EMS-LECTURE 7: Economic Dispatch and Optimal Power Flow

Introduction:

Economic Dispatch forms the important analysis functions dealing with Operation in an EMS. Economic Dispatch (ED) is defined as the process of allocating generation levels to the generating units in the mix, so that the system load is supplied entirely and most economically. In static economic dispatch, the objective is to calculate, for a single period of time, the output power of every generating unit so that all the demands are satisfied at the minimum cost, while satisfying different technical constraints of the network and the generators.

Economic Dispatch is the process of determination of the output power generated by the unit or units to supply the specified load in a manner that will minimize the total cost of fuel. Each generating unit has a unique production cost defined by its fuel cost coefficients (a, b, c of a+bp+cp2). Economic dispatch is also defined as the coordination of the production costs of all the participating units in supplying the total load. The purpose of economic dispatch is to determine the optimal power generation of the units participating in supplying the load. The sum of the total power generation should equal to the load demand at the station. In a simplified case, the transmission losses are neglected. This makes the task of solution procedure easier. In actual practice, the transmission losses are to be considered. The inclusion of transmission losses makes the task of economic dispatch more complicated. A different solution procedure has to be employed to arrive at the solution.

Economic Dispatch models the electric power system (with one or more control areas) and dispatches the available generation resources to supply a given load for each control area in the most economic manner in real-time operation. The objective is to minimize the total generation cost (including fuel cost, plus emission cost, plus operation/maintenance cost, plus network loss cost) by meeting the following operational constraints:

- System load demand
- Downward-and-upward regulating margin requirements of the system
- Lower and upper economic limits of each generating unit
- Maximum ramping rate of each generating unit
- Unit's restricted operating zones (up to three restricted zones per unit)
- Emission allowance of the system (SO2, CO2, NOx)
- Network security constraints (maximum mW power flows of transmission lines)
- Supporting multiple I/O curves (incremental Heat Rate) and emission cost curves for different fuels.

The program operates in real-time, according to a schedule you determine Automatic Generation Control (AGC), e.g. every two (2) minutes. It can also be scheduled more frequently by AGC under various circumstances, such as changing the control mode of a unit in AGC, or the system base load deviation exceeding a pre-specified threshold, or a request to execute on demand by an operator.

Economic dispatch is the method of determining the most efficient, low-cost and reliable operation of a power system by dispatching the available electricity generation resources to supply the load on the system. The primary objective of economic dispatch is to minimize the total cost of generation while honoring the operational constraints of the available generation resources.

Economic Dispatch is the process of allocating the required load demand between the available generation units such that the cost of operation is minimized. There have been many algorithms proposed for economic dispatch: Merit Order Loading, Range Elimination, Binary Section, Secant Section, Graphical/Table Look-Up, Convex Simplex, Dantzig-Wolf Decomposition, Separable Convex Linear Programming, Reduced Gradient with Linear Constraints, Steepest

Descent Gradient, First Order Gradient, Merit Order Reduced Gradient, etc. The close similarity of the above techniques can be shown if the solution steps are compared. These algorithms are well documented in the literature. We will use only the graphical (LaGrangian Relaxation) techniques.

Generation Models

The electric power system representation for Economic Dispatch consists of models for the generating units and can also include models for the transmission system. The generation model represents the cost of producing electricity as a function of power generated and the generation capability of each unit. We can specify it as:

1. Unit cost function:

$$F_i = F_i(P_i) \tag{7.1}$$

where F_i = production cost

$F_i(.)$ = energy to cost conversion curve

P_i = production power

2. Unit capacity limits:

$$\begin{aligned} P_i &\leq P_{i\max} \\ P_i &\geq P_{i\min} \end{aligned} \tag{7.2}$$

3. System constraints (demand – supply balance)

$$\sum_{i=1}^{N} P_i = D \tag{7.3}$$

Formulation of the LaGrangian

We are now in a position to formulate our optimization problem. Stated in words, we desire to minimize the total cost of generation subject to the inequality constraints on individual units (2) and the power balance constraint (3). Stated analytically, we have:

Minimize: $\sum_{i=1}^{N} F_i(P_i)$

Subject to: (1)~(3)

The equality constraint $\underline{h}(\underline{x}) = \underline{b}$ for the general case was allowed to contain multiple constraints. Here, in the EDC problem, we see that there is only one equality constraint, i.e., $\underline{h}$ and $\underline{b}$ are both scalars. This implies that lambda is a scalar also. The Lagrangian function, then, is:

$$\ell = \sum_{i=1}^{N} F_i(P_i) + \lambda(D - \sum_{i=1}^{N} P_i) \tag{7.4}$$

KKT Conditions

Application of the KKT conditions to the LaGrangian function of (4) results in:

$$\partial F_i(P_i)/\partial P_i = \lambda \quad i=1,2\ldots N \tag{7.5}$$

$$D - \sum_{i=1}^{N} P_i = 0 \tag{7.6}$$

The KKT conditions provide us with a set of equations that can be solved. The unknowns in these equations include the generation levels P_1, P_2, …, P_n and the LaGrange multipliers λ, a total of (n+1) unknowns. We note that (5) provides n equations, (6) provides one equation. Thus, we have a total of (n+1) equations.

