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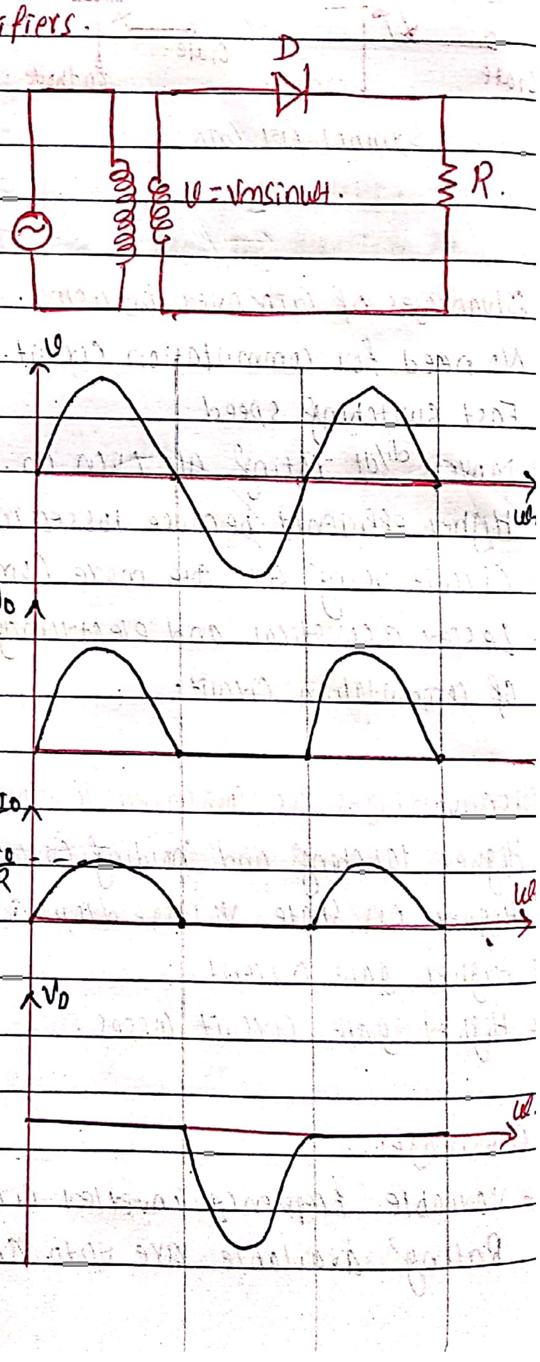
Rectifiers:

The conversion of ac to dc is known as rectification. Diode rectifiers are known as uncontrolled rectifiers because for a fixed value of ac input voltage, the output dc voltage is fixed and cannot be changed.

Rectifiers circuit using thyristors are known as controlled rectifiers. By changing the firing angle of thyristor the output dc voltage can be controlled.

Single phase half wave uncontrolled rectifiers.



$$V_{dc} = \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\omega t \, d(\omega t)$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin\omega t \, d\omega t \int_{\pi}^{2\pi} 0 \, d(\omega t)\right]$$

$$= \frac{V_m}{2\pi}\left(-\cos\omega t\right)\Big|_0^{\pi}$$

$$= \frac{V_m}{2\pi}(1+1)$$

$$= \frac{V_m}{\pi}$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{V_m}{\pi R}$$

$$V_{rms} = \left[\frac{1}{2\pi}\int_0^{\pi}(V_m \sin\omega t)^2 d(\omega t) + \int_{\pi}^{2\pi} 0 \cdot d(\omega t)\right]$$

$$= \left[\frac{V_m^2}{2\pi}\int_0^{\pi}\left(\frac{1-\cos 2\omega t}{2}\right) d(\omega t)\right]^{1/2}$$

$$= \left[\frac{V_m^2}{4\pi}\left|\omega t - \frac{\sin \omega t}{2}\right|_0^{\pi}\right]^{1/2}$$

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$$= \left[\frac{Vm^2}{4\pi}\left(\pi - \frac{\sin\pi}{2} - 0 + \frac{\sin 0}{2}\right)\right]^{1/2}.$$

$$= \left[\frac{Vm^2}{4\pi}\cdot\pi\right]^{1/2} = \frac{Vm}{2}$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{V_m}{2R}$$

$$\text{DC Power } (P_{dc}) = V_{dc}\, I_{dc} = \frac{V_m^2}{\pi^2 R}$$

$$\text{AC Power } (P_{ac}) = V_{rms}\, I_{rms} = \frac{V_m^2}{4R}.$$

$$\text{Rectification efficiency } (\eta) = \frac{P_{dc}}{P_{ac}} = \frac{V_m^2}{\pi^2 R} \times \frac{4R}{V_m^2} = \frac{4}{\pi^2} = 40.5\%$$

Effective (rms) value of ac component of output voltage is

$$V_{ac} = \sqrt{V_{rms}^2 - V_{dc}^2} = \sqrt{\frac{V_m^2}{4} - \frac{V_m^2}{\pi^2}} = V_m\sqrt{\frac{1}{4} - \frac{1}{\pi^2}}$$

$$= 0.3855\, V_m$$

$$\text{Form factor } (FF) = \frac{V_{rms}}{V_{dc}} = \frac{V_m}{2} \times \frac{\pi}{V_m} = \frac{\pi}{2} = 1.57$$

$$\text{Ripple factor } (RF) = \frac{V_{ac}}{V_{dc}} = \frac{0.3855\, V_m \times \pi}{V_m} = 1.21$$

$$= \sqrt{F.F^2 - 1} = \sqrt{1.57^2 - 1} = 1.21$$

If V_s and I_s are the rms voltage and current at the secondary of the transformer feeding the rectifier circuit, transformer utilization factor (TUF) is given by,

$$TUF = \frac{P_{dc}}{V_s I_s} = \frac{\frac{V_m}{\pi} \times \frac{V_m}{\pi R}}{\frac{V_m}{\sqrt{2}} \times \frac{V_m}{2R}} \Rightarrow \frac{2\sqrt{2}}{\pi^2} = 0.2865 = 28.65\%$$

$$\text{Input power factor } (P.F) = \frac{I_{s1}}{I_s} \cdot \cos\theta.$$

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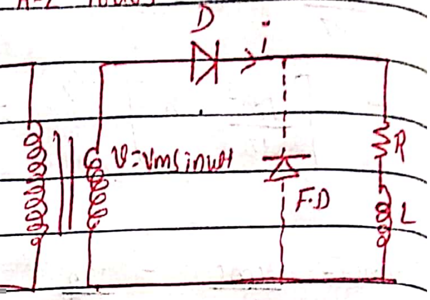
Single phase half wave uncontrolled rectifier with R-L loads :

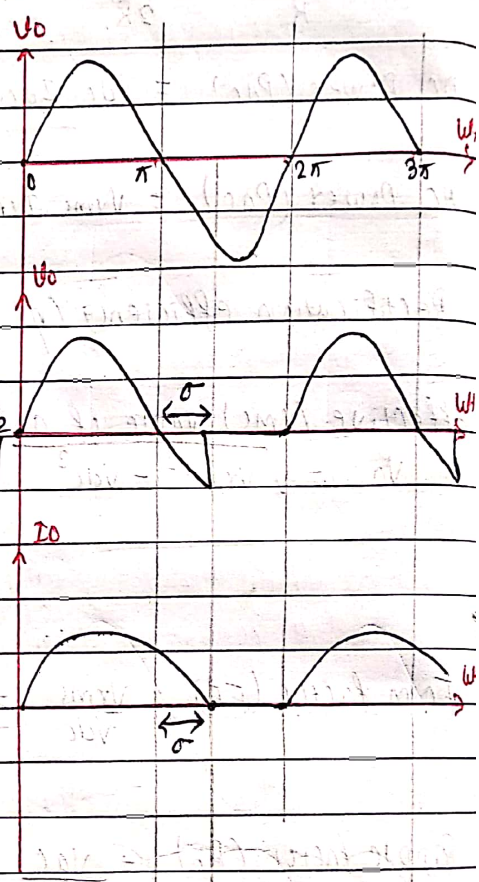


$$V_{dc} = \frac{1}{2\pi}\left[\int_0^{\pi+\sigma} V_m \sin\omega t \, d\omega t + \int_{\pi+\sigma}^{2\pi} 0 \, d\omega t\right]$$

$$= \frac{V_m}{2\pi} \left. -\cos\omega t \right|_0^{\pi+\sigma}$$

$$= \frac{V_m}{2\pi}\left[-\cos(\pi+\sigma) + \cos 0\right]$$

$$= \frac{V_m}{2\pi}(1+\cos\sigma)$$



$$I_{dc} = \frac{V_{dc}}{R} = \frac{V_m}{2\pi R}(1+\cos\sigma)$$

$$V_{rms} = \left[\frac{1}{2\pi}\int_0^{\pi+\sigma} (V_m \sin\omega t)^2 \, d(\omega t) + \int_{\pi+\sigma}^{2\pi} 0 \cdot d\omega t\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi}\int_0^{\pi+\sigma}\left(\frac{1-\cos 2\omega t}{2}\right) \cdot d\omega t\right]^{1/2}$$

$$= \left[\frac{V_m^2}{4\pi}\left|\omega t - \frac{\sin 2\omega t}{2}\right|_0^{\pi+\sigma}\right]^{1/2}$$

$$= \left[\frac{V_m^2}{4\pi}\left((\pi+\sigma) - \frac{\sin(\pi+\sigma)}{2}\right)\right]^{1/2}$$

$$= \left[\frac{V_m^2}{4\pi}\left((\pi+\sigma) + \frac{\sin\sigma}{2}\right)\right]^{1/2}$$

$$= \frac{V_m}{2}\sqrt{\frac{1}{\pi}\left[(\pi+\sigma) + \frac{\sin\sigma}{2}\right]}$$

The value of σ depend on the ratio $\frac{R}{L}$.

