a given rating, excitation required, efficiency of operation etc. Some of the properties that a good magnetic material should possess are listed below.

- Low reluctance or should be highly permeable or should have a high value of relative permeability μr.
- High saturation induction (to minimize weight and volume of iron parts)
- High electrical resistivity so that the eddy EMF and the hence eddy current loss is less
- Narrow hysteresis loop or low Coercivity so that hysteresis loss is less and efficiency of operation is high
- A high curie points. (Above Curie point or temperature the material loses the magnetic property or becomes paramagnetic, that is effectively non-magnetic

Magnetic materials can broadly be classified as Diamagnetic, Paramagnetic, Ferromagnetic, Antiferromagnetic and Ferrimagnetic materials. Only ferromagnetic materials have properties that are well suitable for electrical machines. Ferromagnetic properties are confined almost entirely to iron, nickel and cobalt and their alloys. The only exceptions are some alloys of manganese and some of the rare earth elements.

The relative permeability μr of ferromagnetic material is far greater than 1.0. When ferromagnetic materials are subjected to the magnetic field, the dipoles align themselves in the direction of the applied field and get strongly magnetized.

Further the Ferromagnetic materials can be classified as Hard or Permanent Magnetic materials and Soft Magnetic materials.

a) Hard or permanent magnetic materials have large size hysteresis loop (obviously hysteresis loss is more) and gradually rising magnetization curve. Example carbon steel, tungsten steal, cobalt steel, alnico, hard ferrite etc.

b) Soft magnetic materials have small size hysteresis loop and a steep magnetization curve. Example cast iron, cast steel, rolled steel, forged steel etc., (in the solid form).

Generally used for yokes poles of dc machines, rotors of turbo alternator etc., where steady or dc flux is involved.

Silicon steel (Iron + 0.3 to 4.5% silicon) in the laminated form. Addition of silicon in proper percentage eliminates ageing & reduce core loss. Low silicon content steel or dynamo grade steel is used in rotating electrical machines and are operated at high flux density. High content silicon steel (4 to 5% silicon) or transformer grade steel (or high resistance steel) is used in transformers.

Further sheet steel may be hot or cold rolled. Cold rolled grain-oriented steel (CRGOS) is costlier and superior to hot rolled. CRGO steel is generally used in transformers.

c) Special purpose Alloys: Nickel iron alloys have high permeability and addition of molybdenum or chromium leads to improved magnetic material. Nickel with iron in different proportion leads to

- High nickel permalloy (iron +molybdenum +copper or chromium), used in current transformers, magnetic amplifiers etc.,
- Low nickel Permalloy (iron +silicon +chromium or manganese), used in transformers, induction coils, chokes etc.
- Perminvor (iron +nickel +cobalt)
- Pemendur (iron +cobalt +vanadium), used for microphones, oscilloscopes, etc.
- Mumetal (Copper + iron)

7.2 PERMANENT MAGNET BRUSHLESS MACHINE

The stator of the BLPM dc motor is made up of silicon steel stampings with slots in its interior surface. These slots accommodate either a closed or opened distributed armature winding usually it is closed. This winding is to be wound for a specified number of poles. This winding is suitably connected to a dc supply through a power electronic switching circuitry (named as electronic commutator).

Rotor is made of forged steel. Rotor accommodates permanent magnet. Number of poles of the rotor is the same as that of the stator. The rotor shaft carries a rotor position sensor. This position sensor provides information about the position of the shaft at any instant to the controller which sends suitable signals to the electronic commutator.

When dc supply is switched on to the motor the armature winding draws a current. The current distribution within the stator armature winding depends upon rotor position and the devices turned on. An emf perpendicular to the permanent magnet field is set up. Then the armature conductors experience a force. The reactive force develops a torque in the rotor. If this torque is more than the opposing frictional and load torque the motor starts. It is a self-starting motor.

As the motor picks up speed, there exists a relative angular velocity between the permanent magnet field and the armature conductors. AS per faradays law of electromagnetic induction, an emf is dynamically induced in the armature conductors. This back emf as per Lenz ‘s law opposes the cause armature current and is reduced. As a result, the developed torque reduces. Finally, the rotor will attain a steady speed when the developed torque is exactly equal to the opposing frictional load torque. Thus, the motor attains a steady state condition.

When the load – torque is increased, the rotor speed tends to fall. As a result, the back emf generated in the armature winding tends to get reduced. Then the current drawn from the mains is increased as the supply voltage remains constant. More torque is developed by the motor. The motor will attain a new dynamic equilibrium position when the developed torque is equal to the new torque. Then the power drawn from the mains V*I is equal to the mechanical power delivered $\frac{2\pi NT}{60} = P_m = \omega T$ and the various losses in the motor and in the electronic switching circuitry.

7.2.1 CLASSIFICATION OF BLPM DC MOTOR

BLPM dc motors can be classified on the basis of the flux density distribution in the air gap of the motor. They are

(a). BLPM Square wave dc motor [BLPM SQW DC Motor]

(b).BLPM sinusoidal wave dc motor [BLPM SINE WAVE DC Motor]

(a) BLPM Square wave motor

These are two types: 180° pole arc and 120° pole arc. Air gap flux density distribution in 180° and 120° BLMP SWM is shown in figure a and b respectively.

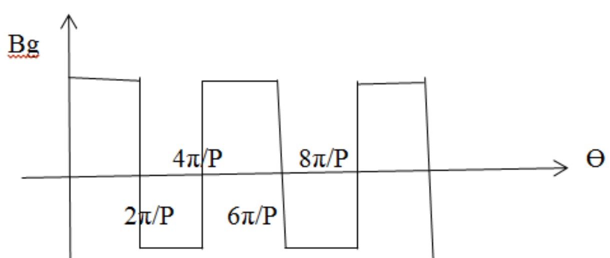


Fig: a

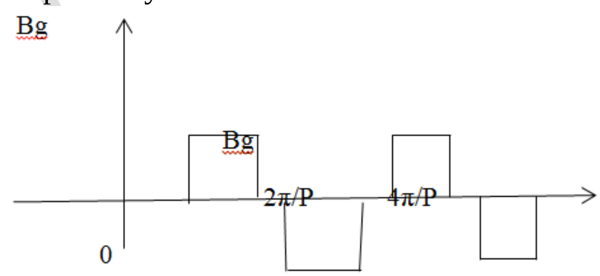
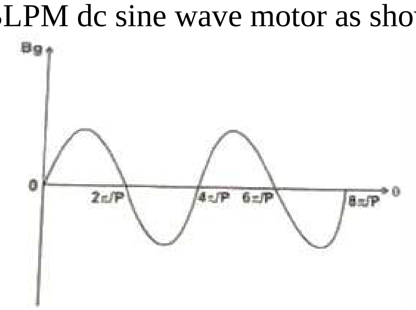


Fig: b

(b) BLPM Sine wave DC Motor

Air gap density distribution of BLPM dc sine wave motor as shown in fig.



7.2.2 EMF EQUATION

The basic torque emf equations of the brushless dc motor are quite simple and resemble those of the dc commutator motor. The co-ordinate axis has been chosen so that the center of a north pole

of the magnetic is aligned with the x-axis at $\theta = 0$. The stator has 12 slots and a three-phasing winding. Thus, there are two slots per pole per phase. Consider a BLPM SQW DC MOTOR
Let p be the number of poles (PM)
B_g be the flux density in the air gap in wb/m[2].
B_k is assumed to be constant over the entire pole pitch in the air gap (180° pole arc)
r be the radius of the airgap in m.
l be the length of the armature in m.
T_c be the number of turns per coil.
ω_m be the uniform angular velocity of the rotor in mechanical rad/sec.
$\omega_m = \frac{2\pi N}{60}$ where N is the speed in rpm.
Flux density distribution in the air gap is as shown in fig above. At t=0(it is assumed that the axis of the coil coincides with the axis of the permanent magnet at time t=0).
Let at ω_{mt}=0, the center of N-pole magnet is aligned with x-axis.
At ω_{mt}==0, x-axis is along PM axis.

Therefore flux enclosed by the coli is

Φ_{max}=B x 2πr/p x l ……………….(1)

=flux/pole

$\Phi_{max} = rl\int_0^{\pi} B(\theta)d\theta$

$= B_g rl[\theta]_0^{\pi}$

$= B_g rl[\pi]$

At ω_{mt}=0,the flux linkage of the coil is

Λ_{max}= (B_g x 2πr/p x l)T_c ωb-T ……………………(2)

Let the rotor rotating in ccw direction and when $\omega_{mt}=\pi/2$, the flux enclosed by the coil Φ, Therefore λ=0.

The flux linkages of the coil vary with θ variation of the flux linkage is as shown above.

The flux linkages of the coil changes from $B_g rlTc\pi/p$ at ω_{mt}=0 (i.e) t= 0 t0 θ at $t=\pi/p\omega_m$.

Change of flux linkage of the coil (i.e) Δλ is

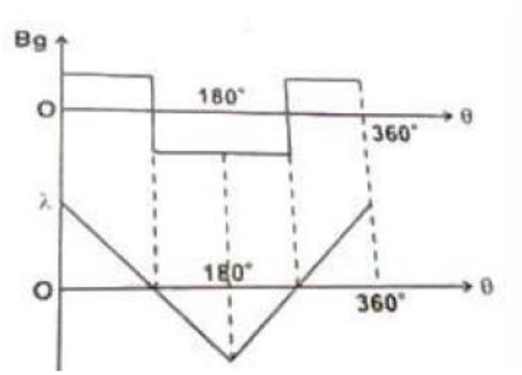
Δλ/Δt =Final flux linkage – Initial flux linkage/time.