KKT Conditions for a 2-unit system

To illustrate more concretely, let's consider a simple system having only two generating units. The LaGrangian function, from (4), is:

$$\ell = F_1(P_1) + F_2(P_2) + \lambda(D - P_1 - P_2)$$

The KKT conditions, from (5) and (6) become:

$$\begin{aligned} \partial F_1(P_1)/\partial P_1 &= \lambda \\ \partial F_2(P_2)/\partial P_2 &= \lambda \end{aligned} \qquad (7.7)$$

$$D = P_1 + P_2 \qquad (7.8)$$

Graphical Solution

Recall the first KKT condition when applied to the generation system, if we assume that all binding inequality constraints have been converted to equality, then the equations becomes

$$\partial F_1(P_1)/\partial P_1 = \partial F_2(P_2)/\partial P_2 = \lambda \qquad (7.9)$$

This equation implies that for all regulating generators (i.e. units not at their limits.) each generators incremental costs are the same and are equal to lambda.

This very important principle provides the basis on which to apply the graphical solution method. The graphical solution is illustrated in Figure 1 (note that "ICC" means incremental-cost-curve). The unit's data are simply plotted adjacent to each other. Then, a value for lambda is chosen (judiciously) and the generations are added. If the total generation is equal to the total demand "P_T" then the optimal solution has been found. Otherwise, a new value for lambda is chosen and the process repeated. The limitations of each unit are included as vertical lines since the rulers must not include generation beyond unit capabilities. The unit is simply fixed at the value crossed.

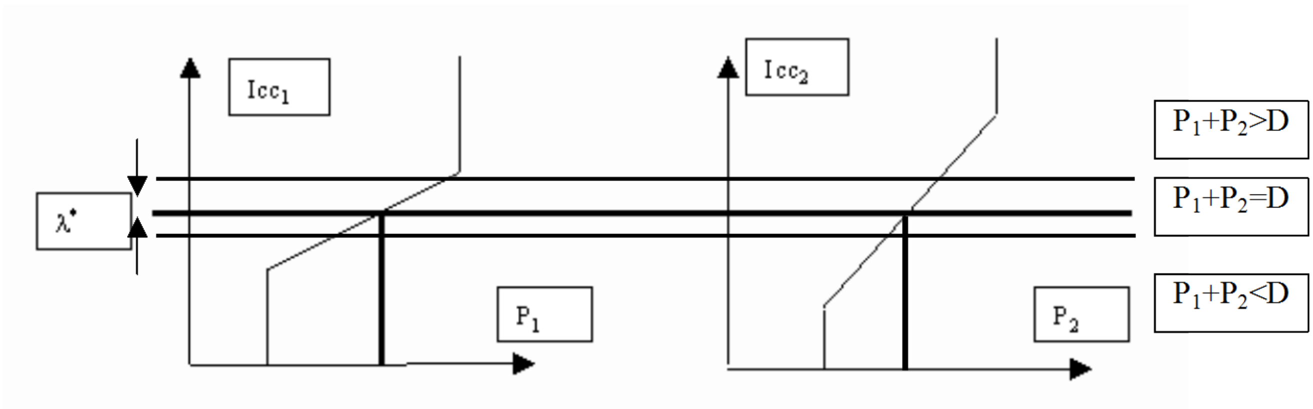

Figure 7.1. Graphical Solution of EDC

Example:

There is a simple two units system including two very similar units that have the following input-output cost function and incremental cost function:

$$F_1 = 8P_1 + 0.024P_1^2 + 80$$
$$F_2 = 8.2P_2 + 0.025P_2^2 + 82 \qquad (7.10)$$

$$\partial F_1 / \partial P_1 = 8 + 0.048P_1$$
$$\partial F_2 / \partial P_2 = 8.2 + 0.05P_2 \qquad (7.11)$$

$$0 \le P_1 \le 80$$
$$0 \le P_2 \le 80 \qquad (7.12)$$

Discretize the I/O curve and incremental cost curve with 10MW space each, 9.92 is the mean value among the discrete points of the incremental cost curve for unit 1, 10.2 is the mean value for unit 2.

Suppose the system lambda value is known, then apply the equal lambda criteria (graphical solution) to the two units, corresponding to each specific value of lambda within the minimal and maximal of incremental cost, there is optimal output level for each unit. From the spreadsheet, we can see that the correlation coefficient between the optimal output levels is

$\rho_{X,Y} = \frac{COV(X,Y)}{\sigma_X . \sigma_Y} = 1$; for the lambda value outside of the minimal or maximal incremental cost of any unit, the output level for that unit is at minimal or maximal, another unit bears the left demand, their correlation coefficient is also 1 (perfect correlation). According to the correlation defined in pearson, the whole correlation is 1 in the whole data set.

Suppose the system demand level is known, then apply the graphical method to the economic dispatch problem of the two units, and determine the corresponding lambda for the demand level. If value of lambda is within the minimal and maximal of incremental cost, the specific output level of each unit is determined, from the spreadsheet, we can see that the correlation between the optimal output levels is 1, if the lambda value outside of the minimal or maximal incremental cost of any unit, the output level for that unit is at minimal or maximal, another unit bears the left demand, their correlation coefficient is also 1 (perfect correlation). According to the correlation defined in pearson, the whole correlation is 1 in the whole data set.

