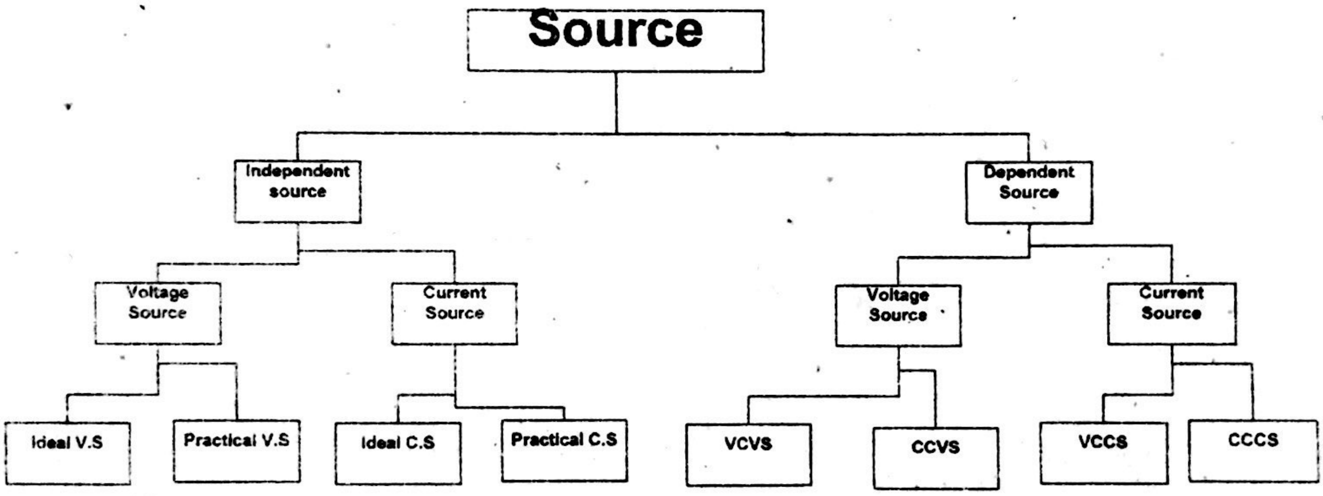
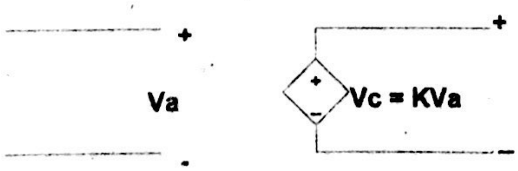
Network Analysis of Ac Circuit and Dependent Sources

An Electrical Network is an interconnection of electrical elements such as resistors, inductors, capacitors, source, current and switches. An electrical circuit is a network consisting of a closed loop, given a return path for the current. Linear electrical networks, a special type consisting only of sources (voltage or current), linear lumped elements (resistors, capacitors, inductors), and linear distrusted elements (transmission lines), have the property that signals are linearly. They are thus more easily analyzed, using powerful frequency domain methods such as Laplace transforms, to determine DC response, Ac response, and transient response.



***Dependent Sources**: The Source that can be controlled by a voltage or current existing at some other place of the circuit is known as dependent source.

***VCVS** = Voltage controlled Voltage Source



***CCVS** = Current Controlled Voltage Source

Pdf by: Sachin Lamsal

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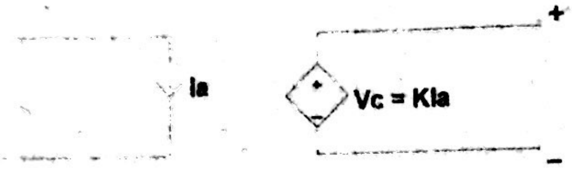
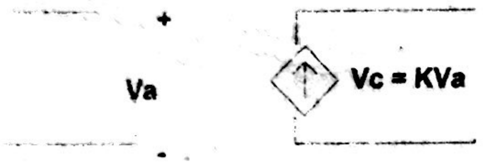
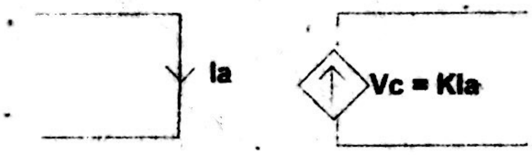


Note: Consider all outgoing current while using KCL at a particular node except given current source direction.

*VCCS = Voltage controlled Current Source



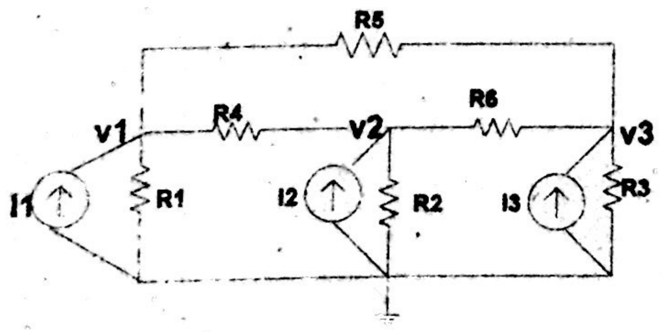
*CCCS = Current controlled Current Source



1.1. Nodal Analysis:

- Steps to be applied for solving any circuit by Nodal analysis
  1. Find out the possible no. of nodes.
  2. Select one as reference node and assign the node voltage to the remaining nodes such as V1, V2, and V3 and so on.
  3. See the types of source present on the circuit and solve accordingly.

Type1:



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Solution:

> **Note:** Consider all outgoing current while using KCL at a particular node except given current source direction.

Using KCL at node 1:

$$I1 = \frac{V1-0}{R1} + \frac{V1-V2}{R4} + \frac{V1-V3}{R5}$$

$$\text{Or, } I1 = \left(\frac{1}{R1} + \frac{1}{R4} + \frac{1}{R5}\right) V1 + \left(\frac{-1}{R4}\right) V2 + \left(\frac{-1}{R5}\right) V3$$

$$\text{Or, } I1 = G11\ V1 + G12\ V2 + G13\ V3 \text{-----------} (i)$$

Where,

$$G11 = \left(\frac{1}{R1} + \frac{1}{R4} + \frac{1}{R5}\right)$$

$$\text{Or, } G12 = \left(\frac{-1}{R4}\right)$$

$$\text{Or, } G13 = \left(\frac{-1}{R5}\right)$$

Using KCL at node 2:

$$I2 = \frac{V2-0}{R2} + \frac{V2-V1}{R4} + \frac{V2-V3}{R6}$$

$$\text{Or, } I2 = \left(\frac{1}{R2} + \frac{1}{R4} + \frac{1}{R6}\right) V2 + \left(\frac{-1}{R4}\right) V1 + \left(\frac{-1}{R6}\right) V3$$

$$\text{Or, } I2 = G21\ V1 + G22\ V2 + G23\ V3 \text{-----------------} (ii)$$

And Using KCL at node 3:

$$I3 = \frac{V3-0}{R3} + \frac{V3-V2}{R6} + \frac{V3-V1}{R5}$$

$$\text{Or, } I3 = \left(\frac{1}{R3} + \frac{1}{R5} + \frac{1}{R6}\right) V3 + \left(\frac{-1}{R6}\right) V2 + \left(\frac{-1}{R5}\right) V1$$

$$\text{Or, } I3 = G31\ V1 + G32\ V2 + G33\ V3 \text{-----------------} (iii)$$

Now, Putting equation (i),(ii)and (iii) in matrix form

$$[G]\ [V] = [I]$$

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$$\begin{bmatrix} G11 & G12 & G13 \\ G21 & G22 & G23 \\ G31 & G32 & G33 \end{bmatrix} * \begin{bmatrix} V1 \\ V2 \\ V3 \end{bmatrix} = \begin{bmatrix} I1 \\ I2 \\ I3 \end{bmatrix}$$

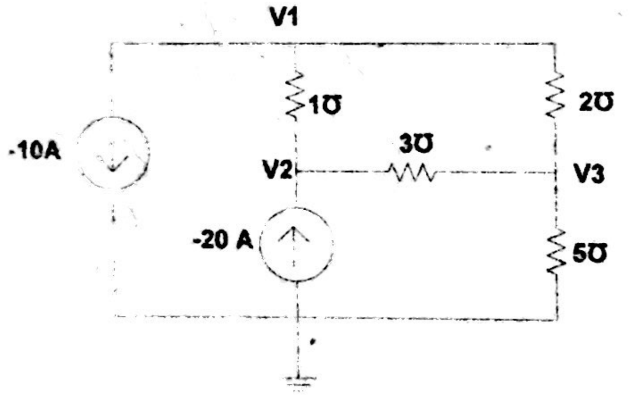
Thus using Cramer's Rule calculate V1, V2 and V3

$V1=\frac{\Delta 1}{\Delta}$ $V2=\frac{\Delta 2}{\Delta}$ $V3=\frac{\Delta 3}{\Delta}$

Where, $\Delta = \begin{bmatrix} G11 & G12 & G12 \\ G21 & G22 & G23 \\ G31 & G32 & G33 \end{bmatrix}$ $\Delta 1 = \begin{bmatrix} I1 & G12 & G12 \\ I2 & G22 & G23 \\ I3 & G32 & G33 \end{bmatrix}$

$\Delta 2 = \begin{bmatrix} G11 & I1 & G12 \\ G21 & I2 & G23 \\ G31 & I3 & G33 \end{bmatrix}$ $\Delta 3 = \begin{bmatrix} G11 & G12 & I1 \\ G21 & G22 & I2 \\ G31 & G32 & I3 \end{bmatrix}$

Example .1.1: In the Network shown, find current through each resistor using Nodal analysis



Solution:

Let us assign nodal Voltage V1, V2, V3 and reference node as shown in the figure

Using KCL at node 1:

$-10 + (V1-V2)*1 + (V1-V3)*2 = 0$

Or, $3V1-V2-2V3-10 = 0$ ----------------- (i)

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Using KCL at node 2:

$-20 = (V2-V1)*1 + (V2-V3)*3$

Or, $-20 = 4V2-V1-3V3$ ------------------ (ii)

Using KCL at node 3:

$(V3-V2)*3 + (V3-0)*5 + (V3-V1)*2 = 0$

Or, $10V3 -2V1 -3V2 = 0$ ------------------ (iii)

Solving equation (i), (ii) and (iii) we have

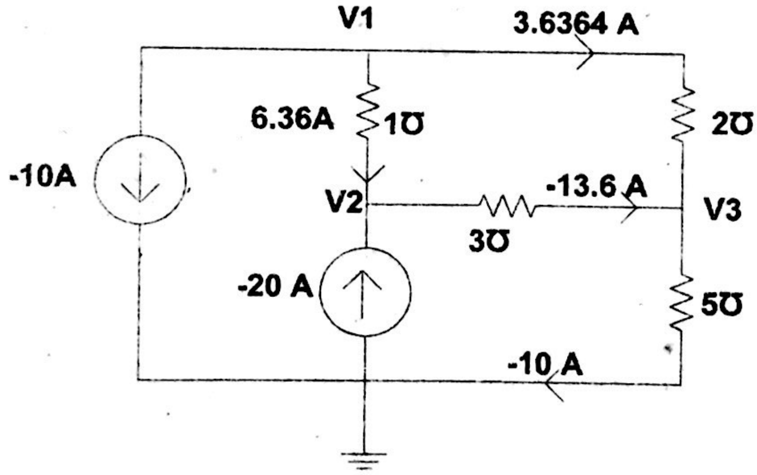
V1 = -0.1818 Volt

V2 = -6.545 Volt

V3 = -2 Volt

Note: Use calculator for solving three equations, most of the student make errors while solving the equation by themselves plus most importantly it save your time.

Current through each resistor are:



Type 2: Circuit Containing Voltage source in addition with current source

2. a. Voltage source in the circuit that can transformable into current source

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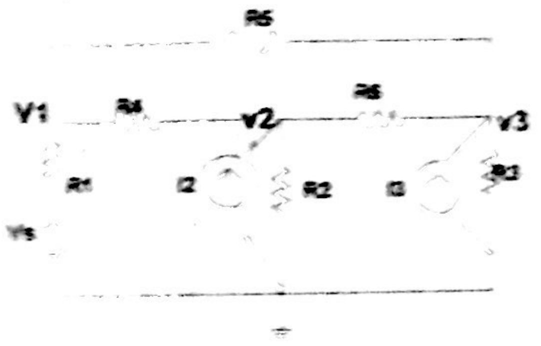
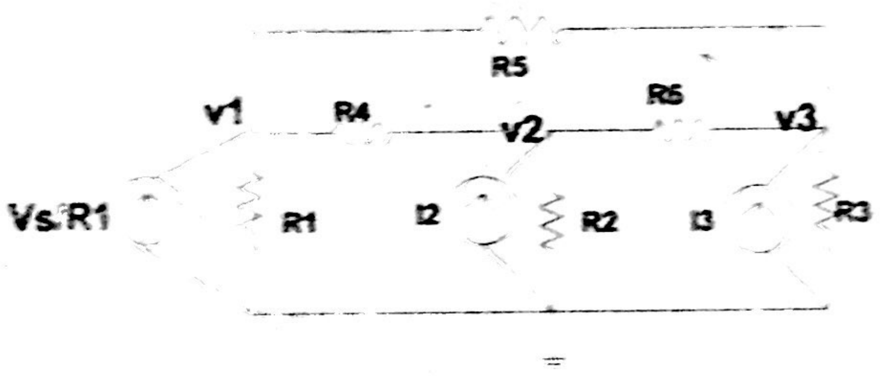
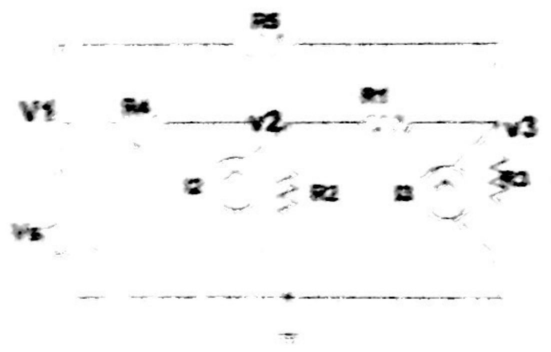




* Then apply KCL on the particular node and solve accordingly

2. b. Voltage source not transformable into current source

2. b.i. The voltage source involving reference node



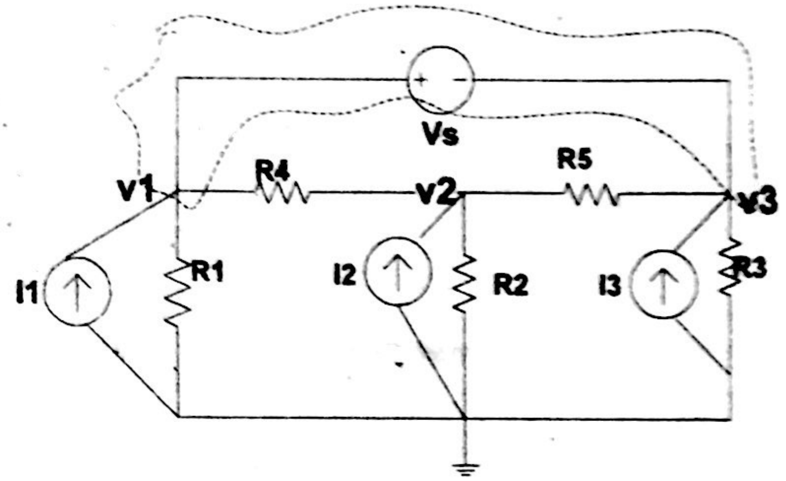
At node 1: The value of V1 is directly known as

$V1 - Vs = 0$

$V1 = Vs$

And for other node solve accordingly

2. b.ii. The voltage source not involving reference node:



Apply super node concept: If there is no resistor between the two nodes' except voltage source, apply super node.

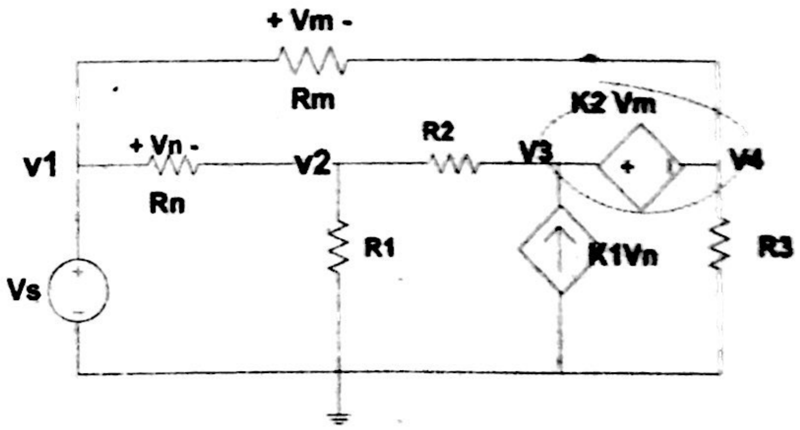
* From super node :(node V1 and V3 are now same point)

$$I3 + I1 = \frac{V1-0}{R1} + \frac{V1-V2}{R4} + \frac{V3-V2}{R5} + \frac{V3-0}{R3} \text{ ------------ (i)}$$

Also (from supernode branch), V1-VS-V3 = 0 ------------- (ii)

And solve accordingly

Type 3: Dependent source in circuit:



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Here, from node 1

$V1 - Vs = 0$

$V1 = Vs$

From super node

$$K1\,V1 = \frac{V4-0}{R3} + \frac{V4-V1}{Rm} + \frac{V3-V2}{R2}$$

Or, $$K1\,(V1 - V2) = \frac{V4-0}{R3} + \frac{V4-V1}{Rm} + \frac{V3-V2}{R2} \text{ -------------- (i)}$$

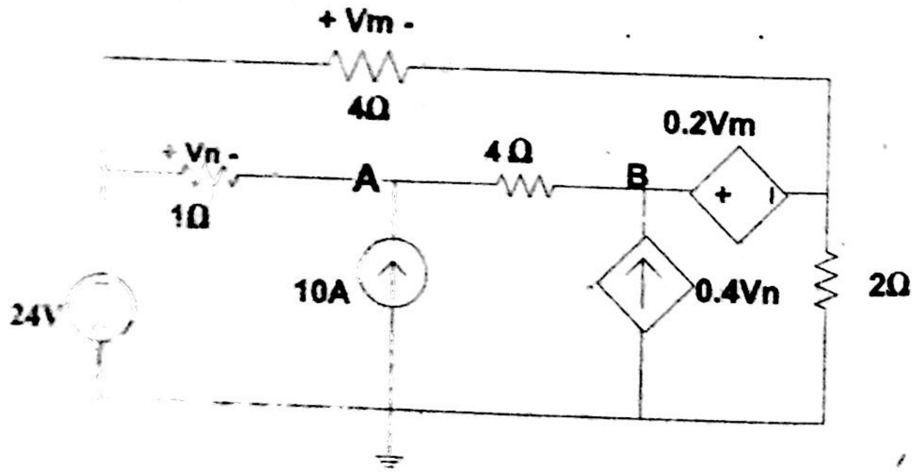
Also,

$V3 - V4 = K2\,Vm$

Or, $V3 - V4 = K2\,(V1 - V4)$ -------------- (ii)

And solve accordingly.

Example .1.2: Using nodal analysis, determine the current through 4 Ω resistor connected **between terminals A and B for the network of following figure.**



Solution:

Considering nodal voltage Va, Vb, Vc, Vd and reference node as shown in figure

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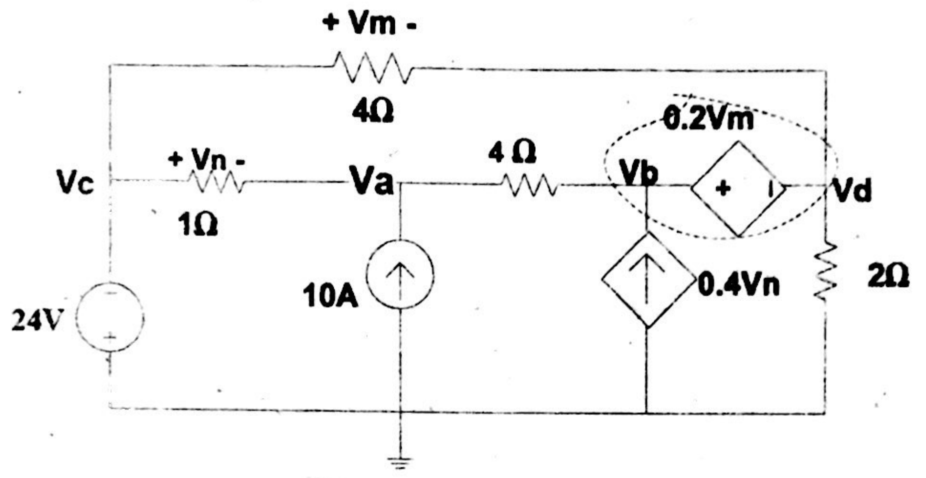

From node C,

$Vc + 24 = 0$

Or, $Vc = -24$ Volt

From super node,

$$0.4\,Vn = \frac{Vb-Va}{4} + \frac{Vd-0}{2} + \frac{Vd-Vc}{4}$$

$1.6\,Vn = Vb - Va + 2Vd + Vd + 24$ $[Vc = -24]$

$1.6\,(Vc - Va) = Vb - Va + 3Vd + 24$

$62.4 = -0.6\,Va - Vb - 3Vd$ ----------------- (i)

Also,

$Vb - Vd = 0.2\,Vm$

Or, $Vb - Vd = 0.2\,(Vc - Vd)$

Or, $Vb - Vd + 0.2\,Vd = -4.8$

Or, $Vb - 0.8\,Vd = -4.8$ ----------------- (ii)

From node A:

$$10 = \frac{Va-Vc}{1} + \frac{Va-Vb}{4}$$

Or, $40 = 4Va + 96 + Va - Vb$

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Or, $-56 = 5\,Va - Vb$ -------- (iii)

Solving equation (i), (ii) and (iii) we get

$Va = -14.22$ Volt

$Vb = -15.129$ Volt

$Vc = -12.91$ Volt

The current through R2 resister is $= \frac{Va-Vb}{4}$

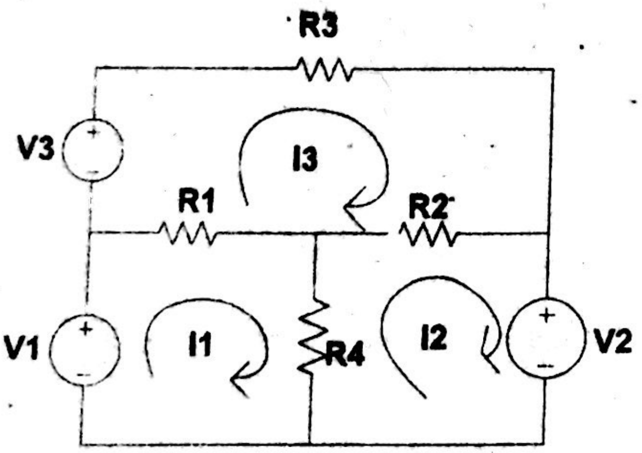
$= \frac{-14.22+15.129}{4}$

$= 0.22725$ Amp

1.2. Mesh Analysis:

- Steps to be applied for solving any circuit by Mesh analysis
  1. Find out the possible no. of loops
  2. Assign the loop current along with their direction
  3. Check the type of the source and classify the circuit on the basis of source present and solve accordingly

Type 1:



Applying KVL on loop (1):

$R1\,(I1-I3) + R4\,(I1-I2) = V1$

OR, $(R1+R4)\,I1 + (-R4)\,I2 + (-R1)\,I3 = V1$

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OR, $R11\ I1 + R12\ I2 + R13\ I3 = V1$ ------------------ (i)

Where,

$R11 = R1 + R4$, $R12 = -R4$ and $R13 = -R1$

Applying KVL on the loop (2):

$R2\,(I2-I3) + R4\,(I2-I1) = -V2$

Or, $(-R4)\,I1 + (R2+R4)\,I2 + (-R2)\,I3 = -V2$

Or, $R21\ I1 + R22\ I2 + R23\ I3 = -V2$------------------ (ii)

Applying KVL on loop (3):

$R31\ I1 + R32\ I2 + R33\ I3 = V3$ ------------------ (iii)

Now, Putting equation (i),(ii)and (iii) in matrix form

[R][I]=[V]

$$\begin{bmatrix} R11 & R12 & R13 \\ R21 & R22 & R23 \\ R31 & R32 & R33 \end{bmatrix} * \begin{bmatrix} I1 \\ I2 \\ I3 \end{bmatrix} = \begin{bmatrix} V1 \\ V2 \\ V3 \end{bmatrix}$$

Then, using Cramer's Rule calculate I1, I2 and I3

$$I1 = \frac{\Delta 1}{\Delta}$$

$$I2 = \frac{\Delta 2}{\Delta}$$

$$I3 = \frac{\Delta 3}{\Delta}$$

Where, $\Delta = \begin{bmatrix} R11 & R12 & R13 \\ R21 & R22 & R23 \\ R31 & R32 & R33 \end{bmatrix}$ $\quad \Delta 1 = \begin{bmatrix} V1 & R12 & R13 \\ V2 & R22 & R23 \\ V3 & R32 & R33 \end{bmatrix}$

$\Delta 2 = \begin{bmatrix} R11 & V1 & R13 \\ R21 & V2 & R23 \\ R31 & V3 & R33 \end{bmatrix}$ $\quad \Delta 3 = \begin{bmatrix} R11 & R12 & V1 \\ R21 & R22 & V2 \\ R31 & R32 & V3 \end{bmatrix}$

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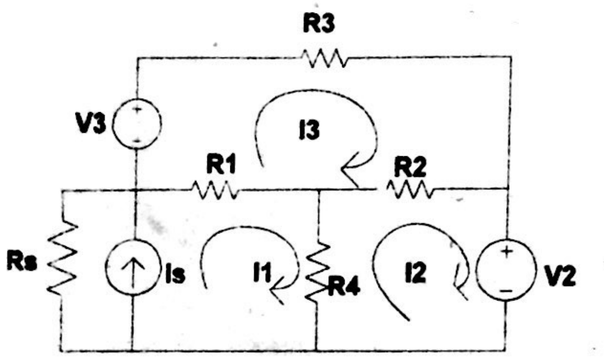
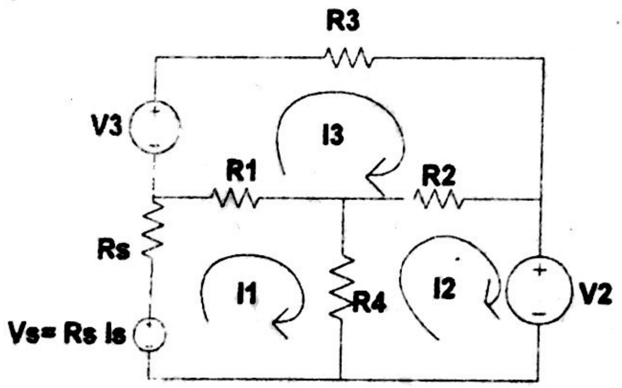
Type 2: Circuit containing Current source in addition to voltage source.

2.a. Current source transformable into voltage source.

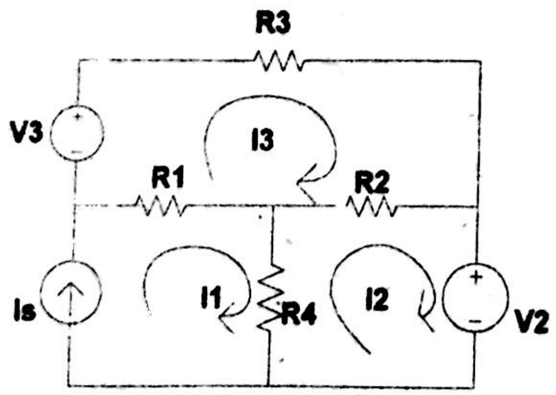




Then apply KVL on the particular loop and solve accordingly

2.b. Current source not transformable into voltage source

2.b.i. Current source present in the perimeter of any individual loop:



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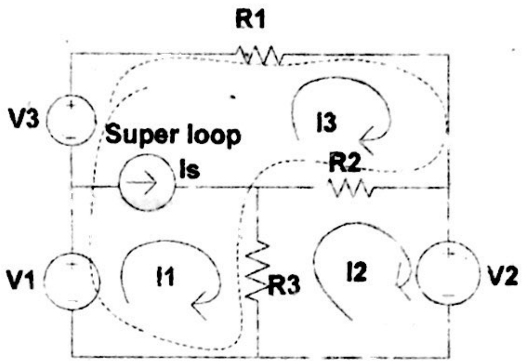
From loop (1):

I1 = Is -------------------------- (i)

And for other loop solve accordingly

2. b.ii. Current source present in the common branch between any two loops



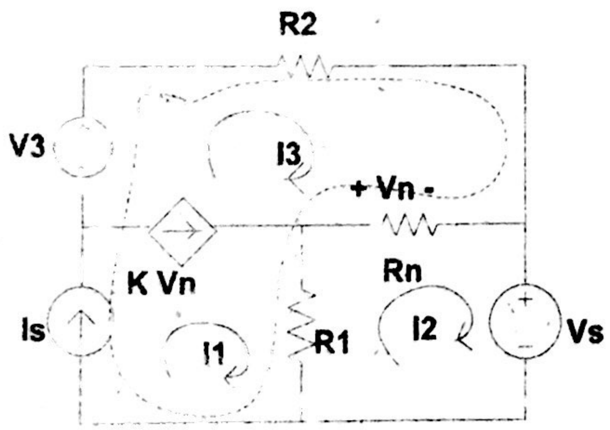
From Super loop: (loop (1) and loop (2) are now same loop)

I3 R1+ (I3- I2) R2 + (I1 –I2) R3 = V1 +V3 -------------------------- (i)

Also, I1 –I3 = Is -------------------------- (i)

And solve accordingly

Type 3: Dependent Source in circuit:



From loop (1):

I1 =Is -------------------------- (i)

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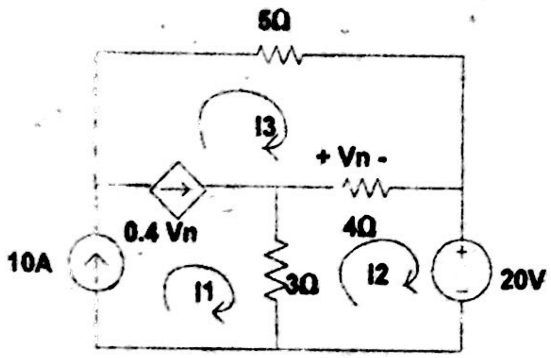
From super loop:

$I1 - I3 = K\ Vn$

Or, $I1 - i3 = K\ (I2 - I3)\ Rn$ ------------------------ (i)

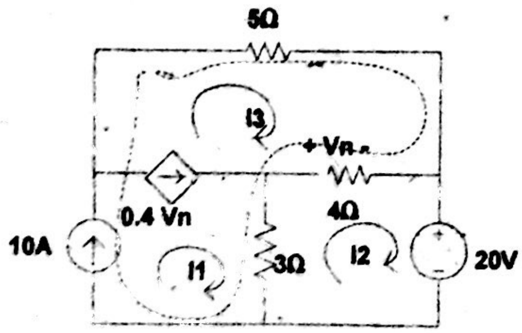
And solve accordingly

Example.1.3: Using mesh analysis, determine the loop current for the network of following figure.



Solution:

Considering loop current as shown in the figure,



From loop (1):

$I1 = 10\ A$ ------------------ (i)

From super mesh:

$I1 - I3 = 0.4\ Vn$

Or, $I1 - I3 = 0.4\ (I2 - I3)\ 4$

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Or, $I1 - 1.6* I2 + 0.6 *I3 = 0$-------------------- (ii)

And from loop (2):

$20 = 4 (I2- I3) + 3 (I2- I1)$

Or, $20 = 4 I2 - 4 I3 + 3 I2 -3 I1$

Or, $20 = 7 I2 - 3 I1 - 4 I3$ -------------------- (iii)

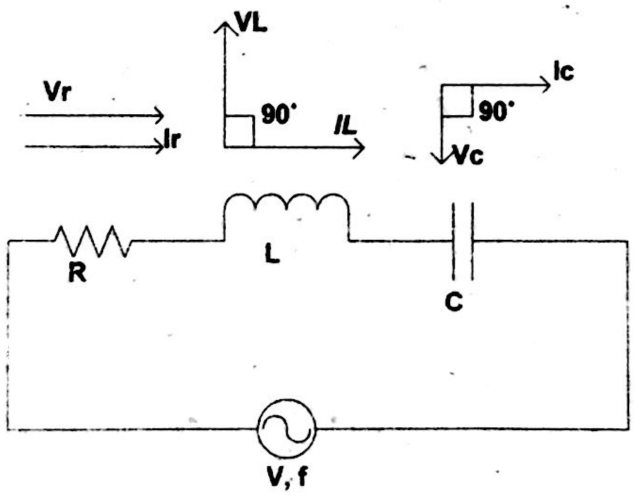
Solving equation (i), (ii) and (iii), we get

I2 = 4.545 A

I3 = - 4.545 A

1.3 Series and Parallel resonance in RLC Circuits:

 # Resonance in series RLC circuit:



Here,

$$I = \frac{V}{Z}$$

$$I = \frac{V}{\sqrt{R^2 + (Xl - XC)^2}}$$

We know,

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$X_L = 2\pi fL$ i.e $X_L \propto f$, Linear function of 'f'

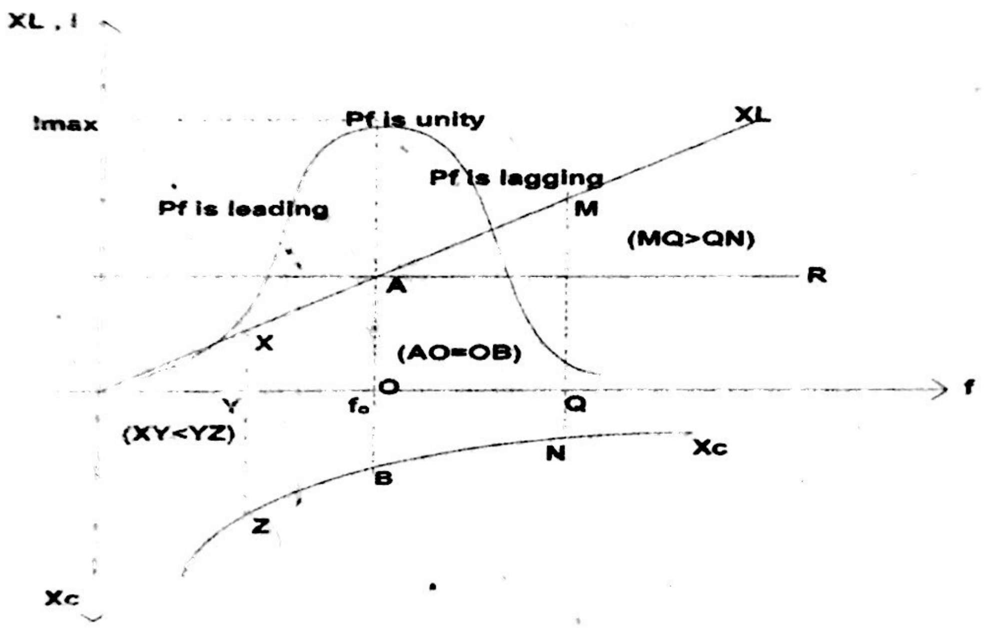
$X_C = \frac{1}{2\pi fC}$ i.e $X_C \propto \frac{1}{f}$, rectangular hyperbolic function.

And R, independent of 'f'

For lower frequency of the input AC voltage, $X_L << X_C$ and for higher frequency of the input AC voltage, $X_L >> X_C$. Thus there exist an intermediate frequency such that

$$X_L = X_C$$

And this particular frequency is known as resonant frequency graphically, we can draw the relation between X_L, X_C and R with reference to input frequency as shown in figure.



At certain frequency f_o,

AO = OB

i.e $X_L = X_C$

The resonant frequency can be evaluated as

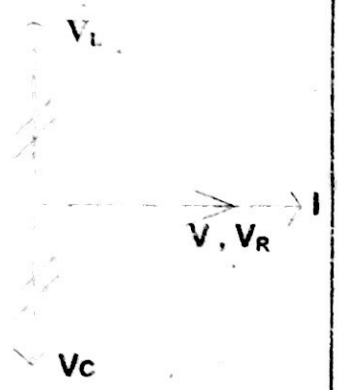
$X_L = X_C$

$2\pi fL = \frac{1}{2\pi fC}$

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

At this frequency of AC input voltage, the current is maximum i.e. $I_m = \frac{V}{R}$, and the current is in phase with the applied voltage. At resonance, $X_L = X_C$ or, $V_L = V_C$

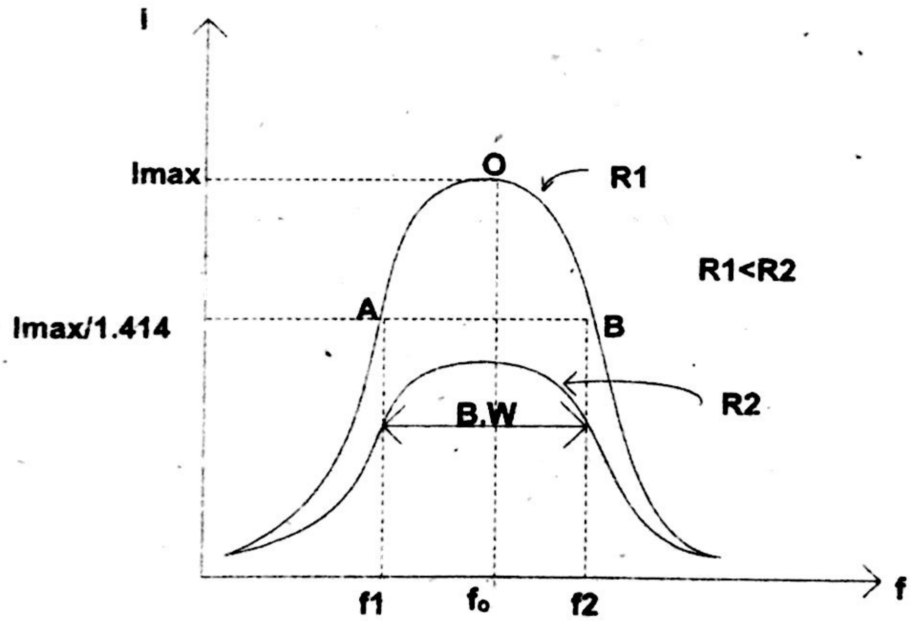
In phase representation



I and V are in phase

> **Note:**
>
> - At resonant frequency voltage across L and C are very high in magnitude then input voltage but V_L and V_C is equal and opposite. i.e. $V_L + V_C = 0$. So, the series LCR circuit is also known as Voltage resonance circuit.
> - At half power point, the phase angle between I and V is 45°

 # Resonance Curve:



Point A and B on the curve are known as half power point. At point A and B power delivered on R is equal to half the power deliver on R at resonant Frequency.

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i.e. $P_A = \frac{1}{2} P_O$

or, $I^2 R = (1/2)\, I^2_{max} R$

or, $I = \frac{I_{max}}{\sqrt{2}} = \frac{I_{max}}{1.414}$

Bandwidth of the circuit is defined as the frequency band for which power delivered on Resistance R is more than half of the maximum power.

$$B.W = f_2 - f_1$$

Where, f_1 and f_2 are half power frequencies

f_1 = lower power frequency

f_2 = higher power frequency

Now for derivation of half power frequency, let us consider at any instant in series RLC circuit.

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

At resonant frequency (f_0)

$$I_m = \frac{V}{R}$$

Now, $I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$

$$I = \frac{V}{R\sqrt{1 + \left(\frac{X_L - X_C}{R}\right)^2}}$$

$$I = \frac{I_m}{\sqrt{1 + \left(\frac{X_L - X_C}{R}\right)^2}} \qquad \left[I_m = \frac{V}{R}\right]$$

At any instant,

$$I = \frac{I_m}{\sqrt{1 + \left(\frac{X_L - X_C}{R}\right)^2}}$$

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But at frequencies f_1 and f_2;

$$I = \frac{Im}{\sqrt{2}}$$

So, $\frac{Im}{\sqrt{2}} = \frac{Im}{\sqrt{1+(\frac{Xl-XC}{R})^2}}$ (at f_1 and f_2)

Now, for $\frac{Im}{\sqrt{1+(\frac{Xl-XC}{R})^2}}$ to be $\frac{Im}{\sqrt{2}}$, then value of $\frac{X_L-X_C}{R}$ should be equal to 1.

i.e. $\frac{X_L-X_C}{R} = 1$

or, $X_L - X_C = R$

But, at resonance

$X_L - X_C = 0$

$X_L - X_C = R$ (at f_1 or f_2)

Conclude if we increase frequency from f_o to f_2; X_L increase by $\frac{R}{2}$ and X_C decreases by $\frac{R}{2}$ so that X_L- X_C = R (at f_2)

So, at resonance frequency (f_0)

$X_{L0} = 2\pi f_0 L$

At frequency f_2, $X_{L2} = 2\pi f_2 L$

$$X_{L2} - X_{L0} = \frac{R}{2}$$

$$2\pi f_2 L - 2\pi f_0 L = \frac{R}{2}$$

$$f_2 - f_0 = \frac{R}{4\pi L}$$

$$f_2 = f_0 + \frac{R}{4\pi L}$$

Similarly,

$$f_1 = f_0 - \frac{R}{4\pi L}$$

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Thus, $B.W = f_2 - f_1 = \frac{R}{2\pi L}$

Quality Factor:

It is a measure of voltage magnification produced in the circuit at resonance.

i.e. $Q = \frac{V_{L0}}{V}$, V_{L0} = Voltage across L at resonance

$V_{L0} = I_{max} X_{L0}$

And $V = I_{max} R$

$Q = \frac{X_{L0}}{R}$ or, $Q = \frac{X_{C0}}{R}$ [$X_{L0} = X_{C0}$ at resonance]

Higher the value of R, Lower is the quality factor.

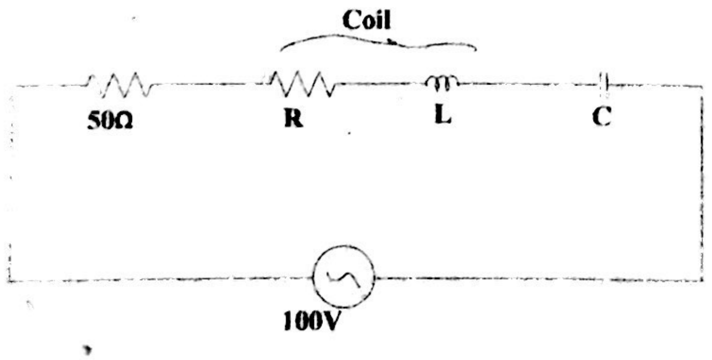
$$Q = \frac{X_{L0}}{R}$$

$$Q = \frac{2\pi fL}{R}$$

$$Q = \frac{f}{\frac{R}{2\pi L}}$$

$$Q = \frac{f}{B.W}$$

Q.1. A 50Ω resistor is connected in series with a coil having resistance R and inductance L, a Capacitor C and 100V variable frequency supply as shown in figure below. At a frequency of 200Hz, the maximum current of 0.7 Amp flows through the circuit and voltage across the capacitor is 200V. Determine the value of R,L and C.



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Solution:

From the question: $f_0 = 200Hz$

And $Z = \sqrt{(50 + R)^2 + (X_L - X_C)^2}$

Now, at Resonant $X_L = X_C$

So, $Z = 50 + R$

Here, $I = \frac{100}{50+R}$

$0.7 = \frac{100}{50+R}$

$R = 92.85\Omega$

Now,

$V_C = I * X_C$

$200 = 0.7 * \frac{1}{2\pi * 200 * C}$

$C = 27.48\mu F$

Also,

$f_0 = \frac{1}{2\pi\sqrt{LC}}$

$200 = \frac{1}{2\pi\sqrt{L * 27.48 * 10^{-6}}}$

$L = 23.04$ mH

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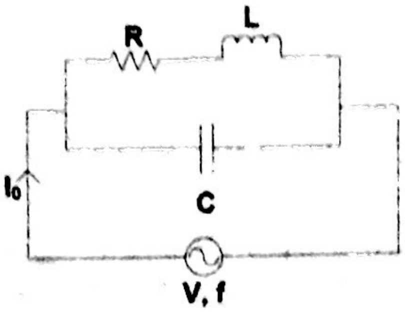
Resonance in parallel RLC circuit:



Resonance of a parallel ac circuit will take place when an inductive reactance and a capacitive reactance are connected in parallel as shown in the figure below, thus conditions may reach under which current resonance.

The parallel RLC circuit is said to be in resonance when impedance (Z) is maximum.

We have,

$$\frac{1}{Z} = \frac{1}{R+jX_L} - \frac{1}{jX_C}$$

Or, $$\frac{1}{Z} = \frac{R-jX_L}{R^2+X_L^2} - \frac{1}{jX_C}$$

Or, $$\frac{1}{Z} = \frac{R}{R^2+X_L^2} + j\left(\frac{1}{X_C} - \frac{X_L}{R^2+X_L^2}\right)$$

For $\frac{1}{Z}$ to be minimum

$$\frac{1}{X_C} - \frac{x_L}{R^2+X_L^2} = 0$$

Or, $$R^2 + \omega^2 L^2 = \frac{L}{C}$$

Or, $$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$\therefore f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

Here, f_0 is the resonant frequency for parallel RLC circuit.

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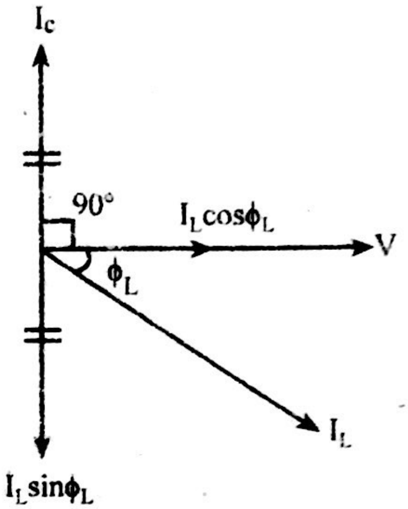
Phasor Diagram for parallel RLC circuit:

So at resonance $I_C = I_L \sin\phi_L$ and the resultant current or net current is again in phase with voltage.

At resonant frequency, a very small in phase current is drawn by circuit i.e. $I_L \cos\phi_L$. Here I_C and $I_L \sin\phi_L$ may be very greater than $I_L \cos\phi_L$ but since they act in opposite direction they get cancel out.



At resonant,

$$I_c = I_L \sin\phi_L$$

Or, $$\frac{V}{X_C} = \frac{V}{Z_{RL}} * \frac{X_L}{Z_{RL}}$$

Or, $$Z^2_{RL} = X_L * X_C$$

Or, $$R^2 + X^2_L = X_L * X_C$$

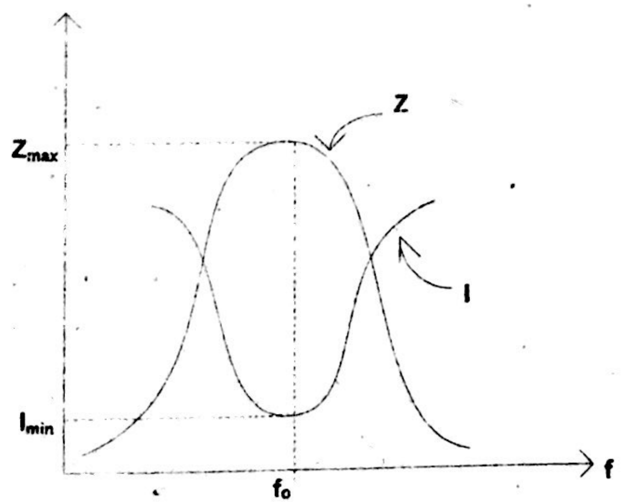
Or, $$R^2 + \omega^2 L^2 = \frac{L}{C}$$

Or, $$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$\therefore f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

Note: I_L and I_C are very larger than I_0 at resonance so, the parallel resonance circuit is also called current resonance circuit.

Resonance Curve:



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Quality factor in parallel circuit is measure of current magnification,

i.e. $Q = \frac{I_C}{I_0}$

Here, $I_C = \frac{V}{X_C} = V\omega C$

Or, $I_0 = I_L \cos\phi_L = \frac{V}{Z_{RL}} * \frac{R}{Z_{RL}}$

Or, $I_0 = \frac{VR}{Z^2_{RL}} = \frac{VR}{\frac{L}{C}}$

Or, $I_0 = \frac{VRC}{L}$

So, $Q = \frac{V\omega C}{\frac{VRC}{L}}$

$\therefore \mathbf{Q = \frac{\omega L}{R}}$

Note: At resonance of parallel RLC circuit.

- $Z^2_{RL} = X_L * X_C$

  $\mathbf{Z^2_{RL}} = \frac{L}{C}$
- Line current is minimum and equal to $\frac{VCR}{L}$ and is in phase with applied voltage.

  i.e. $\mathbf{I_0} = \frac{VCR}{L}$
- The total Impedance at resonant is:

  $\frac{1}{Z} = \frac{R}{R^2 + X_L^2}$

  $Z = \frac{R^2 + X_L^2}{R}$

  $Z = R + \frac{\omega_0^2 L^2}{R}$

  $Z = R + \frac{\frac{L}{C} - R^2}{R}$, [$R^2 + \omega^2 L^2 = \frac{L}{C}$]

  $\mathbf{Z} = \frac{L}{CR}$

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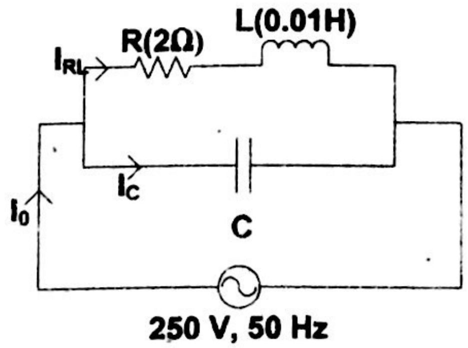
Comparison between voltage resonance and current resonance:

S.N	Particulars	Voltage Resonance	Current resonance
1.	Impedance under resonance condition	Z=R, the minimum	Z=L/RC, the maximum
2.	Power factor at resonance	Unity	Unity
3.	Resonant frequency	$f_o = \frac{1}{2\pi\sqrt{LC}}$	$f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
4.	Quantity magnified	Voltage	Current
5.	Magnification(Quality factor)	ωL/R	ωL/R
6.	Bandwidth	R/2πL	R/2πL
7.	Magnitude of current under resonance condition	I=V/R, the maximum	I=V/(L/RC), minimum

Example.1. 4: An inductive circuit of resistance 2Ω and inductance 0.01 H is connected parallel with capacitor C and with 250V, 50 Hz supply.

- **(i)** **What value of capacitance placed in parallel will produce resonance?**
- **(ii)** **Determine the total current takes from the supply and the current in the branch circuit?**

Solution:



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For parallel RLC circuit.

(i) $Z_{RL} = \sqrt{R^2 + X_L^2}$

$X_L = 2\pi f L$

$X_L = 2\pi * 50 * 0.01$

$X_L = 3.14\ \Omega$

$\therefore Z_{RL} = \sqrt{2^2 + 3.14^2} = 3.72\ \Omega$

At resonance

$Z_{RL}^2 = \frac{L}{C}$

Or, $C = \frac{L}{Z_{RL}^2}$

Or, **$C = 722.6\ \mu F$**

(ii) Total current (I_0)

$I_0 = I_L \cos\phi_L$

$I_0 = \frac{VCR}{L}$

$I_0 = \frac{250 * 722.6 * 10^{-6} * 2}{0.01}$

$I_0 = 36.13$ A

Branch currents are:

$I_{RL} = \frac{V}{Z_{RL}} = \frac{250}{3.72}$

$I_{RL} = 67.2$ A

And $\tan\phi_L = \frac{X_L}{R} = \frac{3.14}{2}$

$\phi_L = 57.5°$

Also,

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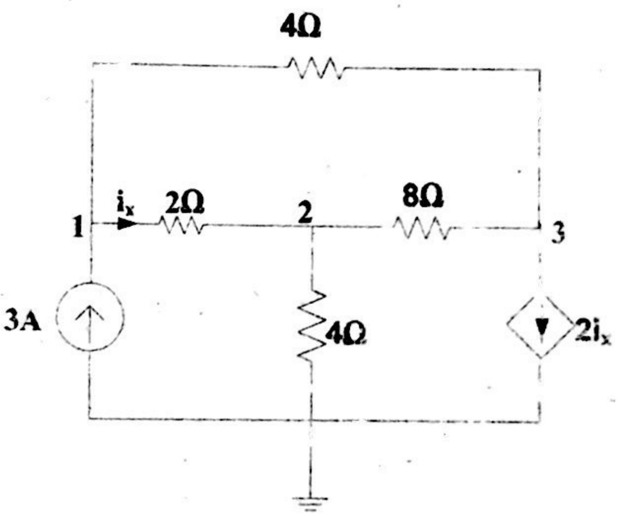
$$I_C = \frac{V}{X_C} = V\omega C = 250*2\pi*50*722.6*10^{-6}$$

$$\mathbf{I_C = 56.75\ A < 90°}$$

Problems:

#Determine the nodal Voltages of the figure below:

Figure.1



[Ans: $V_1 = 4.8V$, $V_2 = 2.4V$, $V_3 = -2.4V$]

Figure.2

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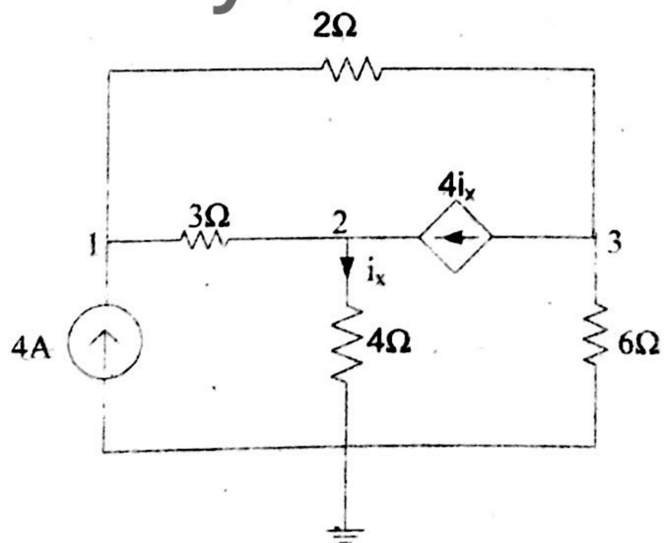

[Ans: $V_1 = 32V$, $V_2 = -25.6V$, $V_3 = 62.4V$]

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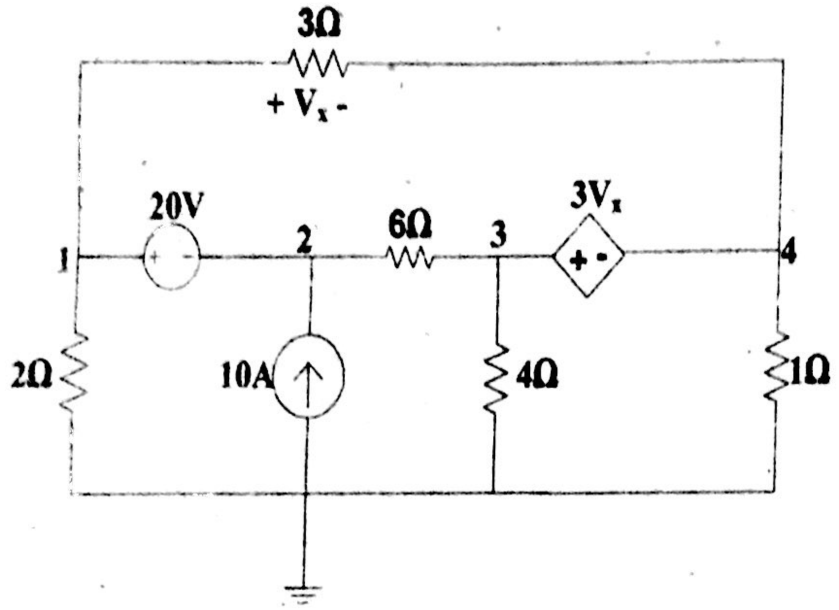
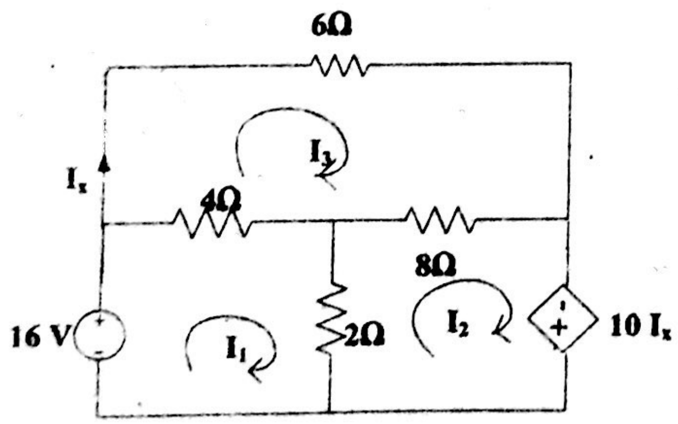
Figure.3



[Ans: $V_1 = 26.67V$, $V_2 = 6.667V$, $V_3 = 173.33V$, $V_3 = -46.67V$]

#Use mesh analysis to find the current I_x from the figure below:

Figure.4



[Ans: I_x= −4A]

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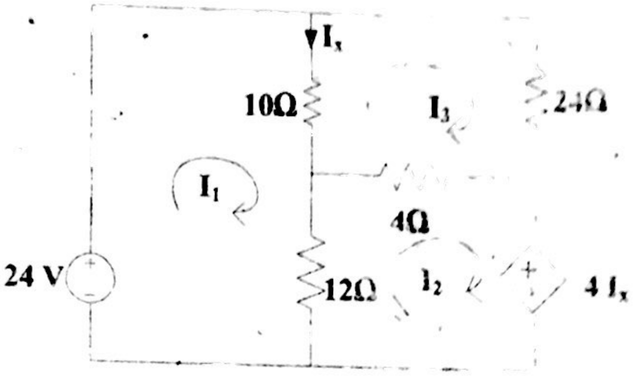
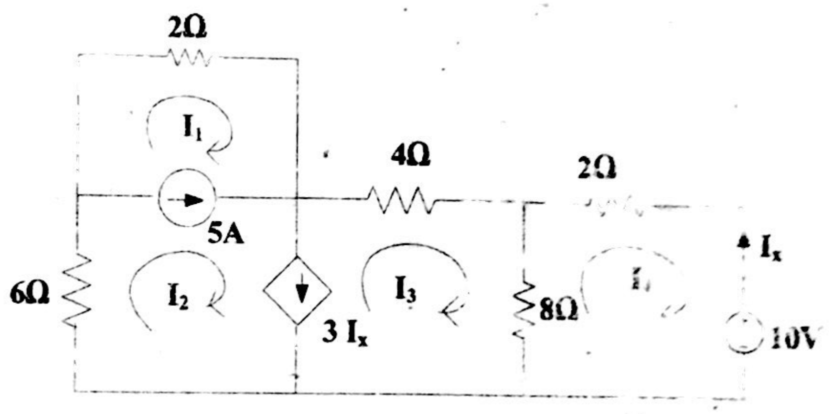
Figure.5



[Ans: I_x= 1.5 A]

Figure.6



[Ans: I_x= -2.143A]

Resonance

<u>Series:</u>

Q.1: A coil of resistance 40Ω and inductance 0.75H forms part of a series circuit for which the resonant frequency is 55Hz. If the supply is 250V, 50Hz, find (i) the line current, (ii) circuit power factor, (iii) Voltage across the coil.

[Ans: 3.93A, 0.63 leading, 939V]

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Q.2: A 50µF Capacitor, when connected in series with a coil having 40Ω resistor, resonates at 1000Hz. Find the inductance of the coil. Also obtain the circuit current if the applied voltage is 100V. Also calculate the voltage across the capacitor and the coil at resonance.

[Ans: 0.5mH, 2.5A, 7.96V, 100.31V]

Q.3: A 220V, 100Hz AC source supplies a series RLC circuit with a capacitor and a coil. If a coil has 50mΩ resistance and 5mH inductance, find the value of capacitor for which resonance frequency is 100Hz. Also calculate the Q-factor and half power frequencies of the circuit.

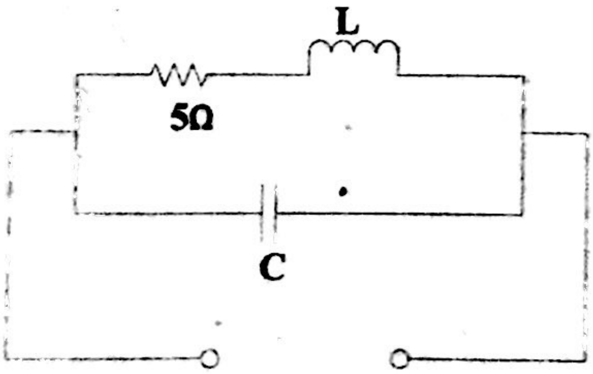
[Ans: 507µF, 62.83, 100.795Hz, 99.205Hz]

Parallel

Q.4: The dynamic impedance of a parallel resonant circuit is 500KΩ. The circuit consists of a 250pF capacitor in parallel with a coil of resistance 10Ω.Calculate (i) the coil inductance (ii) the resonant frequency, (iii) the Q- factor of the circuit.

[Ans: 1.25mH,284.7KHz,223.6]

Q.5: Calculate the impedance of parallel- tuned circuit shown in figure below at a frequency of 500KHz and for bandwidth of operation equal to 20KHz



[Ans: 3KΩ]

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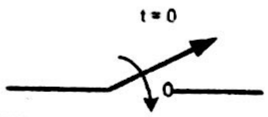
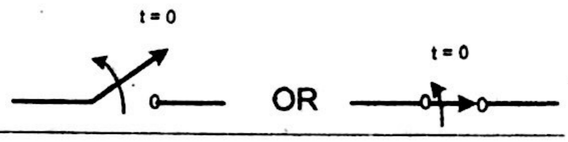
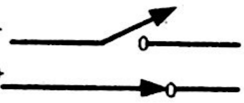
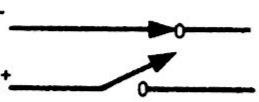
Initial Conditions

 # Initial Conditions: The value of dependent variables and its derivatives just after any change is made is known as Initial condition. Example, $i(0^+)$ = ' i ' is the dependent value at $t = 0^+$ condition.

 # Changing Element [Switch]:

	
(i) Initially it was opened at $t = 0^-$ and going for closed at $t = 0^+$	(i) Initially it was closed at $t = 0^-$ and going open for at $t = 0^+$
At $t = 0^-$ At $t = 0^+$ 	At $t = 0^-$ At $t = 0^+$ 
(ii) $t = 0^+$ the time just after the switch is closed.	(ii) $t = 0^+$ the time just after the switch is opened.
(iii) $t = 0^-$ the time just before the switch is closed.	(iii) $t = 0^-$ the time just before the switch is opened.
(iv) $i(0^+)$the value of the current just after the switch is closed.	(iv) $i(0^+)$the value of the current just after the switch is opened.

<u># Initial condition for different circuit elements:</u>

While evaluating initial condition for different circuit elements, we should know about two conditions for better understanding (a) De-energized condition (b) Energized condition.

(a).<u>De- energized condition:</u> If a current doesn't pass through the inductor and there is no voltage across capacitor at $t = 0^-$ then it is termed as De-energized element.

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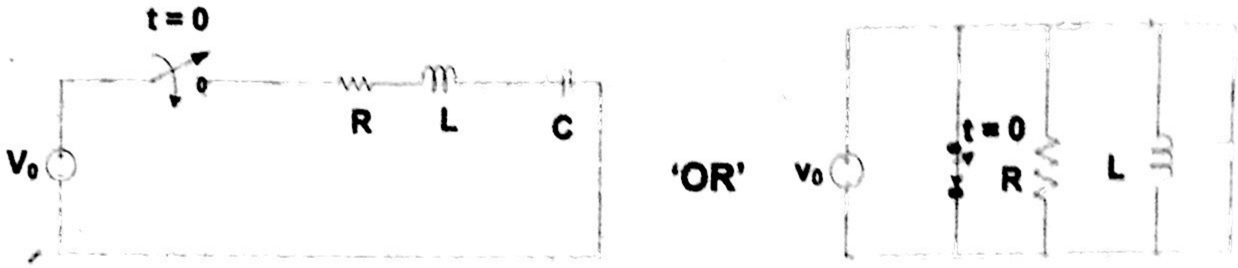
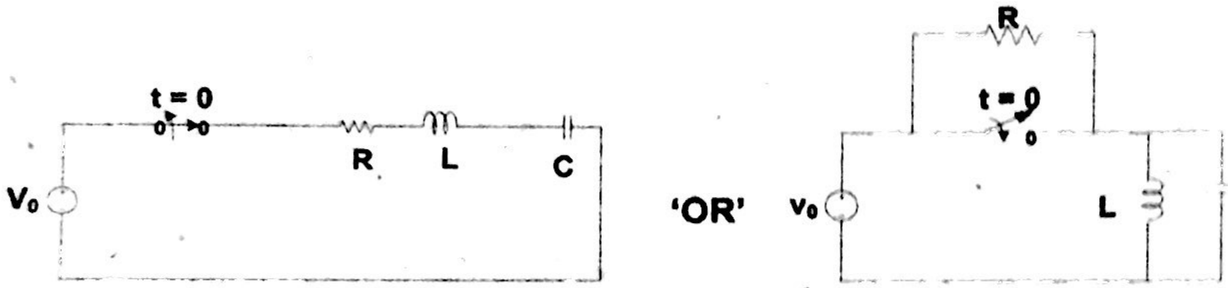


At, $t = 0^-$ there is zero value of current passing through inductor and capacitor for both circuit shown in above figures.

(b). Energized condition: If a current pass through the inductor and there is voltage across capacitor at $t = 0^-$ then it is termed as energized element.



At, $t = 0^-$ there is some value of current passing through inductor and capacitor for both circuit shown in above figures.

1. **For Inductor:**

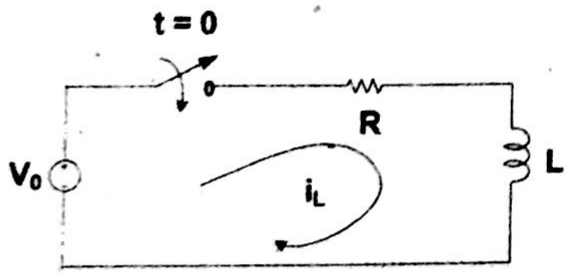


Fig .(i)

From the circuit, (De- energized)

$i_L(0^-) = 0$

Now let us consider the current through conductor can change instantaneously (i.e. $i_L(0^+)$ have some current value)

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$i_L(0^-) = 0$

$i_L(0^+) =$ Constant Value

We know,

$$V_L = L\frac{di_L}{dt}$$

$di_L = i_L(0^+) - i_L(0^-) =$ Constant Value

$dt = 0^+ - 0^- \approx 0$

Thus,

$$V_L = L * \frac{\text{Constant}}{0}$$

$$\mathbf{V_L = \infty}$$

This is impossible because there is no any such practical voltage source. Thus our assumption of inductor current change instantaneously is wrong. Hence we can conclude that current through inductor cannot change instantaneously.

i.e. $i_L(0^-) = i_L(0^+)$ ---------------------- (i)

Equation (i) is known as continuity relation for Inductor.

Note: Current through inductor cannot change instantaneously but voltage does.

I.e. $\mathbf{i_L(0^-) = i_L(0^+)}$

And $\mathbf{V_L(0^-) \neq V_L(0^+)}$

Now,

From continuity relation of inductor in Fig.(i)

$i_L(0^-) = i_L(0^+) = 0$

[This shows that at $t = 0^+$ there should be a zero current in inductor, and to be zero current there must be an open circuit in place of Inductor.]

Also, $V_L(0^-) = 0$

So, circuit of Fig.(i) at $t= 0^+$ is

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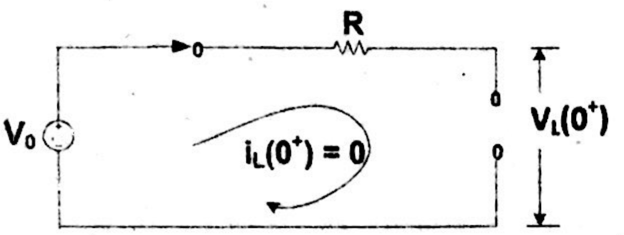

Thus $V_L(0^+) = V_0$

$i_L(0^+) = 0$

Thus we conclude that the De-energized inductor can be replaced by open circuit at $t = 0^+$

And

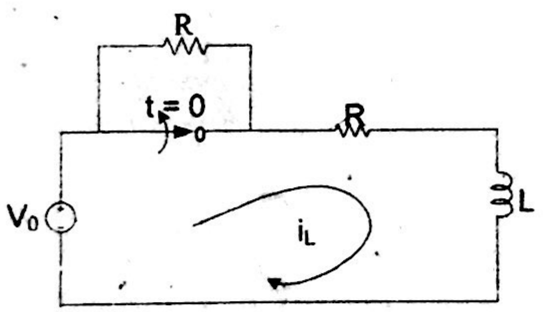

Fig (ii)

From the circuit, (Energized)

At this circuit for time at $t = 0^-$ all the voltage and current attains their maximum value or final value as switch was in this position (ON) for very long time. Thus $i_L(0^-) = I_0 = \frac{V_0}{R}$(constant)

$$V_L(0-) = L\frac{dI_L}{dt} = L\frac{dI_0}{dt} = L * 0$$

$$V_L(0-) = 0$$

[This shows that at $t = 0^-$ there should be a zero voltage across inductor, and to be zero voltage across there must be a short circuit in place of inductor.]

So, Circuit at $t = 0^-$ is

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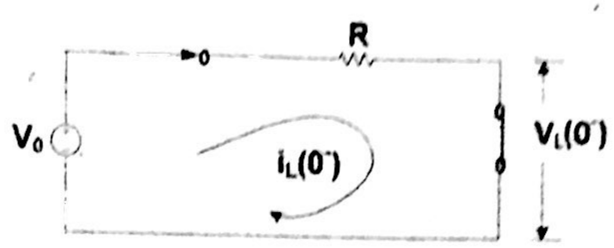


From the circuit $I_L(0^-) = \frac{V_0}{R} = I_0$

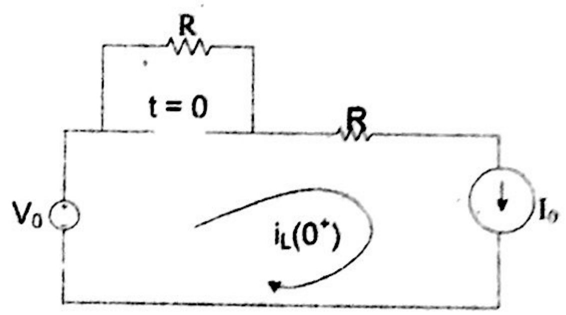
And $V_L(0-) = 0$

Thus we conclude that the inductance of energized condition is replaced by short circuit at $t= 0^-$ or $t = \infty$.

Note: $t = 0^-$ and $t = \infty$ are same

From continuity relation for inductor, $i_L(0^-) = i_L(0^+) = I_0$

Now, circuit at $t= 0^+$ is



From the circuit $i_L(0^+) = I_0$

And $V_L(0^+) = V_0 - I_0 * 2R$

Thus we conclude that the energized inductor is replaced by current source at $t = 0^+$

2. **For Capacitor :**

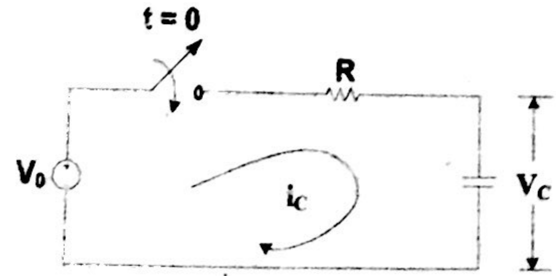


Fig (iii)

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From the circuit, (De- energized)

$V_C(0^-) = 0$

Now let us assume that voltage across capacitor can change instantaneously (i.e. $V_C(0^+)$ have some current value)

$V_C(0^-) = 0$

$V_C(0^+) = \text{Constant Value}$

We know,

$$i_c = C\frac{dV_C}{dt}$$

$dV_C = V_C(0^+) - V_C(0^-) = \text{Constant Value}$

$dt = 0 - 0^- \approx 0$

Thus,

$$i_c = C * \frac{\text{Constant}}{0}$$

$i_C = \infty$

Note: Voltage across capacitor cannot change instantaneously but current does.

I.e. $V_C(0^-) = V_C(0^+)$

And $i_C(0^-) \neq i_C(0^+)$

This is impossible thus our assumption of voltage across capacitor can change instantaneously is wrong. Hence we can conclude that voltage across capacitor cannot change instantaneously.

i.e. $V_C(0^-) = V_C(0^+)$ ------------------ (i)

Equation (i) is known as continuity relation for capacitor.

Now,

From continuity relation of capacitor in Fig.(iii)

$V_C(0^-) = V_C(0^+) = 0$

[This shows that at $t = 0^+$ there should be a zero voltage across capacitor, and to be zero voltage there must be a short circuit in place of capacitor.]

Also, $i_C(0^-) = 0$

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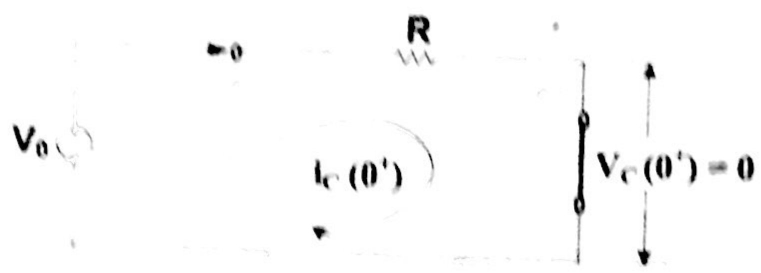
So, circuit of Fig (iii) at $t = 0^+$ is



Thus $V_C(0^+) = 0$

$i_C(0^+) = \frac{V_0}{R} = I_0$

Thus we conclude that the De-energized capacitor can be replaced by short circuit at $t = 0^+$

And

Fig (iv)

From the circuit, (Energized)

At this circuit for time at $t = 0^-$ all the voltage and current attains their maximum value or final value as switch was in this position (ON) for very long time. Thus $V_C(0^-) = V_0$ (constant)

$i_C(0-) = C\frac{dV_C}{dt} = C\frac{dV_0}{dt} = C * 0$

$i_C(0-) = 0$

[This shows that at $t = 0^-$ there should be a zero current through capacitor, and to be zero current there must be an open circuit in place of capacitor.]

