4.1 Operational Amplifier:

The operational amplifier most commonly referred to as 'Op-amp' was introduced in 1940s. In those days it was used in the analog computer to perform a variety of mathematical operations such as addition, subtraction, multiplication etc. Due to the use of vacuum tubes, the early op-amps were bulky, power consuming and expensive.

An operation amplifier (or Op-amp) is a high gain differential amplifier having very high input impedance (typically a few mega ohms) and low output impedance (less than 100 π)

Symbol of the Op-amp is shown below.

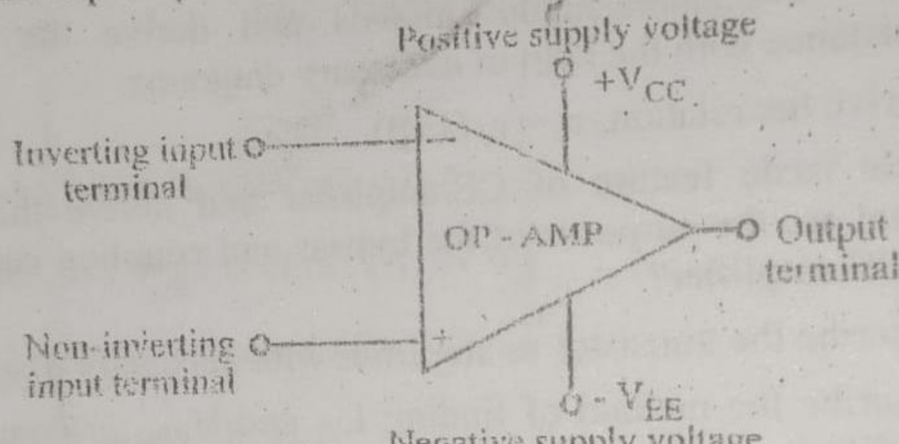
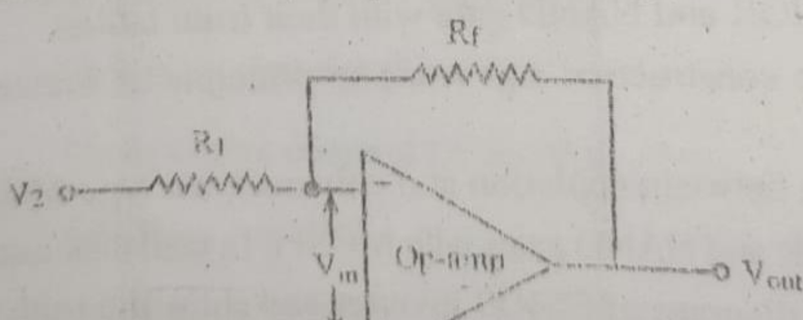


Fig.: OP-amp symbol

4.1.1 Basic Op-amp circuit:

The basic circuit of an op-amp is shown in figure:



(a) *Basic circuit Diagram*

Characteristics of an Ideal Op-Amp

The various characteristics or properties of an ideal op-amp are:

(i) The open loop voltage gain is infinite, i.e., $A_V = \infty$.

(ii) The input impedance (R_{in}) is infinite, i.e., $R_{in} = \infty$.

(iii) The output impedance (R_o) is zero, i.e., $R_o = 0$

(iv) The range of frequency over which the amplifier performance is satisfactory is called its bandwidth. The bandwidth of an ideal op-amp is infinite, i.e., the gain of an op-amp remains constant over the frequency range from dc to infinite frequency.

(v) The output offset voltage is zero. The output offset voltage is the presence of small output voltage even if $V_1 = V_2 = 0$.

(vi) The common mode rejection ratio ($CMRR = \left|\frac{A_d}{A_c}\right|$) is infinite, i.e., the common mode noise output voltage is zero for an ideal op-amp.

(vii) The slew rate is infinite. This ensures that the changes in the input voltage occur simultaneously with the changes in the output voltage.

The characteristics of ideal op-amp are shown in Table 7.1.

Table 7.1 Characteristics of an ideal op-amp

S.N.	Characteristics	Representation	Values
1.	Open loop voltage gain	A	∞
2.	Input impedance	R_i	∞
3.	Bandwidth	B.W.	∞
4.	CMRR	ρ	∞
5.	Slew rate	S.R.	∞
6.	Output impedance	R_o	0
7.	Offset voltage	V_{oos}	0

7.6. Characteristics of the Practical Op-Amp

The practical op-am characteristics are a little bit different from the ideal op-amp characteristics. The equivalent circuit of the practical op-amp is shown in Figure 7.4.