The result is coincident with the fact that in the competitive market, the optimal bid for identical or very similar units are strongly correlated, which is an important aspect that should be considered carefully in making decision on optimal bidding strategy.

This is mathematically given as

$$\min F = \sum_{i=1}^{N} F_i(P_i) \qquad (7.13)$$

$$\sum_{i=1}^{N} P_i - P_D = 0 \qquad (7.14)$$

$$P_{imin} \le P_i \le P_{imax} \qquad i = 1, \ldots\ldots\ldots, N \qquad (7.15)$$

where

F is the operating cost,

N is the number of generating units,

P_i is the power output of i th generating unit,

$F_i\ (P_i)$ is the individual fuel cost function of i th generating unit,

P_D is the demand,

P_{imin} is the i th generating unit’s minimum output,

P_{imax} is the i th generating unit's maximum output.

The fuel cost function or input-output characteristic of the generator maybe obtained from design calculations or from heat rate tests. The fuel cost function of generator that usually used in power system operation and control problem is represented with a second-order polynomial.

$$F_i(P_i) = a_i + b_i P_i + c_i P_i^2 \tag{7.16}$$

where a_i , b_i and c_i are non-negative constants of the i th generating unit. For some generator such as large steam turbine generators, however, the input-output characteristic is not always smooth. Large steam turbine generators will have a number of steam admission values that are opened in sequence to obtain ever-increasing output of the unit. This kind of unit's input-output curve is shown in Fig. 1. The fuel cost function of this kind of unit can be expressed as

$$F_i(P_i) = a_i + b_i P_i + c_i P_i^2 + \left| e_i \sin(f_i(P_{i\,\min} - P_i)) \right| \tag{7.17}$$

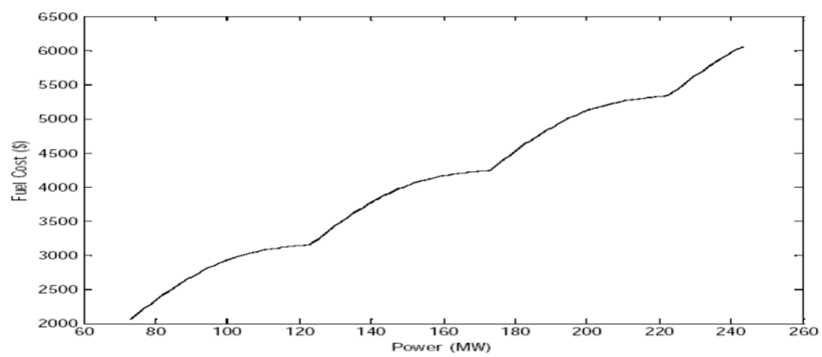

Fig. 1 Valve Point Loading

The economic dispatch problem assumes that the amount of power to be supplied by a given set of units is constant for a given interval of time and attempts to minimize the cost of supplying this energy subject to constraints on the static behavior of the generating units. However, plant operators, to avoid shortening the life of their equipments, try to keep thermal gradients inside the turbine within safe limits. This mechanical constraint is usually translated into a limit on the rate of increase/decrease of the power output. Such ramp rate constraints distinguish the dynamic economic dispatch from the traditional, static economic dispatch. Since these ramp rates constraints involve the evolution of the output of the generators, the dynamic economic dispatch cannot be solved for a single value of the load. Instead it attempts to minimize the cost of producing a given profile of demand.

The dynamic economic dispatch is one of the main functions of power system operation and control. It is a method to schedule the online generator outputs with the predicted load demands over a certain period of time so as to operate an electric power system most economically while the system is operating within its security limits. This problem is a dynamic optimization problem taking into account the constraints imposed on the system operation by generator ramping rate limits. The dynamic economic dispatch is not only the most accurate formulation of the ED problem but also the most difficult to solve because of its large dimensionality.

The problem can be mathematically formulated as follows.

$$\text{Min F} = \sum_{t=1}^{T}\sum_{i=1}^{N} F_{it}(P_{it}) \quad t = 1,\ldots\ldots.,T \tag{6}$$

Subject to

(i) Power Balance constraint

$$\sum_{i=1}^{N} P_{it} - P_D = 0 \qquad i = 1,\ldots\ldots\ldots,N \tag{7}$$

(ii) Unit capacity Constraints

$$P_{it\,\min} \le P_{it} \le P_{it\,\max} \qquad i = 1,\ldots\ldots..,N \quad t = 1,\ldots\ldots.,\mathrm{T} \tag{8}$$

(iii) Ramp Rate constraints

$$\begin{aligned} P_{it} - P_{i(t-1)} &\le UR_i \\ P_{i(t-1)} - P_{it} &\le DR_i \end{aligned} \qquad i = 1,\ldots\ldots..,N \quad t = 1,\ldots\ldots.,T \tag{9}$$

Since dynamic economic dispatch was introduced, several optimization methods have been used to solve this problem.

Economic Dispatch:

1) Consider a three generator system

$H_1 = 510.0 + 7.21\ P_1 + 0.0142\ P^2{}_1$ MBtu/hr.

$H_2 = 310.0 + 7.85\ P_2 + 0.0194\ P^2{}_2$ MBtu/hr.