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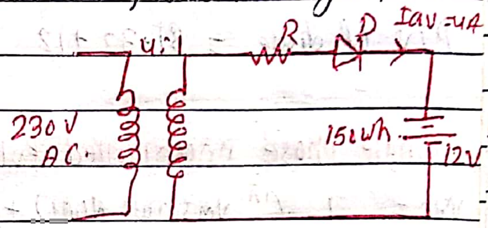
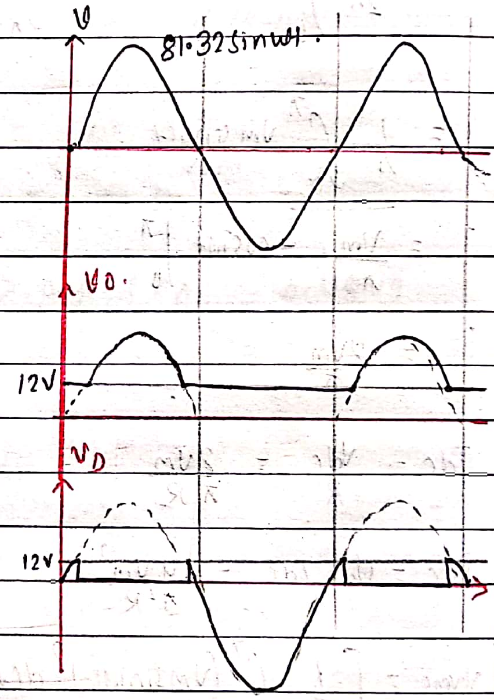
The average Voltage and current can be increased by making $\sigma = 0$. If a free wheeling diode (FD) is connected across the load this can be achieved. The free wheeling diode FD does not allow the negative Voltage to appear across the load. At $\omega t = \pi$ the current from diode is transferred to FD.
From $\omega t = \pi$ to $\omega t = \pi$ to the load current flows through F.D.



3.3)
⇒ Solution
The peak value of input secondary voltage

$$= \frac{230 \times \sqrt{2}}{4} = 81.32\ V$$



Now, the diode starts to conduct when

$81.32 \sin\omega t = 12$

$$\omega t = \sin^{-1}\left(\frac{12}{81.32}\right) = 8.49^\circ$$

Here, the diode conducts 8.49° to $(180 - 8.49^\circ)$
$= 8.49^\circ$ to 171.51°

$$V_{dc} = \frac{1}{2\pi}\int_{8.49}^{171.51^\circ} (81.32 \sin\omega t - 12)\, d(\omega t)$$

$= 20.167\ V$

$$I_{dc} = \frac{V_{dc}}{R} \Rightarrow R = \frac{V_{dc}}{I_{dc}} = \frac{20.167}{4} = 5.04\ \Omega$$

$$V_{rms} = \left[\frac{1}{2\pi}\int_{8.49}^{171.51^\circ} (81.32 \sin\omega t - 12)\, d\omega t\right]^{1/2} = 33.192\ V$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{33.192}{5.04} = 6.585\ A.$$

$P_{dc} = 12 \times 4 = 48$

Pac = power rating of $R = I_{rms}^2 R = (6.585)^2 \times 5.04 = 218.6\ W$.

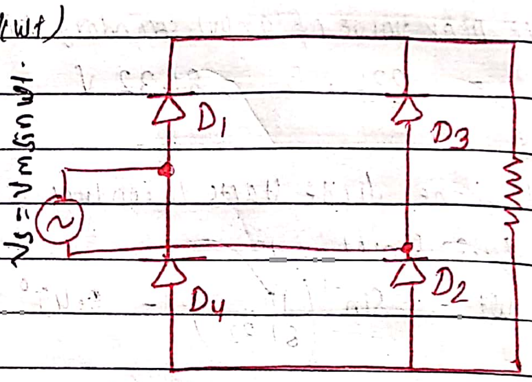
Here, $48 \times$ Charging time $= 150$

$\therefore$ Charging time $= \frac{150}{48} = 3.125$ hours.

Rectifier efficiency $= \frac{P_{dc}}{P_{dc} + I_{rms}^2 R} = \frac{48}{48 + 218.6} \times 100\% = 18\%$

PIV of diode $= 81.32 + 12 = 93.32$ V

Single phase uncontrolled full wave Rectifier :-

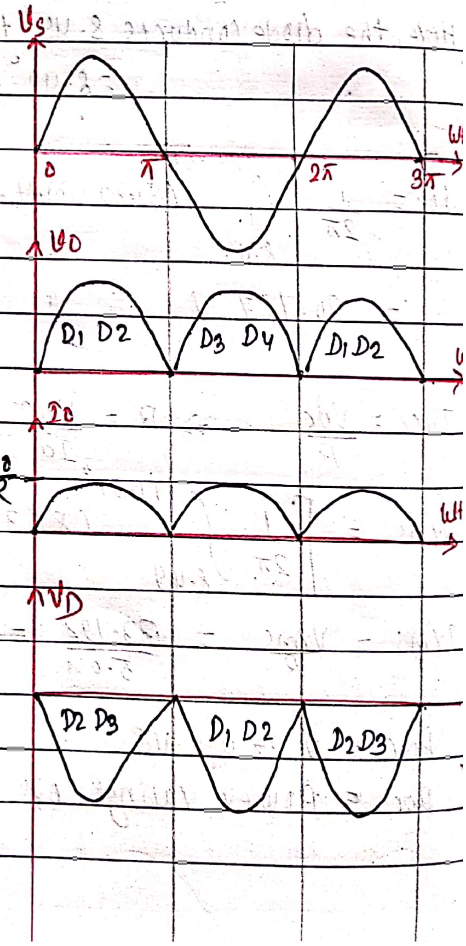


$$V_{dc} = \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin \omega t \cdot d(\omega t) + \int_{\pi}^{2\pi} V_m \sin \omega t \cdot d(\omega t)\right]$$

$$= \frac{1}{\pi}\int_0^{\pi} V_m \sin \omega t$$

$$= \frac{V_m}{\pi} \left. -\cos \omega t \right|_0^{\pi}$$

$$= \frac{2V_m}{\pi}$$



$$I_{dc} = \frac{V_{dc}}{R} = \frac{2V_m}{\pi R}$$

$$P_{dc} = V_{dc} I_{dc} = \frac{4V_m^2}{\pi^2 R}$$

$$V_{rms} = \left[\frac{1}{\pi}\int_0^{\pi} (V_m \sin \omega t)^2 \, d(\omega t)\right]^{1/2}$$

$$= \left[\frac{V_m^2}{\pi}\int_0^{\pi}\left(\frac{1 - \cos 2\omega t}{2}\right) d\omega t\right]^{1/2}$$

$$\left[\frac{V_m^2}{2\pi} \left| \omega t - \frac{\sin 2\omega t}{2} \right|_0^{\pi}\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi}(\pi - 0 + 0 - 0)\right]^{1/2}$$

$$= \frac{V_m}{\sqrt{2}}$$

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$$I_{rms} = \frac{V_{rms}}{R} = \frac{V_m}{\sqrt{2}R}$$

$$P_{ac} = V_{rms} I_{rms} = \frac{V_m}{\sqrt{2}} \times \frac{V_m}{\sqrt{2}R} = \frac{V_m^2}{2R}.$$

$$\text{Rectification efficiency } (\eta) = \frac{P_{dc}}{P_{ac}} = \frac{4V_m^2}{\pi^2 R} \times \frac{2R}{V_m^2} = \frac{8}{\pi^2} = 81.05\%$$

$$V_{ac} = \sqrt{V_{rms}^2 - V_{dc}^2} = \sqrt{\frac{V_m^2}{2} - \frac{4V_m^2}{\pi^2}} = V_m\sqrt{\frac{1}{2} - \frac{4}{\pi^2}}$$

$$= 0.3078\, V_m.$$

$$\text{Form factor} = \frac{V_{rms}}{V_{dc}} = \frac{V_m}{\sqrt{2}} \times \frac{\pi}{2V_m} = \frac{V_m}{\sqrt{2}} \times \frac{\pi}{2V_m} = \frac{\pi}{2\sqrt{2}} = 1.11$$

$$\text{Ripple factor} = \frac{V_{ac}}{V_{dc}} = \frac{0.3078\, V_m}{\frac{2V_m}{\pi}} \Rightarrow \frac{0.3078 \times \pi}{2} = 0.4849$$

$PIV = V_m.$

Advantages of bridge rectifier circuit over centre tapped transformer rectifier

→ It does not require a centre tapped transformer.
→ The peak inverse voltage in bridge circuit is half of that in the full wave circuit using centre tapped transformer.

Disadvantages of bridge rectifier circuit.