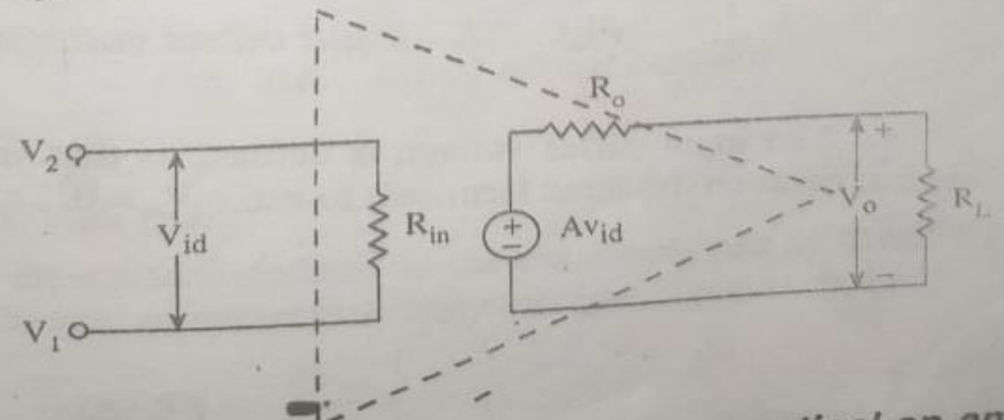


Fig. 7.4 Equivalent circuit of the practical op-amp

The characteristics of the practical op-amp are:

(i) The open loop voltage gain is very high. Practically it is several thousands.

(ii) The input impedance is very large, i.e., $R_{in} > 1\ M\Omega$. If we use FETs for the input stage, R_{in} can be increased upto several hundreds $M\Omega$.

(iii) The output impedance is very low, i.e., a few hundred ohms. With the application of negative feedback, it can be reduced to a very small value like $1\ \Omega$ or $2\ \Omega$.

(iv) The bandwidth is not very large. But it can be extended using negative feedback.

(v) The practical op-amp always shows a small offset voltage.

(vi) The input bias current is very small. The input offset current is very small, of the order of 20 nA to 60 nA.

gain forces the voltage at the inverting and non-inverting inputs to be approximately equal.

7.8. Parameters of an Op-Amp

The common mode and differential mode gain, CMRR, output offset voltage, input offset voltage, input offset current, input bias current, slew rate and power rejection ratio are important parameters of the op-amp.

7.8.1 *Difference mode gain (A_{id})*

$$V_o = A_{id}(V_1 - V_2).$$

$$\therefore \quad A_{id} = \frac{V_o}{(V_1 - V_2)} = \frac{V_o}{V_{id}}.$$

7.8.2 *Common mode gain (A_{cm})*

If $V_1 = V_2$, then, $V_1 - V_2 = 0$, i.e., $V_o = 0$.

But the output voltage of the practical differential amplifier not only depends on the difference voltage but also on the average of the two input signals.

$$\text{Thus, } V_{cm} = \frac{V_1 + V_2}{2}.$$

$$\text{Then, } V_o = A_{cm} \times V_{cm}.$$

Now, the total output of any differential amplifier can be expressed as

$$V_o = A_{id}V_{id} + A_{cm}V_{cm}$$

For an ideal differential amplifier, A_{id} must be infinite and A_{cm} must be zero.

7.8.3 *CMRR*

The common mode rejection ratio (ρ) is defined as $\rho = \frac{|A_d|}{|A_c|}$ where A_d is the difference mode gain and A_c is the common mode gain.

Ideally, $\rho = \infty$. This means, the common signals like noises are perfectly rejected by the amplifier.

7.8.4 *Output offset voltage*

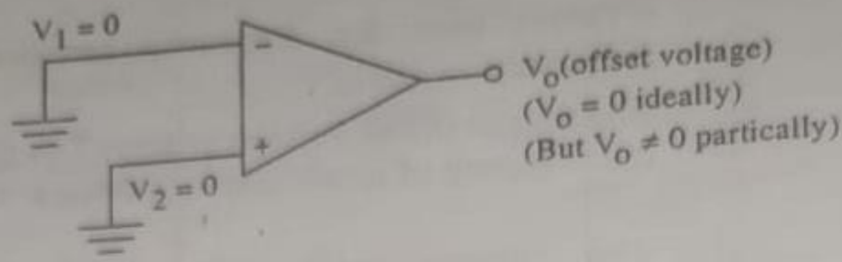


Fig. 7.6. **Output offset voltage**

The output offset voltage is the dc voltage (positive or negative) present at the output terminal when the two input terminals are grounded. Ideally, the offset voltage = 0. Practically, there exists a small output voltage even though both the inputs are grounded.

The polarity of the output offset voltage depends on the mismatching between the two input terminals. For example, for a 741 op-amp, V_{io} = 6 mV maximum means that the maximum potential difference between the two input terminals can be as large as 6 mV dc. This input offset voltage gives rise to an output offset voltage V_{oo}. Thus, we need to apply a differential input voltage of specific amplitude and correct polarity in order to reduce the output offset voltage V_{oo} to zero. This voltage is referred to as the input offset voltage V_{io}.