$= 0 - (2B_g rlTc\pi/p)/(\pi/p\omega_m)$

$= -(2B_g rlTc\omega_m)$ ……………………………(3)

The emf induced in the coil $e_c = -d\lambda/dt$

$e_c = 2B_g rlTc\omega_m$ ………………………………(4)



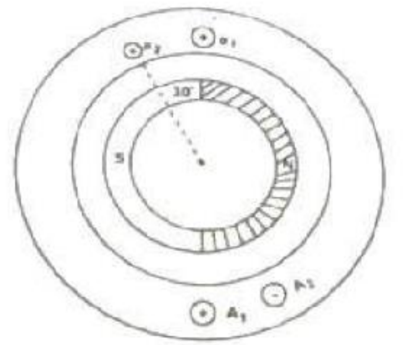


Fig Magnetic Flux Density around the Air gap. Fig Motor Showing two Coils of One Phase.

It is seen that the emf waveform is rectangular and it toggles between + e_c to - e_c. The period of the wave is $2\pi r/p\omega_m$ sec and magnitude of e_c is

$e_c = 2B_g rlTc\omega_m$ volts ....(5)

Consider two coils a1A1 and a2A2 as shown in fig .Coil a2A2 is adjacent to a1A1 is displaced from a1A1 by an angle 30^0(i.e.) slot angle ϒ .

The magnitude of emf induced in the coil a1A1

$e_{c2} = B_g rlTc\omega_m$ volts ....(6)

The magnitude of emf induced in the coil a2A2

$e_{c2} = B_g rlTc\omega_m$ volts ....(7)

Its emf waveform is also rectangular but displaced by the emf of waveform of coil e_{c1} by slot angle ϒ .

If the two coils are connected in series, the total phase voltage is the sum of the two separate coil voltages.

$e_{c1} + e_{c2} = 2B_g rlTc\omega_m$ ....(8)

Let n_c be the number of coils that are connected in series per phase $n_cT_c = T_{ph}$ be the number of turns/phase.

$e_{ph} = n_c\,[2B_g rlTc\omega_m]$ ....(9)

$e_{ph} = 2B_g rlTph\omega_m$ volts ....(10)

e_{ph}=resultant emf when all n_c coils are connected in series.

The waveforms are as shown in fig.

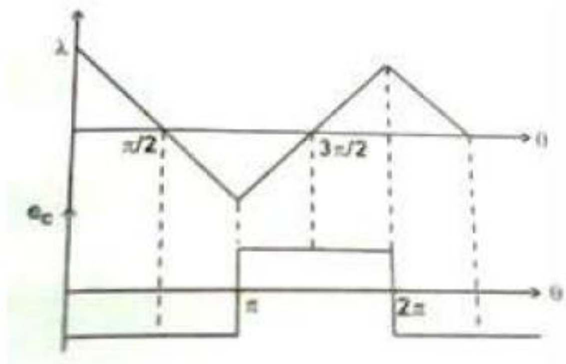


Fig Emf waveform of coil 1

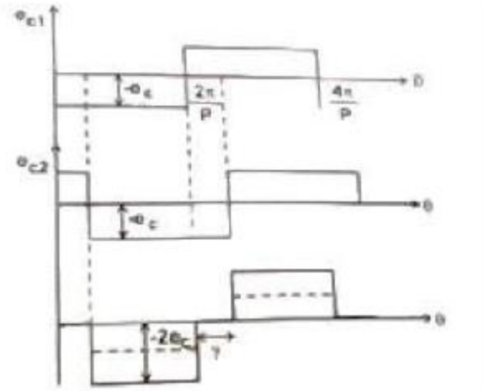


Fig Emf waveform of phase a

The waveform of e_{ph} is stepped and its amplitude is $2B_g rlT_{ph}\omega_m$ volts.

7.2.3 TORQUE EQUATION

Power input = VI

=[2 eph + 2 I Rph + 2 Vdd] I ....(1)

VI=[2 eph + 2 I Rph + 2 Vdd] I ....(2)

VI= electrical power input

2 eph I = power converted as mechanical

2 I2 Rph = power loss in the armature winding

2 Vdd I = power loss in the device

Mechanical power developed= 2 eph I .... (3)

eph= 2(2BgrlTphωm)I

eph= 4BgrlTphωm .... (4)

Mechanical power = (2πN/60)T .... (5)

= ωmT ....(6)

Where N=Speed in rpm

T=Torque in N-m

ωm=Speed in rad/sec

Therefore T=4BgrlTphI ....(7)

=KtT .... (8)

Where Kt = 4BgrlTph=Ke ....(9)

(a) Case1: Starting Torque

$\omega m=0$

$Istg=(V/2Rph)$ ....(10)

$Tstg=4BgrlTph(V/2Rph)$ ....(11)

$Tstg=Kt(V/2Rph)$ ....(12)

Starting torque or stalling torque depends upon V.

To vary the starting torque the supply voltage is to be varied.

(b) Case 2: On load condition

$T = K_t I$ ....(13)

$= 4 B_g r l T_{ph} I$

$I = (V-2e_{ph})/(2R_{ph})$ ....(14)

$2e_{ph} = V-2I R_{ph}$

$4 B_g r l T_{ph} \omega_m = V-2I R_{ph}$ ....(15)

$K_e \omega_m = V-2I R_{ph}$

$\omega_m = (V-2I R_{ph})/ K_e$ ....(16)

$\omega_{m0} = V/ K_e$ ....(17)

$\omega_m/ \omega_{m0} = ((V-2I R_{ph})/ K_e) (V/ K_e)$

$=(V-2I R_{ph})/ V$

$\omega_m/ \omega_{m0} = 1- ((V-2I R_{ph})/ V)$ ....(18)

$I/(T_{stg}) = (K_t I)/(K_t I_{stg})$

$= I.(2R_{ph}/V)$

$T/ T_{stg} = 2I R_{ph}/V$ ....(19)

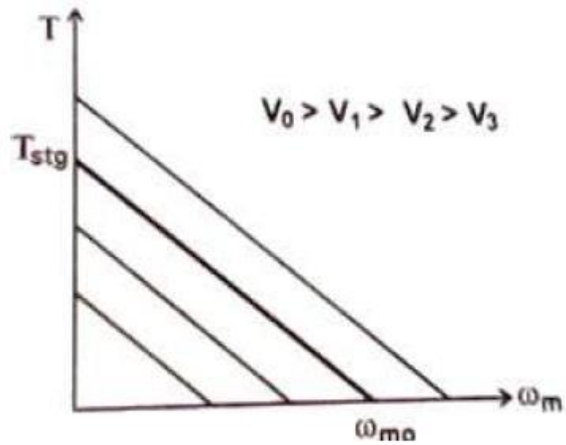
Substituding eqn. 19 in eqn. 18

$\omega_m/ \omega_{m0} = 1-(T/ T_{stg})$ ....(20)

$\omega_m/ \omega_{m0} = 1-(I-I_{stg})$ ....(21)

7.2.4 TORQUE- SPEED CHARACTERSISTICS

Let the supply voltage V be constant. A family of torque speed characteristics for various constant supply voltages is as shown in figure below.



Merits

- There is no field winding. Therefore, there is no field cu loss.
- The length of the motor is less as there is no mechanical commutator.
- Size of the motor becomes less.
- It is possible to nave very high speeds.
- It is self-starting motor. Speed can be controlled.
- Motor can be operated in hazardous atmospheric condition.
- Efficiency is better.

Demerits

- Field cannot be controlled.
- Power rating is restricted because of the maximum available size of permanent
- magnets.
- A rotor position sensor is required.
- A power electronic switch circuitry is required.

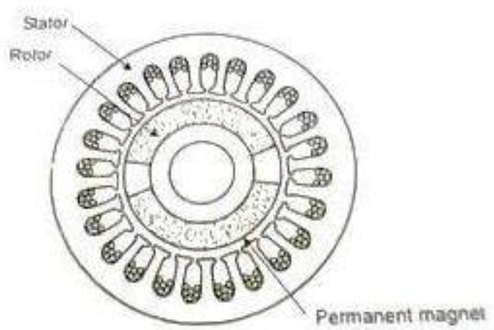
7.3 PERMANENT MAGNET SYNCHRONOUS MOTOR

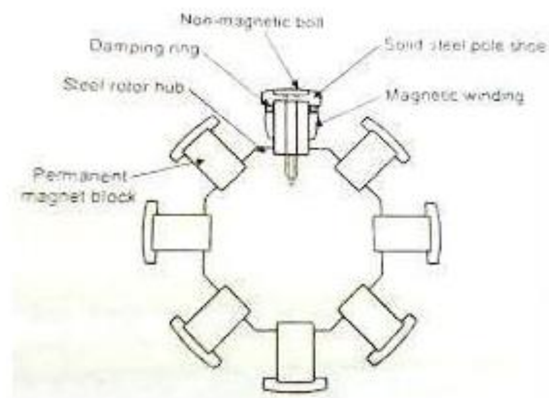
A permanent magnet synchronous motor is also called as brushless permanent magnet sine wave motor. Permanent magnet synchronous machines generally have same operating and performance characteristics as synchronous machines. A permanent magnet machine can have a configuration almost identical to that of the conventional synchronous machines with absence of slip rings and a field winding.

Fig. shows a cross section of simple permanent magnet synchronous machines. It consists of the stationary member of the machine called stator. Stator laminations for axial air gap machines are often formed by winding continuous strips of soft steel. Various parts of the laminations are the teeth slots which contain the armature windings. Yoke completes the magnetic path. Lamination thickness depends upon the frequency of the armature source voltage and cost.

Armature windings are generally double layer (two coil side per slot) and lap wound. Individual coils are connected together to form phasor groups. Phasor groups are connected together in series/parallel combinations to form star, delta, two phase (or) single windings. AC windings are generally short pitched to reduce harmonic voltage generated in the windings. Coils, phase groups and phases must be

insulated from each other in the end-turn regions and the required dielectric strength of the insulation will depend upon the voltage ratings of the machines.
In a permanent magnet machines the air gap serves an role in that its length largely determines the operating point of the permanent magnet in the no-load operating condition of the machines. Also, longer air gaps reduce machines windage losses. The permanent magnets form the poles equivalent to the wound field pole of conventional synchronous machines. Permanent magnet poles are inherently _salient_ and there is no equivalent to the cylindrical rotor pole configurations used in many convectional synchronous machines.
Many permanent magnet synchronous machines may be cylindrical or _smooth rotor_ physically but electrically the magnet is still equivalent to a salient pole structure. Some of the PMSM rotors have the permanent magnets directly facing the air gap as in fig.