→ It requires 4 diodes as compared to 2 in the other circuit.
→ In a bridge circuit two diodes are always in circuit (in both positive and negative half cycles). In the circuit using centre tapped transformer only one diode is in circuit at one time. Therefore, bridge circuit has more voltage drop, more losses, poor efficiency and power voltage regulation as compared to the circuit using centre tapped transformer. Therefore bridge circuit is more suitable for high voltages.

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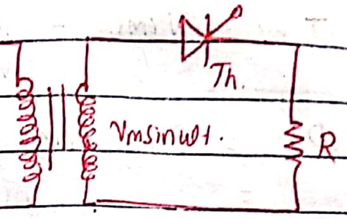
Single phase half wave controlled rectifier.



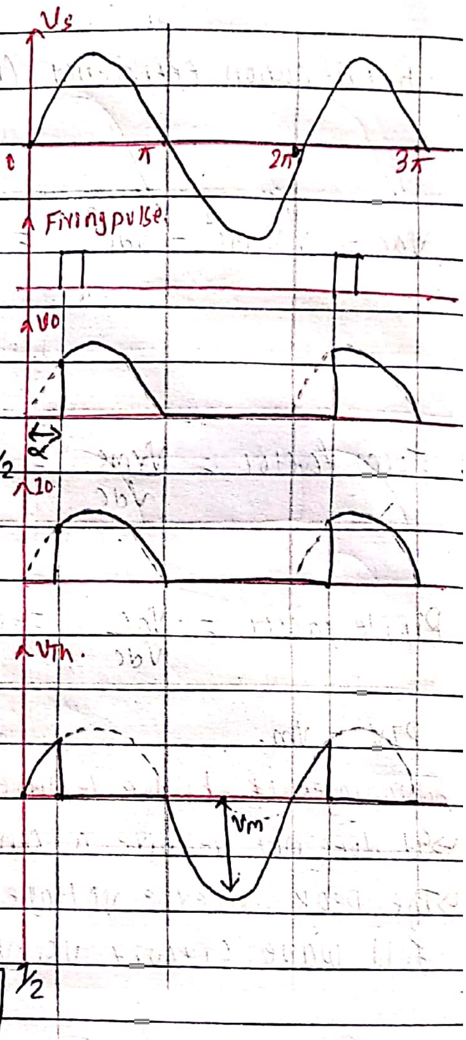
$$V_{dc} = \frac{1}{2\pi}\left[\int_{\alpha}^{\pi} V_m \sin\omega t \, d\omega t + \int_{\pi}^{2\pi} 0 \cdot d\omega t\right]$$

$$= \frac{V_m}{2\pi}\int_{\alpha}^{\pi} \sin\omega t \cdot d\omega t.$$

$$= \frac{V_m}{2\pi}\left. -\cos\omega t \right|_{\alpha}^{\pi}$$

$$= \frac{V_m}{2\pi}(1+\cos\alpha)$$

$$\therefore V_{dc} = \frac{V_m}{2\pi}(1+\cos\alpha)$$



$$I_{dc} = \frac{V_{dc}}{R}$$

$$V_{rms} = \left[\frac{1}{2\pi}\left[\int_{\alpha}^{\pi}(V_m \sin\omega t)^2 d\omega t + \int_{\pi}^{2\pi} 0 \, d\omega t\right]\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi}\int_{\alpha}^{\pi}\left(\frac{1-\cos 2\omega t}{2}\right) d\omega t\right]^{1/2}$$

$$= \left[\frac{V_m^2}{4\pi}\left|\omega t - \frac{\sin 2\omega t}{2}\right|_{\alpha}^{\pi}\right]^{1/2}$$

$$= \left[\frac{V_m^2}{4\pi}\left[(\pi-\alpha) - \frac{\sin 2\pi}{2} + \frac{\sin 2\alpha}{2}\right]\right]^{1/2}$$

$$= \left[\frac{V_m^2}{4\pi}(\pi-\alpha) + \frac{\sin 2\alpha}{2}\right]^{1/2}$$

$$= \frac{V_m}{2}\sqrt{\frac{1}{\pi}\left[(\pi-\alpha) + \frac{\sin 2\alpha}{2}\right]}$$

$$I_{rms} = \frac{V_{rms}}{R}.$$

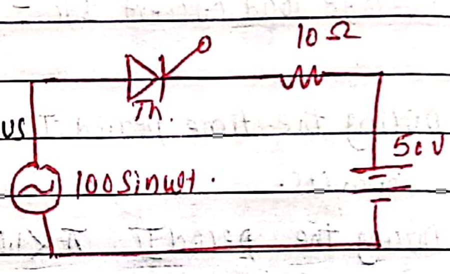
If firing angle (α) $= 0°$, then.

$$V_{dc} = \frac{V_m}{2\pi}(1+1) = \frac{V_m}{\pi}.$$

$$V_{rms} = \frac{V_m}{2}\sqrt{\frac{1}{\pi}\left((\pi - 0) + \frac{\sin 2\times 0}{2}\right)} = \frac{V_m}{2}$$

If firing angle $\alpha = 180°$ both V_{dc} & V_{rms} are zero.
The input current is the same as rms load current.

3.9) =>



The thyristor will conduct when the instantaneous value of ac voltage is more than 50 V.

$$100 \sin\omega t = 50$$

$$\omega t = \sin^{-1}\left(\frac{50}{100}\right) = 30° = \frac{\pi}{6}$$

Conduction angle is from $30°$ to $(180 - 30°) = 150°$ i.e $\frac{\pi}{6}$ to $\frac{5\pi}{6}$

Now, current through the circuit $= \frac{1}{2\pi}\int_{\pi/6}^{5\pi/6}\left(\frac{100\sin\omega t - 50}{10}\right) d\omega t.$

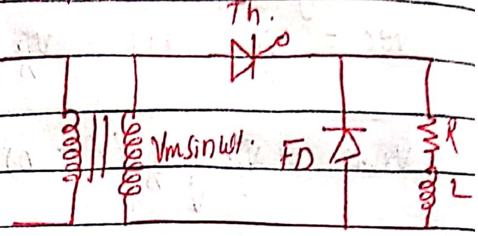
$$= 1.089 \text{ A}.$$

Single phase half wave controlled rectifier with R-L load.

$$V_{dc} = \frac{1}{2\pi}\left[\int_{\alpha}^{\pi} V_m \sin\omega t \; d(\omega t)\right]$$

$$= \frac{V_m}{2\pi}\left[-\cos\omega t\right]_{\alpha}^{\pi}$$

$$= \frac{V_m}{2\pi}(1+\cos\alpha)$$



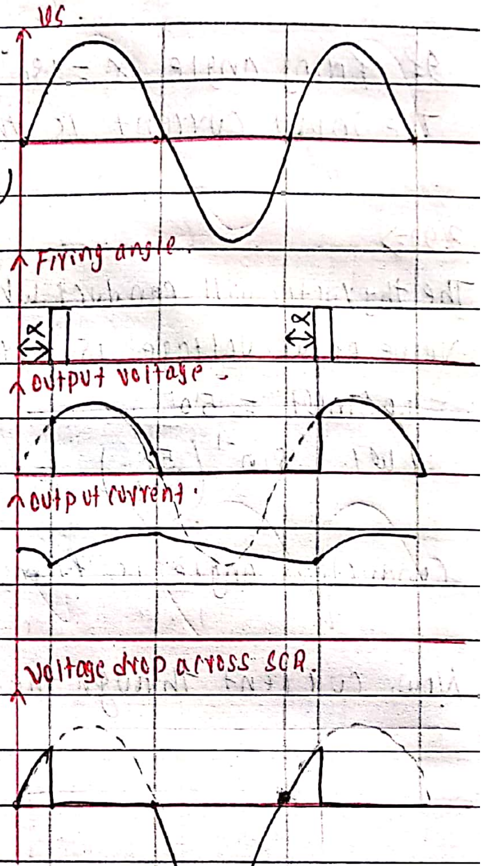
Average load current $I_{dc} = \frac{V_{dc}}{R} = \frac{V_m}{2\pi R}(1+\cos\alpha)$

During the time period I $\alpha < \omega t < \pi$, SCR conducts.

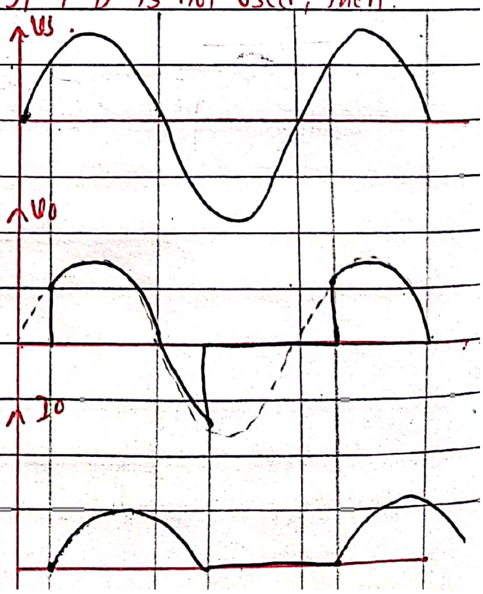
During the period II $\pi < \omega t < 2\pi + \alpha$, FD conducts.

During the interval $\alpha < \omega t < \pi$, the source supplies power to the load.

During the interval $\pi < \omega t < (2\pi + \alpha)$, the energy stored in inductance is supplied to load resistance.