$H_3 = 78.0 + 7.97\ P_3 + 0.0048\ P^2{}_3$ MBtu/hr.

The fuel cost are:

Unit 1: fuel cost = 1.1 Rs/MBtu $150 \leq P_1 \leq 600$

Unit 2: fuel cost = 1.0 Rs/MBtu $100 \leq P_1 \leq 400$

Unit 3: fuel cost = 1.0 Rs/MBtu $50 \leq P_1 \leq 200$

$$F_1 = H_1 \times \text{fuel cost}_1$$
$$= (510 + 7.2\,P_1 + 0.00142\,P_1^{\,2}) \times 1.1$$
$$F_1 = 561 + 7.92\,P_1 + 0.001562\;P_1^{\,2}) \qquad \rightarrow (1)$$

$$F_2 = H_2 \times \text{fuel cost}_2$$
$$F_2 = 310.0 + 7.85\,P_2 + 0.00194\,P_2^{\,2} \qquad \rightarrow (2)$$

$$F_3 = H_3 \times \text{fuel cost}_3$$
$$F_3 = 78 + 7.97\,P_3 + 0.00482\;P_3^{\,2} \qquad \rightarrow (3)$$

The total operating load $P_D = 850\,\text{Mw}$

$$\therefore\; P_1 + P_2 + P_3 = 850 \qquad \rightarrow (4)$$

By lambda iteration method,

$$\boxed{\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2} = \frac{dF_3}{dP_3} = \lambda}$$

for optimal generation scheduling

$$\frac{dF_1}{dP_1} = 7.92 + 0.003124\,P_1 \;=\; \lambda$$
$$\frac{dF_2}{dP_2} = 7.85 + 0.00388\,P_2 \;=\; \lambda$$
$$\frac{dF_3}{dP_3} = 7.97 + 0.00964\,P_3 \;=\; \lambda$$

$$P_1 = \frac{\lambda - 7.92}{0.003124}$$
$$P_2 = \frac{\lambda - 7.85}{0.00388}$$
$$P_3 = \frac{\lambda - 7.97}{0.00964}$$

Sub for P_1, P_2, P_3 in equation (4)

$$\frac{\lambda - 7.92}{0.003124} + \frac{\lambda - 7.85}{0.00388} + \frac{\lambda - 7.97}{0.00964} = 850$$
$$681.57\lambda - 5385.6706 = 850$$
$$\lambda = \frac{850 + 5385.6706}{681.57}$$
$$\boxed{\lambda = 9.148 \ \text{Rs / Mwhr}}$$

$$\therefore P_1 = \frac{9.148 - 7.92}{0.003124} = 393.08\,\text{Mw}$$
$$P_2 = \frac{9.148 - 7.85}{0.00388} = 334.5\,\text{Mw}$$
$$P_3 = \frac{9.148 - 7.97}{0.00964} = 122.2\,\text{Mw}$$
$$\boxed{P_1 = 393.08\,\text{Mw}, P_2 = 334.5\,\text{Mw}, P_3 = 122.2\,\text{Mw}}$$

2) Suppose the price of unit 1 is decreased to 0.9 Rs/Mwhr. Obtain the optimum schedule

Soln: The fuel cost function of unit 1 becomes:

$$F_1(P_1) = (510 + 7.2P_1 + 0.00142P_1^2) \times 0.9$$
$$F_1(P_1) = 459 + 6.48P_1 + 0.001278P_1^2$$

$$\therefore \frac{dF_1}{dP_1} = 6.48 + 0.00256P_1 = \lambda$$

$$P_1 = \frac{\lambda - 6.48}{0.00256};\ P_2 = \frac{\lambda - 7.85}{0.00388};\ P_3 = \frac{\lambda - 7.97}{0.00964}$$

$\text{Sub for } P_1, P_2, P_3 \text{ in equality constrain}$

$$P_1 + P_2 + P_3 = P_D = 850$$

$$\frac{\lambda - 6.48}{0.00256} + \frac{\lambda - 7.85}{0.00388} + \frac{\lambda - 7.97}{0.00964} = 850$$

$$752.1\lambda - 5381.21 = 850$$

$$\lambda = \frac{850 + 5381.21}{752.1} = 8.285$$

$$\boxed{\lambda = 8.285 \text{ Rs / Mwhr}}$$

Schedule of units :

$$P_1 = \frac{\lambda - 6.48}{0.00256} = 705 \text{ Mw}$$

$$P_2 = \frac{\lambda - 7.85}{0.00388} = 112.1 \text{ Mw}$$

$$P_3 = \frac{\lambda - 7.97}{0.00964} = 32.67 \text{ Mw}$$

Both units P_1 and P_2 are found to violate the inequality constraints

$$P_1 = 600 \text{ Mw} ; \ P_3 = 50 \text{ Mw}$$

$$P_2 = 850 - (650 + 50) = 200 \text{ Mw}$$

$$\boxed{P_1 = 600 \text{ Mw} ; \ P_2 = 200 \text{ Mw}; P_3 = 50 \text{ Mw}}$$

$$\left.\frac{dF_1}{dP_1}\right|_{P_1 = 600} = 6.48 + 0.00256 \times 600$$

$$= 8.016 \text{ Rs / Mwhr}$$

$$\left.\frac{dF_3}{dP_3}\right|_{P_3 = 50} = 7.97 + 0.00964 \times 50$$

$$= 8.452 \text{ Rs / Mwhr}$$

$$\lambda_{new} = \frac{dF_2}{dP_2} = 7.85 + 0.00388 \times 200$$

$$= 8.626 \text{ Rs / Mwhr}$$

For unit 1 and unit 3, the following conditions needs to be satisfied.