7.3.1 EMF Equation

Flux density can be expressed as $B=|B|\sin\theta$ or $|B|\cos p\theta$ or $B\sin(P\theta+a)$ or $\cos(\rho\theta+\alpha)$, $2p=p$, (i.e) p-no of pole pairs depending upon the position of the reference axis as shown in figure.

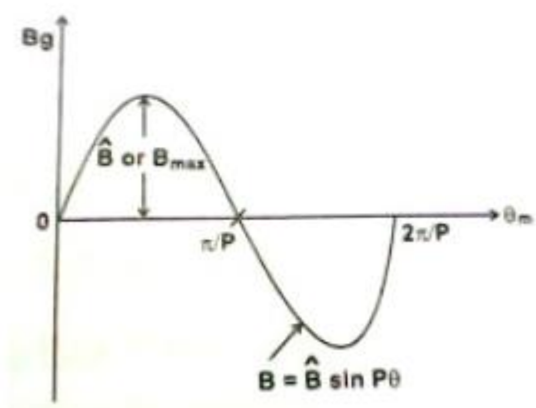


Fig flux density distribution

Consider a full pitched single turn armature coil as shown in fig . Let the rotor be revolving with a uniform angular velocity of $\dot{\omega}_m$ mech.rad/sec.
At time t = 0, let the axis of the single turn coil be along the polar axis.

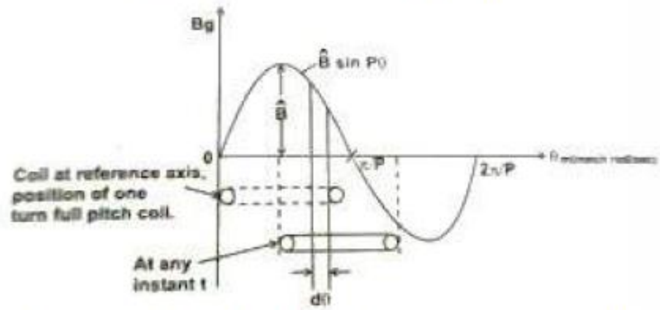


Fig   full pitched single turn armature coil

Consider a small strip of dθ mech.radians at a position θ from the reference.

Flux density at the strip B= B□sin pθ

Incremental flux in the strip dθ = B X area swept by the conductor

dθ = B□ sin pθ X lr dθ

B lr dθ weber

Where

L – Length of the armature in m

r – Radians of the armature

dθ = B□ sin pθ X lr dθ

= B□ lr sin pθ X dθ

Flux enclosed by the coil after lapes of t sec is

$$\Phi = \int_0^{\omega m + \frac{\pi}{p}} B^{\wedge}\, lr \sin P\,\theta\, d\,\theta \qquad \ldots\ldots\ (1)$$

$$\Phi = (2\, B\square\ \ lr/p) \cos p\theta\ \omega_{mt}$$

As per faradays law of electromagnetic induction, emf induction in the single turn coil.

$e = -N\, d\,\Phi\,/dt$

$-d\Phi\ /dt$ as N=1

$= -\, d\Phi\ /dt\ \ ((2\, B\square\ lr/p) \cos p\theta\ \omega_{mt})$

$= (2\, B\square\ lr/p)\ p\ \omega_m \sin p\ \omega_{mt}$

$$e = 2\, B\square\ lr\ \omega_m \sin p\ \omega_{mt} \qquad \ldots\ldots (2)$$

let the armature winding be such that all turns of the phase are concentrated full pitched and located with respect to pole axis in the same manner.

Let T_{ph} be the number of turns connected in series per phase. Then the algebraic addition of the emfs of the individual turns gives the emf induced per phase as all the emf are equal and in phase.

$e_{ph} = (2 B□lr\, \omega_m \sin p\, \omega_{mt})T_{ph}$ ……..(3)

$= 2 B□\, lr\, \omega_m T_{ph} \sin p\, \omega_{mt}$

$= Ĕ_{ph} \sin p\, \omega_{mt}$ where $p\, \omega_{mt} = \omega_e$ angular frequency in red/sec

$= Ĕ_{ph} \sin \omega_{et}$

$Ĕ_{ph} = 2 B□\, lr\, \omega_m T_{ph} \omega_m$ ……..(4)

$Ĕ_{ph}$ = rms value of the phase emf

$= Ĕ_{ph}/ \sqrt{2}$

$= \sqrt{2} B□\, lr\, \omega_m T_{ph} \omega_m$

$\omega_m = \omega_e/\rho$

Φ_m – sinusoidal distributedflux / pole

$\Phi = B_{av}\, \tau\, l$ …….(5)

$= B_{av}$ X $(2\pi r / 2p)$ X l

Average value of flux density for sinewave $=2/\pi$

$= (2/\pi) B□$

$\Phi_m = (2/\pi) B□$ X $(\pi r / P)$. l

$\Phi_m = (2 B□\, r\, l / P)$

$B□\, r\, l = (P \Phi_m / 2)$ …….(6)

$E_{ph} = \sqrt{2} B□\, lr\, \omega_m T_{ph}$ .volt

Sub equ

$E_{ph} = \sqrt{2} (P \Phi_m / 2)\omega_m T_{ph}$

$= \sqrt{2} (P \Phi_m / 2) (\omega/p) T_{ph}$

$= \sqrt{2} (P \Phi_m / 2) (2\pi f/p) T_{ph}$

$\mathbf{E_{ph} = 4.44\, f\, \Phi_m T_{ph}}$. **Volt** ……. (7)

7.3.2 Torque Equation

Ampere conductor density distribution

Let the figure shows the ampere conductor density distribution in the air gap due to the current carrying armature winding be sinusoidal distributed in the airgap space.

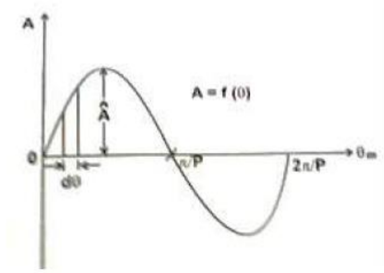


Fig. Ampere conductor density distribution

$A = A^\wedge \sin p\Theta$

Where A = ampere conductor density

= ampere conductor/degree

Consider a strip of d θ at an angle θ from the reference axis.

Ampere conductor in the strip d θ = A d θ ...... (1)

$= A^\wedge \sin P\theta\, d\theta$

Ampere conductor per pole $= \int_0^{\frac{\pi}{P}} A^\wedge \sin P\theta\, d\theta$ .... (2)

$= -A^\wedge \left[\frac{\cos P\theta}{P}\right]$

$= -\frac{A^\wedge}{P}[\cos\pi - \cos 0]$

$= \frac{2A^\wedge}{P}$

Let T_{ph} be the number of full pitched turns per phase.

Let i be the current

i T_{ph} be the total ampere turns which is assumed to be θ sine distributed.

Total ampere conductors [sine distributed] = 2i T_{ph}

Sine distributed ampere conductors/pole $= \frac{2i\,Tph}{2P}$

Equating eqn.

$\frac{2A^\wedge}{P} = \frac{2i\,Tph}{2P}$

$A^{\wedge} = \frac{I\,Tph}{2}$ …….. (3)

Torque equation of an ideal BLPM sine wave motor:

Let the ampere conductor distribution of ideal BLPM sine wave motor be given by

$$A = A^{\wedge} \sin P\,\theta$$

Let the flux density distribution set up by the rotor permanent magnet be also sinusoidal.

Let the axis of armature ampere conductor distribution be displaced from the axis of the flux density distribution by an angle $(\frac{\pi}{2} - \alpha)$ as shown in fig 5.6

$$[\, B = B^{\wedge} \sin \left(P\,\theta + \left(\frac{\pi}{2} - \alpha\right)\right) \quad …….. (4)$$

$$= B^{\wedge} \sin \left[\frac{\pi}{2} = (P\,\theta - \alpha)\right]$$

$$= B^{\wedge} \cos (P\,\theta - \alpha) \quad ……..(5)$$

Consider a small strip of width d θ at an angle $\theta$from the reference axis.

Flux density at the strip $B = B^{\wedge} \cos(p\Theta - \alpha)$

Ampere conductors in the strip $= Ad\Theta$

$$= A \sin P\Theta\, d\Theta \quad ……. (6)$$

Force experienced by the armature conductors in the strip $d\Theta = BIAd\Theta$

$dF = B^{\wedge} \cos(P\Theta\text{-}\alpha) 1.A^{\wedge}B^{\wedge}1.A^{\wedge} \sin P\Theta.d\Theta$

$dF = A^{\wedge}B^{\wedge}I \sin P\Theta \cos(P\Theta\text{-}\alpha)\, d\Theta.$

Let _r' be the radial distance of the conductors from the axis of the shaft.

Torque experienced by the ampere conductors of the strip=dF*r

$dT = AB\ \ r1 \sin P\Theta \cos(P\Theta\text{-}\alpha)$ De N-m

Torque experienced by the ampere conductors/pole T/Pole = $\int_0^{\pi/p} dT$

$$T = \int_0^{\pi} A\,B\ rl \sin P\,\theta \cos (P\,\theta - \alpha)\, d\,\theta \quad …….(7)$$

$$= A\ B\ rl/2 \int_0^{\pi/p} (\sin P\,\theta + P\,\theta - \alpha + \sin \alpha)\, d\,\theta$$

$$= A\ B\ rl/2 \left[-\frac{\cos(2P\theta - \alpha)}{2p} + \theta \sin\alpha\right]$$

$=A\ B\ rl/2\left[-\frac{\cos\alpha}{2p}+\frac{\cos\alpha}{2p}+\frac{\pi}{p}\sin\alpha\right]$

$T=A\ B\ rl/2.\frac{\pi}{p}\sin\alpha$ N-m ……(8)

The total torque experienced by all the armature conductors

=2P x torque/pole

$=2P \times \frac{\pi}{P} \times \frac{ABrl}{2}\sin\alpha$

T= π A B rl sin α N-m… …..(9)

As the armature conductors are located in stator of the BLPM SNW motor, the rotor experiences an equal and opposite torque.

