Calculations by Trapezoidal Speed-Time Curve

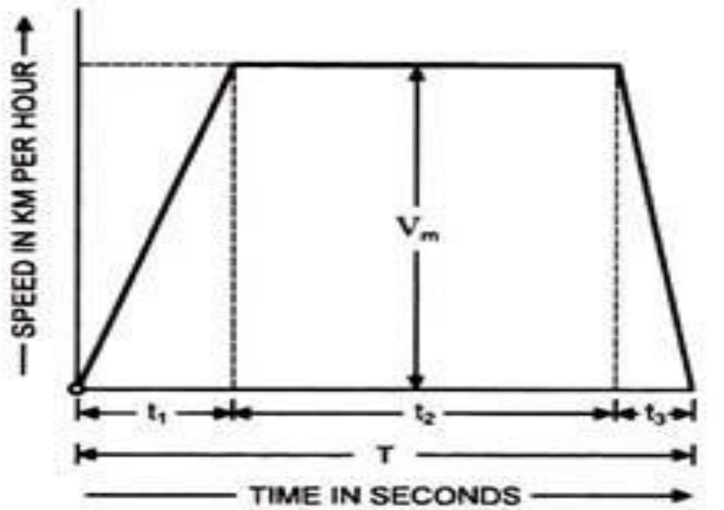

Fig. 11.5. ***Trapezoidal Speed-Time Curve***

Let α = Acceleration in kmphps

β = Retardation in kmphps

V_m = Crest speed in kmph
T = Total time of run in seconds.

Time for acceleration in seconds, $t_1 = V_m/\alpha$
Time for retardation in seconds, $t_3 = V_m/\beta$
Time for free running in seconds, $t_2 = T - (t_1 + t_3) = T - (V_m/\alpha + V_m/\beta)$
Total distance of run in km,

S = Distance travelled during acceleration

+ distance travelled during free run

+ distance travelled during braking

$$= \frac{1}{2} V_m \frac{t_1}{3{,}600} + V_m \frac{t_2}{3{,}600} + \frac{1}{2} V_m \frac{t_3}{3{,}600}$$

Substituting $t_1 = \frac{V_m}{\alpha}$, $t_3 = \frac{V_m}{\beta}$ and $t_2 = T - \left(\frac{V_m}{\alpha} + \frac{V_m}{\beta}\right)$ we have

$$S = \frac{V_m^2}{7{,}200\alpha} + \frac{V_m}{3{,}600}\left[T - \left(\frac{V_m}{\alpha} + \frac{V_m}{\beta}\right)\right] + \frac{V_m^2}{7{,}200\beta}$$

or $$S = \frac{V_m^2}{7{,}200\alpha} + \frac{V_m}{3{,}600}T - \frac{V_m^2}{3{,}600\alpha} - \frac{V_m^2}{3{,}600\beta} + \frac{V_m^2}{7{,}200\beta}$$

$$= \frac{V_m T}{3{,}600} - \frac{V_m^2}{7{,}200\alpha} - \frac{V_m^2}{7{,}200\beta} \qquad ...(11.1)$$

or $$\frac{V_m^2}{3{,}600}\left(\frac{1}{2\alpha} + \frac{1}{2\beta}\right) - \frac{V_m T}{3{,}600} + S = 0$$

or $$V_m^2\left(\frac{1}{2\alpha} + \frac{1}{2\beta}\right) - V_m T + 3{,}600S = 0$$

This is a quadratic equation for V_m. Substituting $\frac{1}{2\alpha} + \frac{1}{2\beta} = K$, we get

$$KV_m^2 - V_m T + 3{,}600\,S = 0$$

or $$V_m = \frac{T \pm \sqrt{T^2 - 4K \times 3{,}600S}}{2K}$$

$$= \frac{T}{2K} \pm \sqrt{\frac{T^2}{4K^2} - \frac{3{,}600\,S}{K}}$$

The +ve sign cannot be adopted, as value of V_m obtained by using +ve sign will be much higher than that is possible in practice. Hence -ve sign will be used and, therefore, we have

$$V_m = \frac{T}{2K} - \sqrt{\frac{T^2}{4K^2} - \frac{3,600\,S}{K}} \qquad \text{...(11.2)}$$

From the above equation unknown quantity can be determined by substituting the value of known quantities.