So, Circuit at $t = 0^-$ is

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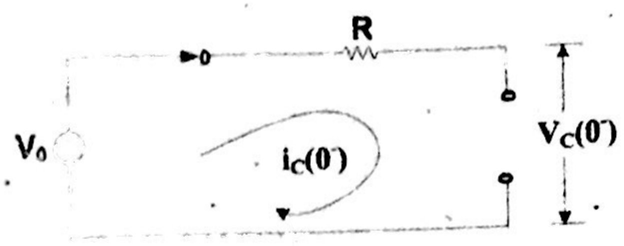


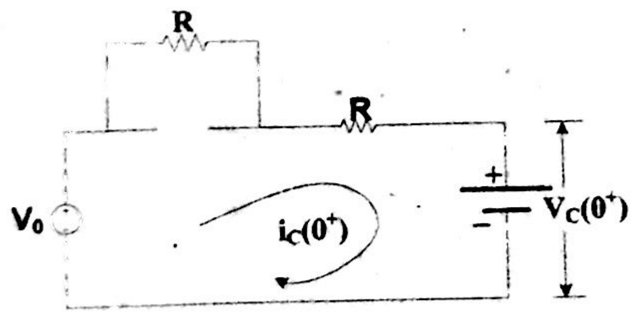
From the circuit $V_C(0^-) = V_0$

And $i_C(0-) = 0$

Thus we conclude that the capacitor of energized condition is replaced by open circuit at t= 0^- or $t = \infty$.

From continuity relation for inductor, $V_C(0^-) = V_C(0^+) = V_0$

Now, circuit at t= 0 is



From the circuit $V_C(0^+) = V_0$

And $i_C(0^+) = \frac{V_0 - V_C}{2R} = 0$

Thus we conclude that the energized capacitor is replaced by Voltage source at $t = 0^+$

3. **For Resistor:**

And for resistor there will be:

. i.e. $i_R(0^-) \neq i_R(0^+)$

And $V_R(0^-) \neq V_R(0^+)$

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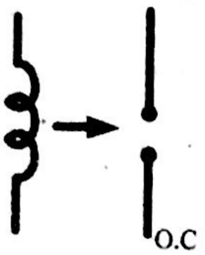
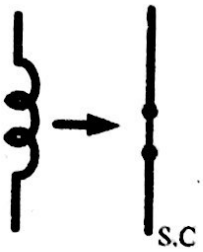
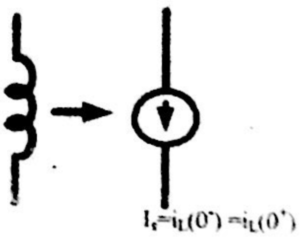
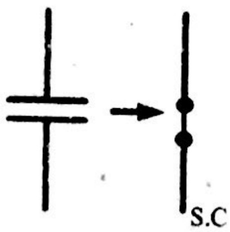
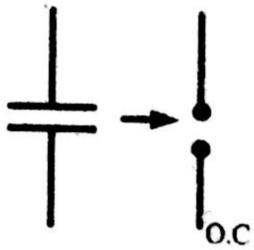
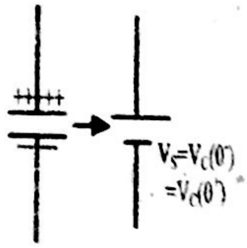
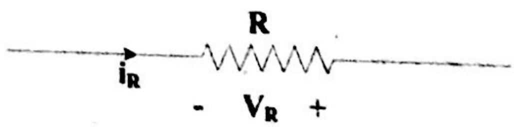
Summary:

Circuit Elements	De-energized		Energized	
	At $t=0^-$ or $t=\infty$	At $t=0^+$	At $t=0-$ or $t=\infty$	At $t=0+$
Inductor (L)	No need to change circuit elements			
Capacitor (C)	No need to change circuit elements			

 # Voltage – current (V-I) relation of different circuit elements:

(a) Resistance or Resistor (R)

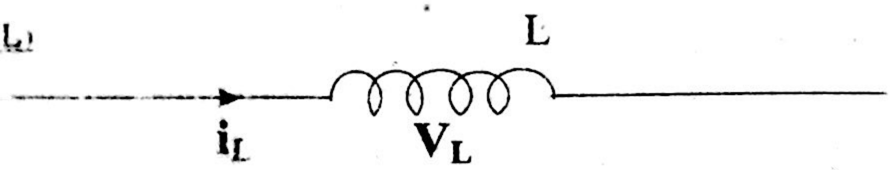
Here. $V_R = i_R * R$



$$i_R = \frac{V_R}{R}$$

The voltage across a resistance change instantaneously the current through it also changes instantaneously and vice versa.

(b) Inductor (L)



Here, $V_L = L\frac{di_L}{dt}$

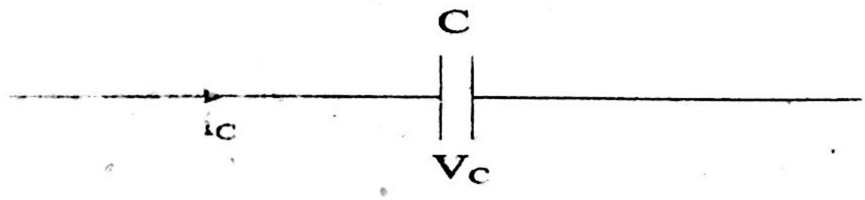
Or, $di_L = \frac{1}{L} V_L \, dt$

Integrating

$i_L = \frac{1}{L}\int V_L \, dt$

The current through an inductor cannot change instantaneously. Thus inductor acts as open circuit for the newly applied energy at the instant of switching, but if I_0 current is already flowing in the inductor before switching action, the inductor may be considered as a current source I_0.

(c) Capacitor (C)



Here, $i_C = \frac{dq}{dt}$ [q = VC]

Or, $i_C = \frac{dCV_c}{dt}$

Or, $i_C = C\frac{dV_c}{dt}$

Or, $dV_c = \frac{1}{C} i_C \, dt$

Integrating

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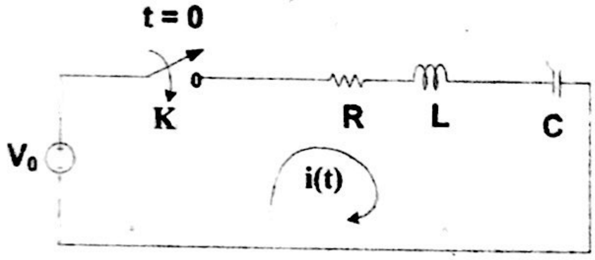
$$V_C = \frac{1}{C}\int i_C \, dt$$

For a system of fixed capacitance, the voltage cannot change instantaneously. Thus for a suddenly applied energy source a capacitor is equivalent to a short circuit. Hence on connecting an uncharged capacitor to an energy source, a current flows instantaneously.

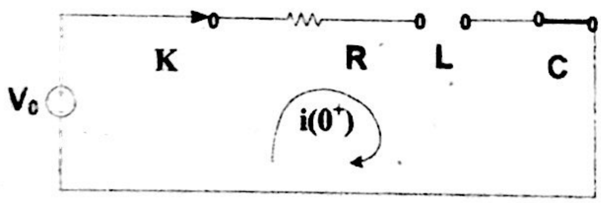
Initial conditions in the case of a series RLC network:

In the circuit shown below switch K is open for a long time at t = 0 switch is going to close then find $i(0^+)$, $\frac{di}{dt}(0^+)$ and $\frac{d^2i}{dt^2}(0^+)$.



Solution:

It's a de-energized condition, so at $t = 0^+$ inductor is replace with open circuit and capacitor is replace with short circuit. Circuit at $t = 0^+$ is shown below



So we know by inspection

$i_L(0^-) = 0 = i_L(0^+)$, $V_C(0^-) = 0 = V_C(0^+)$.

$\mathbf{i(0^+) = i_L(0^+) = 0}$ **Amp**

Now, to find $\frac{di}{dt}(0^+)$, apply KVL at $t > 0$ in the circuit,

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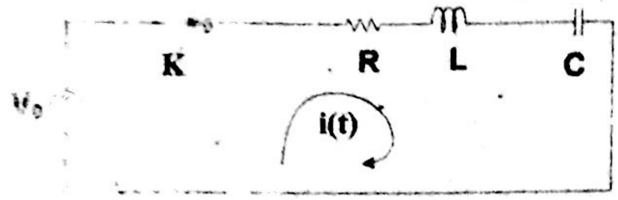

$V_0 = V_R + V_L + V_C$

Or, $V_0 = i * R + L\frac{di}{dt} + V_C$ ------------------- (i)

Put $t = 0^+$

Or, $V_0 = i(0+) * R + L\frac{di}{dt}(0^+) + V_C(0^+)$

Or, $V_0 = 0 + L\frac{di}{dt}(0^+) + 0$

$\frac{di}{dt}(0^+) = \frac{V_0}{L}$ Amp/ sec

Now, to find $\frac{d^2i}{dt^2}(0^+)$, differential equation(i) w.r.t 't', we get

$V_0 = i * R + L\frac{di}{dt} + V_C$

Or, $V_0 = i * R + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$

Differential

Or, $0 = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i$

Put $t = 0^+$

Or, $0 = R\frac{di}{dt}(0^+) + L\frac{d^2i}{dt^2}(0^+) + \frac{1}{C}i(0^+)$

Or, $0 = R * \frac{V_0}{L} + L\frac{d^2i}{dt^2}(0^+) + 0$

$\frac{d^2i}{dt^2}(0^+) = -\frac{RV_0}{L}$ Amp/ sec^2

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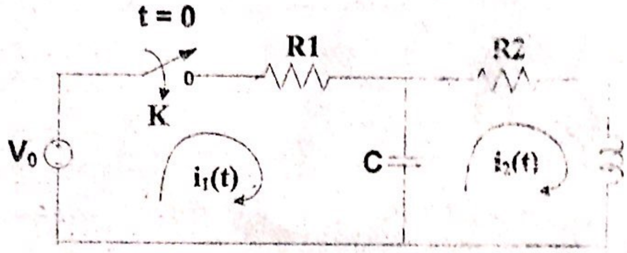
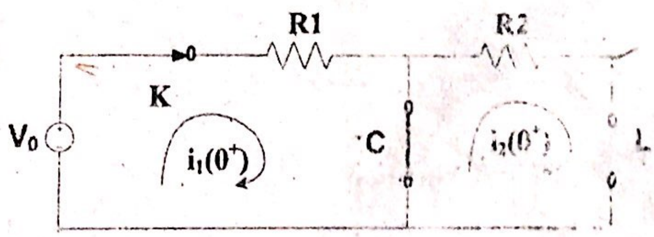
Initial conditions in the case of a two-loop RLC network:

In the circuit shown below switch K is open for a long time at t = 0 switch is closed then find $i_1(0^+)$, $i_2(0^+)$, $\frac{di_1}{dt}(0^+)$, $\frac{di_2}{dt}(0^+)$ $\frac{d^2i_1}{dt^2}(0^+)$ and $\frac{d^2i_1}{dt^2}(0^+)$.



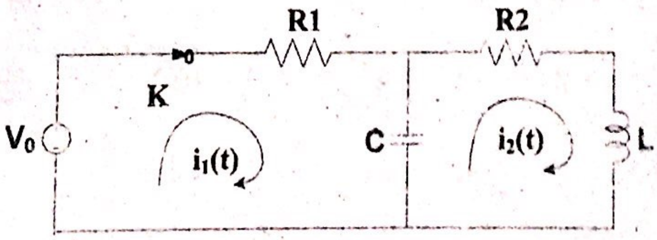
Solution:

It's a de-energized condition, so at $t = 0^+$ inductor is replace with open circuit and capacitor is replace with short circuit. Circuit at $t = 0^+$ is shown below



By inspection, $i_L(0^-) = i_2(0^-) = i_2(0^+) = 0$ and $i_1(0^+) = \frac{V_0}{R_1}$ Also, $V_C(0^-) = V_C(0^+) = 0$

Now, to find $\frac{di_1}{dt}(0^+)$, $\frac{di_2}{dt}(0^+)$ apply KVL at $t > 0$ in the circuit.



From loop (1): $V_0 = V_{R1} + V_C$

Or, $V_0 = R1 * i_1 + \frac{1}{C}\int(i_1 - i_2)dt$ Differential w.r.t 't'

Or, $0 = R1\frac{di_1}{dt} + \frac{1}{C}(i_1 - i_2)$ ---------------- (i)

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Put $t = 0^+$

Or, $0 = R1\frac{di_1}{dt}(0^+) + \frac{1}{C}i_1(0^+) - \frac{1}{C}i_2(0^+)$

Or, $0 = R1\frac{di_1}{dt}(0^+) + \frac{1}{C} * \frac{V_0}{R1} - \frac{1}{C} * 0$

$\frac{di_1}{dt}(0^+) = -\frac{1}{CR1} * \frac{V_0}{R1}$ **Amp/sec**

From loop (2): $V_C + V_{R2} + V_L = 0$

Or, $\frac{1}{C}\int(i_2 - i_1)dt + R2 * i_2 + L\frac{di_2}{dt} = 0$ ------------------- (ii)

Or, $V_C + R2 * i_2 + L\frac{di_2}{dt} = 0$

Put $t = 0^+$

Or, $V_C(0^+) + R2 * i_2(0^+) + L\frac{di_2}{dt}(0^+) = 0$

Or, $0 + R2 * 0 + L\frac{di_2}{dt}(0^+) = 0$

$\frac{di_2}{dt}(0^+) = 0$ **Amp/sec**

Now, to find $\frac{d^2i_1}{dt^2}(0^+)$ and $\frac{d^2i_1}{dt^2}(0^+)$, differential equation (i) and (ii),

From equation (i) after differential, $0 = R1\frac{d^2i_1}{dt^2} + \frac{1}{C}\frac{di_1}{dt} - \frac{1}{C}\frac{di_2}{dt}$

Put $t = 0^+$

Or, $0 = R1\frac{d^2i_1}{dt^2}(0^+) + \frac{1}{C}\frac{di_1}{dt}(0^+) - \frac{1}{C}\frac{di_2}{dt}(0^+)$

Or, $0 = R1\frac{d^2i_1}{dt^2}(0^+) + \frac{1}{C}(-\frac{1}{CR1} * \frac{V_0}{R1}) - \frac{1}{C} * 0$

$\frac{d^2i_1}{dt^2}(0^+) = \frac{V_0}{C^2R1^3}$ **Amp/sec²**

From equation (ii) after differential, $\frac{1}{C}i_2 - \frac{1}{C}i_1 + R2\frac{di_2}{dt} + L\frac{d^2i_2}{dt^2} = 0$

Put $t = 0^+$

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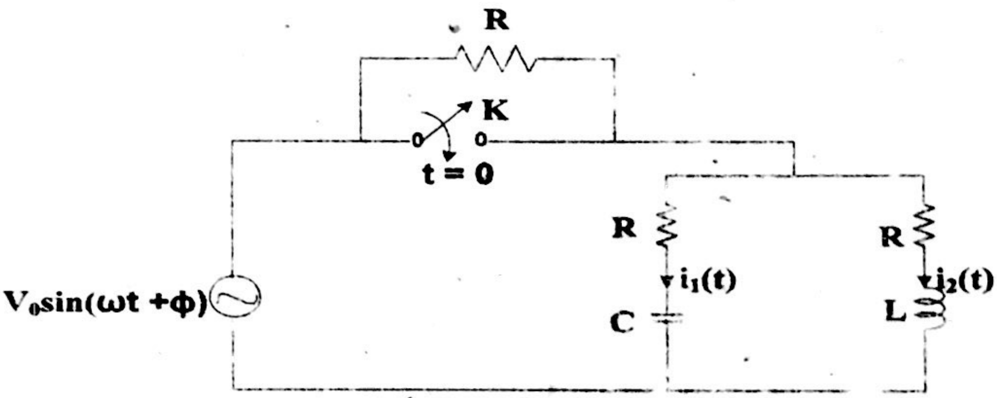
Or, $\frac{1}{C} i_2(0^+) - \frac{1}{C} i_1(0^+) + R2\frac{di_2}{dt}(0^+) + L\frac{d^2 i_2}{dt^2}(0^+) = 0$

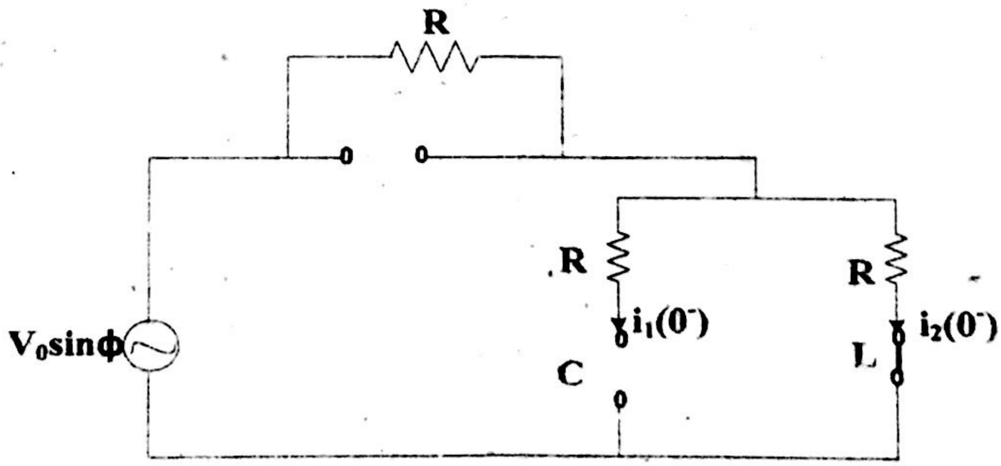
Or, $\frac{1}{C} * 0 - \frac{1}{C} * \frac{V_0}{R_1} + R2 * 0 + L\frac{d^2 i_2}{dt^2}(0^+) = 0$

$$\frac{d^2 i_2}{dt^2}(0^+) = \frac{V_0}{LCR1} \quad \text{Amp/sec}^2$$

Example 2.1: In the given circuit, switch K is closed at time t = 0, find $i_1(0^+)$, $i_2(0^+)$, $\frac{di_1}{dt}(0^+)$ and $\frac{di_2}{dt}(0^+)$



Solution: It's an energized condition, so at $t = 0^-$ inductor is replace with short circuit and capacitor with open circuit. Circuit at $t = 0^-$ is



From the circuit, $i_2(0^-) = i_2(0^+) = \frac{V_0 \sin}{2R}$

$$V_C(0^-) = V_C(0^-) = R * i_2(0^+) = \frac{V_0 \sin}{2}$$

Now, at $t = 0^+$ inductor is replace with current source and capacitor with voltage source.

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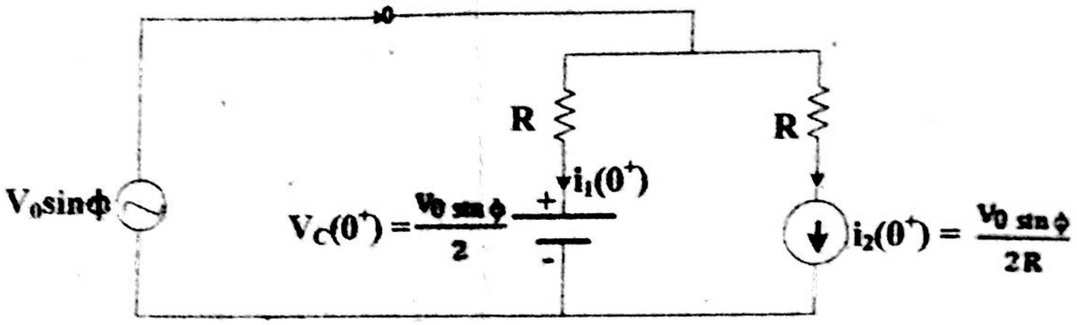
Circuit at $t = 0^+$ is

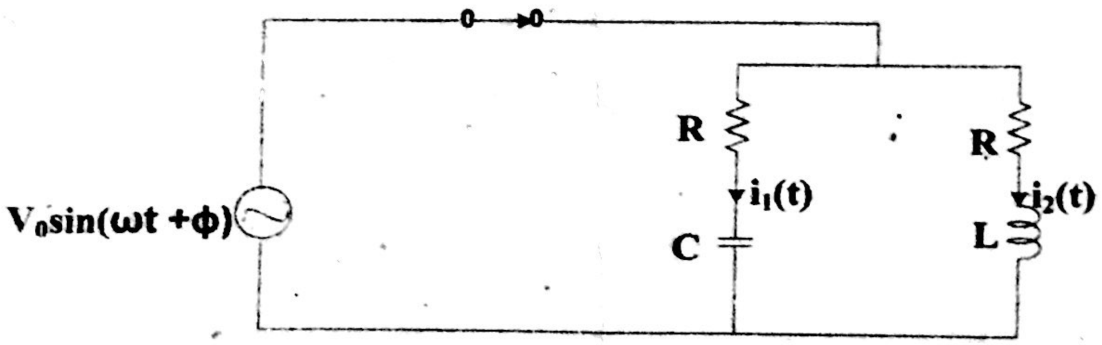


From the circuit,

$$i_1(0^+) = \frac{V_0 \sin - \frac{V_0 \sin}{2}}{R}$$

$$i_1(0^+) = \frac{V_0 \sin}{2R}$$

Now, to find $\frac{di_1}{dt}(0^+)$ and $\frac{di_2}{dt}(0^+)$, apply KVL at $t > 0$ in the circuit



From the circuit,

$$V_0\sin(\omega t + \phi) = V_R + V_C$$

Or, $V_0\sin(\omega t + \phi) = i_1 * R + \frac{1}{C}\int i_1 dt$

Differential

Or, $V_0 * \omega \cos(\omega t + \phi) = R\frac{di_1}{dt} + \frac{1}{C} i_1$

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Put $t = 0^+$

Or, $V_0 * \omega \cos(\omega * 0 + \phi) = R \frac{di_1}{dt}(0^+) + \frac{1}{C} i_1(0^+)$

Or, $V_0 * \omega \cos \phi = R \frac{di_1}{dt}(0^+) + \frac{1}{C} * \frac{V_0 \sin}{2R}$

$$\frac{di_1}{dt}(0^+) = \frac{V_0 * \omega \cos - \frac{V_0 \sin}{2CR}}{R} \text{ Amp/sec}$$

Again, from the circuit , (Outer loop)

$V_0 \sin(\omega t + \phi) = V_R + V_L$

Or, $V_0 \sin(\omega t + \phi) = R * i_2 + L \frac{di_2}{dt}$

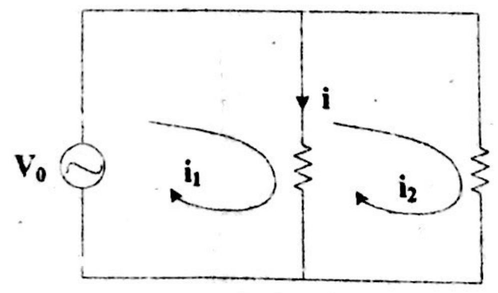
Put $t = 0^+$

Or, $V_0 \sin(\omega * 0 + \phi) = R * i_2(0^+) + L \frac{di_2}{dt}(0^+)$

Or, $V_0 \sin \phi = R * \frac{V_0 \sin}{2R} + L \frac{di_2}{dt}(0^+)$

$$\frac{di_2}{dt}(0^+) = \frac{V_0 \sin}{2L} \text{ Amp/sec}$$

Note: Remember there is a difference between loop current and branch current. They are not same current.



$i_1 - i_2 = i$

Here, $i_1 - i_2 = i$, as i_1 and i_2 are loop current where as 'i' is branch current.

The above Example 2.1 is of branch current.

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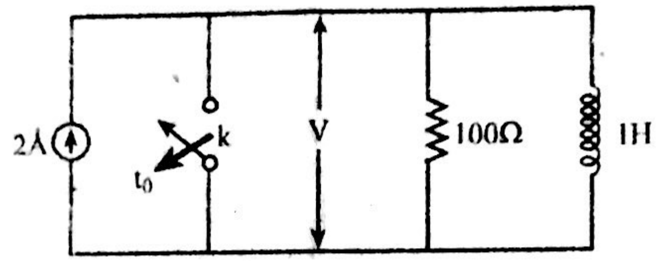
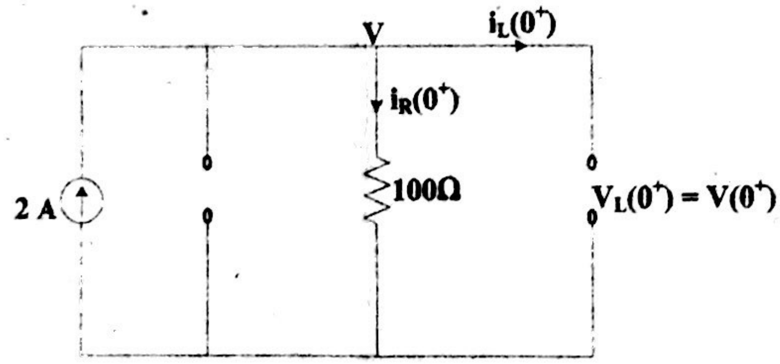
Example.2.2: In the given circuit, switch k is opened at t =0, find the value of V, $\frac{dv}{dt}$, $\frac{d^2v}{dt^2}$ at $t = 0^+$.



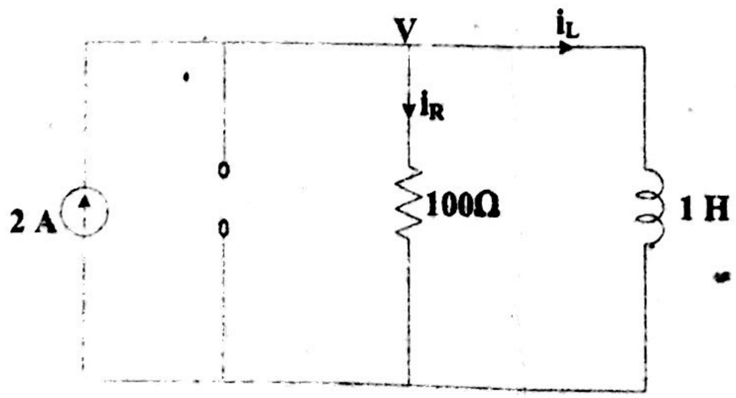
Solution:

It's a de-energized condition [since switch (ON at $t=0^-$) make short circuit and all current pass through it]. so at $t = 0^+$ inductor is replace with open circuit.



$$V(0^+) = 200 \text{ Volt}$$

Now to find $\frac{dv}{dt}$, $\frac{d^2v}{dt^2}$ at $t = 0^+$ take circuit diagram at $t > 0$ is as shown below



Applying KCL at $t > 0$

$2 = i_R + i_L$

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$$2 = \frac{V}{100} + \frac{1}{1}\int V\,dt$$

Different w.r.t 't'

$$0 = \frac{1}{100}\frac{dv(t)}{dt} + V(t) \quad \text{............ (i)}$$

Put $t = 0^+$

$$0 = \frac{1}{100}\frac{dv}{dt}(0^+) + V(0^+).$$

$$\frac{dv}{dt}(0^+) = -200\times 100$$

$$\frac{dv}{dt}(0^+) = -2\times 10^4 \text{ volt/sec}$$

Again from equation (i)

$$0 = \frac{1}{100}\frac{dv}{dt} + v$$

Different w.r.t 't'

$$0 = \frac{1}{100}\frac{d^2v}{dt^2} + \frac{dv}{dt}$$

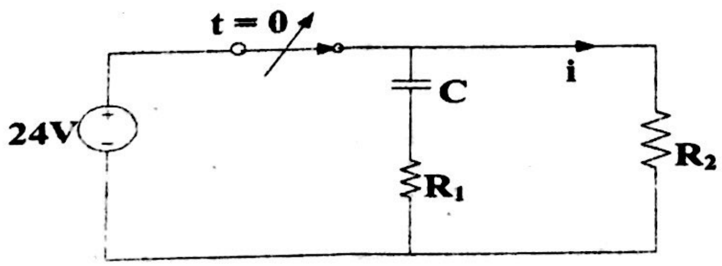
Put $t = 0^+$

$$\frac{d^2v}{dt^2}(0^+) = 2\times 10^6 \text{ volt/sec}^2$$

Note: Here student get confuse whether to use KVL at $t > 0$ 'OR' KCL at $t > 0$, it depends upon your own practice, But in my suggestion if there is pure parallel circuit along with current source like this example.2.2 use KCL at $t > 0$ and for other circuit use KVL at $t > 0$.

Problems:

Q.1: The circuit of figure below was under steady sate before the switch was opened at time t = 0. If $R_1 = 1\Omega$, $R_2 = 2\Omega$, C = 0.167F, determine $V_C(0^-)$, $V_C(0^+)$ and $i(0^+)$.



[Ans: $V_C(0^-)$=24V, $V_C(0^+)$=24, $i(0^+)$= 8A]

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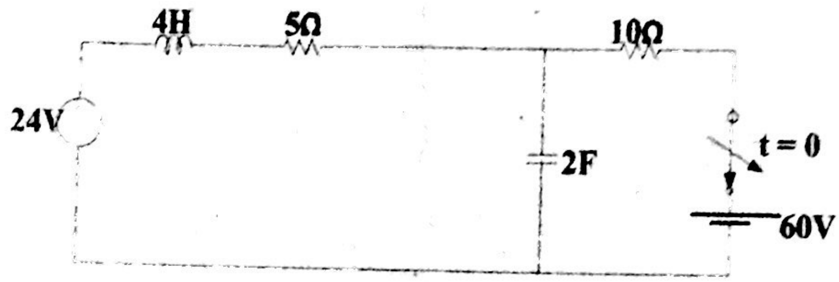
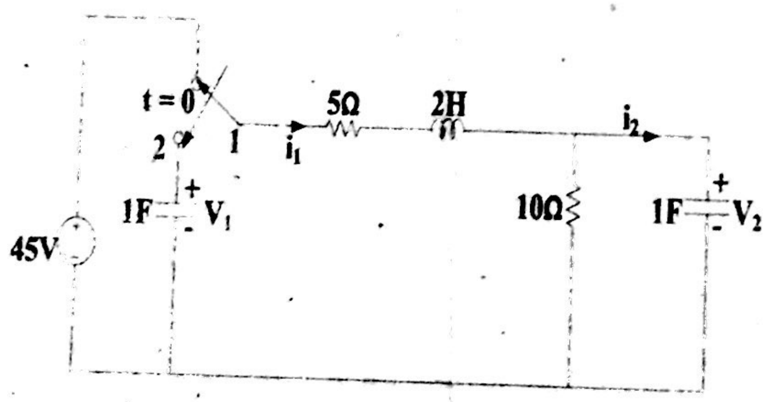
Q.2: The circuit of figure below was under steady state before the switch was opened at t = 0. Find the voltage and current for all four circuit elements at time $t = 0^+$



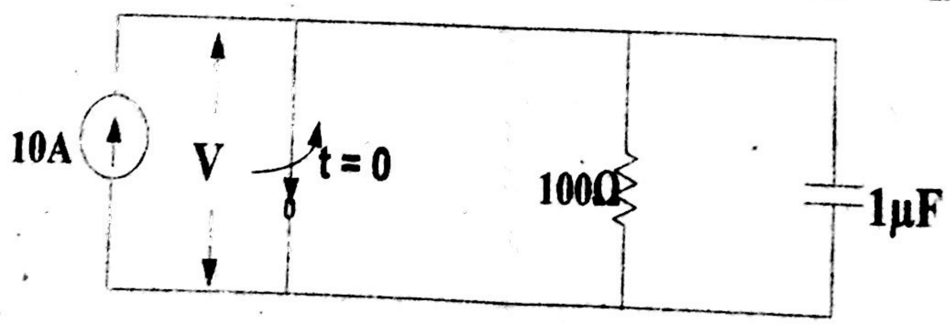
[Ans: Current: 3A,3A,3A,0A and Voltage:0V,15V,30V,0V]

Q.3: The switch was as position 1 under steady state. The switch was put to position 2 at t = 0. Find V_1, V_2, i_1 and i_2 at $t = 0^+$



[Ans: $V_1(0^+) = 0V$, $V_2(0^+) = 30V$, $i_1(0^+) = 3A$, $i_2(0^+) = 0A$]

Q.4: In the given circuit switch is opened at t = 0. Find the value of V, $\frac{dV}{dt}$ and $\frac{d^2V}{dt^2}$ at $t = 0^+$



[Ans: $V(0^+) = 0$Volt, $\frac{dV}{dt}(0^+) = 10^7$V/s and $\frac{d^2t}{dt^2}(0^+) = -10^{11}$ V/s^2]

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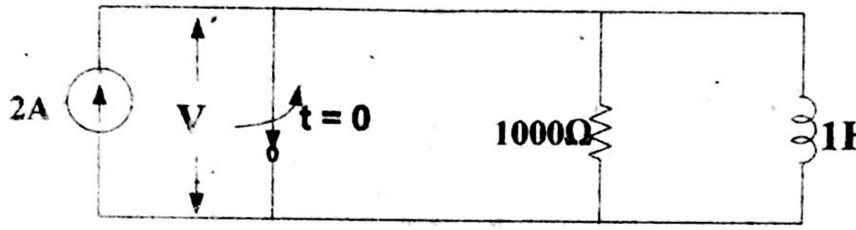
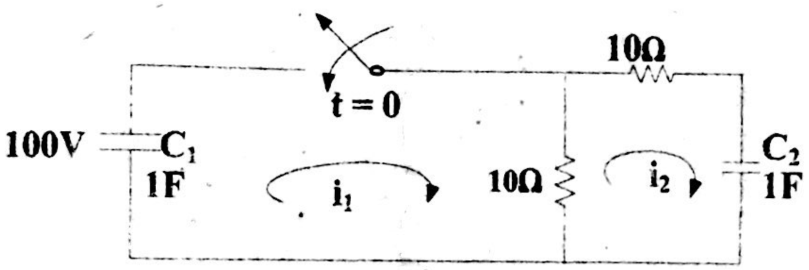
Q.5: In the given circuit, Switch is opened at t =0. Find the values of V, $\frac{dV}{dt}$ and $\frac{d^2V}{dt^2}$ at t = 0^+



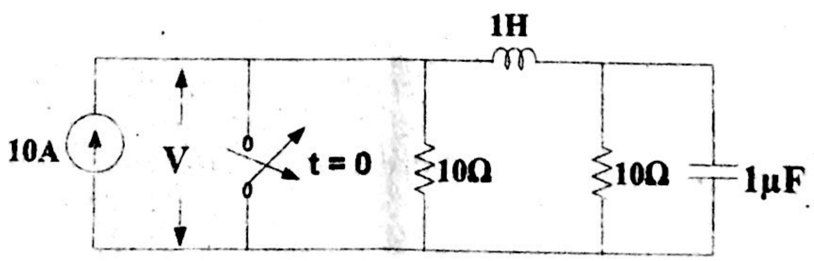
[Ans: $V(0^+)$= 2000Volt, $\frac{dV}{dt}(0^+)$= $-2*10^6$V/s and $\frac{d^2V}{dt^2}(0^+)$= $2*10^9$ V/s^2]

Q.6: In the given circuit, C_1 is charged to 100Volts in the polarity shown. Switch is closed at t =0. Find the value of i_1, i_2, $\frac{di_1}{dt}$, $\frac{d^2i_1}{dt^2}$, $\frac{di_2}{dt}$ and $\frac{d^2i_2}{dt^2}$ at t = 0^+



[Ans: $i_1(0^+)$= 20A, $i_2(0^+)$= 10A, $\frac{di_1}{dt}(0^+)$= −5 A/s, $\frac{d^2i_1}{dt^2}(0^+)$= 1.3 A/s^2, $\frac{di_2}{dt}(0^+)$= −3 A/s, $\frac{d^2i_2}{dt^2}(0^+)$= 0.8A/s^2]

Q.7: In the given circuit, Switch is opened at t= 0.Find the values of V_1,V_2, $\frac{dV_1}{dt}$, $\frac{dV_2}{dt}$ at t = 0^+



[Ans: $V_1(0^+)$= 100Volt, $V_2(0^+)$= 0Volt, $\frac{dV_1}{dt}(0^+)$= 1000 V/s, $\frac{dV_2}{dt}(0^+)$= 0 V/s]

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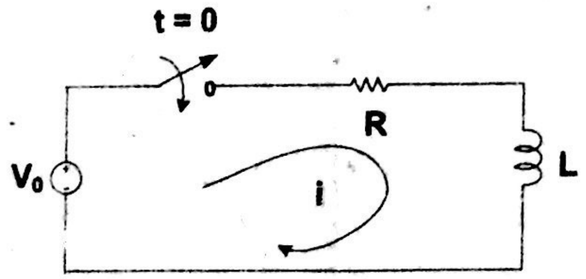
Transient Analysis By Direct Solution

Pdf by: Sachin Lamsal

 # Steps (procedure) to be applied for solving any circuit:

1. Recognize the circuit elements and remember the V-I relationship of the circuit element.
2. Apply KVL or KCL at $t > 0$ in the circuit.
3. After the application of KVL or KCL we will get the differential equation so solve that differential equation to obtain the value of response.

Example :(how to find out differential equation from the given circuit)



Applying KVL at $t > 0$

$$V_0 = V_R + V_L$$

$$\text{Or, } V_0 = i * R + L\frac{di}{dt}$$

$$\text{Or, } \frac{di}{dt} + \frac{R}{L} * i = \frac{V_0}{L} \quad \text{-------------- (i)}$$

Equation (i) is first order differential equation of **current.**

If question is asked to find out the first order differential equation of inductor voltage then,

Applying KVL at $t > 0$

$$V_0 = V_R + V_L$$

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$$Or, V_0 = i * R + V_L$$

$$Or, V_0 = \frac{1}{L}\int V_L\, dt * R + V_L \qquad [\text{since } i = i_L = \frac{1}{L}\int V_L\, dt\,]$$

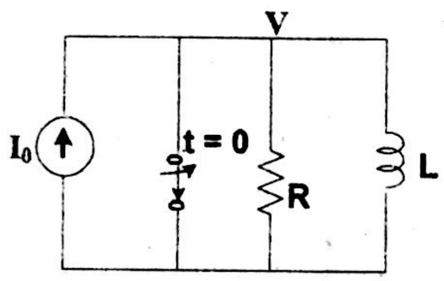
Differentiating

$$Or,\ 0 = \frac{R}{L} * V_L + \frac{dV_L}{dt}$$

$$Or,\ \frac{dV_L}{dt} + \frac{R}{L} * V_L = 0 \text{ ----------------- (ii)}$$

Equation (ii) is first order differential equation of **inductor voltage**

Example:



Applying KCL at t > 0

$$I_0 = i_R + i_L$$

$$Or, I_0 = \frac{V}{R} + \frac{1}{L}\int V\, dt$$

Differentiating

$$Or, 0 = \frac{1}{R} * \frac{dV}{dt} + \frac{1}{L} * V$$

$$Or,\ \frac{dV}{dt} + \frac{R}{L} * V = 0 \text{ ----------------- (iii)}$$

Equation (iii) is first order differential equation of **inductor voltage.**

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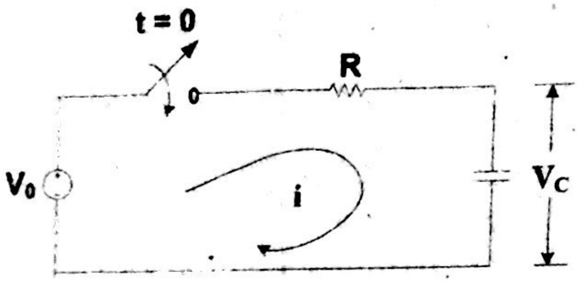
Example:



Applying KVL at $t > 0$

$V_0 = V_R + V_C$

Or, $V_0 = i * R + V_C$

Or, $V_0 = C\frac{dV_c}{dt} * R + V_C$ [since $i = i_C = C\frac{dV_c}{dt}$]

Or, $\frac{dV_c}{dt} + \frac{1}{CR} * V_C = \frac{V_0}{CR}$ ------------------ (iv)

Equation (iv) is first order differential equation of **voltage across capacitor.**

If question is asked to find out the first order differential equation of current then,

Applying KVL at $t > 0$

$V_0 = V_R + V_C$

Or, $V_0 = i * R + V_C$

Or, $V_0 = i * R + \frac{1}{C}\int i\,dt$

Differentiating

Or, $0 = R\frac{di}{dt} + \frac{1}{C} * i$

Or, $\frac{di}{dt} + \frac{1}{CR} * i = 0$ ————————— (v)

Equation (v) is first order differential equation of **current.**

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Also if question is asked to find out the first order differential equation of charge (q) then,

Applying KVL at t > 0

$$V_0 = V_R + V_C$$

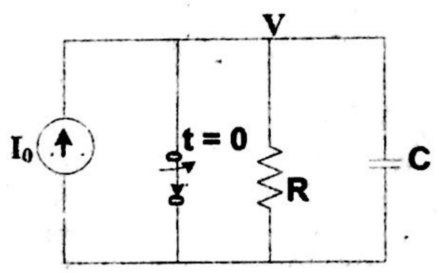
$$\text{Or, } V_0 = i * R + \frac{q}{C}$$

$$\text{Or, } V_0 = \frac{dq}{dt} * R + \frac{q}{C} \qquad [\text{since } i = \frac{dq}{dt}]$$

$$\text{Or, } \frac{dq}{dt} + \frac{1}{CR}q = \frac{V_0}{R} \quad \text{------------------- (vi)}$$

Equation (vi) is first order differential equation of **charge (q).**

Example:



Applying KCL at t > 0

$$I_0 = i_R + i_C$$

$$\text{Or, } I_0 = \frac{V}{R} + C\frac{dV}{dt}$$

$$\text{Or, } \frac{I_0}{C} = \frac{V}{CR} + \frac{dV}{dt}$$

$$\text{Or, } \frac{dV}{dt} + \frac{1}{CR} * V = \frac{I_0}{C} \quad \text{----------------- (vii)}$$

Equation (vii) is first order differential equation of **inductor voltage.**

Here, General first order differential equation is given as:

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$$\frac{dy}{dt} + P * y = Q$$

In general the time is always independent variable in circuit analysis while current through inductor. capacitor and voltage across inductor; capacitor is taken as dependent variable.

Differential Equations:

For study of any circuit transient analysis, it is necessary to be familiar with the mathematical concept of differential equations and the solution techniques. The order of the differential equation represents the highest derivative involved and is equal to the number of energy storing elements.

Type 1: (first order non-homogeneous differential equation):

$$\frac{dy}{dt} + P * y = Q$$

To obtain the solution of this differential equation, let us multiply both sides by e^{pt}

$$e^{pt} * \frac{dy}{dt} + P * y * e^{pt} = e^{pt} * Q$$

$$\frac{d(y * e^{pt})}{dt} = e^{pt} * Q$$

Integrating both sides w.r.t 't'

$$y * e^{pt} = \int e^{pt} * Q\, dt + K$$

$$y(t) = e^{-pt} * Q * \frac{e^{pt}}{p} + K e^{-pt}$$

$$y(t) = \frac{Q}{P} + K e^{-pt}$$

Where K is constant

Note: Remember that this solution is only for first order linear differential equation .In easy sense if **Q = constant value**, for example **Q = 100.** And for the **Q = 10 e^{-4t} or Q = 100sin300t** this solution is not valid.

This gives general solution of 1st order linear differential equation.

Type 2: (first order homogeneous differential equation):

$$\frac{dy(t)}{dt} + P * y(t) = 0$$

Thus solution of this differential equation is given as

$$y(t) = Ke^{-pt}$$

Where K is constant

Type 3: (second order homogeneous differential equation):

$$A\frac{d^2y(t)}{dt^2} + B\frac{dy(t)}{dt} + C * y(t) = 0$$

The general solution of the above second order differential equation is

$$y(t) = K_1e^{S_1t} + K_2 e^{S_2t}$$

Where K_1 and K_2 are constants.

And, S1 and S2 are the roots of the quadratic equation

$AS^2 + BS + C = 0$, and are given by

$$S_1, S_2 = -\frac{B}{2A} \pm \frac{1}{2A}\sqrt{B^2 - 4AC}$$

Case I: If roots are real and $S_1 \neq S_2$ then the solution is given as

$$y(t) = K_1e^{S_1t} + K_2 e^{S_2t}$$

Case II: If roots are real and $S_1 = S_2$ then the solution is given as

$$y(t) = (K_1 + K_2t)\, e^{St}$$

Case III: If roots are complex then the solution is given as

$$S_1, S_2 = -\alpha \pm j\,\omega_d$$

$$y(t) = e^{-\alpha t}[K_1 \cos\omega_d t + K_2 \sin\omega_d t]$$

Type 4: (second order non-homogeneous differential equation):

$$A\frac{d^2y(t)}{dt^2} + B\frac{dy(t)}{dt} + C * y(t) = Q \text{ ———— } [1]$$

The general solution of the above second order differential equation is

$$y(t) = \text{Natural response} + \text{Forced response}$$

$$y(t) = y_N(t) + y_F(t)$$

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To find $y_N(t)$ put $Q = 0$ in equation [1], thus

$$A\frac{d^2y(t)}{dt^2} + B\frac{dy(t)}{dt} + C * y(t) = 0$$

And solve according as homogeneous differential equation solution

And to find $y_F(t)$ the method of undetermined coefficient is used, in this method the trial solution for $y_F(t)$ is selected on the basis of source in the circuit as given in the following table:

Source or Excitation	Trial solution for $y_F(t)$
a) DC Source (V_0)	$y_F(t) = A$
b) Exponential Source ($V_0e^{-\alpha t}$)	$y_F(t) = Ae^{-\alpha t}$ 'or' $y_F(t) = Ate^{-\alpha t}$
c) V_m sinωt or V_m cosωt	$y_F(t) = A\cos\omega t + B\sin\omega t$

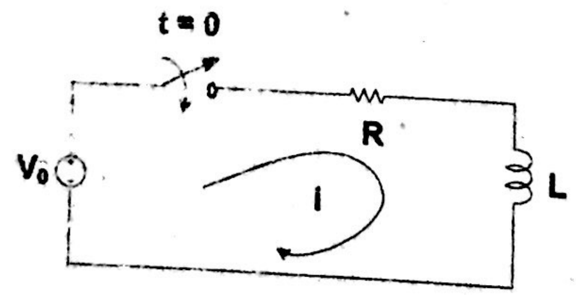
#Procedure to find out the value of undetermined coefficient of $y_F(t)$

- Select the trial solution for $y_F(t)$ on the basis of source as given in the above table.
- Replace the dependent variable of differential equation by the trial solution of $y_F(t)$ and solve for undetermined coefficient.

 # Response of R-L Circuit with:

1. **DC excitation**

Find the expression of current i (t) using classical method



Applying KVL at t > 0

$V_0 = V_R + V_L$

$$Or, V_0 = i * R + L\frac{di}{dt}$$

$$Or, \frac{di}{dt} + \frac{R}{L} * i = \frac{V_0}{L}$$

This is first order differential equation, so the solution is

$$i(t) = \frac{Q}{P} + K e^{-pt}$$

Where, $Q = \frac{V_0}{L}$ and $P = \frac{R}{L}$

$$i(t) = \frac{V_0}{R} + K e^{-\frac{R}{L}t} \text{ ------------------(i)}$$

Now to find the value of K, put $t = 0^+$ in above equation

$$i(0^+) = \frac{V_0}{R} + K e^{-\frac{R}{L}*0}$$

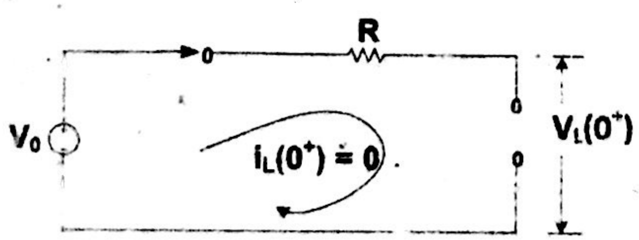
$$K = i(0^+) - \frac{V_0}{R} \text{ ------------- (a)}$$

Now to find $i(0^-)$ use initial condition, the given circuit is de-energized circuit so $i(0^-) = 0$

From continuity relation of inductor

$i_L(0^-) = i_L(0^+) = 0$

Circuit at $t = 0^+$ is



$i(0^+) = 0$

From equation (a) $K = -\frac{V_0}{R}$, so the equation (i) becomes as

$$i(t) = \frac{V_0}{R} - \frac{V_0}{R} e^{-\frac{R}{L}t}$$

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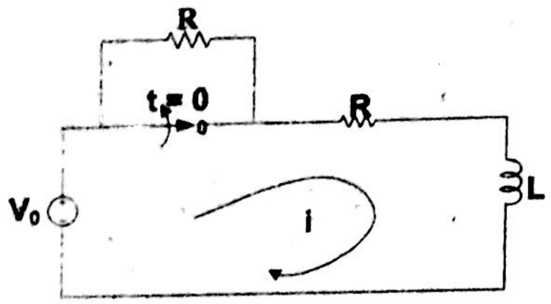
$$[\ i(t) = \frac{V_0}{R}(1 - e^{-\frac{R}{L}t})\ \text{amp}\ ]$$

Find the expression of voltage across inductor $V_L(t)$ using classical method



Applying KVL at t > 0

$V_0 = 2V_R + V_L$

Or, $V_0 = i * 2R + V_L$

Or, $V_0 = \frac{1}{L}\int V_L\, dt * 2R + V_L$ [since $i = i_L = \frac{1}{L}\int V_L\, dt$]

Differentiating

Or, $0 = \frac{2R}{L} * V_L + \frac{dV_L}{dt}$

Or, $\frac{dV_L}{dt} + \frac{2R}{L} * V_L = 0$

This is first order differential equation, so the solution is

$V_L(t) = \frac{Q}{P} + K e^{-pt}$

Where, $Q = 0$ and $P = \frac{2R}{L}$

$V_L(t) = K e^{-\frac{2R}{L}t}$ ------------------ (i)

Now to find the value of K, put $t = 0^+$ in above equation

$V_L(0^+) = K e^{-\frac{2R}{L}*0}$

$K = V_L(0^+)$ ------------ (a)

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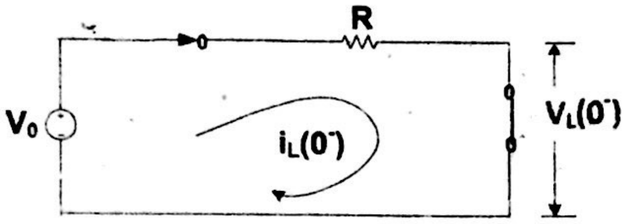
Now to find $V_L(0^+)$ use initial condition, the given circuit is energized so circuit at $t = 0^-$

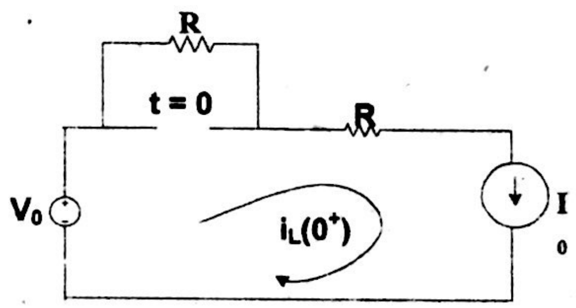


From the circuit $i_L(0^-) = \frac{V_0}{R}$

And $V_L(0^-) = 0$

- From continuity relation for inductor, $i_L(0^-) = i_L(0^+) = \frac{V_0}{R} = I_0$

Now, circuit at $t = 0^+$ is



From the circuit $i_L(0^+) = \frac{V_0}{R}$

And $V_L(0^+) = V_0 - i_L(0^+) * 2R$

Or, $V_L(0^+) = V_0 - \frac{V_0}{R} * 2R$

Or, $V_L(0^+) = -V_0$

From equation (a) $K = -V_0$, so the equation (i) becomes as

$$[V_L(t) = -V_0\, e^{-\frac{2R}{L}t} \text{ Volt}]$$

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'Alternative way'

Applying KVL at $t > 0$

$V_0 = 2V_R + V_L$

Or, $V_0 = i * 2R + L\frac{di}{dt}$

Or, $\frac{di}{dt} + \frac{2R}{L} * i = \frac{V_0}{L}$

This is first order differential equation, so the solution is

$i(t) = \frac{Q}{P} + K e^{-pt}$

Where, $Q = \frac{V_0}{L}$ and $P = \frac{2R}{L}$

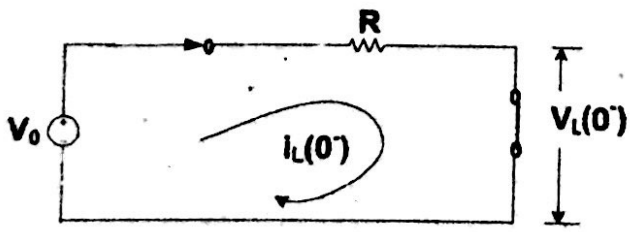
$i(t) = \frac{V_0}{2R} + K e^{-\frac{2R}{L}t}$ ----------------(i)

Now to find the value of K, put $t = 0^+$ in above equation

$i(0^+) = \frac{V_0}{2R} + K e^{-\frac{2R}{L}*0}$

$K = i(0^+) - \frac{V_0}{2R}$ ------------ (a)

Now to find $i(0^+)$ use initial condition, the given circuit is energized so circuit at $t = 0^-$



From the circuit $i_L(0^-) = \frac{V_0}{R}$

And $V_L(0^-) = 0$

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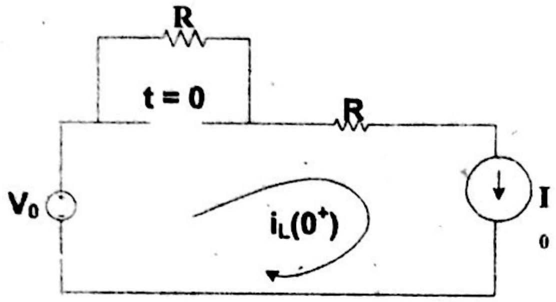
From continuity relation for inductor, $i_L(0^-) = i_L(0^+) = \frac{V_0}{R} = I_0$

Now, circuit at $t = 0^+$ is



From the circuit $i_L(0^+) = \frac{V_0}{R}$

From equation (a) $K = \frac{V_0}{R} - \frac{V_0}{2R} = \frac{V_0}{2R}$, so the equation (i) becomes as

$$i(t) = \frac{V_0}{R} - \frac{V_0}{2R} e^{-\frac{2R}{L}t} \text{ Amp}$$

But we have to find the $V_L(t)$ so we know

$$V_L(t) = L\frac{di}{dt}$$

$$V_L(t) = L * \frac{d[\frac{V_0}{R} - \frac{V_0}{2R} e^{-\frac{2R}{L}t}]}{dt}$$

$$V_L(t) = L * (0 - \frac{V_0}{2R} * \frac{2R}{L} e^{-\frac{2R}{L}t})$$

$$[\, V_L(t) = -V_0\, e^{-\frac{2R}{L}t} \text{ volt} \,]$$

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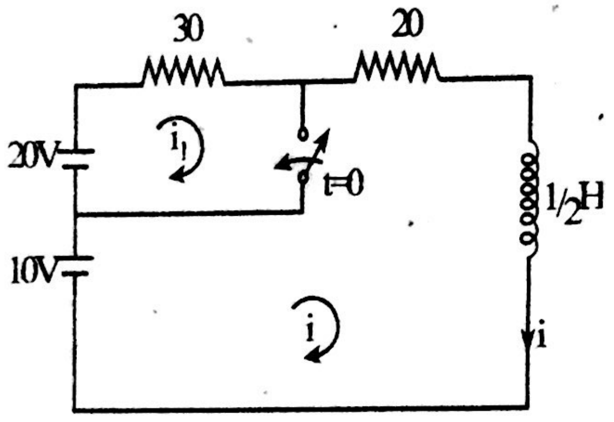
Q.1: In the circuit shown switch is opened for a long time & then it is suddenly closed at t = 0. Obtain the expression for current through inductor for t > 0. Also calculate the voltage across inductor after 10 ms. Use classical method.



Solution:

Apply KVL in upper loop:

$20 = 30\, i_1$

$\therefore i_1 = \frac{2}{3}$ .....(1)

Apply KVL in lower loop:

$$10 = 20i + \frac{1}{2}\frac{di}{dt}$$

Or, $\frac{di}{dt} + 40i = 20$ .....(2)

$$\therefore i = \frac{20}{40} + Ke^{-40t}$$

Or, $i = \frac{1}{2} + Ke^{-40t}$ ......(3)

Put $t = 0^+$

$i(0^+) = \frac{1}{2} + K$

Or, $K = i(0^+) - \frac{1}{2}$ ......(4)

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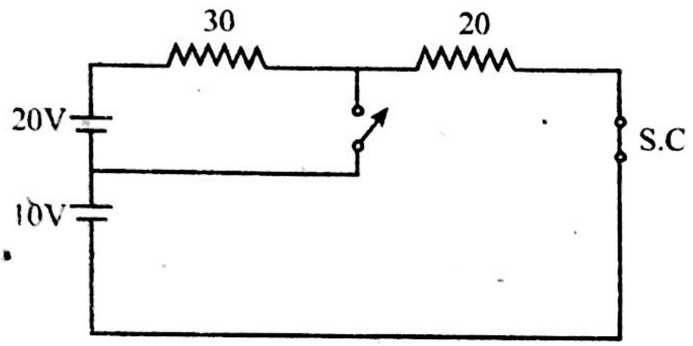

at $t=0^-$

$$i(0^-) = \frac{10+20}{30+20} = \frac{30}{50} = \frac{3}{5}$$

$$\therefore\ i(0^+) = i(0^-) = \frac{3}{5} \ \ldots\ldots (5)$$

From (4) & (5)

$$K = \frac{3}{5} - \frac{1}{2}$$

$$\therefore\ K = \frac{6-5}{10} = \frac{1}{10} \ \ldots..(6)$$

From (6) & (3)

$$i = \frac{1}{2} + \frac{1}{10}e^{-40t} \ \ldots..(7).$$

$$V_L = \frac{1}{2}\frac{di}{dt}$$

$$\text{Now, } V_L = \frac{1}{2}\frac{d}{dt}\left[\frac{1}{2} + \frac{1}{10}e^{-40t}\right]$$

$$\text{Or, } V_L = \frac{1}{2}\left[0 + \frac{1}{10}(-40)\,e^{-40t}\right]$$

$$\mathbf{V_L = -2e^{-40t}}$$

$$\therefore V_L\,(10\text{ ms}) = -2\,e^{-40\times 10\times 10^{-3}}$$

$$= -2\,e^{-0.4}$$

$$\mathbf{= -1.34\ Volt}$$

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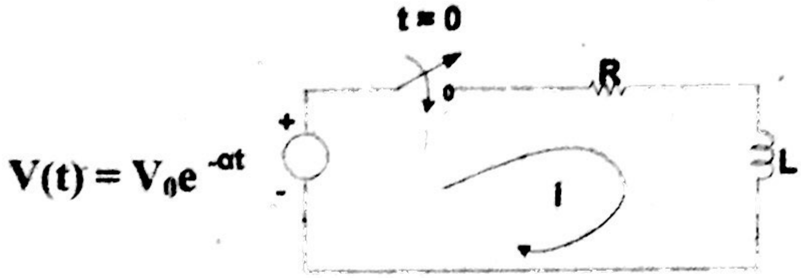
2. Exponential Excitation:

Find the expression of current i (t) using classical method



Applying KVL at $t > 0$

$$V_0 e^{-\alpha t} = V_R + V_L$$

$$\text{Or, } V_0 e^{-\alpha t} = i * R + L\frac{di}{dt}$$

$$\text{Or, } \frac{di}{dt} + \frac{R}{L} * i = \frac{V_0 e^{-\alpha t}}{L} \text{ ---------------- (1)}$$

This is first order non-homogenous differential equation, so the solution is

$$i(t) = i_N(t) + i_F(t) \text{ ----------------(2)}$$

Now to find $i_N(t)$, the auxiliary equation of (1) is

$$\frac{di}{dt} + \frac{R}{L} * i = 0$$

$$i_N(t) = K e^{-\frac{R}{L}t} \text{ ---------------- (3)}$$

Now to find $i_F(t)$, take a trial solution according to source present.

Case I **If $\alpha \neq \frac{R}{L}$, then the trail solution is taken as**

$$i_F(t) = Ae^{-\alpha t} \text{ --------------- (4)}$$

Now, replacing the value of dependent variable of equation (1) by the value of trial solution of $i_F(t)$, then

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$$\frac{d[Ae^{-\alpha t}]}{dt} + \frac{R}{L} * [Ae^{-\alpha t}] = \frac{V_0\ e^{-\alpha t}}{L}$$

$$\text{Or,} -A\ \alpha e^{-\alpha t} + \frac{R}{L} * [Ae^{-\alpha t}] = \frac{V_0\ e^{-\alpha t}}{L}$$

$$\text{Or,} A[-\alpha + \frac{R}{L}] = \frac{V_0}{L}$$

$$\text{Or,} A = \frac{V_0/L}{\frac{R}{L} - \alpha} \text{ ------------------- (5)}$$

Thus equation (4) becomes as,

$$i_F(t) = \frac{V_0/L}{\frac{R}{L} - \alpha} * e^{-\alpha t} \text{ ------------------- (6)}$$

Hence from the equation (2),(3) and (6), the complete solution is

$$i\ (t) = i_N(t) + i_F(t)$$

$$\text{Or,} i\ (t) = K\ e^{-\frac{R}{L}t} + \frac{V_0/L}{\frac{R}{L} - \alpha} * e^{-\alpha t} \text{ ------------- (7)}$$

Now to find the value of K, put $t = 0^+$ in above equation

$$i\ (0^+) = K\ e^{-\frac{R}{L}*0} + \frac{V_0/L}{\frac{R}{L} - \alpha} * e^{-\alpha * 0}$$

$$\text{Or,} i\ (0^+) = K + \frac{V_0/L}{\frac{R}{L} - \alpha}$$

$$\text{Or,} K = i\ (0^+) - \frac{V_0/L}{\frac{R}{L} - \alpha} \text{ --------------- (8)}$$

Now to find $i\ (0^+)$ use initial condition, the given circuit is de-energized $i_L(0^-) = 0$

From continuity relation for inductor, $i_L(0^-) = i_L(0^+) = 0$

From equation (8) $K = -\frac{V_0/L}{\frac{R}{L} - \alpha}$, so the equation (7) becomes as

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$$i(t) = -\frac{V_0/L}{\frac{R}{L}-\alpha}\; e^{-\frac{R}{L}t} + \frac{V_0/L}{\frac{R}{L}-\alpha} * e^{-\alpha t} \text{ Amp} \text{ ---------------- (9)}$$

Case II **If $\alpha = \frac{R}{L}$, then the trail solution is taken as**

$$i_F(t) = Ate^{-\alpha t} \text{ ------------------ (10)}$$

Now, replacing the value of dependent variable of equation (1) by the value of trial solution of $i_F(t)$, then

$$\frac{d[Ate^{-\alpha t}]}{dt} + \frac{R}{L} * [Ate^{-\alpha t}] = \frac{V_0\, e^{-\alpha t}}{L}$$

$$\text{Or, } A[e^{-\alpha t} - \alpha te^{-\alpha t}] + \frac{R}{L} * [Ate^{-\alpha t}] = \frac{V_0\, e^{-\alpha t}}{L}$$

$$\text{Or, } A[1-\alpha t] + \frac{R}{L} * [At] = \frac{V_0}{L}$$

$$\text{Or, } A = \frac{V_0/L}{\frac{2R}{L}t - \alpha t + 1}$$

Since we have $\alpha = \frac{R}{L}$

$$\text{Or, } A = \frac{V_0}{L} \text{ ------------------- (11)}$$

Thus equation (4) becomes as,

$$i_F(t) = \frac{V_0}{L}t * e^{-\alpha t} \text{ ------------------- (12)}$$

Hence from the equation (2),(3) and (12), the complete solution is

$$i(t) = i_N(t) + i_F(t)$$

$$\text{Or, } i(t) = K\, e^{-\frac{R}{L}t} + \frac{V_0}{L}t * e^{-\alpha t} \text{ ------------ (13)}$$

Now to find the value of K, put $t = 0^+$ in above equation

$$i(0^+) = K\, e^{-\frac{R}{L}*0} + \frac{V_0}{L} * 0 * e^{-\alpha * 0}$$

$$\text{Or, } i(0^+) = K$$

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$$Or, K = i(0^+) \text{ -------------- } (14)$$

Now to find $i(0^+)$ use initial condition, the given circuit is de-energized $i_L(0^-) = 0$

From continuity relation for inductor, $i_L(0^-) = i_L(0^+) = 0$

From equation (14) K = 0, so the equation (13) becomes as

$$i(t) = \frac{V_0}{L} t * e^{-\alpha t} \text{ Amp} \text{ -------------- } (9)$$

Q.2: An exponential source $V(t) = 10e^{-4t}$ is suddenly applied at time t = 0 to a series R-L circuit comprising R = 1 Ω and L = 0.01 H. Obtain the expression for the current i(t) in the circuit using classical method.

Solution:

Applying KVL at t > 0

$$10e^{-4t} = V_R + V_L$$

$$Or, 10\, e^{-4t} = i * 1 + 0.01 \frac{di}{dt}$$

$$Or, \frac{di}{dt} + \frac{1}{0.01} * i = \frac{10e^{-4t}}{0.01}$$

$$Or, \frac{di}{dt} + 100 * i = 1000\, e^{-4t} \text{ ---------------- } (1)$$

This is first order non-homogenous differential equation, so the solution is

$$i(t) = i_N(t) + i_F(t) \text{ -----------------} (2)$$

Now to find $i_N(t)$, the auxiliary equation of (1) is

$$\frac{di}{dt} + 100 * i = 0$$

$$i_N(t) = K\, e^{-100t} \text{ ---------------- } (3)$$

Now to find $i_F(t)$, take a trial solution according to source present,

$$\text{Here } \alpha = 4 \text{ and } \frac{R}{L} = 100, \text{ so condition is } \alpha \neq \frac{R}{L}$$

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$$i_f(t) = Ae^{-4t} \text{ ------------------ } (4)$$

Now, replacing the value of dependent variable of equation (1) by the value of trial solution of $i_f(t)$, then

$$\frac{d[Ae^{-4t}]}{dt} + 100 * [Ae^{-4t}] = 1000\ e^{-4t}$$

$$\text{Or, } -A\,4\,e^{-4t} + 100 * [Ae^{-4t}] = 1000\ e^{-4t}$$

$$\text{Or, } A\,[-4 + 100] = 1000$$

$$\text{Or, } A = 10.41 \text{ ------------------ } (5)$$

Thus equation (4) becomes as,

$$i_f(t) = 10.41 * e^{-4t} \text{ ------------------ } (6)$$

Hence from the equation (2),(3) and (6), the complete solution is

$$i(t) = i_n(t) + i_f(t)$$

$$\text{Or, } i(t) = K\,e^{-100t} + 10.41 * e^{-4t} \text{ ------------ } (7)$$

Now to find the value of K, put $t = 0^+$ in above equation

$$i(0^+) = K\,e^{-100*0} + 10.41 * e^{-4*0}$$

$$\text{Or, } i(0^+) = K + 10.41$$

$$\text{Or, } K = i(0^+) - 10.41 \text{ ------------ } (8)$$

Now to find $i(0^+)$ use initial condition, the given circuit is de-energized $i_L(0^-) = 0$

From continuity relation for inductor, $i_L(0^-) = i_L(0^+) = 0$

From equation (8) K = - 10.41, so the equation (7) becomes as

$$\mathbf{i(t) = -10.41\,e^{-100t} + 10.41 * e^{-4t}\ Amp} \text{ ------------ } (9)$$

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Q.3: An exponential source $V(t) = 100e^{-4t}$ is suddenly applied at time $t = 0$ to a series R-L circuit comprising $R = 1\ \Omega$ and $L = 0.25$ H. Obtain the particular solution for the current i(t) in the circuit using classical method.

Solution:

Applying KVL for solving at $t > 0$

$$100e^{-4t} = V_R + V_L$$

$$\text{Or, } 100\, e^{-4t} = i * 1 + 0.25 \frac{di}{dt}$$

$$\text{Or, } \frac{di}{dt} + 4 * i = \frac{100e^{-4t}}{0.25}$$

$$\text{Or, } \frac{di}{dt} + 4 * i = 400\, e^{-4t} \text{ ----------------- (1)}$$

This is first order non-homogenous differential equation, so the solution is

$$i(t) = i_N(t) + i_F(t) \text{ -----------------(2)}$$

Now to find $i_N(t)$, the auxiliary equation of (1) is

$$\frac{di}{dt} + 4 * i = 0$$

$$i_N(t) = K\, e^{-4t} \text{ ----------------- (3)}$$

Now to find $i_F(t)$, take a trial solution according to source present,

Here $\alpha = 4$ and $\frac{R}{L} = 4$, so condition is $\alpha = \frac{R}{L}$

$$i_F(t) = Ate^{-4t} \text{ ----------------- (4)}$$

Now, replacing the value of dependent variable of equation (1) by the value of trial solution of $i_F(t)$, then

$$\frac{d[Ate^{-4t}]}{dt} + 4 * [Ate^{-4t}] = 400\, e^{-4t}$$

$$\text{Or, } -At\, 4\, e^{-4t} + Ae^{-4t} + 4\, A\, t\, e^{-4t} = 400\, e^{-4t}$$

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Or,A [-4t + 1 + 4t] = 400

Or,A = 400 ------------------- (5)

Thus equation (4) becomes as,

$i_F(t) = 400t * e^{-4t}$ ------------------- (6)

Hence from the equation (2),(3) and (6), the complete solution is

$i(t) = i_N(t) + i_F(t)$

Or, $i(t) = K e^{-4t} + 400t * e^{-4t}$ ------------- (7)

Now to find the value of K, put $t = 0^+$ in above equation

$i(0^+) = K e^{-4*0} + 400t * e^{-4*0}$

Or, $i(0^+) = K + 0$

Or, $K = i(0^+)$ -------------- (8)

Now to find $i(0^+)$ use initial condition, the given circuit is de-energized $i_L(0^-) = 0$

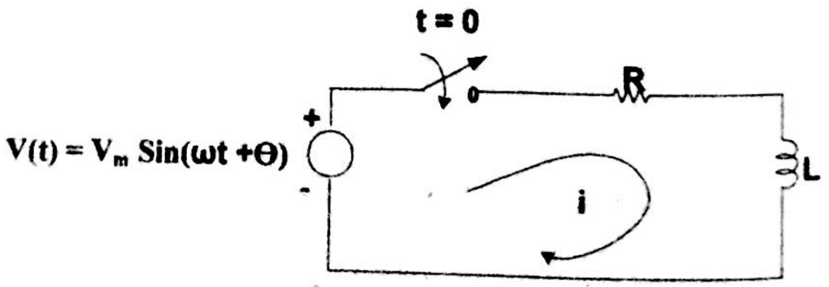
From continuity relation for inductor, $i_L(0^-) = i_L(0^+) = 0$

From equation (8) K = 0, so the equation (7) becomes as

$\mathbf{i(t) = 400t * e^{-4t}}$ **Amp** -------------- (9)

3. **Sinusoidal Excitation:**

Find the expression of current i (t) using classical method



Applying KVL at t > 0

$V_m \text{Sin}(\omega t + \Theta) = V_R + V_L$

$\text{Or}, V_m \text{Sin}(\omega t + \Theta) = i * R + L\frac{di}{dt}$

$\text{Or}, \frac{di}{dt} + \frac{R}{L} * i = \frac{V_m}{L} \text{Sin}(\omega t + \Theta)$ ------------------ (1)

This is first order non-homogenous differential equation, so the solution is

$i(t) = i_N(t) + i_F(t)$ ------------------(2)

Now to find $i_N(t)$, the auxiliary equation of (1) is

$\frac{di}{dt} + \frac{R}{L} * i = 0$

$i_N(t) = K e^{-\frac{R}{L}t}$ ------------------ (3)

Now to find $i_F(t)$, take a trial solution according to source present,

$i_F(t) = A \text{Cos}(\omega t + \Theta) + B \text{Sin}(\omega t + \Theta)$ ------------------ (4)

Now, replacing the value of dependent variable of equation (1) by the value of trial solution of $i_F(t)$, then

$\frac{d[A \text{Cos}(\omega t +) + B \text{Sin}(\omega t +)]}{dt} + \frac{R}{L} * [A \text{Cos}(\omega t + \Theta) + B \text{Sin}(\omega t + \Theta)] = \frac{V_m}{L} \text{Sin}(\omega t + \Theta)$

$\text{Or}, -A\omega \text{Sin}(\omega t + \Theta) + B\omega \text{Cos}(\omega t + \Theta) + \frac{R}{L} * [A \text{Cos}(\omega t + \Theta) + B \text{Sin}(\omega t + \Theta)] = \frac{V_m}{L} \text{Sin}(\omega t + \Theta)$

Equating the coefficient of Cosine and Sine, we get

$B\omega + \frac{R}{L} * A = 0$ ------------- (a)

$-A\omega + \frac{R}{L} * B = \frac{V_m}{L}$ ------------ (b)

Solving equation (a) and (b)

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$$A = -V_m * \frac{L\omega}{L^2\omega^2+R^2}$$

$$B = V_m * \frac{R}{L^2\omega^2+R^2}$$

Thus equation (4) becomes

$$i_F(t) = -V_m * \frac{L\omega}{L^2\omega^2+R^2} Cos\,(\omega t + \Theta) + V_m * \frac{R}{L^2\omega^2+R^2} Sin\,(\omega t + \Theta) \text{------}$$

Hence from the equation (2),(3) and (6), the complete solution is

$$i\,(t) = i_N(t) + i\,(t)$$

Or,
$$i\,(t) = K\,e^{-\frac{R}{L}t} - V_m * \frac{L\omega}{L^2\omega^2+R^2} Cos\,(\omega t + \Theta) + V_m * \frac{R}{L^2\omega^2+R^2} Sin\,(\omega t + \Theta) \text{------}$$

Now to find the value of K, put $t = 0^+$ in above equation

$$i\,(0^+) = K\,e^{-\frac{R}{L}*0} - V_m * \frac{L\omega}{L^2\omega^2+R^2} Cos\,(\omega*0 + \Theta) + V_m * \frac{R}{L^2\omega^2+R^2} Sin\,(\omega*0 + \Theta)$$

$$\text{Or, } i\,(0^+) = K - V_m * \frac{L\omega}{L^2\omega^2+R^2} Cos\,(\Theta) + V_m * \frac{R}{L^2\omega^2+R^2} Sin\,(\Theta)$$

Or,
$$K = i\,(0^+) + Vm * \frac{L\omega}{L^2\omega^2+R^2} Cos\,() - Vm * \frac{R}{L^2\omega^2+R^2} Sin\,() \text{-----} (8)$$

Now to find $i\,(0^+)$ use initial condition, the given circuit is de-energized $i_L(0^-) = 0$

From continuity relation for inductor, $i_L(0^-) = i_L(0^+) = 0$

From equation (8) $K = Vm * \frac{L\omega}{L^2\omega^2+R^2} Cos\,() - Vm * \frac{R}{L^2\omega^2+R^2} Sin\,()$, so the equation (becomes

$$i\,(t) = \Big(Vm * \frac{L\omega}{L^2\omega^2+R^2} Cos\,() - Vm * \frac{R}{L^2\omega^2+R^2} Sin\,(\,)\,e^{-\frac{R}{L}t} - V_m * \frac{L\omega}{L^2\omega^2+R^2} Cos\,(\omega t + \Theta) + V_m * \frac{R}{L^2\omega^2+R^2} Sin\,(\omega t + \Theta)\Big) \quad \textbf{Amp}$$

Q.4: Determine the complete Solution for the current when the switch k is closed at t= 0 in the circuit shown in figure below using classical approach.