Torque experienced by the rotor

= Torque developed by the rotor

= -π A B rl sin α

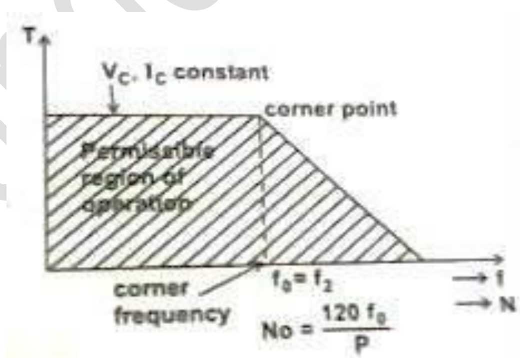
= π A B rl sin β where β= -α ……(10)

Β is known as power angle or torque angle.

T= π A B rl sin β in an ideal motor.

7.3.3 Torque Speed Characteristics

The torque-speed characteristics of BLPM sine wave motor is shown in figure.



For a given V_c and I_c(i.e) maximum permissible voltage and maximum permissible current, maximum torque remains constant from a low frequency to f_o(i.e) corner frequency.

Any further increase in frequency decreases the maximum torque. At f_D(i.e.) f_{max} the torque Developed is zero. Shaded pole represents the permissible region of operation in torque speed characteristics.

7.4 RELUCTANCE MOTOR

The *variable reluctance motor* is based on the principle that an unrestrained piece of iron will move to complete a magnetic flux path with minimum *reluctance*, the magnetic analog of electrical resistance.

Synchronous reluctance

If the rotating field of a large synchronous motor with salient poles is de-energized, it will still develop 10 or 15% of synchronous torque. This is due to variable reluctance throughout a rotor revolution. There is no practical application for a large synchronous reluctance motor. However, it is practical in small sizes.

If slots are cut into the conductor less rotor of an induction motor, corresponding to the stator slots, a synchronous reluctance motor result. It starts like an induction motor but runs with a small amount of synchronous torque. The synchronous torque is due to changes in reluctance of the magnetic path from the stator through the rotor as the slots align. This motor is an inexpensive means of developing a moderate synchronous torque. Low power factor, low pullout torque, and low efficiency are characteristics of the direct power line driven variable reluctance motor. Such was the status of the variable reluctance motor for a century before the development of semiconductor power control.

Switched reluctance

If an iron rotor with poles, but without any conductors, is fitted to a multi-phase stator, a *switched reluctance motor*, capable of synchronizing with the stator field results. When a stator coil pole pair is energized, the rotor will move to the lowest magnetic reluctance path. A switched reluctance motor is also known as a variable reluctance motor. The reluctance of the rotor to stator flux path varies with the position of the rotor.

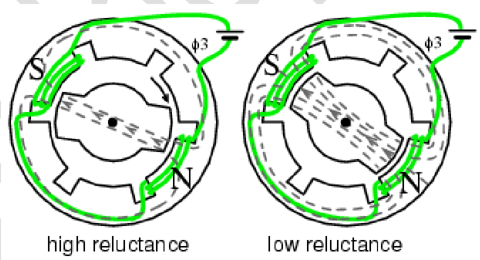


Fig: Reluctance is a function of rotor position in a variable reluctance motor.

Sequential switching of the stator phases moves the rotor from one position to the next. The magnetic flux seeks the path of least reluctance, the magnetic analog of electric resistance. This is an over simplified rotor and waveforms to illustrate operation.

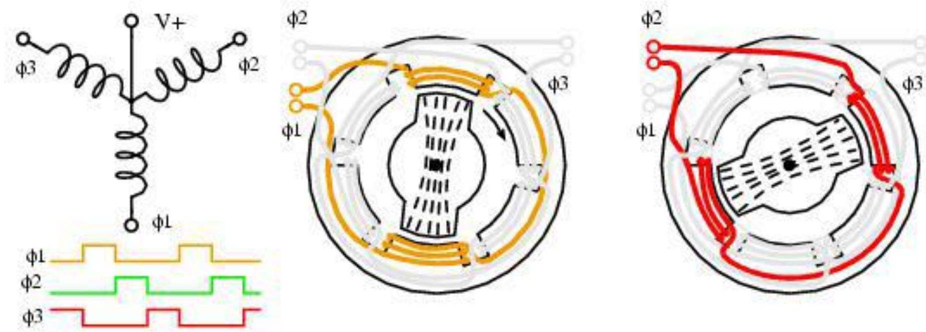


Fig: Sequential switching of stator phases of the reluctance motor

If one end of each 3-phase winding of the switched reluctance motor is brought out via a common lead wire, we can explain operation as if it were a stepper motor. The other coil connections are successively pulled to ground, one at a time, in a *wave drive* pattern. This attracts the rotor to the clockwise rotating magnetic field in 60_o increments.

Various waveforms may drive variable reluctance motors. Wave drive (a) is simple, requiring only a single ended unipolar switch. That is, one which only switches in one direction. More torque is provided by the bipolar drive (b), but requires a bipolar switch. The power driver must pull alternately high and low. Waveforms (a & b) are applicable to the stepper motor version of the variable reluctance motor. For smooth vibration free operation, the 6-step approximation of a sine wave (c) is desirable and easy to generate. Sine wave drive (d) may be generated by a pulse width modulator (PWM), or drawn from the power line.

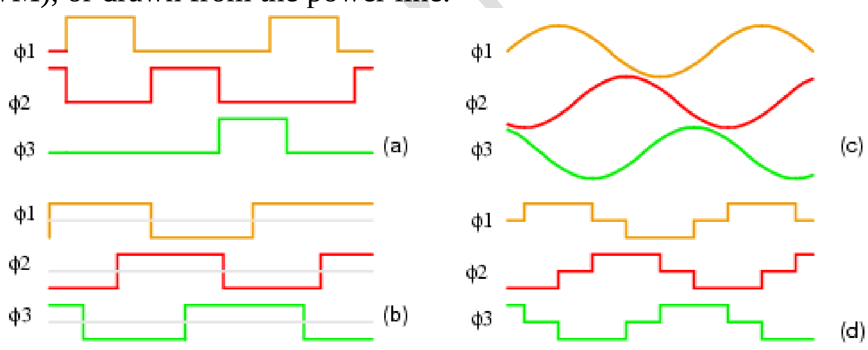


Fig: Variable reluctance motor drive waveforms: (a) unipolar wave drive, (b) bipolar full step (c) sine wave (d) bipolar 6-step.

Doubling the number of stator poles decreases the rotating speed and increases torque. This might eliminate a gear reduction drive. A variable reluctance motor intended to move in discrete steps, stop, and start is a *variable reluctance stepper motor*, covered in another section. If smooth rotation is the goal, there is an electronic driven version of the switched reluctance motor. Variable reluctance motors or steppers actually use rotors like those in Fig.

Electronic driven variable reluctance motor

Variable reluctance motors are poor performers when direct power line driven. However, microprocessors and solid-state power drive makes this motor an economical high performance solution in some high volume applications.

Though difficult to control, this motor is easy to spin. Sequential switching of the field coils creates a rotating magnetic field which drags the irregularly shaped rotor around with it as it seeks out the lowest magnetic reluctance path. The relationship between torque and stator current is highly nonlinear– difficult to control.

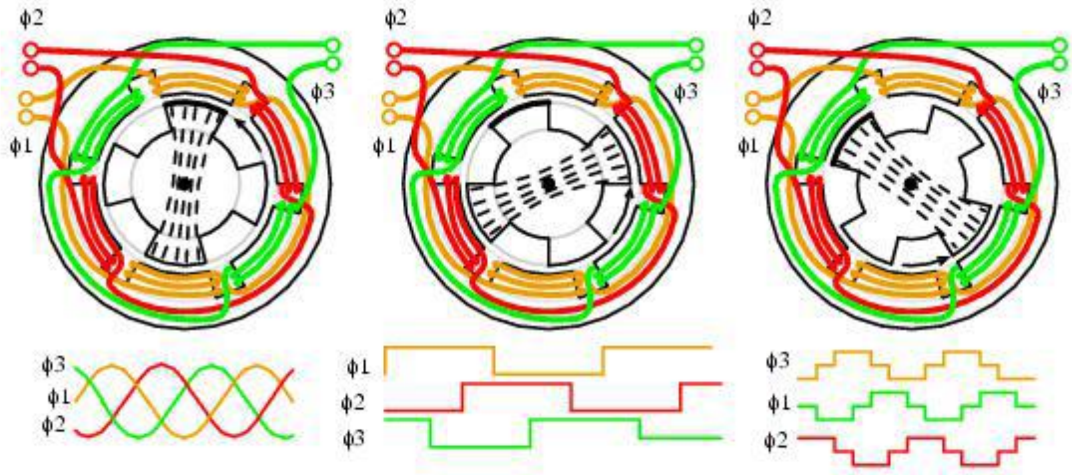


Fig: Electronic driven variable reluctance motor

An electronic driven variable reluctance motor resembles a brushless DC motor without a permanent magnet rotor. This makes the motor simple and inexpensive. However, this is offset by the cost of the electronic control, which is not nearly as simple as that for a brushless DC motor. Electronic control makes it practical to drive the motor well above and below the power line frequency. A variable reluctance motor driven by a *servo*, an electronic feedback system, controls torque and speed, minimizing ripple torque.