Input offset voltage

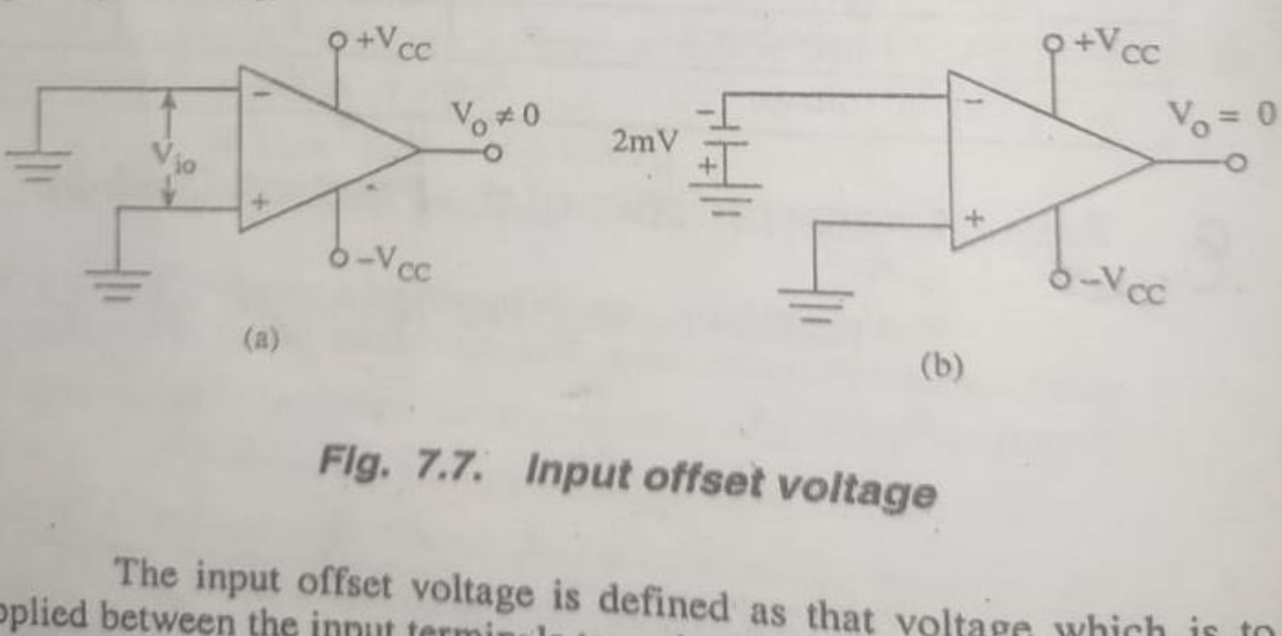


Fig. 7.7. **Input offset voltage**

The input offset voltage is defined as that voltage which is to be applied between the input terminals to make $V_o = 0$, i.e., to remove the offset voltage.

7.8.6 *Input bias current*

The input bias current (I_B) is defined as

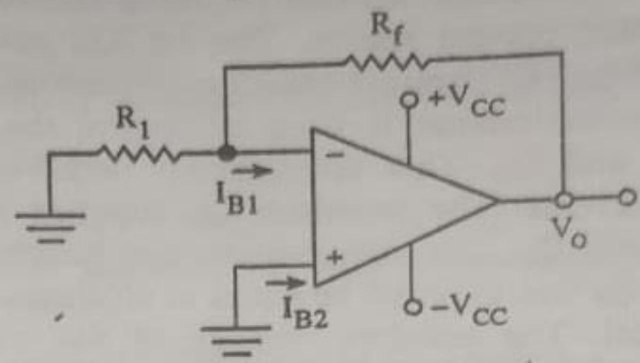


Fig. 7.8. Input bias current

$$I_B = \frac{I_{B1} + I_{B2}}{2}.$$

The input bias current is very very small (~ nA). Although the input bias current is very small, it may cause a significant output offset voltage.

7.8.7. *Input offset current*

The input offset current ($I_{inoffset}$) is defined as the algebraic difference between the two input bias currents I_{B1} and I_{B2},

i.e., $I_{in(offset)} = |I_{B2} - I_{B1}|$.

For the 741 op-amp, the maximum input offset current is 200 nA dc.