If F.D is not used, then



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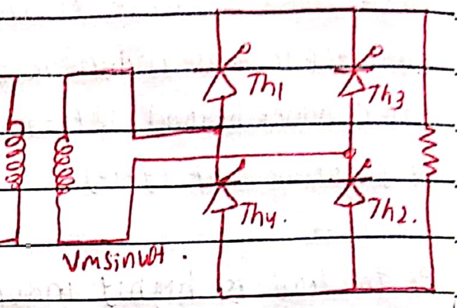
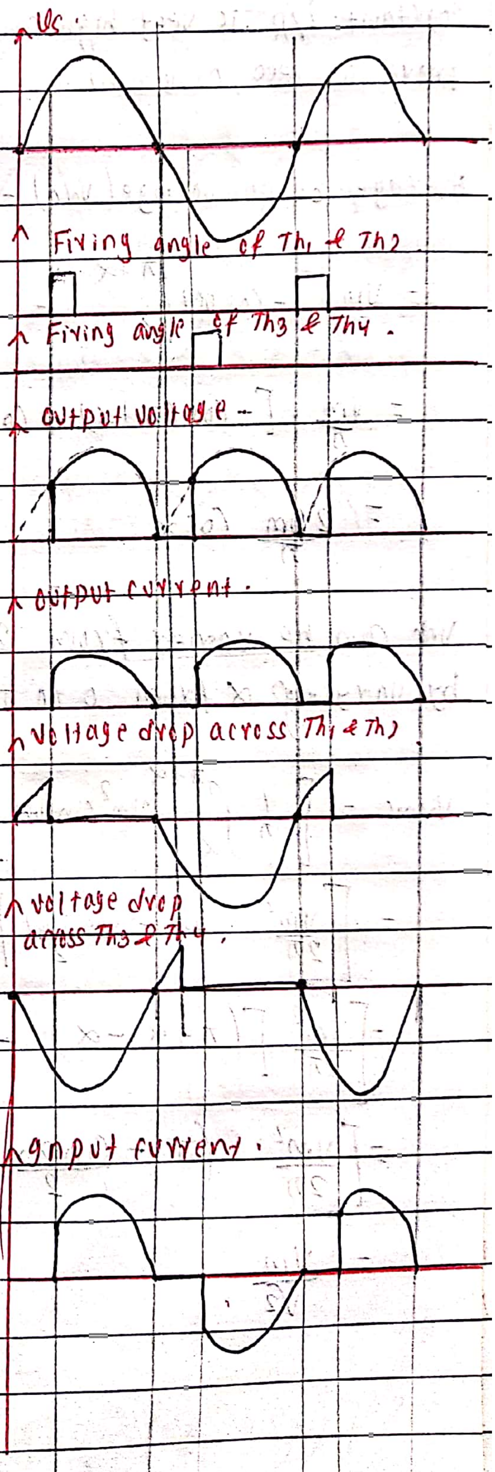
Single phase full wave controlled rectifier



It is also known as full converter. During the positive half cycle thyristor Th_1 & Th_2 are forward biased. When Th_1 & Th_2 are fired at $wt = \alpha$, they start conducting and supplying the load current. At $wt = \pi$, current is zero, the supply voltage reverses and Th_1 & Th_2 are turned off by natural commutation.



In negative half cycle Th_3 & Th_4 conduct from $wt = \pi + \alpha$ to $wt = 2\pi$. At $wt = 2\pi$, current is zero, the supply voltage reverses and Th_3 & Th_4 are turned off by natural commutation.

$$V_{dc} = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin wt \, d(wt)$$

$$= \frac{V_m}{\pi}(1 + \cos\alpha)$$

$$V_{rms} = \left[\frac{1}{\pi}\int_{\alpha}^{\pi} V_m^2 \sin^2 wt \cdot dwt\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi}\left(wt - \frac{\sin 2wt}{2}\right)\Big|_{\alpha}^{\pi}\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi}(\pi - \alpha) - 0 + \frac{\sin 2\alpha}{2}\right]^{1/2}$$

$$= V_m\sqrt{\frac{1}{2\pi}\left[(\pi - \alpha) + \frac{\sin 2\alpha}{2}\right]}$$

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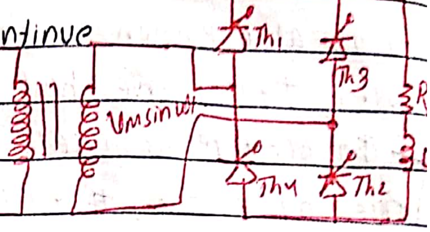
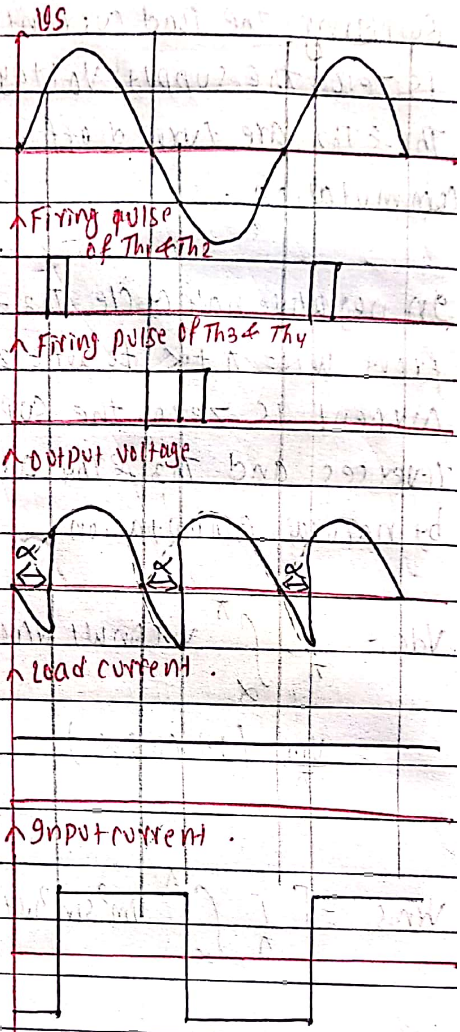
Single phase full wave controlled Rectifier with R-L load.

Since the load is inductive thyristor Th_1 & Th_2 continue to conduct beyond $\omega t = \pi$ (i.e during part of negative half cycle).



If the load is highly inductive, the time constant L/R is very high and i_0 remains, more or less, constant.



$$\text{Average output voltage}(V_{dc}) = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin\omega t \cdot d\omega t$$

$$= \frac{V_m}{\pi}\left[-\cos\omega t\right]_{\alpha}^{\pi+\alpha}$$

$$= \frac{V_m}{\pi}\left[-\cos(\pi+\alpha) + \cos\alpha\right]$$

$$= \frac{2V_m}{\pi}\cos\alpha$$

V_{dc} can be varied from $\frac{2V_m}{\pi}$ to $-\frac{2V_m}{\pi}$ by varying α from 0 to π.

$$V_{rms} = \left[\frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} V_m^2 \sin^2\omega t \; d\omega t\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi}\left(\omega t - \frac{\sin 2\omega t}{2}\right)\Big|_{\alpha}^{\pi+\alpha}\right]^{1/2}$$

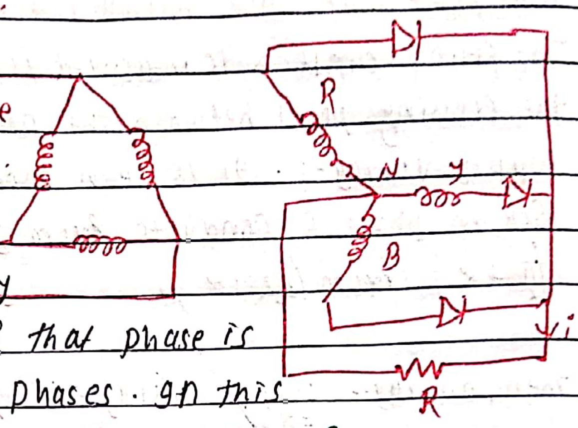
$$= \left[\frac{V_m^2}{2\pi}\left[(\pi+\alpha-\alpha) - \frac{\sin 2(\pi+\alpha)}{2} + \frac{\sin 2\alpha}{2}\right]\right]^{1/2}$$

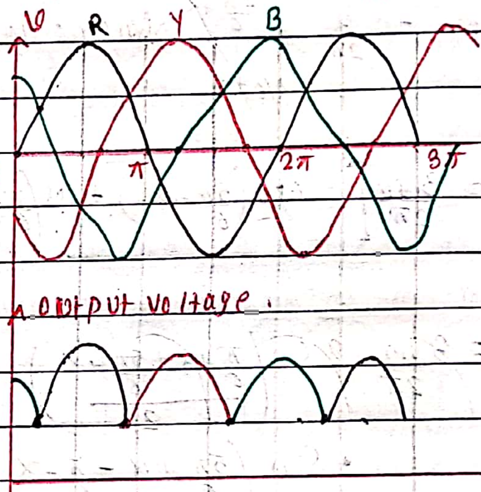
$$= \left[\frac{V_m^2}{2\pi}\left(\pi - \frac{\sin 2\alpha}{2} + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

$$= \frac{V_m}{\sqrt{2}}$$

Three phase half wave diode rectifier.

It is fed by a delta-star transformer from 3-phase mains supply. It can be considered as three single phase half wave rectifiers combined together. The diode in any phase conducts when the voltage of that phase is higher than that on the other two phases. In this rectifier, each diode conducts for 120°.