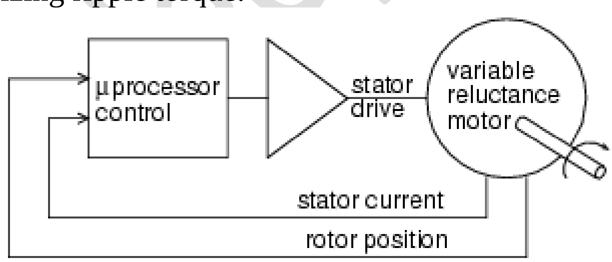


Fig: Electronic driven variable reluctance motor.

This is the opposite of the high ripple torque desired in stepper motors. Rather than a stepper, a variable reluctance motor is optimized for continuous high-speed rotation with minimum ripple torque. It is necessary to measure the rotor position with a rotary position sensor like an optical or magnetic encoder, or derive this from monitoring the stator back EMF. A microprocessor performs complex calculations for switching the windings at the proper time with solid state devices. This must be done precisely to minimize audible noise and ripple torque. For lowest ripple torque, winding current must be monitored and controlled. The strict drive requirements make this motor only practical for high volume applications like energy efficient vacuum cleaner motors, fan motors, or pump motors. One such vacuum cleaner uses a compact high efficiency electronic driven 100,000 rpm fan motor. The simplicity of the motor compensates for the drive electronics cost. No brushes, no commutator, no rotor windings, no permanent magnets, simplifies motor manufacture. The efficiency of this electronic driven motor can be high. But, it requires considerable optimization, using specialized design techniques, which is only justified for large manufacturing volumes.

Advantages

- Simple construction- no brushes, commutator, or permanent magnets, no Cu or Al in the rotor.
- High efficiency and reliability compared to conventional AC or DC motors.
- High starting torque.
- Cost effective compared to brushless DC motor in high volumes.
- Adaptable to very high ambient temperature.
- Low cost accurate speed control possible if volume is high enough.

Disadvantages

- Current versus torque is highly nonlinear
- Phase switching must be precise to minimize ripple torque
- Phase current must be controlled to minimize ripple torque
- Acoustic and electrical noise
- Not applicable to low volumes due to complex control issues

7.5 STEPPER MOTOR

It is an incremental drive (digital) actuator and is driven in fixed angular steps. This means that a digital signal is used to drive the motor and every time it receives a digital pulse it rotates a specific number of degrees in rotation.

- Each step of rotation is the response of the motor to an input pulse (or digital command).
- Step-wise rotation of the rotor can be synchronized with pulses in a command pulse train, assuming that no steps are missed, thereby making the motor respond faithfully to the pulse signal in an open-loop manner.
- Stepper motors have emerged as cost-effective alternatives for DC servomotors in high-speed, motion-control applications (except the high torque-speed range) with the improvements in permanent magnets and the incorporation of solid-state circuitry and logic devices in their drive systems.
- Today stepper motors can be found in computer peripherals, machine tools, medical equipment, automotive devices, and small business machines, etc.

Stepper motors are usually operated in open loop mode.

A stepper motor rotates by a specific number of degrees like 2°, 2.5°,5°,7.5°……….15° and so on in response to an input electrical pulse. So, it basically converts digital pulse inputs into analog output shaft of the machine. Typical applications of stepper motors requiring incremental motors are printers, tape-drives, disk drives, machine tools, process controls, X-Y recorders, robotics etc.
Two types of stepper motor are used:
i) Variable Reluctance type
ii) Permanent magnet type

i) Variable Reluctance Type: Figure shows a basic circuit configuration of a 4-phase 2-pole variable reluctance stepper motor. When the stator phases are excited with dc current in proper sequence, the resultant air gap field steps around and the rotor follows the axis of this air gap field. The rotor intact tries to align itself with the direction of the resultant magnetic field.

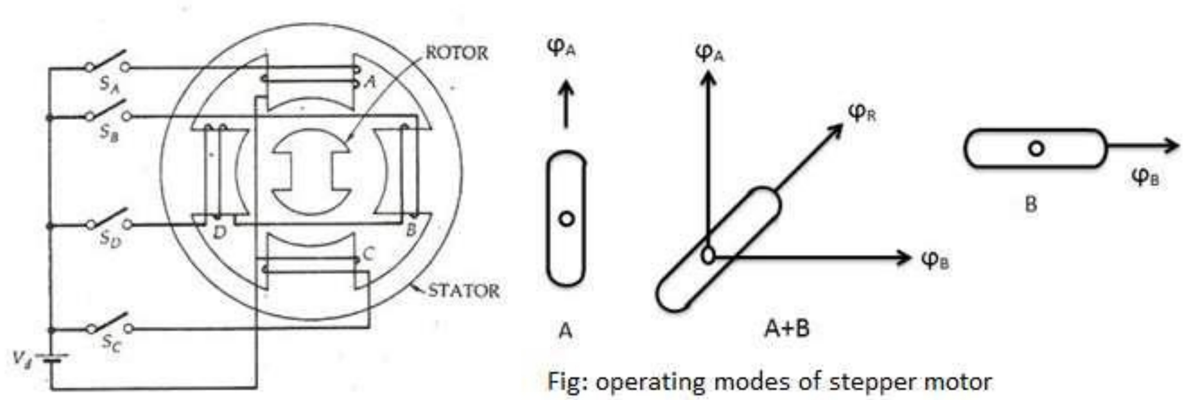
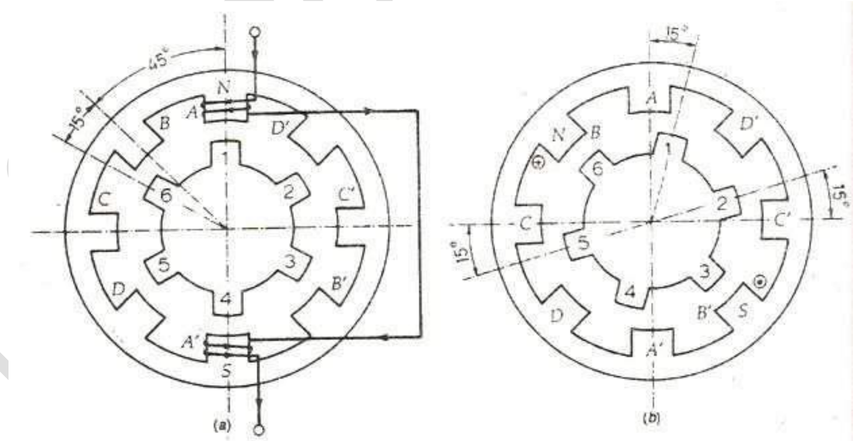


Fig: operating modes of stepper motor

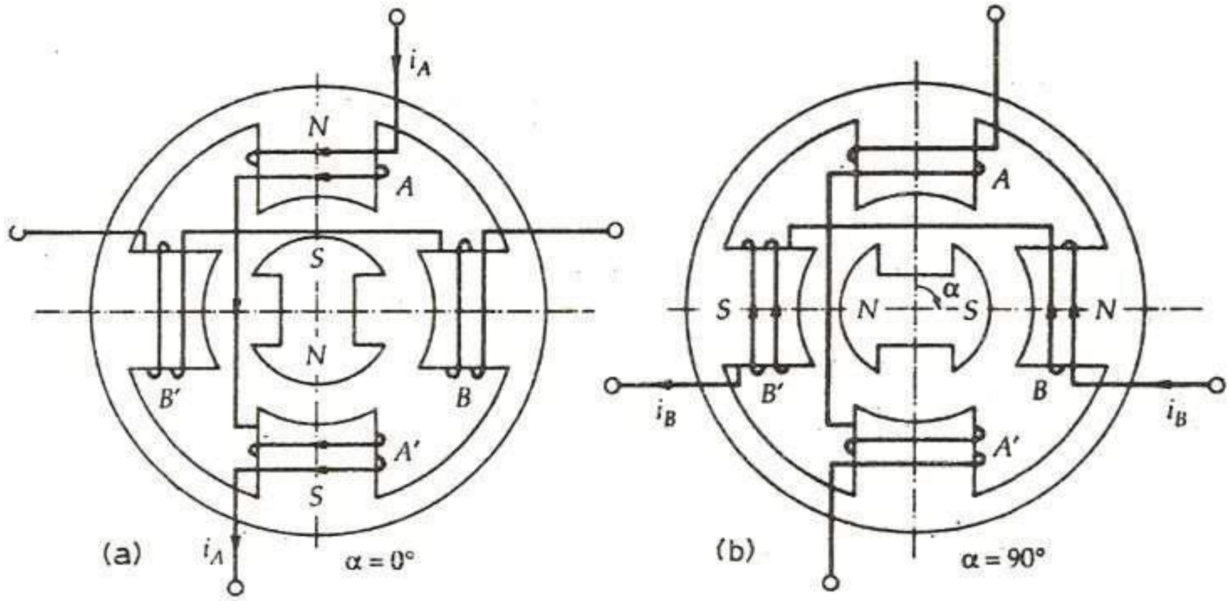
Fig above shows the modes of stepper motor for a 45° step in the clockwise direction. The windings are excited in the sequence A, A+B, B, B+C and so on. When winding A is excited, the rotor aligns with axis of phase A. Next both windings A and B are excited which makes the resultant mmf axis move 45° in the clockwise direction. The rotor thus now aligns with this resultant mmf axis. In this way, the rotor can be rotated in steps of 45°. The direction of rotation can be reversed by reversing the sequence of switching the windings A, A+ D, D, D+C etc.

How can smaller step rotation be achieved?