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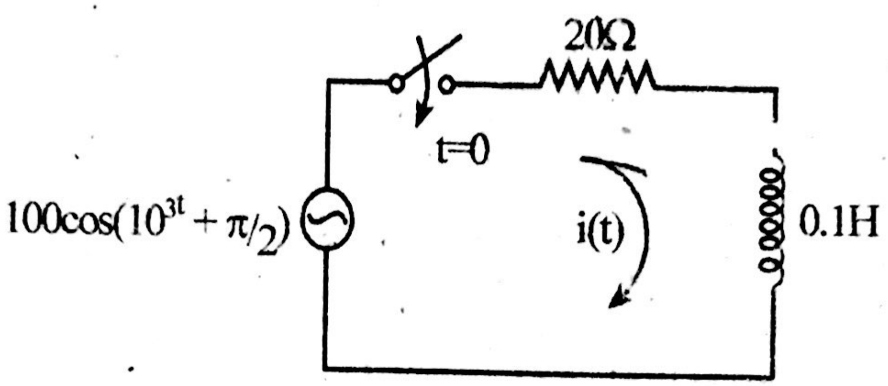

Solution:

Applying KVL at t > 0

$$100\cos(10^3t+\pi/2) = i \times 20 + 0.1\frac{di}{dt}$$

$$\frac{di}{dt} + 200i = 1000\cos(10^3t+\pi/2) \text{ .... (i)}$$

Now, the complete Solution of equation is

$$i(t) = i_N + i_F$$

and to find i_N make $1000\cos(10^3t+\frac{\pi}{2}) = Q = 0$ in eq^n......(i)

$$\frac{di}{dt} + 200i = 0$$

$$i_N = k\,e^{-200t}$$

Also, to find i_F, taking trial Solution

$$i_F = A\cos(10^3t+\pi/2) + B\sin(10^3t+\pi/2)$$

From equation (i)

Or $\frac{d}{dt}[A\cos(10^3t+\pi/2) + B\sin(10^3t+\pi/2)] + 200[A\cos(10^3t+\pi/2) + B\sin(10^3t+\pi/2)]$

$= 1000\cos(10^3t+\pi/2)$

Or $-10^3 A\sin(10^3t+\pi/2) + 10^3 B\cos(10^3t+\pi/2) + 200A\cos(10^3t+\pi/2) + 200B\sin(10^3t+\pi/2) = 1000\cos(10^3t+\pi/2)$

Comparing $\sin(10^3t+\pi/2)$ and $\cos(10^3t+\pi/2)$ on both sides we get

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$0 = -10^3 A + 200B$

$1000 = 10^3 B + 200A$

Solving above equation

$A = 0.1923$

$B = 0.9615$

And, $X = \sqrt{A^2 + B^2} = 0.9805$

$\phi = \tan^{-1}\left(\frac{A}{B}\right) = 11.30°$

Thus,

$i_F = X \sin\ (10^3 t + \pi/2 + \phi)$

$i_F = 0.9805 \sin\ (10^3 t + 90° + 11.30°)$

$i_F = 0.98 \sin (10^3 t + 101.30°)$

Solution is $i(t) = i_N + i_F$

$i(t) = k\, e^{-200t} + 0.98 \sin (10^3 t + 101.30°)$

put $t = 0^+$

$i(0^+) = k + 0.96$

$0 = k + 0.96$

$\therefore\ k = -0.96$

$\therefore$ $i(t) = -0.96\, e^{-200t} + 0.98 \sin(10^3 t + 101.30°)$ Amp

Q.5: In a series R-L circuit the applied voltage is $V(t) = 10 \sin\left(10^4 t + \frac{\pi}{6}\right)$ with $R = 2\Omega$, $L = 0.01$ H, V (t) is applied at t = 0. Obtain the particular Solution for current i(t) through the circuit. Assume zero initial current through the inductor. (Use classical method).

Solution:

Applying KVL at $t > 0$

$V(t) = V_R + V_L$

$10 \sin\left(10^4 t + \frac{\pi}{6}\right) = i \times 2 + 0.01 \frac{di}{dt}$

$\frac{di}{dt} + 200\, i = 1000 \sin\left(10^4 t + \frac{\pi}{6}\right)$ .... (i)

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Now,

To find i_N,

$$\frac{di}{dt} + 200\, i = 0$$

$$i_N(t) = \frac{Q}{P} + k\, e^{-pt}$$

$$i_N = ke^{-200t} \text{ ....(i)}$$

Also,

To find i_F, taking trial solution

$$i_F = A \cos\left(10^4 t + \frac{\pi}{6}\right) + B \sin\left(10^4 t + \frac{\pi}{6}\right)$$

Put $i(t) = i_F$, in equation (i)

$$\frac{d}{dt}\left[A \cos\left(10^4 t + \frac{\pi}{6}\right) + B \sin\left(10^4 t + \frac{\pi}{6}\right)\right] + 200\left[A \cos\left(10^4 t + \frac{\pi}{6}\right) + B.\sin\left(10^4 t + \frac{\pi}{6}\right)\right]$$
$$= 1000 \sin\left(10^4 t + \frac{\pi}{6}\right)$$

$$\text{or, } -A\, 10^4.\sin\left(10^4 t + \frac{\pi}{6}\right) + B.\,10^4 \cos\left(10^4 t + \frac{\pi}{6}\right) + 200\, A \cos\left(10^4 t + \frac{\pi}{6}\right) + 200\, B \sin\left(10^4 t + \frac{\pi}{6}\right) = 1000 \sin\left(10^4 t + \frac{\pi}{6}\right)$$

Comparing $\sin\left(10^4 t + \frac{\pi}{6}\right)$ and $\cos\left(10^4 t + \frac{\pi}{6}\right)$ on both sides we get,

$$1000 = -10^4\, A + 200\, B \text{ ....(ii)}$$

$$0 = 10^4\, B + 200\, A \text{ ....(iii)}$$

Solving equation (ii) & (iii)

$A = -0.09996$

$B = 0.001992$

$$k = \sqrt{A^2 + B^2} = 0.09997 \text{ and } \phi = \tan\left(\frac{A}{B}\right) = -88.85°$$

Thus,

$$i_F = K \sin\left(10^4 t + \frac{\pi}{6} + \phi\right)$$

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$i_F = 0.09997 \sin(10^4t + 30° - 88.85°)$

$i_F = 0.09997 \sin(10^4t - 58.85°)$

Now,

The complete Solution is

$i(t) = i_N + i_F$

$i(t) = k\,e^{-200t} + 0.09997 \sin(10^4t - 58.85°)$

put $t = 0^+$

$i(0+) = k + 0.09997 \sin(0 - 58.85°)$

$i(0+) = k - 0.08555$

From question $i(0^-) = 0$

$\therefore\ k = 0.08555$

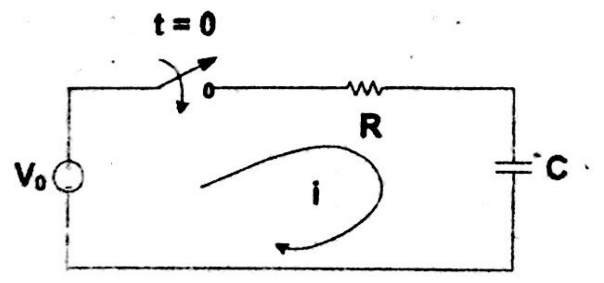
So,

$$\mathbf{i(t) = 0.08555\, e^{-200t} + 0.09997 \sin(10^4t - 58.85°)\ Amp.}$$

 # Response of R-C Circuit with:

1. **DC excitation:**

Find the expression of voltage across $V_C(t)$ using classical method



Applying KVL at $t > 0$

$V_0 = V_R + V_C$

$Or, V_0 = i * R + V_C$

$Or, V_0 = C\frac{dV_C}{dt} * R + V_C$ [since $i = i_C = C\frac{dV_C}{dt}$]

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Or, $\frac{dV_C}{dt} + \frac{1}{RC} V_C = \frac{V_0}{RC}$

This is first order differential equation, so the solution is

$$V_C(t) = \frac{Q}{P} + K e^{-pt}$$

Where, $Q = \frac{V_0}{RC}$ and $P = \frac{1}{RC}$

$$V_C(t) = V_0 + K e^{-\frac{1}{RC}t} \text{ ------------------ (i)}$$

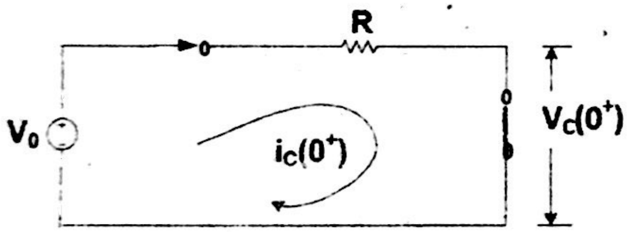
Now to find the value of K, put $t = 0^+$ in above equation

$$V_C(0^+) = V_0 + K e^{-\frac{1}{RC}*0}$$

$$K = V_C(0^+) - V_0 \text{ ------------- (a)}$$

Now to find $V_C(0^+)$ use initial condition, the given circuit is de-energized circuit

Circuit at $t = 0^-$ is



From the circuit, $V_C(0^+) = 0$

From equation (a) $K = -V_0$, so the equation (i) becomes as

$$\mathbf{V_C(t) = V_0 - V_0\ e^{-\frac{2R}{L}t}\ \ Volt}$$

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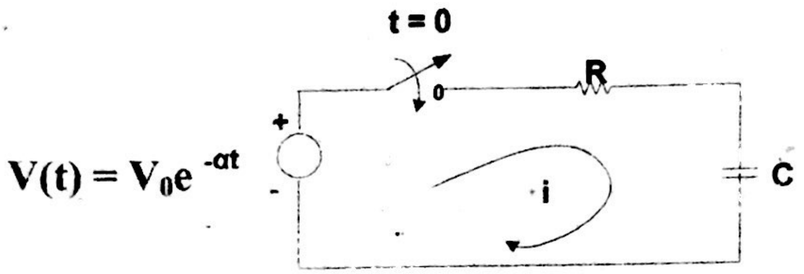
2. Exponential Excitation

Find the expression of current i (t) using classical method



Applying KVL at t > 0

$V_0\, e^{-\alpha t} = V_R + V_C$

Or, $V_0\, e^{-\alpha t} = i * R + \frac{1}{C}\int i_C\, dt$

Differentiating

Or, $-V_0\, \alpha\, e^{-\alpha t} = \frac{di}{dt} * R + \frac{1}{C} * i$

Or, $\frac{di}{dt} + \frac{1}{CR} * i = -\frac{V_0\, \alpha\, e^{-\alpha t}}{R}$ ----------------- (1)

This is first order non-homogenous differential equation, so the solution is

$i(t) = i_N(t) + i_F(t)$ -----------------(2)

Now to find $i_N(t)$, the auxiliary equation of (1) is

$\frac{di}{dt} + \frac{1}{CR} * i = 0$

$i_N(t) = K\, e^{-\frac{1}{CR}t}$ ----------------- (3)

Now to find $i_F(t)$, take a trial solution according to source present.

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Case I **If $\alpha \neq \frac{1}{CR}$, then the trail solution is taken as**

$$i_F(t) = Ae^{-\alpha t} \text{ ------------------ (4)}$$

Now, replacing the value of dependent variable of equation (1) by the value of trial solution of $i_F(t)$, then

$$\frac{d[Ae^{-\alpha t}]}{dt} + \frac{1}{CR} * [Ae^{-\alpha t}] = -\frac{V_0 \alpha e^{-\alpha t}}{R}$$

$$\text{Or, } -A\alpha e^{-\alpha t} + \frac{1}{CR} * [Ae^{-\alpha t}] = -\frac{V_0 \alpha e^{-\alpha t}}{R}$$

$$\text{Or, } A[-\alpha + \frac{1}{CR}] = -\frac{V_0 \alpha}{R}$$

$$\text{Or, } A = \frac{-\frac{V_0 \alpha}{R}}{[-\alpha + \frac{1}{CR}]} \text{ ------------------- (5)}$$

Thus equation (4) becomes as,

$$i_F(t) = \frac{-\frac{V_0 \alpha}{R}}{[-\alpha + \frac{1}{CR}]} * e^{-\alpha t} \text{ ------------------- (6)}$$

Hence from the equation (2),(3) and (6), the complete solution is

$$i(t) = i_N(t) + i_F(t)$$

$$\text{Or, } i(t) = K e^{-\frac{1}{CR}t} + \frac{\frac{V_0 \alpha}{R}}{[\alpha - \frac{1}{CR}]} * e^{-\alpha t} \text{ ------------ (7)}$$

Now to find the value of K, put $t = 0^+$ in above equation

$$i(t) = K e^{-\frac{1}{CR}*0} + \frac{\frac{V_0 \alpha}{R}}{[\alpha - \frac{1}{CR}]} * e^{-\alpha * 0}$$

$$\text{Or, } i(0^+) = K + \frac{\frac{V_0 \alpha}{R}}{[\alpha - \frac{1}{CR}]}$$

$$\text{Or, } K = i(0^+) - \frac{\frac{V_0 \alpha}{R}}{[\alpha - \frac{1}{CR}]} \text{ -------------- (8)}$$

Now to find $i(0^+)$ use initial condition, the given circuit is De-energized $i_C(0^-) = 0$

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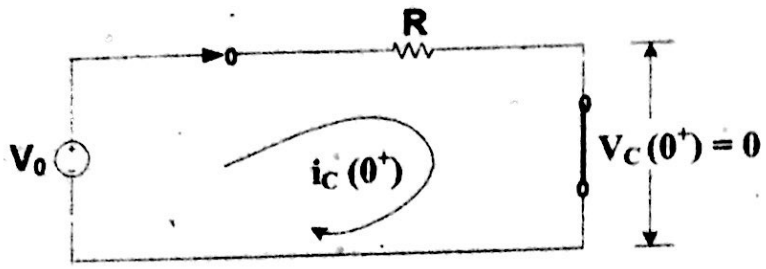
From circuit at $t = 0^-$



Thus $V_C(0^+) = 0$

$i_C(0^+) = \frac{V_0}{R}$

From equation (8) $K = \frac{V_0}{R} - \frac{\frac{V_0\alpha}{R}}{[\alpha - \frac{1}{CR}]}$, so the equation (7) becomes as

$$i(t) = [\frac{V_0}{R} - \frac{\frac{V_0\alpha}{R}}{[\alpha - \frac{1}{CR}]}]e^{-\frac{R}{L}t} + \frac{\frac{V_0\alpha}{R}}{[\alpha - \frac{1}{CR}]} * e^{-\alpha t} \text{ Amp} \text{------------ (9)}$$

Case II **If $\alpha = \frac{R}{L}$, then the trail solution is taken as**

$$i_F(t) = Ate^{-\alpha t} \text{ ---------------- (10)}$$

Now, replacing the value of dependent variable of equation (1) by the value of trial solution of $i_F(t)$, then

$$\frac{d[Ate^{-\alpha t}]}{dt} + \frac{1}{CR} * [Ate^{-\alpha t}] = -\frac{V_0\alpha e^{-\alpha t}}{R}$$

$$\text{Or, } A[e^{-\alpha t} - \alpha te^{-\alpha t}] + \frac{1}{CR} * [Ate^{-\alpha t}] = -\frac{V_0\alpha e^{-\alpha t}}{R}$$

$$\text{Or, } A[1 - \alpha t] + \frac{1}{CR} * [At] = -\frac{V_0\alpha}{R}$$

$$\text{Or, } A = \frac{-\frac{V_0\alpha}{R}}{\frac{1}{CR}t - \alpha t + 1}$$

Since we have $\alpha = \frac{1}{CR}$

$$\text{Or, } A = -\frac{V_0\alpha}{R} \text{ ---------------- (11)}$$

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Thus equation (4) becomes as,

$$i_F(t) = -\frac{V_0\,\alpha}{R}\, t * e^{-\alpha t} \quad \text{-------------------- (12)}$$

Hence from the equation (2),(3) and (12), the complete solution is

$$i\,(t) = i_N(t) + i_F(t)$$

$$\text{Or, } i\,(t) = K\, e^{-\frac{1}{CR}t} \; - \frac{V_0\,\alpha}{R}\, t * e^{-\alpha t} \quad \text{------------- (13)}$$

Now to find the value of K, put $t = 0^+$ in above equation

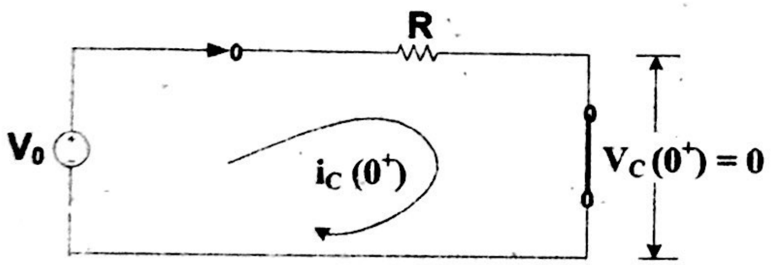
$$i\,(t) = K\, e^{-\frac{1}{CR}*0} \; - \frac{V_0\,\alpha}{R} * 0 * e^{-\alpha * 0}$$

$$\text{Or, } i\,(0^+) = K$$

$$\text{Or, } K = i\,(0^+) \text{ -------------- (14)}$$

Now to find $i\,(0^+)$ use initial condition, the given circuit is de-energized $i_L(0^-) = 0$

From circuit at $t = 0^-$



Thus $V_C(0^+) = 0$

$$i_C(0^+) = \frac{V_0}{R}$$

From equation (14) $K = \frac{V_0}{R}$, so the equation (13) becomes as

$$\mathbf{i\,(t)} = \frac{V_0}{R} e^{-\frac{1}{CR}t} \; - \frac{V_0\,\alpha}{R}\, t * e^{-\alpha t} \;\; \textbf{Amp} \text{ -------------- (9)}$$

Q.6: An exponential source $V(t) = 10e^{-4t}$ is suddenly applied at time $t = 0$ to a series R-C circuit comprising $R = 1\ \Omega$ and $C = 0.1$ F. Obtain the expression for the current i(t) in the circuit using classical method.

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Solution:

Applying KVL at $t > 0$

$$10e^{-4t} = V_R + V_C$$

$$\text{Or, } 10\, e^{-4t} = i * 1 + \frac{1}{0.1} \int i\, dt$$

Differentiating

$$\text{Or, } -4 * 10\, e^{-4t} = \frac{di}{dt} * 1 + \frac{1}{0.1}\, i_C$$

$$\text{Or, } \frac{di}{dt} + 10 * i = -40\, e^{-4t} \text{ ---------------- (1)}$$

This is first order non-homogenous differential equation, so the solution is

$$i'(t) = i_N(t) + i_F(t) \text{ ----------------(2)}$$

Now to find $i_N(t)$, the auxiliary equation of (1) is

$$\frac{di}{dt} + 10 * i = 0$$

$$i_N(t) = K\, e^{-10t} \text{ ---------------- (3)}$$

Now to find $i_F(t)$, take a trial solution according to source present,

Here, $\alpha = 4$ and $\frac{1}{CR} = 10$, so condition is $\alpha \neq \frac{R}{L}$

$$i_F(t) = Ae^{-4t} \text{ ---------------- (4)}$$

Now, replacing the value of dependent variable of equation (i) by the value of trial solution of $i_F(t)$, then

$$\frac{d[Ae^{-4t}]}{dt} + 10 * [Ae^{-4t}] = -40\, e^{-4t}$$

$$\text{Or, } -A\, 4\, e^{-4t} + 10 * [Ae^{-4t}] = -40\, e^{-4t}$$

$$\text{Or, } A\, [-4 + 10] = -40$$

$$\text{Or, } A = -6.667 \text{ ------------------ (5)}$$

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Thus equation (4) becomes as,

$i_F(t) = -6.667 * e^{-4t}$ -------------------- (6)

Hence from the equation (2),(3) and (6), the complete solution is

$i(t) = i_N(t) + i_F(t)$

Or, $i(t) = K e^{-10t} - 6.667 * e^{-4t}$ ------------- (7)

Now to find the value of K, put $t = 0^+$ in above equation

$i(0^+) = K e^{-10*0} - 6.667 * e^{-4*0}$

Or, $i(0^+) = K - 6.667$

Or, $K = i(0^+) + 6.667$ --------------- (8)

Now to find $i(0^+)$ use initial condition, the given circuit is de-energized

Thus $V_C(0^-) = 0$

$i_C(0^+) = 10$ amp

From equation (8) K = 16.667, so the equation (7) becomes as

$$i(t) = 16.667\, e^{-10t} - 6.667 * e^{-4t} \text{ Amp}$$

Q.7: An exponential source $V(t) = 20e^{-4t}$ is suddenly applied at time $t = 0$ to a series R-C circuit comprising $R = 1\ \Omega$ and $C = 0.25F$. Obtain the particular solution for the current i(t) in the circuit using classical method.

Solution:

Applying KVL at $t > 0$

$10e^{-4t} = V_R + V_C$

Or, $10\, e^{-4t} = i * 1 + \frac{1}{0.25}\int i\, dt$

Differentiating

Or, $-4 * 10\, e^{-4t} = \frac{di}{dt} * 1 + \frac{1}{0.25} i$

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$$Or, \frac{di}{dt} + 4 * i = -40\, e^{-4t} \text{ ---------------- } (1)$$

This is first order non-homogenous differential equation, so the solution is

$$i(t) = i_N(t) + i_F(t) \text{ ---------------- } (2)$$

Now to find $i_N(t)$, the auxiliary equation of (1) is

$$\frac{di}{dt} + 4 * i = 0$$

$$i_N(t) = K\, e^{-4t} \text{ ---------------- } (3)$$

Now to find $i_F(t)$, take a trial solution according to source present,

Here, $\alpha = 4$ and $\frac{R}{L} = 4$, so condition is $\alpha = \frac{1}{CR}$

$$i_F(t) = Ate^{-4t} \text{ ---------------- } (4)$$

Now, replacing the value of dependent variable of equation (i) by the value of trial solution of $i_F(t)$, then

$$\frac{d[Ate^{-4t}]}{dt} + 4 * [Ate^{-4t}] = -40\, e^{-4t}$$

$$Or, -At\, 4\, e^{-4t} + Ae^{-4t} + 4\, A\, t\, e^{-4t} = -40\, e^{-4t}$$

$$Or, A\,[-4t + 1 + 4t] = -40$$

$$Or, A = -40 \text{ ---------------- } (5)$$

Thus equation (4) becomes as,

$$i_F(t) = -40t * e^{-4t} \text{ ---------------- } (6)$$

Hence from the equatSion (2),(3) and (6), the complete solution is

$$i(t) = i_N(t) + i_F(t)$$

$$Or, i(t) = K\, e^{-4t} - 40t * e^{-4t} \text{ ------------ } (7)$$

Now to find the value of K, put $t = 0^+$ in above equation

$$i(0^+) = K\, e^{-4*0} - 40t * e^{-4*0}$$

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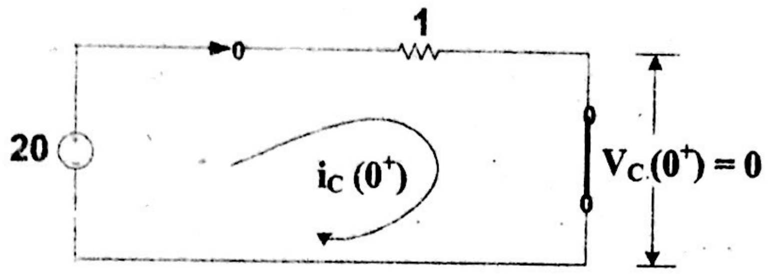
Or, $i(0^+) = K + 0$

Or, $K = i(0^+)$ ---------------- (8)

Now to find $i(0^+)$ use initial condition, the given circuit is energized $i_L(0^-) = 0$

From circuit at $t = 0^-$



Thus $V_C(0^+) = 0$

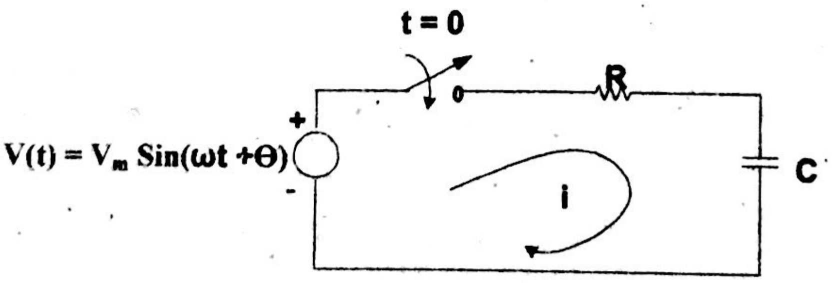
$i_C(0^+) = 20$ amp

From equation (8) K = 20, so the equation (7) becomes as

$$\mathbf{i(t) = 20 * e^{-4t} - 40t * e^{-4t} \text{ Amp}}$$

3. <u>Sinusoidal Excitation:</u>

Find the expression of current i (t) using classical method



Applying KVL at $t > 0$

$V_m \sin(\omega t + \Theta) = V_R + V_C$

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$$\text{Or, } V_m \sin(\omega t + \Theta) = i * R + \frac{1}{C}\int i\,dt$$

Differentiating

$$\text{Or, } \frac{di}{dt} + \frac{1}{CR} * i = \frac{V_m}{R}\,\omega \cos(\omega t + \Theta) \text{ ------------------ (1)}$$

This is first order non-homogenous differential equation, so the solution is

$$i(t) = i_N(t) + i_F(t) \text{ ------------------(2)}$$

Now to find $i_N(t)$, the auxiliary equation of (1) is

$$\frac{di}{dt} + \frac{1}{CR} * i = 0$$

$$i_N(t) = K\,e^{-\frac{1}{CR}t} \text{ ------------------ (3)}$$

Now to find $i_F(t)$, take a trial solution according to source present,

$$i_F(t) = A\cos(\omega t + \Theta) + B\sin(\omega t + \Theta) \text{ ------------------ (4)}$$

Now, replacing the value of dependent variable of equation (1) by the value of trial solution of $i_F(t)$, then

$$\frac{d[A\cos(\omega t +) + B\sin(\omega t +)]}{dt} + \frac{1}{CR} * [A\cos(\omega t + \Theta) + B\sin(\omega t + \Theta)] = \frac{V_m}{R}\,\omega\cos(\omega t + \Theta)$$

$$\text{Or, } -A\,\omega \sin(\omega t + \Theta) + B\,\omega\cos(\omega t + \Theta) + \frac{1}{CR} * [A\cos(\omega t + \Theta) + B\sin(\omega t + \Theta)] = \frac{V_m}{R}\,\omega\cos(\omega t + \Theta)$$

Equating the coefficient of Cosine and Sine, we get

$$B\,\omega + \frac{1}{CR} * A = \frac{V_m}{R}\,\omega \text{ -------------- (a)}$$

$$-A\,\omega + \frac{1}{CR} * B = 0 \text{ ------------ (b)}$$

Solving equation (a) and (b)

$$A = V_m * \frac{\omega C}{L^2\omega^2R^2+1}$$

$$B = V_m * \frac{C^2\omega^2 R}{L^2\omega^2R^2+1}$$

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Thus equation (4) becomes

$$i_F(t) = V_m * \frac{\omega C}{L^2\omega^2R^2+1}\text{Cos}\,(\omega t + \Theta) + V_m * \frac{C^2\omega^2R}{L^2\omega^2R^2+1}\text{Sin}\,(\omega t + \Theta) \text{ ------------- (6)}$$

Hence from the equation (2),(3) and (6), the complete solution is

$$i\,(t) = i_N(t) + i_F(t)$$

Or, $$i\,(t) = K\,e^{-\frac{1}{CR}t} + V_m * \frac{\omega C}{L^2\omega^2R^2+1}\text{Cos}\,(\omega t + \Theta) + V_m * \frac{C^2\omega^2R}{L^2\omega^2R^2+1}\text{Sin}\,(\omega t + \Theta) \text{ ---- (7)}$$

Now to find the value of K, put $t = 0^+$ in above equation

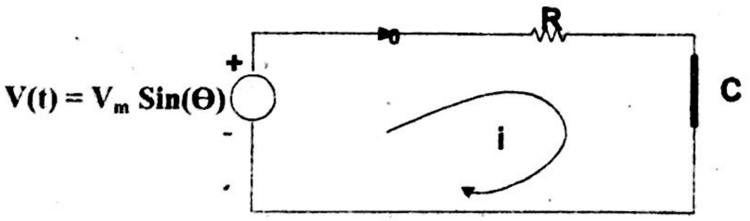
$$i\,(t) = K\,e^{-\frac{1}{CR}*0} + V_m * \frac{\omega C}{L^2\omega^2R^2+1}\text{Cos}\,\Theta + V_m * \frac{C^2\omega^2R}{L^2\omega^2R^2+1}\text{Sin}\,\Theta$$

Or, $$i\,(0^+) = K \;+ V_m * \frac{\omega C}{L^2\omega^2R^2+1}\text{Cos}\,\Theta + V_m * \frac{C^2\omega^2R}{L^2\omega^2R^2+1}\text{Sin}\,\Theta$$

Or, $$K = i\,(0^+) - Vm * \frac{\omega C}{L^2\omega^2R^2+1}\text{Cos} \;+ Vm * \frac{C^2\omega^2R}{L^2\omega^2R^2+1}\text{Sin} \text{ ------ (8)}$$

Now to find $i\,(0^+)$ use initial condition, the given circuit is de-energized $i_C(0^-) = 0$

Circuit at $t = 0^-$



$$i\,(0^+) = \frac{V_m\text{Sin}}{R}$$

From equation (8) $K = \frac{V_m\text{Sin}}{R} \;- Vm * \frac{\omega C}{L^2\omega^2R^2+1}\text{Cos} \;+ Vm * \frac{C^2\omega^2R}{L^2\omega^2R^2+1}\text{Sin}$, so the equation (7) becomes

$$i\,(t) = \left(\frac{V_m\text{Sin}}{R} - Vm * \frac{\omega C}{L^2\omega^2R^2+1}\text{Cos} + Vm * \frac{C^2\omega^2R}{L^2\omega^2R^2+1}\text{Sin}\right) e^{-\frac{1}{CR}t} + V_m * \frac{\omega C}{L^2\omega^2R^2+1}\text{Cos}\,(\omega t + \Theta) + V_m * \frac{C^2\omega^2R}{L^2\omega^2R^2+1}\text{Sin}\,(\omega t + \Theta) \quad \textbf{Amp}$$

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Q.8: In a series R-L circuit the applied voltage is $V(t) = 10 \sin\left(10^4 t + \frac{\pi}{6}\right)$ with R =2Ω, C = 0.25F, V (t) is applied at t = 0. Obtain the Complete Solution for current i(t) through the circuit. Assume zero initial current through the inductor. (Use classical method).

Solution:

Applying KVL at t = 0

$V(t) = V_R + V_C$

$10 \sin\left(10^4 t + \frac{\pi}{6}\right) = i \times 2 + * R + \frac{1}{0.25} \int i\, dt$

Differentiating

$\frac{di}{dt} + 2i = 5 \sin\left(10^4 t + \frac{\pi}{6}\right)$ .... (i)

Now,

To find i_N.

$\frac{di}{dt} + 2i = 0$

$i_N(t) = \frac{Q}{P} + k e^{-pt}$

$i_N = ke^{-2t}$ ....(i)

Also,

To find i_F, taking trial solution

$i_F = A \cos\left(10^4 t + \frac{\pi}{6}\right) + B \sin\left(10^4 t + \frac{\pi}{6}\right)$

Put $i(t) = i_F$, in equation (i)

$\frac{d}{dt}[A \cos\left(10^4 t + \frac{\pi}{6}\right) + B \sin\left(10^4 t + \frac{\pi}{6}\right)] + 2[A \cos\left(10^4 t + \frac{\pi}{6}\right) + B. \sin\left(10^4 t + \frac{\pi}{6}\right)] = 5 \sin\left(10^4 t + \frac{\pi}{6}\right)$

or, $-A\, 10^4. \sin\left(10^4 t + \frac{\pi}{6}\right) + B.\, 10^4 \cos\left(10^4 t + \frac{\pi}{6}\right) + 2A \cos\left(10^4 t + \frac{\pi}{6}\right) + 2B \sin\left(10^4 t + \frac{\pi}{6}\right) = 5 \sin\left(10^4 t + \frac{\pi}{6}\right)$

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Comparing $\sin\left(10^4t+\frac{\pi}{6}\right)$ and $\cos\left(10^4t+\frac{\pi}{6}\right)$ on both sides we get,

$5 = -10^4\ A + 2\ B$ ....(ii)

$0 = 10^4\ B + 2\ A$ ....(iii)

Solving equation (ii) & (iii)

$A = -5 * 10^{-4}$

$B = 10 * 10^{-8}$

Thus,

$i_F = -5 * 10^{-4}\ \mathrm{Cos}\left(10^4t+\frac{\pi}{6}\right) + 10 * 10^{-8}\ \sin\left(10^4t+\frac{\pi}{6}\right)$

Now,

The complete Solution is

$i\ (t) = i_N + i_F$

$i(t) = k\ e^{-2t} - 5 * 10^{-4}\ \mathrm{Cos}\left(10^4t+\frac{\pi}{6}\right) + 10 * 10^{-8}\ \sin\left(10^4t+\frac{\pi}{6}\right)$

put $t = 0^+$

$i\ (0+)\ k - 5 * 10^{-4}\ \mathrm{Cos}\left(\frac{\pi}{6}\right) + 10 * 10^{-8}\ \mathrm{Sin}\left(\frac{\pi}{6}\right)$

$i(0+) = k - 4.329 * 10^{-4}$

From question $i\ (0^+) = \frac{10\mathrm{Sin}30°}{2} = 2.5$

$\therefore\ k = 2.5004329$

So,

$$\mathbf{i(t) = 2.5004329\ e^{-2t} + -5 * 10^{-4}\ Cos\left(10^4t+\frac{\pi}{6}\right) + 10 * 10^{-8}\ sin\left(10^4t+\frac{\pi}{6}\right)}$$

Amp.

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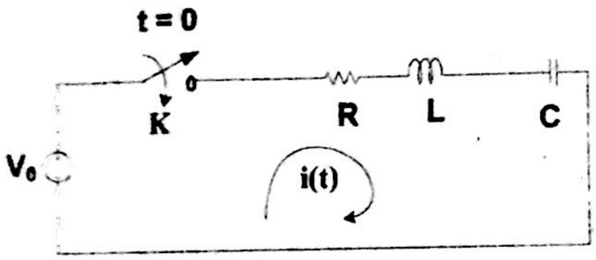
Response of RLC circuit with

1. **DC excitation**

Solution for 2nd order differential equation of RLC series circuit



Applying KVL at t > 0 in above circuit

$$V_R + V_L + V_C = V_0$$

$$\text{Or, } i * R + L\frac{di}{dt} + \frac{1}{C}\int i\, dt = V_0 \text{ ----------- (1)}$$

Differentiating

$$\text{Or, } R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C} * i = 0$$

$$\text{Or, } \frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{CR} * i = 0 \text{ -------------- (2)}$$

Which is 2nd order differential equation and its auxiliary equation is,

$$S^2 + \frac{R}{L}S + \frac{1}{LC} = 0$$

Which has two roots S_1 and S_2

$$S_1, S_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4*1*\frac{1}{LC}}}{2*1}$$

$$S_1, S_2 = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}$$

$$S_1, S_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_n^2} \text{ ------- (3)}$$

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Where, $\alpha = \frac{R}{2L}$ = Damping Coefficient

$\omega_n = \frac{1}{\sqrt{LC}}$ = Natural Frequency

The solution of equation (2) depends upon the nature of roots:

Case I: If $\alpha < \omega_n$ (under damped)

The roots will be complex, then the system is said to be under damped and the roots are given as

$$S_1, S_2 = -\alpha \pm \sqrt{(-1)[\omega_n^2 - \alpha^2]}$$

$$S_1, S_2 = -\alpha \pm j\,\omega_d \quad \text{where } \omega_d = \sqrt{\omega_n^2 - \alpha^2}$$

Hence the solution of equation (2) is given by :

$$\mathbf{i(t) = e^{-\alpha t}[K_1 \cos \omega_d t + K_2 \sin \omega_d t]} \text{ ---------------------- } (4)$$

Put = 0^+

$$i(0+) = e^{-\alpha * 0}[K_1 \cos(\omega_d * 0) + K_2 \sin(\omega_d * 0)]$$

$$i(0+) = K_1$$

We know that it's a de-energized circuit so inductor current is $i(0^+) = i(0^-) = 0$

$$K_1 = i(0^+) = 0$$

Equation (4) becomes

$$i(t) = e^{-\alpha t}[K_2 \sin \omega_d t] \text{ -------------------- } (5)$$

Differentiating equation (5) , we get

$$\frac{di}{dt} = K_2[(-\alpha)e^{-\alpha t} \sin \omega_d t + \omega_d e^{-\alpha t} \cos \omega_d t] \text{ ------- } (6)$$

Put $t = 0^+$

$$\frac{di}{dt}(0^+) = K_2\, \omega_d$$

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$$K_2 = \frac{\frac{di}{dt}(0^+)}{\omega_d} \text{ ---------------- } (7)$$

Now to find $\frac{di}{dt}(0^+)$ use equation (1)

$$i R + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = V_0$$

$$i R + L\frac{di}{dt} + V_C = V_0$$

Put $t = 0^+$

$$i(0^+)R + L\frac{di}{dt}(0^+) + V_C(0^+) = V_0$$

As the circuit is de-energized so that current through inductor $i(0^-)=i(0^+)=0$ and voltage across capacitor $V_C(0^-)=V_C(0^+) = 0$

$$0{*}R + L\frac{di}{dt}(0^+) + 0 = V_0$$

$$\frac{di}{dt}(0^+) = \frac{V_0}{L}$$

Thus equation (7) becomes as:

$$K_2 = \frac{\frac{V_0}{L}}{\omega_d}$$

Hence the solution under damped condition is given as

$$i(t) = e^{-\alpha t}\left[\frac{\frac{V_0}{L}}{\omega_d} \sin \omega_d t\right] \quad \textbf{Amp} \text{ ———— } (8)$$

Case II: If $\alpha = \omega_n$ (critically damped)

In this case, $\frac{R}{2L} = \frac{1}{\sqrt{LC}}$

$$R = 2 * \sqrt{\frac{L}{C}}$$

If $\alpha = \omega_n$ then roots will be real and equal and the roots are given as

$$S_1, S_2 = -\alpha$$

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Hence the solution of equation (2) is

$$\mathbf{i(t) = (K_1 + K_2 t)\, e^{-\alpha t}} \text{ ------------ (9)}$$

Put $t = 0^+$

$$i(0^+) = K_1$$

From continuity relationship $i(0^-) = i(0^+) = 0$, $K_1 = 0$

$$i(t) = K_2 t\, e^{-\alpha t} \text{ --------------- (10)}$$

Taking derivative of equation (10)

$$\frac{di}{dt} = K_2\,[e^{-\alpha t} - \alpha\, t e^{-\alpha t}] \text{ ---------------- (11)}$$

Put $t = 0^+$

$$\frac{di}{dt}(0^+) = K_2$$

Now to find $\frac{di}{dt}(0^+)$ use equation (1)

$$i\,R + L\frac{di}{dt} + \frac{i}{C}\int i\,dt = V_0$$

$$i\,R + L\frac{di}{dt} + V_C = V_0$$

Put $t = 0^+$

$$i(0^+)R + L\frac{di}{dt}(0^+) + V_C(0^+) = V_0$$

As the circuit is de-energized so that current through inductor $i(0^-)=i(0^+)=0$ and voltage across capacitor $V_C(0^-) = V_C(0^+) = 0$

$$0*R + L\frac{di}{dt}(0^+) + 0 = V_0$$

$$\frac{di}{dt}(0^+) = \frac{V_0}{L}$$

Thus $K_2 = \frac{V_0}{L}$

So equation (9) becomes

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$$i(t) = \frac{V_0}{L}\, t\, e^{-\alpha t} \text{ ———— } (12)$$

Case III: If $\alpha > \omega_0$ (over damped)

In this case roots are real and unequal and the roots are given as

$$S_1 = -\alpha_1$$

$$S_2 = -\alpha_2$$

Solution of equation (2) is given by

$$i(t) = K_1 e^{-\alpha_1 t} + K_2 e^{-\alpha_2 t} \text{ ———— } (13)$$

Put $t = 0^-$

$i(0^-) = K_1 + K_2$

$K_1 + K_2 = 0$ ——— (a)

Now differentiating equation (13) we get

$$\frac{di(t)}{dt} = K_1(-\alpha_1)e^{-\alpha_1 t} + K_2(-\alpha_2)e^{-\alpha_2 t}$$

Put $t = 0^-$

$$\frac{di(0^+)}{dt} = K_1(-\alpha_1) + K_2(-\alpha_2) \text{ ———— } (14)$$

Now to find $\frac{di}{dt}(0+)$ use equation (1)

$$iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = V_0$$

$$iR + L\frac{di}{dt} + V_C = V_0$$

Put $t = 0^-$

$$i(0^-)R + L\frac{di}{dt}(0^-) + V_C(0^-) = V_0$$

As the circuit is de-energized so that current through inductor $i(0^-)=i(0^+)=0$ and voltage across capacitor $V_C(0^-)=V_C(0^+)=0$

$$0*R + L\frac{di}{dt}(0^+) + 0 = V_0$$

$$\frac{di}{dt}(0^+) = \frac{V_0}{L}$$

So equation (13) becomes as

$$\frac{V_0}{L} = K_1(-\alpha_1) + K_2(-\alpha_2) \text{ ---------------- (b)}$$

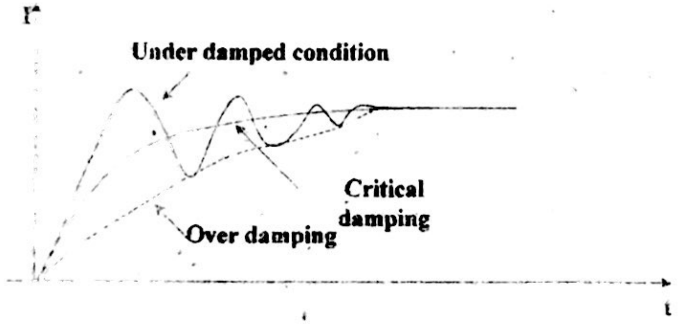
Solving equation (a) and (b) we get:

$$K_2 = \frac{V_0}{L(\alpha_1 - \alpha_2)}$$

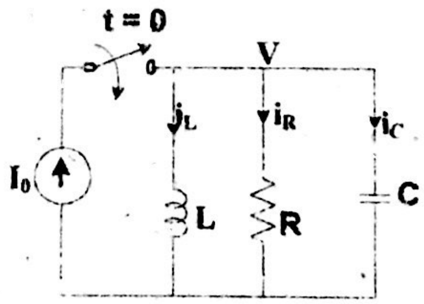
$$K_1 = \frac{-V_0}{L(\alpha_1 - \alpha_2)}$$

Hence equation (2) becomes

$$i(t) = \frac{-V_0}{L(\alpha_1 - \alpha_2)} e^{-\alpha_1 t} + \frac{V_0}{L(\alpha_1 - \alpha_2)} e^{-\alpha_2 t} \text{ Amp}$$



Solution for 2nd order differential equation of RLC parallel circuit



Here, $G = \frac{1}{R}$

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Applying KCL at $t > 0$ in above circuit

$$i_R + i_L + i_C = I_0$$

$$\text{Or, } \frac{V}{R} + \frac{1}{L}\int V\,dt + C\frac{dV}{dt} = I_0 \text{ ----------- (1)}$$

Differentiating

$$\text{Or, } G\frac{dV}{dt} + C\frac{d^2V}{dt^2} + \frac{1}{L} * v = 0$$

$$\text{Or, } \frac{d^2V}{dt^2} + \frac{G}{C}\frac{dV}{dt} + \frac{1}{CL} * V = 0 \text{ -------------- (2)}$$

Which is 2nd order differential equation and its auxiliary equation is,

$$S^2 + \frac{G}{C}S + \frac{1}{LC} = 0.$$

Which has two roots S_1 and S_2

$$S_1, S_2 = \frac{-\frac{G}{C} \pm \sqrt{\left(\frac{G}{C}\right)^2 - 4*1*\frac{1}{LC}}}{2*1}$$

$$S_1, S_2 = \frac{-G}{2C} \pm \sqrt{\left(\frac{G}{2C}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}$$

$$S_1, S_2 = -\beta \pm \sqrt{\beta^2 - \omega_n^2} \text{ ------- (3)}$$

Where, $\beta = \frac{G}{2C}$ = Damping Coefficient

$\omega_n = \frac{1}{\sqrt{LC}}$ = Natural Frequency

The solution of equation (2) depends upon the nature of roots:

Case I: If $\beta < \omega_n$ (under damped)

The roots will be complex, then the system is said to be under damped and the roots are given as

$$S_1, S_2 = -\beta \pm \sqrt{(-1)[\omega_n^2 - \alpha^2]}$$

$$S_1, S_2 = -\beta \pm j\,\omega_d \quad \text{where } \omega_d = \sqrt{\omega_n^2 - \alpha^2}$$

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Hence the solution of equation (2) is given by :

$$V(t) = e^{-\beta t}[K_1 \cos \omega_d t + K_2 \sin \omega_d t] \text{ ---------------------- (4)}$$

Put = 0^+

$$V(0+) = e^{-\beta * 0}[K_1 \cos(\omega_d * 0) + K_2 \sin(\omega_d * 0)]$$

$$V(0+) = K_1$$

We know that it's a de-energized circuit so voltage across capacitor is $V_C(0^+) = V_C(0^-) = V(0^+) = 0$

$$K_1 = V(0^+) = 0$$

Equation (4) becomes

$$V(t) = e^{-\beta t}[K_2 \sin \omega_d t] \text{ -------------------- (5)}$$

Differentiating equation (5) , we get

$$\frac{dV}{dt} = K_2\left[(-\beta)e^{-\beta t} \sin \omega_d t + \omega_d e^{-\beta t} \cos \omega_d t\right] \text{ ------- (6)}$$

Put $t = 0^+$

$$\frac{dV}{dt}(0^+) = K_2\, \omega_d$$

$$K_2 = \frac{\frac{dV}{dt}(0^+)}{\omega_d} \text{ ---------------- (7)}$$

Now to find $\frac{dV}{dt}(0^+)$ use equation (1)

$$\frac{V}{R} + \frac{1}{L}\int V\,dt + C\frac{dV}{dt} = I$$

$$VG + i_L + C\frac{dV}{dt} = I_0$$

Put $t = 0^+$

$$V(0^+)G + C\frac{dV}{dt}(0^+) + i_L(0^+) = I_0$$

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As the circuit is de-energized so that current through inductor $i(0^-)=i(0^+)=0$ and voltage across capacitor $V_C(0^-)=V_C(0^+)=0$

$$0*G + C\frac{dV}{dt}(0^+) + 0 = I_0$$

$$\frac{dV}{dt}(0^+) = \frac{I_0}{C}$$

Thus equation (7) becomes as:

$$K_2 = \frac{\frac{I_0}{C}}{\omega_d}$$

Hence the solution under damped condition is given as

$$V(t) = e^{-\beta t}\left[\frac{\frac{I_0}{C}}{\omega_d}\sin \omega_d t\right] \quad \text{Amp} \text{ ——————— (8)}$$

Case II: If $\beta = \omega_n$ (critically damped)

In this case, $\frac{G}{2C} = \frac{1}{\sqrt{LC}}$

$$R = \frac{1}{2} * \sqrt{\frac{L}{C}}$$

If $\alpha = \omega_n$ then roots will be real and equal and the roots are given as

$$S_1, S_2 = -\beta$$

Hence the solution of equation (2) is

$$V(t) = (K_1 + K_2 t)\, e^{-\beta t} \text{ ----------- (9)}$$

Put $t = 0^+$

$$V(0^+) = K_1 = 0$$

As the circuit is de-energized so that current through inductor $i(0^-)=i(0^+)=0$ and voltage across capacitor $V_C(0^-)=V_C(0^+)=0$

$$V(t) = K_2 t\, e^{-\beta t} \text{ -------------- (10)}$$

Differentiating

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$$\frac{dV}{dt} = K_2\,[e^{-\beta t} - \beta\, t e^{-\beta t}]$$

Put $t = 0^+$

$$\frac{dV}{dt}(0^+) = K_2$$

Now to find $\frac{dV}{dt}(0^+)$ use equation (1)

$$\frac{V}{R} + \frac{1}{L}\int V\,dt + C\,\frac{dV}{dt} = I_0$$

$$V\,G + i_L + C\,\frac{dV}{dt} = I_0$$

Put $t = 0^+$

$$V(0^+)G + C\,\frac{dV}{dt}(0^+) + i_L(0^+) = I_0$$

As the circuit is de-energized so that current through inductor $i(0^-)=i(0^+)=0$ and voltage across capacitor $V_C(0^-)=V_C(0^+)=0$

$$0*G + C\,\frac{dV}{dt}(0^+) + 0 = I_0$$

$$\frac{dV}{dt}(0^+) = \frac{I_0}{C}$$

Thus $K_2 = \frac{I_0}{C}$

Hence the solution critically damped condition is given as

$$\mathbf{V(t) = \frac{I_0}{C}\, t\, e^{-\alpha t}} \text{ ---------------------- } (11)$$

Case III: If $\beta > \omega_n$ (over damped)

In this case roots are real and unequal and the roots are given as

$$S_1 = -\beta_1$$

$$S_2 = -\beta_2$$

Solution of equation (2) is given by

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$$V(t) = K_1 e^{-\beta_1 t} + K_2 e^{-\beta_2 t} \text{ ---------------- } (12)$$

Put $t = 0^+$

$$V(0^+) = K_1 + K_2$$

$$K_1 + K_2 = 0 \text{ ---------- (a)}$$

Now differentiating equation (12) we get

$$\frac{dV(t)}{dt} = K_1(-\beta_1)e^{-\beta_1 t} + K_2(-\beta_2)e^{-\beta_2 t}$$

Put $t = 0^+$

$$\frac{dV(0^+)}{dt} = K_1(-\beta_1) + K_2(-\beta_2) \text{ --------------- } (13)$$

Now to find $\frac{dV}{dt}(0^+)$ use equation (1)

$$\frac{V}{R} + \frac{1}{L}\int V\,dt + C\frac{dV}{dt} = I_0$$

$$V * G + i_L + C\frac{dV}{dt} = I_0$$

Put $t = 0^+$

$$V(0^+) * G + C\frac{dV}{dt}(0^+) + i_L(0^+) = I_0$$

Pdf by: Sachin Lamsal

As the circuit is de-energized so that current through inductor $i(0^-) = i(0^+) = 0$ and voltage across capacitor $V_C(0^-) = V_C(0^+) = 0$

$$0 * G + C\frac{dV}{dt}(0^+) + 0 = I_0$$

$$\frac{dV}{dt}(0^+) = \frac{I_0}{C}$$

So equation (13) becomes as

$$\frac{I_0}{C} = K_1(-\beta_1) + K_2(-\beta_2) \text{ ---------------- (b)}$$

Solving equation (a) and (b) we get:

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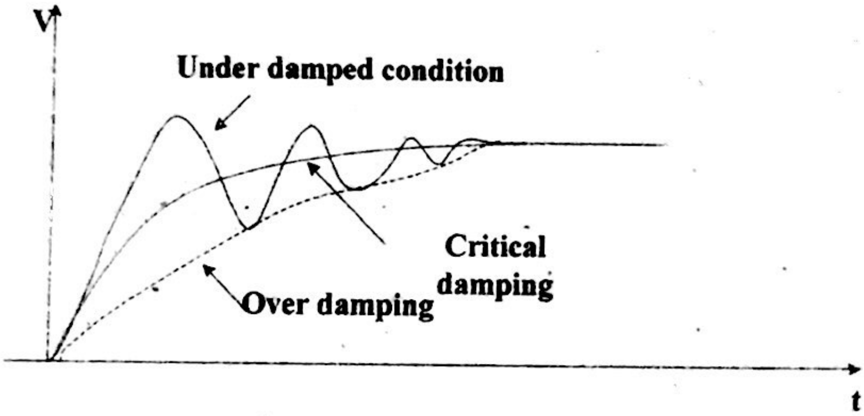
$$K_2 = \frac{I_0}{C(\beta_1 - \beta_2)}$$

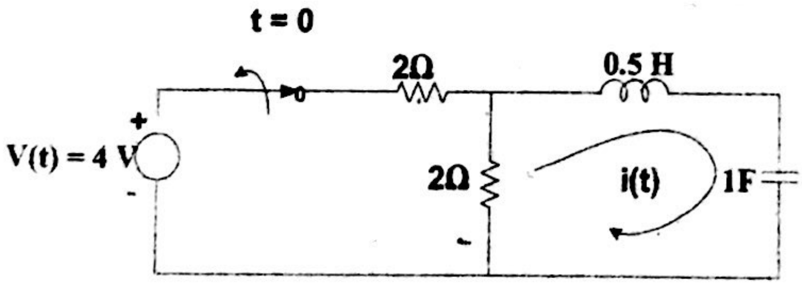
$$K_1 = -\frac{I_0}{C(\beta_1 - \beta_2)}$$

Hence the solution over damped condition is given as

$$V(t) = -\frac{I_0}{C(\beta_1 - \beta_2)} e^{-\beta_1 t} + \frac{I_0}{C(\beta_1 - \beta_2)} e^{-\beta_2 t} \text{ Amp}$$

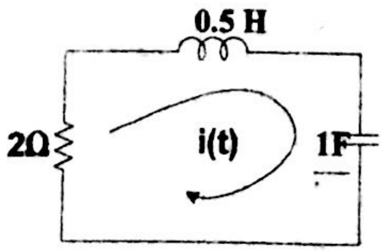


Q.9: Determine i(t) in the circuit shown by classical method.



Solution:

Circuit at t > 0



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Applying KVL at $t > 0$ in above circuit

$V_R + V_L + V_C = 0$

Or, $i * 2 + 0.5 \frac{di}{dt} + \frac{1}{1} \int i\, dt = 0$ ------------ (1)

Differentiating

Or, $2 \frac{di}{dt} + 0.5 \frac{d^2i}{dt^2} + i = 0$

Or, $\frac{d^2i}{dt^2} + 4 \frac{di}{dt} + 2 * i = 0$ --------------- (2)

Which is 2nd order differential equation and its auxiliary equation is,

$S^2 + 4S + 2 = 0$

Which has two roots S_1 and S_2

$S_1 = -0.58$

$S_2 = -3.41$

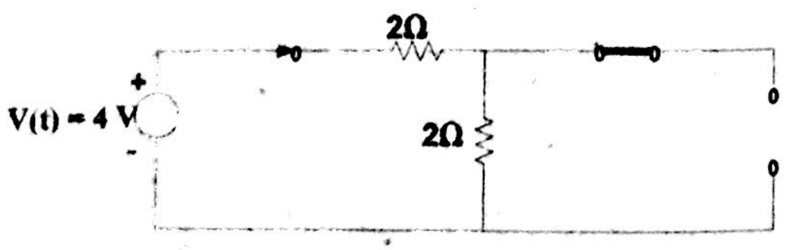
Solution of equation (2) is given by

$i(t) = K_1 e^{-0.58t} + K_2 e^{-3.41t}$ --------------- (3)

Put $t = 0^+$

$i(0^+) = K_1 + K_2$

now to find $i(0^-)$, it's an energized condition circuit so circuit at $t = 0^-$



From the circuit, $i_L(0^-) = i_L(0^+) = i(0^+) = 0$

But voltage across capacitor $V_C(0^-) = V_C(0^+) = 2 * \frac{4}{4} = 2$ Volt

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$$K_1 + K_2 = 0 \text{ ---------- (a)}$$

Now differentiating equation (3) we get

$$\frac{di(t)}{dt} = K_1(-0.58)e^{-0.58t} + K_2(-3.41)e^{-3.41t}$$

Put $t = 0^+$

$$\frac{di(0^+)}{dt} = K_1(-0.58) + K_2(-3.41) \text{ --------------- (4)}$$

Now to find $\frac{di}{dt}(0+)$ use equation (1)

$$2 * i + 0.5\frac{di}{dt} + \frac{1}{1}\int i\,dt = 0$$

$$2 * i + 0.5\frac{di}{dt} + V_C = 0$$

Put $t = 0^+$

$$i(0^+) * 2 + 0.5\frac{di}{dt}(0^+) + V_C(0^+) = 0$$

As the circuit is de-energized so that current through inductor $i(0^-)=i(0^+)=0$ and voltage across capacitor $V_C(0^-)=V_C(0^+) = 2$ Volt

$$0 * R + 0.5\frac{di}{dt}(0^+) + 2 = 0$$

$$\frac{di}{dt}(0^+) = -4$$

So equation (4) becomes as

$$-4 = K_1(-0.58) + K_2(-3.41) \text{ ----------------- (b)}$$

Solving equation (a) and (b) we get:

$$K_2 = -1.41$$

$$K_1 = 1.41$$

Hence the complete solution is:

$$\mathbf{i(t) = -1.41e^{-0.58t} + 1.41e^{-3.41t}\ Amp}$$

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Q.10: A voltage V(t) = 20 is suddenly applied at time t = 0 to a series RLC circuit comprising R = 2Ω , L = 1H and C = 1F. Obtain the expression for the current i(t) in the circuit using classing method.

Solution: Applying KVL at t > 0

$$V_R + V_L + V_C = V(t)$$

$$i(t)*2 + 1\frac{di}{dt} + \frac{1}{1}\int i\,dt = 20 \text{ ------------ (1)}$$

Differentiating

$$\text{Or, } 2\frac{di}{dt} + \frac{d^2i}{dt^2} + i = 0$$

$$\text{Or, } \frac{d^2i}{dt^2} + 2\frac{di}{dt} + i = 0 \text{ ------------ (2)}$$

Which is 2nd order differential equation and its auxiliary equation is.

$$S^2 + 2S + 1 = 0$$

Which has two roots S_1 and S_2

$$S_1, S_2 = -1$$

Hence the solution of equation (2) is

$$i(t) = (K_1 + K_2t)\,e^{-t} \text{ ------------ (3)}$$

Put $t = 0^+$

$$i(0^+) = K_1$$

From continuity relationship $i(0^-) = i(0^+) = 0$, $K_1 = 0$

$$i(t) = K_2 t\, e^{-t} \text{ ------------ (4)}$$

Taking derivative of equation (4)

$$\frac{di}{dt} = K_2\left[e^{-t} - te^{-t}\right] \text{ ------------ (5)}$$

Put $t = 0^+$

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$$\frac{di}{dt}(0^+) = K_2$$

Now to find $\frac{di}{dt}(0^+)$ use equation (1)

$$i*2 + \frac{di}{dt} + V_c = 20$$

$$i*2 + \frac{di}{dt} + V_C = 20$$

Put $t = 0^+$

$$i(0^+)*2 + \frac{di}{dt}(0^+) + V_C(0^+) = 20$$

As the circuit is de-energized so that current through inductor $i(0^-)=i(0^+)=0$ and voltage across capacitor $V_C(0^-)=V_C(0^+) = 0$

$$0*2 + \frac{di}{dt}(0^+) + 0 = 20$$

$$\frac{di}{dt}(0^+) = 20$$

Thus $K_2 = 20$

So equation (4) becomes

$$\mathbf{i(t) = 20t\, e^{-\alpha t}\ Amp}$$

Q.11: A voltage V(t) = 100 is suddenly applied at time t = 0 to a series RLC circuit comprising R = 4Ω , L =1H and C = $\frac{1}{5}$F. Obtain the expression for the current i(t) in the circuit using classing method.

Solution: Applying KVL at t > 0

$$V_R + V_L + V_C = V(t)$$

$$\text{Or, } i*4 + 1\frac{di}{dt} + 5\int i\, dt = 100 \text{ ----------- (1)}$$

Differentiating

$$\text{Or, } 4\frac{di}{dt} + \frac{d^2i}{dt^2} + 5*i = 0$$

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$$Or. \frac{d^2i}{dt^2} + 4\frac{di}{dt} + 5 * i = 0 \text{ -------------- } (2)$$

Which is 2nd order differential equation and its auxiliary equation is,

$$S^2 + 4S + 5 = 0$$

Which has two roots S_1 and S_2

$$S_1, S_2 = -2 \pm j1$$

Hence the solution of equation (2) is given by :

$$i(t) = e^{-2t}[K_1 \cos t + K_2 \sin t] \text{ -------------------- } (3)$$

Put = 0^-

$$i(0+) = e^{-2*0}[K_1 \cos(0) + K_2 \sin(0)]$$

$$i(0+) = K_1$$

We know that it's a de-energized circuit so inductor current is $i(0^+) = i(0^-) = 0$

$$K_1 = i(0^+) = 0$$

Equation (3) becomes

$$i(t) = e^{-2t}[K_2 \sin t] \text{ ------------------ } (4)$$

Differentiating equation (4) , we get

$$\frac{di}{dt} = K_2[(-2)e^{-2t} \sin t + e^{-2t} \cos t] \text{ ------- } (5)$$

Put $t = 0^+$

$$\frac{di}{dt}(0^+) = K_2$$

Now to find $\frac{di}{dt}(0^+)$ use equation (1)

$$I*4 + 1\frac{di}{dt} + 5\int i\, dt = 100$$

$$i*4 + \frac{di}{dt} + V_C = 100$$

Put $t = 0^+$

$$i(0^+)*4 + \frac{di}{dt}(0^+) + V_C(0^+) = 100$$

As the circuit is de-energized so that current through inductor $i(0^-)=i(0^+)=0$ and voltage across capacitor $V_C(0^-)=V_C(0^+) = 0$

$$0*R + \frac{di}{dt}(0^+) + 0 = 100$$

$$\frac{di}{dt}(0^+) = 100$$

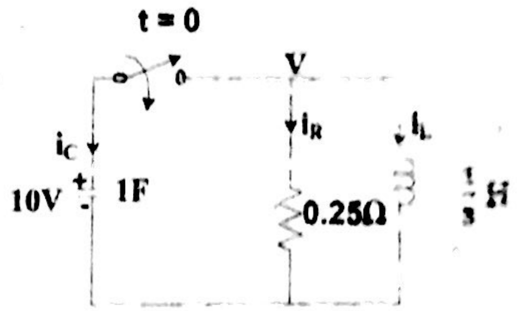
Thus equation becomes as:

$$K_2 = 100$$

Hence the solution is given as

$$\mathbf{i(t) = 100e^{-2t}Sint \quad Amp}$$

Q.12 Using classical method, obtain the expression for the voltage across the inductor in the circuit shown in the figure below.



Solution: Applying KCL at $t > 0$ in above circuit

$$i_R + i_L + i_C = 0$$

$$Or, \frac{V}{0.25} + 3\int V\,dt + 1*\frac{dV}{dt} = 0 \text{ ------------ (1)}$$

Differentiating

$$Or, 4*\frac{dV}{dt} + \frac{d^2V}{dt^2} + 3*V = 0$$

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$$\text{Or,} \frac{d^2V}{dt^2} + 4\frac{dV}{dt} + 3 * V = 0 \text{ --------------- (2)}$$

Which is 2nd order differential equation and its auxiliary equation is,

$$S^2 + 4S + 3 = 0$$

Which has two roots S_1 and S_2

$$S_1 = -1$$

$$S_2 = -3$$

.Solution of equation (2) is given by

$$V(t) = K_1e^{-t} + K_2e^{-3t} \text{ -------------- (3)}$$

Put $t = 0^+$

$$V(0^+) = K_1 + K_2$$

Now to find $V(0^+)$, we have from the question $V_C(0^-) = V_C(0^+) = V(0^+) = 10$ V, But $i_L(0^-) = i_L(0^+) = 0$

$$K_1 + K_2 = 10 \text{ --------- (a)}$$

Now differentiating equation (3) we get

$$\frac{dV(t)}{dt} = K_1(-1)e^{-t} + K_2(-3)e^{-3t}$$

Put $t = 0^+$

$$\frac{dV(0^+)}{dt} = K_1(-1) + K_2(-3) \text{ -------------- (4)}$$

Now to find $\frac{dV}{dt}(0^+)$ use equation (1)

$$\frac{V}{0.25} + 3\int V\,dt + 1\frac{dV}{dt} = 0$$

$$V * 4 + i_L + \frac{dV}{dt} = 0$$

Put $t = 0^+$

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$$V(0^+)*4 + \frac{dV}{dt}(0^+) + i_L(0^+) = 0$$

As the circuit is de-energized so that current through inductor $i(0^-)=i(0^+)=0$ and voltage across capacitor $V_C(0^-)=V_C(0^+) = 10$ V

$$10*4 + \frac{dV}{dt}(0^+) + 0 = 0$$

$$\frac{dV}{dt}(0^+) = -40$$

So equation (4) becomes as

$$-40= K_1(-1) + K_2(-3) \text{ ------------------ (b)}$$

Solving equation (a) and (b) we get:

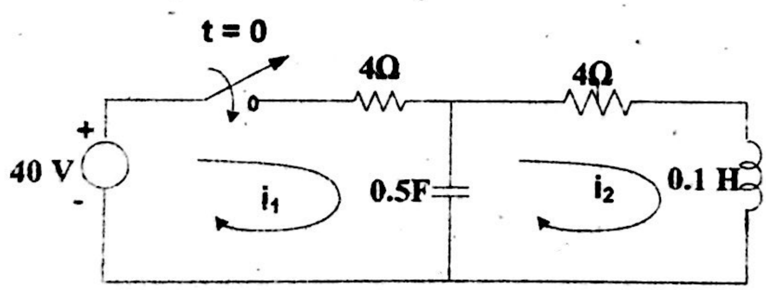
$$K_2 = 15$$

$$K_1 = -5$$

Hence the complete solution is :

$$\mathbf{V(t) = -\ 5e^{-t} + 15e^{-3t}\ Amp}$$

Q.13: In the two mesh network shown in figure, find the current i_1 and i_2 when the switch is closed at t= 0, using classical approach.



Solution:

Circuit at t > 0

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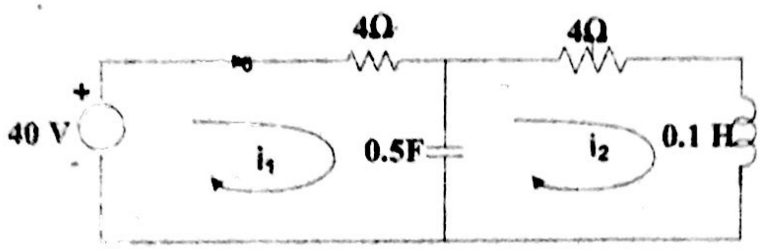

Applying KVL at loop (i)

$40 = V_R + V_C$

$40 = 4 * i_1 + 2\int(i_1 - i_2)\,dt$

Differentiating

$0 = 4\frac{di_1}{dt} + 2i_1 + 2i_2$

$i_2 = 2\frac{di_1}{dt} + i_1$ ------------- (1)

Now again applying KVL at outer loop of the circuit

$40 = V_R + V_R + V_C$

$40 = 4 * i_1 + 4 * i_2 + 0.1\frac{di_2}{dt}$

From equation (1)

$40 = 4 * i_1 + 4\left(2\frac{di_1}{dt} + i_1\right) + 0.1\frac{d}{dt}\left(2\frac{di_1}{dt} + i_1\right)$

$40 = 4i_1 + 8\frac{di_1}{dt} + 4i_1 + 0.2\frac{d^2i_1}{dt^2} + 0.1\frac{di_1}{dt}$

$40 = 0.2\frac{d^2i_1}{dt^2} + 8.1\frac{di_1}{dt} + 8i_1$

$\frac{d^2i_1}{dt^2} + 40.5\frac{di_1}{dt} + 40\,i_1 = 200$ ----------- (2)

Which is 2nd order non-homogeneous differential equation so solution is given as

$i_1(t) = i_{1N} + i_{1F}$

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Now to find i_N, take auxiliary equation

$$S^2 + 40.5S + 40 = 0$$

$$S_1 = -1.01$$

$$S_2 = -39.49$$

So $i_{1N} = K_1 e^{-1.01t} + K_2 e^{-39.49t}$

To find i_{1F}, take trial solution as source present and put it in equation (2)

$$i_{1F} = A \text{ (constant)}$$

$$\frac{d^2A}{dt^2} + 40.5\frac{dA}{dt} + 40A = 200$$

$$0 + 0 + 40A = 200$$

$$A = 5$$

Thus, $i_{1F} = 5$

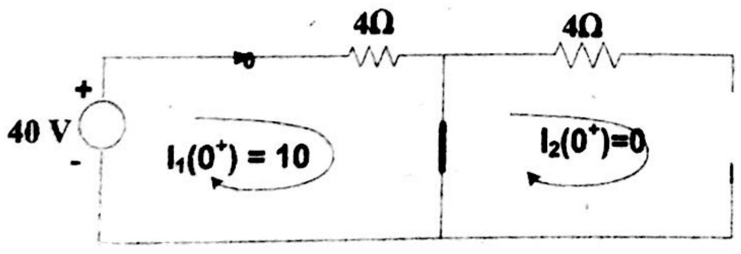
Thus the complete solution is

$$i_1(t) = K_1 e^{-1.01t} + K_2 e^{-39.49t} + 5 \text{ ------------------------ (3)}$$

Put $t = 0^+$

$$i_1(0^+) = K_1 + K_2 + 5$$

As circuit is de-energized, so circuit at $t = 0^+$



So, $10 = K_1 + K_2 + 5$

$$5 = K_1 + K_2 \text{ --------------- (a)}$$

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Now differentiating equation (3) we get

$$\frac{di_1}{dt} = K_1(-1.01)e^{-1.01t} + K_2(-39.49)e^{-39.49t} + 0$$

Put $t = 0^+$

$$\frac{di_1}{dt}(0^+) = K_1(-1.01) + K_2(-39.49)$$

To find $\frac{di_1}{dt}(0^+)$ take equation (1)

$$i_2 = 2\,\frac{di_1}{dt} + i_1$$

Put $t = 0^+$

$$i_2(0^+) = 2\,\frac{di_1}{dt}(0^+) + i_1(0^+)$$

$$0 = 2\,\frac{di_1}{dt}(0^+) + 10$$

$$\frac{di_1}{dt}(0^+) = -5$$

Then

$$-5 = K_1(-1.01) + K_2(-39.49) \text{ ---------------- (b)}$$

Solving equation (a) and (b) we get

$$K_1 = 5$$

$$K_2 = -1.30 * 10^{-3}$$

Hence the solution for $i_1(t)$ of the circuit is

$$\mathbf{i_1(t) = 5\,e^{-1.01t} - 1.30 * 10^{-3}e^{-39.49t} + 5\ Amp}$$

Also, from the equation (1)

$$i_2(t) = 2\,\frac{di_1(t)}{dt} + i_1(t)$$

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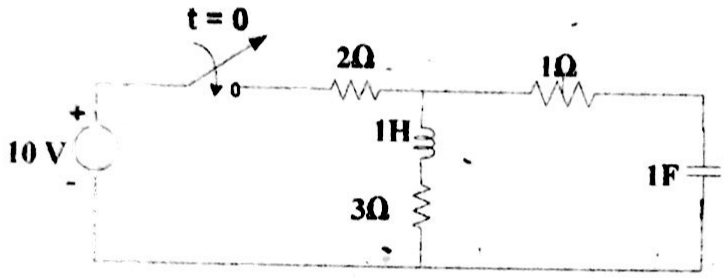
$$i_2(t) = 2\,\frac{d}{dt}[5e^{-1.01t} - 1.30 * 10^{-3}e^{-39.49t} + 5] + 5\,e^{-1.01t} - 1.30 * 10^{-3}e^{-39.49t} + 5$$

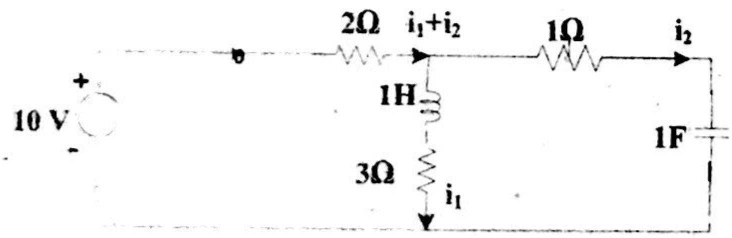
$$\mathbf{i_2(t) = -5.1\,e^{-1.01t} + 0.101\,e^{-39.49t} + 5\ Amp}$$

Q.14: Using classical method, find the current through inductor in the network.



Solution:

Circuit at t > 0



Note: If the question didn't give you any current direction and asked you to find branch current like in this question Q.14 (current through inductor), then it's better to use branch current as shown in the circuit t >0 because it helps you to find direct branch current expression.