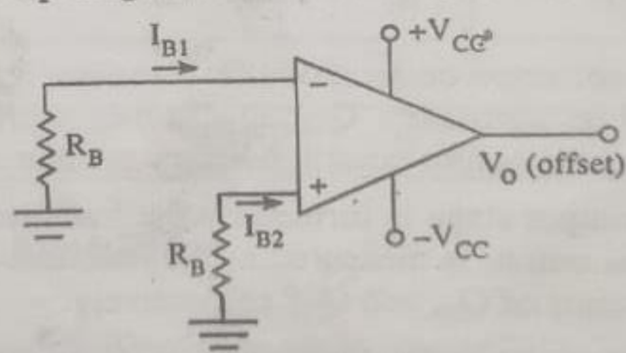


Fig. 7.9. Input offset current

7.8.8 *Slew rate (SR)*

The slew rate is defined as the maximum rate at which the output voltage can change, no matter how large an input signal is applied,

$$\text{i.e., } SR = \left.\frac{dV_{out}}{dt}\right|_{max}.$$

Inverting Amplifier

The inverting amplifier is shown in Figure 7.11.

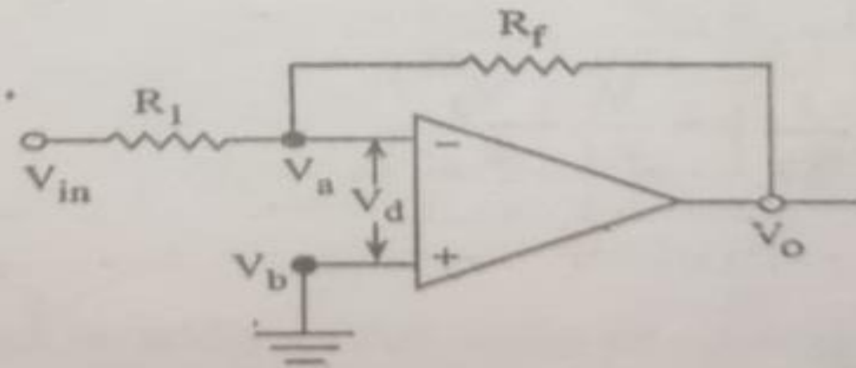


Fig. 7.11. Inverting amplifier

Write Kcl at 'a',

$$\frac{V_a - V_{in}}{R_1} + \frac{V_a - V_o}{R_f} = 0.$$

For an ideal op-amp, $V_a = 0$.

$$-\frac{V_{in}}{R_1} = \frac{V_o}{R_f};$$

$$\text{Then, } V_o = -\frac{V_f}{R_1} V_{in}.$$

For a non-ideal op-amp $V_a \neq 0$. Let the open-loop voltage gain of op-amp be A.

$$\text{Then, } A = -\frac{V_o}{V_d} \Rightarrow V_d = -\frac{V_o}{A}$$

or,

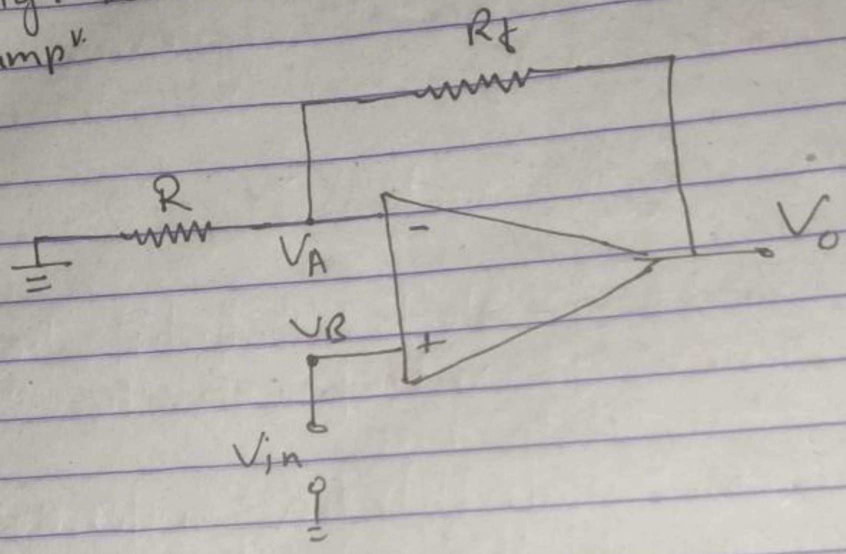
$$V_a - V_b = -\frac{V_o}{A}.$$

∴

$$V_a = -\frac{V_o}{A} \quad [\because V_b = 0].$$

(*) Non-Inverting Ampr.:-

⇒ fig. below shows the non-inverting ampr.