To achieve smaller steps, a multi-pole rotor construction is required. The construction of 4-phase, 6-pole stepper motor as shown in fig. Again when phase A winding is excited, pole P_1 is aligned with the axis of A as shown in fig Now, phase A and B windings are excited. The resultant mmf moves in clockwise direction by 45°. Pole P_2 is nearest to the new resultant field axis, is pulled to align with it. The motor therefore steps in the anticlockwise direction by 15°. Next phase A is de-excited and excitation of phase B windings pulls the nearest poles P_3 to align with the axis of phase B. Therefore, if the windings are excited in the sequence A, A+B, B, B+C…… the rotor rotates in anticlockwise direction as steps of 15°.



ii) Permanent magnet type stepper motor: It has construction similar to variable reluctance motor. The fig below shows the alignment of the rotor if phase A winding is excited. Now if the excitation is switched to phase B the rotor moves by a step of 90°. However, the direction in which the rotor will move is governed by the direction of current in B phase. Fig above illustrates the rotor position for positive current in phase A (A →A’). A switch over to positive current in phase B (B→ B’) will produce a clockwise step. If direction of currents B→ B’, the rotor will step in the anticlockwise direction. These motors are usually limited to steps range from 30°-90° only as it is difficult to make a permanent magnet rotor with large number of poles.



ADVANTAGES

- Position error is noncumulative. A high accuracy of motion is possible, even under open loop control.
- Large savings in sensor (measurement system) and controller costs are possible when the open-loop mode is used.
- Because of the incremental nature of command and motion, stepper motors are easily adaptable to digital control applications.
- No serious stability problems exist, even under open-loop control.
- Torque capacity and power requirements can be optimized and the response can be controlled by electronic switching. Brushless construction has obvious advantages.

DISADVANTAGES

- They have low torque capacity (typically less than 2,000 oz-in) compared to DC motors.
- They have limited speed (limited by torque capacity and by pulse-missing problems due to faulty switching systems and drive circuits).
- They have high vibration levels due to stepwise motion.
- Large errors and oscillations can result when a pulse is missed under open-loop control.

UNIT 8: TEMPERATURE RISE IN ELECTRICAL MACHINE

Heat is developed in all electrical machine due to the losses in the various parts,

- copper loss (I^2R loss) in conductor,
- Hysteresis losses and eddy current losses in iron and
- mechanical losses (in rotating machine only) due to friction of the bearing, air friction or windage causing the temperature of that part to rise.

The temperature rise continues until all the heat generated is dissipated to the surroundings by one or more of the natural modes of heat transfer i.e Conduction, Convection and radiation). The temperature rise depends upon

1. The amount of heat produced and
2. The amount of heat dissipated per 1°C rise of the surface of a machine.

According to Newtons law of cooling the rate of loss of energy of a hot body is proportional to the difference in temperature between that body and its surroundings. This law is only valid for the moderate temperature difference(up to 100°C) and for the bodies dissipating heat by radiation and natural convection.

The amount of heat dissipated per 1°C rise of the surface of a machine depends on the surface area of cooling. The size of motor for any service is governed by the maximum temperature rise when operating under the given load conditions and the maximum torque required. Electrical machine is designed for a limited temperature rise.

[Continuous rating of machine is that rating for which the final temperature rise is equal to or just below the permissible value of temperature rise for the insulating material used in protection of motor windings]

When the machine is overloaded for long time that its final temperature rise exceeds the permissible limit, it is likely to be damaged. In worst cases, it will result in thermal breakdown of the insulating material which will cause a short circuit in the motor. The short circuit may lead to fire. But, in less severe cases, the quality of the insulation will deteriorate such that thermal breakdown with future overloads occur, shortening the useful life of machine.

8.1 HEATING-TIME CURVE

For determination of an expression for the temperature rise of an electrical machine after time t seconds from the instant of switching it on,

Power converted into heat=P joules/s or Watts

Mass of active parts of machine=m Kg

Specific heat of material=Cp joules/kg/°C surface area of cooling=α in watts per metre^2 of surface per °C of difference between surface and ambient cooling temperatures.

Assumptions made

1. Losses or heat produces remains constant during the temperature rise.
2. Heat dissipated is directly proportional to the difference in temperature of the motor and the cooling medium.

Suppose a machine attains a temperature rise of θ° C above ambient temperature after t seconds of switching on the machine and further rise of temperature by dθ in very small time dt.

Energy converted into heat=Pdt joules

Heat absorbed=mCpdθ joules

Heat dissipated=Sθαdt joules

since energy converted into heat=Heat absorbed + heat dissipated

pdt=mCpdθ + Sθαdt

Or, (p-Sαθ)dt=mCpdθ

$$or\ \frac{dt}{mC_p/s\alpha} = \frac{d\theta}{\frac{p}{s\alpha} - \theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(i)$$

When the final temperature is reached, there is no absorption of heat. The heat generated is dissipated.

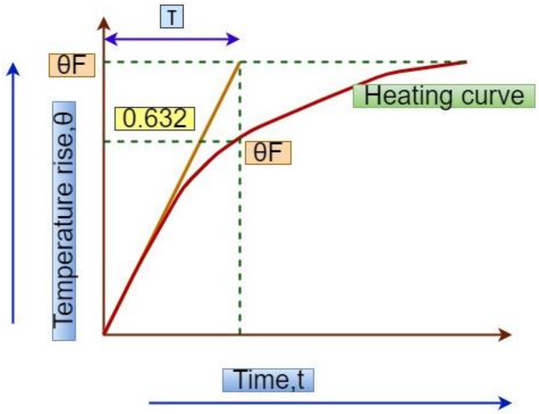


Fig:Temperature Rise-Time curve

$$pdt = S\theta_F \alpha dt$$

or Final temperature rise,

$$\theta_F = \frac{p}{S\alpha}$$

Substituting vale of θ_F *in equation (i) we get,*

$$\frac{S\alpha dt}{mC_p} = \frac{d\theta}{\theta_F - \theta}$$

Integrating both sides we get,

$$\frac{S\alpha}{mC_p} t = -\log_e(\theta_F - \theta) + K_1 \quad \text{.................}(ii)$$

where $K1$ *is a constant of integration.*

Substituting $t = o, \theta = \theta_1$*;initial temperature from initial conditions;we have,*

$$0 = -\log_e(\theta_F - \theta_1) + K_1$$

$$or\ K_1 = \log_e(\theta_F - \theta_1)$$

substituting $K_1 = \log_e(\theta_F - \theta_1)$ *in equation ii we get,*

$$\frac{S\alpha}{mC_p} t = -\log_e(\theta_F - \theta) + \log_e(\theta_F - \theta_1) = \log_e\left(\frac{\theta_F - \theta_1}{\theta_F - \theta}\right)$$

$$or, e^{\frac{S\alpha}{mC_p} t} = \frac{\theta_F - \theta_1}{\theta_F - \theta}$$

$$or, \theta_F - \theta = (\theta_F - \theta_1) e^{-\frac{S\alpha}{mC_p} t}$$

$$or, \theta = \theta_F - (\theta_F - \theta_1) e^{-\frac{S\alpha}{mC_p} t} = \theta_F - (\theta_F - \theta_1) e^{-\frac{t}{\zeta}}$$

where,

$$\zeta = \frac{mC_p}{S\alpha} \text{ and is known as heating constant.}$$

If motor is started from ambient temperature $\theta_1 = 0$ *we have,*

$$\theta = \theta_F - \theta_F^{-\frac{t}{\zeta}}$$

$$= \theta_F (1 - e^{-\frac{t}{\zeta}}) \quad \text{.....................}(iii)$$

Heating time constant is defined as the time taken by the motor in attaining the final steady value if the initial rate of rise of temperature were maintained through the operation.

Substituting t=τ in equation iii we get,

$$\theta = \theta_F(1 - e^{-1})$$
$$= 0.632\theta_F$$

Heating time constant is also defined as the time duration which the machine will attain 63.2% of its final temperature rise above ambient temperature.

θF is directly proportional to the power losses and inversely proportional to the surface area S and specific heat dissipation α.

Note: Typical values of heating time constant lie between about 3/2 hour for small motors (7.5Kw to 15Kw) up to about 5 hours for motors of several hundred kW.

8.2 COOLING-TIME CURVE

Let the machine be switched off after reaching final steady-temperature rise of θF. when the machine is switched off, no heat is produced,

0=Heat absorbed + heat dissipated

or 0=mCpdθ + Sθα'dt

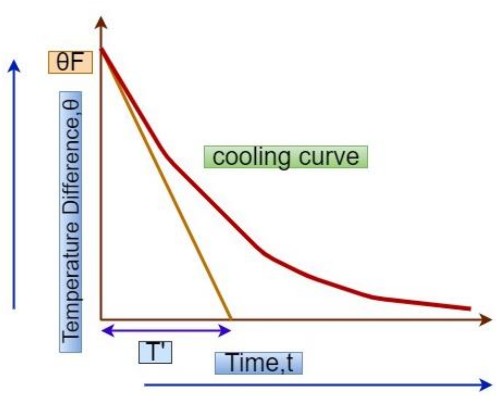
where α'=rate of heat dissipation during cooling

$$or\ dt = \frac{-mC_p d\theta}{S\alpha'\theta}$$

Integrating both sides, we have

$$t = \frac{-mC_p}{S\alpha'}\log_e\theta + K_2 \ ....(iv)$$

where K_2 is a constant of integration.



Substituting t=0,θ=θF from initial conditions in above equation(iv) we get,

$$K_2 = \frac{mC_p}{S\alpha'}\log_e \theta_F$$

substituting the value of K_2in equation iv we get,

$$t = -\frac{mC_p}{S\alpha'}\log_e \theta + \frac{mC_p}{S\alpha'} - \log_e \theta_F$$

$$= -\frac{mC_p}{S\alpha'}\log_e \frac{\theta}{\theta_F}$$

$$or\ \theta = \theta_F e^{-\frac{S\alpha' t}{mC_p}} = \theta_F e^{-\frac{t}{\zeta'}} \ \ldots\ldots\ldots\ldots\ldots(v)$$

where $\zeta' = \frac{mC_p}{S\alpha'}$ *called cooling time constant.*

substituting $t = \zeta'$ *in equation v we get,*

$$\theta = \theta_F e^{-1} = 0.368\theta_F$$

Cooling time constant may be also defined as the time required to cool the machine down to 0.368 times the initial temperature rise above ambient temperature.