Applying KVL at loop (1)

$$10 = V_{R2} + V_L + V_{R3}$$

$$10 = 2(i_1 + i_2) + 1\,\frac{di_1}{dt} + 3\,i_1$$

$$10 = 5\,i_1 + \frac{di_1}{dt} + 2\,i_2$$

$$i_2 = 5 - 2.5\,i_1 - 0.5\,\frac{di_1}{dt} \text{ -------------- } (1)$$

Now applying KVL at outer loop

$$10 = V_{R2} + V_{R1} + V_C$$

$$10 = 2(i_1 + i_2) + 1\,i_2 + \frac{1}{1}\int i_2 dt$$

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Differentiating

$$0 = 2\frac{di_1}{dt} + 3\frac{di_2}{dt} + i_2$$

From equation (1) put value of i_2 in above equation

$$0 = 2\frac{di_1}{dt} + 3\frac{d}{dt}\left[5 - 2.5\,i_1 - 0.5\,\frac{di_1}{dt}\right] + \left(5 - 2.5\,i_1 - 0.5\,\frac{di_1}{dt}\right)$$

$$0 = 2\frac{di_1}{dt} - 1.5\frac{d^2i_1}{dt^2} - 7.5\frac{di_1}{dt} + 5 - 0.5\frac{di_1}{dt} - 2.5\,i_1$$

$$-5 = -1.5\frac{d^2i_1}{dt^2} - 6\frac{di_1}{dt} - 2.5\,i_1$$

$$\frac{d^2i_1}{dt^2} + 4\frac{di_1}{dt} + 1.66\,i_1 = 3.33$$

Which is 2nd order non-homogeneous differential equation so solution is given as

$$i_1(t) = i_{1N} + i_{1F}$$

Now to find i_{1N}, take auxiliary equation

$$S^2 + 4S + 1.66 = 0$$

$$S_1 = -0.47$$

$$S_2 = -3.53$$

$$\text{So, } i_{1N} = K_1 e^{-0.47t} + K_2 e^{-3.53t}$$

To find i_{1F}, take trial solution as source present and put it in equation (2)

$$i_{1F} = A \text{ (constant)}$$

$$\frac{d^2A}{dt^2} + 4\frac{dA}{dt} + 1.66\,A = 3.33$$

$$0 + 0 + 1.66\,A = 3.33$$

$$A = 2$$

$$\text{Thus, } i_{1F} = 2$$

Thus the complete solution is

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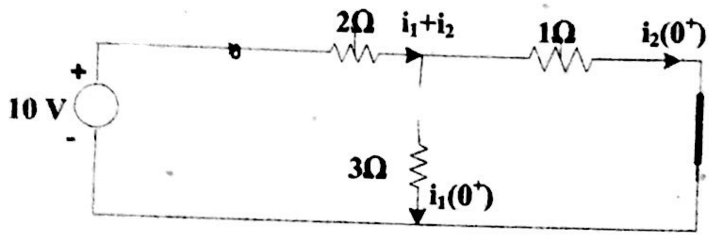
$$i_1(t) = K_1e^{-0.47t} + K_2e^{-3.53t} + 2 \text{ ---------------------------- (3)}$$

Put $t = 0^+$

$$i_1(0^+) = K_1 + K_2 + 2$$

As circuit is de-energized, so circuit at $t = 0^+$



From the circuit, $i_1(0^+)= 0$ and $i_2(0^+) =3.33$ Amp

So, $0 = K_1 + K_2 + 5$

$$-5 = K_1 + K_2 \text{ --------------- (a)}$$

Now differentiating equation (3) we get

$$\frac{di_1}{dt} = K_1(-0.47)e^{-0.47t} + K_2(-3.53)e^{-3.53t} + 0$$

Put $t= 0^+$

$$\frac{di_1}{dt}(0^+) = K_1(-0.47) + K_2(-3.53)$$

To find $\frac{di_1}{dt}(0^+)$ take equation (1)

$$i_2 = 5 - 2.5\, i_1 + 0.5\frac{di_1}{dt}$$

Put $t = 0^+$

$$i_2(0^+) = 5 + 2.5i_1(0^+) + 0.5\,\frac{di_1}{dt}(0^+)$$

$$3.33 = 0.5\,\frac{di_1}{dt}(0^+) + 5$$

$$\frac{di_1}{dt}(0^+) = -3.34$$

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Then

$$-3.34 = K_1(-0.47) + K_2(-3.53) \text{ ------------------ (b)}$$

Solving equation (a) and (b) we get

$$K_1 = -6.85$$

$$K_2 = 1.85$$

Hence the solution for current through inductor $i_1(t)$ in the circuit is

$$\mathbf{i_1(t) = -6.85\, e^{-0.47t} + 1.85\, e^{-3.53t} + 5 \text{ Amp}}$$

2. Exponential excitation

Q.15 An exponential voltage $V(t) = 20e^{-4t}$ is suddenly applied at time $t = 0$ to a series RLC circuit comprising $R = 2\Omega$, $L = 0.5H$ and $C = 1F$. Obtain the expression for the current $i(t)$ in the circuit using classing method.

Solution:

Applying KVL at $t > 0$

$$V(t) = V_R + V_L + V_C$$

$$20e^{-4t} = i * 2 + 0.5\frac{di}{dt} + \frac{1}{1}\int i\, dt \text{ --- (1)}$$

Differentiating

$$\text{Or, } -4 * 20\, e^{-4t} = 2\frac{di}{dt} + 0.5\frac{d^2i}{dt^2} + i$$

$$\text{Or, } 0.5\frac{d^2i}{dt^2} + 2\frac{di}{dt} + i = -80\, e^{-4t}$$

$$\text{Or, } \frac{d^2i}{dt^2} + 4\frac{di}{dt} + 2i = -160\, e^{-4t} \text{ ------------------ (2)}$$

> **Note:** For Exponential Excitation in 2nd order differential equation there are also two cases:
>
> - Case I : if $|\alpha| \neq |S_1|$ or $|S_2|$ then Take $i_F = Ae^{-\alpha t}$
> - Case II : if $|\alpha| = |S_1|$ or $|S_2|$ then Take $i_F = Ate^{-\alpha t}$

This is 2nd order non-homogenous differential equation, so the solution is

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$$i(t) = i_N(t) + i_F(t) \text{ -----------------(3)}$$

Now to find $i_N(t)$, the auxiliary equation of (1) is

$$S^2 + 4S + 2 = 0$$

$$S_1 = -0.58$$

$$S_2 = -3.41$$

$$i_N(t) = K_1 e^{-0.58t} + K_2 e^{-3.41t} \text{ ----------------- (4)}$$

Now to find $i_F(t)$, take a trial solution according to source present,

Here, $\alpha = 4$ and $S_1 = 0.58$ and $S_2 = 3.41$, so condition is $\alpha \neq S_1$ or S_2

$$i_F(t) = Ae^{-4t} \text{ ----------------- (5)}$$

Now, replacing the value of dependent variable of equation (1) by the value of trial solution of $i_F(t)$, then

$$\frac{d^2[Ae^{-4t}]}{dt^2} + 4\frac{d[Ae^{-4t}]}{dt} + 2[Ae^{-4t}] = -160\, e^{-4t}$$

$$\text{Or,} A*4*4\, e^{-4t} - 16*Ae^{-4t} + 2\, Ae^{-4t} = -160\, e^{-4t}$$

$$\text{Or,} 2\, Ae^{-4t} = -160\, e^{-4t}$$

$$\text{Or,} A = -80$$

Thus equation (5) becomes as,

$$i_F(t) = -80e^{-4t} \text{ ------------------ (6)}$$

Hence from the equation (3),(4) and (6), the complete solution is

$$i(t) = i_N(t) + i_F(t)$$

$$\text{Or,} i(t) = K_1 e^{-0.58t} + K_2 e^{-3.41t} - 80e^{-4t} \text{ ------------ (7)}$$

Now to find the value of K, put $t = 0^+$ in above equation

$$i(0^+) = K_1 + K_2 - 80$$

Now to find $i(0^+)$ use initial condition, the given circuit is de-energized $i(0^+) = 0$

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$80 = K_1 + K_2$ ---------------------- (a)

Now differentiating equation (7) we get

$$\frac{di_1}{dt} = K_1(-0.58)e^{-0.58t} + K_2(-3.41)e^{-3.41t} + 80 * 4\, e^{-4t}$$

Put t= 0^+

$$\frac{di_1}{dt}(0^+) = K_1(-0.58) + K_2(-3.41) + 320$$

To find $\frac{di_1}{dt}(0^+)$ take equation (1)

$$20e^{-4t} = i * 2 + 0.5\frac{di}{dt} + V_C$$

Put $t = 0^+$

$\frac{di_1}{dt}(0^+) = 40$

Thus $40 = K_1(-0.58) + K_2(-3.41) + 320$ ----------- (b)

Solving equation (a) and (b) we get,

$K_2 = 82.42$

$K_1 = -2.42$

Hence the equation (7) becomes

$$i(t) = -2.42\, e^{-0.58t} + 82.42e^{-3.41t} - 80e^{-4t} \quad \textbf{Amp}$$

Q.16 An exponential voltage $V(t) = 20e^{-3t}$ is suddenly applied at time $t = 0$ to a series RLC circuit comprising $R = 4\Omega$, $L = 1H$ and $C = \frac{1}{3}F$. Obtain the expression for the current i(t) in the circuit using classing method.

Solution:

Applying KVL at $t > 0$

$V(t) = V_R + V_L + V_C$

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$$20e^{-3t} = i * 4 + \frac{di}{dt} + 3\int i\,dt \text{ ------------------- } (1)$$

Differentiating

$$\text{Or,} -3 * 20\, e^{-3t} = 4\frac{di}{dt} + \frac{d^2i}{dt^2} + 3\,i$$

$$\text{Or,} \frac{d^2i}{dt^2} + 4\frac{di}{dt} + 3\,i = -60\, e^{-3t} \text{ ------------------ } (2)$$

This is 2^{nd} order non-homogenous differential equation, so the solution is

$$i\,(t) = i_N(t) + i_F(t) \text{ ------------------}(3)$$

Now to find $i_N(t)$, the auxiliary equation of (1) is

$$S^2 + 4S + 3 = 0$$

$$S_1 = -1$$

$$S_2 = -3$$

$$i_N(t) = K_1e^{-t} + K_2e^{-3t} \text{ ------------------ } (4)$$

Now to find $i_F(t)$, take a trial solution according to source present,

Here, $\alpha = 3$ and $S_1 = 1$ and $S_2 = 3$, so condition is $\alpha = S_1$ or S_2

$$i_F(t) = A\,te^{-4t} \text{ ------------------ } (5)$$

Now, replacing the value of dependent variable of equation (1) by the value of trial solution of $i_F(t)$, then

$$\frac{d^2[Ate^{-3t}]}{dt^2} + 4\frac{d[Ate^{-3t}]}{dt} + 3[Ate^{-3t}] = -60\,e^{-3t}$$

$$\text{Or,} \frac{d[Ae^{-3t} - 3Ate^{-3t}]}{dt} + 4[Ae^{-3t} - 3Ate^{-3t}] + 3[Ate^{-3t}] = -60\,e^{-3t}$$

Or, $$-3Ae^{-3t} - 3[Ae^{-3t} - 3Ate^{-3t}] + 4[Ae^{-3t} - 3Ate^{-3t}] + 3[Ate^{-3t}] = -60\,e^{-3t}$$

Or, $$-3Ae^{-3t} - 3Ae^{-3t} + 9Ate^{-3t} + 4Ae^{-3t} - 12Ate^{-3t} + 3Ate^{-3t} = -60\,e^{-3t}$$

Or, $$-2Ae^{-3t} = -60\,e^{-3t}$$

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$A = 30$

Thus equation (5) becomes as,

$i_F(t) = 30te^{-4t}$ -------------------- (6)

Hence from the equation (3),(4) and (6), the complete solution is

$i(t) = i_N(t) + i_F(t)$

Or, $i(t) = K_1e^{-t} + K_2e^{-3t} + 30te^{-3t}$------------ (7)

Now to find the value of K, put $t = 0^+$ in above equation

$i(0^+) = K_1 + K_2 + 0$

Now to find $i(0^+)$ use initial condition, the given circuit is de-energized $i(0^+) = 0$

$0 = K_1 + K_2$ ---------------------- (a)

Now differentiating equation (7) we get

$$\frac{di_1}{dt} = K_1(-1)e^{-t} + K_2(-3)e^{-3t} + 30e^{-3t} - 90te^{-3t}$$

Put $t = 0^+$

$$\frac{di_1}{dt}(0^+) = K_1(-1) + K_2(-3) + 30 - 0$$

To find $\frac{di_1}{dt}(0^+)$ take equation (1)

$$20e^{-3t} = i * 4 + \frac{di}{dt} + V_C$$

Put $t = 0^+$

$$\frac{di_1}{dt}(0^+) = 20$$

Thus, $-10 = K_1(-1) + K_2(-3)$ ---------- (b)

Solving equation (a) and (b) we get,

$K_1 = -5$

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$K_2 = 5$

Hence the equation (7) becomes

$$i(t) = -5e^{-t} + 5e^{-3t} + 30te^{-3t} \text{ Amp}$$

Sinusoidal Excitation:

Q.17: In a RLC series circuit containing R = 5Ω, L = 1H and C = 0.25 F a sinusoidal voltage V(t) = 25 Sin10t is suddenly applied at time t = 0. Using classical method, find the resultant current i(t) for t > 0. Assume zero current through inductor and zero voltage across capacitor before application of voltage.

Solution:

Applying KVL at t > 0

$$V(t) = V_R + V_L + V_C$$

$$25\ \text{Sin}10t = i * 5 + 1\frac{di}{dt} + 4\int i\, dt \text{ ---------------- (1)}$$

Differentiating

$$\text{Or, } 250\,\text{Cos}10t = 5\frac{di}{dt} + \frac{d^2i}{dt^2} + 4i$$

$$\text{Or, } \frac{d^2i}{dt^2} + 5\frac{di}{dt} + 4i = 250\ \text{Cos}10t \text{ ------------------ (2)}$$

This is 2nd order non-homogenous differential equation, so the solution is

$$i(t) = i_N(t) + i_F(t) \text{ ------------------(3)}$$

Now to find $i_N(t)$, the auxiliary equation of (1) is

$$S^2 + 5S + 4 = 0$$

$$S_1 = -1$$

$$S_2 = -4$$

$$i_N(t) = K_1e^{-t} + K_2e^{-4t} \text{ ------------------ (4)}$$

Now to find $i_F(t)$, take a trial solution according to source present,

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$i_F(t) = A\ Cos10t + B\ Sin10t$ ----------------- (5)

Now, replacing the value of dependent variable of equation (1) by the value of trial solution of $i_F(t)$, then

Or. $\frac{d^2[A\ Cos10t + B\ Sin10t]}{dt^2} + 4\frac{d[A\ Cos10t + B\ Sin10t]}{dt} + 3[A\ Cos10t + B\ Sin10t] =$
$250\ Cos10t$

Or. $-100A\ Cos10t - 100B\ Sin10t - 50A\ Sin10t + 50B\ Cos10t + 4A\ Cos10t + 4B\ Sin10t =$
$250\ Cos10t$

Or. $Sin10t\{-100B - 50A + 4B\} + Cos10t\{-100A + 50B + 4A\} = 250\ Cos10t$

Equating the coefficient of Cosine and Sine, we get

$-50A - 96B = 0$ ------------ (a)

$-96A + 50B = 250$ ----------- (b)

Solving equation (a) and (b)

$A = -2.04$

$B = 1.06$

Thus equation (5) becomes as,

$i_F(t) = -2.04Cos10t + 1.06Sin10t$ ----------------- (6)

Hence from the equation (3),(4) and (6), the complete solution is

$i\ (t) = i_N(t) + i_F(t)$

Or, $i\ (t) = K_1e^{-t} + K_2e^{-4t} - 2.04\ Cos10t + 1.06\ Sin10t$ ---------
(7)

Now to find the value of K, put $t = 0^+$ in above equation

$i\ (0^+) = K_1 + K_2 - 2.04$

Now to find $i\ (0^+)$ use initial condition, the given circuit is de-energized $i(0^+) = 0$

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$2.04 = K_1 + K_2$ ---------------------- (a)

Now differentiating equation (7) we get

$$\frac{di_1}{dt} = K_1(-1)e^{-t} + K_2(-4)e^{-4t} + 20.4\ \mathrm{Sin}10t + 10.6\ \mathrm{Cos}10t$$

Put $t = 0^+$.

$$\frac{di_1}{dt}(0^+) = K_1(-1) + K_2(-4) + 0 + 10.6,$$

To find $\frac{di_1}{dt}(0^+)$ take equation (1)

$$25\ \mathrm{Sin}10t = i * 5 + 1\frac{di}{dt} + V_C$$

Put $t = 0^+$

$$\frac{di_1}{dt}(0^+) = 0$$

Thus, $-10.6 = K_1(-1) + K_2(-4)$ ----------- (b)

Solving equation (a) and (b) we get,

$K_1 = -0.82$

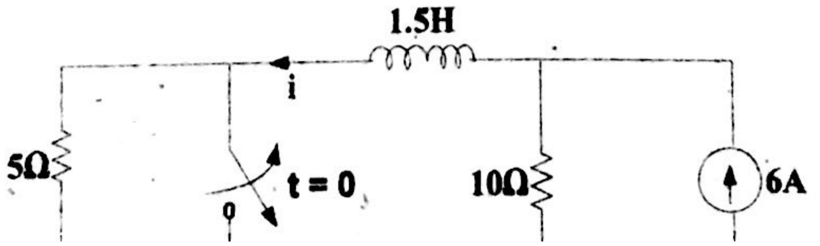
$K_2 = 2.87$

Hence the equation (7) becomes

$$\mathbf{i(t) = -0.82e^{-t} + 2.87\,e^{-4t} - 2.04\ Cos10t + 1.06\ Sin10t \quad Amp}$$

Problems:

Q.1: The switch in figure has been closed for a long time. It opens at t = 0. Find i(t) at t > 0. Use classical method.



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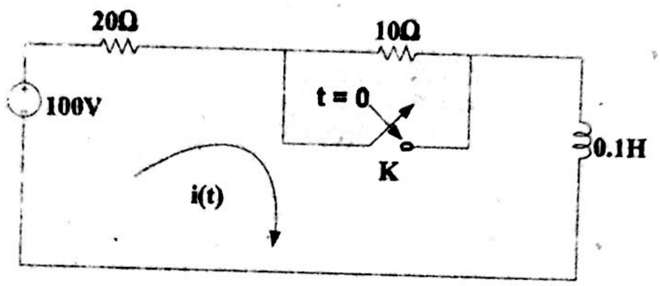
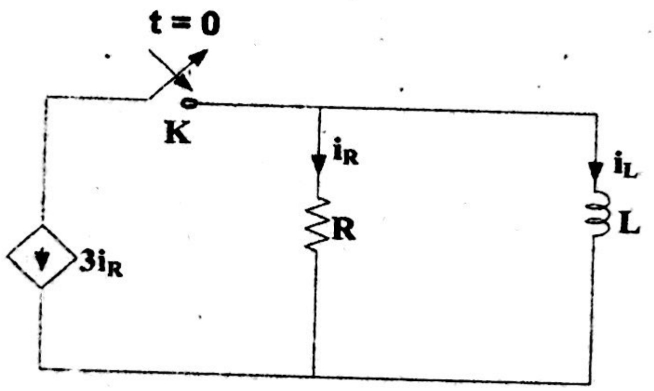
[Ans: $i(t) = \left(4 + 2e^{-10t}\right)$]

Q.2: The switch in figure has been opened for a long time. It is closed at t = 0. Find the complete expression for the current using classical method.



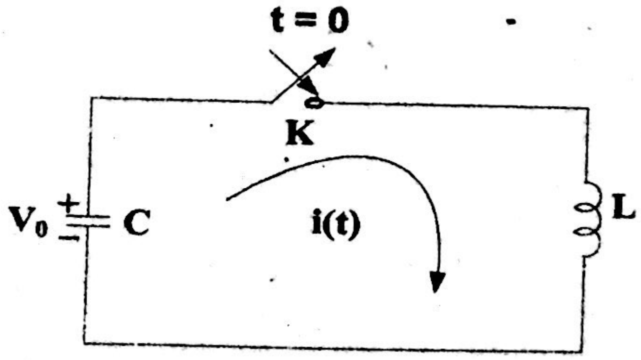
[Ans: i(t)= $(-1.67e^{-200t} + 5)$Amp]

Q.3: If the initial current through the inductor be 1A, find $i_L(t)$ at t > 0 in the circuit below using classical method.



[Ans: $i_L(t) = e^{-\frac{R}{4L}*t}$ Amp

Q.4: In the network shown, C is initially charged to V_0, the switch K is closed at t = 0. Solve for the current i(t), Using Classical method.



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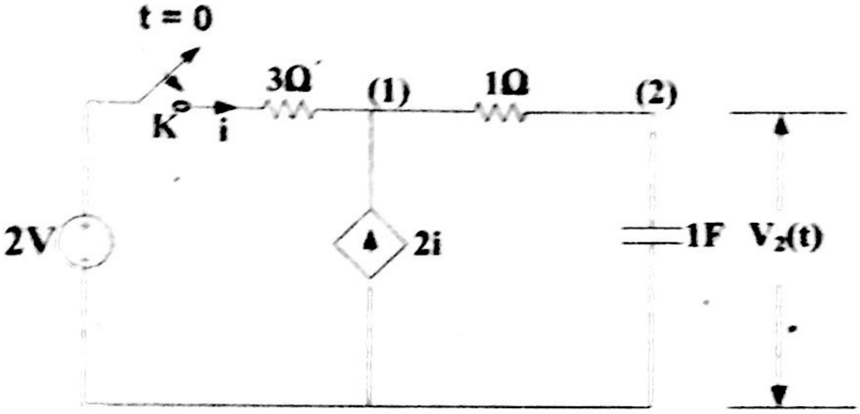
[Ans: $i(t) = \sqrt{\frac{C}{L}} \sin \frac{1}{\sqrt{LC}} t$ Amp]

Q.5: Find the voltage across the capacitor at t > 0 using classical method.



Hint: At node 1: $\frac{V_1 - V_2}{1} = i + 2i = 3i = 3\left(\frac{2 - V_1}{3}\right)$

$$V_1 - V_2 = 2 - V_1$$

$$V_1 = \frac{2 - V_2}{2} \text{ ------- } (1)$$

At node 2: $\frac{V_2 - V_1}{1} + 1\frac{dV_2}{dt} = 0$

$$V_2 - \left(\frac{2 - V_2}{2}\right) + \frac{dV_2}{dt} = 0$$

$$\frac{dV_2}{dt} + \frac{1}{2}V_2 = 1$$

[Ans: $V_2(t) = (2 - 2e^{-0.5t})$ Volt]

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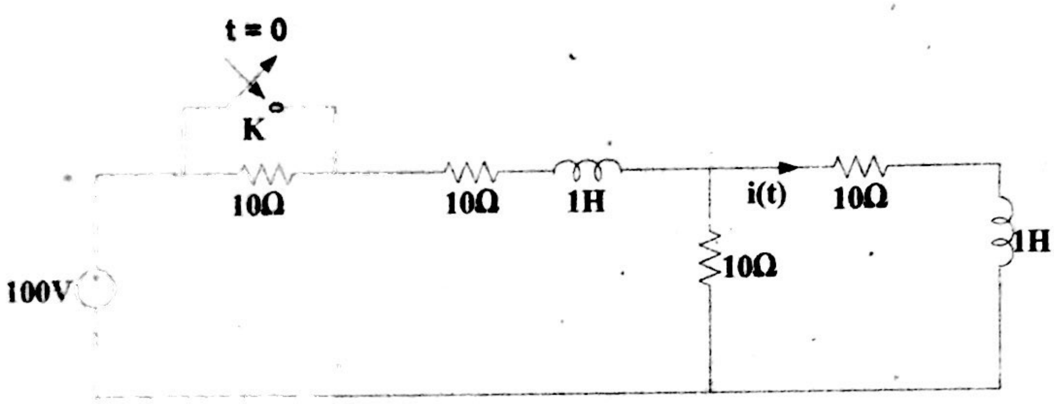
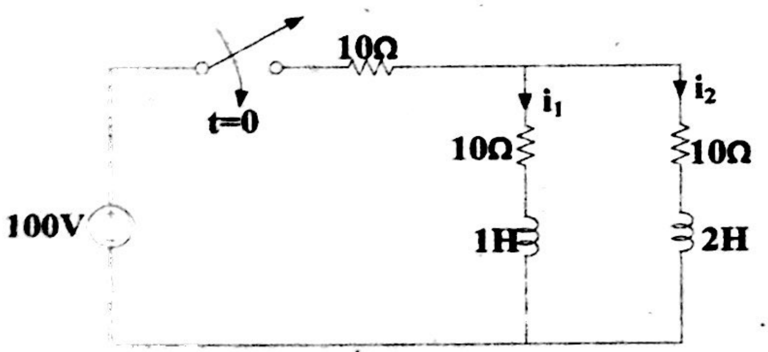
#Q.6:The network shown in figure below is under steady state condition. The switch is closed at t = 0. Determine the current through 10 ohm resistor connected between terminals AB.[Use classical method]



[Ans: $i(t) = (-2.495e^{-10t} + 0.412e^{-30t} + 3.3)$ Amp]

Q.7:In the circuit shown in figure below the switch is closed at t= 0, find $i_1(t)$ using classical method.



[Ans: $i(t) = (-4.5e^{-23.65t} + 1.121e^{-6.35t} + 3.3)$ Amp]

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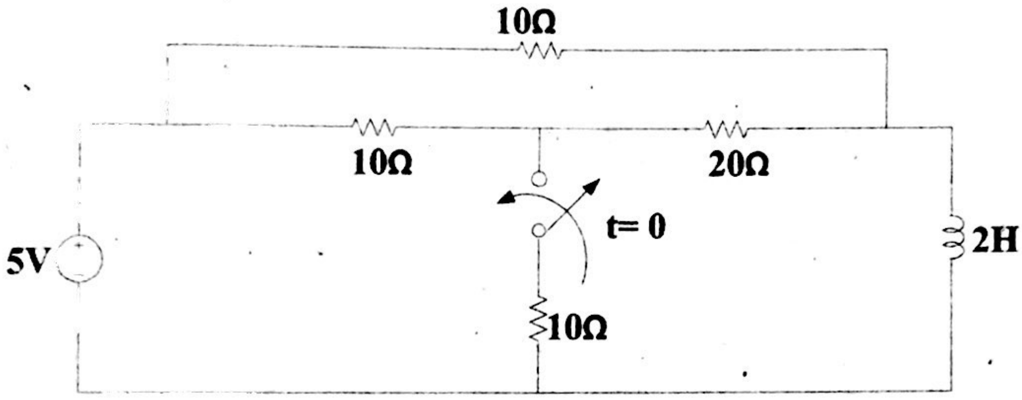
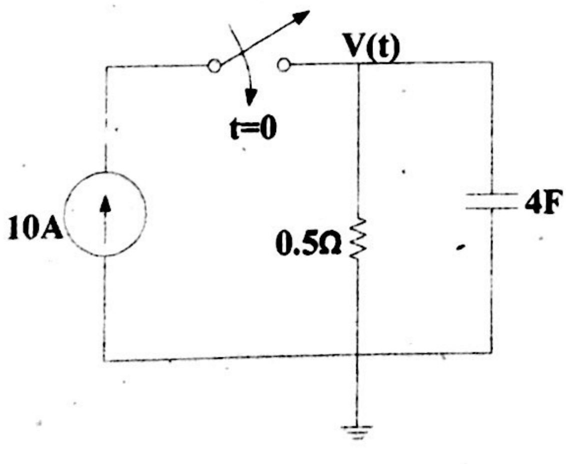
Q.8: In the circuit of the figure below, the switch is open and the circuit reaches a steady state. At t=0, the switch is closed. Find the current in the inductor for $t > 0$ using classical method.



[Hint: Use $\Delta - Y$ transformation]

[Ans: $i(t) = (0.067e^{-3.57t} + 0.6)$ Amp]

Q.9: In the circuit of the figure below, the switch is close at t=0, find V (t) for $t > 0$ if the initial capacitor voltage be 2V, use classical method.



[Ans: $V(t) = (3e^{-0.5t} + 5)$ Volt]

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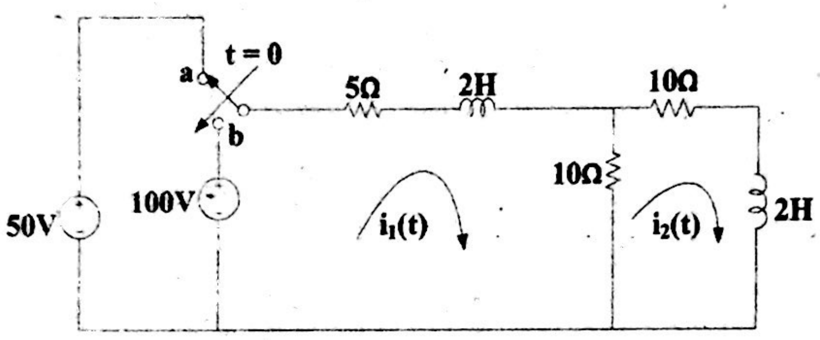
Q.10:In the circuit shown, switch is changed from position "a" to "b" at t = 0. Find the expression for current i1(t) and i2(t) using classical metod.



[Ans: $i_1(t) = (10 - 4.31e^{-3.59t} + 0.68e^{-13.90t})$ A,

$i_2(t) = (5 - 3.37e^{-3.59t} + 0.87e^{-13.90t})$ A]

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Transient Analysis By Laplace Transform

Some Important Formula:

- $L[i(t)] = I$
- $L[v(t)] = V$
- $L[1] = \frac{1}{S}$
- $L[\frac{di}{dt}] = SI - i(0^+)$
- $L[\frac{d^2i}{dt^2}] = S^2I - Si(0^+) - i^{\prime}(0^+)$
- $L[\int_0^t i\, dt] = \frac{I}{S}$
- $L[e^{-at}] = \frac{1}{(S+a)}$
- $L[Sinbt] = \frac{b}{b^2+S^2}$
- $L[Cosbt] = \frac{S}{b^2+S^2}$
- $L[te^{-at}] = \frac{1}{(S+a)^2}$
- $L[e^{-at}Sinbt] = \frac{b}{(S+a)^2+b^2}$
- $L[e^{-at}Cosbt] = \frac{S+a}{(S+a)^2+b^2}$

#Steps to be applied for solving any circuit by Laplace Transform

- Apply KVL or KCL in the circuit at $t > 0$
- After application of KVL or KCL we will always get differential equation, so take the transform on both sides of the equation.
- After that we will get a equation containing initial condition, so substitute the value of initial condition
- Keep the dependent variable on the one side of equation and remaining term on the other side of the equation then apply **Heaviside's partial fraction**.
- Finally take the inverse Laplace Transform to get the dependent variable as a function of time

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#Heaviside's Partial Fraction Expression:

Let any function, $I(S) = \frac{P(S)}{Q(S)}$

1) If all the roots of Q(S) are non repeated

$$I(S) = \frac{P(S)}{(S+S_1)(S+S_2)......(S+S_n)}$$

$$I(S) = \frac{K_1}{S+S_1} + \frac{K_2}{S+S_2} + \text{--------} + \frac{K_n}{S+S_n}$$

To find K_1

$$K_1 = \frac{P(S)}{(S+S_1)(S+S_2)......(S+S_n)} * (S + S_1)$$

$$K_1 = \frac{P(S)}{(S+S_2)......(S+S_n)} \quad \text{then put } S = -S_1$$

And so on.

$$K_n = \frac{P(S)}{(S+S_1)(S+S_2)......(S+S_n)} * (S + S_n)$$

$$K_n = \frac{P(S)}{(S+S_1)(S+S_2)......} \text{ then put } S = -S_n$$

For Example 4.1: $I = \frac{(S+2)}{S(S+5)(S+6)(S+1)}$

$$I = \frac{K_1}{S} + \frac{K_2}{S+5} + \frac{K_3}{S+6} + \frac{K_4}{S+1}$$

For K_1

$$K_1 = \frac{(S+2)}{S(S+5)(S+6)(S+1)} * S$$

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$K_1 = \frac{(S+2)}{(S+5)(S+6)(S+1)}$ then put $S = 0$

$K_1 = \frac{(2)}{(5)(6)(1)}$

$K_1 = \frac{1}{15}$

And For K_2

$K_2 = \frac{(S+2)}{S(S+5)(S+6)(S+1)} * (S + 5)$

$K_2 = \frac{(S+2)}{S(S+6)(S+1)}$ then put $S = -5$

$K_2 = \frac{(-5+2)}{(-5)(-5+6)(-5+1)}$

$K_2 = \frac{-3}{20}$

And For K_3

$K_3 = \frac{(S+2)}{S(S+5)(S+6)(S+1)} * (S + 6)$

$K_3 = \frac{(S+2)}{S(S+5)(S+1)}$ then put $S = -6$

$K_3 = \frac{(-6+2)}{(-6)(-6+5)(-6+1)}$

$K_3 = \frac{2}{15}$

And For K_4

$K_4 = \frac{(S+2)}{S(S+5)(S+6)(S+1)} * (S + 1)$

$K_4 = \frac{(S+2)}{S(S+5)(S+6)}$ then put $S = -1$

$K_4 = \frac{(-1+2)}{(-1)(-1+5)(-1+6)}$

$K_4 = \frac{-1}{20}$

Thus

$$I = \frac{1}{15} * \frac{1}{S} - \frac{3}{20} * \frac{1}{S+5} + \frac{2}{15} * \frac{1}{S+6} - \frac{1}{20} * \frac{1}{S+1}$$

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2) **If some of roots of Q(s) has power**

$$I = \frac{P(S)}{(S+S_i)^r}$$

$$I = \frac{P(S)}{(S+S_i)^1(S+S_i)^2 \ldots\ldots (S+S_i)^r}$$

$$I = \frac{K_{11}}{(S+S_i)^1} + \frac{K_{12}}{(S+S_i)^2} + --- + \frac{K_{1r}}{(S+S_i)^r} + ---$$

To find K_{1r}

$$K_{1r} = \frac{P(S)}{(S+S_i)^1(S+S_i)^2 \ldots\ldots (S+S_i)^r} * (S + S_i)^r$$

$$K_{1r} = \frac{P(S)}{(S+S_i)^1(S+S_i)^2 \ldots\ldots} \text{ then put } S = -S_i$$

To find $K_{1(r-1)}$

$$K_{1(r-1)} = \frac{1}{1!}\frac{d}{dS}\left[\frac{P(S)}{(S+S_i)^1(S+S_i)^2 \ldots\ldots}\right] \text{ after doing differentiating then put } S = -S_i$$

To find $K_{1(r-2)}$

$$K_{1(r-2)} = \frac{1}{2!}\frac{d^2}{dS^2}\left[\frac{P(S)}{(S+S_i)^1(S+S_i)^2 \ldots\ldots}\right] \text{ after doing double differentiating then put } S = -S_i$$

And so on up to

$$K_{11} = \frac{1}{(r-1)!}\frac{d^{(r-1)}}{dS^{(r-1)}}\left[\frac{P(S)}{(S+S_i)^1(S+S_i)^2 \ldots\ldots}\right] \text{ after doing (r-1) times differentiating then put } S = -S_i$$

For Example 4.2: $I(S) = \frac{S}{(S+1)^3(S+4)}$

$$I = \frac{K_{11}}{(S+1)^1} + \frac{K_{12}}{(S+1)^2} + \frac{K_{13}}{(S+1)^3} + \frac{K}{S+4}$$

To find K

$$K = \frac{S}{(S+1)^3(S+4)} * (S + 4)$$

$$K = \frac{S}{(S+1)^3} \text{ then put } S = -4$$

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$K = \frac{-4}{(-4+1)^3}$

$K = \frac{4}{27}$

To find K_{13}

$K_{13} = \frac{S}{(S+1)^3(S+4)} * (S+1)^3$

$K_{13} = \frac{S}{(S+4)}$ then put $S = -1$

$K_{13} = \frac{-1}{(-1+4)}$

$K_{13} = \frac{-1}{3}$

To find K_{12}

$K_{12} = \frac{S}{(S+1)^3(S+4)} * (S+1)^3$

$K_{12} = \frac{1}{1!}\frac{d}{dS}\left[\frac{S}{(S+4)}\right]$

$K_{12} = \frac{4}{(S+4)^2}$ then put $S = -1$

$K_{12} = \frac{4}{(-1+4)^2} = \frac{4}{9}$

To find K_{11}

$K_{11} = \frac{S}{(S+1)^3(S+4)} * (S+1)^3$

$K_{11} = \frac{1}{2!}\frac{d^2}{dS^2}\left[\frac{S}{(S+4)}\right]$

$K_{11} = \frac{1}{2!}\frac{d}{dS}\left[\frac{4}{(S+4)^2}\right]$

$K_{11} = \frac{(S+4)^2-8(S+4)}{(S+4)^4}$ then put $S = -1$

$K_{11} = \frac{-24)}{9}$

> **NOTE:** It is not necessarily important to use **Heaviside's partial fraction** for solving partial fraction on Laplace Transform if student know another process for solving partial fraction it is relevant.

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3) If some of roots of Q(S) are Complex

$$I = \frac{P(S)}{(S+\alpha-J\beta)(S+\alpha+J\beta)}$$

$$I = \frac{K}{S+\alpha-J\beta} + \frac{K^*}{S+\alpha+J\beta}$$

To find K

$$K = \frac{P(S)}{(S+\alpha-J\beta)(S+\alpha+J\beta)} * (S+\alpha-J\beta)$$

$$K = \frac{P(S)}{(S+\alpha+J\beta)} \quad \text{then put } S = -\alpha + J\beta$$

For Example 4.3: $I = \frac{S}{(S+2-J3)(S+2+J3)}$

$$I = \frac{K}{S+2-J3} + \frac{K^*}{S+2+J3}$$

To find K

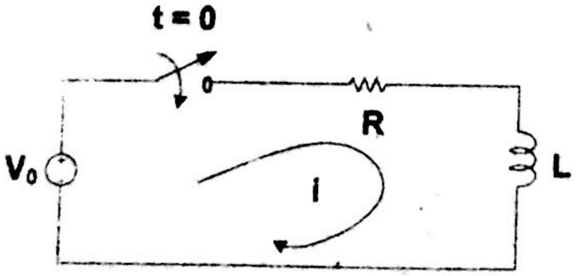
$$K = \frac{S}{(S+2-J3)(S+2+J3)} * (S+2-J3)$$

$$K = \frac{S}{(S+2+J3)} \quad \text{Then put } S = -2 + J3$$

$$K = \frac{-2+J3}{(-2+J3+2+J3)}$$

$$K = \frac{-2+J3}{J6} = \frac{1}{2} + J\frac{1}{3} \text{ and } K^* = \frac{1}{2} - J\frac{1}{3}$$

Find the expression of current i (t) using classical method



Applying KVL at t > 0

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$$V_0 = V_R + V_L$$

$$\text{Or,} V_0 = i * R + L\frac{di}{dt}$$

$$\text{Or,} \frac{di}{dt} + \frac{R}{L} * i = \frac{V_0}{L}$$

Taking Laplace Transform on both sides

$$\text{Or,} SI - i(0^+) + \frac{R}{L} * I = \frac{V_0}{SL}$$

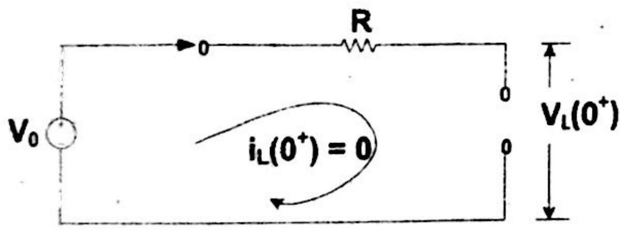
Note: For solving homogenous differential equation or non-homogenous differential equation from Laplace Transform method the procedures are same, procedure doesn't depend upon source present like classical method. I.e. solve every problem with same procedure.

Now to find i (0^+) use initial condition, the given circuit is de-energized circuit so $i(0^-) = 0$

From continuity relation of inductor

$i_L(0^-) = i_L(0^+) = 0$

Circuit at $t= 0^+$ is



$i(0^+) = 0$

$$\text{Or,} SI + \frac{R}{L} * I = \frac{V_0}{SL}$$

$$\text{Or,} I \left[S + \frac{R}{L}\right] = \frac{V_0}{SL}$$

$$\text{Or,} I\left[\frac{SL+R}{L}\right] = \frac{V_0}{SL}$$

$$\text{Or,} I = \frac{V_0/L}{S\left[S+\frac{R}{L}\right]}$$

Now applying partial fraction,

$$I = \frac{V_0/L}{S\left[S+\frac{R}{L}\right]} = \frac{K_1}{S} + \frac{K_2}{S+\frac{R}{L}} \text{ -------------- (i)}$$

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For K_1

$$K_1 = \frac{V_0/L}{S\left[S+\frac{R}{L}\right]} * S$$

$$K_1 = \frac{V_0/L}{\left[S+\frac{R}{L}\right]} \text{ then put } S = 0$$

$$K_1 = \frac{V_0/L}{\left[0+\frac{R}{L}\right]}$$

$$K_1 = \frac{V_0/L}{\frac{R}{L}}$$

$$K_1 = \frac{V_0}{R}$$

And For K_2

$$K_2 = \frac{V_0/L}{S\left[S+\frac{R}{L}\right]} * \left(S + \frac{R}{L}\right)$$

$$K_2 = \frac{V_0/L}{S} \text{ then put } S = -\frac{R}{L}$$

$$K_2 = \frac{V_0/L}{-\frac{R}{L}}$$

$$K_2 = -\frac{V_0}{R}$$

Thus equation (i) becomes as

$$I = \frac{\frac{V_0}{R}}{S} + \frac{-\frac{V_0}{R}}{S+\frac{R}{L}}$$

$$I = \frac{V_0}{RS} - \frac{1}{R} * \frac{V_0}{S+\frac{R}{L}}$$

Now taking Inverse Laplace Transform, we get

$$i(t) = \frac{V_0}{R} - \frac{V_0}{R} \quad e^{-\frac{R}{L}t}$$

$$\left[\, i(t) = \frac{V_0}{R}\left(1 - e^{-\frac{R}{L}t}\right) \text{Amp} \,\right]$$

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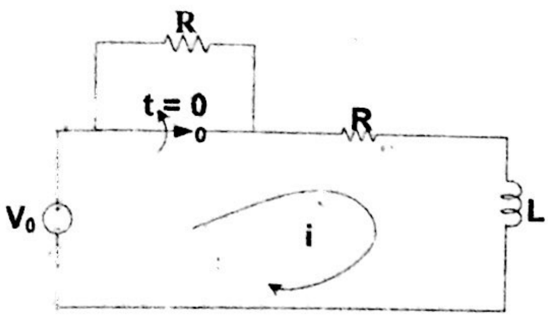
Find the expression of voltage across inductor V_L (t) using Laplace Transform method



Note: While using L.T on integration there must be limit of '0' to 't' otherwise L.T method cannot be used.

Applying KVL at t > 0

$V_0 = 2V_R + v_L$

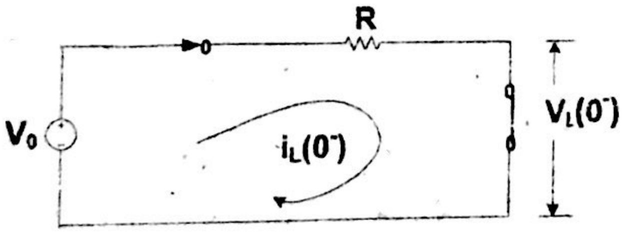
Or, $V_0 = i * 2R + v_L$

Or, $V_0 = [\frac{1}{L}\int_{-\infty}^{t} v_L\, dt]*2R + v_L$ [since $i = i_L = \frac{1}{L}\int V_L\, dt$]

Or, $V_0 = [\frac{1}{L}\int_{-\infty}^{0} v_L\, dt + \frac{1}{L}\int_{0}^{t} v_L\, dt]* 2R + v_L$

Or, $V_0 = [\, i_L(0^-) + \frac{2R}{L}\int_{0}^{t} v_L\, dt]*2R + v_L$ [since $i_L(0^-) = \frac{1}{L}\int_{-\infty}^{0} v_L\, dt$]

Now to find $i_L(0^+)$ use initial condition, the given circuit is energized so circuit at $t = 0^-$



From the circuit $i_L(0^-) = \frac{V_0}{R}$

And $V_L(0^-) = 0$

From continuity relation for inductor, $i_L(0^-) = i_L(0^+) = \frac{V_0}{R}$

Now, circuit at $t = 0^+$ is

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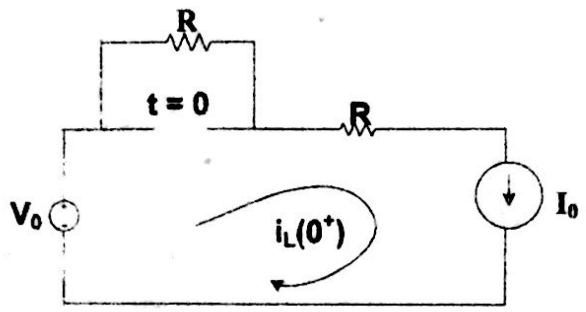

From the circuit $i_L(0^+) = \frac{V_0}{R}$

Thus equation becomes as

Or. $V_0 = [\frac{V_0}{R} + \frac{2R}{L}\int_0^t v_L\, dt]*2R + v_L$

Taking Laplace Transform on both sides

Or, $\frac{V_0}{S} = \frac{V_0}{SR} * 2R + \frac{V_L}{SL} * 2R + V_L$

Or, $V_L\, [\frac{2R+SL}{SL}] = -\frac{V_0}{S}$

Or, $V_L = \frac{-V_0 * L}{(SL+2R)}$

Or, $V_L = \frac{-V_0 * L}{L\left(S+\frac{2R}{L}\right)}$

Or, $V_L = \frac{-V_0}{\left(S+\frac{2R}{L}\right)}$

Now taking Inverse Laplace Transform, we get

$[V_L(t) = -V_0\ e^{-\frac{2R}{L}t}$ Volt]

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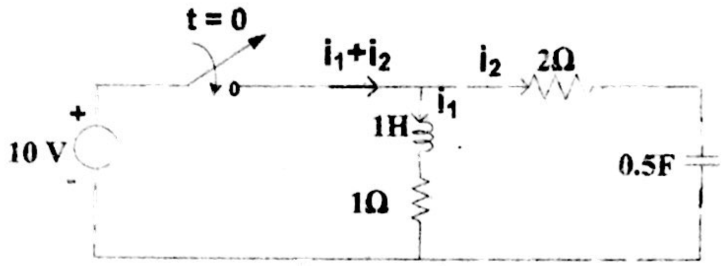
Q.1: In the network shown, the switch is closed at t =0. Find the current supplied by the source using Laplace Transform method.



Solution:

Applying KVL at outer loop at t > 0

$$10 = 2\,i_2 + \frac{1}{0.5}\int_{-\infty}^{t} i_2 dt$$

$$10 = 2\,i_2 + \frac{1}{0.5}\int_{-\infty}^{0} i_2 dt + \frac{1}{0.5}\int_{0}^{t} i_2 dt$$

$$10 = 2\,i_2 + V_C(0^-) + \frac{1}{0.5}\int_{0}^{t} i_2 dt$$

Since it's a de-energized condition so $V_C(0^+) = V_C(0^-) = 0$ and also $i_1(0^+) = i_1(0^-) = 0$

$$10 = 2\,i_2 + 0 + \frac{1}{0.5}\int_{0}^{t} i_2 dt$$

Taking Laplace Transform

$$\frac{10}{S} = 2\,I_2 + 2 * \frac{I_2}{S}$$

$$\frac{10}{S} = I_2\left[2 + \frac{2}{S}\right]$$

$$I_2 = \frac{5}{s+1}$$

Taking Inverse Laplace Transform

$$i_2(t) = 5\,e^{-1t} \text{ -------------- (i)}$$

And,

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Now applying KVL at loop (1) at $t > 0$

$$10 = 1i_1 + 1\frac{di_1}{dt}$$

Taking Laplace Transform

$$\frac{10}{S} = I_1 + S\,I_1 - i_1(0^+)$$

$$\frac{10}{S} = I_1 + S\,I_1 - 0$$

$$I_1 = \frac{10}{S(S+1)}$$

Using partial fraction

$$\frac{10}{S(S+1)} = \frac{A}{S} + \frac{B}{(S+1)} \text{ -------- (ii)}$$

To find A

$$A = \frac{10}{S(S+1)} * S$$

$$A = \frac{10}{(S+1)} \quad \text{then put } S = 0$$

$$A = 10$$

To find B

$$B = \frac{10}{S(S+1)} * (S+1)$$

$$B = \frac{10}{S} \quad \text{then put } S = -1$$

$$B = -10$$

Thus equation(ii) becomes

$$I_1 = \frac{10}{S(S+1)} = \frac{10}{S} - \frac{10}{(S+1)}$$

$$I_1 = \frac{10}{S} - \frac{10}{(S+1)}$$

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Taking Inverse Laplace Transform

$i_1(t) = 10 - 10e^{-1t}$ -------------- (iii)

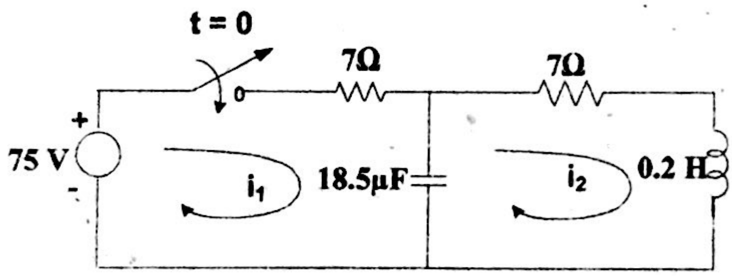
Thus, according to the question the current supplied by the source is

$i = i_1 + i_2$

$i(t) = 10 - 10e^{-1t} + 5\,e^{-1t}$

$\mathbf{i(t) = 10 - 5e^{-1t}}$ **Amp**

Q.2: In the network shown in figure below the switch K is closed at t = 0. Using Laplace Transformation method find the two mesh currents i_1 and i_2.



Solution:

Applying KVL at outer loop at t > 0

$$75 = 7i_1 + 7i_2 + 0.2\frac{di_2}{dt}$$

Taking L.T on both sides

$$\frac{75}{S} = 7I_1 + 7I_2 + 0.2\left(SI_2 - i_2(0^+)\right)$$

Since it's a de-energized condition so $i_2(0^+) = i_2(0^-) = 0$ and also $V_C(0^+) = V_C(0^-) = 0$

$$\frac{75}{S} = 7I_1 + 7I_2 + 0.2(SI_2 - 0)$$

$$\frac{75}{S} = 7I_1 + I_2(7 + 0.2S) \text{ ---------------- (i)}$$

Applying KVL at loop (1) at t > 0

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$$75 = 7i_1 + \frac{1}{18.5*10^{-6}}\int_{-\infty}^{t}(i_1 - i_2)dt$$

$$75 = 7i_1 + \frac{1}{18.5*10^{-6}}\int_{-\infty}^{0}(i_1 - i_2)dt + \frac{1}{18.5*10^{-6}}\int_{0}^{t}(i_1 - i_2)dt$$

$$75 = 7i_1 + V_C(0^-) + \frac{1}{18.5*10^{-6}}\int_{0}^{t}(i_1 - i_2)dt$$

$$75 = 7i_1 + 0 + \frac{1}{18.5*10^{-6}}\int_{0}^{t}(i_1 - i_2)dt$$

Taking L.T on both sides

$$\frac{75}{S} = 7I_1 + \frac{1}{18.5*10^{-6}} * \frac{I_1}{S} - \frac{1}{18.5*10^{-6}} * \frac{I_2}{S}$$

$$\frac{75}{S} = 7I_1 + 54054.05 * \frac{I_1}{S} - 54054.05 * \frac{I_2}{S}$$

$$\frac{75}{S} = I_1\left(\frac{7S+54054.05}{S}\right) - 54054.05 * \frac{I_2}{S}$$

$$75 = I_1(7S + 54054.05) - I_2 * 54054.05 \text{ ------------- (ii)}$$

Putting this two equation in matrix form and solve from Cramer's Rule, we get

$$\begin{bmatrix} \frac{75}{S} \\ 75 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} * \begin{bmatrix} 7 & (7 + 0.2S) \\ (7S + 54054.05) & -54054.05 \end{bmatrix}$$

Thus

$$\Delta = \begin{bmatrix} 7 & (7 + 0.2S) \\ (7S + 54054.05) & -54054.05 \end{bmatrix}$$

$$\Delta = -7 * 54054.05 - (7 + 0.2S) * (7S + 54054.05)$$

$$\Delta = -378378.35 - (49S + 378378.35 + 1.4S^2 + 10810.81S)$$

$$\Delta = -378378.35 - (378378.35 + 1.4S^2 + 10859.81S)$$

$$\Delta = -378378.35 - 378378.35 - 1.4S^2 - 10859.81S$$

$$\Delta = -1.4S^2 - 10859.81S - 756756.7$$

$$\Delta = -(1.4S^2 + 10859.81S + 756756.7)$$

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And, $\Delta_1 = \begin{bmatrix} \frac{75}{S} & (7+0.2S) \\ 75 & -54054.05 \end{bmatrix}$

$$\Delta_1 = \frac{-4054053.75}{S} - (525 + 15S)$$

$$\Delta_1 = \frac{-(15S^2+525S+4054053.75)}{S}$$

Also, $\Delta_2 = \begin{bmatrix} 7 & \frac{75}{S} \\ (7S+54054.05) & 75 \end{bmatrix}$

$$\Delta_2 = 525 - \frac{(525S-4054053.75)}{S}$$

$$\Delta_2 = \frac{-4054053.75}{S}$$

Now, $I_1 = \frac{\Delta_1}{\Delta} = \frac{\frac{-(15S^2+525S+4054053.75)}{S}}{-(1.4S^2+10859.81S+756756.7)} = \frac{15S^2+525S+4054053.75}{S*1.4*(S^2+7757S+540540.5)}$

$$I_1 = \frac{15S^2+525S+4054053.75}{1.4*S(S+70.32)(S+7686.67)} \text{------------- (iii)}$$

Using partial fraction

$$\frac{15S^2+525S+4054053.75}{1.4*S(S+70.32)(S+7686.67)} = \frac{A}{S} + \frac{B}{(S+70.32)} + \frac{C}{(S+7686.67)}$$

To find A

$$A = \frac{15S^2+525S+4054053.75}{1.4*S(S+70.32)(S+7686.67)} * S$$

$$A = \frac{15S^2+525S+4054053.75}{1.4*(S+70.32)(S+7686.67)} \quad \text{then put } S = 0$$

$$A = \frac{15*0+525*0+4054053.75}{1.4*(0+70.32)(0+7686.67)}$$

$$A = 5.35$$

To find B

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$$B = \frac{15S^2+525S+4054053.75}{1.4*S(S+70.32)(S+7686.67)} * (S + 70.32)$$

$$B = \frac{15S^2+525S+4054053.75}{1.4*S(S+7686.67)} \text{ then put } S = -70.32$$

$$B = \frac{15(-70.32)^2+525*(-70.32)+4054053.75}{-1.4*70.32(-70.32+7686.67)}$$

$$B = -5.45$$

To find C

$$C = \frac{15S^2+525S+4054053.75}{1.4*S(S+70.32)(S+7686.67)} * (S + 7686.67)$$

$$B = \frac{15S^2+525S+4054053.75}{1.4*S(S+70.32)} \text{ then put } S = -7686.67$$

$$B = \frac{15(-7686.67)^2+525*(-7686.67)+4054053.75}{-1.4*7686.67(-7686.67+70.32)}$$

$$B = 10.80$$

Thus equation (iii) becomes as

$$I_1 = \frac{15S+4054578.75}{S(S+70.32)(S+7686.67)} = \frac{5.35}{S} - \frac{5.45}{(S+70.32)} + \frac{10.80}{(S+7686.67)}$$

$$I_1 = \frac{5.35}{S} - \frac{5.45}{(S+70.32)} + \frac{10.80}{(S+7686.67)}$$

Taking I.L.T

$$\mathbf{i_1(t) = 5.35 - 5.45e^{-70.32t} + 10.80e^{-7686.67t}\ Amp}$$

$$\text{Again, } I_2 = \frac{\Delta_2}{\Delta} = \frac{\frac{-4054053.75}{S}}{-(1.4S^2+10859.81S+756756.7)} = \frac{4054053.75}{S*1.4*(S^2+7757S+540540.5)}$$

$$I_1 = \frac{4054053.75}{1.4*S(S+70.32)(S+7686.67)} \text{ ------------ (iii)}$$

Using partial fraction

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$$\frac{4054053.75}{1.4*S(S+70.32)(S+7686.67)} = \frac{A}{S} + \frac{B}{(S+70.32)} + \frac{C}{(S+7686.67)}$$

To find A

$$A = \frac{4054053.75}{1.4*S(S+70.32)(S+7686.67)} * S$$

$$A = \frac{4055053.75}{1.4*(S+70.32)(S+7686.67)} \quad \text{then put } S = 0$$

$$A = \frac{4054053.75}{1.4*(0+70.32)(0+7686.67)}$$

$$A = 5.35$$

To find B

$$B = \frac{4054053.75}{1.4*S(S+70.32)(S+7686.67)} * (S + 70.32)$$

$$B = \frac{4054053.75}{1.4*S(S+7686.67)} \quad \text{then put } S = -70.32$$

$$B = \frac{4054053.75}{-1.4*70.32(-70.32+7686.67)}$$

$$B = -5.4$$

To find C

$$C = \frac{4054053.75}{1.4*S(S+70.32)(S+7686.67)} * (S + 7686.67)$$

$$B = \frac{4054053.75}{1.4*S(S+70.32)} \quad \text{then put } S = -7686.67$$

$$B = \frac{4054053.75}{-1.4*7686.67(-7686.67+70.32)}$$

$$B = 0.049$$

Thus equation (iii) becomes as

$$I_2 = \frac{4054053.75}{S(S+70.32)(S+7686.67)} = \frac{5.35}{S} - \frac{5.4}{(S+70.32)} + \frac{0.049}{(S+7686.67)}$$

$$I_2 = \frac{5.35}{S} - \frac{5.4}{(S+70.32)} + \frac{0.049}{(S+7686.67)}$$

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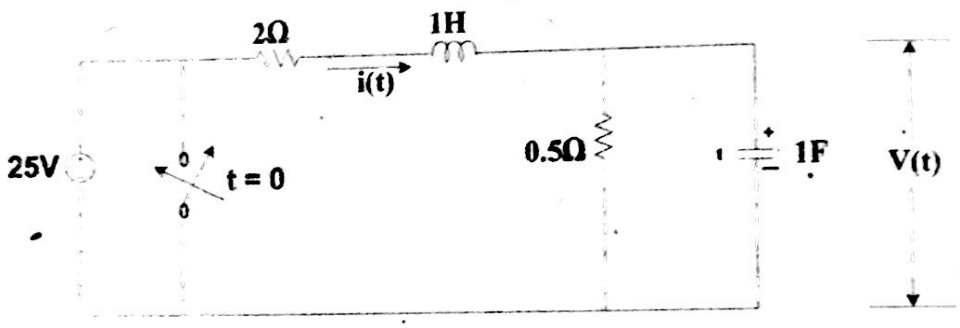
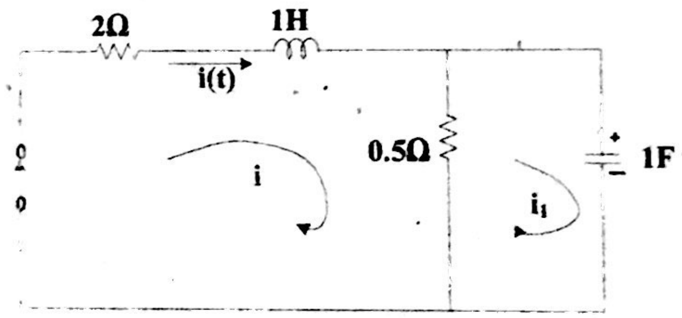
Taking I.L.T

$$i_2(t) = 5.35 - 5.4e^{-70.32t} + 0.049e^{-7686.67t} \text{ Amp}$$

Q.4: In the network shown in figure steady state is reached with the switch open. Using Laplace Transform method, determine the value of i(t) for t > 0 when the switch is closed at t = 0



Solution:



Applying KVL in loop (1) at $t > 0$

$$2i + 1\frac{di}{dt} + 0.5(i - i_1) = 0$$

Taking L.T on both sides

$$2I + SI - i(0^+) + 0.5I - 0.5I_1 = 0$$

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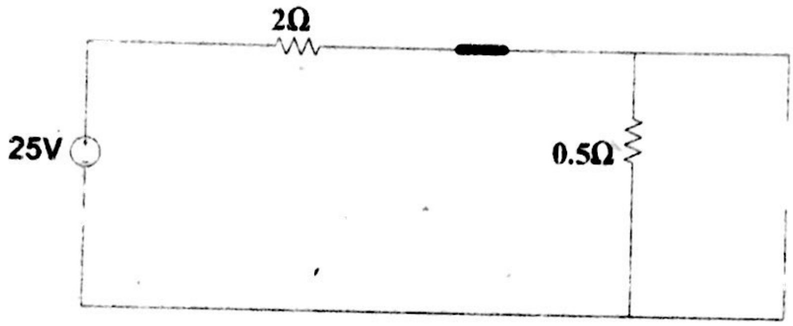

Circuit at $t = 0^-$

Since it's a Energized condition so $i(0^+) = i(0^-) = \frac{25}{2+0.5} = 10A$

And also $V_C(0^+) = V_C(0^-) = 5$ V

$$2I + SI - 10 + 0.5I - 0.5I_1 = 0$$

$$2.5I + SI - 0.5I_1 = 10$$

$$(2.5 + S)I - 0.5I_1 = 10 \text{ -------------- (i)}$$

Applying KVL in Loop (2) at $t > 0$

$$0.5(i_1 - i) + \frac{1}{1}\int_{-\infty}^{t} i_1 dt = 0$$

$$0.5(i_1 - i) + \frac{1}{1}\int_{-\infty}^{0} i_1 dt + \frac{1}{1}\int_{0}^{t} i_1 dt = 0$$

$$0.5(i_1 - i) + V_C(0^-) + \frac{1}{1}\int_{0}^{t} i_1 dt = 0$$

$$0.5(i_1 - i) + 5 + \frac{1}{1}\int_{0}^{t} i_1 dt = 0$$

Taking L.T on both sides

$$0.5I_1 - 0.5I + \frac{5}{S} + \frac{I_1}{S} = 0$$

$$\left(0.5 + \frac{1}{S}\right) I_1 - 0.5I = -\frac{5}{S}$$

$$-0.5SI + (0.5S + 1)I_1 = -5 \text{ ------- (ii)}$$

Putting two equations in matrix form and solve from Cramer's Rule, we get

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$$\begin{bmatrix}10\\-5\end{bmatrix} = \begin{bmatrix}I\\I_1\end{bmatrix} * \begin{bmatrix}(2.5+S) & -0.5\\ -0.5S & (0.5S+1)\end{bmatrix}$$

Thus

$$\Delta = \begin{bmatrix}(2.5+S) & -0.5\\ -0.5S & (0.5S+1)\end{bmatrix}$$

$$\Delta = (2.5+S) * (0.5S+1) - 0.5 * 0.5S$$

$$\Delta = 1.25S + 2.5 + 0.5S^2 + S - 0.25S$$

$$\Delta = 0.5S^2 + 2S + 2.5$$

And, $\Delta_1 = \begin{bmatrix}10 & -0.5\\ -5 & (0.5S+1)\end{bmatrix}$

$$\Delta_1 = 10(0.5S+1) - 2.5$$

$$\Delta_1 = 5S + 10 - 2.5$$

$$\Delta_1 = 5S + 7.5$$

Now, $I_1 = \frac{\Delta_1}{\Delta} = \frac{5S+7.5}{0.5S^2+2S+2.5} = \frac{5S+7.5}{0.5(S^2+4S+5)} = \frac{10S+15}{(S^2+4S+5)}$

$I_1 = \frac{10S+15}{(S+2-J)(S+2+J)}$ ------------- (iii)

Using partial fraction

$$\frac{10S+15}{(S+2-J)(S+2+J)} = \frac{K}{(S+2-J)} + \frac{K^*}{(S+2+J)}$$

To find K

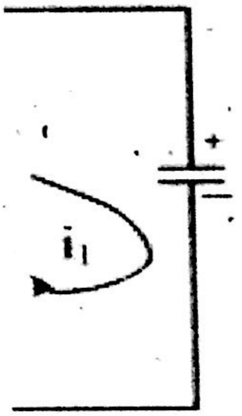
$$K = \frac{10S+15}{(S+2-J)(S+2+J)} * (S+2-J)$$

$K = \frac{10S+15}{(S+2+J)}$ then put $S = -2 + J$

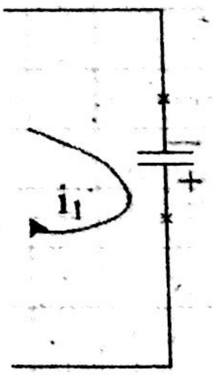
$$K = \frac{10(-2+J)+15}{(-2+J+2+J)}$$

$$K = \frac{10J-5}{2J}$$

NOTE: This is very important to know that $V_C(0^-)$ could have positive or negative value when direction of current and voltage polarity are same likewise in this example take $V_C(0^-) = +Volt$



And if direction and voltage polarity are different likewise as shown below then take $V_C(0^-) = -Volt$



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$$K = \frac{-J(10J-5)}{2}$$

$$K = \frac{10+J5}{2}$$

$$K = 5 + J\frac{5}{2}$$

And

$$K^* = 5 - J\frac{5}{2}$$

Thus equation (iii) becomes as

$$I_1 = \frac{5+J\frac{5}{2}}{(S+2-J)} + \frac{5-J\frac{5}{2}}{(S+2+J)}$$

Taking I.L.T

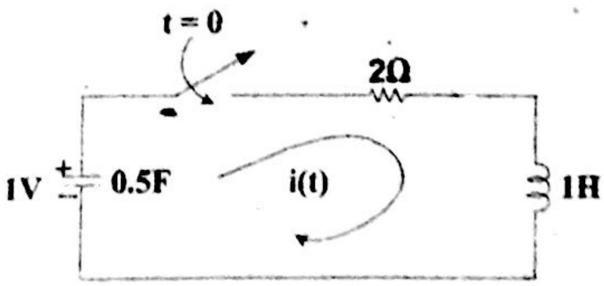
$$i_1 = \left(5 + j\frac{5}{2}\right)e^{(-2+J)t} + \left(5 - j\frac{5}{2}\right)e^{(-2-J)t}$$

$$i_1 = 5e^{-2t}\left(e^{Jt} + e^{-Jt}\right) + j\frac{5}{2}e^{-2t}\left(e^{Jt} - e^{-Jt}\right)$$

$$i_1 = 10e^{-2t}\frac{(e^{Jt}+e^{-Jt})}{2} - 5e^{-2t}\frac{(e^{Jt}-e^{-Jt})}{2J}$$

$$\mathbf{i_1 = 10e^{-2t}Cos1t - 5e^{-2t}Sin1t \quad Amp}$$

Q.5: The capacitor initially charged to voltage of 1V as indicated in figure. Find the expression for i(t).



Solution: Applying KVL at t > 0

$$V_C + V_R + V_L = 0$$

$$\therefore \frac{1}{0.5}\int_{-\infty}^{t} i\,dt + i * 2 + 1\frac{di}{dt} = 0$$

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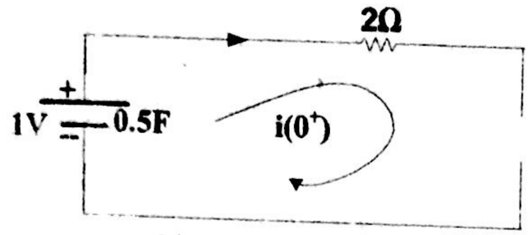
$$\frac{1}{0.5}\int_{-\infty}^{0} i\,dt + \frac{1}{0.5}\int_{0}^{t} i\,dt + i * 2 + \frac{di}{dt} = 0$$

$$V_C(0^-) + \frac{1}{0.5}\int_{0}^{t} i\,dt + i * 2 + \frac{di}{dt} = 0$$



Circuit at $t = 0^+$

From the circuit at $t = 0^+$, $i(0^+) = 0$(as inductor is de-energized) and $V_C(0^+) = V_C(0^-) =$ 1V.

$$-1 + \frac{1}{0.5}\int_{0}^{t} i\,dt + i * 2 + \frac{di}{dt} = 0$$

Taking L.T in both sides

$$\frac{-1}{s} + \frac{2}{s}I + 2I + SI - i(0^+) = 0$$

$$\frac{2}{s}I + 2I + Si - 0 = \frac{1}{s}$$

$$(\frac{2}{s} + 2 + S)I = \frac{1}{s}$$

$$\frac{(S^2+2S+2)}{S}I = \frac{1}{S}$$

$$I = \frac{1}{S^2+2S+2}$$

$$I = \frac{1}{S^2+2S+1+1}$$

$$I = \frac{1}{(S+1)^2+1^2}$$

Taking I.L.T on both sides

$$\mathbf{i = e^{-t}\sin 1t \text{ Amp}}$$

Q.6: Solve for i(t) in circuit as shown in figure below in which 3F capacitor is initially charged to 20 volts, the 6F capacitor to 10 volts and the switch is closed at t = 0

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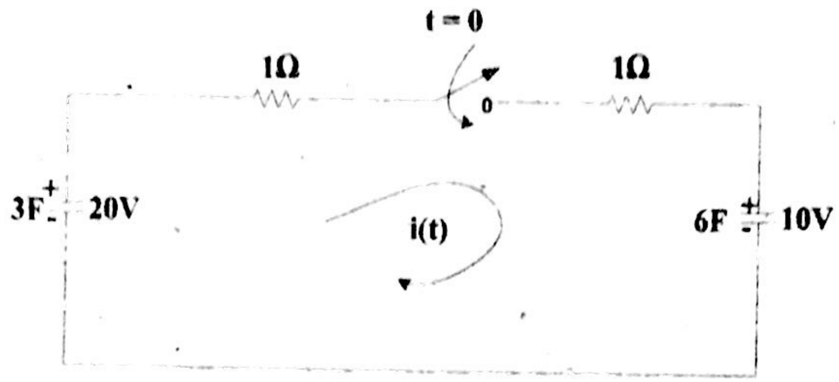

Solution:

Applying KVL at t > 0

$V_{C1}+V_{R1}+V_{R2}+V_{C2} = 0$

$\frac{1}{3}\int_{-\infty}^{t} i\,dt + 1*i + 1*i + \frac{1}{6}\int_{-\infty}^{t} i\,dt = 0$

$$\frac{1}{3}\int_{-\infty}^{0} i\,dt + \frac{1}{3}\int_{0}^{t} i\,dt + 1*i + 1*i + \frac{1}{6}\int_{-\infty}^{0} i\,dt + \frac{1}{6}\int_{0}^{t} i\,dt = 0$$

$V_{C1}(0^-) + \frac{1}{3}\int_{0}^{t} i\,dt + 1*i + 1*i + V_{C2}(0^-) + \frac{1}{6}\int_{0}^{t} i\,dt = 0$

From the question it is given that $V_{C1}(0^-) = 20$ volt and $V_{C2}(0^-) = 10$ volt

$-20 + \frac{1}{3}\int_{0}^{t} i\,dt + 1*i + 1*i + 10 + \frac{1}{6}\int_{0}^{t} i\,dt = 0$

Taking L.T in both sides

$\frac{-20}{S} + \frac{I}{3S} + I + I + \frac{10}{S} + \frac{I}{6S} = 0$

$\frac{I}{3S} + 2I + \frac{I}{6S} = \frac{10}{S}$

$\left(\frac{1}{3S} + 2 + \frac{1}{6S}\right) I = \frac{10}{S}$

$\frac{(2+2*6S+1)}{6S} I = \frac{10}{S}$

$I = \frac{60}{(12S+3)}$

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$$I = \frac{5}{(S+0.25)}$$

Taking I.L.T on both sides

$$i(t) = 5e^{-0.25t}\ \text{Amp}$$

Q.7: An exponential voltage $4e^{-3t}$ is applied at time t = 0 to a series RLC circuit comprising R = 4Ω, L = 1H and C = $\frac{1}{3}$F. Obtain complete solution for the current. Assume zero current through the inductor and zero voltage across the capacitor before application of the voltage. [Use transform method.]

Solution:

Applying KVL at t > 0

$$4e^{-3t} = 4i + 1\frac{di}{dt} + \frac{1}{\frac{1}{3}}\int_{-\infty}^{t} i\,dt$$

$$4e^{-3t} = 4i + \frac{di}{dt} + \frac{1}{\frac{1}{3}}\int_{-\infty}^{0} i\,dt + \frac{1}{\frac{1}{3}}\int_{0}^{t} i\,dt$$

$$4e^{-3t} = 4i + \frac{di}{dt} + V_C(0^-) + 3\int_{0}^{t} i\,dt$$

From the question $V_C(0^-) = 0$ and $i(0^-) = 0$

$$4e^{-3t} = 4i + \frac{di}{dt} + 0 + 3\int_{0}^{t} i\,dt$$

Taking L.T on both sides

$$\frac{4}{S+3} = 4I + SI - i(0^+) + 3\frac{I}{S}$$

$$\frac{4}{S+3} = 4I + SI - 0 + 3\frac{I}{S}$$

$$\frac{4}{S+3} = I\left(4 + S + \frac{3}{S}\right)$$

$$I = \frac{4S}{(S+3)(S^2+4S+3)}$$

$$I = \frac{4S}{(S+3)(S+1)(S+3)}$$

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$$I = \frac{4S}{(S+1)(S+3)^2} \text{ ------------------ (i)}$$

Using partial fraction

$$I = \frac{4S}{(S+1)(S+3)^2} = \frac{K}{(S+1)} + \frac{K_{11}}{(S+3)} + \frac{K_{12}}{(S+3)^2}$$

To find K

$$K = \frac{4S}{(S+1)(S+3)^2} * (S+1)$$

$$K = \frac{4S}{(S+3)^2} \text{ then put } S = -1$$

$$K = -1$$

To find K_{12}

$$K_{12} = \frac{4S}{(S+3)^2(S+1)} * (S+3)^2$$

$$K_{12} = \frac{4S}{(S+1)} \text{ then put } S = -3$$

$$K_{12} = \frac{-4*3}{(-3+1)}$$

$$K_{12} = 6$$

To find K_{11}

$$K_{11} = \frac{4S}{(S+3)^2(S+1)} * (S+3)^2$$

$$K_{11} = \frac{1}{1!}\frac{d}{dS}\left[\frac{4S}{(S+1)}\right]$$

$$K_{11} = \frac{4}{(S+1)^2} \text{ then put } S = -3$$

$$K_{11} = \frac{4}{(-3+1)^2}$$

$$K_{11} = 1$$

Thus equation (i) becomes as

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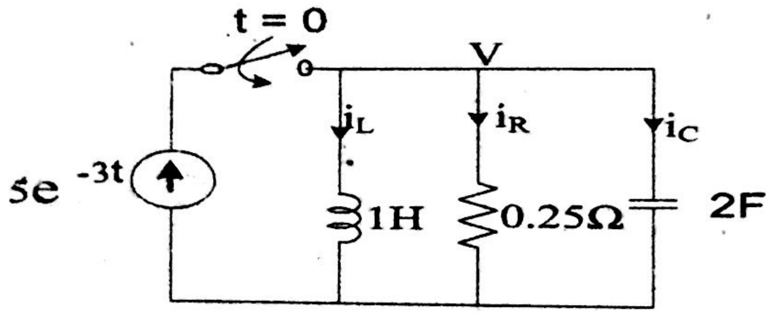
$$I = \frac{4S}{(S+1)(S+3)^2} = \frac{-1}{(S+1)} + \frac{1}{(S+3)} + \frac{6}{(S+3)^2}$$

$$I = \frac{-1}{(S+1)} + \frac{1}{(S+3)} + \frac{6}{(S+3)^2}$$

Taking I.L.T on both sides we get

$$\mathbf{i(t) = -e^{-1t} + e^{-3t} + 6te^{-3t} \text{ Amp}}$$

Q.8: Find the voltage across the capacitor of the circuit shown below using Lapla Transform method.