Applying KCL at A.

$$\frac{V_A}{R} + \frac{V_A - V_o}{R_f} = 0$$

or $$\frac{V_A}{R} + \frac{V_A}{R_f} - \frac{V_o}{R_f} = 0$$

or $$\frac{V_o}{R_f} = V_A\left(\frac{1}{R} + \frac{1}{R_f}\right)$$

or $$V_o = V_A . R_f\left(\frac{1}{R} + \frac{1}{R_f}\right)$$

$$V_o = V_A\left(\frac{R_f}{R} + \frac{R_f}{R_f}\right)$$

$$V_o = V_A\left(1 + \frac{R_f}{R}\right) \quad \text{---(i)}$$

At point B,

$$V_B = V_{in}$$

$$V_{in} = V_B = V_A \,. \quad (\because \text{Virtual ground})$$

Hence,

$$V_o = V_{in}\left(1 + \frac{R_f}{R}\right)$$

$$\frac{V_o}{V_{in}} = \left(1 + \frac{R_f}{R}\right) \quad \#$$

$$\frac{V_o}{V_{in}} = A = \text{closed loop voltage gain}$$

or $\Rightarrow$ overall voltage gain.

Summer or adder circuit

The adder or the summer circuit is shown in Figure 7.12.

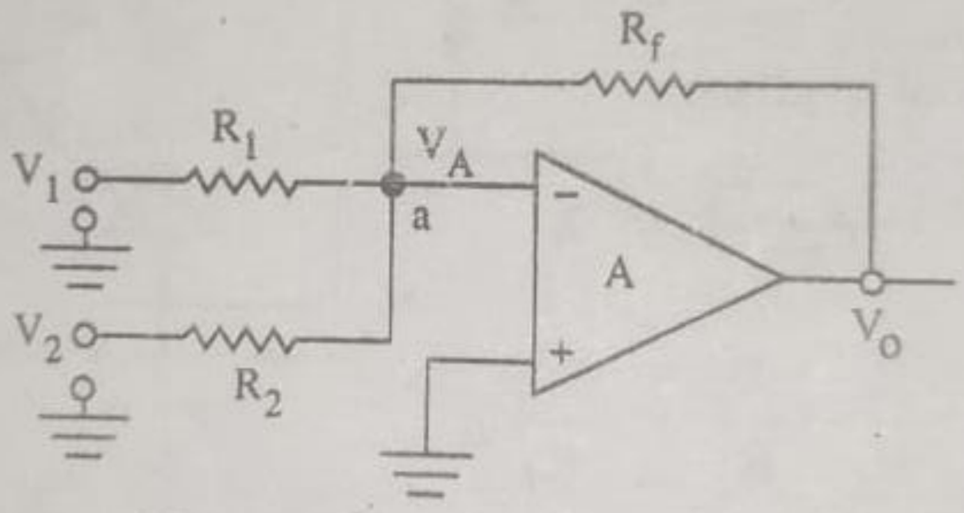


Fig. 7.12. Summer circuit

Writing Kcl at 'a', we get,

$$\frac{V_A - V_1}{R_1} + \frac{V_A - V_2}{R_2} + \frac{V_A - V_o}{R_f} = 0.$$

But, $V_A = 0$ [∵ Virtual ground]

$$-\frac{V_1}{R_1} - \frac{V_2}{R_2} = \frac{V_o}{R_f},$$

$$\therefore \quad V_o = -\left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2\right)$$

If $R_1 = R_2 = R_f$, then $V_o = -(V_1 + V_2)$.

Thus, the magnitude of the output voltage is the sum of the input voltages, and, hence, the circuit is called a summer or 'adder circuit. Since the output has a negative sign, it is called an inverting adder circuit.

7.13.2 *Non-inverting summing amplifier*

c.c

The non-inverting summing amplifier is shown in Figure 7.13.

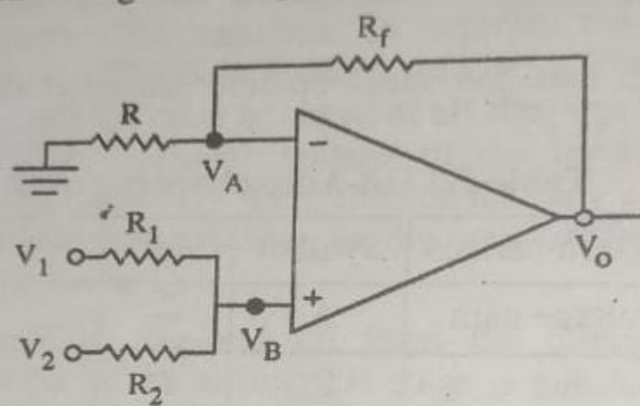


Fig. 7.13.