Note: The cooling time constant of a rotating machine is usually larger than its heating time constant. In self cooled rotating machines, the cooling time constant is about 2-3 times greater than its heating time constant.

8.3 DUTY CYCLE

The term duty defines the load cycle to which the machine is subjected, including, if applicable, starting, electric braking, no-load, and rest de-energized periods, and including their durations and sequence in time. Duty considered as a generic term, for example, can be classified as a continuous duty, short-time duty, or periodic duty.

The percentage ratio between the period of loading and the total duration of the duty cycle is defined cyclic duration factor. It is the responsibility of the purchaser to declare the duty.

Where the purchaser does not declare the duty, the manufacturer shall assume that duty type S1 (continuous running duty) applies. The duty type shall be designated by the appropriate abbreviation and the purchaser may describe the type of duty based on the classifications according to the indications given below.

When the rating is assigned to the motor (values declared, usually by the manufacturer, for a specified working condition of a machine), the manufacturer must select one of the rating classes. If no designation is given, the rating relevant to the continuous running duty shall be applied.

Continuous running duty (S1): For a motor suitable to this duty type, the rating at which the machine may be operated for an unlimited period is specified. This class of rating corresponds to the duty type whose appropriate abbreviation is **S1.**

Short time duty(S2): For a motor suitable to this duty type, the rating at which the machine, starting at ambient temperature, may be operated for a limited period is specified. This class of rating corresponds to the duty type whose appropriate abbreviation is **S2.**

Periodic duty (S3-S8): For a motor suitable to this duty type, the rating at which the machine may be operated in a sequence of duty cycles is specified. With this type of duty, the loading cycle does not allow the machine to reach thermal equilibrium.

Intermittent periodic duty (S3): The duty type S3 is defined as a sequence of identical duty cycles, each including a time of operation at constant load and a time de-energized and at rest. The contribution to the temperature-rise given by the starting phase is negligible. A complete designation provides the abbreviation of the duty type followed by the indication of the cyclic duration factor (S3 30%).

Intermittent periodic duty with starting (S4): The duty type S4 is defined as a sequence of identical duty cycles, each cycle including a significant starting time, a time of operation at constant load and a time de-energized and at a rest.

Intermittent periodic duty with electric braking (S5): The duty type S5 is defined as a sequence of identical duty cycles, each cycle consisting of a starting time, a time of operation at constant load, a time of electric braking and a time de-energized and at a rest.

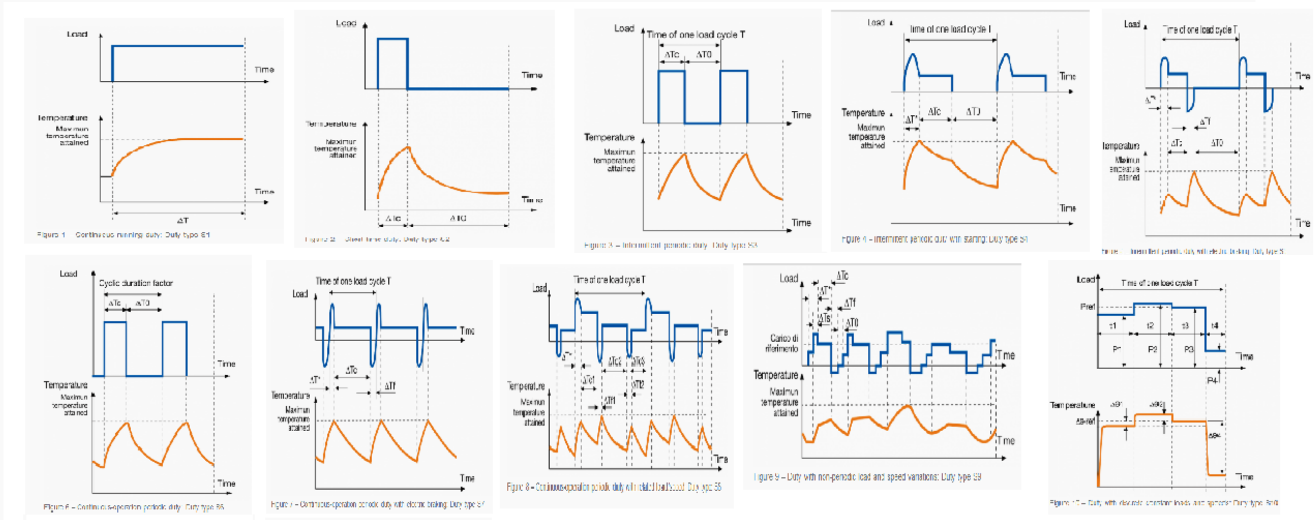
Continuous-operation periodic duty (S6): The duty type S6 is defined as a sequence of identical duty cycles, each cycle consisting of a time of operation at constant load and a time of operation at no-load. There is no time de-energized and at rest.

Continuous-operation periodic duty with electric braking (S7): The duty type S7 is defined as a sequence of identical duty cycles, each cycle consisting of a starting time, time of operation at constant load and a time of electric braking. There is no time de-energized and at rest.

Continuous-operation periodic duty with related load / speed (S8): The duty type S8 is defined as a sequence of identical duty cycles, each consisting of a time of operation at constant load corresponding to a predetermined speed of rotation, followed by one or more times of operation at other constant loads corresponding to different speeds of rotation.

Duty with non-periodic load and speed variations (S9): For a motor suitable to this duty type, the rating at which the machine may be operated non-periodically is specified. This class of rating corresponds to the duty type whose appropriate abbreviation is S9.

Duty with discrete constant load and speed (S10): For a motor suitable to this duty type, the rating at which the machine may be operated with a specific number of discrete loads for a sufficient time to allow the machine to reach thermal equilibrium is specified. The maximum permissible load within one cycle shall take into consideration all parts of the machine (the insulation system, bearings or other parts with respect to thermal expansion).



8.4 SELECTION OF POWER RATING OF MOTORS

From the point of view of motor rating for various duty cycles in section 1.6 can be broadly classified as:

- Continuous duty and constant load
- Continuous duty and variable load
- Short time rating

1. Continuous duty and constant load

If the motor has load torque of T N-m and it is running at *w* radians/seconds, if efficiency in *h*, then power rating of the motor is

$$P = \frac{T\omega}{1000} KW$$

Power rating is calculated and then a motor with next higher power rating from commercially available rating is selected. Obviously, motor speed should also match load's speed requirement. It is also necessary to check whether the motor can fulfill starting torque requirement also.

2. Continuous duty and variable load: The operating temperature of a motor should never exceed the maximum permissible temperature, because it will result in deterioration and breakdown of insulation and will shorten the service life of motors. It is general practice to base the motor power ratings on a standard value of temperature, say 35° c. Accordingly, the power given on the name plate of a motor corresponds to the power which the motor is capable of delivering without overheating at an ambient temperature of 35 c. the duty cycle is closely related to temperature and is generally taken to include the environmental factors also. The rating of a machine can be determined from heating considerations. However, the motor so selected should be checked for its overload capacity and starting torque. This is because, the motor selected purely on the basis of heating may not be able to meet the mechanical requirements of the basis of heating may not be able to meet the mechanical requirements of the load to be driven by it. The majority of electric machines used in drives operate continuously at a constant or only slightly variable load. For the determination of ratings of machines whose load characteristics have not been thoroughly studied, it becomes necessary to determine the load diagram i.e., diagram shown the variation of power output versus time. The temperature of the motor changes continuously when the load is variable. On account of this, it becomes difficult to select the motor rating as per heating. The analytical study of heating becomes highly complicated if the load diagram is irregular in shape or when it has a large number of steps. Therefore, it becomes extremely difficult to select the motor capacity through analysis of the load diagram due to select the motor capacity through analysis of the load diagram due to lack of accuracy of this method.

On the other hand, it is not correct to select the motor according to the lowest or highest load because the motor would be overloaded in the first case and under loaded in the second case. Therefore, it becomes necessary to adopt suitable methods for the determination of motor ratings.

The four commonly used methods are:

1 Methods of average losses

2 Equivalent current method

3 Equivalent torque method

4 Equivalent power method

1. Methods of average losses

- The method consists of finding average losses Q *av* in the motor when it operates according to the given load diagram.
- These losses are then compared with the Q , the losses corresponding to the continuous duty of the machine when operated at its normal rating.
- The method of average losses presupposes that when Q *av* = Q *nomn* , the motor will operate without temperature rise going above the maximum permissible for the particular class of insulation.

The figure shows a simple power load diagram and loss diagram for variable load conditions. The losses of the motor are calculated for each portion of the load diagram by referring to the efficiency curve of the motor.

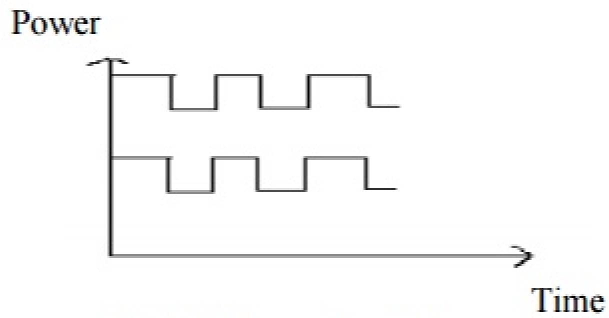


Fig 1.4 Average Load Losses

The average losses are given by

$$Q_{av} = \frac{Q_1t_1 + Q_2t_2 + Q_3t_3 + \ldots\ldots\ldots\ldots + Q_nt_n}{t_{1+}t_2 + \ldots\ldots + t_n}$$

- In case, the two losses are equal or differ by a small amount ,the motor is selected .if the losses differ considerably ,another motor is selected and the calculations repeated till a motor having almost the same losses as the average losses is found.
- It should be checked that the motor selected has a sufficient overload capacity and starting torque.
- The method of average losses does not take into account, the maximum temperature rise under variable load conditions. However, this method is accurate and reliable for determining the average temperature rise of the motor during one work cycle.