Solution:

Applying KCL at $t > 0$

$$5e^{-3t} = i_L + i_R + i_C$$

$$5e^{-3t} = \frac{1}{1}\int_{-\infty}^{t} v\,dt + \frac{v}{0.25} + 2\frac{dv}{dt}$$

$$5e^{-3t} = \frac{1}{1}\int_{-\infty}^{0} v\,dt + \frac{1}{1}\int_{0}^{t} v\,dt + \frac{v}{0.25} + 2\frac{dv}{dt}$$

$$5e^{-3t} = i_L(0^-) + \frac{1}{1}\int_{0}^{t} v\,dt + \frac{v}{0.25} + 2\frac{dv}{dt}$$

Since it's a de-energized condition so $i_L(0^-) = i_L(0^+) = 0$ and $v_C(0^-) = v_C(0^+) = 0$

$$5e^{-3t} = 0 + \frac{1}{1}\int_{0}^{t} v\,dt + \frac{v}{0.25} + 2\frac{dv}{dt}$$

Taking L.T on both sides

$$\frac{5}{(S+3)} = \frac{V}{S} + 4V + 2(SV - v(0^+))$$

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$$\frac{5}{(S+3)} = \frac{V}{S} + 4V + 2(SV - 0)$$

$$\frac{5}{(S+3)} = (\frac{1}{S} + 4 + 2S)V$$

$$V = \frac{5S}{(S+3)(2S^2+4S+1)}$$

$$V = \frac{5S}{2*(S+3)(S^2+2S+0.5)}$$

$$V = \frac{2.5S}{(S+3)(S+0.29)(S+1.70)}$$

Using partial fraction

$$V = \frac{2.5S}{(S+3)(S+0.29)(S+1.70)} = \frac{K_1}{(S+3)} + \frac{K_2}{(S+0.29)} + \frac{K_3}{(S+1.70)} \text{ ---------------- (i)}$$

To find K_1

$$K_1 = \frac{2.5S}{(S+3)(S+0.29)(S+1.70)} * (S+3)$$

$$K_1 = \frac{2.5S}{(S+0.29)(S+1.70)} \text{ then put } S = -3$$

$$K_1 = -2.12$$

To find K_2

$$K_2 = \frac{2.5S}{(S+3)(S+0.29)(S+1.70)} * (S+0.29)$$

$$K_2 = \frac{2.5S}{(S+3)(S+1.70)} \text{ then put } S = -0,29$$

$$K_2 = -0.1$$

To find K_3

$$K_3 = \frac{2.5S}{(S+3)(S+0.29)(S+1.70)} * (S+1.70)$$

$$K_3 = \frac{2.5S}{(S+3)(S+0.29)} \text{ then put } S = -1.70$$

$$K_3 = 2.31$$

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Thus equation (i) becomes as

$$V = \frac{2.5S}{(S+3)(S+0.29)(S+1.70)} = \frac{-2.12}{(S+3)} + \frac{-0.1}{(S+0.29)} + \frac{2.31}{(S+1.70)}$$

$$V = \frac{-2.12}{(S+3)} + \frac{-0.1}{(S+0.29)} + \frac{2.31}{(S+1.70)}$$

Taking I.L.T on both sides we get

$$\mathbf{v(t) = -2.12e^{-3t} - 0.1e^{-0.29t} + 2.31e^{-1.7t}\ Amp}$$

Q.9: A Sinusoidal Voltage 25Sin10t is applied at time t = 0 to a series RL circuit comprising resistor R= 5Ω and inductor L = 1H. By the method of Laplace Transformation find current i(t), Assume zero current through inductor before application of voltage.

Solution:

Applying KVL at t > 0

$$25\text{Sin}10t = i * 5 + 1\frac{di}{dt}$$

Taking L.T on both sides

$$\frac{25*10}{S^2+100} = 5I + SI - i(0^+)$$

From the question $i(0^+) = i(0^-) = 0$

$$I = \frac{250}{(S^2+100)(S+5)}$$

Using partial fraction

$$\frac{250}{(S^2+100)(S+5)} = \frac{K_1}{(S+5)} + \frac{K}{(S+10J)} + \frac{K^*}{(S-10J)} \text{ ------------ (i)}$$

To find K_1

$$K_1 = \frac{250}{(S^2+100)(S+5)} * (S+5)$$

$$K_1 = \frac{250}{(S^2+100)} \text{ then put } S = -5$$

$$K_1 = 2$$

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To find K

$$K = \frac{250}{(S+10J)(S-10J)(S+5)} * (S+10J)$$

$$K = \frac{250}{(-10J-10J)(-10J+5)} \quad \text{then put } S = -10J$$

$$K = -1 + J\frac{1}{2}$$

And

$$K^* = -1 - J\frac{1}{2}$$

Thus equation (i) becomes as

$$I = \frac{2}{(S+5)} + \frac{(-1+J\frac{1}{2})}{(S+10J)} + \frac{(-1-J\frac{1}{2})}{(S-10J)}$$

Taking I.L.T

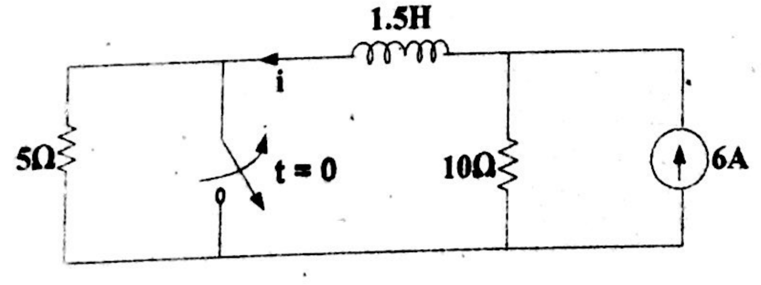
$$i = 2e^{-5t} + (-1 + J\frac{1}{2})e^{(-10J)t} + (-1 - J\frac{1}{2})e^{(10J)t}$$

$$i = 2e^{-5t} - 2\frac{(e^{10Jt}+e^{-10Jt})}{2} + 1 * \frac{(e^{10Jt}-e^{-10Jt})}{2J}$$

$$\mathbf{i = 2e^{-5t} + 2Cos10t + Sin10t \quad Amp}$$

Problems:

Q.1:The switch in figure has been closed for a long time. It opens at t = 0. Find i(t) at t > 0. Use Laplace Transformation method.



[Ans: $i(t) = (4 + 2e^{-10t})$]

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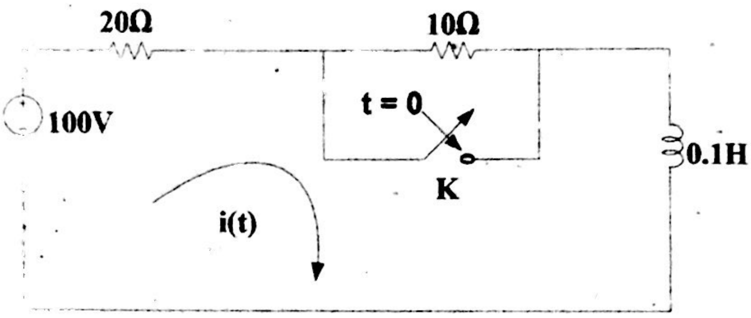
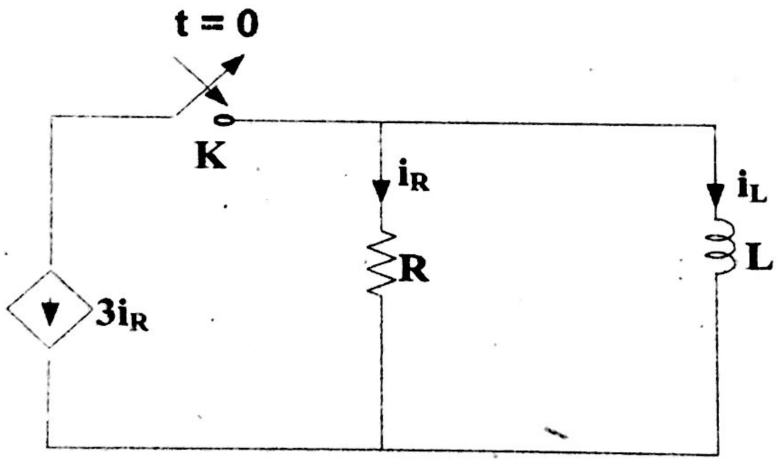
Q.2: The switch in figure has been opened for a long time. It is closed at t = 0. Find the complete expression for the current using Laplace Transformation method



[Ans: $i(t) = (-1.67e^{-200t} + 5)$ Amp]

Q.3: If the initial current through the inductor be 1A, find $i_L(t)$ at t > 0 in the circuit below using Laplace Transformation method.



[Ans: $i_L(t) = e^{-\frac{R}{4L}*t}$ Amp]

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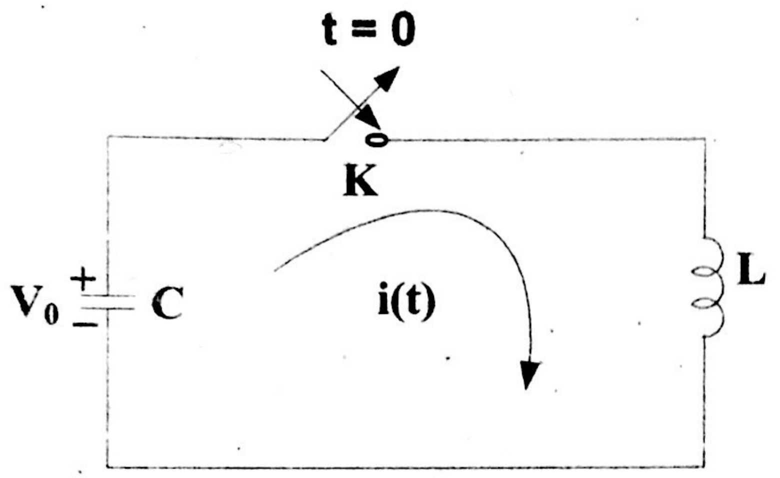
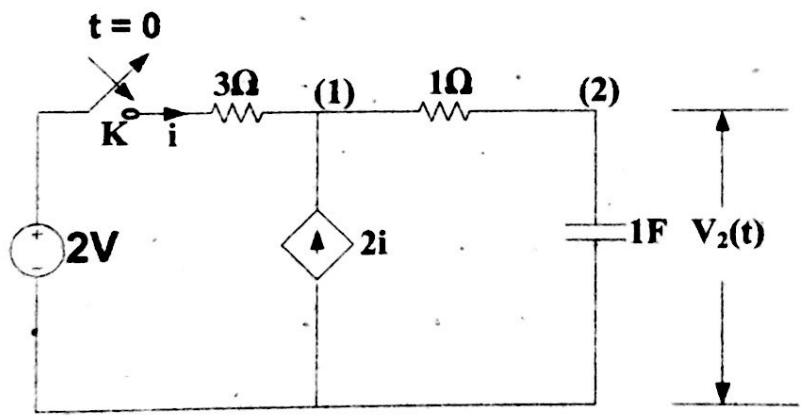
Q.4:In the network shown, C is initially charged to V_0, the switch K is closed at t = 0. Solve for the current i(t), using Laplace Transformation method.



[Ans: $i(t) = \sqrt{\frac{C}{L}} \sin \frac{1}{\sqrt{LC}} t$ Amp]

Q.5:Find the voltage across the capacitor at t > 0 using Laplace Transformation method.



[Ans: $V_2(t) = (2 - 2e^{-0.5t})$ Volt]

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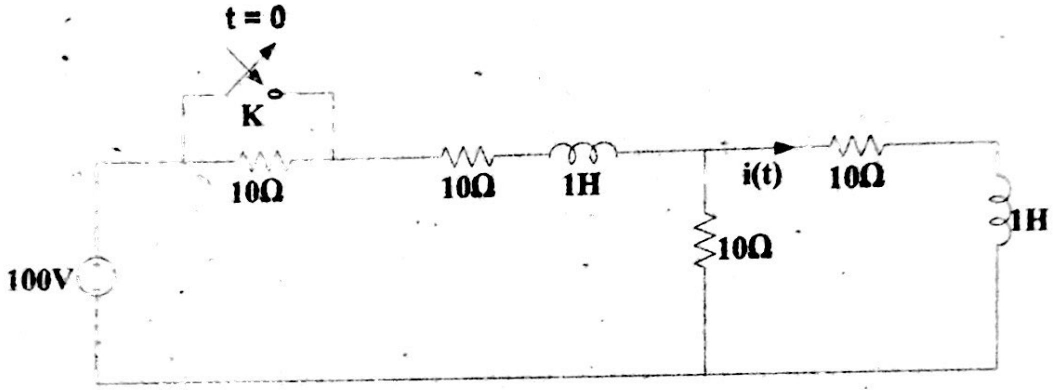
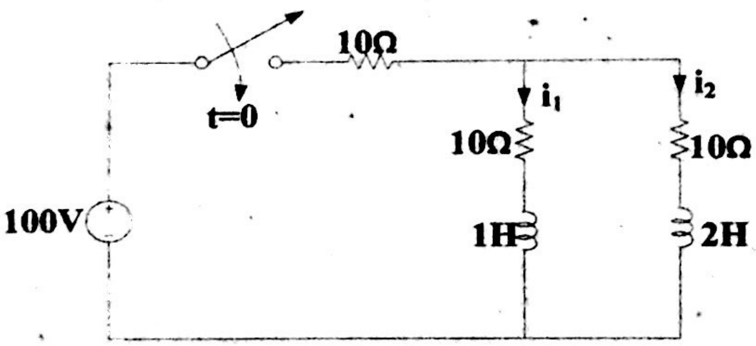
#Q.6:The network shown in figure below is under steady state condition. The switch is closed at t = 0. Determine the current through 10 ohm resistor connected between terminals AB.[Use Laplace Transformation method]



[Ans: $i(t) = (-2.495e^{-10t} + 0.412e^{-30t} + 3.3)$ Amp]

Q.7:In the circuit shown in figure below the switch is closed at t= 0, find $i_1(t)$ using Laplace Transformation method.



[Ans: $i(t) = (-4.5e^{-23.65t} + 1.121e^{-6.35t} + 3.3)$ Amp

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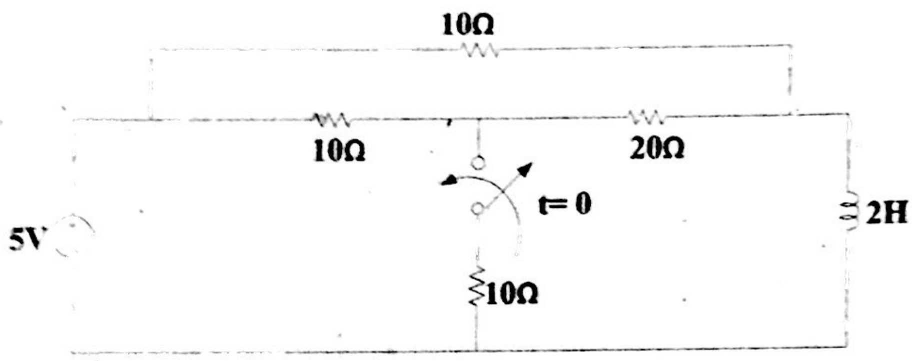
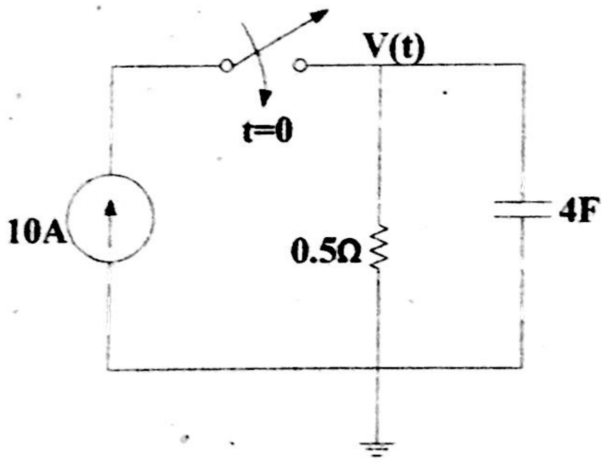
Q.8: In the circuit of the figure below, the switch is open and the circuit reaches a steady state. At t=0, the switch is closed. Find the current in the inductor for t > 0 using Laplace method.



[Hint: Use Δ – Y transformation]

[Ans: $i(t) = (0.067e^{-3.57t} + 0.6)$ Amp]

Q.9: In the circuit of the figure below, the switch is close at t=0, find V(t) for t > 0 if the initial capacitor voltage be 2V, use Laplace Transformation method.



[Ans: $V(t) = (3e^{-0.5t} + 5)$ Volt]

Q.10:In the circuit shown, switch is changed from position "a" to "b" at t = 0. Find the expression for current i1(t) and i2(t) using Laplace Transformation method.

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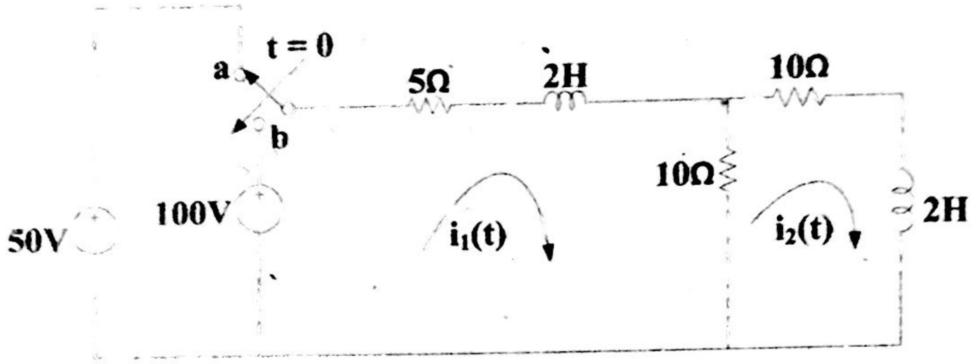
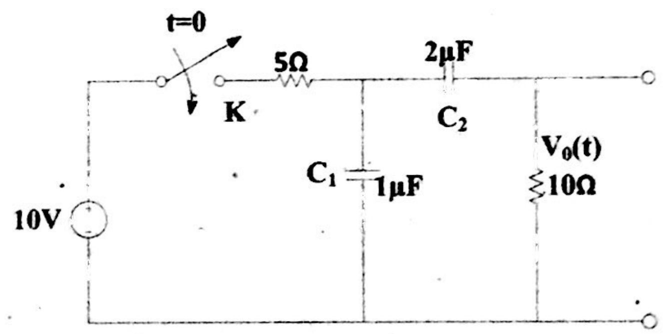


[Ans: $i_1(t) = (10 - 4.31e^{-3.59t} + 0.68e^{-13.90t})$ A, $i_2(t) = (5 - 3.37e^{-3.59t} + 0.87e^{-13.90t})$ A]

Q.11: In figure below, the capacitors C_1 and C_2 are initially discharged. The switch K is closed at t = 0. Find the Voltage $V_0(t)$ for t > 0 using Laplace Transformation method.



[Ans: $V_0(t) = (6.74e^{-6591.78t} - 6.74e^{-303408.22t})$ Volt]

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Frequency Response of Network

Frequency Response:

Network function can be represented in rectangular form in steady state as follows:

$N(j\omega) = R(\omega) + jX(\omega)$

Where, $R(\omega)$ is real part of $N(j\omega)$

$X(\omega)$ is imaginary part of $N(j\omega)$

Or, $N(j\omega) = |N(j\omega)| < \emptyset(\omega)$

Where,

$N(j\omega) = \sqrt{R^2(\omega) + X^2(\omega)}$ -------- (1)

$< \emptyset(\omega) = \tan^{-1}\left(\frac{X(\omega)}{R(\omega)}\right)$--------- (2)

From (1) and (2), we see that both magnitude and phase angle of network function depends upon the frequency.

Poles and Zeroes:

A network function is always ratio of two - polynomial in "S". i.e.

$$N(s) = \frac{P(s)}{Q(s)} = \frac{a_0S^n + a_1S^{n-1} + \ldots + a_ns^0}{b_0s^m + b_1S^{m-1} + \ldots + b_ms^0}$$

$$= \frac{K(S - Z_1)(S - Z_2)\ldots(s - Z_n)}{(s - P_1)(S - P_2)\ldots(S - P_m)}$$

Where

$K = \frac{a_0}{b_0}$ = Scale factor.

$Z_1, Z_2 \ldots Z_n$ are the roots of $P(s)$ and

$P_1, P_2 \ldots P_m$ are the roots of $Q(s)$

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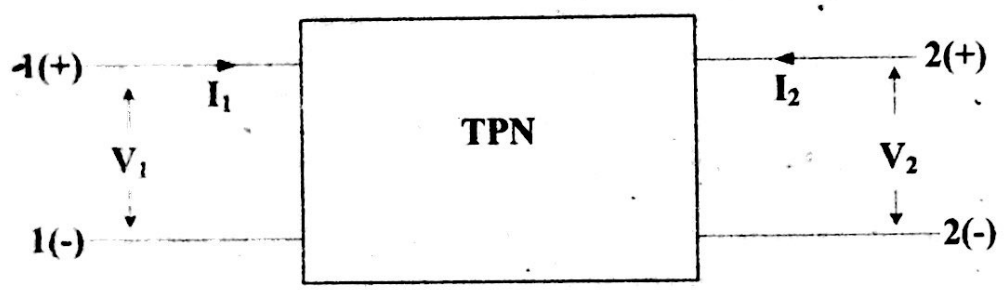
If the complex frequency "S" takes a value equal to any one of the roots of P(s) i.e either $S = Z_1$ or Z_2, ... or Z_m, then, the network function becomes zero. Hence these roots are known as zero of the network function and the factor of the network function and the factor $(S - Z_1)$, $(S - Z_2)$ ... $(S - Z_n)$ are known as zeroes factors.

If the complex frequency "S' takes a value equal to any one of the roots of Q(S) i. e if either $S = P_1$ or P_2 or .... or P_m, then the network function n becomes ∞, Hence these roots are known as poles of network functions and the factors $(s - P_1)$ .... $(S - P_m)$ are known as poles factor.

Transfer function or Network function of two port Network:



Two port Network consists of four terminals.

Terminals pair 1-1' makes input port 1.

Terminals pair 2-2' makes output port 2.

$V_1(s)$ is the voltage at port 1

$V_2(s)$ is the voltage at port 2

$I_1(s)$ is the current at port 1

$I_2(s)$ is the current at port 2

Reference polarities and direction of current are taken as shown in above figure.

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Transform Impedance and admittance of different circuit elements:

Circuit Elements	Transform impedance in S - domain	Transform admittance in S- domain
Resistor(R); in Ω	**R**	$\frac{1}{R}$
Inductor (L); in H	**SL**	$\frac{1}{SL}$
Capacitor (C); in F	$\frac{1}{SC}$	**SC**

Network Function of Two port Network (TPN):

I) Driving point network function:

- The ratio of Quantities of same port.

i)Driving point input impedance and admittance (Z_{11} and Y_{11})

$$Z_{11}(S) = \frac{V_1(S)}{I_1(S)}$$

Here, $Z_{11} = \frac{1}{Y_{11}}$

$$Y_{11}(S) = \frac{I_1(S)}{V_1(S)}$$

ii)Driving point output impedance and admittance (Z_{22} and Y_{22})

$$Z_{22}(S) = \frac{V_2(S)}{I_2(S)}$$

Here, $Z_{22} = \frac{1}{Y_{22}}$

$$Y_{22}(S) = \frac{I_2(S)}{V_2(S)}$$

II) Transfer Network Function:

- Ratio of quantities of different port
- Ratio of quantity of output port to quantity of input port is called forward transfer network function.
- Ratio of quantity of input port to quantity of output port is called reverse transfer network function.

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i)Forward transfer impedance and admittance (Z_{21} and Y_{21})

$$Z_{21}(S) = \frac{V_2(S)}{I_1(S)}$$

Here, $Z_{21} \neq \frac{1}{Y_{21}}$

$$Y_{21}(S) = \frac{I_2(S)}{V_1(S)}$$

ii)Reverse transfer impedance and admittance (Z_{12} and Y_{12})

$$Z_{12}(S) = \frac{V_1(S)}{I_2(S)}$$

Here, $Z_{12} \neq \frac{1}{Y_{12}}$

$$Y_{12}(S) = \frac{I_1(S)}{V_2(S)}$$

iii)Forward voltage ratio transfer function (G_{21}) and Forward current ratio transfer function (α_{21})

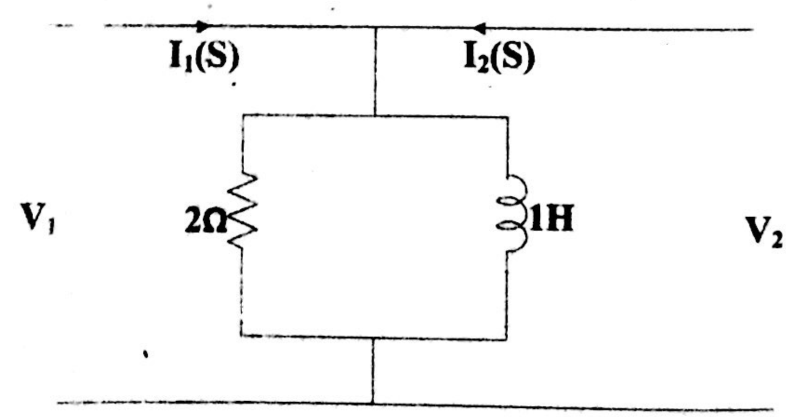
$$G_{21}(S) = \frac{V_2(S)}{V_1(S)}$$

$$\alpha_{21}(S) = \frac{I_2(S)}{I_1(S)}$$

 # Steps to find transfer function of given circuit:

1) Write the given network in S-domain
2) Consider port-1 as source having voltage V_1 and port-2 as given according to question
3) Perform series-parallel reduction of possible. Reduction should not involve any of the two ports.
4) Write KVL for all loops.
5) Find the value of required transfer function.

Example.1.Find Z_{21} for circuit below:



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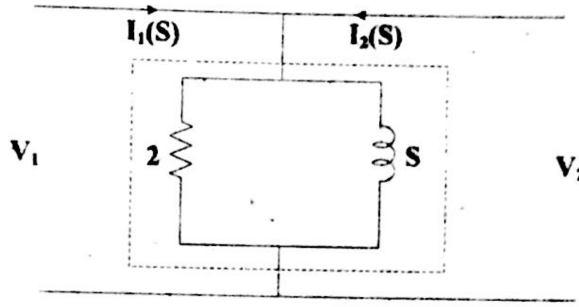
Solution:

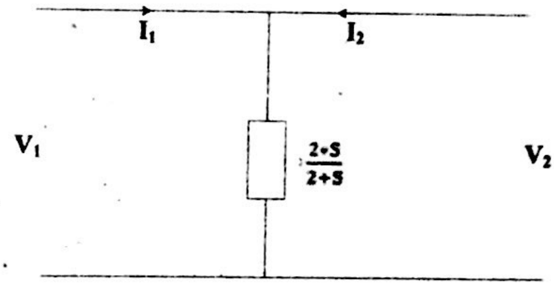
Writing in S-domain,



Resistor and Inductor are in parallel, so reducing as

$$= 2 // S$$

$$= \frac{2*S}{2+S}$$



$$Z_{21} = \frac{V_2}{I_1}s$$

Here, port-2 is open circuited, i.e. $I_2 = 0$.

Applying KVL in right loop;

$$V_2 = \left(\frac{2S}{S+2}\right) I_1$$

$$\frac{V_2}{I_1} = \frac{2S}{S+2}$$

$$Z_{21} = \frac{V_2}{I_1} = \frac{2S}{S+2}$$

Example.2.Find Y_{21} and G_{21} for circuit below:

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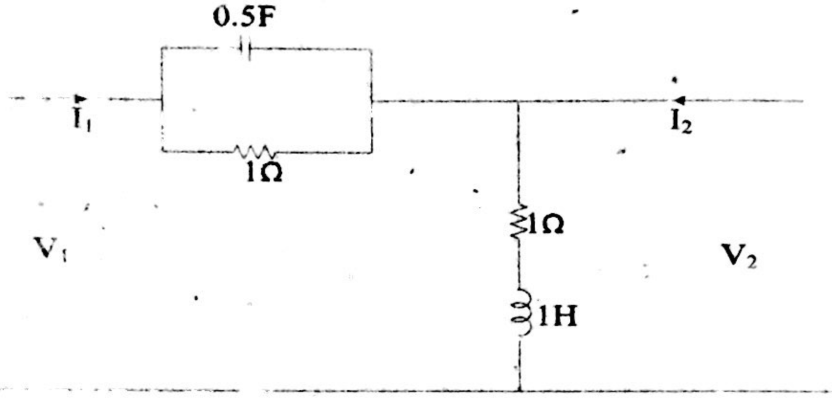
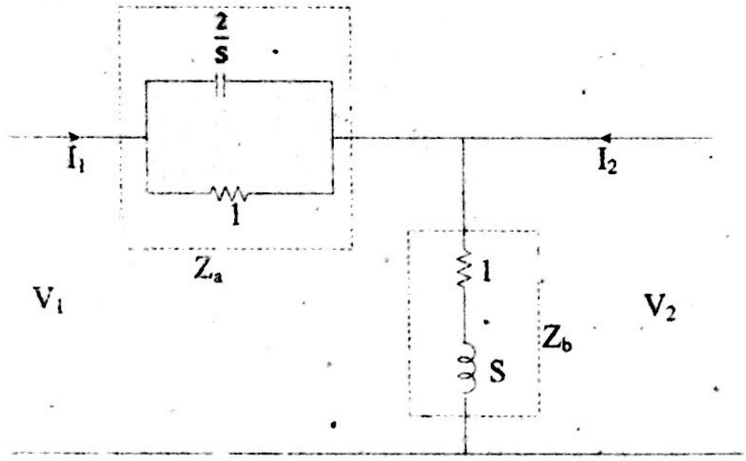


Solution:

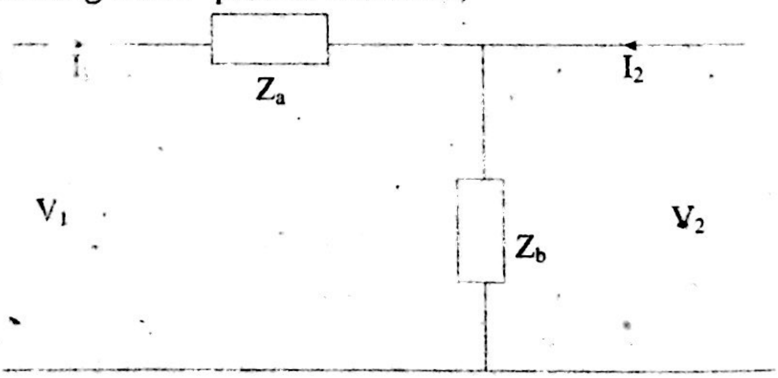
Writing in S-domain,



$$Z_a = \frac{2}{S} \mathbin{//} 1 = \frac{2}{S+2}$$

$$Z_b = 1 + S$$

Performing Series- parallel reduction;



$$Y_{12} = \frac{I_1}{V_2}$$

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$$G_{21} = \frac{V_2}{V_1}$$

Applying KVL in loop 1

$$V_1 = \left(\frac{2}{S+2} + S + 1\right) I_1$$

$$V_1 = \left(\frac{S^2+3S+4}{S+2}\right) I_1$$

Applying KVL in loop 2

$$V_2 = (s+1)I_1 \qquad [\text{As } I_2 = 0]$$

$$\frac{I_1}{V_2} = \frac{1}{S+1}$$

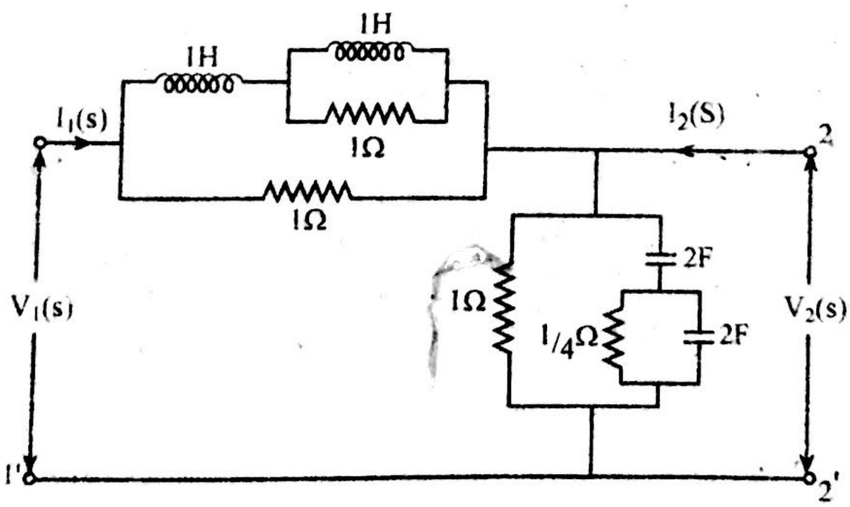
$$\mathbf{Y_{21}} = \frac{1}{S+1}$$

Also,

$$G_{21} = \frac{V_2}{V_1} = \frac{(s+1)I_1}{\left(\frac{S^2+3S+4}{S+2}\right)I_1}$$

$$G_{21} = \frac{(s+1)(S+2)}{S^2+3S+4}$$

Example.3 Find driving point input impedance and voltage ratio transfer function of following circuit.



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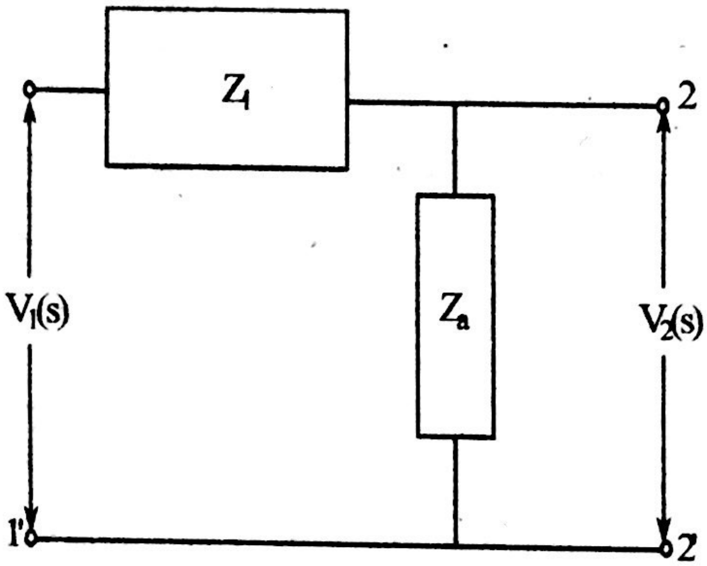
Solution:

Writing in S-domain,

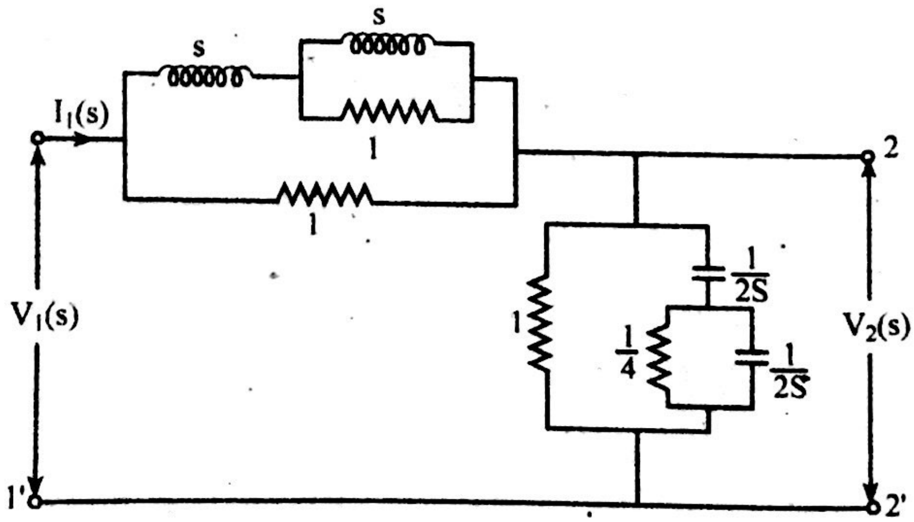
The driving impedance of port-1 is given as Z_{11}



Thus,

$$Z_{11} = Z_1 + Z_a$$

And,



$$Z_1 = ((s\ //\ 1) + s)\ //\ 1$$

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$$Z_2 = \left(\frac{s}{s+1} + s\right) // 1$$

$$Z_1 = \frac{\dfrac{s^2+2s}{s+1}}{\dfrac{s^2+2s+s+1}{s+1}}$$

$$Z_1 = \frac{s^2+2s}{s^2+3s+1}$$

And,

$$Z_a = \left\{\left(\frac{1}{2S} // \frac{1}{4}\right) + \frac{1}{2S}\right\} //1$$

$$Z_a = \left\{\left(\frac{\frac{1}{2s} \times \frac{1}{4}}{\frac{1}{4} + \frac{1}{2s}} + \frac{1}{2s}\right)\right\} // 1$$

$$Z_a = \left\{\frac{1}{2s+4} + \frac{1}{2s}\right\} // 1$$

$$Z_a = \left\{\frac{4s+4}{4s^2+8s}\right\} // 1$$

$$Z_a = \frac{\dfrac{s+1}{s^2+2s}}{\dfrac{s+1+s^2+2s}{s^2+2s}}$$

$$Z_a = \frac{s+1}{s^2+3s+1}$$

Hence,

$$Z_{11} = Z_1 + Z_a$$

$$Z_{11} = \frac{s^2+2s}{s^2+3s+1} + \frac{s+1}{s^2+3s+1}$$

$$Z_{11} = \frac{s^2+3s+1}{s^2+3s+1}$$

$$\mathbf{Z_{11} = 1}$$

And the voltage ratio transfer function is given as,

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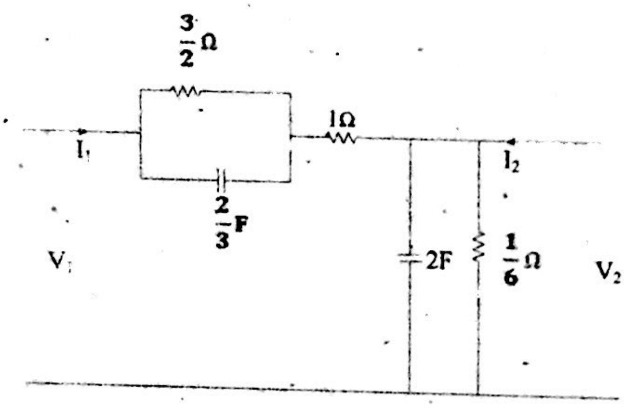
$$G_{21}(s) = \frac{V_2(s)}{V_1(s)}$$

$$G_{21}(s) = \frac{Z_a\, I_1(s)}{Z_{11}\, I_1(s)}$$

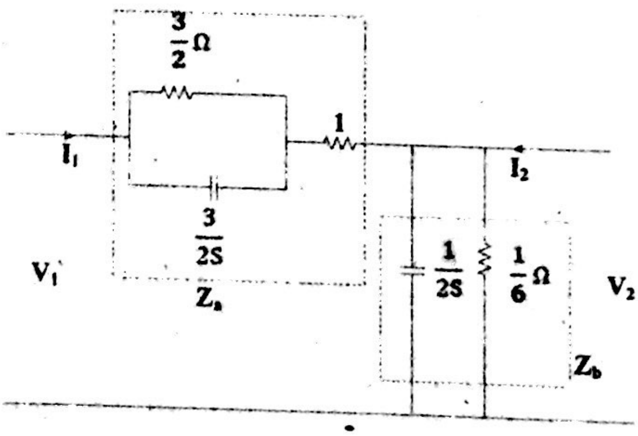
$$G_{21}(s) = \frac{s+1}{s^2+3s+1}$$

Example.4 Find voltage ratio transfer function



Solution:

Writing in S-domain,



Performing series-parallel reduction;

$$Z_a = \left(\frac{3}{2} // \frac{3}{2S}\right) + 1$$

$$Z_b = \left(\frac{1}{2S} // \frac{1}{6}\right)$$

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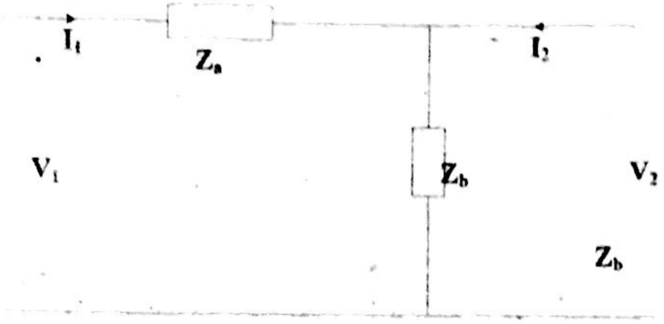

Applying KVL in left loop;

$$V_1 = (Z_a + Z_b)I_1$$

$$= \left(\frac{2S+5}{2S+2} + \frac{1}{2S+6}\right) I_1$$

$$= \left(\frac{S^2+6S+8}{(S+1)(s+3)}\right) I_1$$

Applying KVL in right loop;

$$V_2 = Z_b I_1 = \frac{1}{2S+6} I_1$$

$$G_{21} = \frac{V_2}{V_1}$$

$$= \frac{\frac{1}{2S+6} I_1}{\left(\frac{S^2+6S+8}{(S+1)(S+3)}\right) I_1}$$

$$= \frac{S+1}{2(S+2)(S+4)}$$

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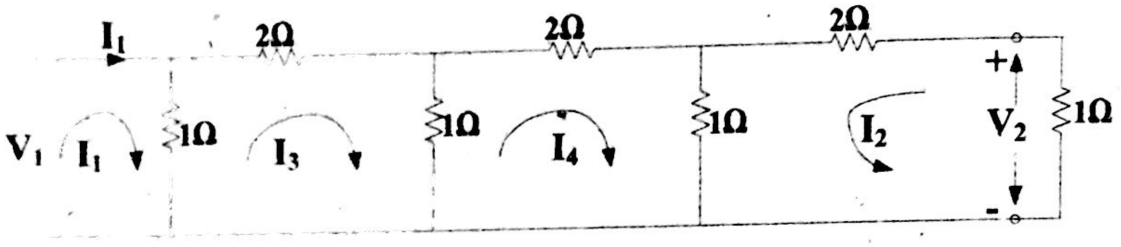
Example.5 For the given resistance network, determine the value of (i) Z_{11} (ii) G_{21} (iii) Z_{21} (iv) Y_{21} (v) α_{21}



Solution:

> **Note:** Always take direction of I_1 and I_2 clockwise and anticlockwise respectively as shown in figure. For other loops, you can consider any direction.

Writing KVL for each loop;

Loop-1:

$V_1 = I_1 - I_3$

Loop-2:

$2I_2 + I_2 + I_2 + I_4 = 0$ [Here, V_2 is voltage across 1ohm resister. It is not source]

> **Note:** For network involving two or more than two loops, write KVL in matrix form and solve by Cramer's rule by doing so, you can find all variables in terms of the variable in RHS (here V_1)

Loop-3:

$2I_3 + I_3 - I_1 + I_3 - I_4 = 0$

Loop-4:

$2I_4 + I_4 + I_2 + I_4 - I_3 = 0$

Writing above equations in matrix form,

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 4 & 0 & 1 \\ -1 & 0 & 4 & -1 \\ 0 & 1 & -1 & 4 \end{bmatrix} * \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{------------------------- (1)}$$

You can also write KVL in matrix form directly as,

$$\begin{bmatrix} R11 & R12 & R13 & R14 \\ R21 & R22 & R23 & R24 \\ R31 & R32 & R33 & R34 \\ R41 & R42 & R43 & R44 \end{bmatrix} * \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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R_{ii} = Sum of all resistor through which current I_i flows.

R_{if} = Sum of all resistor through which both current I_i and I_f flows.

–ve value , If both I_i and I_j in clockwise direction.

-ve value, If both I_i and I_j in anticlockwise direction.

+ve value, If one in clockwise and other in anticlockwise direction.

For above equation,

R11 = 1

R12 = 0

R13 = −1 [I_1 and I_2 both are clockwise, so –ve]

R14 = 0

R22 = 2 + 1 + 1 = 4

R24 = 1 [I_2 is anticlockwise and I_4 is clockwise direction. So +ve]

And so on……

Thus matrix is

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 4 & 0 & 1 \\ -1 & 0 & 4 & -1 \\ 0 & 1 & -1 & 4 \end{bmatrix} * \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

This equation is same as equation (1)

(i) $Z_{11} = \frac{V_1}{I_1}$

Solving by Cramer's Rule;

$$\Delta = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 4 & 0 & 1 \\ -1 & 0 & 4 & -1 \\ 0 & 1 & -1 & 4 \end{bmatrix} = 41$$

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$$\Delta_1 = \begin{bmatrix} V_1 & 0 & -1 & 0 \\ 0 & 4 & 0 & 1 \\ 0 & 0 & 4 & -1 \\ 0 & 1 & -1 & 4 \end{bmatrix} = V_1 \begin{bmatrix} 4 & 0 & 1 \\ 0 & 4 & -1 \\ 1 & -1 & 4 \end{bmatrix} = 56V_1$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{56V_1}{41}$$

Thus

$$Z_{11} = \frac{V_1}{I_1} = \frac{41}{56}\Omega$$

Since V_2 is voltage across 1Ω resistor. For resistor current flows from + to – Here, direction of I_2 is opposite of that suggested by polarity of V_2. Hence, -ve sign.

(ii) $G_{21} = \frac{V_2}{V_1}$

$V_2 = -I_2 * 1$

$$\Delta_2 = \begin{bmatrix} 1 & V_1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 4 & -1 \\ 0 & 0 & -1 & 4 \end{bmatrix} = -V_1 \begin{bmatrix} 0 & 0 & 1 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{bmatrix} = -V_1$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-V_1}{41}$$

$$V_2 = -I_2 = \frac{V_1}{41}$$

$$G_{21} = \frac{V_2}{V_1} = \frac{V_1}{41} * \frac{1}{V_1} = \frac{1}{41}$$

(iii) $Z_{21} = \frac{V_2}{I_1}$

$$Z_{21} = \frac{V_1}{41} * \frac{41}{56V_1}$$

$$Z_{21} = \frac{1}{56}$$

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(iv) $Y_{21} = \frac{I_2}{V_1}$

$$Y_{21} = \frac{-V_1}{41} * \frac{1}{V_1}$$

$$Y_{21} = -\frac{1}{41}$$

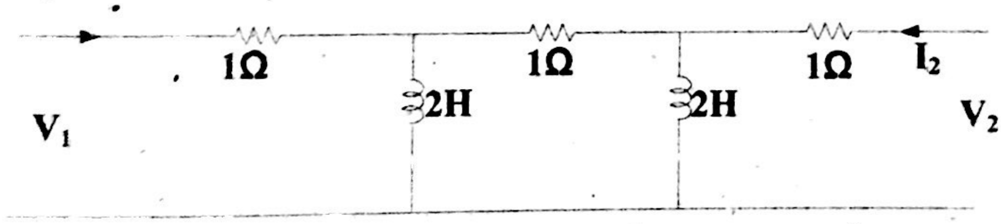
(v) $\alpha_{21} = \frac{I_2}{I_1}$

$$\alpha_{21} = -\frac{V_1}{41} * \frac{41}{56V_1}$$

$$\alpha_{21} = -\frac{1}{56}$$

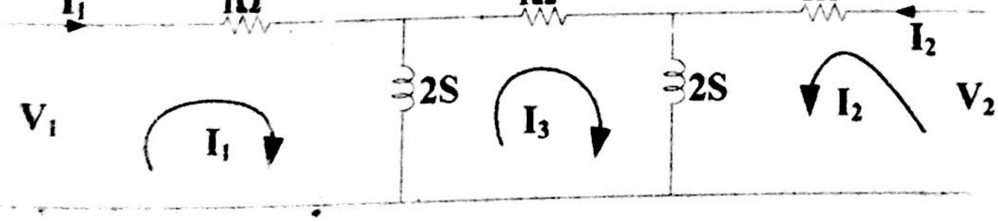
Example.6: For the given two-port network, determine voltage ratio transfer function (a) $G_{21}(S)$ and driving point input impedance (b) $Z_{11}(S)$. If this network is terminated at port-2 with a 2Ω resistor, then for this terminated network, find the following network functions.

(i) $Z_{11}(S)$ (ii) $Z_{21}(S)$ (iii) $Y_{21}(S)$ (v) $\alpha_{21}(S)$



Solution:

Writing in S-domain



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> **Note:*** If port-2 not-terminated, $I_2 = 0$. So, no need to include KVL for loop2 in matrix.
>
> *If port-2 terminated, $I_2 \neq 0$, Include KVL for loop2 in network.

For given network, port-2 is not terminated. So writing KVL for loop 1 and 3 in matrix form,

$$\begin{bmatrix} 2S+1 & -2S \\ -2S & 4S+1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \end{bmatrix}$$

> Port-2 is open. So no current flows through 1Ω resistor. Hence, no voltage across it.

(a) $G_{21} = \frac{V_2}{V_1}$

$V_2 = 2S(I_3)$

Here,

$$\Delta = \begin{bmatrix} 2S+1 & -2S \\ -2S & 4S+1 \end{bmatrix} = 4S^2 + 6S + 1$$

$$\Delta_2 = \begin{bmatrix} 2S+1 & V_1 \\ -2S & 0 \end{bmatrix} = 2S * V_1$$

$$I_3 = \frac{\Delta_2}{\Delta} = \frac{2S * V_1}{4S^2 + 6S + 1}$$

$$V_2 = 2S * I_3 = \frac{4S^2 * V_1}{4S^2+6S+1}$$

$$G_{21} = \frac{\frac{4S^2 * V_1}{4S^2+6S+1}}{V_1} = \frac{4S^2}{4S^2+6S+1}$$

Also

(b) $Z_{11} = \frac{V_1}{I_1}$

$$\Delta_1 = \begin{bmatrix} V_1 & -2S \\ 0 & 4S+1 \end{bmatrix} = (4S+1)V_1$$

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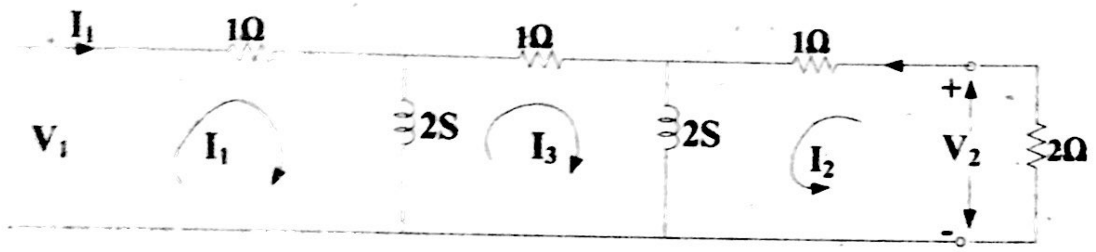
$$I_1 = \frac{\Delta_1}{\Delta} = \frac{(4S+1)V_1}{4S^2+6S+1}$$

$$Z_{11} = \frac{V_1}{I_1} = \frac{V_1}{\frac{(4S+1)V_1}{4S^2+6S+1}} = \frac{4S^2+6S+1}{4S+1}$$

Now,

If this network is terminated at port-2 with 2Ω resistor;



Writing KVL for loop1,2 and 3, and putting in matrix form

$$\begin{bmatrix} 2S+1 & 0 & -2S \\ 0 & 2S+3 & 2S \\ -2S & 2S & 4S+1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \\ 0 \end{bmatrix}$$

Here, port-2 terminated, $I_2 \neq 0$ so write KVL for loop-2 as well.

$$\Delta = \begin{bmatrix} 2S+1 & 0 & -2S \\ 0 & 2S+3 & 2S \\ -2S & 2S & 4S+1 \end{bmatrix}$$

$$=(2S+1)\begin{bmatrix} 2S+3 & 2S \\ 2S & 4S+1 \end{bmatrix} - 2S\begin{bmatrix} 0 & 2S+3 \\ -2S & 2S \end{bmatrix}$$

$$= 20S^2 + 20S + 3$$

(i) $Z_{11} = \frac{V_1}{I_1}$

Here,

$$\Delta_1 = \begin{bmatrix} V_1 & 0 & -2S \\ 0 & 2S+3 & 2S \\ 0 & 2S & 4S+1 \end{bmatrix}$$

$$=V_1\begin{bmatrix} 2S+3 & 2S \\ 2S & 4S+1 \end{bmatrix}$$

$$=(4S^2+14S+3)V_1$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{(4S^2+14S+3)V_1}{20S^2+20S+3}$$

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Thus,

$$Z_{11}=\frac{V_1}{I_1}=\frac{V_1}{\frac{(4S^2+14S+3)V_1}{20S^2+20S+3}}=\frac{20S^2+20S+3}{4S^2+14S+3}$$

(ii) $Z_{21}=\frac{V_2}{I_1}$

Here, $V_2=-2I_2$

$$\Delta_2=\begin{bmatrix}2S+1 & V_1 & -2S\\ 0 & 0 & 2S\\ -2S & 0 & 4S+1\end{bmatrix}$$

$$=V_1\begin{bmatrix}0 & 2S\\ -2S & 4S+1\end{bmatrix}$$

$$=(-4S^2)V_1$$

$$I_2=\frac{\Delta_2}{\Delta}=\frac{(-4S^2)V_1}{20S^2+20S+3}$$

$$V_2=\frac{(8S^2)V_1}{20S^2+20S+3}$$

Thus,

$$Z_{21}=\frac{V_2}{I_1}=\frac{\frac{(8S^2)V_1}{20S^2+20S+3}}{\frac{(4S^2+14S+3)V_1}{20S^2+20S+3}}=\frac{8S^2}{4S^2+14S+3}$$

(iii) $$Y_{21}=\frac{I_2}{V_1}=\frac{\frac{(-4S^2)V_1}{20S^2+20S+3}}{V_1}=\frac{(-4S^2)}{20S^2+20S+3}$$

(iv) $$\alpha_{21}=\frac{I_2}{I_1}=\frac{\frac{(-4S^2)V_1}{20S^2+20S+3}}{\frac{(4S^2+14S+3)V_1}{20S^2+20S+3}}=\frac{(-4S^2)}{4S^2+14S+3}$$

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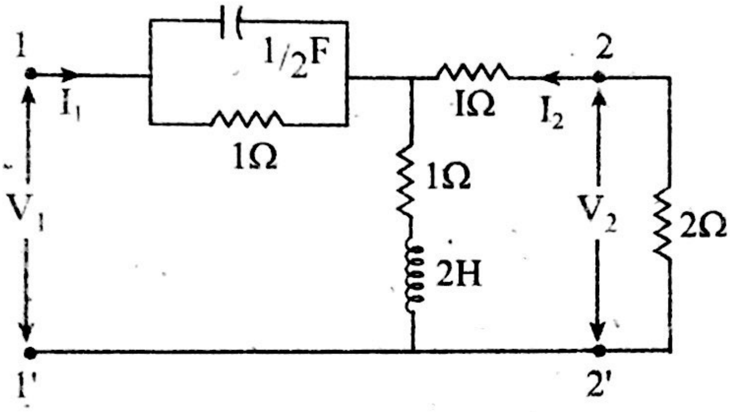
Example.7: In the network shown in fig, find the following network function.

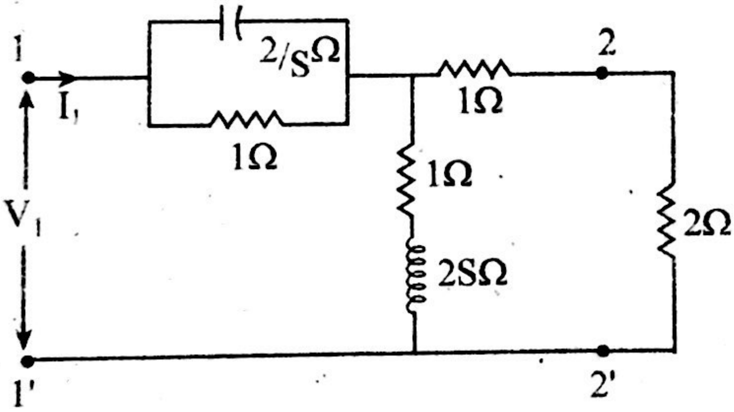
i. Voltage ratio transfer function $G_{21}(s)$

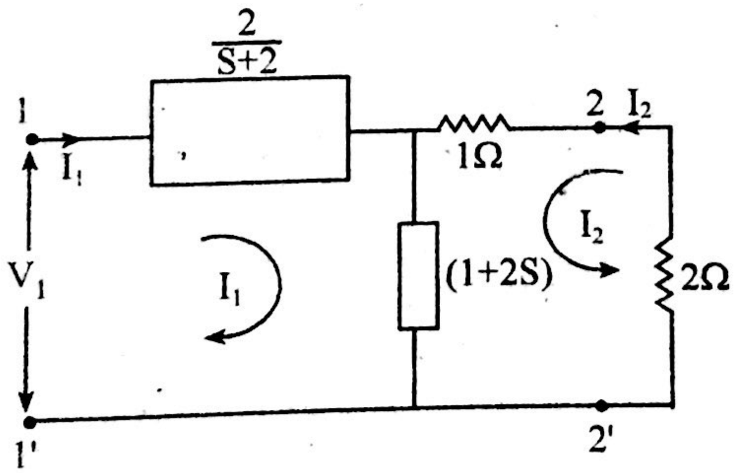
ii. Transfer admittance $[Y_{21}(s)]$



Solution:

The transform Network is shown in fig (1)





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$$\text{Here. } \frac{\left(\frac{2}{S}\right)\times 1}{\frac{2}{S}+1} = \frac{2/S}{\frac{2+S}{S}} = \frac{2}{S+2}$$

Now applying KVL in loop (1) and (2) and then putting them in matrix from, we get

$$\begin{bmatrix} \left(\frac{2}{S+2}+1+2S\right) & (1+2S) \\ (1+2S) & (3+1+2S) \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \end{bmatrix}$$

$$\therefore I_1 = \frac{\Delta_1}{\Delta} = \frac{V_1(4+2S)}{\left(\frac{2}{S+2}+1+2S\right)(2S+4)-(1+2S)^2}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-V_1(1+2S)}{\left(\frac{2}{S+2}+1+2S\right)(2S+4)-(1+2S)^2}$$

Now,

$$\mathbf{Y_{21}(s) = \frac{I_2}{V_1} = \frac{-(1+2S)}{\left(\frac{2}{S+2}+1+2S\right)(2S+4)-(1+2S)^2}}$$

Also,

$$V_2 = -2I_2$$

$$\therefore V_2 = \frac{2V_1(1+2S)}{\left(\frac{2}{S+2}+1+2S\right)(2S+4)-(1+2S)^2}$$

Now,

$$\mathbf{G_{21} = \frac{V_2}{V_1} = \frac{2(1+2S)}{\left(\frac{2}{S+2}+1+2S\right)(2S+4)-(1+2S)^2}}$$

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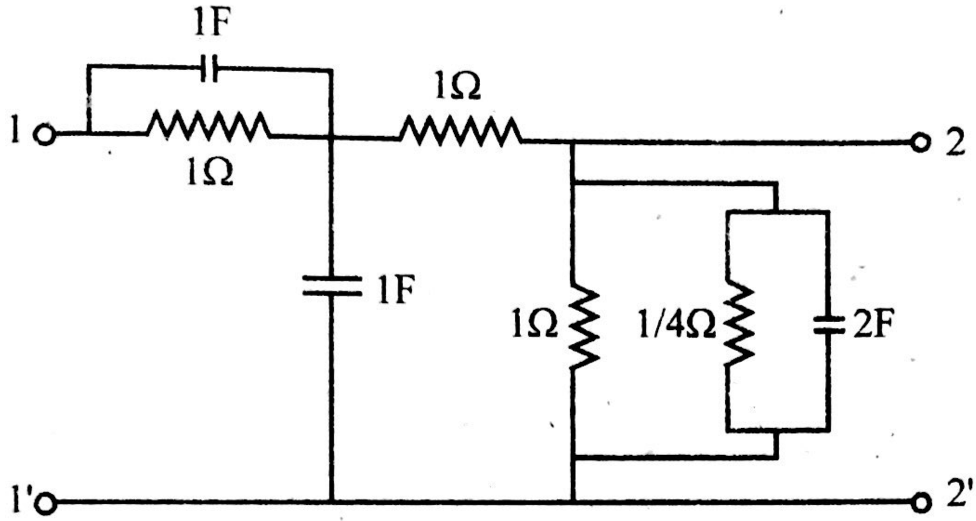
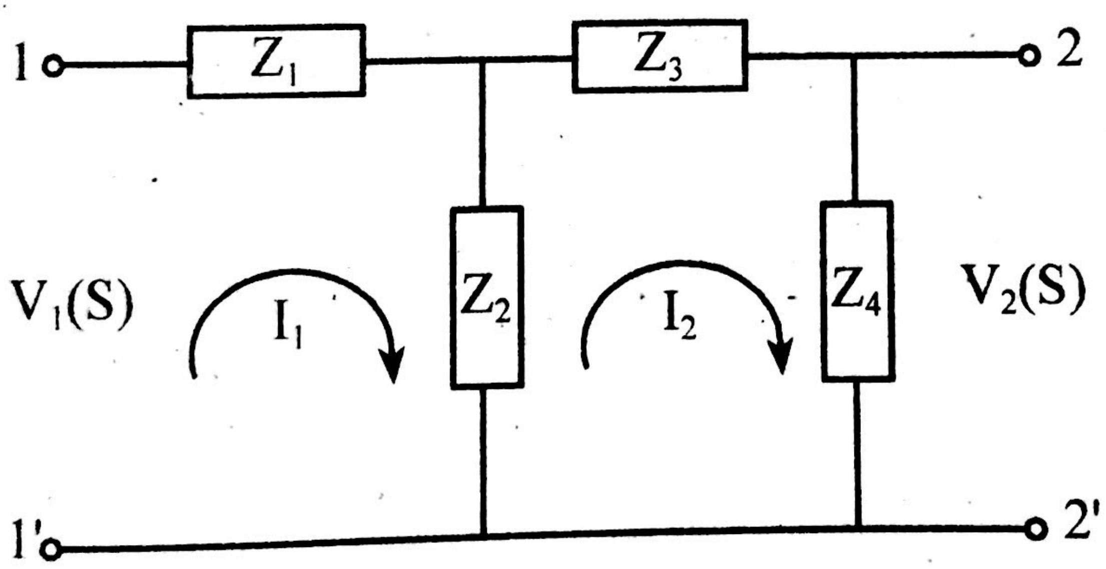
Example.8: For the network shown in figure below, determine the voltage ratio transfer function & the transfer impedance.



Solution:

The voltage ratio transfer function is given as

$$G_{21} = \frac{V_2}{V_1}$$



$$\text{Thus, } Z_1 = \frac{1}{s} // 1 = \frac{\frac{1}{s} \times 1}{1 + \frac{1}{s}} = \frac{\frac{1}{s}}{s + \frac{1}{s}} = \frac{1}{s+1}$$

Here, $Z_3 = 1$

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$$Z_4 = \frac{1}{s}$$

$$Z_4 = \left[\left(\frac{1}{2s} // \frac{1}{4}\right) // 1\right] = \left[\frac{\frac{1}{2s} \times \frac{1}{4}}{\frac{1}{2s} + \frac{1}{4}} // 1\right]$$

$$Z_4 = \frac{\frac{1}{2s+4} \cdot 1}{1 + \frac{1}{2s+4}} = \frac{\frac{1}{2s+4}}{\frac{2s+4+1}{2s+4}} = \frac{1}{2s+5}$$

Then,

Taking KVL at loop: $V_1(s) = Z_1 I_1 + Z_2 (I_1 - I_2)$

$$V_1(s) = (Z_1 + Z_2) I_1 - Z_2 I_2 \;\ldots\ldots(i)$$

And,

loop II: $(Z_2 + Z_3 + Z_4) I_2 - Z_2 I_2 = 0 \;\ldots\ldots (ii)$

Also, $V_2(s) = Z_4 I_2(s) \;\ldots\ldots\ldots (iii)$

Using Cramer's rule,

$$I_1(s) = \frac{\begin{vmatrix} V_1 & -Z_2 \\ 0 & (Z_2+Z_3+Z_4) \end{vmatrix}}{Z_1 Z_2 + Z_1 Z_3 + Z_1 Z_4 + Z_2 Z_3 + Z_2 Z_4}$$

$$I_1(s) = \frac{(Z_2+Z_3+Z_4) V_1}{Z_1Z_2+Z_1Z_3+Z_1Z_4+Z_2Z_3+Z_2+Z_4}$$

$$\text{And } I_2(s) = \frac{Z_2V_1}{Z_1Z_2+Z_1Z_3+Z_1Z_4+Z_2Z_3+Z_2Z_4}$$

Thus,

The transfer impedance $Z_{21} = \frac{V_2}{I_1}$

$$Z_{21} = \frac{(Z_4 \times Z_2 V_1)(Z_1Z_2+Z_1Z_3+Z_1Z_4+Z_2Z_3+Z_2Z_4)}{(Z_2+Z_3+Z_4)V_1 (Z_1Z_2+Z_1Z_3+Z_1Z_4+Z_2Z_3+Z_2Z_4)}$$

$$Z_{21} = \frac{\frac{1}{(2s+5)} \times \frac{1}{s}}{\frac{1}{s} + 1 + \frac{1}{2s+5}}$$

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$$Z_{21} = \frac{1}{2s^2 + 8s + 5}$$

And,

$$\text{Voltage ratio transfer function } G_{21} = \frac{V_2(s)}{V_1(s)}$$

$$G_{21} = \frac{\dfrac{Z_4 \times Z_2 V_1}{(Z_1Z_2+Z_1Z_3+Z_1Z_4+Z_2Z_3+Z_2Z_4)}}{V_1}$$

$$G_{21} = \frac{\dfrac{1}{(2s+5)} \times \dfrac{1}{s}}{\dfrac{1}{(s+1)} \times \dfrac{1}{s} + \dfrac{1}{(s+1)} + \dfrac{1}{(s+1)} \times \dfrac{1}{(2s+5)} + \dfrac{1}{s} \times \dfrac{1}{2s+5}}$$

$$G_{21} = \frac{\dfrac{1}{s(2s+5)}}{\dfrac{1}{s(s+1)} + \dfrac{1}{(s+1)} + \dfrac{1}{(s+1)(2s+5)} + \dfrac{1}{s} + \dfrac{1}{s(2s+5)}}$$

Example.9: If the two port network is terminated with 2Ω resistor at port -2 then for this terminated network find following network functions.

(i) G_{21} **(ii)** α_{21}

Solution:

Writing in S-domain;

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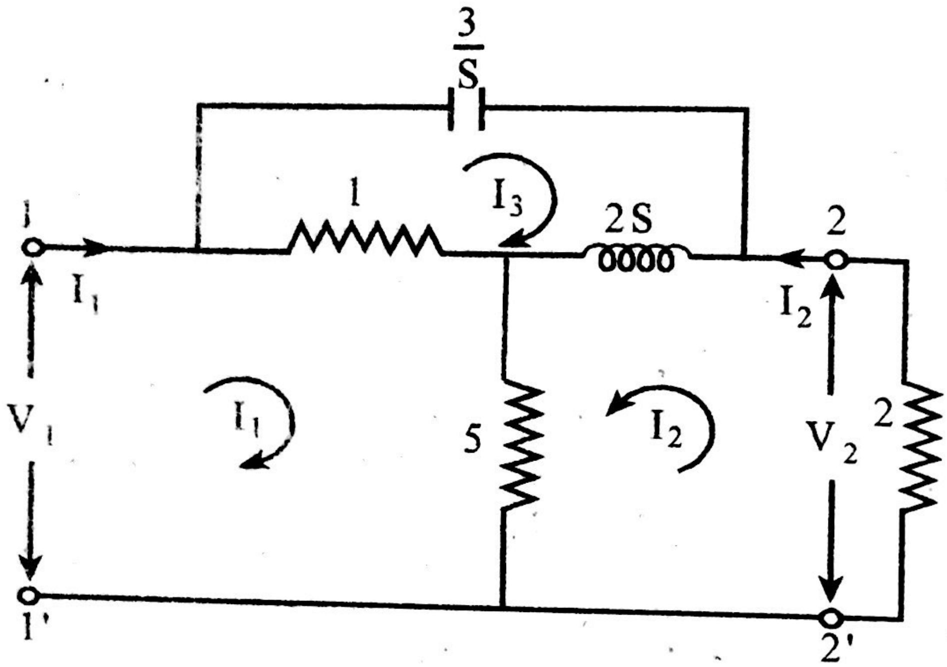

$$\begin{bmatrix} 6 & 5 & -1 \\ 5 & 2S+7 & 2S \\ -1 & 2S & \left(1+2S+\frac{3}{S}\right) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \\ 0 \end{bmatrix}$$

$$I_1 = \frac{\Delta_1}{\Delta} \text{......(1)}$$

$$\Delta_1 = \begin{bmatrix} V_1 & 5 & -1 \\ 0 & (2S+7) & 2S \\ 0 & 2S & (1+2S+\frac{3}{S}) \end{bmatrix}$$

$$= V_1 \left\{(2S+7)\left(1+2S+\frac{3}{S}\right) - 4S^2\right\}$$

$$I_2 = \frac{\Delta_2}{\Delta} \text{.........(2)}$$

$$\Delta_2 = \begin{bmatrix} 6 & V_1 & -1 \\ 5 & 0 & 2S \\ 1 & 0 & (1+2S+\frac{3}{S}) \end{bmatrix}$$

$$= -V_1\left[5(1.+2S+\frac{3}{S}) - 2S\right]$$

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$$\alpha_{21} = \frac{I_2}{I_1} = \frac{\frac{\Delta_2}{\Delta}}{\frac{\Delta_1}{\Delta}} = \frac{\Delta_2}{\Delta_1} = \frac{-[5\,(1+2S+\frac{3}{S})-2S\,]}{(2S+7)\,(1+2S+\frac{3}{S})-4S^2}$$

$$G_{21} = \frac{V_2}{V_1} = \frac{-2I_2}{V_1} \quad ....(3)$$

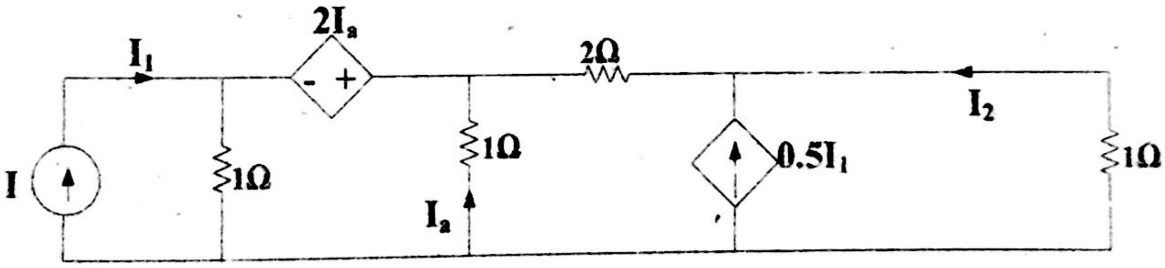
$$\Delta = 6\left[(2S+7)\,(1+2S+\frac{3}{S})-4S^2\right]-5\left[5(1+2S+\frac{3}{S})+2S\right]-1[10S+(2S+7)]$$

$$\therefore I_2 = \frac{\Delta_2}{\Delta}$$

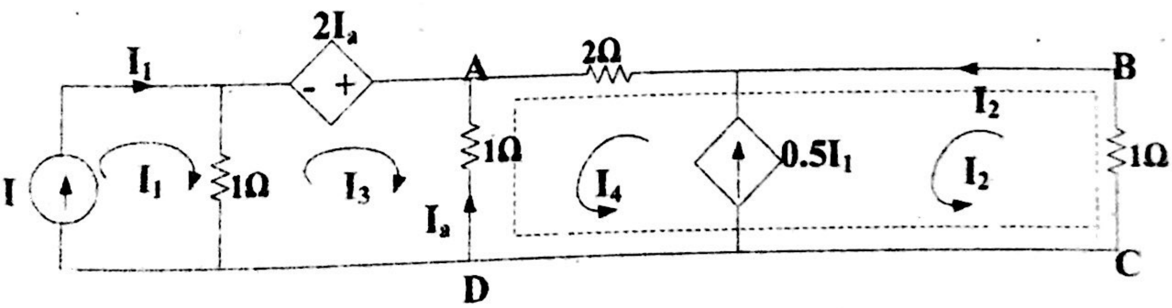
Hence,

$$G_{21} = \frac{-2\left[-5(1+2S+\frac{3}{S})-2S\right]}{6[(2s+7)(1+2s+\frac{3}{s})-4s^2]-5[5(1+2s+\frac{3}{s})+2s]-1[10s+(2s+7)]}$$

Example.10: For the network of following figure, find the current ratio transfer function.



Solution:



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From the figure $I_a = -(I_3 + I_4)$---- (1)

Note: Incase of presence of Voltage or current source (dependent or independent), don't write KVL directly in matrix form

Writing KVL;

Loop3:

$$2I_a = I_3 + I_4 + I_3 - I_1$$

$$-2(I_3 + I_4) = I_3 + I_4 + I_3 - I_1$$

$$3I_4 + 4I_3 - I_1 = 0 \text{------------} (2)$$

Applying Supermesh ABCD;

$$2I_4 + I_4 + I_3 + I_2 = 0$$

$$3I_4 + I_3 + I_2 = 0 \text{------------} (3)$$

Also

$$I_4 - I_2 = \frac{I_1}{2} \qquad \text{----------------} (4)$$

Substituting I_3 from equation (3) in equation (2)

$$3I_4 + 4(-3I_4 - I_2) - I_1 = 0$$

$$9I_4 + 4I_2 + I_1 = 0 \text{--------------} (5)$$

Now, Substituting Value of I_4 from equation (4) in equation (5);

$$9(I_2 + \frac{I_1}{2}) + 4I_2 + I_1 = 0$$

$$\frac{I_2}{I_1} = -\frac{11}{26}$$

$$\alpha_{21} = \frac{I_2}{I_1} = -\frac{11}{26}$$

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Bode diagram:

General procedure for constructing bode plot:

- Rewrite the Transfer function given by the question in time-constant form.
- Identify the corner frequencies (types) associated with each factor of the transfer functions.
- Knowing the corner frequencies, draw the asymptotic magnitude plot. This plot consists of straight line segments with line slope changing at each corner frequency by +20dB/decade for a zero and – 20 dB/decade for a pole. For a complex conjugate zero or pole the slope changes by± 40dB/decade.
- Calculate the phase angle for different frequencies and draw the asymptotic phase angle curve.