$$\frac{dF_1}{dP_1} \leq \lambda_{new} \text{ as } P_1 = P_{1max}$$

$$\frac{dF_3}{dP_3} \leq \lambda_{new} \text{ as } P_3 = P_{3min}$$

The first is satisfied, but 2 condition i.e. $\frac{dF_3}{dP_3} = 8.452 < \lambda_{new}$. Therefore, it cannot be fixed at the lower (minimum) limit.

Now let us fix $P_1 = 600 \text{ Mw}$, do the economic dispatch for units 2 and units 3.

$$P_{D\,new} = P_D - P_1 = 850 - 600 = 250 \text{ Mw}$$

$$P_2 = \frac{\lambda - 7.85}{0.00388} \;;\quad P_3 = \frac{\lambda - 7.97}{0.00964}$$

$$P_2 + P_3 = P_{D\,new}$$

$$\frac{\lambda - 7.85}{0.00388} + \frac{\lambda - 7.97}{0.00964} = 250$$

$$361.46\lambda - 2849.95 = 250$$

$$\boxed{\lambda = 8.576 \text{ Rs / Mwhr}}$$

$$P_2 = \frac{8.576 - 7.85}{0.00388} = 187.17 \text{Mw}$$

$$P_3 = \frac{8.576 - 7.97}{0.00964} = 62.86 \text{Mw}$$

$$\left.\frac{dF_1}{dP_1}\right|_{P_1 = 600} = 8.016 \leq 8.576(\lambda)$$

$\therefore$ Final gen :

$$\boxed{P_1 = 600 \text{ Mw}; \; P_2 = 187.17 \text{ Mw}; P_3 = 62.86 \text{ Mw}}$$

Economic dispatch with losses

$$F_1 = 561 + 7.92 P_1 + 0.001562 P_1^2$$

$$F_2 = 310 + 7.85 P_2 + 0.00194 P_2^2$$

$$F_1 = 78 + 7.97 P_3 + 0.00482 P_3^2$$

$$P_{loss} = 0.00003 P_1^2 + 0.00009 P_2^2 + 0.00012 P_3^2$$

$$\frac{dF_1}{dP_1} = 7.92 + 0.003124\ P_1$$
$$\frac{dF_2}{dP_2} = 7.85 + 0.00388\,P_2$$
$$\frac{dF_3}{dP_3} = 7.97 + 0.00964\ P_3$$

$$\frac{\partial P_{loss}}{\partial P_1} = 0.00006\,P_1;\quad \frac{\partial P_{loss}}{\partial P_2} = 0.00018\,P_2;\quad \frac{\partial P_{loss}}{\partial P_3} = 0.00024\,P_3$$

The Coordination equation is:

$$\boxed{\frac{dF_i}{dP_i} + \lambda\frac{\partial P_{loss}}{\partial P_i} = \lambda} \qquad i = 1, 2, 3...$$

$$\left.\begin{aligned} 7.92 + 0.003124\ P_1 + \lambda(0.00006\,P_1) = \lambda \\ 7.85 + 0.00388\,P_2 + \lambda(0.00018\,P_2) = \lambda \\ 7.97 + 0.00964\ P_3 + \lambda(0.00024\,P_3) = \lambda \end{aligned}\right\} \rightarrow A$$

<u>Iteration - 1</u> :

Let us start with values of P_1, P_2, P_3 as

$P_1 = 400\,Mw$

$P_2 = 300\,Mw$

$P_3 = 150\,Mw$

The total losses are

$$\frac{\partial P_{loss}}{\partial P_1} = 0.024;\quad \frac{\partial P_{loss}}{\partial P_2} = 0.054;\quad \frac{\partial P_{loss}}{\partial P_3} = 0.036$$

$\therefore$ Total losses are $P_{loss} = 15.6\ Mw$

Equation A becomes

$$7.92 + 0.003124\ P_1 + \lambda(0.024) = \lambda$$
$$7.85 + 0.00388\,P_2 + \lambda(0.054) = \lambda$$
$$7.97 + 0.00964\ P_3 + \lambda(0.036) = \lambda$$

$$P_1 + P_2 + P_3 = 850 + 15.6$$
$$\boxed{P_1 + P_2 + P_3 = 865.6} \rightarrow 2$$

$7.92 + 0.003124\ P_1 = \lambda(0.976)$

$P_1 = \dfrac{0.976\lambda - 7.92}{0.003124}$

$P_2 = \dfrac{0.946\lambda - 7.85}{0.00388}$

$P_3 = \dfrac{0.964\lambda - 7.97}{0.00964}$

Sub in 2,

$$\frac{0.976\lambda - 7.92}{0.003124} + \frac{0.946\lambda - 7.85}{0.00388} + = \frac{0.964\lambda - 7.97}{0.00964} = 865.6$$

$$656.23\lambda = 5385.17 = 865.6$$

$$\boxed{\lambda = 9.5252\ \mathrm{Rs / Mw\ hr}}$$

The values of P_1, P_2, P_3 are:

$$\boxed{P_1 = 440.65\ \mathrm{Mw}\ ;\ P_2 = 299.18\ \mathrm{Mw}\ ;\ P_3 = 125.76\ \mathrm{Mw}}$$

Since the values obtained are found to be far different from start values assumed, the second iteration is proceede with this new values of generations.