The diode in phase R conducts from $\frac{\pi}{6}$ to $\frac{5\pi}{6}$. If line to neutral voltage of phase R is given by $V_m \sin\theta$, then average output voltage V_{dc} is.

$$V_{dc} = \frac{1}{\left(\frac{2\pi}{3}\right)} \int_{\pi/6}^{5\pi/6} V_m \sin\theta \cdot d\theta.$$

$$= \frac{3V_m}{2\pi} \left. -\cos\theta \right|_{\pi/6}^{5\pi/6}$$

$$= \frac{3}{2\pi} V_m (\sqrt{3})$$

$$= \frac{3\sqrt{3}\, V_m}{2\pi}$$

$$V_{rms} = \left[\frac{3}{2\pi} \int_{\pi/6}^{5\pi/6} V_m^2 \sin^2\theta \cdot d\theta \right]^{1/2}$$

$$= \left[\frac{3V_m^2}{4\pi} \left. \theta - \frac{\sin 2\theta}{\theta} \right|_{\pi/6}^{5\pi/6} \right]^{1/2}$$

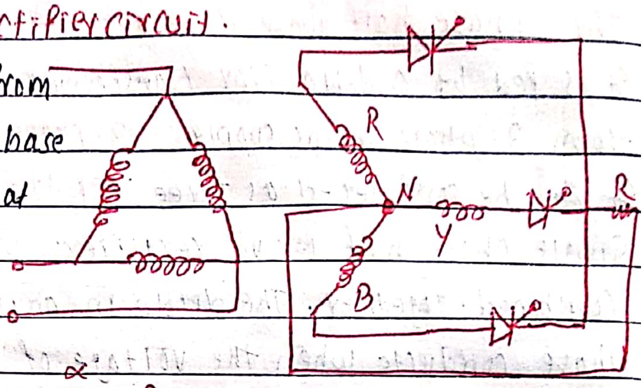
$$= \left[\frac{3V_m^2}{4\pi} \left(\frac{5\pi}{6} - \frac{\pi}{6} \right) + \left(\frac{\sin 2\pi/6}{2} - \frac{\sin 2\times\frac{5\pi}{6}}{2} \right) \right]^{1/2}$$

$$= \left[\frac{3V_m^2}{4\pi} \left(\frac{2\pi}{3} + \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} \right) \right]^{1/2} = \left[\frac{3V_m^2}{4\pi} \left(\frac{2\pi}{3} + \frac{\sqrt{3}}{2} \right) \right]^{1/2}$$

$$= V_m \sqrt{\frac{1}{2} + \frac{3\sqrt{3}}{8\pi}} = 0.84 V_m$$

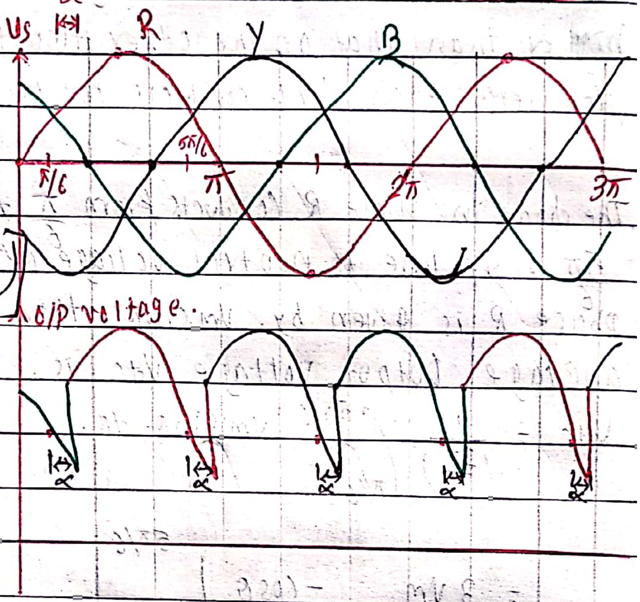
Three phase half wave controlled rectifier circuit.

The firing angle α is measured from the crossing point between the phase supply voltages. It is seen that SCR in phase R conducts from $\left(\frac{\pi}{6}+\alpha\right)$ from $\left(\frac{5\pi}{6}+\alpha\right)$.



Then, average output voltage

$$V_{dc} = \frac{3}{2\pi}\int_{\pi/6+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin\theta \, d\theta$$



$$= \frac{3V_m}{2\pi}\left[-\cos\left(\frac{5\pi}{6}+\alpha\right)+\cos\left(\frac{\pi}{6}+\alpha\right)\right]$$

$$= \frac{3V_m}{2\pi}\, 2\sin\left(\frac{\frac{\pi}{6}+\alpha+\frac{5\pi}{6}+\alpha}{2}\right)\cdot\sin\left(\frac{\frac{5\pi}{6}+\alpha-\frac{\pi}{6}-\alpha}{2}\right)$$

$$= \frac{3V_m}{2\pi} * 2\sin\left(\frac{\pi+2\alpha}{2}\right)\cdot\sin\left(\frac{2\pi}{3*2}\right)$$

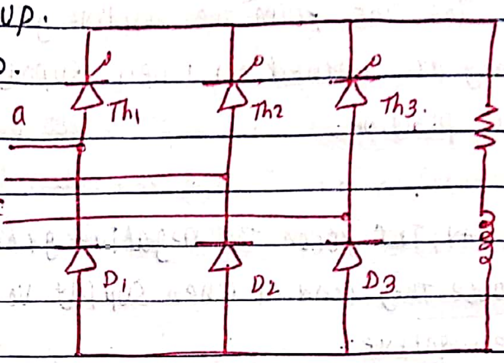
$$= \frac{3V_m}{2\pi} * 2\sin\left(\frac{\pi}{2}+\alpha\right)\cdot\sin\left(\frac{\pi}{3}\right)$$

$$= \frac{3V_m}{\pi}\cos\alpha \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{3\sqrt{3}\,V_m\cos\alpha}{2\pi}.$$

Three phase full wave rectifier circuit.

Th1, Th2 & Th3 form the positive group.
D1, D2 & D3 form the negative group.
The thyristor are fired at intervals of 120°. At one time thyristor and one diode conduct.



Let line to neutral voltages be written as,

$V_{an} = V_m \sin\omega t$.

$V_{bn} = V_m \sin(\omega t - 120°)$

$V_{cn} = V_m \sin(\omega t - 240°)$.

The line to line voltages are.

$V_{ac} = V_{an} - V_{cn}$

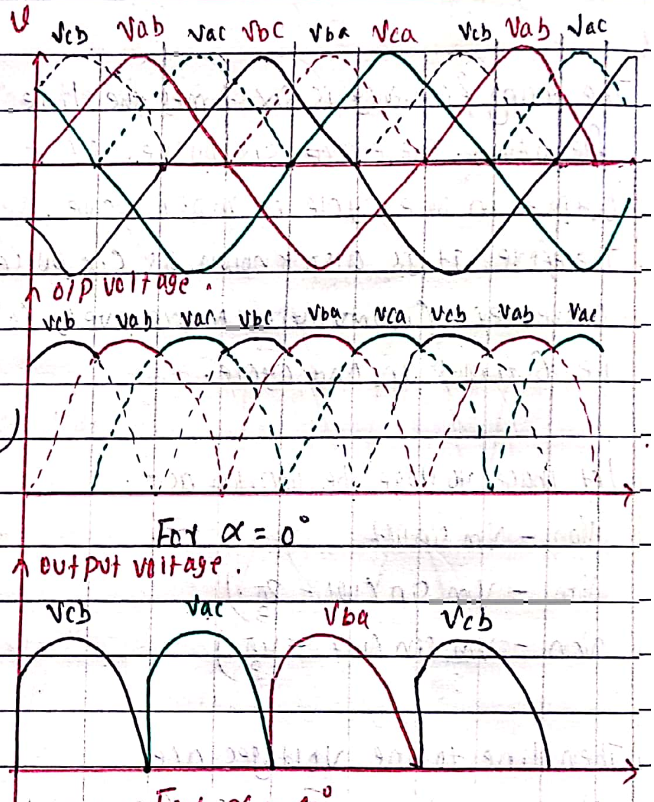
$= V_m \sin\omega t - V_m \sin(\omega t - 240°)$

$= \sqrt{3}\, V_m \sin(\omega t - 30°)$

$V_{ba} = \sqrt{3}\, V_m \sin(\omega t - 150°)$

$V_{cb} = \sqrt{3}\, V_m \sin(\omega t + 90°)$

$V_{ba} = \sqrt{3}\, V_m \sin(\omega t + 30°)$



For $\alpha = 0°$

For $\alpha = 60°$

* When $\alpha \le \pi/3$, the output voltage wave is continuous.