Type 1:

Sketch the asymptotic Bode plots for the transfer function given by,

$$G(s) = \frac{20(s+5)}{(s+10)(s^2+21s+20)}$$

Solution:

Transfer function is given by,

$$G(s) = \frac{20(s+5)}{(s+10)\,(s^2+21s+20)}$$

$$G(s) = \frac{20(s+5)}{(s+10)\,(s+1)(s+20)}$$

$$G(s) = \frac{20*5*\left(1+\frac{s}{5}\right)}{10\times1\times20\times\left(1+\frac{s}{10}\right)(1+s)\left(1+\frac{s}{20}\right)}$$

$$G(s) = \frac{0.5\left(1+\frac{s}{5}\right)}{(1+s)\left(1+\frac{s}{10}\right)\left(1+\frac{s}{20}\right)}$$

Put $j\omega \rightarrow s$

$$G[j\omega] = \frac{0.5\,[1+j\omega/5]}{[1+j\omega]\,[1+j\omega/10]\,[1+j\omega/20]}$$

So its corner frequencies are 1(-), 5(+), 10(-), 20(-)

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Magnitude plot:

Its starting point is = 20 log [0.5] = −6.020 dB

To find slope of different corner frequencies:

Factors	corner frequencies	initial slope	final slope
[0.5]	low	–	0dB/decade
$[1+j\omega]^{-1}$	1(-)	-20dB/decade	-20dB/decade
$\left[1+\frac{j\omega}{5}\right]^{+1}$	5(+)	+20dB/decade	0dB/decade
$\left[1+\frac{j\omega}{10}\right]^{-1}$	10(-)	-20dB/decade	-20dB/decade
$\left[1+\frac{j\omega}{20}\right]^{-1}$	20(-)	-20dB/decade	-40dB/decade

Phase plot:

It starting point is = 0°

To find slope of different corner frequencies:

Factors	corner freq^n	effective freq^n	initial slope	final slope
[0.5]	low	Low	–	0°/decade
$[1+j\omega]^{-1}$	1(-)	0.1(-)	-45°/decade	-45°/decade
$\left[1+\frac{j\omega}{5}\right]^{+1}$	5(+)	0.5(+)	+45°/decade	0°/decade
$\left[1+\frac{j\omega}{10}\right]^{-1}$	10(-)	1(-)	-45°/decade	-45°/decade
$\left[1+\frac{j\omega}{20}\right]^{-1}$	20(-)	2(-)	-45°/decade	-90°/decade

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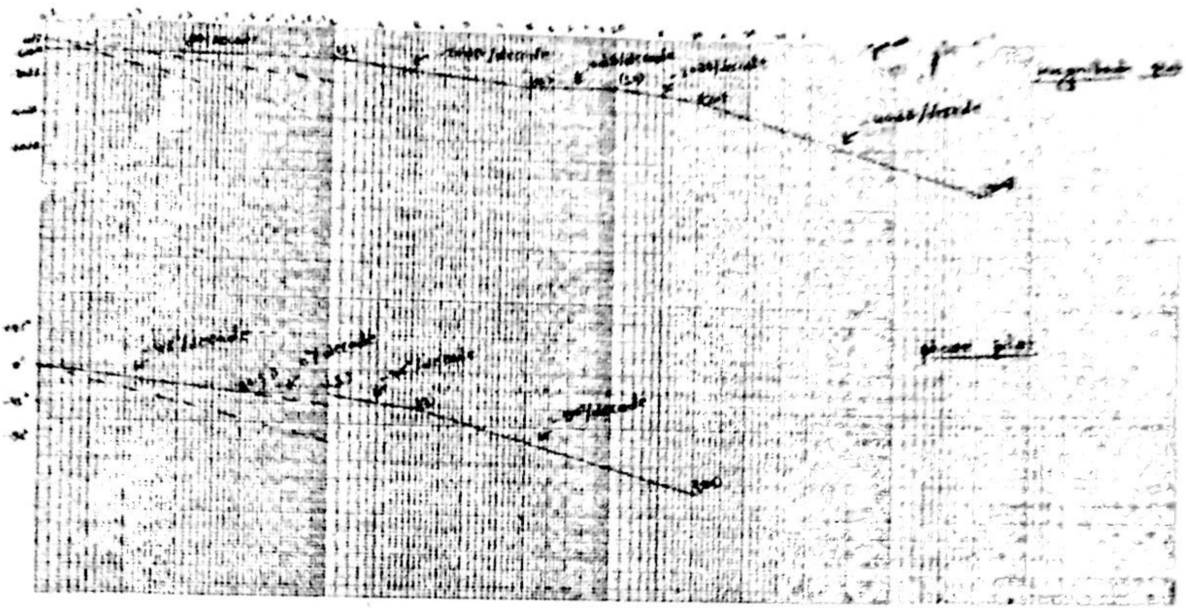

Type 2:

For the given transfer function, drawn the asymptotic bode - plot

$$G(s) = \frac{250(s+1)}{s(s^2 + 15s + 50)}$$

Solution:

$$G(s) = \frac{250(s+1)}{s(s+5)(s+10)}$$

$$G(s) = \frac{250(1+s)}{5*10*s(1 + s/5)(1 + s/10)}$$

$$G(s) = \frac{5(1 + s)}{s(1+s/5)(1 + s/10)}$$

Put $s \to jw$

$$G(jw) = \frac{5[1+jw/1]}{[jw][1 + jw/5][1+jw/10]}$$

Its corner frequencies are 1(+), 5(-), 10(-)

Magnitude plot:

Its starting point $= 20 \log\left[\frac{5}{w}\right]$, $w = 0.1$ rad/sec

$= 33.9794$ dB

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To find slope of different corner frequencies:

Factors	Corner frequencies	initial slope	final slope
$5[jw]^{-1}$	low (-)	–	-20dB/decade
$\left[1+\frac{jw}{1}\right]^{+1}$	1(+)	+20dB/decade	0dB/decade
$\left[1+\frac{jw}{5}\right]^{-1}$	5(-)	-20dB/decade	-20dB/decade
$\left[1+\frac{jw}{10}\right]^{-1}$	10(-)	-20dB/decade	-40dB/decade

Phase plot:

Its starting point is = -90°

To find slope of different corner frequencies:

Factors	corner frequencies	effective frequencies	initial slope	final slope
$5[jw]^{-1}$	low (-)	low (-)	–	0°/decade
$\left[1+\frac{jw}{1}\right]^{+1}$	1(+)	0.1(+)	+45°/decade	+45°/decade
$\left[1+\frac{jw}{5}\right]^{-1}$	5(-)	0.5(-)	-45°/decade	0°/decade
$\left[1+\frac{jw}{10}\right]^{-1}$	10(-)	1(-)	-45°/decade	-45°/decade

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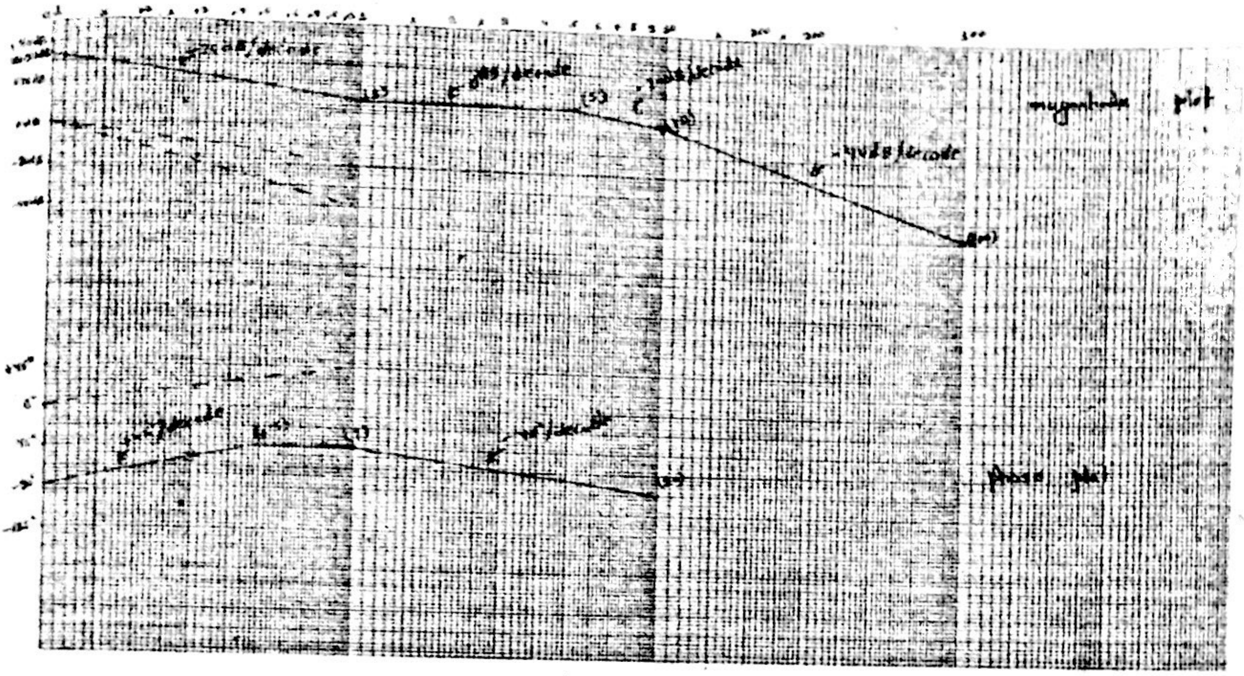

Type 3:

Sketch the asymptotic Bode plots for the transfer function given by,

$$\mathbf{H(s)} = \frac{20*S(S+5)}{(S+20)(S^2+41S+40)}$$

Solution:

$$H(s) = \frac{20*S(S+5)}{(S+20)(S+40)(S+1)}$$

$$H(s) = \frac{20*5*S(\frac{S}{5}+1)}{20*40*\left(\frac{S}{20}+1\right)\left(\frac{S}{40}+1\right)(S+1)}$$

$$H(s) = \frac{0.125*S(1+\frac{S}{5})}{\left(1+\frac{S}{20}\right)\left(1+\frac{S}{40}\right)(1+S)}$$

Put s $\rightarrow$ jw

$$H(jw) = \frac{0.125*[jw]*\left[1+\frac{jw}{5}\right]}{\left[1+\frac{jw}{20}\right]*\left[1+\frac{jw}{40}\right]*[1+jw]}$$

Its corner frequencies are 1(-), 5(+), 20(-), 40(-)

Magnitude plot:

Its starting point = 20 log[0.125 * w], w = 0.1 rad/sec

= –38.0617 dB

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To find slope of different corner frequencies:

Factors	corner frequencies	initial slope	final slope
$0.125[jw]^{+1}$	low(+)	–	+20dB/decade
$[1+j\omega]^{-1}$	1(-)	-20dB/decade	0dB/decade
$\left[1+\frac{j\omega}{5}\right]^{+1}$	5(+)	+20dB/decade	+20dB/decade
$\left[1+\frac{j\omega}{20}\right]^{-1}$	20(-)	-20dB/decade	0dB/decade
$\left[1+\frac{j\omega}{40}\right]^{-1}$	40(-)	-20dB/decade	-20dB/decade

Phase plot:

It starting point is = +90°

To find slope of different corner frequencies:

Factors	corner frequencies	effective frequencies	initial slope	final slope
$0.125[jw]^{+1}$	low (+)	low (+)	–	0°/decade
$[1+j\omega]^{-1}$	1(-)	0.1(-)	-45°/decade	-45°/decade
$\left[1+\frac{j\omega}{5}\right]^{+1}$	5(+)	0.5(+)	+45°/decade	0°/decade
$\left[1+\frac{j\omega}{20}\right]^{-1}$	20(-)	2(-)	-45°/decade	-45°/decade
$\left[1+\frac{j\omega}{40}\right]^{-1}$	40(-)	4(-)	-45°/decade	-90°/decade

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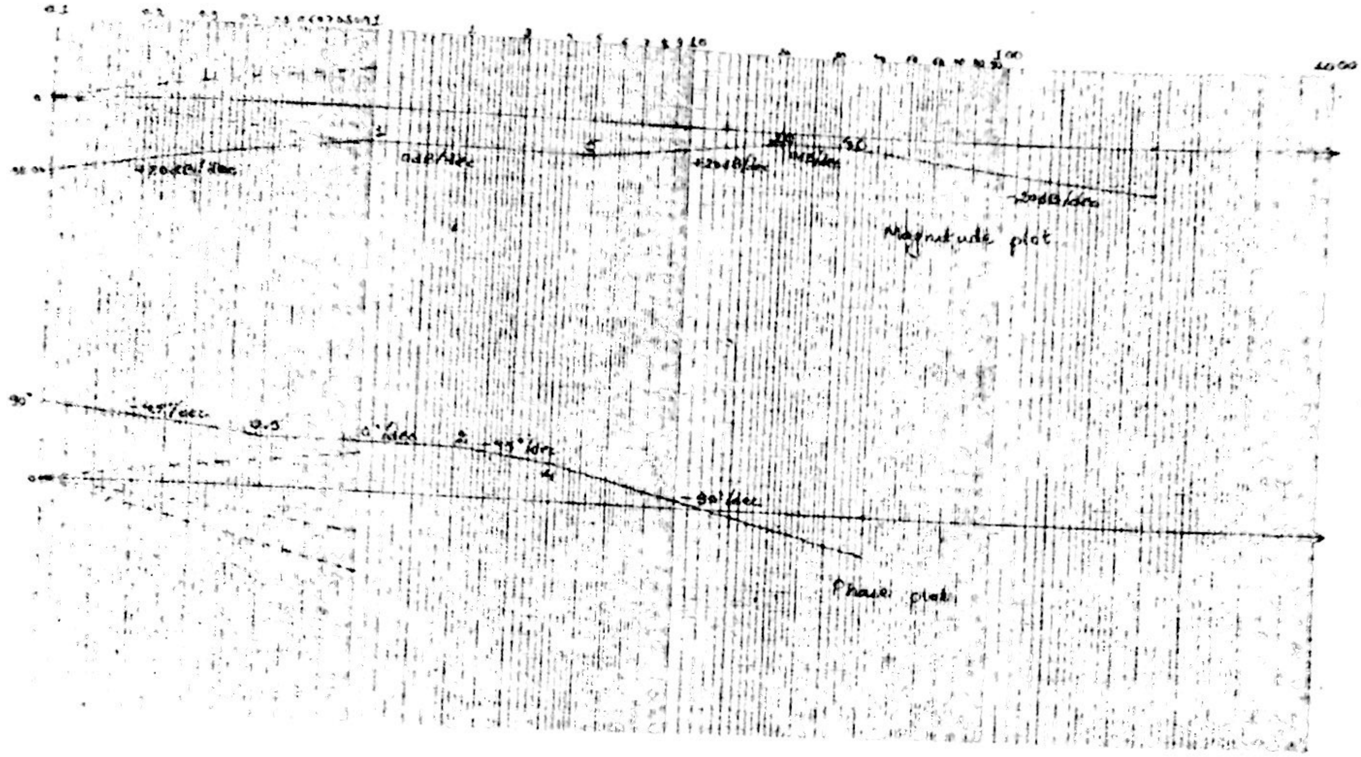

Type 4:

Sketch the Bode-plot for the transfer function given by $G(s) = \dfrac{20(s+5)}{s^2(s^2+41s+40)\,(s+20)}$

Solution:

- Transfer function is given by,

$$G(s) = \frac{20(s+5)}{s^2(s+1)\,(s+40)\,(s+20)}$$

$$G(s) = \frac{20*5*(1+s/5)}{s^2*40*20*(s+1)(1+s/20)(1+s/40)}$$

$$G(s) = \frac{0.125\,(1+s/5)}{s^2(1+s)\,(1+s/20)\,(1+s/40)}$$

put $s \to j\omega$

$$G(j\omega) = \frac{0.125[1+j\omega/5]}{[j\omega]^2\,[1+j\omega]\,[1+j\omega/20]\,[1+j\omega/40]}$$

So, its corner frequencies are 1(-), 5(+), 20(-), 40(-)

- **Magnitude plot:**

Its starting point is $= 20 \log \left[\frac{0.125}{w^2}\right]$ at w = 0.1 rad/sec

$= 21.938$ dB

To find slope of different corner frequencies:

Factors	corner frequencies	initial slope	final slope
$0.125[jw]^{-2}$	low(- -)	–	-40dB/decade
$[1+j\omega]^{-1}$	1(-)	-20dB/decade	-60dB/decade
$\left[1+\frac{j\omega}{5}\right]^{+1}$	5(+)	+20dB/decade	-40dB/decade
$\left[1+\frac{j\omega}{20}\right]^{-1}$	20(-)	-20dB/decade	-60dB/decade
$\left[1+\frac{j\omega}{40}\right]^{-1}$	40(-)	-20dB/decade	-80dB/decade

Phase plot:

It starting point is = -180°

To find slope of different corner frequencies:

Factors	corner frequencies	effective frequencies	initial slope	final slope
$0.125[jw]^{-2}$	low (- -)	low (- -)	–	0°/decade
$[1+j\omega]^{-1}$	1(-)	0.1(-)	-45°/decade	-45°/decade
$\left[1+\frac{j\omega}{5}\right]^{+1}$	5(+)	0.5(+)	+45°/decade	0°/decade
$\left[1+\frac{j\omega}{20}\right]^{-1}$	20(-)	2(-)	-45°/decade	-45°/decade
$\left[1+\frac{j\omega}{40}\right]^{-1}$	40(-)	4(-)	-45°/decade	-90°/decade

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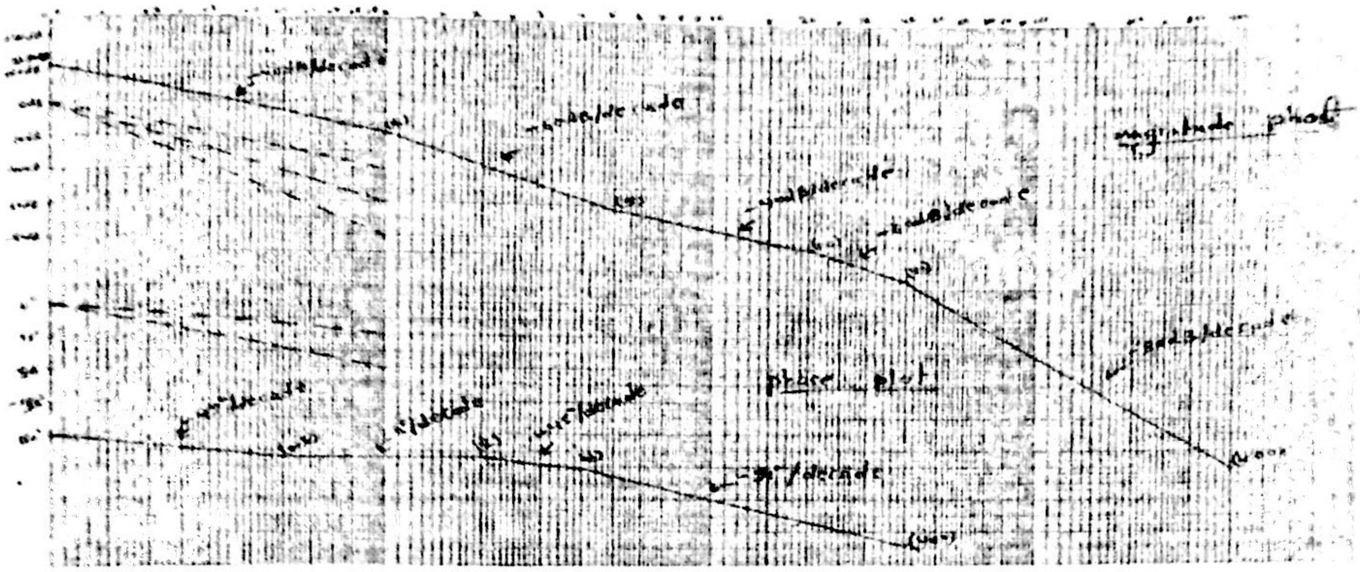

Type 5:

Sketch the asymptotic Bode plots for the transfer function given by,

$$H(s) = \frac{20*S^2(S+5)}{(S+20)(S^2+41S+40)}$$

Solution:

Transfer function is given by,

$$H(s) = \frac{20*S(S+5)}{(S+20)(S+40)(S+1)}$$

$$H(s) = \frac{20*5*S(\frac{S}{5}+1)}{20*40*\left(\frac{S}{20}+1\right)\left(\frac{S}{40}+1\right)(S+1)}$$

$$H(s) = \frac{0.125*S(1+\frac{S}{5})}{\left(1+\frac{S}{20}\right)\left(1+\frac{S}{40}\right)(1+S)}$$

Put s $\rightarrow$jw

$$H(jw) = \frac{0.125*[jw]^2*\left[1+\frac{jw}{5}\right]}{\left[1+\frac{jw}{20}\right]*\left[1+\frac{jw}{40}\right]*[1+jw]}$$

So, its corner frequencies are 1(-), 5(+), 20(-), 40(-)

Magnitude plot:

Its starting point is = 20 log$[0.125 * \omega^2]$at w = 0.1 rad/sec

= -58.061 dB

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To find slope of different corner frequencies:

Factors	corner frequencies	initial slope	final slope
$0.125[jw]^{+2}$	low(+ +)	–	+40dB/decade
$[1+j\omega]^{-1}$	1(-)	-20dB/decade	+20dB/decade
$\left[1+\frac{j\omega}{5}\right]^{+1}$	5(+)	+20dB/decade	+40dB/decade
$\left[1+\frac{j\omega}{20}\right]^{-1}$	20(-)	-20dB/decade	+20dB/decade
$\left[1+\frac{j\omega}{40}\right]^{-1}$	40(-)	-20dB/decade	0dB/decade

Phase plot:

It starting point is = +180°

To find slope of different corner frequencies:

Factors	corner frequencies	effective frequencies	initial slope	final slope
$0.125[jw]^{+2}$	low (+ +)	low (+ +)	–	0°/decade
$[1+j\omega]^{-1}$	1(-)	0.1(-)	-45°/decade	-45°/decade
$\left[1+\frac{j\omega}{5}\right]^{+1}$	5(+)	0.5(+)	+45°/decade	0°/decade
$\left[1+\frac{j\omega}{20}\right]^{-1}$	20(-)	2(-)	-45°/decade	-45°/decade
$\left[1+\frac{j\omega}{40}\right]^{-1}$	40(-)	4(-)	-45°/decade	-90°/decade

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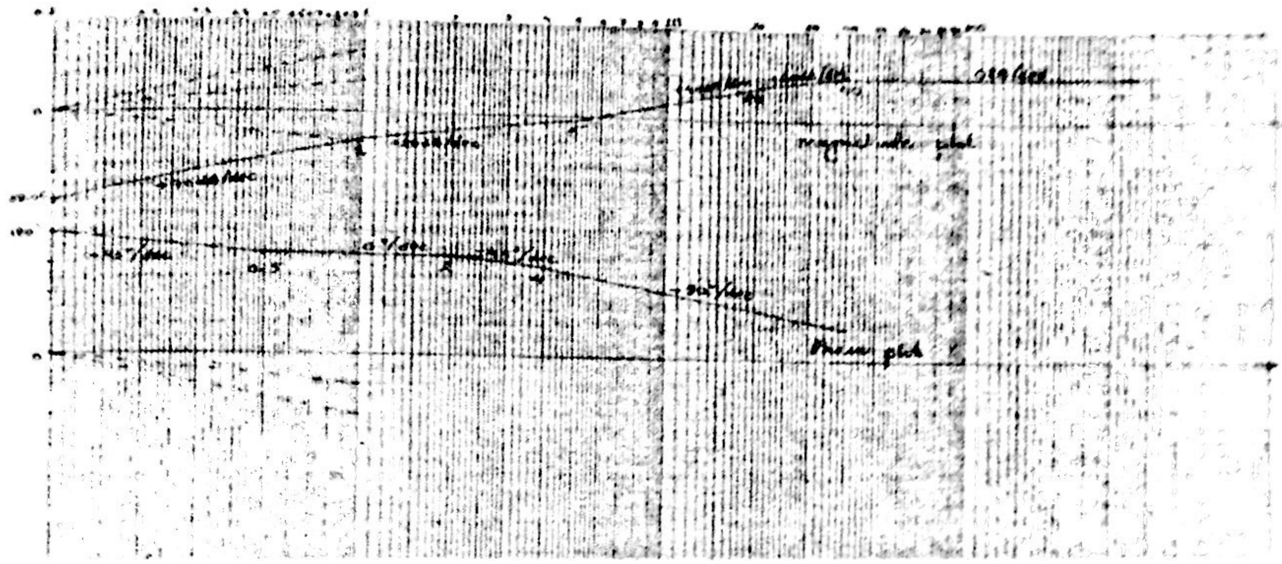

Type 6:

Draw the asymptotic bode magnitude plot of the transfer function,

$$H(s) = \frac{20(s+1)}{(s^2+2s+10)(s^2+5s)}$$

Solution:

$$H(s) = \frac{20(s+1)}{s(s+5)(s^2+2s+10)}$$

$$H(s) = \frac{20(1+s)}{s*5*(1+s/5)*10*\left(1+\frac{s}{5}+\left(\frac{s}{\sqrt{10}}^2\right)\right)}$$

put $s \rightarrow j\omega$

$$H(j\omega) = \frac{20(1+j\omega/1)}{50\,j\omega\left[1+\frac{j\omega}{5}\right]\left[1+\frac{j\omega}{5}+\left(\frac{j\omega}{\sqrt{10}}^2\right)\right]}$$

$$H(j\omega) = \frac{0.4\,[1+j\omega/1]}{j\omega\left[1+\frac{j\omega}{5}\right]\left[1+\frac{j\omega}{5}+\left(\frac{j\omega}{\sqrt{10}}^2\right)\right]}$$

So its corner frequencies are: 1(+), $\sqrt{10}$ (- -), 5(+),

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Magnitude plot:

Its starting point is $= 20 \log \left[\frac{0.4}{\omega}\right]$ at $\omega = 0.1$ rad/sec

$= 12.041$ dB.

To find slope of different corner frequencies:

Factors	Corner frequencies	Initial slope	Final slope
$0.4\,[j\omega]^{-1}$	low (-)	–	-20dB/dec
$\left[1+\frac{j\omega}{1}\right]^{+1}$	1(+)	+20dB/decade	0dB/decade
$\left[1+\frac{j\omega}{5}+\left(\frac{j\omega}{\sqrt{10}}^{2}\right)\right]^{-1}$	$\sqrt{10}$ (- -)	-40dB/decade	-40dB/dec
$\left[1+\frac{j\omega}{5}\right]^{-1}$	5(-)	-20dB/decade	-60dB/dec

Phase plot:

Its starting point is $= -90°$

To find slope of different corner frequencies:

Factors	Corner frequencies	Effective frequencies	Initial slope	Final slope
$0.4\,(j\omega)^{-1}$	low (-)	low (-)	–	0°/decade
$\left[1+\frac{j\omega}{1}\right]^{1}$	1(+)	0.1(+)	+45°/decade	+45°/decade
$\left[1+\frac{j\omega}{5}+\left(\frac{j\omega}{\sqrt{10}}^{2}\right)\right]^{-1}$	$\sqrt{10}$ (- -)	0.316(- -)	–90°/decade	–45°/decade
$\left[1+\frac{j\omega}{5}\right]^{-1}$	5(-)	0.5 (–)	–45°/decade	–90°/decade

Pdf by: Sachin Lamsal

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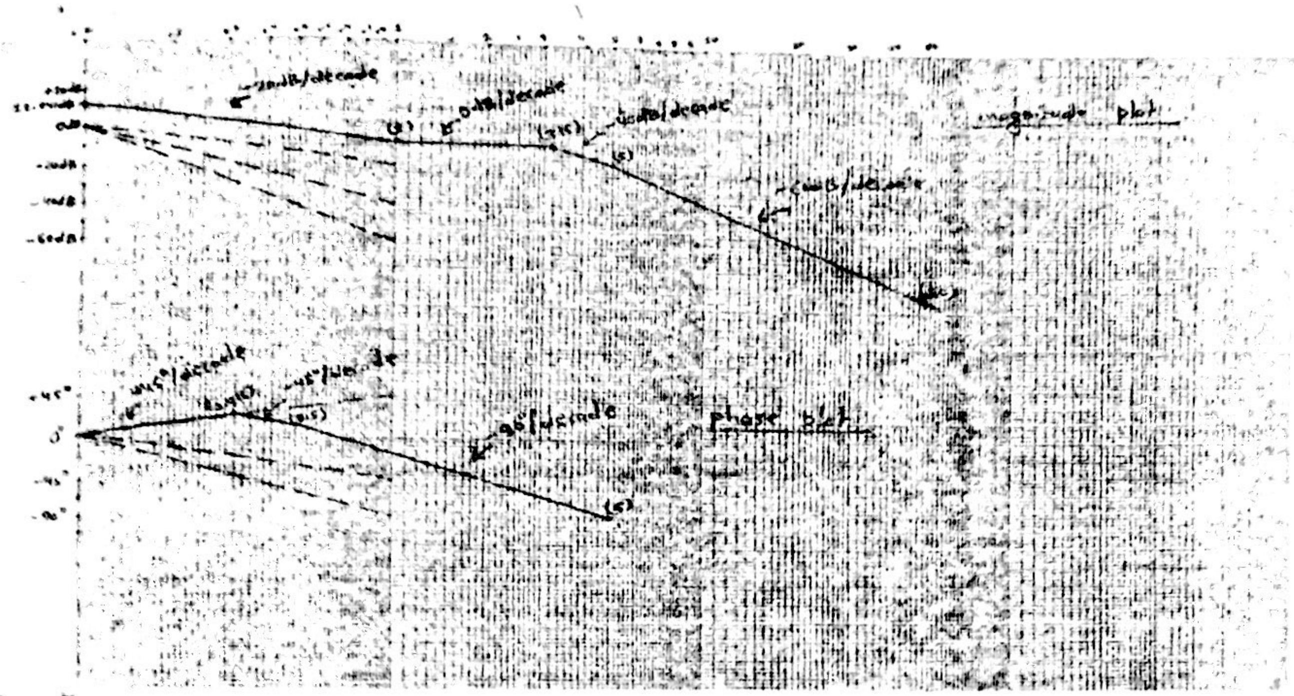

Type 7:

Sketch the asymptotic Bode plots for the transfer function given by,

$$H(s) = \frac{20S(S^2+2S+10)}{(S^2+25S+100)}$$

Solution:

Transfer function is given by,

$$H(s) = \frac{20S(S^2+2S+10)}{(S^2+25S+100)}$$

$$H(s) = \frac{20*S*10\left[1+\frac{S}{5}+\left(\frac{S}{\sqrt{10}}\right)^2\right]}{20*5*\left(\frac{S}{20}+1\right)\left(\frac{S}{5}+1\right)}$$

Put $s \rightarrow jw$

$$H(jw) = \frac{2*[jW]*\left[1+\frac{jW}{5}+\left(\frac{jW}{\sqrt{10}}\right)^2\right]}{\left(\frac{jW}{20}+1\right)\left(\frac{jW}{5}+1\right)}$$

Note: While calculating starting point for Magnitude plot, the value of 'ω' should be taken as $= \frac{\text{least value of coarner frequecy}}{10}$

For example in this question, the value of 'ω' should take as $= \frac{5}{10} = 0.5$rad/sec. but it is taken as 0.1 rad/sec it's because in bode graph, frequency (ω) axis starts either 0.001, 0.01, 0.1,1,10,100 and so on.

So, its corner frequencies are 5(-), 20(-), $\sqrt{10}$ (- -)

Magnitude plot:

Its starting point is $= 20\log[0.125 * \omega]$at w = 0.1 rad/sec

$= -38.06$ dB

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To find slope of different corner frequencies:

Factors	Corner frequencies	Initial slope	Final slope
$0.4\,[j\omega]^{+1}$	low (+)	–	+20dB dec
$\left[\frac{jW}{5}+1\right]^{-1}$	$\sqrt{10}$ (+ +)	+40dB/decade	+60dB decade
$\left[1+\frac{jW}{5}+\left(\frac{jW}{\sqrt{10}}\right)^2\right]^{+1}$	5(-)	-20dB/decade	+40dB dec
$\left[\frac{jW}{20}+1\right]^{-1}$	20(-)	-20dB/decade	+20dB/dec

Phase plot:

Its starting point is = +90°

To find slope of different corner frequencies:

Factors	Corner frequencies	Effective frequencies	Initial slope	Final slope
$0.4\,[j\omega]^{+1}$	low (+)	low (+)	–	0°/decade
$\left[\frac{jW}{5}+1\right]^{-1}$	$\sqrt{10}$ (+ +)	0.316(+ +)	+90°/decade	+90°/decade
$\left[1+\frac{jW}{5}+\left(\frac{jW}{\sqrt{10}}\right)^2\right]^{+1}$	5(-)	0.5(-)	-45°/decade	+45°/decade
$\left[\frac{jW}{20}+1\right]^{-1}$	20(-)	2 (–)	–45°/decade	0°/decade

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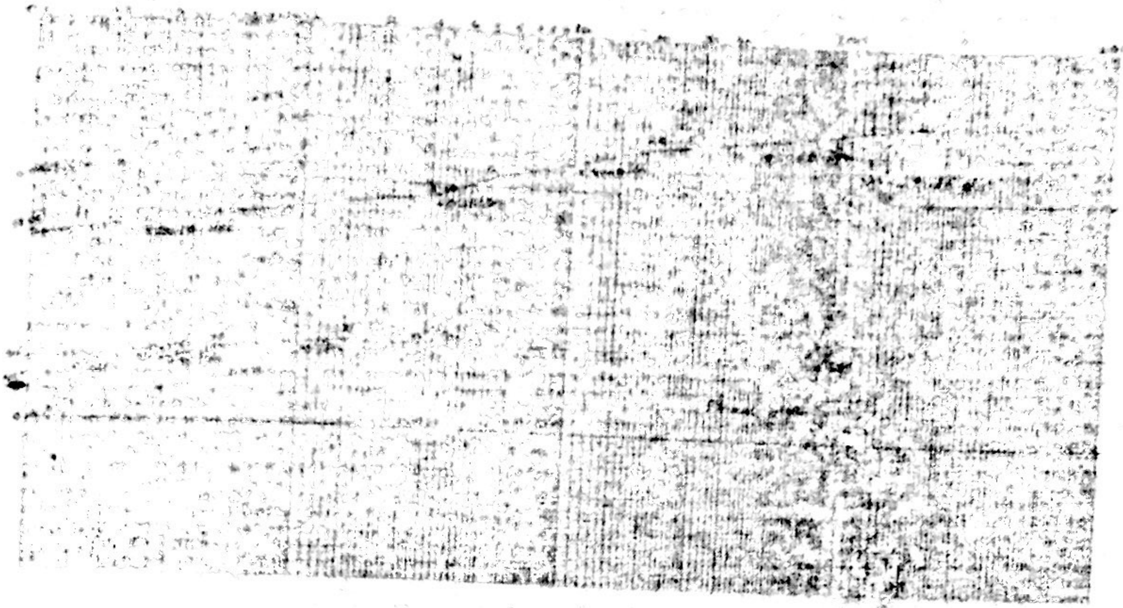
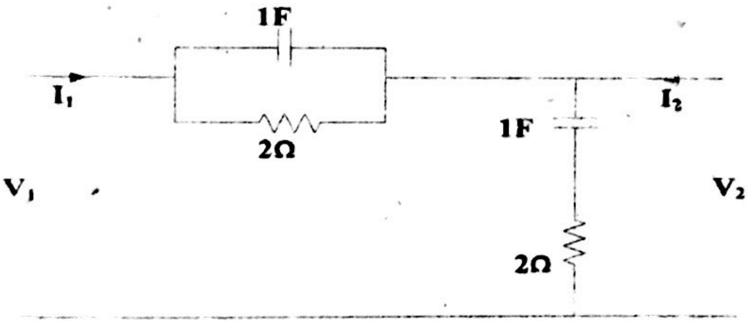


Problems:

Q.1: Find the voltage ratio transfer function of the following network.



[Ans: $G_{21} = \dfrac{(2S+1)^2}{4S^2+6S+1}$ **]**

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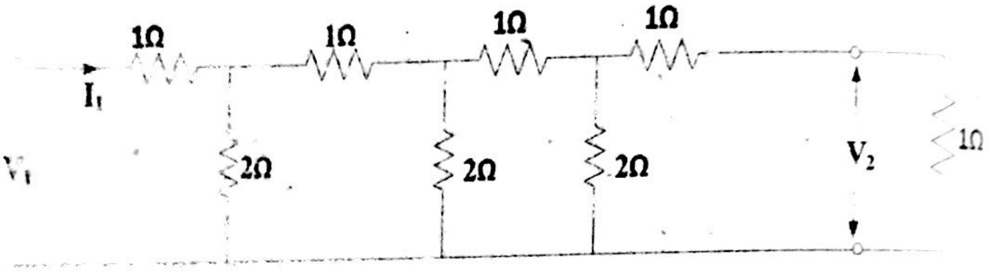
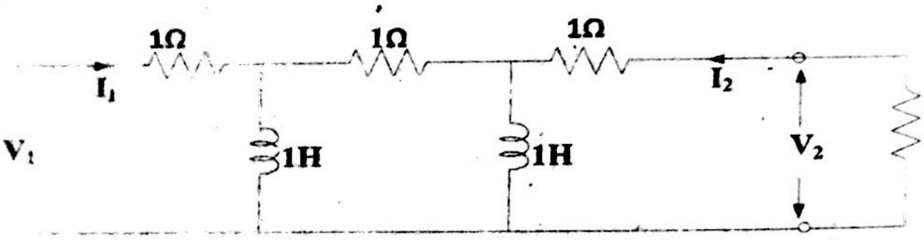
Q.2: For the given resistive network, determine the numerical value of:

(i) Z_{11} (ii) G_{21} (iii) Z_{21} (iv) Y_{21} (v) α_{21}



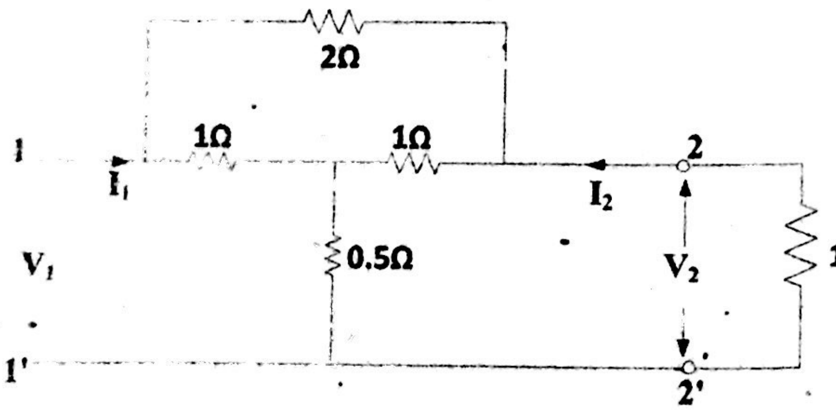
[Ans: $Z_{11}= 2\Omega$, $G_{21} = \frac{1}{16}$, $Z_{21}=\frac{1}{8}\Omega$, $Y_{21}= -\frac{1}{16}\mho$, $\alpha_{21}= -\frac{1}{8}$]

Q.3: For the two port network shown in figure below, find voltage ratio transfer function $G_{21}(S)$ and transfer admittance $Y_{21}(S)$



[Ans: $G_{21}= \frac{s^2}{5S^2+10S+3}$, $Y_{21}= \frac{-s^2}{5S^2+10S+3}$]

Q.4: For the network shown in figure below, find voltage ratio transfer function $G_{21}(S)$ and current ratio transfer function α_{21}



[Ans: $G_{21}= \frac{1}{3}$, $\alpha_{21} = -\frac{1}{3}$]

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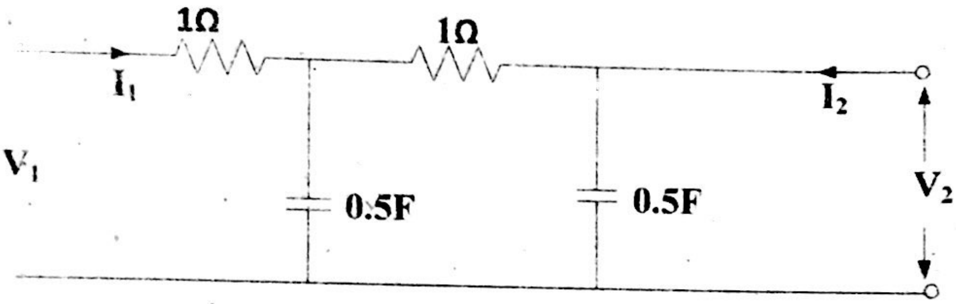
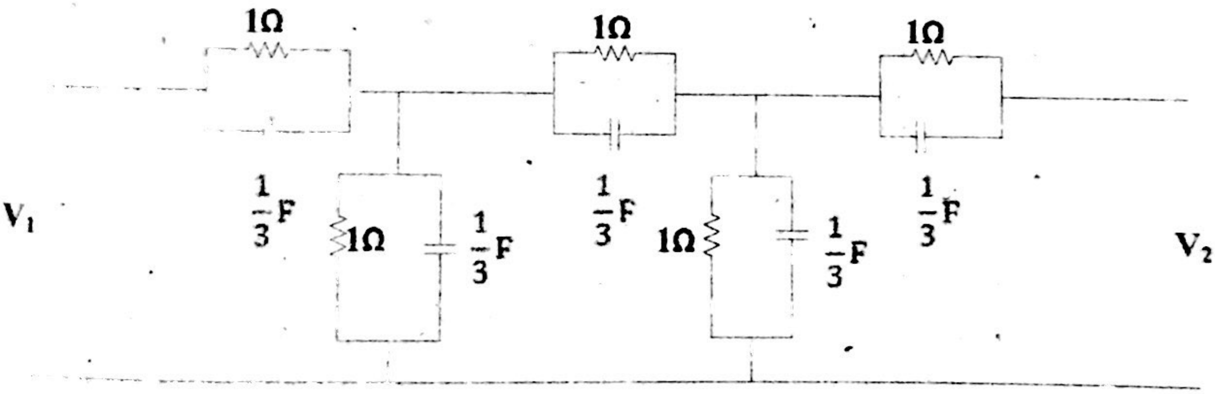
Q.5: For the given two port network, determine voltage ratio transfer function $G_{21}(S)$. If this network is terminated at port-2 with a 2Ω resistor find for this terminated network, the following network functions:

(i) Z_{11} (ii) Z_{21} (iii) Y_{21} (iv) α_{21}



[Ans: $G_{21} = \frac{4}{S^2+6S+4}$, $Z_{11}=\frac{2S^2+14S+16}{2S^2+10S+4}$, $Z_{21}=\frac{4}{S^2+5S+2}$, $Y_{21}=-\frac{2}{S^2+7S+8}$, $\alpha_{21}=-\frac{2}{S^2+5S+2}$]

Q.6: Find the value of $Z_{11}(S)$, $Z_{21}(S)$, $Y_{12}(S)$ for given network.



[Ans:, $Z_{11}=\frac{5S}{3(S+3)}$, $Z_{21}=\frac{S}{3(S+3)}$, $Y_{21}=\frac{3(S+3)}{S}$]

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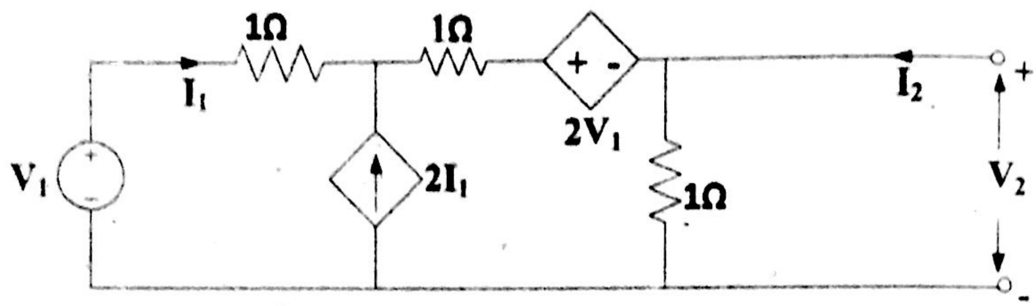
Q.7: For the network shown in figure below, Compute $G_{21}(S) = \frac{V_2(S)}{V_1(S)}$



[Ans: $G_{21} = -\frac{3}{7}$]

Q.8:Sketch the asymptotic Bode plots for the transfer function given by,

$$H(s) = \frac{5S(S^2+5S+4)}{(S^2+25S+100)}$$

Q.9:Sketch the asymptotic Bode plots for the transfer function given by,

$$G(s) = \frac{20(S^2+4S+5)}{(S^2+25S+100)}$$

Q.10:Sketch the asymptotic Bode plots for the transfer function given by,

$$H(s) = \frac{20(S^2+2S+10)}{(S^3+4S^2+5S)}$$

Q.11:Sketch the asymptotic Bode plots for the transfer function given by,

$$G(s) = \frac{20S^2(S^2+4S+5)}{(S+100)}$$

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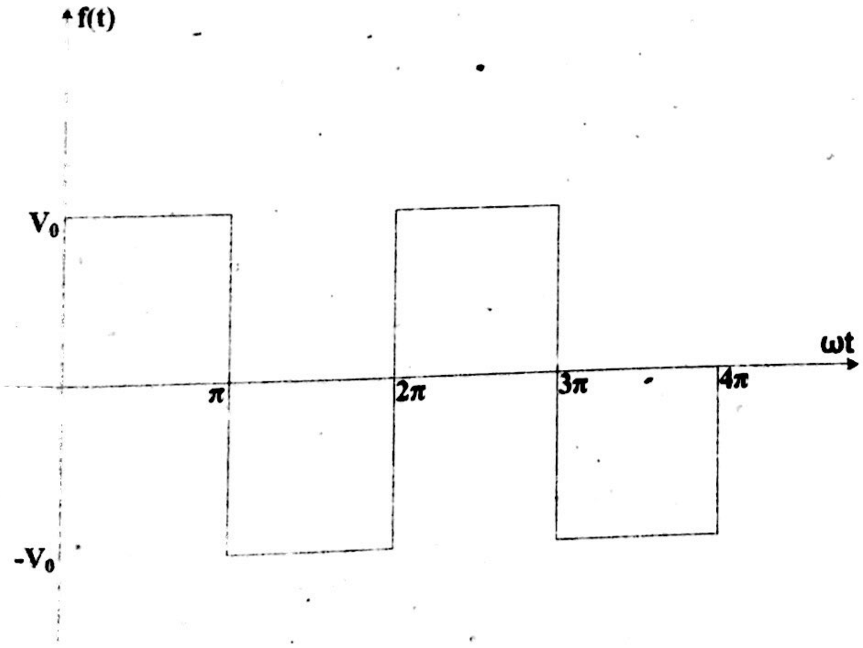
Fourier Series and Transform

Some important formula:

- $\int_0^{2\pi} x\ \sin(nx)\,dx = \left[[x\int \sin(nx)\,dx]_0^{2\pi} - \int_0^{2\pi}\left\{\frac{dx}{dx}\int \sin(nx)\,dx\right\}\right]$
- $\int_a^b \sin(nx)\,dx = -\frac{1}{n}[\cos nx]_a^b$
- $\cos n\pi = (-1)^n$
- $\sin n\pi = \sin 2\pi = 0$
- $\cos 2n\pi = 1$
- Cos = even
- Sin = odd
- Cos * Sin = odd
- Cos * Cos = even

#Trigonometric Form of Fourier series:

A single is periodic if, for some positive value of T, $f(t) = f(t + T)$ for all t.



Here,

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t)\ ,\ 0 \le \omega t \le 2\pi$$

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Taking a definite integral in complete cycle.

$$\int_0^{2\pi} f(t)d(\omega t) = \frac{a_0}{2}\int_0^{2\pi} d(\omega t) + \sum_{n=1}^{\infty}\int_0^{2\pi}(a_n \,\text{Cos}\, n\omega t + b_n \,\text{Sin}\, n\omega t)d(\omega t)$$

So we get,

$$\int_0^{2\pi} f(t)d(\omega t) = \frac{a_0}{2}\int_0^{2\pi} d(\omega t)$$

$$\int_0^{2\pi} f(t)d(\omega t) = \frac{a_0}{2}[2\pi - 0]$$

$$\mathbf{a_0 = \frac{1}{\pi}\int_0^{2\pi} f(t)d(\omega t)}$$

Or,

$$\mathbf{a_0 = \frac{2}{T}\int_0^{T} f(t)d(t) \quad , \text{Where } \omega = \frac{2\pi}{T}}$$

And ,

$$\mathbf{a_n = \frac{1}{\pi}\int_0^{2\pi} f(t)\text{Cos}(n\omega t)\, d(\omega t)}$$

Or,

$$\mathbf{a_n = \frac{2}{T}\int_0^{T} f(t)\text{Cos}(n\omega t)\, d(t) \quad , \text{Where } \omega = \frac{2\pi}{T}}$$

Also,

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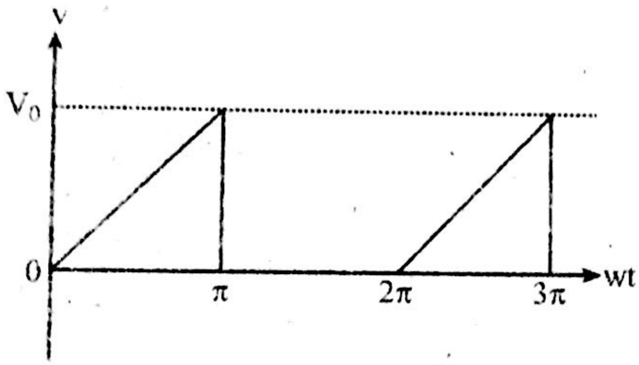
$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(t)\text{Sin}(n\omega t)\, d(\omega t)$$

Or,

$$b_n = \frac{2}{T}\int_0^{T} f(t)\text{Sin}(n\omega t)d(t) \quad \text{, Where } \omega = \frac{2\pi}{T}$$

Find the trigonometric Fourier series for the given waveform shown



Solution:

The trigonometric Fourier series is given as

$$V(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \text{ Cos } n\omega t + b_n \text{ Sin } n\omega t) \; 0 \le wt \le 2\pi$$

From the given waveform we have,

$$V(t) = \frac{V_0}{\pi}\omega t \qquad 0 \le wt \le \pi$$

$$= 0 \qquad \pi \le wt \le 2\pi$$

$$\therefore a_0 = \frac{1}{\pi}\int_0^{2\pi} V(t)\, d(\omega t)$$

$$= \frac{1}{\pi}\int_0^{\pi} \frac{V_0}{\pi}\omega t\, d(\omega t)$$

$$= \frac{V_0}{\pi^2}\left|\frac{(wt)^2}{2}\right|_0^{\pi} = \frac{V_0}{\pi^2}\left[\frac{\pi^2 - 0}{2}\right]$$

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$$= \frac{V_0}{2}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} V(t)\cos n\omega t\, d(\omega t)$$

$$= \frac{1}{\pi}\int_0^{\pi} \frac{V_0}{\pi}\,\omega t \cos n\omega t\, d(\omega t)$$

$$= \frac{V_0}{\pi^2}\left[\left|wt\,\frac{\sin nwt}{x}\right|_0^{\pi} + \left|\frac{1}{(n)^2}\right| |\cos nwt|_0^{\pi}\right]$$

$$= \frac{V_0}{\pi^2}\left[\frac{1}{n}(\pi \sin n\pi - 0) + \frac{1}{n^2}(\cos n\pi - 1)\right]$$

$$= \frac{V_0}{n^2\pi}(\cos n\pi - 1)$$

$\therefore\ a_n = 0$ if n = even.

$a_n = \frac{-2V_0}{n^2\pi}$ if n = odd

$$b_n = \frac{1}{\pi}\int_0^{2\pi} V(t)\, \text{Sin}\, n\omega t\, d(\omega t)$$

$$b_n = \frac{1}{\pi}\int_0^{\pi} \frac{V_0}{\pi}\, wt \sin n\omega t\, d(\omega t)$$

$$\text{or, } b_n = \frac{V_0}{\pi^2}\left[-\left|wt\,\frac{\cos nwt}{n}\right|_0^{\pi} + \frac{1}{n^2}|\sin nwt|_0^{\pi}\right]$$

$$\text{or, } b_n = \frac{V_0}{\pi^2}\left[-\frac{1}{n}(\pi\cos n\pi - 0) - \frac{1}{n^2}[\sin n\pi - 0]\right]$$

$$\text{or, } b_n = \frac{-V0}{n\pi^2}\,\pi \cos n\pi$$

$$\text{or, } b_n = \frac{-V_0}{n\pi}\cos n\pi$$

Hence Fourier series is given by

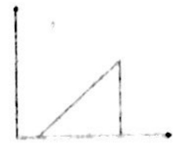
$$V = \frac{V_0}{4} + \sum_{n=1,3,5,\ldots}^{\infty}\left(\frac{-2V_0}{n^2\pi}\right)\cos nwt + \sum_{n=1,2,3\ldots}^{\infty}\left(\frac{-V_0}{n\pi}\right)\cos n\pi . \sin nwt$$

Note: To find $V(t) = \frac{V_0}{\pi} wt$ expression use $y = mx + c$

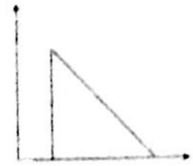
Where, $y = V(t)$

$m = \tan\theta = \text{Slope} = \frac{p}{b}$

Here 'm' can be +ve or −ve, If slope is rising like this figure take m = +ve



And if slope is decreasing like this figure take m = −ve



C = y interception

Here 'C' y interception, the point where slope will cut at y−axies.

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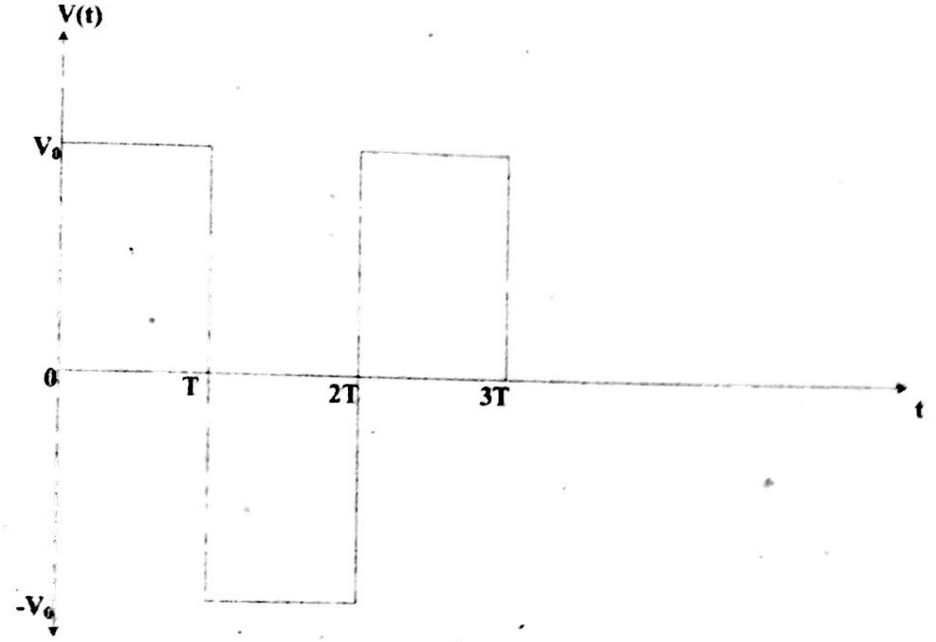
Find the trigonometric Fourier series for the given waveform shown



Solution:

The trigonometric Fourier series is given as

$$V(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \, Cos\, n\omega t + b_n \, Sin\, n\, \omega t]$$

From the question,

$V(t) = V_0$ for $0 < t < T$

$V(t) = -V_0$ for $T < t < 2T$

[$\therefore y = mx + C$, $y = V(t)$ $m = 0$ $C = V_0$ and $-V_0$]

We have, $\omega = \frac{2\pi}{t} = \frac{2\pi}{2T} = \frac{\pi}{T}$

Now,

$$a_0 = \frac{1}{T}\int_0^{2T} V(t)dt$$

$$a_0 = \frac{1}{T}\left[\int_T^0 V_0 dt - \int_{2T}^{T} V_0 dt\right]$$

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$a_0 = \frac{1}{T}[V_0[T - 0] - V_0[2T - T]]$

$a_0 = 0$

And

$a_n = \frac{1}{T}\int_0^{2T} V(t) \text{Cos } n\omega t \, dt$

$a_n == \frac{1}{T}\left[\int_T^0 V_0 \text{Cos } n\frac{\pi}{T}t \, dt - \int_{2T}^T V_0 \text{Cos } n\frac{\pi}{T}t \, dt\right]$

$a_n == \frac{1}{T}\left[\int_T^0 V_0 \text{Cos } n\frac{\pi}{T}t \, dt - \int_{2T}^T V_0 \text{Cos } n\frac{\pi}{T}t \, dt\right]$

$a_n == \frac{1}{T}\left[V_0 * \frac{T}{n\pi}\left[\text{Sin } n\frac{\pi}{T}t\right]_T^0 - V_0 * \frac{T}{n\pi}\left[\text{Sin } n\frac{\pi}{T}t\right]_{2T}^T\right]$

$a_n == \frac{1}{T}\left[V_0 * \frac{T}{n\pi}[\text{Sin } n\pi - \text{Sin } 0] - V_0 * \frac{T}{n\pi}[\text{Sin } 2n\pi - \text{Sin } n\pi]\right]$

$a_n == 0$

Also

$b_n = \frac{1}{\pi}\int_0^{2\pi} V(t) \text{ Sin } n\omega t \, dt$

$b_n == \frac{1}{T}\left[\int_T^0 V_0 \text{Sin } n\frac{\pi}{T}t \, dt - \int_{2T}^T V_0 \text{Sin } n\frac{\pi}{T}t \, dt\right]$

$b_n == \frac{1}{T}\left[\int_T^0 V_0 \text{Sin } n\frac{\pi}{T}t \, dt - \int_{2T}^T V_0 \text{Sin } n\frac{\pi}{T}t \, dt\right]$

$b_n == \frac{1}{T}\left[V_0 * \frac{-T}{n\pi}\left[\text{Cos } n\frac{\pi}{T}t\right]_T^0 + V_0 * \frac{T}{n\pi}\left[\text{Cos } n\frac{\pi}{T}t\right]_{2T}^T\right]$

$b_n == \frac{1}{T}\left[V_0 * \frac{-T}{n\pi}[\text{Cos } n\pi - \text{Cos } 0] + V_0 * n\frac{T}{\pi}[\text{Cos } 2n\pi - \text{Cos } n\pi]\right]$

$b_n == \frac{1}{T}\left[V_0 * \frac{-T}{n\pi}[(-1)^n - 1] + V_0 * \frac{T}{n\pi}[1 - (-1)^n]\right]$

$b_n == \frac{1}{T} * \frac{T}{n\pi} * V_0[-(-1)^n + 1 + 1 - (-1)^n]]$

$b_n == \frac{2}{T} * \frac{T}{n\pi} * V_0[1 - (-1)^n]]$

$b_n == \frac{2}{n\pi} * V_0[1 - (-1)^n]]$

for n = even

$b_n == 0$

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for n = odd

$b_n == \frac{4}{n\pi} * V_0$

Thus the required trigonometric Fourier series is given as:

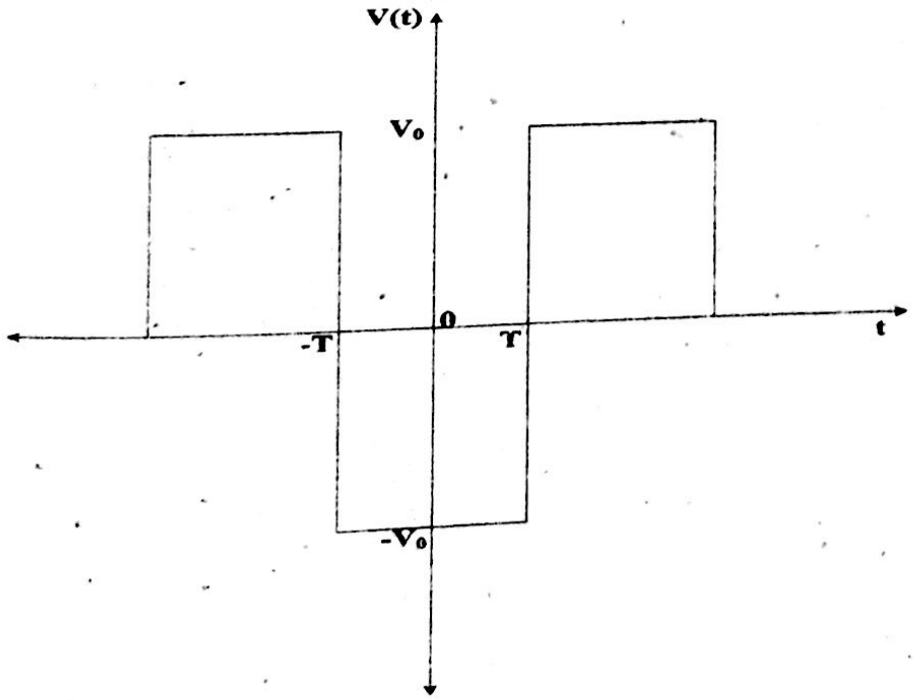
$$V(t) = \sum_{n=1}^{\infty} \left[\frac{4}{n\pi} * V_0 \sin n\frac{\pi}{T}t \right] \qquad \text{for all } n = \text{odd}$$

#Symmetry in Fourier series

For symmetrical signal, simplification procedure can be adopted in Fourier analysis. There are three types of symmetry:

(1) Even function symmetry
(2) Odd function symmetry
(3) Half wave symmetry

(1) Even Function Symmetry:
A function f(t) is said to be even if f(t) = f(-t) , an even function is symmetrical about the vertical axis. Example:



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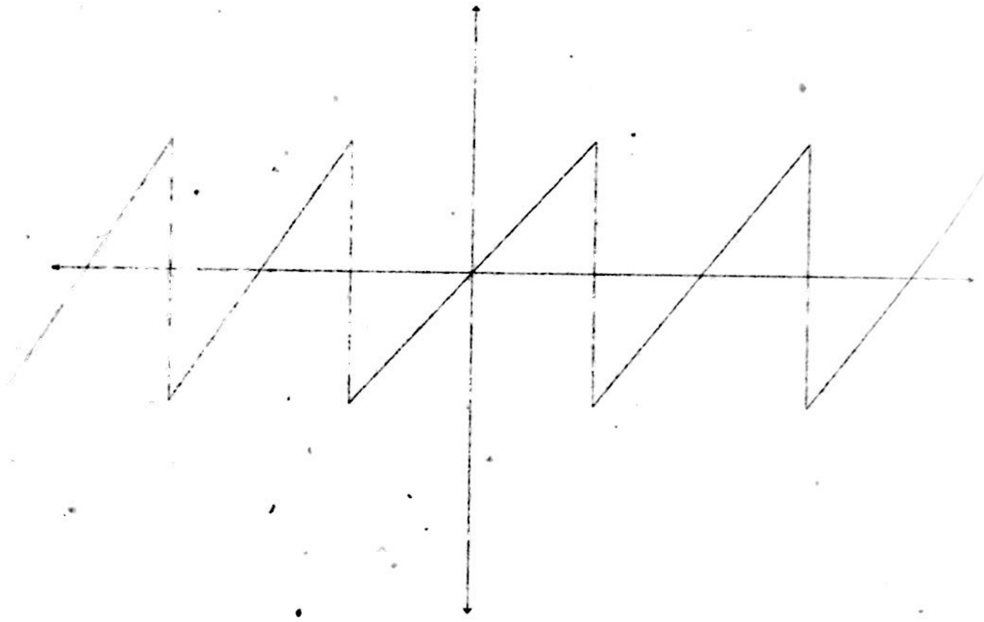
Such waveform is even waveform, all terms of its Fourier series are cosine terms (i.e. terms with 'a' coefficients do exists). But no sine terms are present since ... However, the function does have an average value (a_0)

(2) Odd function symmetry:

A function is said to be odd if f(t) = - f(t).

Example:



Such waveform is odd waveform, the series contains only sine terms. There is no average value (a_0 =0) and no cosine terms since a_n=0. In special cases the ... waveform may be odd only after its average value is subtracted

(3) Half-wave Symmetry:

A function is said to have half wave symmetry if $f(t) = -f(t-\frac{T_0}{2})$ i.e., the half ... is an inverted version of the adjacent half cycle (i.e., if the waveform $-\frac{T_0}{2}$ to ... inverted and if it becomes identical to the waveform from 0 to $\frac{T_0}{2}$, it is said to have ... symmetry).

Example:

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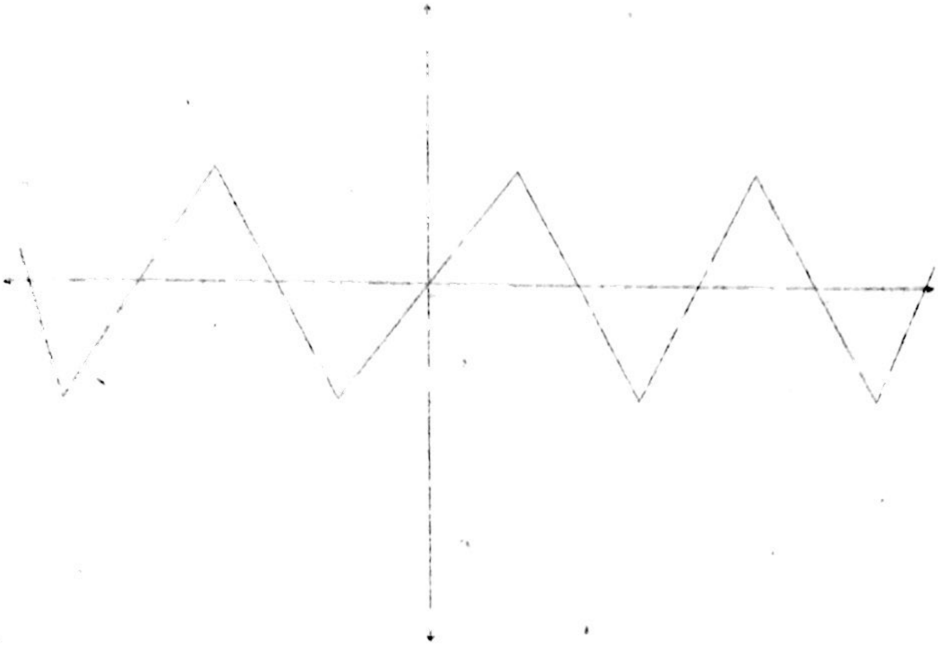
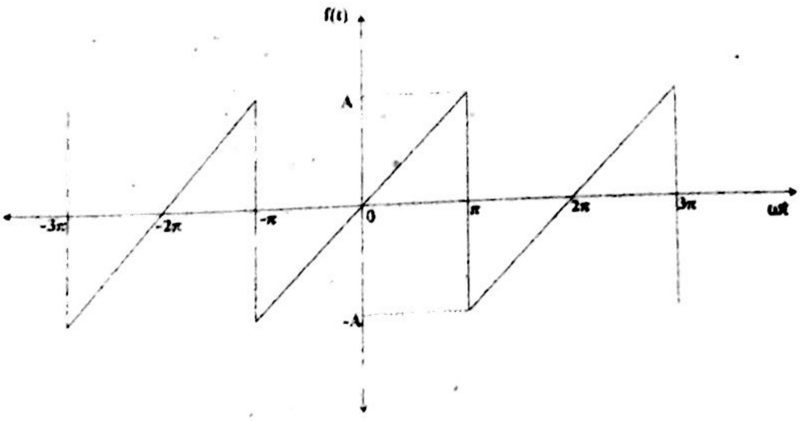


The given waveform is of half-wave symmetry, only odd harmonics are present in the series. The series would contain both sine and cosine terms when n is odd. The average value is zero.

Frequency Spectrum:

The frequency spectrum of the function f(t) expressed as Fourier series consists of a plot of the amplitude of the harmonics versus frequency (commonly known as amplitude spectrum)and the plot of phase of the harmonics versus frequency, which we call the phase spectrum. However, the frequency components being discrete, the spectra are called line spectra and it illustrates the frequency content of the signal. Here $C_0 = a_0$ and C_n is given by $\sqrt{a_n^2 + b_n^2}$

#Find the trigonometric Fourier series for the given waveform shown and also draw line Spectrum



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Solution:

The trigonometric Fourier series is given as

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \text{ Cos } n\omega t + b_n \text{ Sin } n\,\omega t] \qquad -\pi \le \omega t \le \pi$$

By inspection of given waveform, the given function is having odd symmetry which mea there is zero a_0 and a_n.

$$a_0 = 0$$

$$a_n = 0$$

Now, from the given waveform we have,

$$f(t) = \frac{A}{\pi}\omega t \qquad -\pi \le \omega t \le \pi$$

And

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t) \text{ Sin } n\omega t \, d\omega t$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{A}{\pi}\omega t \text{ Sin } n\omega t \, d\omega t$$

$$b_n = \frac{A}{\pi^2}\left[\frac{1}{n^2}\sin(n\omega t) - \frac{\omega t}{n}\cos(n\omega t)\right]_{-\pi}^{\pi}$$

$$b_n = \frac{-2A}{n\pi} * \text{Cos}(n\pi)$$

$$b_n = \frac{-2A}{n\pi} * (-1)^n$$

for n = even

$$b_n = \frac{-2A}{n\pi}$$

for n = odd

$$b_n = \frac{2A}{n\pi}$$

The trigonometric Fourier series is given as

$$f(t) = \sum_{n=1}^{\infty} \frac{-2A}{n\pi} * (-1)^n \text{ Sin } n\,\omega t$$

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$$f(t)=\frac{2A}{\pi}\sin\omega t-\frac{A}{\pi}\sin 2\omega t+\frac{2A}{3\pi}\sin 3\omega t-\frac{2A}{4\pi}\sin 4\omega t+\cdots$$

For line Spectrum

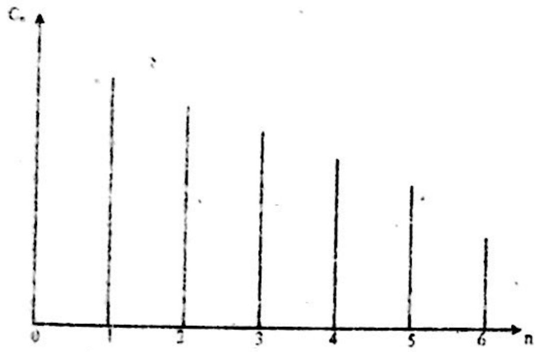
$$C_0 = 0$$

$$C_1=\sqrt{a_1^2+b_1^2}=\sqrt{0+\left(\frac{2A}{\pi}\right)^2}=\frac{2A}{\pi}$$

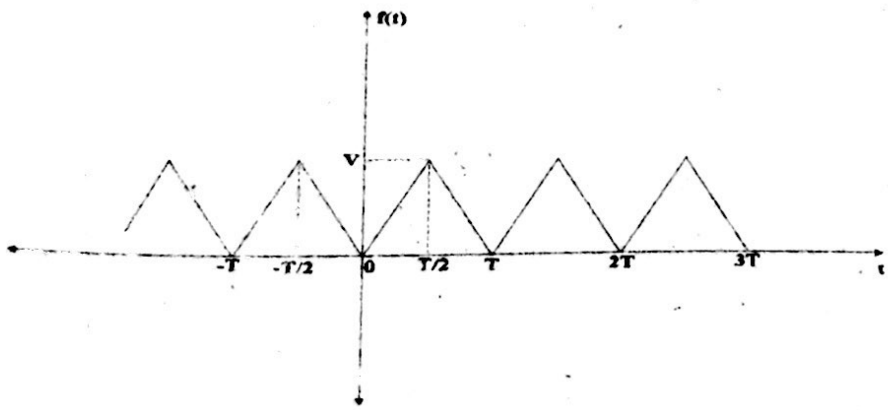
$$C_2=\sqrt{a_2^2+b_2^2}=\frac{A}{\pi}$$

$$C_3=\sqrt{a_3^2+b_3^2}=\frac{2A}{3\pi}$$

so on……



#Find the trigonometric Fourier series for the given waveform shown and also draw line Spectrum



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Solution:

The trigonometric Fourier series is given as

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \text{ Cos } n\omega t + b_n \text{ Sin } n\,\omega t] \qquad -T/2 \le t \le T/2 \quad \text{where } \omega = \frac{2\pi}{t} = \frac{2\pi}{T}$$

By inspection of given waveform, the given function is having even symmetry which mean there is zero b_n.

$$b_n = 0$$

Now, from the given waveform we have,

$$f(t) = \frac{-2V}{T}t \qquad -T/2 \le t \le 0$$

$$= \frac{2V}{T}t \qquad 0 \le t \le T/2$$

Now,

$$a_0 = \frac{2}{T}\int_0^T f(t)dt$$

$$a_0 = \frac{2}{T}\left[\int_{-T/2}^{0}\left(\frac{-2V}{T}t\right)dt + \int_0^{T/2}\left(\frac{2V}{T}t\right)\, dt\right]$$

$$a_0 = \frac{2}{T}\left[\frac{-2V}{T}\left[\frac{-T^2}{8}\right] + \frac{2V}{T}\left[\frac{T^2}{8}\right]\right]$$

$$a_0 = V$$

And

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\text{Cos } n\omega t\ dt$$

$$a_n = \frac{2}{T}\left\{\int_{-T/2}^{0}\left(\frac{-2V}{T}\right) t \text{ Cos } n\frac{2\pi}{T}t\, dt + \int_0^{T/2}\left(\frac{2V}{T}\right) t \text{ Cos } n\frac{2\pi}{T}t\, dt\right\}$$

a_n

$$\frac{2}{T} * \left(\frac{-2V}{T}\right) *$$

$$\left(\frac{2V}{T}\right)\left[\left[t\int \text{Cos } n\frac{2\pi}{T}t\, dt\right]_{-T/2}^{0} - \int_{-\frac{T}{2}}^{0}\left\{\frac{dt}{dt}\int \text{Cos } n\frac{2\pi}{T}t\, dt\right\} + \left[t\int \text{Cos } n\frac{2\pi}{T}t\, dt\right]_{-T/2}^{0} - \int_{-\frac{T}{2}}^{0}\left\{\frac{dt}{dt}\int \text{Cos } n\frac{2\pi}{T}t\, dt\right\}\right]$$

$$a_n = \frac{-4V}{n^2\pi^2}$$

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The trigonometric Fourier series is given as

$$f(t) = \frac{V}{2} + \sum_{n=1}^{\infty} \frac{-4V}{n^2\pi^2} \cos n\frac{2\pi}{T}t$$

$$f(t) = \frac{V}{2} - \frac{4V}{\pi^2}\cos\frac{2\pi}{T}t - \frac{4V}{4\pi^2}\cos\frac{4\pi}{T}t - \frac{4V}{9\pi^2}\cos\frac{6\pi}{T}t - \frac{4V}{16\pi^2}\cos\frac{8\pi}{T}t - \cdots$$

For line Spectrum

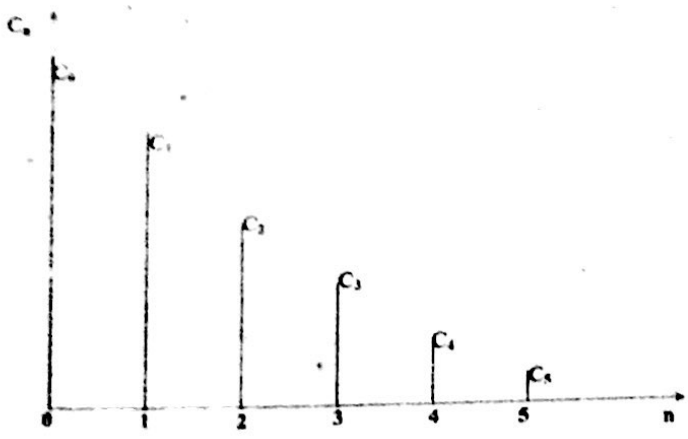
$$C_0 = \frac{V}{2}$$

$$C_1 = \sqrt{a_1^2 + b_1^2} = \sqrt{\left(\frac{4V}{\pi^2}\right)^2 + 0} = \frac{4V}{\pi^2}$$

$$C_2 = \sqrt{a_2^2 + b_2^2} = \frac{V}{\pi^2}$$

$$C_3 = \sqrt{a_3^2 + b_3^2} = \frac{4V}{9\pi^2}$$

so on……



#Exponential Fourier series:

Fourier series can be expressed as:

$$f(t) = \frac{a_0}{2} + a_1\cos\omega t + a_2\cos 2\omega t + --- + b_1\sin\omega t + b_2\sin\omega t + ---$$

Now, sine and cosine terms can be express in exponential as,

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$$f(t)= \frac{a_0}{2} + a_1\left(\frac{e^{j\omega t}+e^{-j\omega t}}{2}\right) + a_2\left(\frac{e^{2j\omega t}+e^{-2j\omega t}}{2}\right) + --- + b_1\left(\frac{e^{j\omega t}+e^{-j\omega t}}{2j}\right) + b_2\left(\frac{e^{2j\omega t}+e^{-2j\omega t}}{2j}\right) ---$$

$$f(t) = --- + \left(\frac{a_2}{2} - \frac{b_2}{2j}\right)e^{-2j\omega t} + \left(\frac{a_1}{2} - \frac{b_1}{2j}\right)e^{-j\omega t} + \frac{a_0}{2} + \left(\frac{a_1}{2} + \frac{b_1}{2j}\right)e^{j\omega t} + \left(\frac{a_2}{2} + \frac{b_2}{2j}\right)e^{2j\omega t} ---$$

Let consider $A_0 = \frac{a_0}{2}$, $A_n = \frac{1}{2}(a_n - jb_n)$, $A_{-n} = \frac{1}{2}(a_n + jb_n)$

Thus equation becomes as

$$f(t) = --- + A_{-2}e^{-2j\omega t} + A_{-1}e^{-j\omega t} + A_0 + A_1 e^{j\omega t} + A_{-2}e^{2j\omega t} + ---$$

This is known as Exponential form of Fourier series.