Writing Kcl at 'A', we get,

$$\frac{V_A}{R} + \frac{V_A - V_o}{R_f} = 0$$

or, $$V_A\left(\frac{1}{R} + \frac{1}{R_f}\right) = \frac{V_o}{R_f}$$

or, $$V_A\left(\frac{R_f + R}{R_f \times R}\right) = \frac{V_o}{R_f}$$

or, $$V_A = \frac{R}{(R_f + R)}V_o.$$

Similarly, with Kcl at 'B', we get,

$$\frac{V_B - V_1}{R_1} + \frac{V_B - V_2}{R_2} = 0$$

or, $$V_B\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V_1}{R_1} + \frac{V_2}{R_2}$$

or, $$V_B\left(\frac{R_2+R_1}{R_1R_2}\right) = \frac{V_1}{R_1}+\frac{V_2}{R_2}$$

or, $$V_B = \frac{R_2}{(R_2+R_1)}V_1 + \frac{R_1}{(R_2+R_1)}V_2.$$

But $V_A = V_B$,

or, $$\frac{R}{(R+R_f)}V_o = \frac{R_2}{(R_2+R_1)}V_1 + \frac{R_2}{(R_2+R_1)}V_2.$$

$$\therefore \quad V_o = \left(\frac{R+R_f}{R}\right)\times\frac{R_2}{(R_2+R_1)}V_1 + \left(\frac{R+R_f}{R}\right)\times\frac{R_1}{(R_2-R_1)}V_2.$$

If $R_1 = R_2 = R = R_f$,

then, $V_o = V_1 + V_2$.

Since there is no phase difference between input and output, it is called a non-inverting summer amplifier.

Example 7.1.

Determine the output voltage for the configuration shown in Figure 7.14.

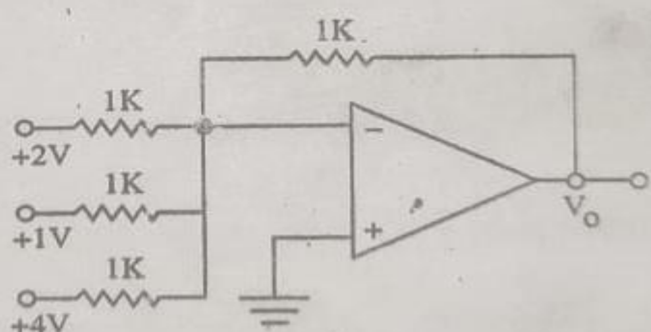


Fig. 7.14. Inverting summer circuit

Solution:

For the inverting summer circuit, we can write

$$V_o = -\left(\frac{R_f}{R_1}V_1+\frac{R_f}{R_2}V_2+\frac{R_f}{R_3}V_3\right)$$

$$\therefore \quad V_o = -\left(\frac{1}{1}\times2+\frac{1}{1}\times1+\frac{1}{1}\times4\right) = -7V$$

Example 7.2.

Design an adder circuit using an op-amp to get the output expression as $V_o = -(V_1 + 10V_2 + 100V_3)$ where V_1, V_2 and V_3 are the inputs.

Assume $R_f = 100\ k\Omega$.

Solution:

The adder circuit is shown in Figure 7.15.

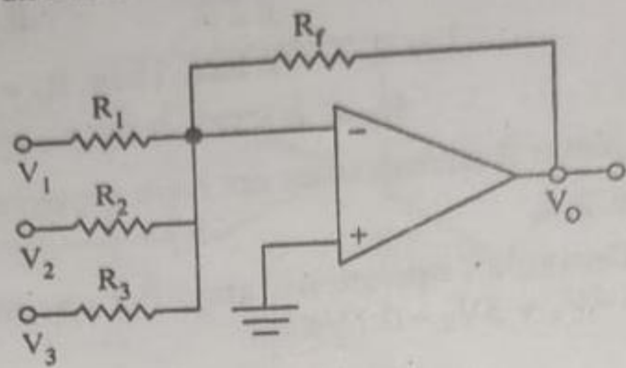


Fig. 7.15.

We know that $V_o = \left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3\right)$.

Comparing this expression with the given expression

$V_o = -(V_1 + 10V_2 + 100V_3)$, we get,

$$\frac{R_f}{R_1} = 1,\ \frac{R_f}{R_2} = 10 \text{ and } \frac{R_f}{R_3} = 100.$$

Since $R_f = 100\ K\Omega$, $R_1 = 100\ K\Omega$, $R_2 = 10\ K\Omega$ and $R_3 = 1\ K\Omega$.

Example 7.3.

Design a summing amplifier to get the output voltage $V_o = -V_1 + 2V_2 - 3V_3$ where V_1, V_2 and V_3 are the inputs.

Solution:

The summing amplifier is shown in Figure 7.16.