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\pi/6+\alpha}^{\pi/2} V_{ab}\, d(\omega t) + \int_{\pi/2}^{\frac{5\pi}{6}+\alpha} V_{ac}\, d(\omega t)\right] = \frac{3\sqrt{3}\, V_m(1+\cos\alpha)}{2\pi}$$

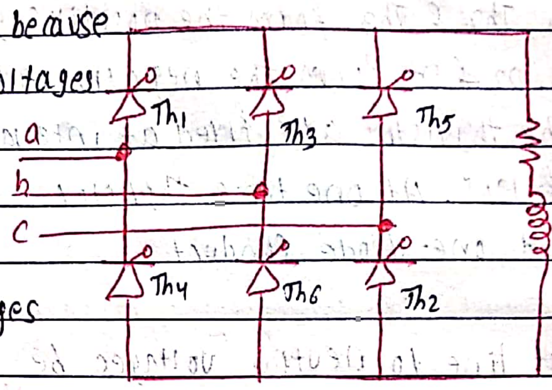
* When $\alpha > \frac{\pi}{3}$ the output voltage wave is discontinuous.

$$V_{dc} = \frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{7\pi}{6}} V_{ac}\, d(\omega t) = \frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{7\pi}{6}} \sqrt{3}\, V_m \sin\left(\omega t - \frac{\pi}{6}\right) d\omega t.$$

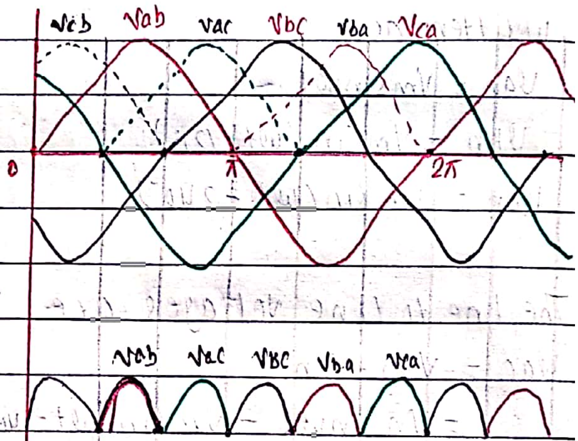
$$= \frac{3\sqrt{3}}{2\pi} V_m(1+\cos\alpha)$$

Three phase fully controlled bridge rectifier:

Th1, Th3, Th5 form the positive group because they are turned on when supply voltages are positive.



Th2, Th4, Th6 form the negative group because they conduct when supply voltages are negative.



The firing frequency is six times the line frequency. There are six pulses of output in one cycle of input wave. Therefore, it is also known as six pulse converter. Commutation occurs every 60° i.e 6 times in one cycle.

For $\alpha = 0°$

Let phase voltage be written as

$V_{an} = V_m \sin \omega t$

$V_{bn} = V_m \sin(\omega t - \frac{2\pi}{3})$

$V_{cn} = V_m \sin(\omega t - \frac{4\pi}{3})$

Then line to line voltages are

$V_{ab} = V_{an} - V_{bn} = \sqrt{3}\, V_m \sin(\omega t + \pi/6)$

$V_{bc} = V_{bn} - V_{cn} = \sqrt{3}\, V_m \sin(\omega t - \pi/2)$

$V_{ca} = V_{cn} - V_{an} = \sqrt{3}\, V_m \sin(\omega t + \frac{5\pi}{6})$

Average output voltage is

$$V_{dc} = \frac{1}{(\pi/3)} \int_{\pi/6+\alpha}^{\frac{\pi}{2}+\alpha} V_{ab}\, d(\omega t) = \frac{3}{\pi} \int_{\pi/6+\alpha}^{\frac{\pi}{2}+\alpha} \sqrt{3}\, V_m \sin(\omega t + \frac{\pi}{6}) \cdot d\omega t .$$

$$\therefore V_{dc} = \frac{3\sqrt{3}}{\pi} V_m \cos\alpha$$

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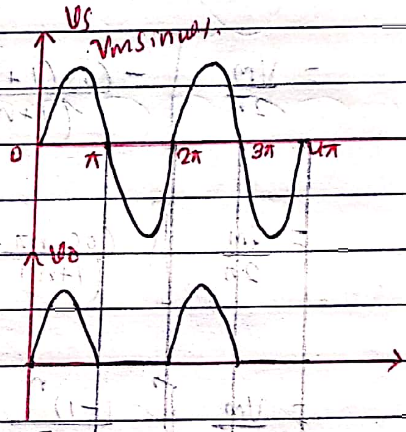
when $\alpha \le 60^\circ$, the instantaneous output voltages will have positive part only.

However, for $\alpha > \pi/3$, the instantaneous output voltages will have negative part also. However, the current through thyristor can flow only in forward direction. Therefore, output current will be positive only.

Fourier Analysis:

The rectifier output voltage $V_L(t)$ can be described by a fourier series as.

$$V_L(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} (A_n \cos n\omega t + B_n \sin n\omega t)$$



where, $A_0 = \frac{2}{T}\int_0^T V_o(t) \cdot dt$.

$$A_n = \frac{2}{T}\int_0^T V_o(t) \cos n\omega t \cdot d\omega t.$$

$$B_n = \frac{2}{T}\int_0^T V_o(t) \sin n\omega t \cdot d\omega t.$$

For Single phase half-wave Rectifier.

$$A_0 = \frac{2}{2\pi}\left[\int_0^{\pi} V_m \sin n\omega t \, d\omega t + \int_{\pi}^{2\pi} 0 \cdot d\omega t\right]$$

$$= \frac{2}{2\pi} V_m \int_0^{\pi} \sin n\omega t \cdot d\omega t.$$

$$= \frac{V_m}{\pi} \left. -\cos\omega t \right|_0^{\pi}$$

$$= \frac{V_m}{\pi}(-(-1)+1)$$

$$= \frac{2V_m}{\pi}$$

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$$A_n = \frac{2}{2\pi}\left[\int_0^{2\pi} V_m \sin\omega t \cdot \cos n\omega t \cdot d\omega t + \int_{\pi}^{2\pi} 0 \cdot \cos n\omega t \cdot d\omega t\right]$$

$$= \frac{2V_m}{2\pi}\int_0^{\pi} \sin\omega t \cdot \cos n\omega t \cdot d\omega t$$

$$= \frac{V_m}{2\pi}\int_0^{\pi} 2\sin\omega t \cdot \cos n\omega t \cdot d\omega t$$

$$= \frac{V_m}{2\pi}\int_0^{\pi}\left[\sin(1+n)\omega t + \sin(1-n)\omega t\right] d\omega t$$

$$= \frac{V_m}{2\pi}\left| -\frac{\cos(1+n)\omega t}{(1+n)} - \frac{\cos(1-n)\omega t}{(1-n)}\right|_0^{\pi}$$

$$= \frac{V_m}{2\pi}\left[-\frac{\cos(1+n)\pi}{(1+n)} + \frac{1}{(1+n)} - \frac{\cos(1-n)\pi}{(1-n)} + \frac{1}{(1-n)}\right]$$

$$= \frac{V_m}{2\pi}\left[\frac{\cos n\pi}{(1+n)} + \frac{\cos n\pi}{1-n} + \frac{1}{(1+n)} + \frac{1}{(1-n)}\right]$$

$$= \frac{V_m}{2\pi}\left[\frac{(-1)^n}{(1+n)} + \frac{(-1)^n}{(1-n)} + \frac{1}{1+n} + \frac{1}{1-n}\right]$$

$$= \frac{V_m}{2\pi}\left[\frac{(-1)^n(1-n) + (-1)^n(1+n)}{1-n^2} + \frac{1-n+1+n}{1-n^2}\right]$$

$$= \frac{V_m}{2\pi}\left[\frac{2(-1)^n + 2}{1-n^2}\right]$$

$$= \frac{V_m}{2\pi}\cdot 2\left[\frac{1+(-1)^n}{1-n^2}\right] = \frac{V_m}{\pi}\left[\frac{1+(-1)^n}{1-n^2}\right]$$

For, n = odd.

$A_n = 0$

For n = even.

$$A_n = \frac{2V_m}{\pi}\frac{1}{(1-n^2)}$$

Similarly,

$$B_n = \frac{2}{2\pi}\left[\int_0^{\pi} V_m \sin\omega t \cdot \sin n\omega t \; d\omega t + \int_{\pi}^{2\pi} 0 \cdot \sin n\omega t \cdot d\omega t\right]$$

$$= \frac{V_m}{2\pi}\left[\int_0^{\pi} 2\sin\omega t \cdot \sin n\omega t \cdot d\omega t\right]$$

$$= \frac{V_m}{2\pi}\left[\int_0^{\pi} \left(\cos(1-n)\omega t - \cos(1+n)\omega t\right) d\omega t\right]$$

$$= \frac{V_m}{2\pi}\left[\left|\frac{\sin(1-n)\omega t}{(1-n)} - \frac{\sin(1+n)\omega t}{1+n}\right|_0^{\pi}\right]$$

$$= \frac{V_m}{2\pi}\left[\frac{\sin(1-n)\pi}{(1-n)} - \frac{\sin(1+n)\pi}{1+n}\right]$$

$$= 0 \quad , \quad B_n = \frac{V_m}{2} \text{ for } n = 1$$

Then, Fourier series becomes,

$$v_o = \frac{A_0}{2} + \sum_{n=1}^{\infty}\left(A_n \cos n\omega t + B_n \sin n\omega t\right)$$

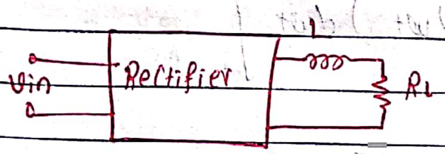
$$= \frac{2V_m}{2\pi} + \sum_{n=2,4,6,\dots}^{\infty} \frac{2V_m}{\pi}\frac{1}{(1-n^2)} \cos n\omega t$$

$$= \frac{V_m}{\pi} - \frac{2V_m}{3\pi}\cos 2\omega t - \frac{2}{15}\frac{V_m}{\pi}\cos 4\omega t - \frac{2V_m}{35\pi}\cos 6\omega t - \dots$$

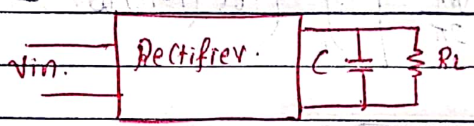
$$+ \frac{V_m}{2}\sin\omega t.$$

Filtering:

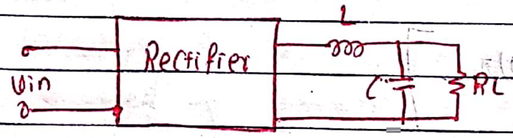
The output voltage waveform of rectifiers are not pure dc, it has harmonics. Filter can be used to smooth out the dc output voltage. The dc filters are usually L, C and LC type. Due to retification action, the input current of rectifier also contains harmonics and ac filter is used to filter out some of the harmonics from the supply system. The ac filters are usually LC type.