The disadvantage of this method is that it is tedious to work with and also many a times the efficiency curve is not readily available and the efficiency has to be calculated by means of empirical formula which may not be accurate.

2. Equivalent Current Method

The equivalent current method is based on the assumption that the actual variable current may be replaced by an equivalent current i *eq* which produces the same losses in the motor as the actual current.

$$I_{eq} = \sqrt{\frac{I_1^2t_1 + I_2^2t_2 + I_3^2t_3 + \ldots\ldots\ldots + I_n^2t_n}{t_1 + t_2 + t_{3+} \ldots\ldots\ldots\ldots + t_n}}$$

The equivalent current is compared with the rated current of the motor selected and the conditions I_{eq} £ I_{nom} should be met. I_{nom} is the rated current of the machine.

The machine selected should also be checked for its overload capacity,

For DC motors,

$$\boxed{\frac{I_{max}}{I_{nom}} \le 2 \text{ to } 2.5 \text{ and for induction motors}, \frac{I_{max}}{nom} \le 1.65 \text{ to } 2.75}$$

I_{max} = maximum current during the work cycle.

T_{max} = maximum load torque

T_{nom} = torque of the motor at rated power and speed

If the over load capacity of the motor selected is not sufficient, it becomes necessary to select a motor of higher power rating. The equivalent current may not be easy to calculate especially in cases where the current load diagram is irregular. The equivalent current in such cases is calculated from the following expression.

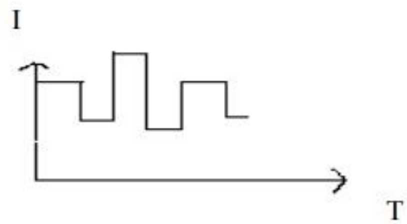


Fig 1.5 Equivalent Current

For a triangular shape diagram,

$$I_{eq} = \sqrt{\frac{I^2}{3}}$$

For a trapezoidal shaped diagram,

$$I_{eq} = \sqrt{\frac{I^2 + I_1 I_2 + I_2^2}{3}}$$

The above method allows the equivalent current values to be calculated with accuracy sufficient for practical purposes.

3. Equivalent torque method

Assuming constant flux and power factor, torque is directly proportional to current.

$$T = \sqrt{\frac{T_1^2 t_1 + T_2^2 t_2 + \dots\dots\dots + T_n^2 t_n}{t_1 + t_2 + \dots\dots + t_n}}$$

4. Equivalent power method

The equation for equivalent power method, power is directly proportional to torque.

At constant speed or where the changes in speed are small, the equivalent power is given by the following relationship,

$$P_{eq} = \sqrt{\frac{P_1^2 t_1 + P_2^2 t_2 + \dots\dots\dots + P_n^2 t_n}{t_1 + t_2 + \dots\dots\dots + t_n}}$$

3. Short time rating of motor

An electric motor of rated power P r subjected to its rated load continuously reaches its permissible temperature rise after due to time. If the same motor is to be used for short time duty, it can take up more load for a short period without increasing the maximum permissible temperature of the motor during this period.

$$\theta = \theta_m(1-e^{\frac{N}{\tau}}) \qquad \theta_m = \theta_m^{'}(1-e^{-\frac{N}{\tau}})$$

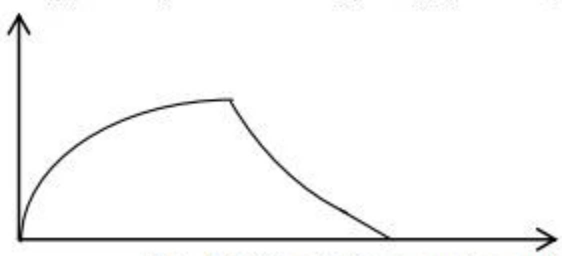


Fig 1.6 Short time motor rating

Where=operating time under rated load

qm =maximum permissible temperature which the motor running on short time rating will reach if run continuously at that rating.

qm = Maximum permissible temperature rise of the motor run continuously at continuous rating.

If it is assumed that the temperature rise is proportional to losses corresponding to the rating of the motor.

$$\frac{\theta_m^{'}}{\theta_m} = \frac{W_x}{W_r} = \frac{1}{(1-e^{-\frac{N}{\tau}})}$$

The ratings of the motor will be proportional to the losses. If P *x* is the short time load P *r* is the continuous rating of the motor, losses for continuous rating are,

$$W_r = W_{const} + W_{cu}$$

$$W_x = W_{const+}(\frac{P_x}{P_r})^2 W_{cu}$$

The ratio of $\frac{P_x}{P_r}$ can be determined.

8.5 COOLING OF ELECTRICAL MACHINES

The removal from electric machines of the heat liberated as a result of magnetic, electric, and other losses. The maximum permissible heating is determined by the heat resistance of the materials insulation, solder, and lubricantused in the machine. The most efficient method of heat removal is to cool the heated parts of themachine with a circulating intermediary substance that may be air, va rious gases (hydrogen, carbon dioxide, helium), or a liquid (transformer oil, water, chlorinated bip henyl).

Air cooling is sufficiently effective in most cases and is the simplest and easiest method. The princi pal air cooling systems are:

(1) natural cooling without the forced circulation of the air around the heated parts;

(2) cooling with air from the surrounding space through forced ventilation provided by one or mor e independent fans or by a single fan mounted on the shaft of the machine (internal self-ventilation)

(3) cooling of an electric machine of an enclosed, or airtight, design, where air circulation in the ho using is maintained by an independent fan or by internal selfventilation.

The first cooling system is used in lowpower (up to several hundred watts) enclosed or open machi nes that do not require intensive cooling. The second system is used chiefly in low- and medium power machines. The third system is used in high- and mediumpower machines and also when the air surrounding an electric machine has been heated to a high temperature, contains explosive gase s, or contain acid vapors that corrode the insulation.

Special gases are used for cooling electric machines where the power consumption for air ventilati on is very great, such as highspeed electric motors, turbogenerators, and synchronous compensator s. When a hydrogen cooling system is used, the possibility of the hydrogen mixing with air and for ming a dangerously explosive mixture must be avoided. If such a danger exists but air cooling is ne vertheless undesirable, as with highpower electric motors located in dangerously explosive places having poor ventilation, a cooling medium such as carbon dioxide or helium is used.

Water is used to cool the stators of highfrequency electric machines, the bearings of highpower ele ctric motors, and the step bearings of generators. Chlorinated biphenyl is used if there is a danger of the water freezing. The windings of heavy-duty transformers are cooled by circulating oil.

8.6 VENTILATION

Cooling and Ventilation In most cases cooling of electrical machines is done by **air streams** and this cooling is called as Ventilation. Depending the origin of the cooing, cooling can be classified into

- Cooling system
- Natural cooling
- Separate cooling

COOLING SYSTEM CLASSIFICATION

Based on manner of cooling:

- Open circuit ventilation
- Surface ventilation
- Closed circuit ventilation
- Liquid cooling
- Inner cooling of windings

Induced and forced ventilation

Both self-ventilation and separate ventilation can be subdivided into two categories:

1) Induced ventilation (with internal fan / external fan)

2) Forced ventilation (with internal fan / external fan)

Induced Ventilation with Internal & External Fan

Internal fan's rotation creates and decreased pressure inside the machines causing the air to be sucked inside the machine as air flows from high pressure to low pressure o Then the air is pushed out by the fan into atmosphere Radial and Axial Ventilation. The ventilating systems can be classified into three types depending upon how the air passes over the heated machine parts are

- Radial
- Axial
- Combined radial and axial

Radial ventilating system

This system is most commonly employed because the movement of rotor induces a natural centrifugal movement of air which may be augmented by provisions of fans if required. This method is suitable for machines about 20kW. For, larger machine, the core is normally divided to provide radial ventilating ducts. Air flowing up the radial ducts only a layer 3mm wide in contact with the core walls is effective in cooling. The middle 4mm wide band contributes very little to cooling of the core.

Advantages

- Minimum energy losses for ventilation.
- Sufficiently uniform temperature rise of machines in the axial direction

Disadvantages

- It makes the machine lengths larger as space for ventilating ducts has to be provided along the core length.
- It may become unstable in respect to quantity of cooling air flowing.

Axial ventilating system

It is suitable for induction machines of medium output and high-speed machines. Axial ducts are provided and two types of axial system:

- Simple axial system
- Double axial system

Advantages & Disadvantages

- It improves cooling but requires a large core diameter for the increased core depth.
- Non uniform heat transfer.
- Increased Iron loss.
- In large number of cases this loss is more than compensated by improved cooling.

Combined Axial and Radial ventilating system

This method is usually employed for large motors and small turbo alternators. Since the area of ducts to carry cooling air becomes large. So, to minimise losses this type is preferred. The air is drawn into the machines from one end is encouraged to pass through the ducts by and finally the rotor mounted fans forces out the air.

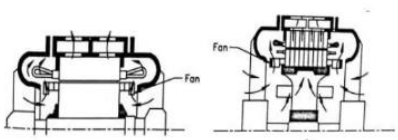


Fig: Radial Ventilation

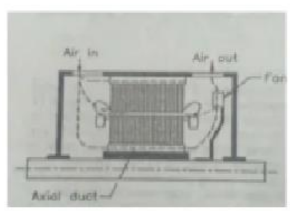


Fig: Axial Ventilation

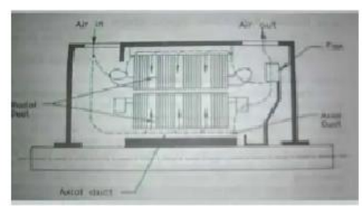


Fig: Combined Ventilation

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