OPTIMAL POWER FLOW PROBLEM FORMULATION

The OPF problem requires the solution of non-linear equations, describing optimal and/or secure operation of a power system. The general OPF problem can be expressed as:

Minimize $F(x,u)$ (1)

Subject to $g(x,u) = 0$ (2)

$h(x,$

where

$$x^T = \left[\delta \quad V_L{}^T\right] \quad (4)$$

$$u^T = [P_G{}^T \quad V_G{}^T \quad t^T \quad Q_{SH}{}^T] \quad (5)$$

Load flow equations are:

$$0 = P_i - V_i \sum_{j=1}^{N_G} V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) \tag{6}$$

$$0 = Q_i - V_i \sum_{j=1}^{N_L} V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) \tag{7}$$

$$F(x,u) \text{ is } \sum_{i=1}^{N_G} (a_i P_{Gi}{}^2 + b_i P_{Gi} + c) \tag{8}$$

$g(x,u)$ is a set of non-linear equality constraints (Power flow equations) and $h(x,u)$ is a set of non-linear inequality constraints of a vector argument x and u. The vector x consists of dependent variables and u consist of control variables. $h(x,u)$ is a system operating constraints that include:

a) Branch flow limits $|S_k| \le S_k{}^{\max}$ $k = 1.....nl$

b)Voltage at load buses $V_{Lk}{}^{\min} \le V_{Lk} \le V_{Lk}{}^{\max}$ $k = 1....N_L$

c) Generator MVAR $Q_{Gk}{}^{\min} \le Q_{Gk} \le Q_{Gk}{}^{\max}$ $k = 1....N_G$

d) Slack bus MW $P_G{}^{\min} \le P_G \le P_G{}^{\max}$

u consist of control variables such as:

a) Generator MW except slack MW $P_{Gk}{}^{\min} \le P_{Gk} \le P_{Gk}{}^{\max}$

b) Generator bus voltage $V_{Lk}{}^{\min} \le V_{Lk} \le V_{Lk}{}^{\max}$ $k = 1....N_G$

c) Transformer tap setting $t_k{}^{\min} \le t_k \le t_k{}^{\max}$ $k = 1....ntran$

The transformer taps are discrete with a change step of 0.0125 p.u.

d) Bus shunt capacitor $b_{SHk}^{\min} \leq b_{SHk} \leq b_{SHk}^{\max} \; k = 1....N_C$

Summary

This section describes the application of Economic Dispatch in power systems.

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Problems in Economic Dispatch

1. The Incremental Fuel costs in rupees per MWh for a plant consisting of two units are:

$$\frac{dC_1}{dP_{G1}} = 0.20P_{G1} + 40.0 \qquad\qquad 20MW \le P_{G1}, P_{G2} \le 125MW$$
$$\frac{dC_2}{dP_{G2}} = 0.25P_{G2} + 30.0$$

Assume that both the units are operating at all times, and total load varies from 40MW to 250MW. How will the load be shared between the two units as the system load varies over the full range? What are the corresponding values of plant incremental costs? Plot the variation of incremental fuel cost versus plant output.

2. For the plant described in Problem 1, find the saving in fuel cost in rupees per hour for the optimal scheduling of a total load of 130MW as compared to equal distribution of the same load between the two units.

3. A constant load of 300 MW is supplied by two 200 MW generators whose incremental fuel costs are given by:

$$\frac{dC_1}{dP_{G1}} = 0.10P_{G1} + 20.0$$
$$\frac{dC_2}{dP_{G2}} = 0.12P_{G2} + 15.0$$

with powers PG in MW and cost C in Rs/hr. Determine a) the most economical division of load between the two generators and b) the saving in Rs/day thereby obtained compared to equal load sharing between the generators.