So we get,

$$A_0 = \frac{1}{2\pi}\int_0^{2\pi} f(t)d(\omega t)$$

Or

$$A_0 = \frac{1}{T}\int_0^{T} f(t)d(t)$$

And,

$$A_n = \frac{1}{2\pi}\int_0^{2\pi} f(t)e^{-jn\omega t}d(\omega t)$$

Also,

$$A_{-n} = \frac{1}{2\pi}\int_0^{2\pi} f(t)e^{jn\omega t}d(\omega t)$$

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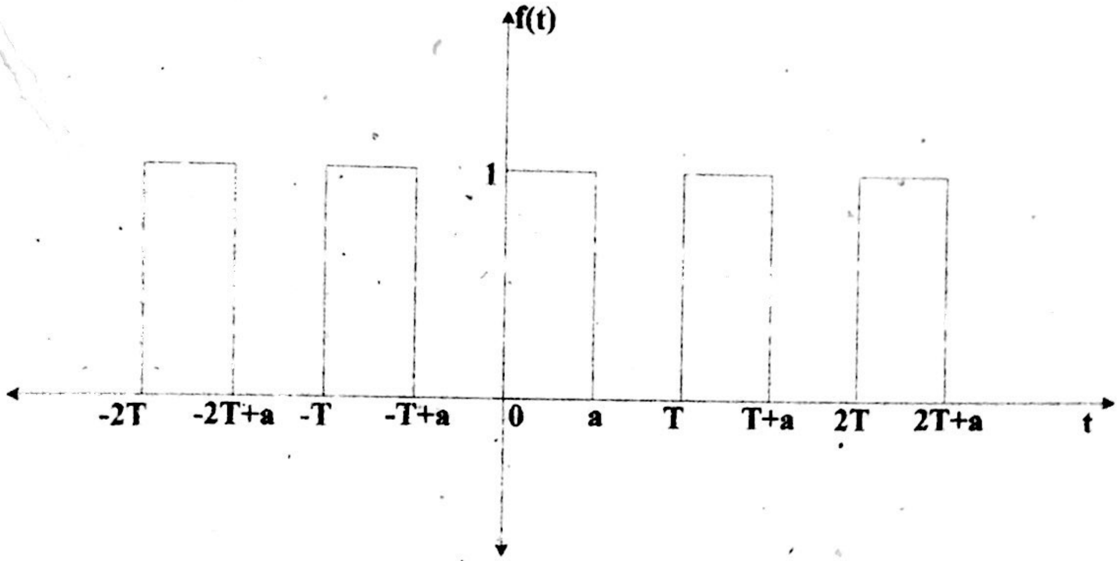
Determine Exponential form of Fourier series expansion



Solution:

Given that

$f(t) = 1$ for $0 < t < a$

$= 0$ for $a < t < T$

Thus then,

$$A_0 = \frac{1}{T}\int_0^T f(t)d(t)$$

$$A_0 = \frac{1}{T}\int_0^a 1d(t)$$

$$A_0 = \frac{1}{T}[a - 0]$$

$$A_0 = \frac{a}{T}$$

And

$$A_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega t}d(t)$$

$$A_n = \frac{1}{T}\int_0^a e^{-jn\omega t}d(t)$$

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$$A_n = \frac{1}{T}\left[\frac{e^{-jn\omega t}}{-jn\omega}\right]_0^a$$

$$A_n = \frac{1}{T}\left[\frac{e^{-jn\omega a}-1}{-jn\omega}\right]$$

So,

$$A_1 = \frac{1}{T}\left[\frac{e^{-j\omega a}-1}{-j\omega}\right]$$

$$A_2 = \frac{1}{T}\left[\frac{e^{-2j\omega a}-1}{-2j\omega}\right]$$

Now,

$$A_{-n} = \frac{1}{T}\int_0^T f(t)e^{jn\omega t}d(t)$$

$$A_{-n} = \frac{1}{T}\int_0^a e^{jn\omega t}d(t)$$

$$A_{-n} = \frac{1}{T}\left[\frac{e^{jn\omega t}}{jn\omega}\right]_0^a$$

$$A_{-n} = \frac{1}{T}\left[\frac{e^{jn\omega a}-1}{jn\omega}\right]$$

So,

$$A_{-1} = \frac{1}{T}\left[\frac{e^{j\omega a}-1}{j\omega}\right]$$

$$A_{-2} = \frac{1}{T}\left[\frac{e^{2j\omega a}-1}{2j\omega}\right]$$

Thus, Exponential Fourier series is:

$$f(t) = ---+\frac{1}{T}\left[\frac{e^{2j\omega a}-1}{2j\omega}\right]e^{-2j\omega t}+\frac{1}{T}\left[\frac{e^{j\omega a}-1}{j\omega}\right]e^{-j\omega t}+\frac{a}{T}+\frac{1}{T}\left[\frac{e^{-j\omega a}-1}{-j\omega}\right]e^{j\omega t}+\frac{1}{T}\left[\frac{e^{-2j\omega a}-1}{-2j\omega}\right]e^{2j\omega t}\pm ----$$

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#Fourier Transform:

The Fourier integral of function f(t) is given by the equation.

$$f(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt \text{ ------ (1)}$$

the function f(jω) is also known the Fourier transform of function f(t). Thus Fourier transform is nothing but the Fourier integral. Hence, we can write,

$$f(j\omega) = \mathcal{F}[f(t)]$$

The symbol $\mathcal{F}$ signifies Fourier transform of equation (1) enables us to obtain f(jω) for a given f(t). The quantity f(t) is in time domain whereas the quantity f(jω) is in frequency domain. Thus equation (1) enables us to convert a quality f(t) in the time domain to the corresponding quantity in the frequency domain.

Now, the inverse Fourier transform is given by,

$$f(t) = \mathcal{F}\text{-}1[\, f(j\omega)]$$

Symbol ss-1 signifies Inverse Fourier transform of which is given by the equation

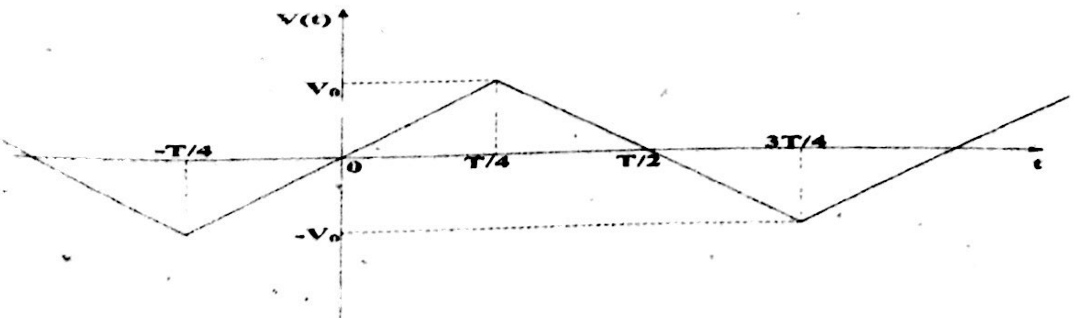
$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(j\omega)\, e^{-j\omega t}dt \text{ ------- (2)}$$

Equation (1) and (2) constitute the Fourier transform pair equations.

Problems:

Find the trigonometric Fourier series for the given waveform shown

(a)



$$\left[\textbf{Ans:}V(t) = \frac{8V_0}{\pi^2}\sin\omega t - \frac{8V_0}{9\pi^2}\sin 3\omega t + \frac{8V_0}{25\pi^2}\sin 5\omega t - \dots\dots\dots\right]$$

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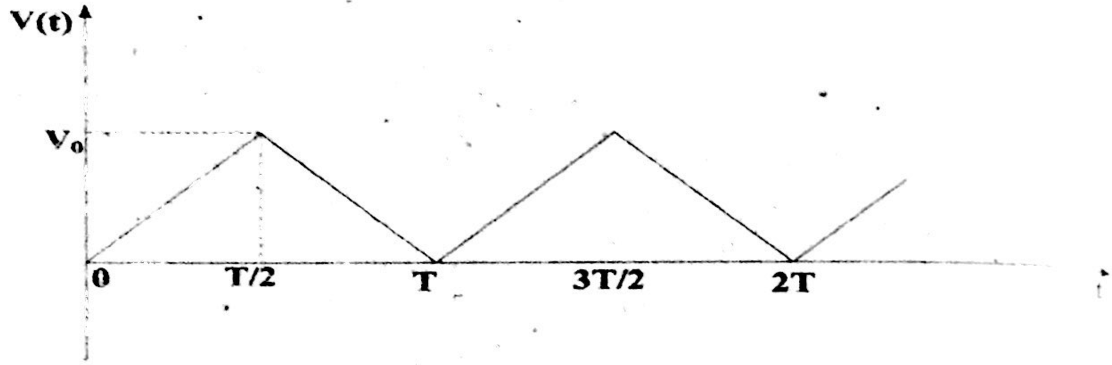
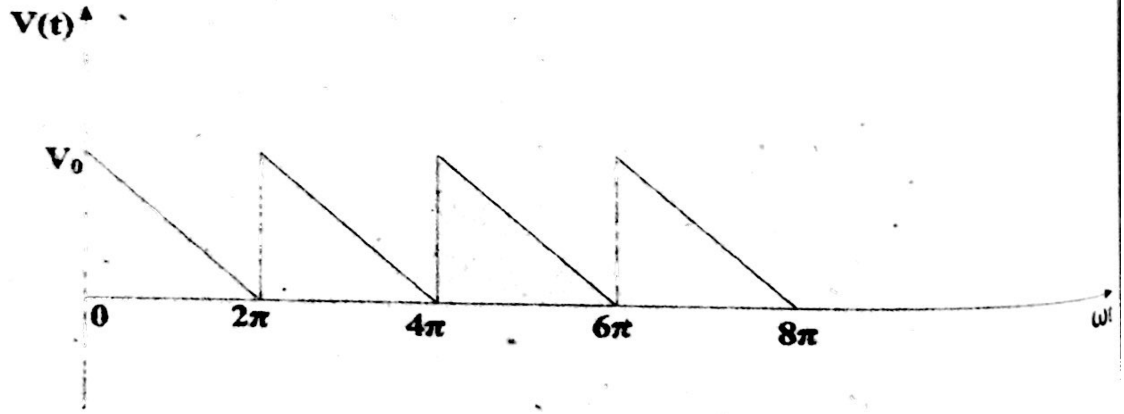
(b)



$$\left[\text{Ans:} V(t) = \frac{V_0}{2} - \frac{4V_0}{\pi^2}\cos\omega t - \frac{4V_0}{9\pi^2}\cos 3\omega t - \frac{4V_0}{25\pi^2}\cos 5\omega t \ldots\ldots\ldots\right]$$

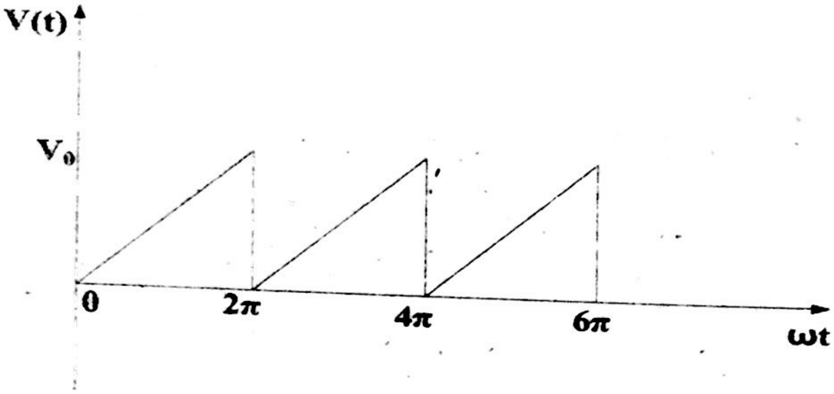
(c)



$$\left[\text{Ans:} \frac{V_0}{2} + \frac{V_0}{\pi}\sin\omega t + \frac{V_0}{2\pi}\sin 2\omega t + \frac{V_0}{3\pi}\sin 3\,\omega t + \ldots\ldots\ldots\right]$$

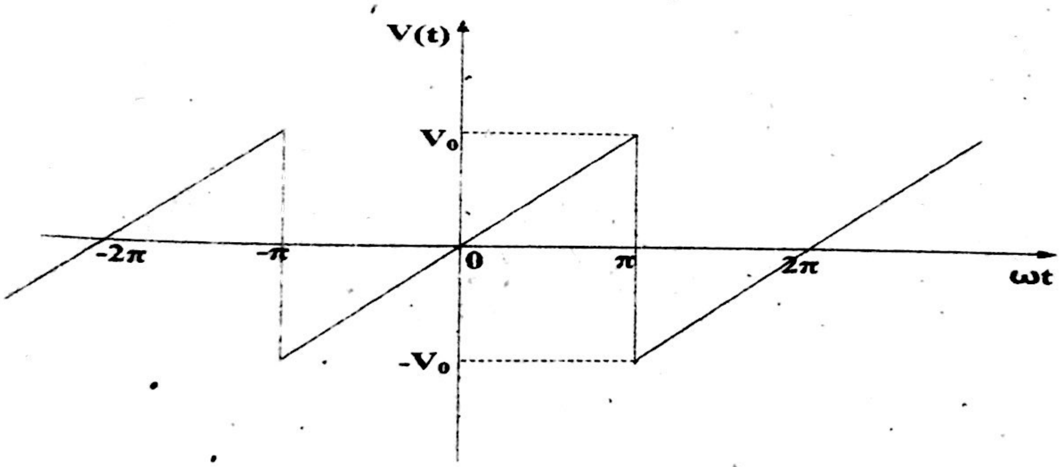
Find the Exponential Fourier series for the given waveform shown

(a)



[Ans: $V(t) = ---+\left[\frac{V_0}{4j\pi}\right]e^{-2j\omega t}+\left[\frac{V_0}{2j\pi}\right]e^{-j\omega t}+\frac{V_0}{2}+\left[\frac{V_0}{-2j\pi}\right]e^{j\omega t}+\left[\frac{V_0}{-4j\pi}\right]e^{2j\omega t}+---$]

(b)



[Ans: $V(t) = \frac{2V_0}{\pi}\sin\omega t - \frac{2V_0}{2\pi}\sin 2\omega t + \frac{2V_0}{3\pi}\sin 3\omega t - \ldots\ldots\ldots\ldots$]

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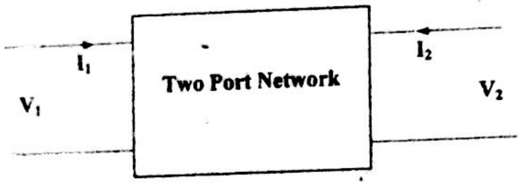
Two- Port Parameter of Networks

#Introduction:



A two port network has four variables

(i) Current I_1 and voltage V_1 at port-1

(ii) Current I_2 and Voltage V_2 at port-2

There are six parameters of a two port network which is obtained by taking two of the variables as dependent variables and the other two as independent variables.

Name of parameters	Dependent Variables	Independent Variables	Equation
1.Z-parameters or O.C Impedance parameter	V_1, V_2	I_1, I_2	$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z11 & Z12 \\ Z21 & Z22 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$
2. Y-parameters or S.C Impedance parameter	I_1, I_2	V_1, V_2	$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y11 & Y12 \\ Y21 & Y22 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$
3. T-parameters or Transmission/ ABCD/chain parameter	V_1, I_1	V_2, I_2	$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$
4. T'-parameters or Inverse Transmission/ A'B'C'D' parameter	V_2, I_2	V_1, I_1	$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$
5. h-parameters or hybrid parameter	V_1, I_2	I_1, V_2	$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h11 & h12 \\ h21 & h22 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$
6. g-parameters or Inverse hybrid parameter	I_1, V_2	V_1, I_2	$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g11 & g12 \\ g21 & g22 \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$

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1) **Open Circuit Impedance (Z) parameters.**

$(V_1, V_2) = f(I_1, I_2)$

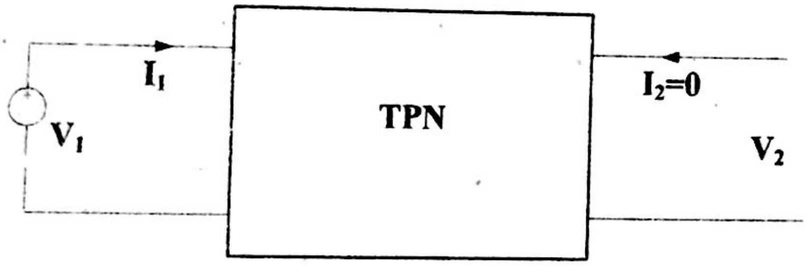
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z11 & Z12 \\ Z21 & Z22 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[V] = [Z][I]$$

i.e. $V_1 = Z_{11} I_1 + Z_{12} I_2$ -------------- (1)

$V_2 = Z_{21} I_1 + Z_{22} I_2$ ------------- (2)

Case 1: Apply Voltage V_1 at port-1 and make port-2 Open circuit

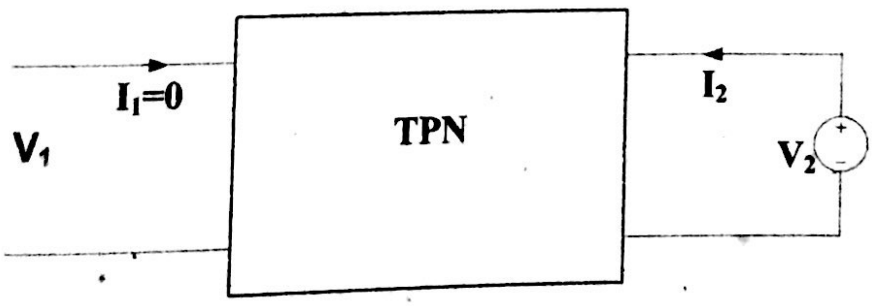


From equation (1) and (2)

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

Case 2: Apply Voltage V_2 at port-2 and make port-1 Open circuit



From equation (1) and (2)

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

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$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

2) **Short circuit Admittance (Y) parameters.**

$(I_1, I_2) = f(V_1, V_2)$

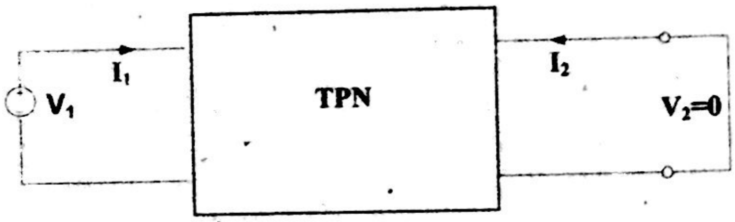
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y11 & Y12 \\ Y21 & Y22 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[I] = [Y][V]$$

i.e. $I_1 = Y_{11} V_1 + Y_{12} V_2$ ------------- (1)

$I_2 = Y_{21} V_1 + Y_{22} V_2$ ------------- (2)

Case 1: Apply Voltage V_1 at port-1 and make port-2 Short circuit

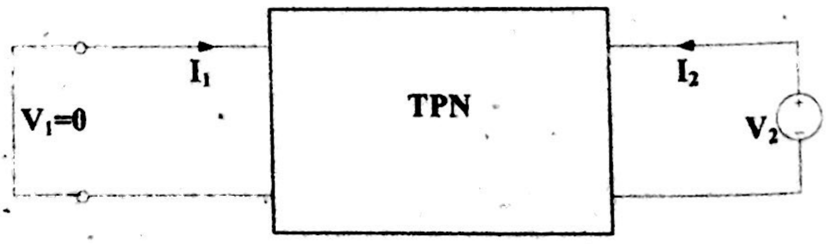


From equation (1) and (2)

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

Case 2: Apply Voltage V_2 at port-2 and make port-1 Short circuit



From equation (1) and (2)

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

Note:

[Short circuit admittance matrix]
=
[Open Circuit impedance matrix]$^{-1}$

i.e. $[Y] = [Z]^{-1}$

$$\begin{bmatrix} Y11 & Y12 \\ Y21 & Y22 \end{bmatrix} = \begin{bmatrix} Z11 & Z12 \\ Z21 & Z22 \end{bmatrix}^{-1}$$

3) **Transmission (T) or Chain or ABCD parameters.**

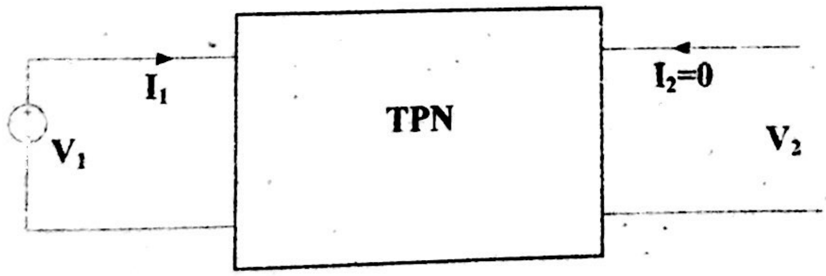
$(V_1, I_1) = f(V_2, -I_2)$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

i.e. $V_1 = AV_2 + B(-I_2)$ -------------- (1)

$I_1 = C V_2 + D(-I_2)$ ------------- (2)

Case 1: Apply Voltage V_1 at port-1 and make port-2 Open circuit



From equation (1) and (2)

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

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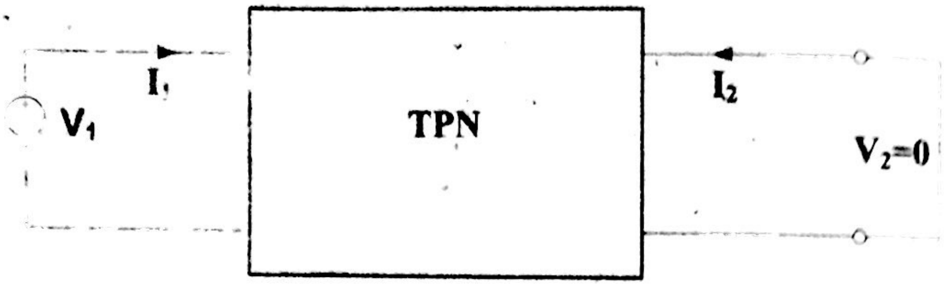
Case 2: Apply Voltage V_1 at port-1 and make port-2 Short circuit



From equation (1) and (2)

$$B = \frac{V_1}{-I_2}\bigg|_{V_2=0}$$

$$D = \frac{I_1}{-I_2}\bigg|_{V_2=0}$$

4) **Transmission (T') or A'B'C'D' parameters.**

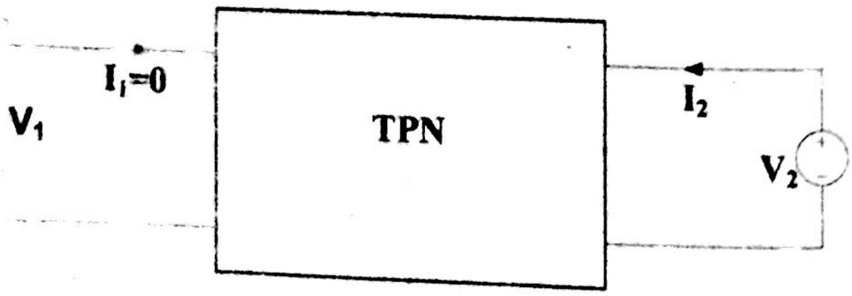
$(V_2, I_2) = f(V_1, -I_1)$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

i.e. $V_2 = A'\,V_1 + B'\,(-I_1)$ -------------- (1)

$I_2 = C'\,V_1 + D'\,(-I_1)$ ------------- (2)

Case 1: Apply Voltage V_2 at port-2 and make port-1 Open circuit



From equation (1) and (2)

$$A' = \frac{V_2}{V_1}\bigg|_{I_1=0}$$

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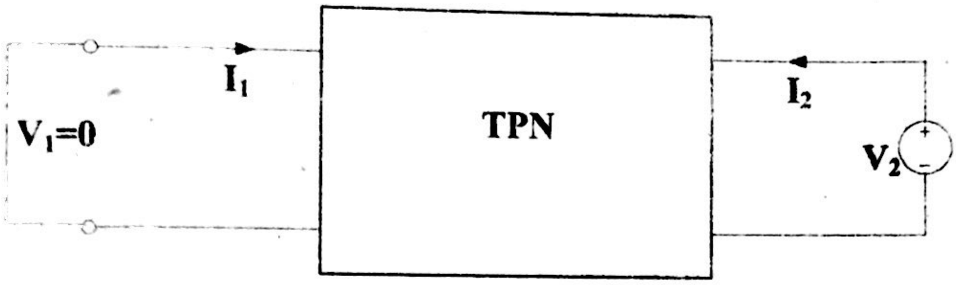
$$C' = \frac{I_2}{V_1}\bigg|_{I_1=0}$$

Case 2: Apply Voltage V_2 at port-2 and make port-1 Short circuit



From equation (1) and (2)

$$B' = \frac{V_2}{-I_1}\bigg|_{V_1=0}$$

$$D' = \frac{I_2}{-I_1}\bigg|_{V_1=0}$$

5) **Hybrid (h) parameters.**

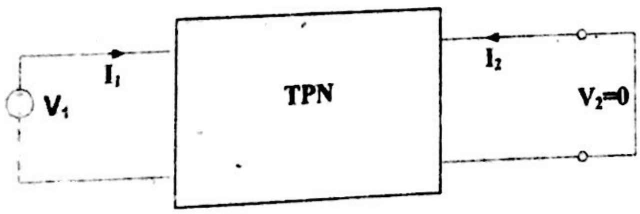
$(V_1, I_2) = f\,(I_1, V_2)$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h11 & h12 \\ h21 & h22 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

i.e. $V_1 = h_{11}\,I_1 + h_{12}\,V_2$ -------------- (1)

$I_2 = h_{21}\,I_1 + h_{22}\,V_2$ ------------- (2)

Case 1: Apply Voltage V_1 at port-1 and make port-2 Short circuit



From equation (1) and (2)

$$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0}$$

$$h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0}$$

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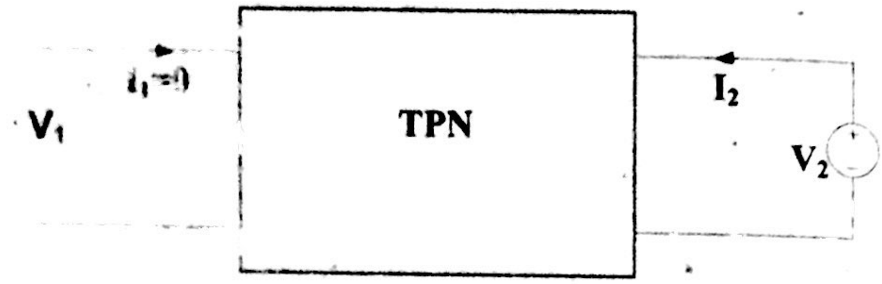
Case 2: Apply Voltage V_2 at port-2 and make port-1 open circuit



From equation (1) and (2)

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

6) **Inverse hybrid (g) parameters.**

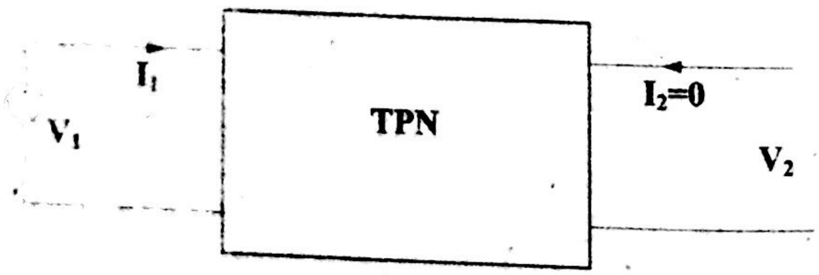
$(I_1, V_2) = f(V_1, I_2)$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g11 & g12 \\ g21 & g22 \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

i.e. $I_1 = g_{11} V_1 + g_{12} I_2$ -------------- (1)

$V_2 = g_{21} V_1 + g_{22} I_2$ -------------- (2)

Case 1: Apply Voltage V_1 at port-1 and make port-2 Open circuit



From equation (1) and (2)

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}$$

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$$

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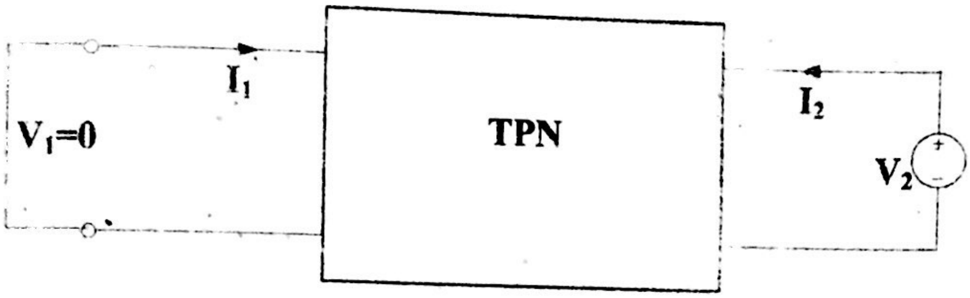
Case 2: Apply Voltage V_2 at port-2 and make port-1 Short circuit



From equation (1) and (2)

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}$$

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$

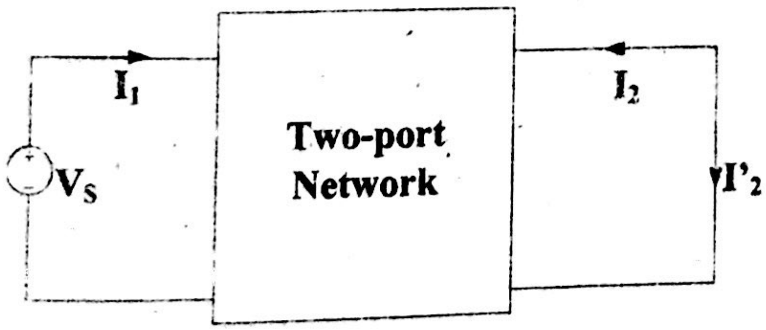
Note:

$[g] = [h]^{-1}$

i.e. $\begin{bmatrix} g11 & g12 \\ g21 & g22 \end{bmatrix} = \begin{bmatrix} h11 & h12 \\ h21 & h22 \end{bmatrix}^{-1}$

#Condition for Reciprocity of a two port Network:

A network is termed as reciprocal if the ratio of response variable to the excitation variable remains identical even if the positions of the response and excitation in the network are interchanged. Mathematically, we can say from figure below:



(a)

$$[V_1 = V_S, I_1 = I_1, V_2 = 0, I_2 = -I_2']$$

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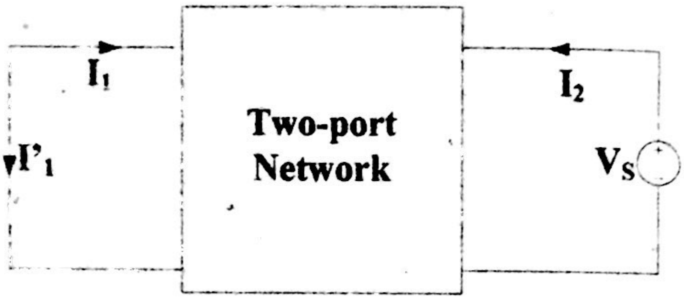

(b)

$$[V_2 = V_S, I_2 = I_2, V_1 = 0, I_1 = -I_1']$$

1.In terms of Z-parameters

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

From figure (a): $[V_1 = V_S, I_1 = I_1, V_2 = 0, I_2 = -I_2']$

Therefore, $V_S = Z_{11} I_1 - Z_{12} I'_2$

and $0 = Z_{21} I_1 - Z_{22} I'_2$

Hence $I_2' = \frac{V_S Z_{21}}{Z_{11} Z_{22} - Z_{21} Z_{12}}$

From figure (b): $[V_2 = V_S, I_2 = I_2, V_1 = 0, I_1 = -I_1']$

Therefore, $0 = -Z_{11} I'_1 + Z_{12} I_2$

and $V_S = -Z_{21} I'_1 + Z_{22} I_2$

Hence $I_1' = \frac{V_S Z_{12}}{Z_{11} Z_{22} - Z_{21} Z_{12}}$

Comparing I'_2 and I'_1; we get

$$\boldsymbol{Z_{12} = Z_{21}}$$

(This is the Condition of reciprocity in terms of Z – parameters)

2.In terms of Y-parameters

$I_1 = Y_{11} V_1 + Y_{12} V_2$

$I_2 = Y_{21} V_1 + Y_{22} V_2$

From figure (a): $[V_1 = V_S, I_1 = I_1, V_2 = 0\ I_2 = -I_2']$

$I'_2 = -Y_{21} V_S$

From figure (b): $[V_2 = V_S, I_2 = I_2, V_1 = 0, I_1 = -I_1']$

$I'_1 = -Y_{12} V_S$

Comparing I'_2 and I'_1, we get

$\mathbf{Y_{12} = Y_{21}}$

(This is the Condition of reciprocity in terms of Y – parameters)

3.In terms of T-parameters

$V_1 = AV_2 + B(-I_2)$
$I_1 = C V_2 + D(-I_2)$
From figure (a): $[V_1 = V_S, I_1 = I_1, V_2 = 0, I_2 = -I_2']$

$I_2' = \frac{V_S}{B}$

From figure (b): $[V_2 = V_S, I_2 = I_2, V_1 = 0, I_1 = -I_1']$

$I_1' = V_S\left(\frac{AD-BC}{B}\right)$

Above discussion leads to the condition of reciprocity,

$\mathbf{AD - BC = 1}$

Or,$\Delta\mathbf{T} = \mathbf{1}$

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4.In terms of T'-parameters

Condition of reciprocity in case of T'-parameters is similar to as in case of T-parameters,

$$\mathbf{A'D' - B'C' = 1}$$

$$\text{Or,}\ \mathbf{\Delta T' = 1}$$

5.In terms of h-parameters

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

From figure (a): $[V_1 = V_S, I_1 = I_1, V_2 = 0, I_2 = -I_2']$

$$I_2' = -V_S \frac{h_{21}}{h_{11}}$$

From figure (b): $[V_2 = V_S, I_2 = I_2, V_1 = 0, I_1 = -I_1']$

$$I_1' = V_S \frac{h_{12}}{h_{11}}$$

Comparing I'_2 and I'_1, we get

$$\mathbf{h_{12}' = -h_{21}}$$

(This is the Condition of reciprocity in terms of g – parameters)

6.In terms of g-parameters

Condition of reciprocity in case of g-parameters is similar to as in case of h-parameters,

$$\mathbf{g_{12} = -g_{21}}$$

#Condition for Symmetry of a two port Network:

A two port network is termed as symmetrical if the input and output ports can be interchanged without altering the port voltages and currents. Mathematically, we can say from figure below:

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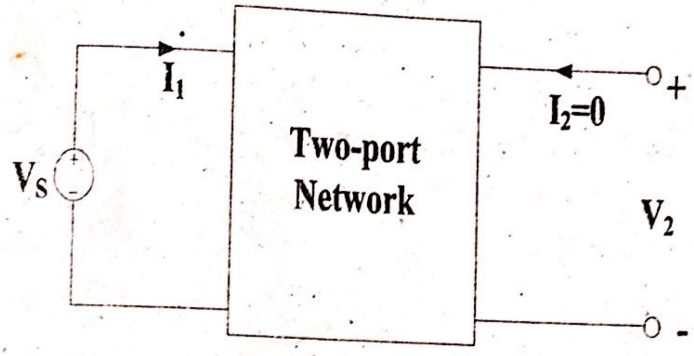
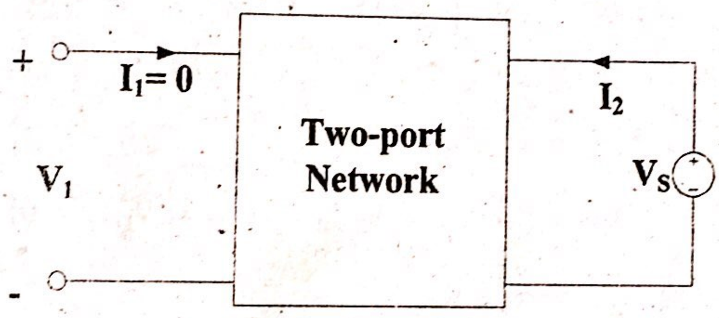


(a)

[$V_1 = V_S, I_1 = I_1, V_2 = V_2, I_2 = 0$]



(b)

[$V_2 = V_S, I_2 = I_2, V_1 = V_1, I_1 = 0$]

1.In terms of Z-parameters

From figure (a): [$V_1 = V_S, I_1 = I_1, V_2 = V_2, I_2 = 0$]

$V_S = Z_{11}\ I_1$

$$Z_{11} = \frac{V_S}{I_1}\bigg|_{I_2=0}$$

From figure (b): [$V_2 = V_S, I_2 = I_2, V_1 = V_1, I_1 = 0$]

$V_S = Z_{22}\ I_2$

$$Z_{22} = \frac{V_S}{I_2}\bigg|_{I_1=0}$$

From the definition of Symmetry, $\frac{V_S}{I_1}\Big|_{I_2=0} = \frac{V_S}{I_2}\Big|_{I_1=0}$ leads to

$$Z_{11} = Z_{22}$$

2.In terms of Y-parameters

From figure (a): $[V_1 = V_S, I_1 = I_1, V_2 = V_2, I_2 = 0]$

$$I_1 = Y_{11}\, V_S + Y_{12}\, V_2$$

$$0 = Y_{21}\, V_S + Y_{22}\, V_2$$

$$I_1 = Y_{11}\, V_S + Y_{12}\left(\frac{-Y_{21}}{Y_{22}}\right) V_S$$

$$\frac{V_S}{I_1} = \frac{Y_{22}}{Y_{11}Y_{22} - Y_{12}Y_{21}}$$

From figure (b): $[V_2 = V_S, I_2 = I_2, V_1 = V_1, I_1 = 0]$

$$0 = Y_{11}\, V_1 + Y_{12}\, V_S$$

$$I_2 = Y_{21}\, V_1 + Y_{22}\, V_S$$

$$\frac{V_S}{I_2} = \frac{Y_{11}}{Y_{11}Y_{22} - Y_{12}Y_{21}}$$

From the definition of Symmetry, $\frac{V_S}{I_1}\Big|_{I_2=0} = \frac{V_S}{I_2}\Big|_{I_1=0}$ leads to

$$Y_{11} = Y_{22}$$

3.In terms of T-parameters

$$V_1 = AV_2 + B\,(-I_2)$$
$$I_1 = C\, V_2 + D\,(-I_2)$$

From figure (a): $[V_1 = V_S, I_1 = I_1, V_2 = V_2, I_2 = 0]$

$$V_S = AV_2$$
$$I_1 = C\, V_2$$

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$$\frac{V_S}{I_1} = \frac{A}{C}$$

From figure (b): $[V_2 = V_S, I_2 = I_2, V_1 = V_1, I_1 = 0]$

$$V_1 = AV_S + B\,(-I_2)$$
$$0 = C\,V_S + D\,(-I_2)$$

$$\frac{V_S}{I_2} = \frac{D}{C}$$

From the definition of Symmetry, $\left.\frac{V_S}{I_1}\right|_{I_2=0} = \left.\frac{V_S}{I_2}\right|_{I_1=0}$ leads to

$$\mathbf{A = D}$$

4.In terms of T'-parameters

Condition of Symmetry in case of T'-parameters is similar to as in case of T-parameters.

$$\mathbf{A' = D'}$$

5.In terms of h-parameters

$$V_1 = h_{11}\,I_1 + h_{12}\,V_2$$
$$I_2 = h_{21}\,I_1 + h_{22}\,V_2$$

From figure (a): $[V_1 = V_S, I_1 = I_1, V_2 = V_2, I_2 = 0]$

$$V_S = h_{11}\,I_1 + h_{12}\,V_2$$
$$0 = h_{21}\,I_1 + h_{22}\,V_2$$

$$\frac{V_S}{I_1} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}$$

From figure (b): $[V_2 = V_S, I_2 = I_2, V_1 = V_1, I_1 = 0]$

$$V_1 = h_{12}\,V_S$$
$$I_2 = h_{22}\,V_S$$
$$\frac{V_S}{I_2} = \frac{1}{h_{22}}$$

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From the definition of Symmetry, $\left.\frac{V_S}{I_1}\right|_{I_2=0} = \left.\frac{V_S}{I_2}\right|_{I_1=0}$ leads to

$$h_{11}h_{22} - h_{12}h_{21} = 1$$

Or, $\Delta h = 1$

6.In terms of g-parameters

Condition of Symmetry in case of g-parameters is similar to as in case of h-parameters.

$$g_{11}g_{22} - g_{12}g_{21} = 1$$

Or, $\Delta g = 1$

Summary:

Parameters	Condition for reciprocity	Condition for symmetry
[Z]	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
[Y]	$Y_{12} = Y_{21}$	$Y_{11} = Y_{22}$
[ABCD]	$AD - BC = 1$	$A = D$
[A'B'C'D']	$A'D' - B'C' = 1$	$A' = D'$
[h]	$h_{12} = -h_{21}$	$h_{11}h_{22} - h_{12}h_{21} = 1$
[g]	$g_{12} = -g_{21}$	$g_{11}g_{22} - g_{12}g_{21} = 1$

 # Relationships between parameters:

1. **Z – parameters in Terms of Other Parameters**
   a. **Z – parameters in terms of Y- parameters**
   As in this case we know,
   [I]=[Y][V]
   And [V]=[Z][I]
   Therefore $[Z]=[Y]^{-1}$

Thus, $Z_{11} = \frac{Y_{22}}{\Delta Y}; Z_{12} = \frac{-Y_{12}}{\Delta Y}; Z_{21} = \frac{-Y_{21}}{\Delta Y}$ and $Z_{22} = \frac{Y_{11}}{\Delta Y}$

b. Z – parameters in terms of T- parameters

As in this case we know,

$V_1 = AV_2 + B(-I_2)$ —— (a)

$I_1 = C V_2 + D(-I_2)$ —— (b)

Rewriting equation (b)

$V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2$ ———— (1)

And from equation (a) and (1)

$V_1 = \frac{A}{C} I_1 + \left(\frac{AD}{C} - B\right) I_2$ ———— (2)

Comparing equation (1) and (2) with that of Z- parameters equation, we get

Thus, $Z_{11} = \frac{A}{C}; Z_{12} = \frac{AD-BC}{C}; Z_{21} = \frac{1}{C}$ and $Z_{22} = \frac{D}{C}$

c. Z – parameters in terms of T'- parameters

As similar to above case,

$Z_{11} = \frac{D'}{C'}; Z_{12} = \frac{1}{C'}; Z_{21} = \frac{\Delta T'}{C'}$ and $Z_{22} = \frac{A'}{C'}$

d. Z – parameters in terms of h- parameters

As in this case we know,

$V_1 = h_{11} I_1 + h_{12} V_2$ ———— (c)

$I_2 = h_{21} I_1 + h_{22} V_2$ ———— (d)

Rewriting equation (d)

$V_2 = -\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2$ ——— (3)

And from equation (b) and (a)

$V_1 = \left[h_{11} - \frac{h_{21}h_{12}}{h_{22}}\right] I_1 + \frac{h_{12}}{h_{22}} I_2$ ———— (4)

Comparing equation (3) and (4) with that of Z- parameters equation, we get

Thus, $Z_{11} = \frac{\Delta h}{h_{22}}; Z_{12} = \frac{h_{12}}{h_{22}}; Z_{21} = -\frac{h_{21}}{h_{22}}$ and $Z_{22} = \frac{1}{h_{22}}$

e. Z – parameters in terms of g - parameters

As similar to above case,

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$$Z_{11} = \frac{1}{g_{11}}; Z_{12} = -\frac{g_{12}}{g_{11}}; Z_{21} = \frac{g_{21}}{g_{11}} \text{ and } Z_{22} = \frac{\Delta g}{g_{11}}$$

2. Y – parameters in Terms of Other Parameters

a. Y – parameters in terms of Z- parameters

As in this case we know,

[I]=[Y][V]

And [V]=[Z][I]

Therefore [Y]=[Z]$^{-1}$

$$\text{Thus, } Y_{11} = \frac{Z_{22}}{\Delta Z}; Y_{12} = \frac{-Z_{12}}{\Delta Z}; Y_{21} = \frac{Z_{21}}{\Delta Z} \text{ and } Y_{22} = \frac{Z_{11}}{\Delta Z}$$

b. Y – parameters in terms of T- parameters

As in this case we know,

$V_1 = AV_2+B(-I_2)$ ----- (e)

$I_1 = C V_2 +D(-I_2)$ ----- (f)

Rewriting equation (e)

$I_2 = -\frac{1}{B}V_1 + \frac{A}{B}V_2$ ---------- (5)

And from equation (f) and (e)

$I_1 = \frac{D}{B}V_1 + \left(C - \frac{AD}{B}\right)V_2$ -------- (6)

Comparing equation (5) and (6) with that of Y- parameters equation, we get

$$\text{Thus, } Y_{11} = \frac{D}{B}; Y_{12} = -\frac{\Delta T}{B}; Y_{21} = -\frac{1}{B} \text{ and } Y_{22} = \frac{A}{B}$$

c. Y – parameters in terms of T'- parameters

As similar to above case,

$$Y_{11} = \frac{A'}{B'}; Y_{12} = -\frac{1}{B'}; Y_{21} = -\frac{\Delta T'}{B'} \text{ and } Y_{22} = \frac{D'}{B'}$$

d. Y – parameters in terms of h- parameters

As in this case we know,

$V_1 = h_{11} I_1+h_{12} V_2$ --------- (g)

$I_2 = h_{21} I_1 +h_{22} V_2$ ---------- (h)

Rewriting equation (g)

$I_1 = \frac{1}{h_{11}}V_1 - \frac{h_{12}}{h_{11}}V_2$ ----------- (7)

And from equation (h) and (g)

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$$I_2 = \frac{h_{21}}{h_{11}} V_1 + \frac{\Delta h}{h_{11}} V_2 \text{ ---------- } (8)$$

Comparing equation (7) and (8) with that of Y- parameters equation, we get

Thus. $Y_{11} = \frac{1}{h_{11}}; Y_{12} = -\frac{h_{12}}{h_{11}}; Y_{21} = \frac{h_{21}}{h_{11}}$ and $Y_{22} = \frac{\Delta h}{h_{11}}$

e. Y – parameters in terms of g - parameters

As similar to above case.

$$Y_{11} = \frac{\Delta g}{g_{22}}; Y_{12} = \frac{g_{12}}{g_{22}}; Y_{21} = -\frac{g_{21}}{g_{22}} \text{ and } Y_{22} = \frac{1}{g_{22}}$$

3. **T – parameters in Terms of Other Parameters**

a. **T – parameters in terms of Z- parameters**

As in this case we know,

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \text{ --------- } (i)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \text{ -------- } (j)$$

Rewriting equation (j)

$$I_1 = \frac{1}{Z_{21}} V_2 + \frac{Z_{22}}{Z_{21}} (-I_2) \text{ ------------ } (9)$$

And from equation (i) and (j)

$$V_1 = \frac{Z_{11}}{Z_{21}} V_2 + \left(\frac{\Delta Z}{Z_{21}}\right) (-I_2) \text{ ----------- } (10)$$

Comparing equation (9) and (10) with that of T- parameters equation, we get

Thus. $A = \frac{Z_{11}}{Z_{21}}; B = \frac{\Delta Z}{Z_{21}}; C = \frac{1}{Z_{21}}$ and $D = \frac{Z_{22}}{Z_{21}}$

b. T – parameters in terms of Y- parameters

As similar to above case,

$$A = -\frac{Y_{22}}{Y_{21}}; B = -\frac{1}{Y_{21}}; C = -\frac{\Delta Y}{Y_{21}} \text{ and } D = -\frac{Y_{11}}{Y_{21}}$$

c. T – parameters in terms of T'- parameters

As we know, the T'- parameters in matrix form,

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

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Rewriting equation

$$\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} A' & -B' \\ -C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

$$\text{Or, } \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A' & -B' \\ -C' & D' \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Thus, $A = -\frac{D'}{\Delta T'}$; $B = -\frac{B'}{\Delta T'}$; $C = -\frac{C'}{\Delta T'}$ and $D = -\frac{A'}{\Delta T'}$

d. T – parameters in terms of h- parameters

As in this case we know,

$V_1 = h_{11} I_1 + h_{12} V_2$ --------- (k)

$I_2 = h_{21} I_1 + h_{22} V_2$ ---------- (l)

Rewriting equation (l)

$I_1 = -\frac{h_{22}}{h_{21}} V_2 + \left(-\frac{1}{h_{21}}\right)(-I_2)$ ------------ (11)

And from equation (k) and (l)

$V_1 = -\frac{\Delta h}{h_{21}} V_2 + \left(-\frac{h_{11}}{h_{21}}\right)(-I_2)$ -------------- (12)

Comparing equation (11) and (12) with that of T- parameters equation, we get

Thus, $A = -\frac{\Delta h}{h_{21}}$; $B = -\frac{h_{11}}{h_{21}}$; $C = -\frac{h_{22}}{h_{21}}$ and $D = -\frac{1}{h_{21}}$

e. T – parameters in terms of g - parameters

As similar to above case,

$A = \frac{1}{g_{21}}$; $B = \frac{g_{22}}{g_{21}}$; $C = \frac{g_{11}}{g_{21}}$ and $D = \frac{\Delta g}{g_{21}}$

4. <u>T' – parameters in Terms of Other Parameters</u>

b. T' – parameters in terms of Z- parameters

As in this case we know,

$V_1 = Z_{11} I_1 + Z_{12} I_2$ ---------- (m)

$V_2 = Z_{21} I_1 + Z_{22} I_2$ --------- (n)

Rewriting equation (m)

$I_2 = \frac{1}{Z_{12}} V_1 + \frac{Z_{11}}{Z_{12}}(-I_1)$ -------------- (13)

And from equation (m) and (n)

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$$V_2 = \frac{Z_{22}}{Z_{12}} V_2 + \left(\frac{\Delta Z}{Z_{12}}\right)(-I_2) \text{ ----------------- } (14)$$

Comparing equation (13) and (14) with that of T'- parameters equation, we get

Thus, $A' = \frac{Z_{22}}{Z_{12}}$; $B' = \frac{\Delta Z}{Z_{12}}$; $C' = \frac{1}{Z_{12}}$ and $D' = \frac{Z_{11}}{Z_{12}}$

b. T' – parameters in terms of Y- parameters

As similar to above case,

$A' = -\frac{Y_{11}}{Y_{12}}$; $B' = -\frac{1}{Y_{12}}$; $C' = -\frac{\Delta Y}{Y_{12}}$ and $D' = -\frac{Y_{22}}{Y_{12}}$

c. T' – parameters in terms of T- parameters

As we know, the T'- parameters in matrix form,

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

Rewriting equation

$$\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} A' & -B' \\ -C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

$$\text{Or,} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A' & -B' \\ -C' & D' \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Thus, $A' = -\frac{D}{\Delta T}$; $B' = -\frac{B}{\Delta T}$; $C' = -\frac{C}{\Delta T}$ and $D' = -\frac{A}{\Delta T}$

d. T' – parameters in terms of h- parameters

As in this case we know,

$V_1 = h_{11} I_1 + h_{12} V_2$ --------- (o)

$I_2 = h_{21} I_1 + h_{22} V_2$ ---------- (p)

Rewriting equation (o)

$$V_2 = -\frac{h_{11}}{h_{12}} I_2 + \frac{1}{h_{12}} V_1 \text{ -------------- } (15)$$

And from equation (o) and (p)

$$I_2 = \left(\frac{h_{22}}{h_{12}}\right) V_1 + \frac{\Delta h}{h_{12}}(-I_1) \text{ -------------- } (16)$$

Comparing equation (15) and (16) with that of T'- parameters equation, we get

Thus, $A' = \frac{1}{h_{12}}$; $B' = \frac{h_{11}}{h_{12}}$; $C' = \frac{h_{22}}{h_{12}}$ and $D' = \frac{\Delta h}{h_{12}}$

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e. T' – parameters in terms of g - parameters

As similar to above case,

$$A' = -\frac{\Delta g}{g_{12}};\ B' = -\frac{g_{22}}{g_{12}};\ C' = -\frac{g_{11}}{g_{12}} \text{ and } D' = -\frac{1}{g_{12}}$$

5. <u>h – parameters in Terms of Other Parameters</u>

c. h – parameters in terms of Z - parameters

As in this case we know,

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \text{ ---------- (q)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \text{ --------- (r)}$$

Rewriting equation (r)

$$I_2 = -\frac{Z_{21}}{Z_{22}} I_1 + \frac{1}{Z_{22}} V_2 \text{ ---------------- (17)}$$

And from equation (q) and (r)

$$V_1 = \left(\frac{\Delta Z}{Z_{22}}\right) I_1 + \frac{Z_{12}}{Z_{22}} V_2 \text{ ------------------- (18)}$$

Comparing equation (17) and (18) with that of h- parameters equation, we get

Thus, $h_{11} = \frac{\Delta Z}{Z_{22}};\ h_{12} = \frac{Z_{12}}{Z_{22}};\ h_{21} = -\frac{Z_{21}}{Z_{22}}$ and $h_{22} = \frac{1}{Z_{22}}$

b. h – parameters in terms of Y- parameters

As similar to above case,

$$h_{11} = \frac{1}{Y_{11}};\ h_{12} = -\frac{Y_{12}}{Y_{11}};\ h_{21} = \frac{Y_{21}}{Y_{11}} \text{ and } h_{22} = \frac{\Delta Y}{Y_{11}}$$

c. h – parameters in terms of T- parameters

As in this case we know,

$$V_1 = AV_2 + B(-I_2) \text{ ----- (s)}$$

$$I_1 = C V_2 + D(-I_2) \text{ ------ (t)}$$

Rewriting equation (t)

$$I_2 = -\frac{1}{D} I_1 + \frac{C}{D} V_2 \text{ ------------------ (19)}$$

And from equation (s) and (t)

$$V_1 = \frac{B}{D} I_1 + \left(A - \frac{BC}{D}\right) V_2 \text{ ----------------- (20)}$$

Comparing equation (19) and (20) with that of h- parameters equation, we get

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Thus, $h_{11} = \frac{B}{D}; h_{12} = \frac{\Delta T}{D}; h_{21} = -\frac{1}{D}$ and $h_{22} = \frac{C}{D}$

d. **h – parameters in terms of T'- parameters**

As similar to above case,

$$h_{11} = \frac{B'}{A'}; h_{12} = \frac{1}{A'}; h_{21} = -\frac{\Delta T'}{A'} \text{ and } h_{22} = \frac{C'}{A'}$$

e. **h – parameters in terms of g - parameters**

As we know, $[h] = [g]^{-1}$

$$h_{11} = \frac{g_{22}}{\Delta g}; h_{12} = -\frac{g_{12}}{\Delta g}; h_{21} = -\frac{g_{21}}{\Delta g} \text{ and } h_{22} = -\frac{g_{11}}{\Delta g}$$

6. **g – parameters in Terms of Other Parameters**

a. **g – parameters in terms of Z - parameters**

As in this case we know,

$V_1 = Z_{11} I_1 + Z_{12} I_2$ --------- (u)

$V_2 = Z_{21} I_1 + Z_{22} I_2$ --------- (v)

Rewriting equation (u)

$$I_1 = -\frac{1}{Z_{11}} V_1 - \frac{Z_{12}}{Z_{11}} I_2 \text{ ------------- (21)}$$

And from equation (u) and (v)

$$V_2 = \frac{Z_{21}}{Z_{11}} V_1 + \left(\frac{\Delta Z}{Z_{11}}\right) I_2 \text{ ----------------- (22)}$$

Comparing equation (21) and (22) with that of g- parameters equation, we get

Thus, $g_{11} = \frac{1}{Z_{11}}; g_{12} = -\frac{Z_{12}}{Z_{11}}; g_{21} = \frac{Z_{21}}{Z_{11}}$ and $g_{22} = \frac{\Delta Z}{Z_{11}}$

b. **g – parameters in terms of Y- parameters**

As similar to above case,

$$g_{11} = \frac{\Delta Y}{Y_{22}}; g_{12} = \frac{Y_{12}}{Y_{22}}; g_{21} = -\frac{Y_{21}}{Y_{22}} \text{ and } g_{22} = \frac{1}{Y_{22}}$$

c. **g – parameters in terms of T- parameters**

As in this case we know,

$V_1 = AV_2 + B(-I_2)$ ----- (w)

$I_1 = C V_2 + D(-I_2)$ ----- (x)

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Rewriting equation (w)

$$V_2 = -\frac{1}{A}V_1 + \frac{B}{A}I_2 \text{------------} (23)$$

And from equation (w) and (x)

$$I_1 = \frac{C}{A}V_1 + \left(\frac{BC}{A} - D\right)I_2 \text{------------} (24)$$

Comparing equation (23) and (24) with that of g- parameters equation, we get

Thus, $g_{11} = \frac{C}{A}; g_{12} = -\frac{\Delta T}{A}; g_{21} = -\frac{1}{A}$ and $g_{22} = \frac{B}{A}$

d. g – parameters in terms of T'- parameters

As similar to above case,

$$g_{11} = \frac{C'}{D'}; g_{12} = \frac{1}{D'}; g_{21} = -\frac{\Delta T'}{D'} \text{ and } g_{22} = \frac{B'}{D'}$$

e. g – parameters in terms of g - parameters

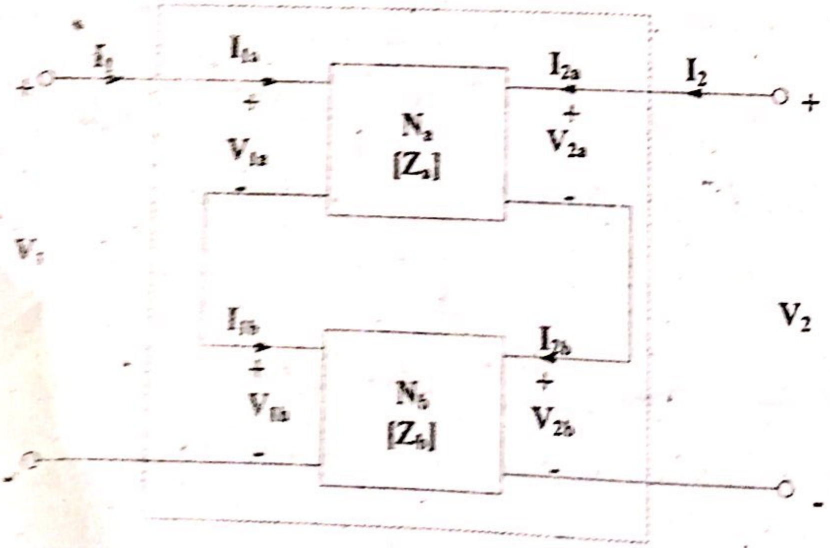
As we know, $[g] = [h]^{-1}$

$$g_{11} = \frac{h_{22}}{\Delta h}; g_{12} = -\frac{h_{12}}{\Delta h}; g_{21} = -\frac{h_{21}}{\Delta h} \text{ and } g_{22} = -\frac{h_{11}}{\Delta h}$$

#Interconnection of two port Networks

1.Series Connection:

(i) Two TPN in series:



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Here, figure shows a series connection of two two-port networks N_a and N_b with open circuit Z- parameters Z_a and Z_b respectively, i.e., for network N_a,

$$\begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} = \begin{bmatrix} Z11a & Z12a \\ Z21a & Z22a \end{bmatrix} \begin{bmatrix} I_{1a} \\ I_{2b} \end{bmatrix}$$

Similarly, for network N_b,

$$\begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = \begin{bmatrix} Z11b & Z12b \\ Z21b & Z22b \end{bmatrix} \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

Then, their series connection requires that

$$I_1 = I_{1a} = I_{1b}, I_2 = I_{2a} = I_{2b}$$

And $V_1 = V_{1a} + V_{1b}, V_2 = V_{2a} + V_{2b}$

Now,

$$V_1 = V_{1a} + V_{1b} = (Z_{11a}I_{1a} + Z_{12a}I_{2a}) + (Z_{11b}I_{1b} + Z_{12b}I_{2b})$$

$$=(Z_{11a} + Z_{11b})I_1 + (Z_{12a} + Z_{12b})I_2$$

(Since $I_1 = I_{1a} = I_{1b}$ and $I_2 = I_{2a} = I_{2b}$)

And similarly,

$$V_2 = V_{2a} + V_{2b} = (Z_{21a} + Z_{21b})I_1 + (Z_{22a} + Z_{22b})I_2$$

So, in matrix form the Z- parameters of the series-connected combination can be written as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z11 & Z12 \\ Z21 & Z22 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Where, $Z_{11} = Z_{11a} + Z_{11b}$

$Z_{12} = Z_{12a} + Z_{12b}$

$Z_{21} = Z_{21a} + Z_{21b}$

$Z_{22} = Z_{22a} + Z_{22b}$

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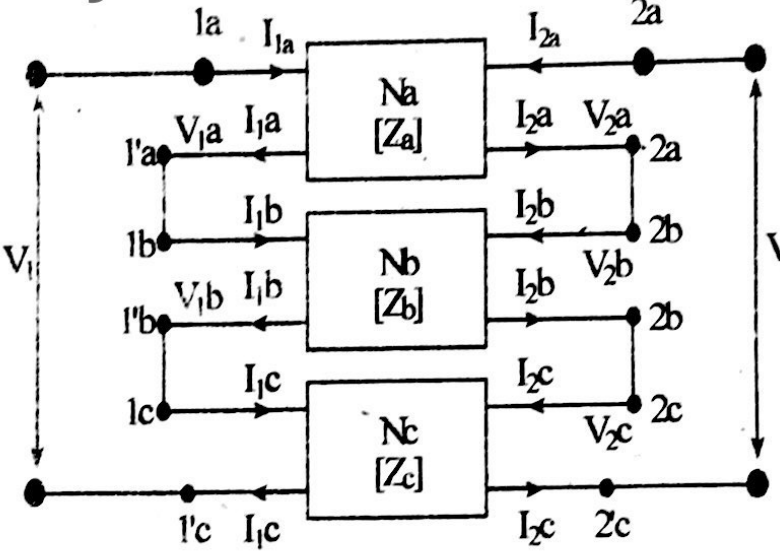
(ii)Three TPN in series:

Pdf by: Sachin Lamsal



Here,

$$\begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} = \begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{21a} & Z_{22a} \end{bmatrix} \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix}$$

$V_{1a} = Z_{11a} I_{1a} + Z_{12a} I_{2a}$ .... (1)

$V_{2a} = Z_{21a} I_{1a} + Z_{22a} I_{2a}$ .... (2)

$$\begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = \begin{bmatrix} Z_{11b} & Z_{12b} \\ Z_{21b} & Z_{22b} \end{bmatrix} \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

$V_{1b} = Z_{11b} I_{1b} + Z_{12b} I_{2b}$ .... (3)

$V_{2b} = Z_{21b} I_{1b} + Z_{22b} I_{2b}$ .... (4)

Also

$$\begin{bmatrix} V_{1c} \\ V_{2c} \end{bmatrix} = \begin{bmatrix} Z_{11c} & Z_{12c} \\ Z_{21c} & Z_{22c} \end{bmatrix} \begin{bmatrix} I_{1c} \\ I_{2c} \end{bmatrix}$$

$V_{1c} = Z_{11c} I_{1c} + Z_{12c} I_{2c}$ .... (5)

$V_{2c} = Z_{21c} I_{1c} + Z_{22c} I_{2c} =$ ..... (6)

Now,

From fig, $I_1 = I_{1a} = I_{1b} = I_{1c}$ and $I_2 = I_{2a} = I_{2b} = I_{2c}$

$V_1 = V_{1a} + V_{1b} + V_{1c}$ .... (7)

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From equ (1), (3), (5) and (7)

$V_1 = Z_{11a}I_{1a} + Z_{12a}I_{2a} + Z_{11b}I_{1b} + Z_{12b}I_{2b} + Z_{11c}I_{1c} + Z_{12c}I_{2c}$

Or, $V_1 = (Z_{11a} + Z_{11b} + Z_{11c}) I_1 + (Z_{12a} + Z_{11b} + Z_{12c}) I_2$

Or, $V_1 = Z_{11} I_1 + Z_{12}I_2$ .... (8)

Where,

$Z_{11} = Z_{11a} + Z_{11b} + Z_{11c}$ and $Z_{12} = Z_{12a} + Z_{11b} + Z_{12c}$

Similarly

$V_2 = Z_{21} I_1 + Z_{22} I_2$ .... (9)

Where,

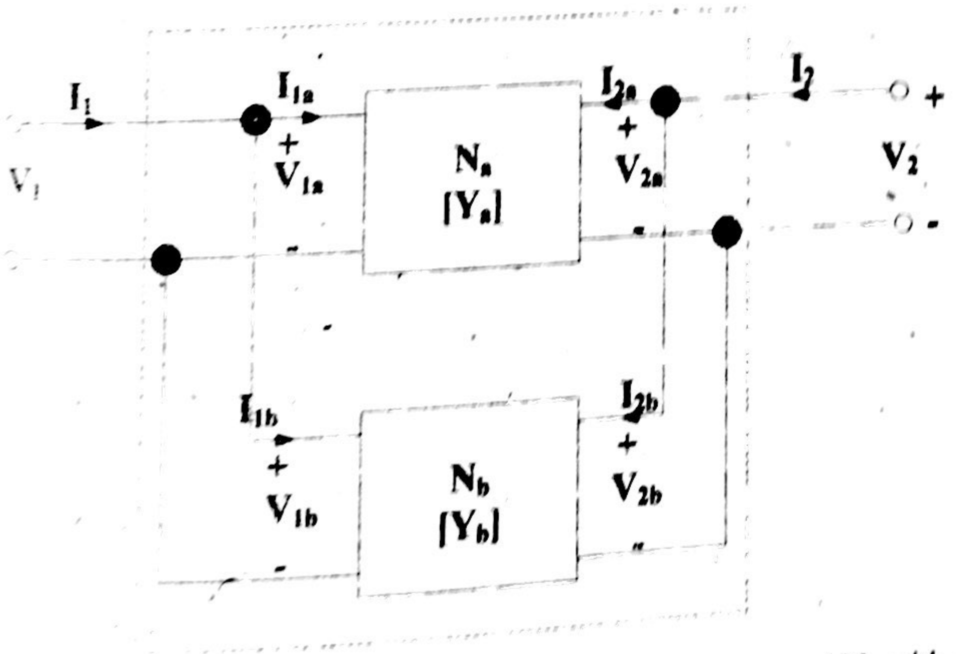
$Z_{21} = Z_{21a} + Z_{21b} + Z_{21c}$

$Z_{22} = Z_{22a} + Z_{22b} + Z_{22c}$

So on...

2.Parallel Connection:

(i) Two TPN in parallel:



Here, figure shows a parallel connection of two two-port networks N_a and N_b with short circuit Z-parameters Y_a and Y_b respectively, i.e., for network N_a,

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$$\begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} = \begin{bmatrix} Y11a & Y12a \\ Y21a & Y22a \end{bmatrix} \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix}$$

Similarly, for network N_b,

$$\begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix} = \begin{bmatrix} Y11b & Y12b \\ Y21b & Y22b \end{bmatrix} \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$$

Then, their parallel connection requires that

$$I_1 = I_{1a} + I_{1b}, I_2 = I_{2a} + I_{2b}$$

And $V_1 = V_{1a} = V_{1b}, V_2 = V_{2a} = V_{2b}$

Now, $I_1 = I_{1a} + I_{1b} = (Y_{11a}V_{1a} + Y_{12a}V_{2a}) + (Y_{11b}V_{1b} + Y_{12b}V_{2b})$

$$=(Y_{11a} + Y_{11b})V_1 + (Y_{12a} + Y_{12b})V_2$$

(Since $V_1 = V_{1a} = V_{1b}$ and $V_2 = V_{2a} = V_{2b}$)

And similarly, $I_2 = I_{2a} + I_{2b} = (Y_{21a} + Y_{21b})V_1 + (Y_{22a} + Y_{22b})V_2$

So, in matrix form the Y- parameters of the parallel-connected combination can be written as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y11 & Y12 \\ Y21 & Y22 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Where, $Y_{11} = Y_{11a} + Y_{11b}$

$$Y_{12} = Y_{12a} + Y_{12b}$$

$$Y_{21} = Y_{21a} + Y_{21b}$$

$$Y_{22} = Y_{22a} + Y_{22b}$$

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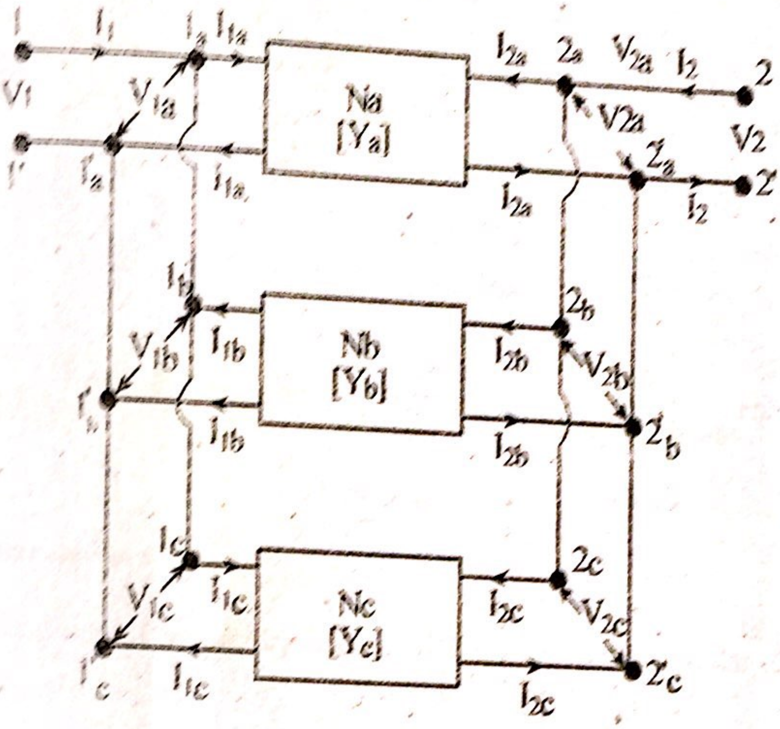
(ii) Three TPN in parallel:



From Figure

$$V_1 = V_{1a} = V_{1b} = V_{1c}$$

$$V_2 = V_{2a} = V_{2b} = V_{2c}$$

For Network "a"

$$\begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} = \begin{bmatrix} y_{11a} & y_{12a} \\ y_{21a} & y_{22a} \end{bmatrix} \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix}$$

$I_{1a} = y_{11a} V_{1a} + y_{12a} V_{2a}$ .... (1)

$I_{2a} = y_{21a} V_{1a} + y_{22a} V_{2a}$ .... (2)

For network "b"

$$\begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix} = \begin{bmatrix} y_{11b} & y_{12b} \\ y_{21b} & y_{22b} \end{bmatrix} \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$$

$I_{1b} = y_{11b} V_{1b} + y_{12b} V_{2b}$ .... (3)

$I_{2b} = y_{21b} V_{1b} + y_{22b} V_{2b} =$ .... (4)

For network "C"

$$\begin{bmatrix} I_{1c} \\ I_{2c} \end{bmatrix} = \begin{bmatrix} y_{11c} & y_{12c} \\ y_{21c} & y_{22c} \end{bmatrix} \begin{bmatrix} V_{1c} \\ V_{2c} \end{bmatrix}$$

$I_{1c} = y_{11c} V_{1c} + y_{12c} V_{2c}$ .... (5)

$I_{2c} = y_{21c} V_{1c} + y_{22c} V_{2c}$ .... (6)

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Also from fig.