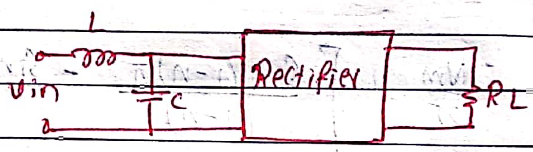
L-Filter.



C-filter

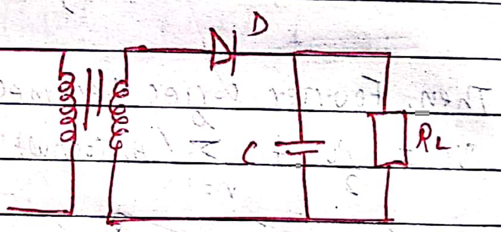


LC filter.

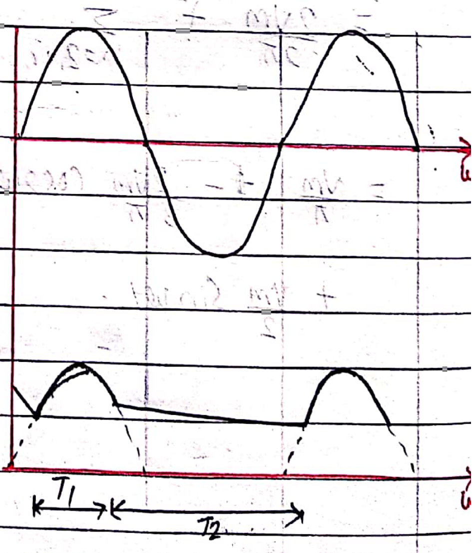


AC filter.

Half wave Rectifier with Capacitor filter.



The capacitor C is charged during the conduction period to peak value of Vm. Therefore energy is stored in the shunt capacitor. The capacitor delivers energy to the load during the time when the input voltage is less than the capacitor voltage.



T_1 = Charging time

T_2 = Discharging time.

The capacitor charges almost simultaneously to Vm.

$\therefore V_c(t_1) = V_m$

The capacitor discharges exponentially through R. It is given by the equation.

$$V_c + Ri = 0 \quad \text{-----} \; (1)$$

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$$i = \frac{dq}{dt} = \frac{d(CV_c)}{dt} = C\frac{dV_c}{dt}$$

From equation ①

$$V_c + RC\frac{dV_c}{dt} = 0$$

$$\text{or, } V_C = -RC\frac{dV_C}{dt}.$$

$$\text{or, } \frac{dt}{RC} = -\frac{dV_C}{V_C}$$

$$\text{or, } \int\frac{dV_C}{V_C} = -\frac{1}{RC}\int dt.$$

$$\text{or, } \ln V_c = -\frac{t}{RC} + K \quad \text{-----} ②$$

At, $t=0$, $V_c = V_m$

Then, $K = \ln V_m$

Now, $\ln V_c = -\frac{t}{RC} + \ln V_m.$

$$\text{or, } \ln\left(\frac{V_c}{V_m}\right) = -\frac{t}{RC}$$

$$\text{or, } V_c = V_m e^{-t/RC}$$

Peak to peak ripple voltage $V_r(P-P) = V_o(t_1) - V_o(t_2)$

$$= V_m - V_m e^{-t_2/RC}.$$

$$= V_m(1 - e^{-t_2/RC})$$

$$= V_m\left[1 - \left(1 - \frac{t_2}{RC}\right)\right]$$

$$\because e^{x} = 1 - x + \underbrace{\frac{x^2}{2!} + \cdots}_{\text{Neglecting}}.$$

$$\therefore V_r(P-P) = \frac{V_m}{RC}\cdot t_2 = \frac{V_m}{fRC}$$

$t_1 + t_2 \approx t_2 = T = \frac{1}{f}$ $\quad t_1 << t_2$

$$V_{dc} = V_m - \frac{V_r(P-P)}{2} = V_m - \frac{V_m}{2fRC} = V_m\left(1 - \frac{1}{2fRC}\right)$$

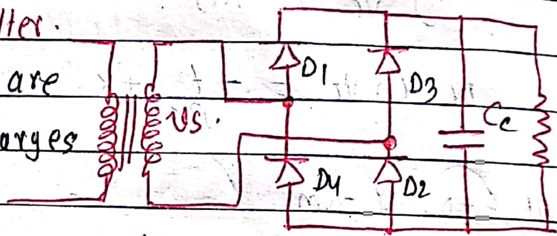
The rms value of ripple voltage $V_r = \frac{V_{r(P-P)}/2}{\sqrt{2}} = \frac{V_m}{2fRC \times \sqrt{2}}$.

$\therefore$ Ripple factor $= \frac{V_r}{V_{dc}} = \frac{V_m}{2\sqrt{2} fRC} \times \frac{2fRC}{V_m(2fRC-1)}$

$$= \frac{1}{\sqrt{2}(2fRC-1)}$$

i.e, RF is independent on the input voltage.

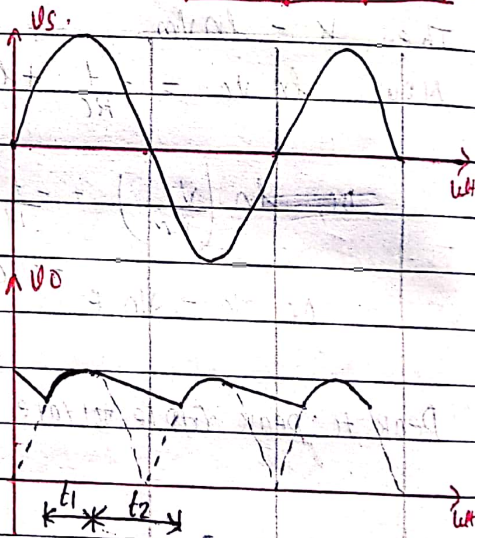
Full wave Rectifier with capacitor filter.



When, $V_s < V_c \Rightarrow D_1 \& D_2$ or $D_3 \& D_4$ are reversed biased and capacitor discharges through the load resistance R.

$t_1 \rightarrow$ charging time.

$t_2 \rightarrow$ discharging time.



At $t = t_1$, $V_c(t) = V_m$.

The capacitor discharges exponentially through R.

$$\frac{1}{C_c}\int i_0 dt + V_c(t=0) + R i_0 = 0$$

which with an initial condition of $V_c(t=0) = V_m$ gives the discharging current as,

$$i_0 = \frac{V_m}{R} = e^{-t/RC}$$

$$V_o(t) = R i_0 = V_m e^{-t/RC}.$$

Peak to peak ripple voltage, $V_{r(p-p)} = V_o(t=t_1) - V_o(t=t_2)$

$$= V_m - V_m e^{-t_2/RC}$$

$$= V_m(1 - e^{-t_2/RC})$$

Since, $e^{n} = 1-n$.

$$V_{r(p-p)} = V_m\left(1-\left(1-\frac{t_2}{RC}\right)\right) = V_m \frac{t_2}{RC} \qquad t_2 \approx \frac{T}{2} = \frac{1}{2f}$$

$$= \frac{V_m}{2fRC}$$

Average load voltage V_{dc} is

$$V_{dc} = V_m - \frac{V_{r(p-p)}}{2} = V_m - \frac{V_m}{4fRC}$$

The rms output ~~r~~ ripple voltage V_{ac} can be found as,

$$V_{ac} = \frac{V_{r(p-p)}}{2\sqrt{2}} = \frac{V_m}{4\sqrt{2}fRC}$$

$$\therefore \text{Ripple factor} = \frac{V_{ac}}{V_{dc}} = \frac{V_m}{4\sqrt{2}fRC} \times \frac{4fRC}{V_m(4fRC-1)}$$

$$= \frac{1}{\sqrt{2}(4fRC-1)}$$

Power factor Improvement.

The input current I_s lags the input voltage V_s in the rectifier circuit. When the firing angle is increased, I_s lags by larger angle with V_s. and input power factor becomes poor.

① Extinction Angle control method.:

In this method, the average value of output voltage of rectifier is controlled by varying the extinction angle (β) rather than the changing firing angle (α). The extinction angle (β) is the angle at which the conduction is forcefully extinguished before the natural zero end of positive half cycle. The angle β is measured back from the zero end of the positive half cycle.