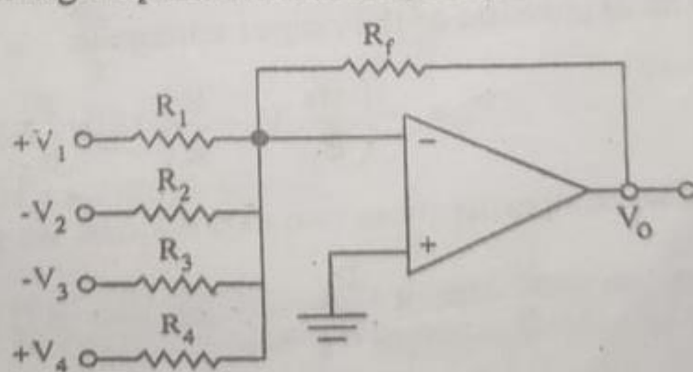


Fig. 7.16. Summing amplifier circuit

For the summing amplifier circuit, we have

7.15. Integrator

The integrator circuit is shown in Figure 7.22.

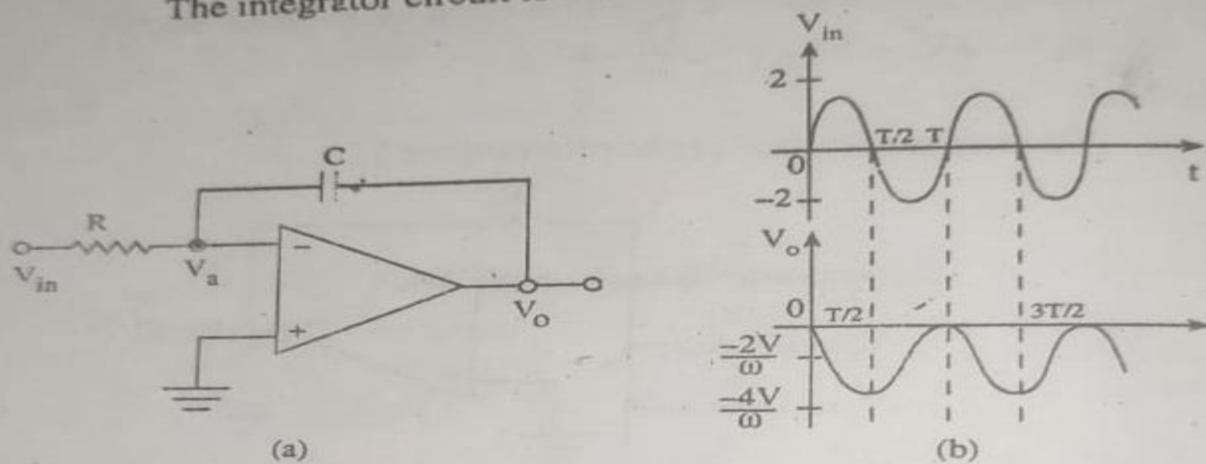


Fig. 7.22. (a) Integrator (b) Input/output waveforms

With Kcl at 'a', we get

$$\frac{V_a - V_{in}}{R} + \frac{V_a - V_o}{\left(\frac{1}{CS}\right)} = 0.$$

$$\text{But, } V_a = 0.$$

$$\therefore \quad -\frac{V_{in}}{R} + CS\,(-V_o) = 0.$$

or,

$$CSV_o = -\frac{1}{R} V_{in}.$$

$$\therefore \quad V_o = -\frac{1}{RCS} V_{in}.$$

In the time domain,

$$V_o(t) = -\frac{1}{RC}\int V_{in}\,dt + A.$$

Case 1. Sine input:

where A is the integration constant.

$$\text{If } V_{in} = 2\sin\omega t,$$

$$V_o(t) = -\frac{1}{RC}\int 2\sin\omega t + A$$

$$V_o(t) = -\frac{2}{RC}\frac{(\cos\omega t)}{\omega} + A.$$

The output is a cosine function for the sine input as shown in Figure 7.22 (b).

7.16. Differentiator

The differentiator circuit is shown in Figure 7.25.