4. Consider a three unit system described by the cost equations as:

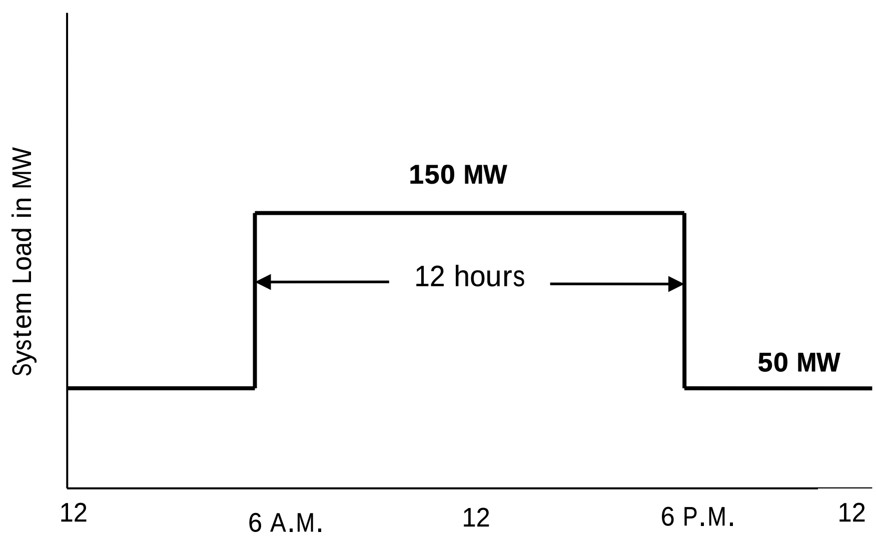
$$F_1 = 225 + 8.4P_1 + 0.0025P_1^2 \ \$/hr$$
$$F_2 = 729 + 6.3P_2 + 0.0081P_2^2 \ \$/hr$$
$$F_3 = 400 + 7.5P_3 + 0.0025P_3^2 \ \$/hr$$
$$45\,MW \le P_1, P_2 \le 350\,MW\ ; \quad 47.5\,MW \le P_3 \le 450\,MW$$

Use the lambda iteration method to find the economic dispatch for a total load demand of 450 MW.

5. Consider a two unit system with fuel cost equations given by:

$$F_1 = 0.024P_1^2 + 8P_1 + 80 \ \ Rs/hr$$
$$F_2 = 0.04P_2^2 + 6P_2 + 120 \ \ Rs/hr$$

Assume a daily load cycle as shown in fig. below:



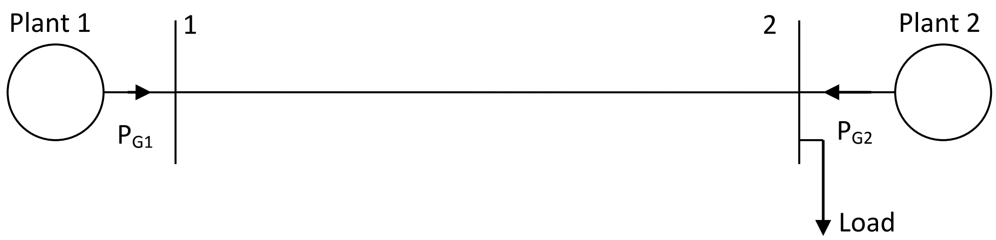
a) Would it be more economical to keep both units in service for the twenty-four hour period?
b) What is the economic schedule for the period from 6.00 A.M. to 6.00 P.M.?
c) What is the economic schedule for the period from 6.00 P.M. to 6.00 A.M.?

6. A two bus system is shown in Fig.1 If 100 MW is transmitted from plant 1 to the load; a transmission loss of 10 MW is incurred. Find the required generation for each plant and the power received by the load when the system λ is Rs. 25 /MWh.

The incremental fuel costs of the two plants are:

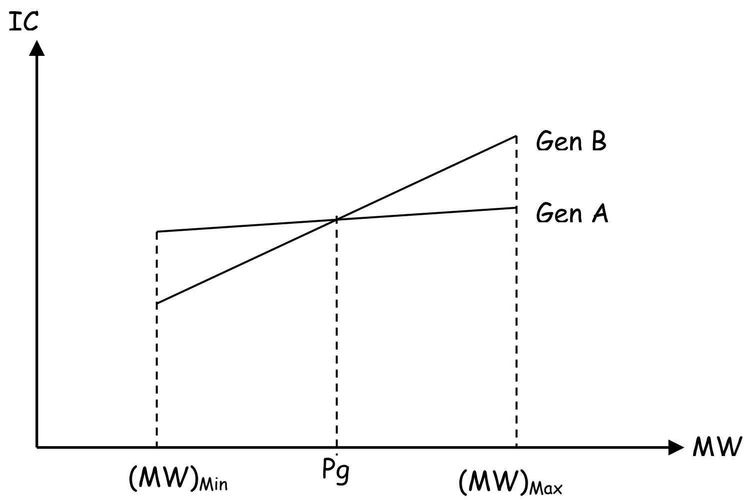
$$\frac{dC_1}{dP_{G1}} = 0.02P_{G1} + 16.0 \quad Rs/MWh$$

$$\frac{dC_2}{dP_{G2}} = 0.04P_{G2} + 20.0 \quad Rs/MWh$$



7. Consider the system of example 6 with a load of 237.04 MW at bus 2 . Find the optimal load distribution between the two plants for (a) when losses are included but not coordinated and (b) when losses are also coordinated. Also find the savings in Rs/hr when losses are coordinated.
8. Fig shows the incremental fuel cost curve of two generators A and B. How would a load
    a. More than 2Pg
    b. Equal to 2Pg
    c. Less than 2Pg

   be shared between A and B, if both the generators are running.



9. The Incremental Fuel costs in rupees per MWh for a plant consisting of two units are:

$$\frac{dF_1}{dP_{G1}} = 0.10P_{G1} + 20$$

$$\frac{dF_2}{dP_{G2}} = 0.12P_{G2} + 16$$

The min and max loads on each unit are 20 MW and 125 MW respectively. Determine the incremental fuel cost and allocation of load between units for minimum cost when loads are (i) 100 MW (ii) 150 MW. Determine the saving in fuel cost in Rs/hr for economic distribution of 200MW load compared with equal distribution at the same total load.