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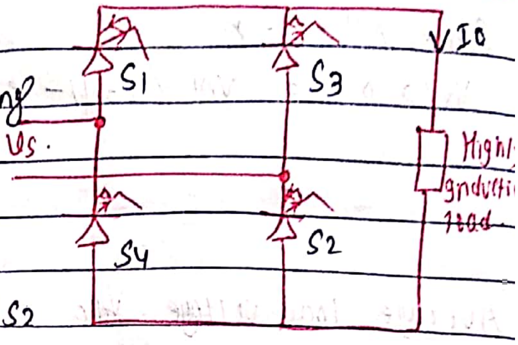
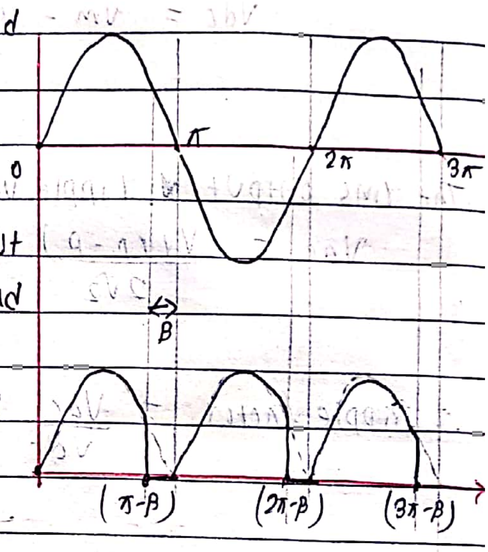
The switches S_1, S_2, S_3 and S_4 are GTO switches which can be turned on by applying positive gate signal and turned off by applying negative gate signals.



During the half cycle of input voltage, S_1 & S_2 are turned on simultaneously at $\omega t = 0$ and at $\omega t = \pi - \beta$, S_2 is turned off and at the same time S_4 is turned on.



When S_2 turns off at $\omega t = \pi - \beta$, the input voltage V_s doesn't appear across the load but the output current I_0 continue to flow due to highly inductive load through the free wheeling path S_1 - load - S_4.

At $\omega t = \pi$, S_1 is turned off and S_3 is turned on so that ~~voltage~~ negative half of the input voltage get rectified and appears as positive output voltage across the load. S_4 is turned off and S_2 is turned on at $\omega t = 2\pi - \beta$.

The average value of output voltage can be changed by varying extinction angle β.

Average value of o/p voltage is $V_{dc} = \frac{1}{\pi}\int_0^{\pi-\beta} V_m \sin\omega t \cdot d\omega t$.

$$= \frac{V_m}{\pi}(1 + \cos\beta)$$

Rms value of o/p voltage is $V_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi-\beta}(V_m \sin\omega t)^2 d\omega t}$

$$= \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[(\pi-\beta) + \frac{\sin 2\beta}{2}\right]}$$

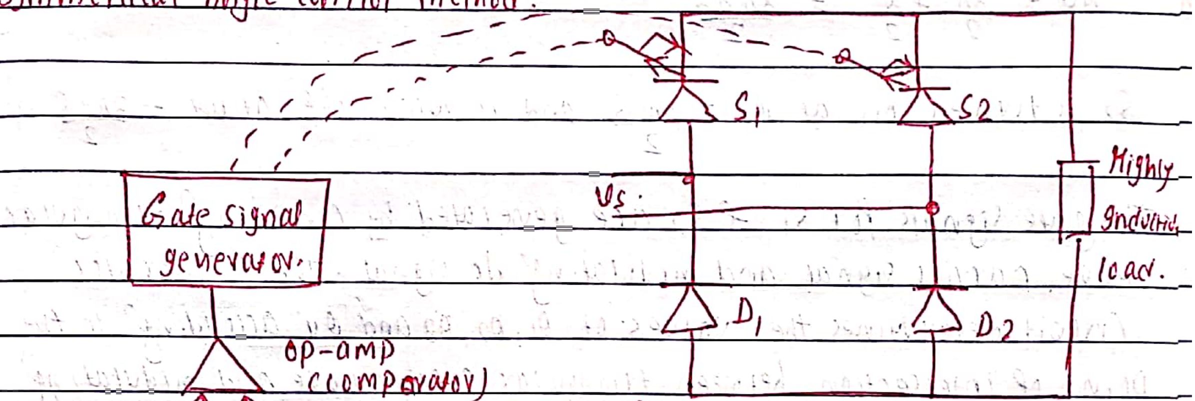
Fourier Analysis.

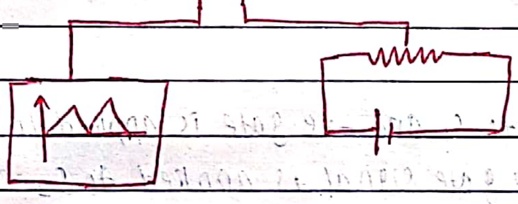
$$A_0 = \frac{2}{\pi}\int_0^{\pi-\beta} V_m \sin\omega t \, d\omega t = \frac{2V_m}{\pi}\left[1+\cos\beta\right]$$

$$A_n = \frac{2}{\pi}\int_0^{\pi-\beta} V_m \sin\omega t \cos n\omega t \cdot d\omega t.$$

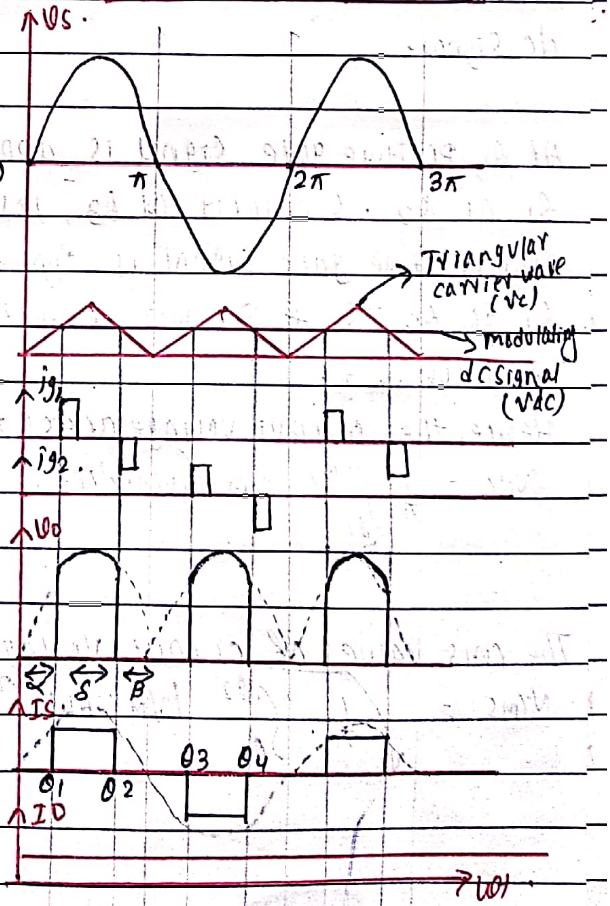
$$B_n = \frac{2}{\pi}\int_0^{\pi-\beta} V_m \sin\omega t \cdot \sin n\omega t \cdot d\omega t.$$

(b) Symmetrical Angle control method.





Triangular wave generator.



In this method, the magnitude of output voltage is controlled by firing as well as extinction angle control.

During +ve half cycle of input voltage Vs, S_1 & D_2 are forward biased and S_2 & D_1 are reversed biased. The switch S_1 is turned on at $\omega t = \frac{\pi-\delta}{2}$ to $\frac{\pi+\delta}{2}$

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Similarly during -ve half cycle of input voltage Vs, S_2 & D_1 are forward biased and S_1 & D_2 are reversed biased.

$$\theta_1 = \frac{\pi}{2} - \frac{\delta}{2} = \frac{\pi - \delta}{2}$$

$$\theta_2 = \frac{\pi}{2} + \frac{\delta}{2} = \frac{\pi + \delta}{2}$$

$$\theta_3 = \frac{3\pi}{2} - \frac{\delta}{2} = \frac{3\pi - \delta}{2}$$

$$\theta_4 = \frac{3\pi}{2} + \frac{\delta}{2} = \frac{3\pi + \delta}{2}$$

S_2 is turned on at $\omega t = \frac{3\pi - \delta}{2}$ and is turned off at $\omega t = \frac{3\pi + \delta}{2}$.

The gate signals for S_1 & S_2 are generated by comparing triangular wave carrier signal and modulating dc signal. The comparator circuit determines the values of $\theta_1, \theta_2, \theta_3$ and θ_4 according to the point of intersection between triangular carrier wave and modulating dc signal.

At θ_1, positive gate signal is applied to S_1 and -ve gate is applied to S_1 at θ_2. Similarly at θ_3, positive gate signal is applied to S_2 and negative gate signal is applied to S_2 at θ_4. Therefore, from θ_1 to θ_2, S_1 & D_2 will conduct and from θ_3 to θ_4, S_2 & D_1 will conduct.

Hence, the output voltage across the load is

$$V_{dc} = \frac{1}{\pi}\int_{\theta_1}^{\theta_2} V_m \sin\omega t \cdot d\omega t = \frac{2V_m}{\pi} \sin\delta/2$$

The rms value of output voltage is

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_{\theta_1}^{\theta_2} (V_m \sin\omega t)^2 \, d\omega t} = \frac{V_m}{\sqrt{2}} \sqrt{\delta + \sin\delta}$$

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