$I_1 = I_{1a} + I_{1b} + I_{1c}$.................... (7)

From equ (1), (3), (5) and (7)

$I_1 = y_{11a}V_{1a} + y_{12a}V_{2a} + y_{11b}V_{1b} + y_{12b}V_{2b} + y_{11c}V_{1c} + y_{12c}V_{2c}$

or, $I_1 = (y_{11a} + y_{11b} + y_{11c})V_1 + (y_{12a} + y_{12b} + y_{12c})V_2$

or, $I_1 = y_{11}V_1 + y_{12}V_2$ .... (8)

Where

$y_{11} = y_{11a} + y_{11b} + y_{11c}$

$y_{12} = y_{12a} + y_{12b} + y_{12c}$

Similarly

$I_2 = y_{21}V_1 + y_{22}V_2$ .... (9)

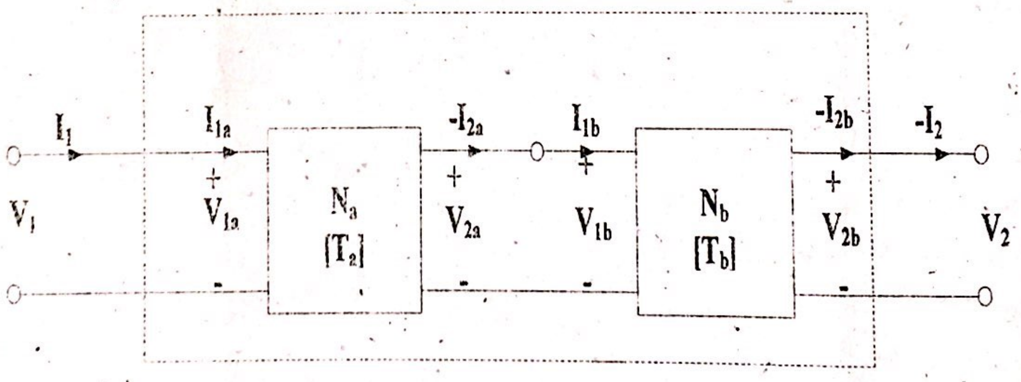
Where,

$y_{21} = y_{21a} + y_{21b} + y_{21c}$

$y_{22} = y_{22a} + y_{22b} + y_{22c}$

3.Cascade or Tandem Connection:

(i) Two TPN in Cascade:



Here, figure shows a cascade connection of two two-port networks N_a and N_b with T-parameters T_a and T_b respectively, i.e., for network N_a.

$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix}$$

Similarly, for network N_b,

$$\begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

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Then, their is cascade connection requires that

$$I_1 = I_{1a}, \quad -I_{2a} = I_{1b} \quad I_{2b} = I_2$$

And $V_1 = V_{1a}, \quad V_{2a} = V_{1b}, \quad V_{2b} = V_2$

Now,
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix}$$

$$= \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} * \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} * \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

So, in matrix form the T- parameters of the Cascade-connected combination can be written as

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} * \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$$

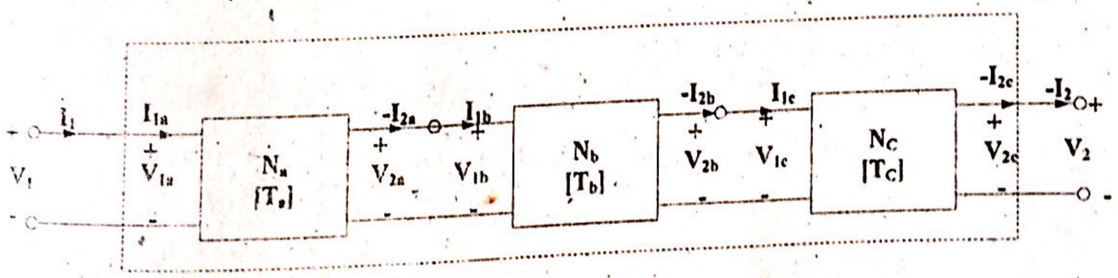
Where, $A = A_a A_b + B_a C_b$

$B = A_a B_b + B_a D_b$

$C = C_a A_b + D_a C_b$

$D = C_a B_b + D_a D_b$

(iI) Three TPN in Cascade:



Here, figure shows a cascade connection of two two-port networks N_a and N_b with T-parameters T_a and T_b respectively, i.e., for network N_a,

$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix}$$

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Similarly, for network N_b,

$$\begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

And for network N_c,

$$\begin{bmatrix} V_{1c} \\ I_{1c} \end{bmatrix} = \begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix} \begin{bmatrix} V_{2c} \\ -I_{2c} \end{bmatrix}$$

Then, their is cascade connection requires that

$I_1 = I_{1a}$, $-I_{2a} = I_{1b}$, $-I_{2b} = I_{1c}$, $I_{2c} = I_2$,

And $V_1 = V_{1a}$, $V_{2a} = V_{1b}$, $V_{2b} = V_{1c}$, $V_{2c} = V_2$

Now,
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix}$$

$$= \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} * \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} * \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{1c} \\ I_{1c} \end{bmatrix}$$

$$= \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} * \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} * \begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix} * \begin{bmatrix} V_{2c} \\ -I_{2c} \end{bmatrix}$$

$$= \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} * \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} * \begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix} * \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

So, in matrix form the Z- parameters of the series-connected combination can be written as

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} * \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} * \begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix}$$

Where, $A = A_aA_bA_c + B_aC_bA_c + A_aB_bC_c + B_aD_bC_c$

$B = A_aB_bB_c + B_aC_bB_c + A_aB_bB_c + B_aD_bD_c$

$C = C_aA_bA_c + D_aC_bC_c + C_aB_bC_c + D_aD_bC_c$

$D = C_aA_bB_c + D_aC_bB_c + C_aB_bD_c + D_aD_bD_c$

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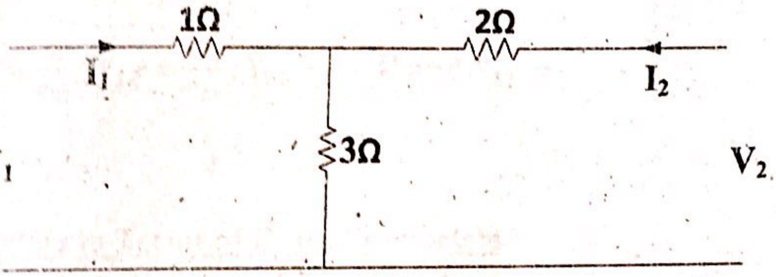
Example.1:

For the network shown in figure, calculate:

(i) Z (ii) Y (iii) T (iv) T' (v) h (vi) g-parameters



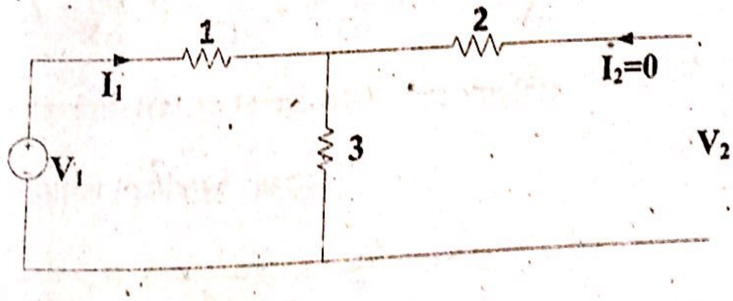
Solution:

(i) Z – parameters

$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} \qquad Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0}$$

$$Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0} \qquad Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0}$$

Case 1: Apply V_1 at port-1 and OC port-2



$V_1 = 4\ I_1$

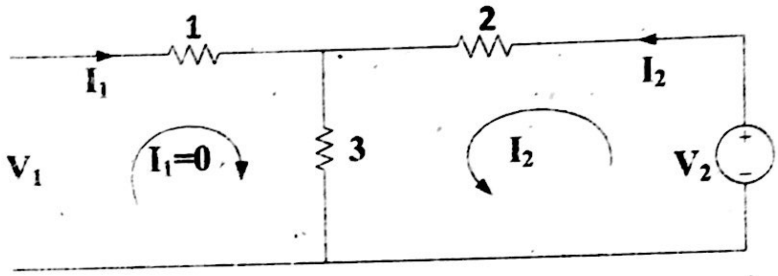
Or,

$$Z_{11} = \frac{V_1}{I_1} = 4\Omega$$

Also, $V_2 = 3\ I_1$

Or, $Z_{21} = \frac{V_2}{I_1} = 3\Omega$

Case 2: Apply V_2 at port-2 and OC port-1



$V_2 = 5\ I_1$

Or, $Z_{22} = \frac{V_2}{I_2} = 5\Omega$

Also, $V_1 = 3\ I_2$

Or, $Z_{12} = \frac{V_1}{I_2} = 3\Omega$

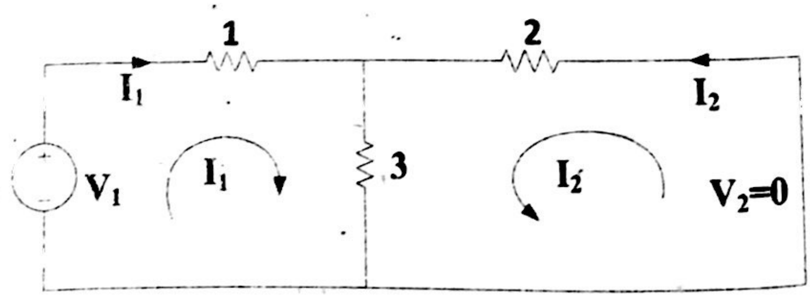
$$[Z] = \begin{bmatrix} 4 & 3 \\ 3 & 5 \end{bmatrix}$$

(ii) Y – parameters

$$Y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} \quad Y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0}$$

$$Y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0} \quad Y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0}$$

Case 1: Apply V_1 at port-1 and SC port-2



Apply KVL for loop-1

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$V_1 = 4I_1 + 3I_2$ ------------ (a)

Also, Apply KVL for loop-2

$3I_1 + 5I_2 = 0$

$I_1 = -\frac{5}{3} I_2$ ------------ (b)

$I_2 = -\frac{3}{5} I_1$ ------------ (c)

From equation (a) and (b);

$V_1 = -\frac{20}{3} I_2 + 3I_2 = -\frac{11}{3} I_2$

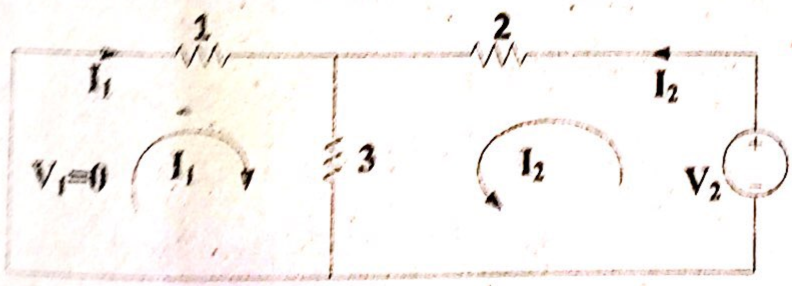
$Y_{21} = \frac{I_2}{V_1} = -\frac{3}{11}\mho$

From equation (a) and (c);

$V_1 = 4I_1 - \frac{9}{5} I_1 = \frac{11}{5} I_1$

$Y_{11} = \frac{I_1}{V_1} = \frac{5}{11}\mho$

Case 2: Apply V_2 at port-2 and SC port-1



Apply KVL for loop-2

$V_2 = 5I_2 + 3I_1$ ------------ (d)

Also, Apply KVL for loop-1

$4I_1 + 3I_2 = 0$

$I_1 = -\frac{3}{4} I_2$ ------------ (e)

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$I_2 = -\frac{4}{3}I_1$-------------- (f)

From equation (d) and (e);

$V_2 = 5I_2 - \frac{9}{4}I_2 = \frac{11}{4}I_2$

$Y_{22} = \frac{I_2}{V_2} = \frac{4}{11}\mho$

From equation (d) and (f);

$V_2 = -\frac{20}{3}I_1 + 3I_1 = -\frac{11}{3}I_1$

$Y_{12} = \frac{I_1}{V_2} = -\frac{3}{11}\mho$

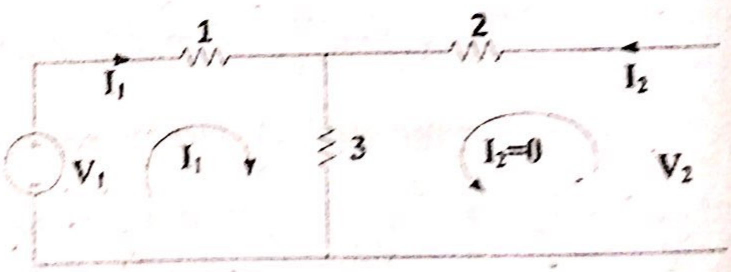
$$[Y] = \begin{bmatrix} 5/11 & -3/11 \\ -3/11 & 4/11 \end{bmatrix}$$

(iii) T – parameters

$A = \frac{V_1}{V_2}\Big|_{I_2=0}$ $C = \frac{I_1}{V_2}\Big|_{I_2=0}$

$B = \frac{V_1}{-I_2}\Big|_{V_2=0}$ $D = \frac{I_1}{-I_2}\Big|_{V_2=0}$

Case 1: Apply V_1 at port-1 and OC port-2



$V_1 = 4\,I_1$

$V_2 = 3\,I_1$

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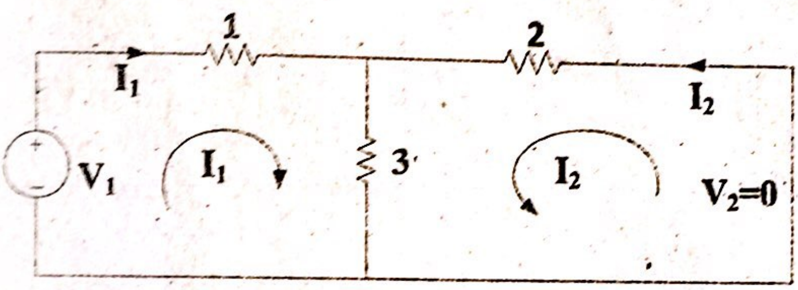
Or.
$$A = \frac{V_1}{V_2} = \frac{4}{3}$$

$$C = \frac{I_1}{V_2} = \frac{1}{3}\mho$$

Case 2: Apply V_1 at port-1 and SC port-2



Apply KVL in left loop

Or.
$V_1 = 4I_1 + 3I_2$ ---------- (g)

Also, Apply KVL in right loop

Or.
$3I_1 + 5I_2 = 0$ ------------- (h)

$I_1 = -\frac{5}{3}I_2$------------- (i)

From equation (g) and (i);

$V_1 = 3I_2 - \frac{20}{3}I_2 = -\frac{11}{3}I_2$

$B = \frac{V_1}{-I_2} = \frac{11}{3}\Omega$

$D = \frac{I_1}{-I_2} = \frac{5}{3}$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 4/3 & 11/3 \\ 1/3 & 5/3 \end{bmatrix}$$

(iv) T' – parameters

$A' = \left.\frac{V_2}{V_1}\right|_{I_1=0}$ $C' = \left.\frac{I_2}{V_1}\right|_{I_1=0}$

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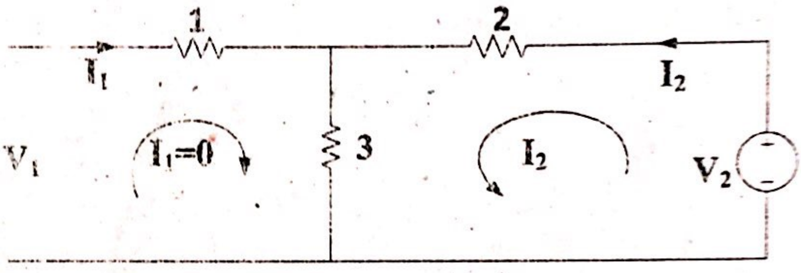
$$B' = \left.\frac{V_2}{-I_1}\right|_{V_1=0} \qquad D' = \left.\frac{I_2}{-I_1}\right|_{V_1=0}$$

Case 1: Apply V_2 at port-2 and OC port-1



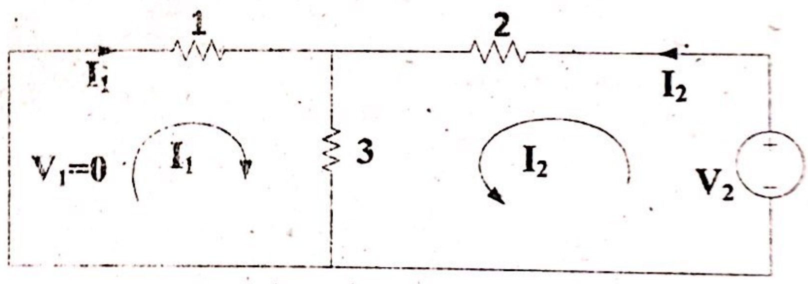
$V_2 = 5\,I_2$

$V_1 = 3\,I_2$

Or, $A' = \frac{V_2}{V_1} = \frac{5}{3}$

$C' = \frac{I_2}{V_1} = \frac{1}{3}\,\mho$

Case 2: Apply V_2 at port-2 and SC port-1



Apply KVL in loop-1

Or, $0 = 4I_1 + 3I_2$ ----------- (j)

$I_2 = -\frac{4}{3} I_1$----------- (k)

Also, Apply KVL in loop-2

Or. $3I_1 + 5I_2 = V_2$

$3I_1 - \frac{20}{3} I_1 = V_2$

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$V_2 = -\frac{11}{3} I_1$

$B' = \frac{V_2}{-I_1} = \frac{11}{3}\,\Omega$

$D' = \frac{I_2}{-I_1} = \frac{4}{3}$

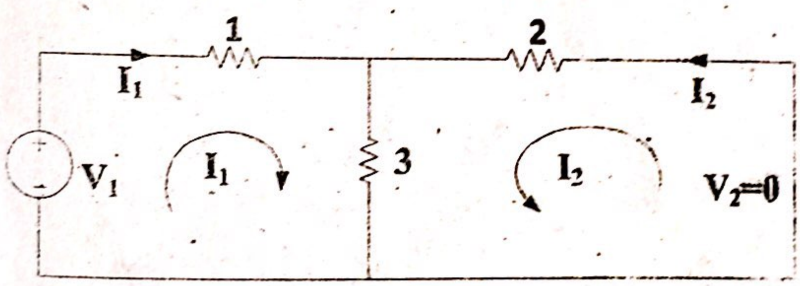
$$\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} 5/3 & 11/3 \\ 1/3 & 4/3 \end{bmatrix}$$

(v) h – parameters

$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$ $h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$

$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$ $h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$

Case 1: Apply V_1 at port-1 and SC port-2



Apply KVL for loop-1

Or, $V_1 = 4I_1 + 3I_2$ ---------- (l)

Also, Apply KVL for loop-2

Or, $3I_1 + 5I_2 = 0$

$I_2 = -\frac{3}{5} I_1$------------ (m)

From equation (l) and (m);

$V_1 = 4I_1 - \frac{9}{5} I_1 = \frac{11}{5} I_1$

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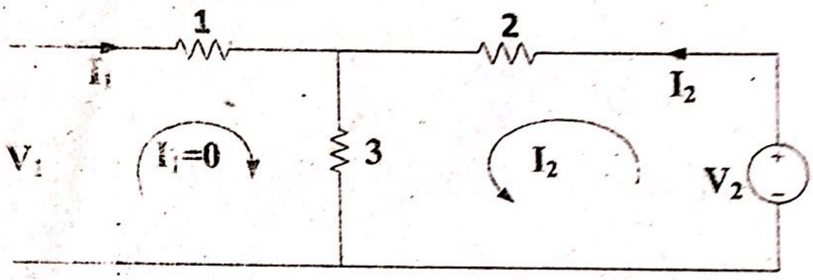
$$h_{11} = \frac{V_1}{I_1} = \frac{11}{5}\,\Omega$$

$$h_{21} = \frac{I_2}{I_1} = -\frac{3}{5}$$

Case 2: Apply V_2 at port-2 and OC port-1



$V_2 = 5\,I_2$

$V_1 = 3\,I_2$.

Or, $$h_{12} = \frac{V_1}{V_2} = \frac{3}{5}$$

Or, $$h_{22} = \frac{I_2}{V_2} = \frac{1}{5}\,\mho$$

$$\begin{bmatrix} h11 & h12 \\ h21 & h22 \end{bmatrix} = \begin{bmatrix} 11/5 & 3/5 \\ -3/5 & 1/5 \end{bmatrix}$$

(vi) g – parameters

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} \quad g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$$

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} \quad g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$

Case 1: Apply V_2 at port-1 and OC port-2

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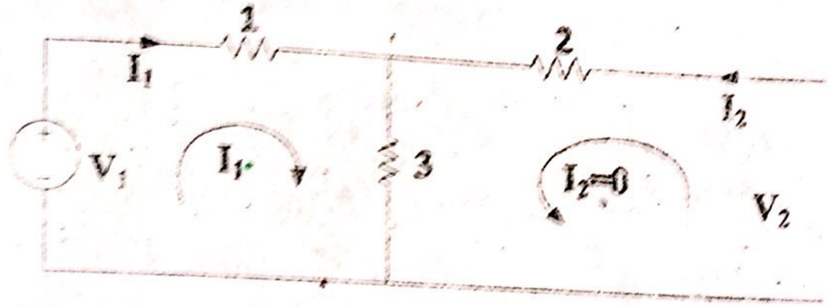


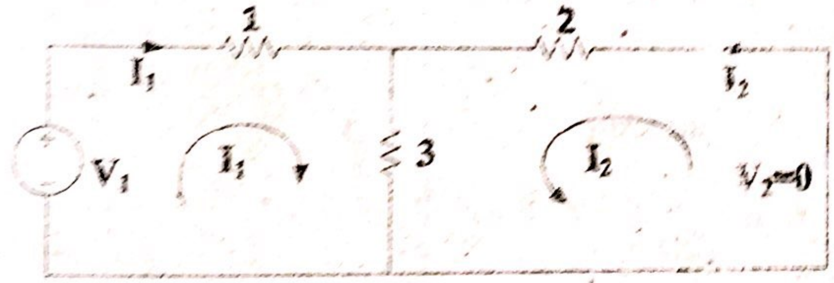
$V_2 = 5 I_2$

$V_1 = 3 I_2$

Or, $g_{11} = \frac{I_1}{V_1} = \frac{1}{4}\mho$

Or, $g_{21} = \frac{V_2}{V_1} = \frac{3}{4}$

Case 2: Apply V_1 at port-2 and SC port-1



Apply KVL for loop-2

Or, $V_2 = 5I_2 + 3I_1$ ------------ (n)

Also, Apply KVL for loop-1

Or, $4I_1 + 3I_2 = 0$

$I_1 = -\frac{3}{4}I_2$-------------- (o)

From equation (n) and (o);

$V_2 = 5I_2 - \frac{9}{4}I_2 = \frac{11}{4}I_2$

$g_{12} = \frac{I_1}{I_2} = -\frac{3}{4}$

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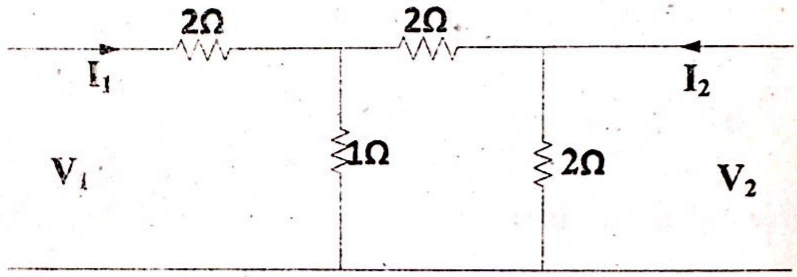
$$g_{22} = \frac{V_2}{I_2} = \frac{11}{4}\,\Omega$$

$$\begin{bmatrix} g11 & g12 \\ g21 & g22 \end{bmatrix} = \begin{bmatrix} 1/4 & -3/4 \\ 3/4 & 11/4 \end{bmatrix}$$

Example.2:

For the network shown in figure, calculate:

(i) Z (ii) Y (iii) T parameters



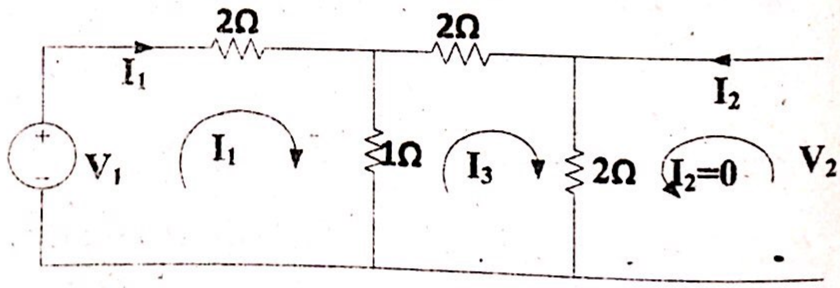
Solution:

(i) Z – parameters

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \qquad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

Case 1: Apply V_1 at port-1 and OC port-2



Apply KVL for loop-1

Or, $V_1 = 3I_1 - I_3$ ----------- (a)

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Also, Apply KVL for loop-3

Or, $-I_1+5I_3=0$

Or, $I_3=\frac{I_1}{5}$ -------------- (b)

From equation (a) and (b);

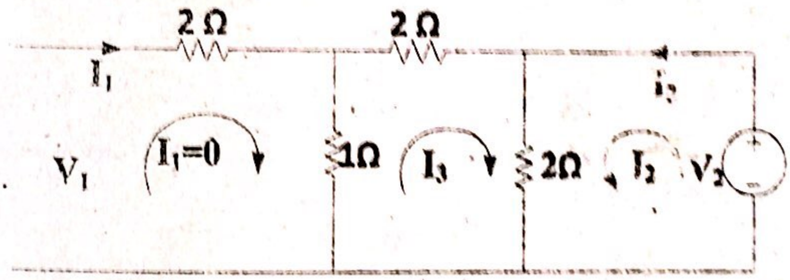
$V_1=3I_1-\frac{I_1}{5}=\frac{14}{5}I_1$

$V_2=2I_3=\frac{2}{5}I_1$

$Z_{11}=\frac{V_1}{I_1}=\frac{14}{5}\Omega$

$Z_{21}=\frac{V_2}{I_1}=\frac{2}{5}\Omega$

Case 2: Apply V_2 at port-2 and OC port-1



Apply KVL for loop-2

Or, $V_2=2I_2+2I_3$ ----------- (c)

Also, Apply KVL for loop-3

Or, $2I_2+5I_3=0$

Or, $I_3=\frac{-2I_2}{5}$ --------------- (d)

From equation (c) and (d);

$V_2=2I_2-\frac{4I_2}{5}=\frac{6}{5}I_2$

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$$V_1 = -I_3 = \frac{2}{5} I_2$$

Or, $$Z_{22} = \frac{V_2}{I_2} = \frac{6}{5}\,\Omega$$

Or, $$Z_{12} = \frac{V_1}{I_2} = \frac{2}{5}\,\Omega$$

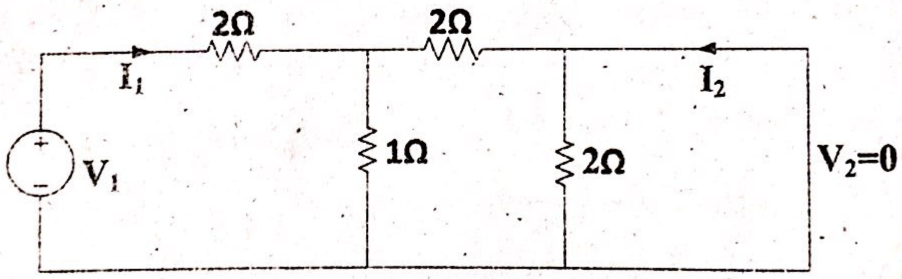
$$[Z] = \begin{bmatrix} 14/5 & 2/5 \\ 2/5 & 6/5 \end{bmatrix}$$

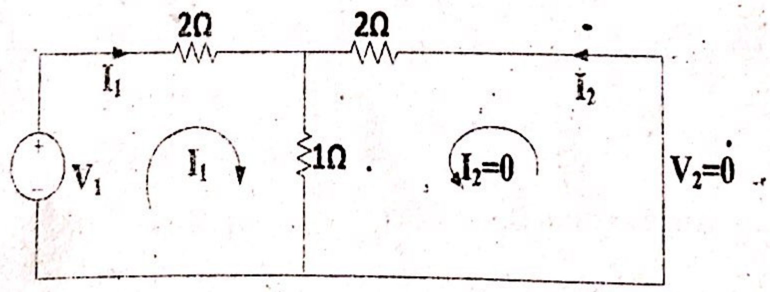
(ii) Y – parameters

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

Case 1: Apply V_1 at port-1 and SC port-2





Apply KVL in loop-1

Or. $V_1 = 3I_1 + I_2$ ---------- (e)

Also, Apply KVL in loop-2

Or. $I_1 + 3I_2 = 0$ ---------- (f)

$I_1 = -3I_2$ ---------- (g)

$I_2 = -\frac{1}{3}I_1$ ---------- (h)

From equation (e) and (g);

$V_1 = -9I_2 + I_2 = -8I_2$

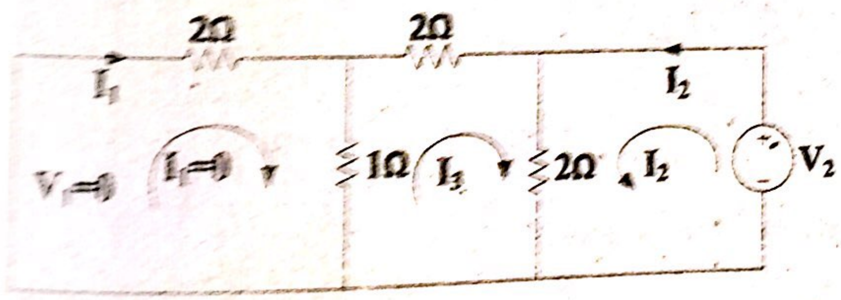
$Y_{21} = \frac{I_2}{V_1} = -\frac{1}{8}℧$

From equation (e) and (h);

$V_1 = 3I_1 - \frac{1}{3}I_1 = \frac{8}{3}I_1$

$Y_{11} = \frac{I_1}{V_1} = \frac{3}{8}℧$

Case 2: Apply V_2 at port-2 and SC port-1



Writing KVL in matrix form;

$$\begin{bmatrix} 3 & 0 & -1 \\ 0 & 2 & 2 \\ -1 & 2 & 5 \end{bmatrix} * \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ V_2 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 3 & 0 & -1 \\ 0 & 2 & 2 \\ -1 & 2 & 5 \end{bmatrix} = 16$$

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$$\Delta_1 = \begin{bmatrix} 0 & 0 & -1 \\ V_2 & 2 & 2 \\ 0 & 2 & 5 \end{bmatrix} = -2V_2$$

$$\Delta_2 = \begin{bmatrix} 3 & 0 & -1 \\ 0 & V_2 & 2 \\ -1 & 2 & 5 \end{bmatrix} = 14V_2$$

$$I_1 = \frac{\Delta_1}{\Delta} = -\frac{1}{8}V_2$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{7}{8}V_2$$

$$Y_{22} = \frac{I_2}{V_2} = \frac{7}{8}\mho$$

$$Y_{12} = \frac{I_1}{V_2} = -\frac{1}{8}\mho$$

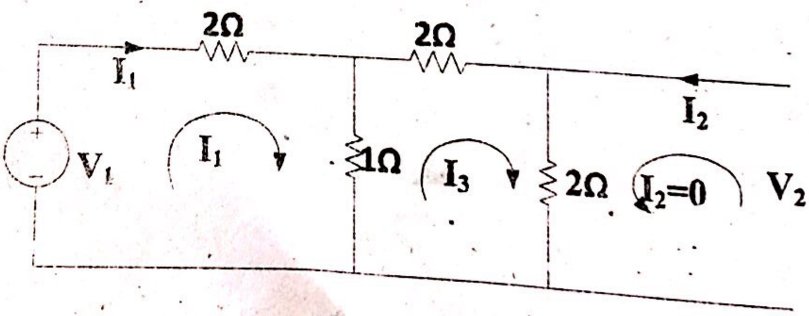
$$[Y] = \begin{bmatrix} 3/8 & -1/8 \\ -1/8 & 7/8 \end{bmatrix}$$

(iii) T – parameters

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \quad C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} \quad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$$

Case 1: Apply V_1 at port-1 and OC port-2



Apply KVL for loop-1

Or, $V_1 = 3I_1 - I_3$ ---------- (i)

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Also, Apply KVL for loop-3

Or: $-I_1+5I_3=0$

Or: $I_3=\frac{I_1}{5}$ ------------- (j)

From equation (i) and (j);

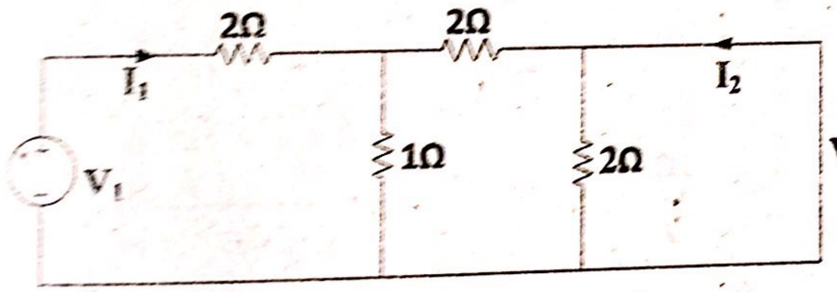
$V_1=3I_1-\frac{I_1}{5}=\frac{14}{5}I_1$

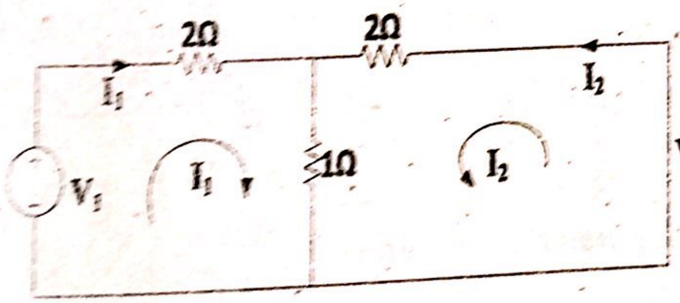
$V_2=2I_3=\frac{2}{5}I_1$

Or: $A=\frac{V_1}{V_2}=\frac{14}{2}=7$

$C=\frac{I_1}{V_2}=\frac{5}{2}\mho$

Case 2: Apply V_1 at port-1 and SC port-2





Apply KVL in loop-1

Or: $V_1=3I_1+I_2$ ---------- (k)

Also, Apply KVL in loop-2

Or: $I_1+3I_2=0$ ---------- (l)

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$I_1 = -3I_2$--------------- (m)

$I_2 = -\frac{1}{3}I_1$--------------- (n)

From equation (k) and (m);

$V_1 = -9I_2 + I_2 = -8I_2$

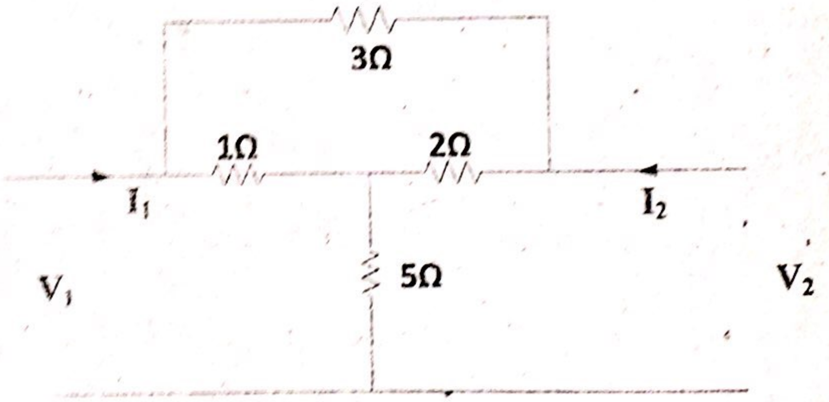
$B = \frac{V_1}{-I_2} = 8\Omega$

$D = \frac{I_1}{-I_2} = 3$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 5/2 & 3 \end{bmatrix}$$

Example.3:

Obtain open circuit parameters of the network shown in figure. Also find Y- parameter.



Solution:

(i) Open circuit parameters (Z – parameters)

$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} \quad Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0}$$

$$Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0} \quad Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0}$$

Case 1: Apply V_1 at port-1 and OC port-2

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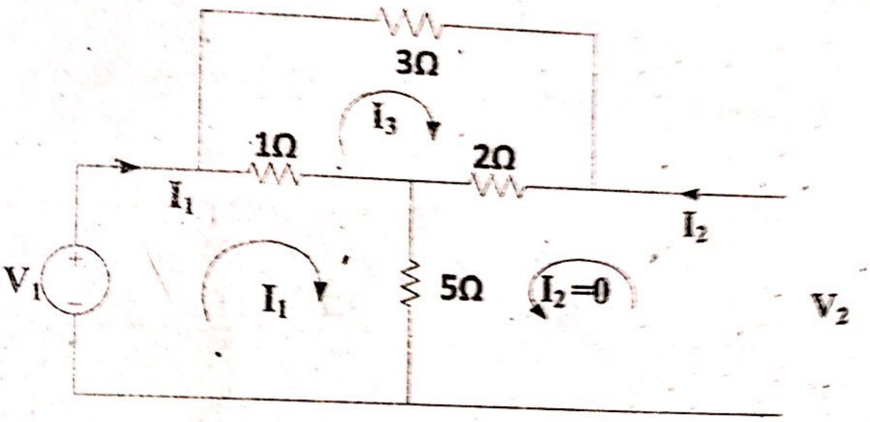

Writing KVL for loop 1 and 3 and writing in matrix form;

$$\begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix} * \begin{bmatrix} I_1 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix} = 35$$

$$\Delta_1 = 6V_1$$

$$\Delta_2 = V_1$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{6}{35}V_1$$

$$I_3 = \frac{\Delta_2}{\Delta} = \frac{1}{35}V_1$$

$$Z_{11} = \frac{V_1}{I_1} = \frac{35}{6}\Omega$$

Also,

$$V_2 = 2I_3 + 5I_1$$

$$V_2 = 2\left(\frac{V_1}{35}\right) + 5\left(\frac{6V_1}{35}\right)$$

$$V_2 = \frac{32}{35}V_1$$

$$Z_{21} = \frac{V_2}{I_1} = \frac{16}{3}\Omega$$

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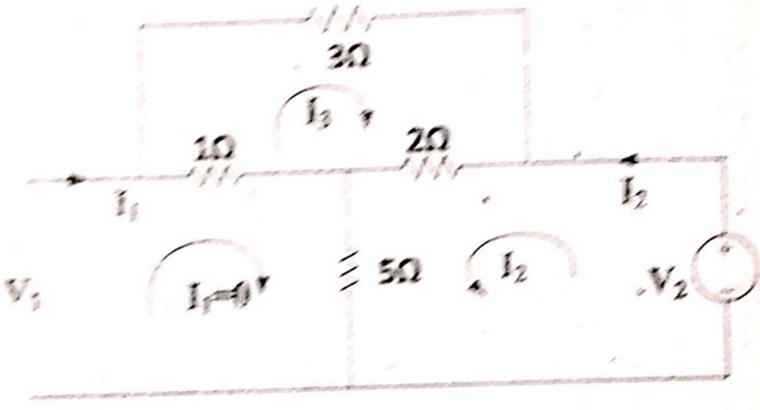
Case 2: Apply V_2 at port-2 and OC port-1



Writing KVL for loop 2 and 3 and writing in matrix form;

$$\begin{bmatrix} 7 & 2 \\ 2 & 6 \end{bmatrix} \cdot \begin{bmatrix} I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_2 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 7 & 2 \\ 2 & 6 \end{bmatrix} = 38$$

$$\Delta_1 = 6V_2$$

$$\Delta_2 = -2V_2$$

$$I_2 = \frac{\Delta_1}{\Delta} = \frac{3}{19}V_2$$

$$I_3 = \frac{\Delta_2}{\Delta} = -\frac{1}{19}V_2$$

Also, $$V_1 = -I_3 + 5I_2 = \frac{V_2}{19} + \frac{15V_2}{19} = \frac{16}{19}V_2$$

Or, $$Z_{22} = \frac{V_2}{I_2} = \frac{19}{3}\Omega$$

Or, $$Z_{12} = \frac{V_1}{I_2} = \frac{16}{3}\Omega$$

$$[Z] = \begin{bmatrix} 35/_6 & 16/_3 \\ 16/_3 & 19/_3 \end{bmatrix}$$

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(ii) Y – parameters

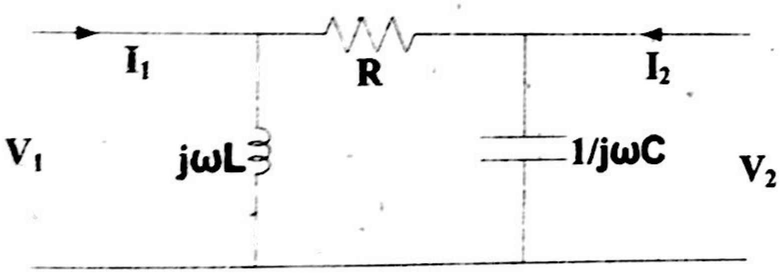
Here.

$$\begin{bmatrix} Y11 & Y12 \\ Y21 & Y22 \end{bmatrix} = \begin{bmatrix} Z11 & Z12 \\ Z21 & Z22 \end{bmatrix}^{-1}$$

$$[Y] = \begin{bmatrix} {}^{38}/_{51} & {}^{-32}/_{51} \\ {}^{-32}/_{51} & {}^{35}/_{51} \end{bmatrix}$$

Example.4:

Determine transmission parameters of the network shown in figure. Also check if the network is symmetrical or not.



Solution:

T – parameters of the network is given as:

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \quad C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} \quad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$$

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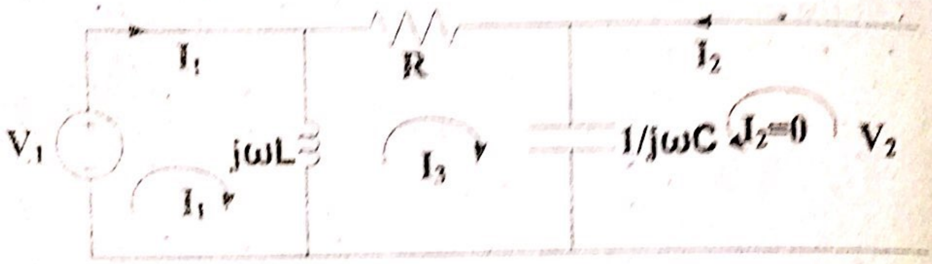
Case 1: Apply V_1 at port-1 and OC port-2



Apply KVL in loop-1

$$V_1 = (j\omega L)(I_1 - I_3) = (j\omega L)I_1 - (j\omega L)I_3 \text{ ------------------ (1)}$$

Also, applying KVL in loop-3

$$\left(R + j\omega L + \frac{1}{j\omega C}\right) I_3 - (j\omega L)I_1 = 0$$

$$\left(\frac{j\omega RC - \omega^2 LC + 1}{j\omega C}\right) I_3 = (j\omega L)I_1$$

$$I_3 = \left(\frac{-\omega^2 LC}{j\omega RC - \omega^2 LC + 1}\right) I_1 \text{ ------------------ (2)}$$

From equation (1) and (2)

$$V_1 = (j\omega L)I_1 - (j\omega L) * \left(\frac{-\omega^2 LC}{j\omega RC - \omega^2 LC + 1}\right) I_1$$

$$V_1 = (j\omega L) * \left(\frac{j\omega RC - \omega^2 LC + 1 + \omega^2 LC}{j\omega RC - \omega^2 LC + 1}\right) I_1$$

$$V_1 = (j\omega L) * \left(\frac{j\omega RC + 1}{j\omega RC - \omega^2 LC + 1}\right) I_1$$

And ,

$$V_2 = \frac{1}{j\omega C} I_3 = \frac{1}{j\omega C} * \left(\frac{-\omega^2 LC}{j\omega RC - \omega^2 LC + 1}\right) I_1 = \left(\frac{j\omega L}{j\omega RC - \omega^2 LC + 1}\right) I_1$$

Or,

$$A = \frac{V_1}{V_2} = \frac{(j\omega L) * \left(\frac{j\omega RC + 1}{j\omega RC - \omega^2 LC + 1}\right) I_1}{\left(\frac{j\omega L}{j\omega RC - \omega^2 LC + 1}\right) I_1} = (1 + j\omega RC)$$

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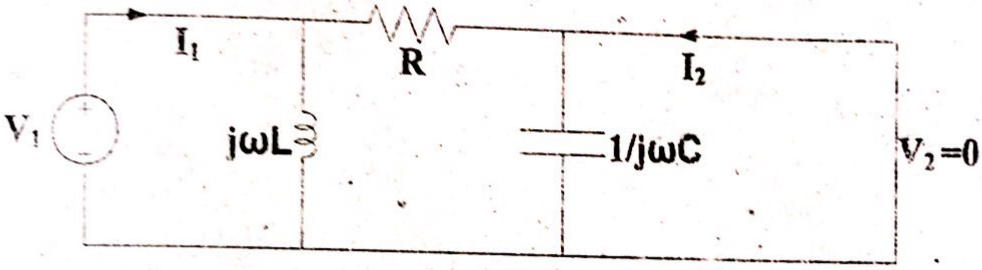
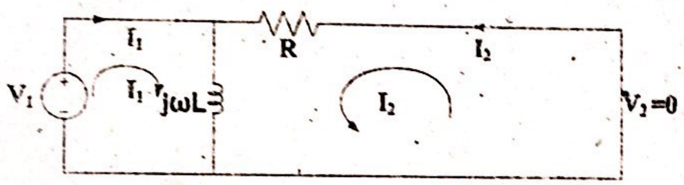
$$C = \frac{I_1}{V_2} = \frac{I_1}{\left(\frac{j\omega L}{j\omega RC - \omega^2 LC + 1}\right) I_1} = \frac{C}{L}\left(j\omega L + R + \frac{1}{j\omega C}\right)$$

Case 2: Apply V_1 at port-1 and SC port-2





Apply KVL in loop-1

Or, $V_1 = j\omega L(I_1 + I_2)$ ----------- (3)

Also, Apply KVL in loop-2

Or, $(R + j\omega L)I_2 + (j\omega L)I_1 = 0$

$$I_1 = \left(-\frac{(R + j\omega L)}{j\omega L}\right) I_2 \text{ -------------- (4)}$$

From equation (3) and (4)

$$V_1 = j\omega L\left(\left(-\frac{(R + j\omega L)}{j\omega L}\right) I_2 + I_2\right)$$

$$V_1 = (-R - j\omega L + j\omega L)I_2 = -RI_2$$

$$B = \frac{V_1}{-I_2} = R$$

$$D = \frac{I_1}{-I_2} = \frac{R}{j\omega L} + 1$$

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$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} ((1 + j\omega RC)) & (R) \\ \left(\frac{C}{L}\left(j\omega L + R + \frac{1}{j\omega C}\right)\right) & \left(\frac{R}{j\omega L} + 1\right) \end{bmatrix}$$

Condition of Symmetry in terms of T- parameter is:

$$A = D$$

Here, $A = 1 + j\omega RC$

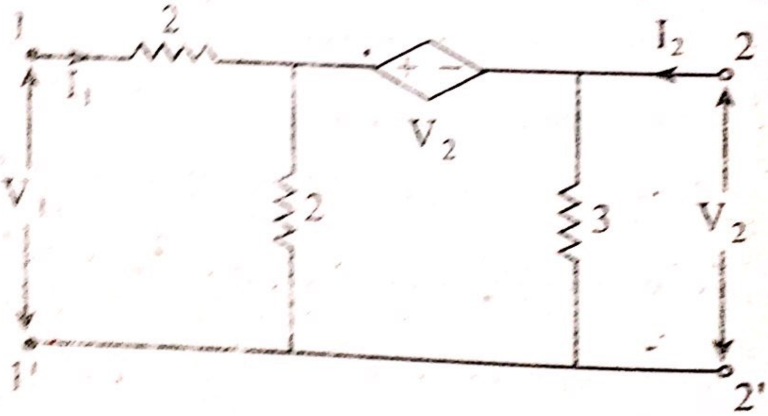
$$D = 1 + \frac{R}{j\omega L}$$

Thus $A \neq D$

Hence, the network is not symmetrical.

Example.5:

Find the Z and Y - parameter for the network shown in figure.



Solution:

Z - parameter

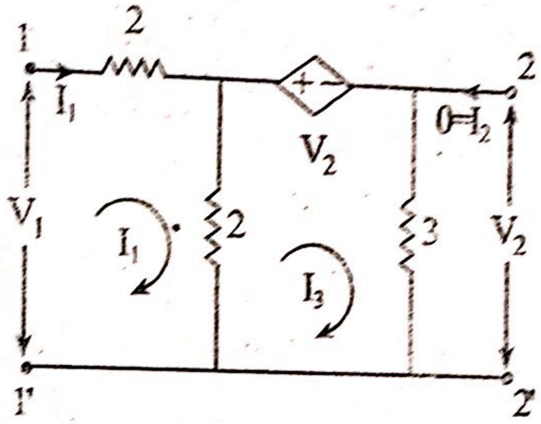
$V_1 = Z_{11} I_1 + Z_{12} I_2$ ..... (1)

$V_2 = Z_{21} I_1 + Z_{22} I_2$ ...... (2)

Applying V_1 at port - 1, and making port -1, open circuited as shown in fig. (1), then from equation (1) and (2) we have,

$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0}$ .... (3) and

$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0}$ .... (4)



KVL in loop (1)

$V_1 = 2I_1 + 2(I_1 - I_3)$

or, $V_1 = 4I_1 - 2I_3$ - (5)

KVL in loop (3)

$- V_2 = 3I_3 + 2(I_3 - I_1)$

or, $- (3I_3) = 5I_3 - 2I_1$

or, $8I_3 - 2I_1 = 0$

or, $I_1 = 4I_3$ ... . (6)

From (5) and (6)

$V_1 = 4(4I_3) - 2I_3$

or, $V_1 = 14I_3$ .... (7)

From (3), (6) and (7)

$$Z_{11} = \frac{V_1}{I_1} = \frac{14I_3}{4I_3} = \frac{7}{2}\,\Omega$$

Now,

$V_2 = 3I_3$ ....(8)

From (4), (6) and (8)

$$Z_{21} = \frac{V_2}{I_1} = \frac{3I_3}{4I_3} = \frac{3}{4}\,\Omega$$

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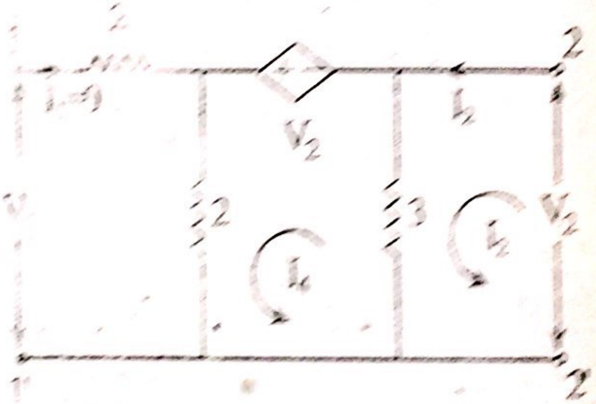
Now, Applying V_2 at port - 2 and making port - 1 open circuited i.e. $I_1 = 0$ as shown in fig. (2), then

From equation (1) and (2), we get

$$Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1 = 0} \;\text{......}\;(9)$$

$$Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1 = 0} \;\text{......}\;(10)$$



KVL in loop (2)

$V_2 = 3I_2 - 3I_4$ ....(11)

KVL in loop (4)

$V_2 = 2I_4 + 3(I_4 - I_2)$

or, $V_2 = 5I_4 - 3I_2$ ....(12)

Now adding (11) and (12)

$2V_2 = 2I_4$

$\therefore V_2 = I_4$ ....(13)

From (11) and (13)

$V_2 = 3I_2 - 3(V_2)$

or, $4V_2 = 3I_2$

$\therefore V_2 = \frac{3}{4} I_2$ ....(14)

Now, from (10), (14)

$$Z_{22} = \frac{V_2}{I_2} = \frac{\left(\frac{3}{4}\right)}{I_2} I_2 = \frac{3}{4}\,\Omega$$

Also from fig (2)

$$V_1 = 2\,I_4 \text{ .... } (15)$$

From (9) and (15)

$$Z_{12} = \frac{V_1}{I_2} = \frac{2I_4}{I_2} = \frac{2V_2}{\frac{4}{3}V_2} = \frac{3}{2}\,\Omega$$

$$\text{Now, } [Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{7}{2} & \frac{3}{2} \\ \frac{3}{4} & \frac{3}{4} \end{bmatrix}.$$

Now,

$$[y] = [Z]^{-1} = \begin{bmatrix} \frac{7}{2} & \frac{3}{2} \\ \frac{3}{4} & \frac{3}{4} \end{bmatrix}^{-1} \text{ .... } (16)$$

$$\text{Now, } |Z| = \begin{vmatrix} \frac{7}{2} & \frac{3}{2} \\ \frac{3}{4} & \frac{3}{4} \end{vmatrix} = \frac{21}{8} - \frac{9}{8} = \frac{21-9}{8} = \frac{12}{8} = \frac{3}{2}$$

$$\therefore \quad [Y] = \begin{bmatrix} \frac{7}{2} & \frac{3}{2} \\ \frac{3}{4} & \frac{3}{4} \end{bmatrix}^{-1} = \begin{bmatrix} \frac{3}{4} \times \frac{2}{3} & -\frac{3}{2} \times \frac{2}{3} \\ -\frac{3}{4} \times \frac{2}{3} & \frac{7}{2} \times \frac{2}{3} \end{bmatrix}$$

$$[Y] = \begin{bmatrix} \frac{1}{2} & -1 \\ -\frac{1}{2} & \frac{7}{3} \end{bmatrix}$$

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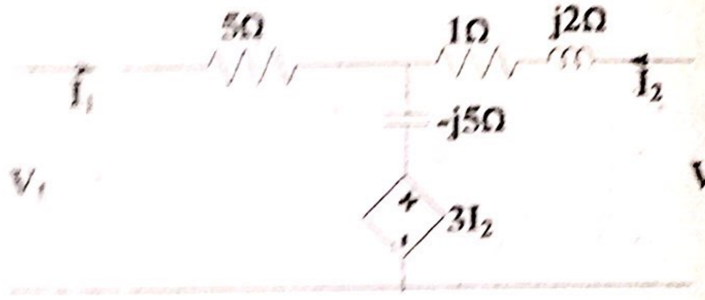
Example.6:

Determine Z – parameters of following network



Solution:

Z – parameters

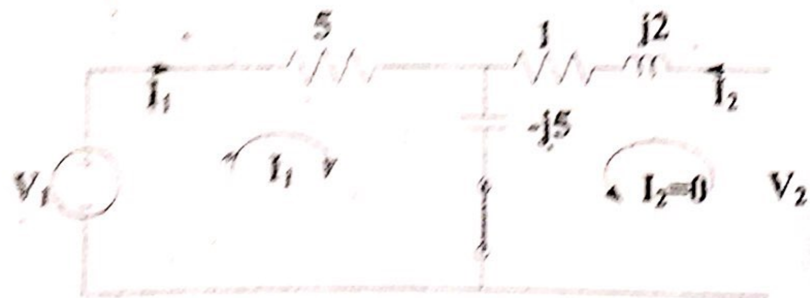
$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} \qquad Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0}$$

$$Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0} \qquad Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0}$$

Note:

- If the value of dependent voltage source is zero, replace it by short circuit.
- If the value of dependent current source is zero, replace it by open circuit.

Case 1: Apply V_1 at port-1 and OC port-2



$[3I_2=0]$

Apply KVL in loop-1

$$V_1 = I_1(5 - j5)$$

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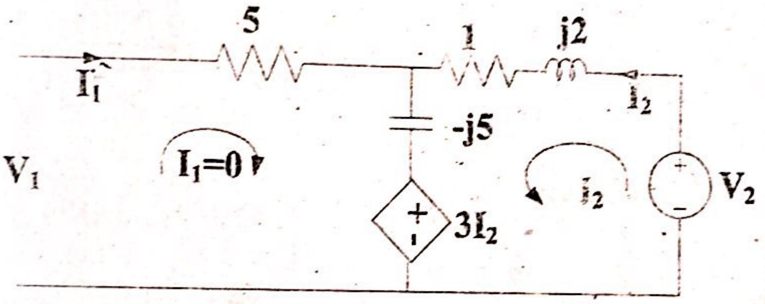
$$Z_{11} = \frac{V_1}{I_1} = 5 - j5\ \Omega$$

$$V_2 = I_1(-j5)$$

$$Z_{21} = \frac{V_2}{I_1} = -j5\ \Omega$$

Case 2: Apply V_2 at port-2 and OC port-1



Apply KVL in loop-2

$$V_2 - 3I_2 = I_2(1 + j2 - j5)$$

$$V_2 = I_2(4 - j3)$$

$$Z_{22} = \frac{V_2}{I_2} = 4 - j3\ \Omega$$

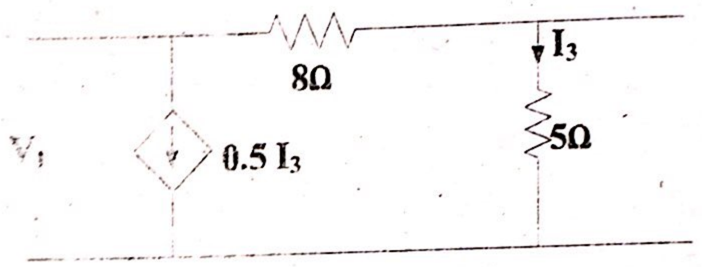
$$V_1 = 3I_2 + (-j5)I_2 = (3 - j5)I_2$$

$$Z_{12} = \frac{V_1}{I_2} = (3 - j5)\ \Omega$$

$$[Z] = \begin{bmatrix} 5 - j5 & (3 - j5) \\ -j5 & 4 - j3 \end{bmatrix}$$

Example.7:

Obtain Z- parameters for the π- circuit model of the network shown in following figure:



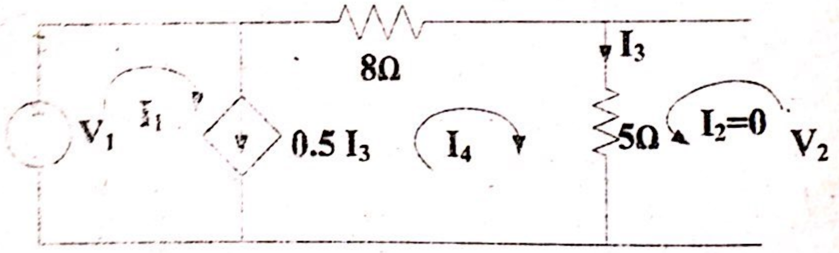
Solution:

Z – Parameters

$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$ $Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$

$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$ $Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$

Case 1: Apply V_1 at port-1 and OC port-2



Here, $I_3=I_4$ ----------- (1) [I_2=0]

$I_1-I_4 = 0.5\ I_3$

$I_1= 1.5\ I_4$ --------- (2) [From (1)]

$V_1 = I_4(5+8) = 13I_4$

$Z_{11} = \frac{V_1}{I_1} = 8.67\ \Omega$

$V_2 = 5I_3 = 5I_4$

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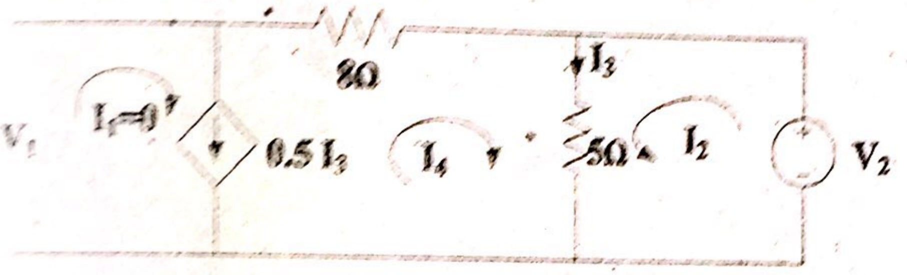
$Z_{21} = \frac{V_2}{I_1} = 3.33\,\Omega$

Case 2: Apply V_2 at port-2 and OC port-1



Here, $I_4 = -0.5I_3$ ----------- (3) $[I_1=0]$

$I_3 = I_4 + I_2$

$I_3 = -0.5I_3 + I_2$ [From (3)]

$I_2 = 1.5I_3$ --------- (4)

Apply KVL in loop-2

$V_2 = 5(I_2 + I_4) = 5(1.5I_3 - 0.5I_3) = 5I_3 = 5 * \frac{I_2}{1.5}$

$Z_{22} = \frac{V_2}{I_2} = 3.33\,\Omega$

$V_1 = 8I_4 + 5I_3 = -4I_4 + 5I_3$ [From equation (3)]

$= I_3$

$= \frac{I_2}{1.5}$ [From equation (4)]

$Z_{12} = \frac{V_1}{I_2} = 0.67\,\Omega$

$$[Z] = \begin{bmatrix} 8.67 & 0.67 \\ 3.33 & 3.33 \end{bmatrix}$$

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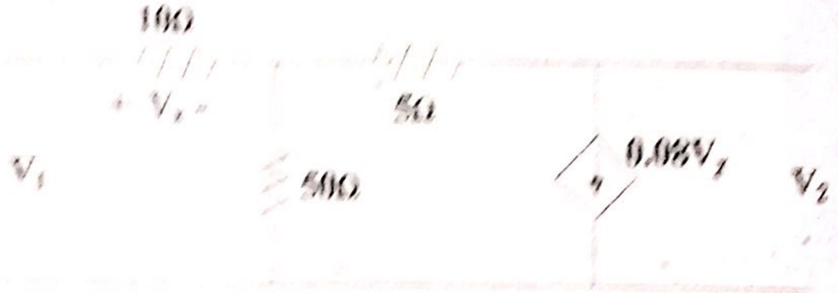
Example.8:

Find T- parameters and Z- parameters of the following network:



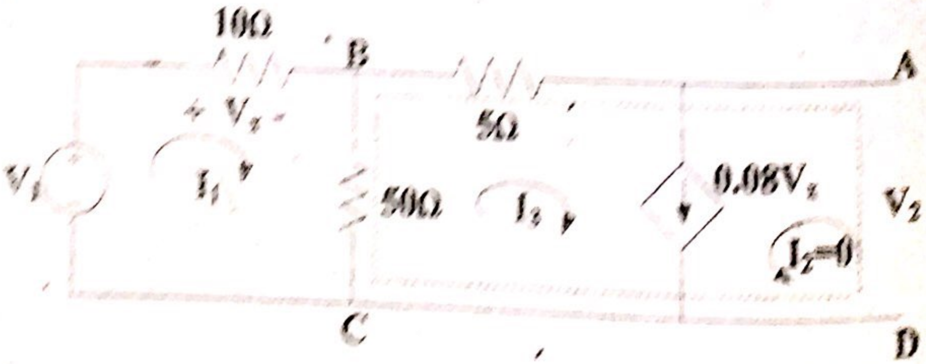
Solution:

T – parameters

$A = \frac{V_1}{V_2}\Big|_{I_2=0}$, $C = \frac{I_1}{V_2}\Big|_{I_2=0}$

$B = \frac{V_1}{-I_2}\Big|_{V_2=0}$, $D = \frac{I_1}{-I_2}\Big|_{V_2=0}$

Case 1: Apply V_1 at port-1 and OC port-2



$V_x = 10\ I_1$ ---------------- (1)

$I_3 = 0.08\ V_x$ $[I_2 = 0]$

$I_3 = 0.8\ I_1$ ------------- (2)

Applying KVL in loop-1

$V_1 = 60I_1 - 50I_3$

$V_1 = 60I_1 - 50 * 0.8I_1$

$V_1 = 20I_1$---------------- (3)

Applying KVL in Supermesh ABCD;

$V_2 = -5I_3 + 50(I_1 - I_3)$

$V_2 = -55I_3 + 50I_1$

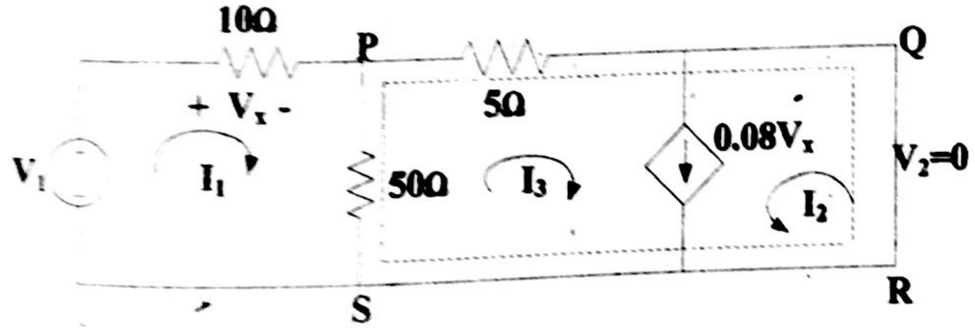
$V_2 = -44I_1 + 50I_1$ [From equation (1)]

$V_2 = 6I_1$

Or, $A = \frac{V_1}{V_2} = \frac{10}{3}$

$C = \frac{I_1}{V_2} = \frac{1}{6}\mho$

Case 2: Apply V_1 at port-1 and SC port-2.



$V_x = 10\ I_1$ ------------- (4)

$I_3 + I_2 = 0.08V_X$

$I_3 + I_2 = 0.8I_1$--------------- (5)

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Apply KVL in Supermesh PQRS

Or, $5I_3 + 50(I_3 - I_1) = 0$

$55I_3 - 50I_1 = 0$

$11I_3 = 10I_1$

$I_3 = \frac{10}{11}I_1$------------------- (6)

Also, Apply KVL in loop-1

Or, $V_1 = 60I_1 - 50I_3$

$V_1 = 60I_1 - 50 * \frac{10}{11}I_1$ From equation (6)

$V_1 = \frac{160}{11}I_1$

From equation (5), (4) ,(6)

$\frac{10}{11}I_1 + I_2 = 0.8I_1$

$I_2 = -\frac{6}{55}I_1$

$B = \frac{V_1}{-I_2} = \frac{400}{3}\Omega$

$D = \frac{I_1}{-I_2} = \frac{55}{6}$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} {}^{10}/_{3} & {}^{400}/_{3} \\ {}^{1}/_{6} & {}^{55}/_{6} \end{bmatrix}$$

Now to find Z- parameters:

$Z_{11} = \frac{A}{C} = 20\ \Omega$

$Z_{12} = \frac{AD-BC}{C} = 50\ \Omega$

$Z_{21} = \frac{1}{C} = 6\ \Omega$

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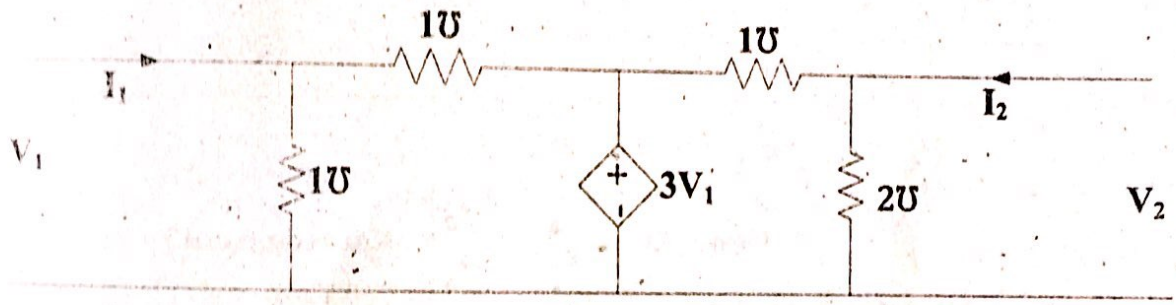
$$Z_{22} = \frac{D}{C} = 55\ \Omega$$

$$[Z] = \begin{bmatrix} 20 & 50 \\ 6 & 55 \end{bmatrix}$$

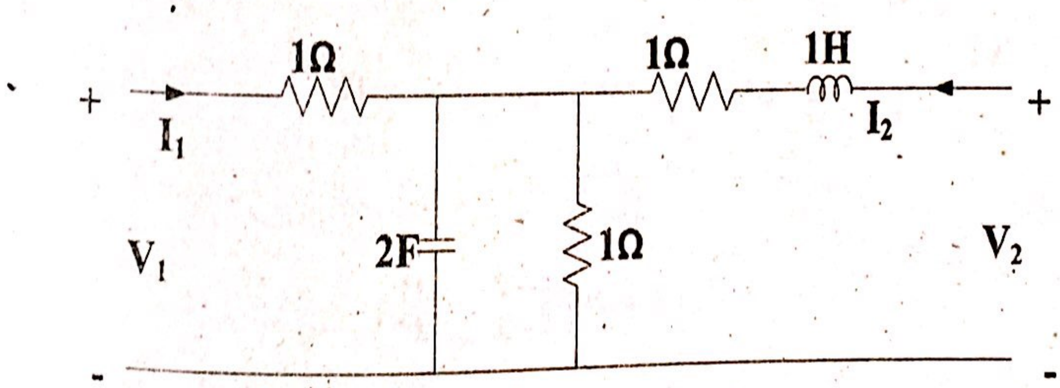
Problems:

Question.1: **Find Y- parameters of the given resister network controlled voltage source**



[Ans;$[Y]=\begin{bmatrix} -1 & -1 \\ -3 & 3 \end{bmatrix}$]

Question.2: **Find Z- parameters of the network containing shown in following figure.**



[Ans:$[Z]=\begin{bmatrix} (S+1)/(S+0.5) & 1/(2S+1) \\ 1/(2S+1) & (2S^2+5S+3)/(2S+1) \end{bmatrix}$]

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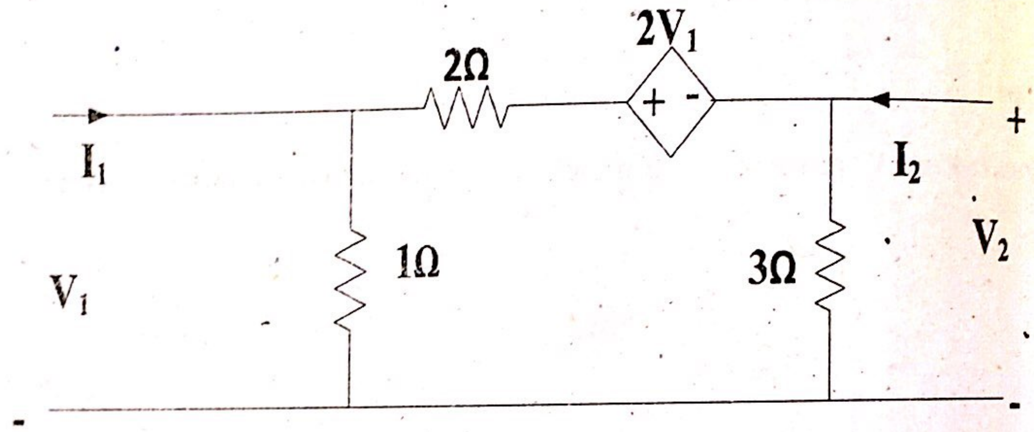
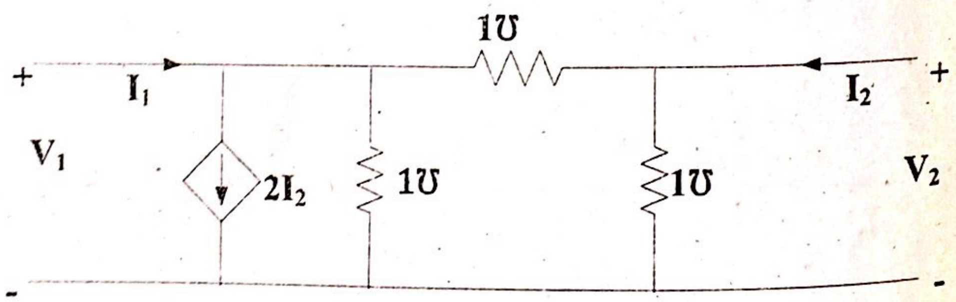
Question.3: Find Z- parameters of the circuit shown in the figure below and also find whether the network is reciprocal or not.



$$[\text{ Ans:}[Z]=\begin{bmatrix} 5/4 & 3/4 \\ -3/4 & 3/4 \end{bmatrix}, \text{Not reciprocal }]$$

Question.4: The given network contains a circuit controlled current source. For this network, find Y – parameters



$$[\text{ Ans:}[Y]=\begin{bmatrix} -2 & -1 \\ -1 & -2 \end{bmatrix}]$$

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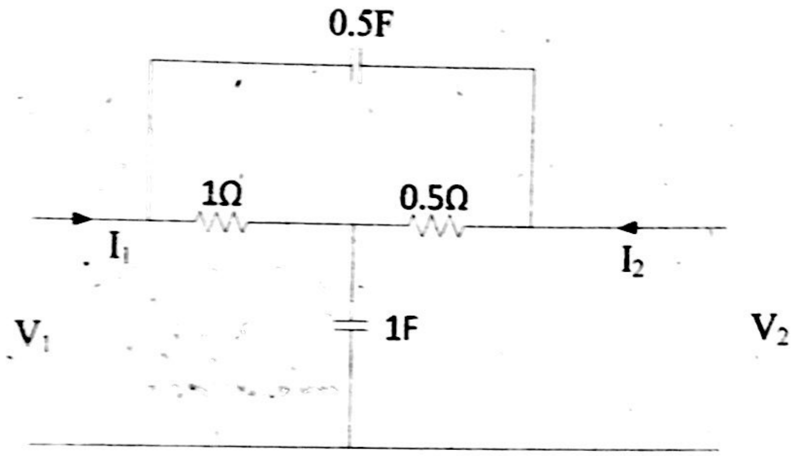
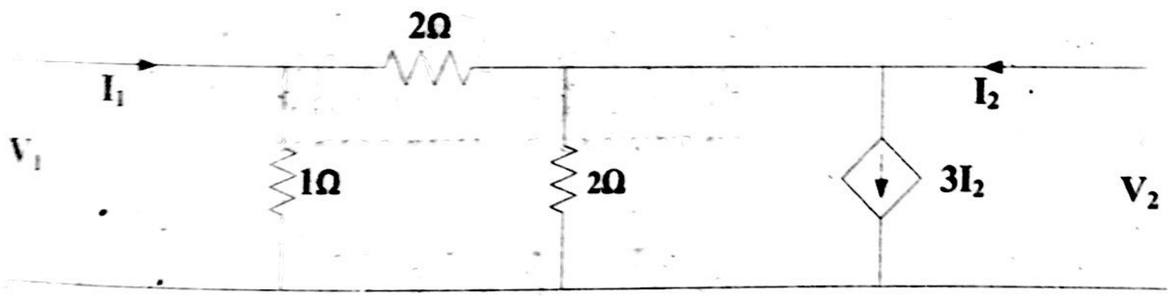
Question.5: For the value given in following network, find Y- parameters



[Ans:[Y]= $\frac{1}{2S+6}\begin{bmatrix} S^2+5S+4 & -(S^2+3S+4) \\ -(S^2+3S+4) & S^2+7S+4 \end{bmatrix}$]

Question.6: Determine Z- parameters of the network shown in following figure.



[Ans:[Z]= $\begin{bmatrix} -0.4 & 0.4 \\ -3.2 & 1.2 \end{bmatrix}$]

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