Calculation by Quadrilateral Speed-Time Curve

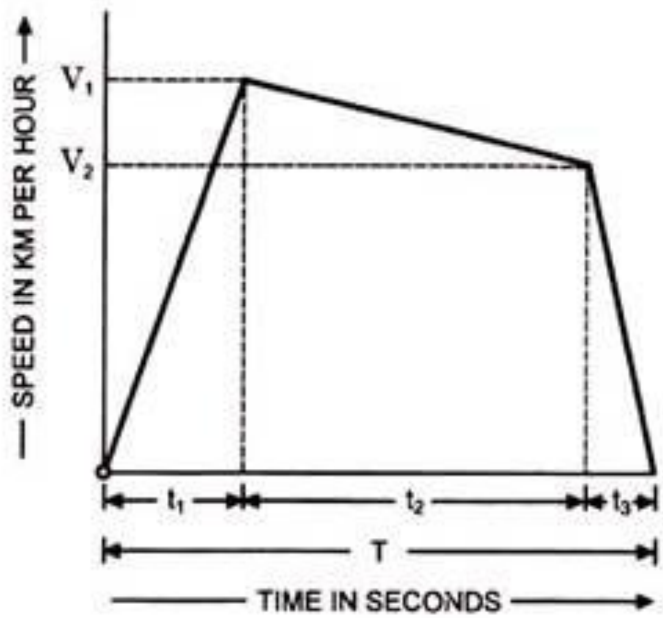

Fig. 11.6. ***Quadrilateral Speed-Time Curve***

Let α = Acceleration in kmphps

β_c = Coasting retardation in kmphps
β = Braking retardation in kmphps

V_1 = Maximum speed at the end of acceleration in kmph
V_2 = Speed at the end of coasting in kmph
T = Total time of run in seconds

Time of acceleration in seconds, $t_1 = V_1/\alpha$
Time of coasting in seconds, $t_2 = (V_1 - V_2) / \beta_c$
Time of braking in seconds, $t_3 = V_2/\beta$
Total distance travelled in km,

S = Distance travelled during acceleration + distance travelled during coasting + distance travelled during retardation

$$= \frac{1}{2}V_1 \times \frac{t_1}{3,600} + \frac{V_1 + V_2}{2} \times \frac{t_2}{3,600} + \frac{1}{2}V_2 \times \frac{t_3}{3,600}$$

$$= \frac{V_1 t_1}{7,200} + \frac{V_1 t_2}{7,200} + \frac{V_2 t_2}{7,200} + \frac{V_2 t_3}{7,200}$$

$$= \frac{V_1}{7,200}(t_1 + t_2) + \frac{V_2}{7,200}(t_2 + t_3)$$

$$\text{or } S = \frac{V_1}{7,200}(T - t_3) + \frac{V_2}{7,200}(T - t_1)$$

Since $t_1 + t_2 + t_3 = T$

$$\text{or } S = \frac{T}{7,200}(V_1 + V_2) - \frac{V_1 t_3}{7,200} - \frac{V_2 t_1}{7,200}$$

$$\text{or } S = \frac{T}{7,200}(V_1 + V_2) - \frac{V_1}{7,200} \times \frac{V_2}{\beta} - \frac{V_2}{7,200} \times \frac{V_1}{\alpha}$$

$$= \frac{T}{7,200}(V_1 + V_2) - \frac{V_1 V_2}{7,200\,\beta} - \frac{V_1 V_2}{7,200\,\alpha}$$

$$\text{or } 7,200S = T(V_1 + V_2) - V_1 V_2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) \qquad \text{...(11.3)}$$

We have $V_2 = V_1 - \beta_c t_2 = V_1 - \beta_c(T - t_1 - t_3)$

$$= V_1 - \beta_c\left(T - \frac{V_1}{\alpha} - \frac{V_2}{\beta}\right)$$

$$\text{or } \left(V_2 - \frac{\beta_c}{\beta}V_2\right) = V_1 - \beta_c\left(T - \frac{V_1}{\alpha}\right)$$

$$\text{or } \quad V_2 = \frac{V_1 - \beta_c T + \frac{\beta_c}{\alpha}V_1}{1 - \frac{\beta_c}{\beta}} \qquad \text{...(11.4)}$$

Solving Eqs. (11.3) and (11.4) values of S, V_1, V_2 etc. can be obtained.