10. A system consists of two plants connected by a tie lie and a load is located at plant 2. when 100 MW is transmitted from plant 1, a loss of 10 MW takes place on the tie line. Determine the generation schedule at both the plants and the power received by the load when λ for the system is Rs 25/ MWhr. The incremental fuel cost equations are

$$\frac{dF_1}{dP_{G1}} = 0.10P_{G1} + 22$$

$$\frac{dF_2}{dP_{G2}} = 0.12P_{G2} + 16$$

11. Consider a two unit system with fuel cost equations given by:

$$F_1 = 0.015P_1^2 + 16P_1 + 50 \quad Rs/hr$$
$$F_2 = 0.025P_2^2 + 12P_2 + 30 \quad Rs/hr$$

The loss coefficients of the system are given by B11 = 0.005 B12 = -0.0012 and B22 = 0.002. The load to be met is 200 MW, determine the economic operating schedule and corresponding cost of generation if (i) transmission losses are coordinated (ii) losses are included but not coordinated.

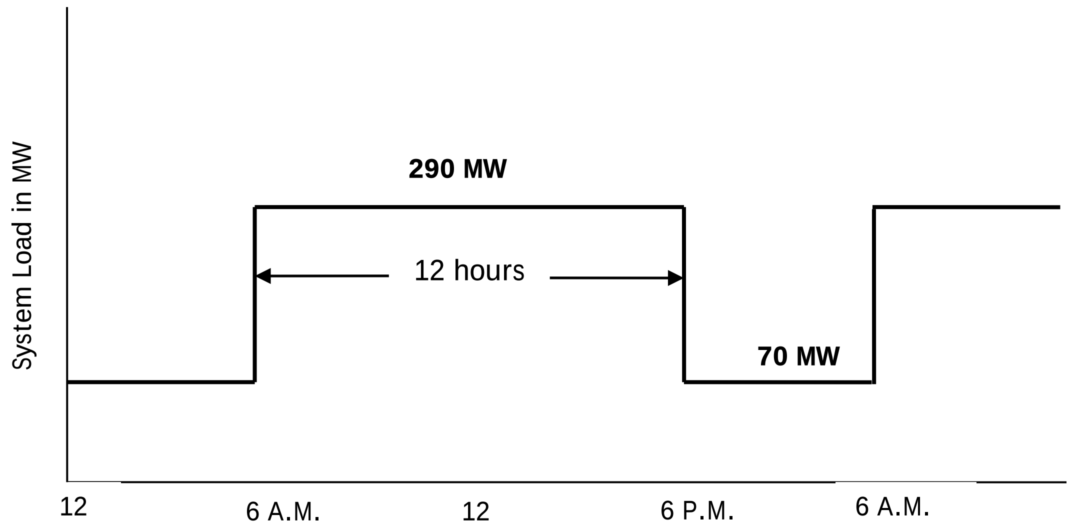
12. To supply a total system load of 310 MW, three plants of total capacity of 500 MW are scheduled for operation. Evaluate the optimum load scheduling if the plants have the following cost characteristics and limitations.

$$\frac{dC_1}{dP_{G1}} = 0.12P_{G1} + 30 \quad Rs/MWh \quad 30 \le P_{G1} \le 150$$
$$\frac{dC_2}{dP_{G2}} = 0.20P_{G2} + 40 \quad Rs/MWh \quad 20 \le P_{G2} \le 100$$
$$\frac{dC_3}{dP_{G3}} = 0.16P_{G3} + 10 \quad Rs/MWh \quad 50 \le P_{G3} \le 250$$

13. The heat rate of a 100MW fuel fired generator is 10 MKCal/MWhr @ 25 % of rating, 9 MKCal/MWhr @ 40 % of rating, 8 MKCal/MWhr @ 100 % of rating. The cost of the fuel is Rs 2/MKCal. Find (a) Cost equation (b) Fuel input rate and fuel cost when 25 % , 50 % and 100 % loaded. (c) Incremental cost in Rs/MWhr (d) the approximate cost and cost using quadratic approximation in Rs /Hr to deliver 101 MW.

14. Let the two units of the system have the following cost curves

$$C_1 = 120 + 40P_1 + 0.1P_1^2 \ Rs/hr$$
$$C_2 = 100 + 30P_2 + 0.2P_2^2 \ Rs/hr$$



Let us assume a daily load cycle as shown in the fig above. Also assume that a cost of Rs 400 is incurred in taking either unit off the line and returning it to service after 12 Hrs. Consider the 24 Hr period from 6 am one morning to 6 am the next morning. Find out whether it would be economical to keep both units in service for this 24 Hr period or to remove one of the units from service for 12 hrs of light load.

15. Consider the following three IC curves

$$P_{G1} = -100 + 50IC_1 - 2.0IC_1^2 \ MW$$
$$P_{G2} = -150 + 60IC_2 - 2.5IC_2^2 \ MW$$
$$P_{G3} = -\ 80 + 40IC_3 - 1.8IC_3^2 \ MW$$

where IC is in Rs/MWhr.

The total load at a certain hour of the day is 400 MW. Neglecting transmission losses obtain optimum generation scheduling within an accuracy of ± 0.05 MW. Note all PGs must be real positive.