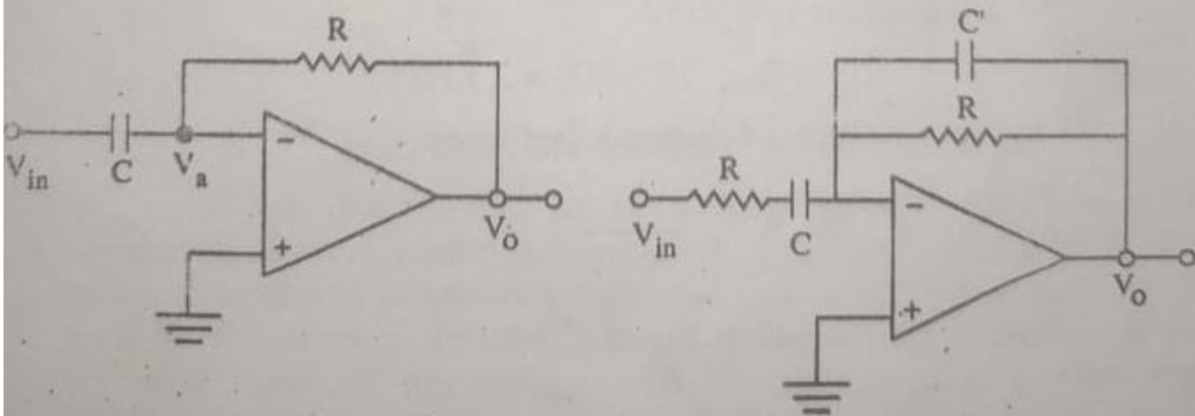


Fig. 7.25. (a) Differentiator (b) Practical differentiator

With Kcl at 'a', we get

$$\frac{V_a - V_{in}}{\left(\frac{1}{CS}\right)} + \frac{V_a - V_o}{R} = 0.$$

$\therefore$

$$CS(-V_{in}) = \frac{V_o}{R} \quad [\because V_a = 0],$$

or,

$$V_o = -RCSV_{in}$$

In the time domain, $V_o(t) = -RC\frac{d(V_{in})}{dt}$

The practical op-amp differentiator is shown in Figure 7.24.

Example 7.9.

If V_{in} is a sinusoidal voltage of a peak value of 10 mV and f = 2 KHz, find V_o. Assume R = 50 KΩ, C = 2 μF.

Solution:

The practical differentiator circuit is shown in Figure 7.26.

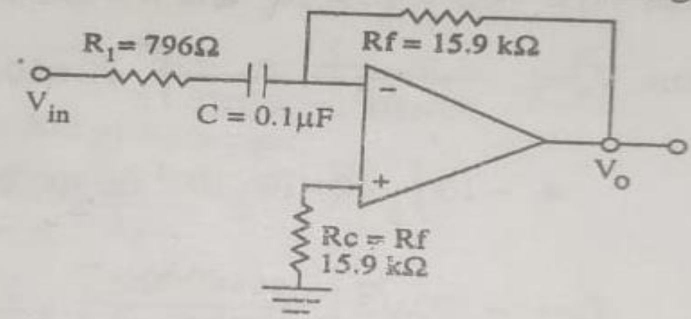


Fig. 7.26. Practical differentiator

We know that $V_o = -50 \times 10^3 \times 2 \times 10^{-6} \frac{d}{dt}[10^{-3} \sin 2\pi \times 2000t]$

$$= 0.1 \times 10^{-3}(\cos 4000\pi t) \times 4000\pi.$$

$\therefore$ $V_o = 12.56 \cos 4000\pi t$ mV.

Let's choose $f_b = 10\, f_h$.

Then, $f_b = 10 \times 200 = 2$ KHz

where f_b is break frequency.

But, $f_b = \frac{1}{2\pi R_1 C}$.

Let C = 0.1 μF. Then,

$$2 \times 10^3 = \frac{1}{2\pi \times 0.9 \times 10^{-6} R_1}.$$

$\therefore$ $R_1 = 796\ \Omega.$

Again, we know that $\frac{|V_o|}{|V_i|} = \omega R_f C = 2\pi f R_f C,$

i.e., $0.1 = 2\pi \times 10 \times R_f \times 0.1 \times 10^{-6}.$

$\therefore$ $R_f = 15.9$ KΩ.

The practical differentiator is shown in the Figure 7.26.

Example 7.10.

Design a practical integrator that

(i) Integrates signals with frequencies down to 100 Hz.

(ii) Produces a peak output of 0.1 V when the input is a 10 V peak sine wave having a frequency 10 KHz.

(iii) Find the dc component in the output when there is a + 50 mV dc input.

Solution:

In order to integrate frequencies down to 100 Hz, we require fc << 100 Hz. Let us choose $f_c = 10$ Hz, where f_c is called the cut-off frequency.

Then, we know that

$$f_c = 10 = \frac{1}{2\pi R_f C}$$

Suppose C = 0.01 μF. Then,

$$10 \times 10^3 \times 2\pi \times 0.01 \times 10^{-6} = \frac{1}{R_f}.$$

$$R_f = 1.59\ M\Omega.$$

The gain at 10 KH is

$$\frac{|V_o|}{|V_{in}|} = \frac{0.1}{10} = 0.01.$$

Also, we know that,

$$\frac{|V_o|}{|V_{in}|} = \frac{1}{\omega R_1 C}$$

$$\text{i.e., } 0.01 = \frac{1}{2\pi\times10\times10^3\times0.01\times10^{-6}\times R_1}.$$

$$\therefore \quad R_1 = 159\ K\Omega.$$

$$\text{Now, } R_C = R_f\,||\,R_1 = 1.59\ M\Omega\,||\,159\ K\Omega = 145\ K\Omega.$$

The practical integrator circuit is shown in the Figure 7.27.

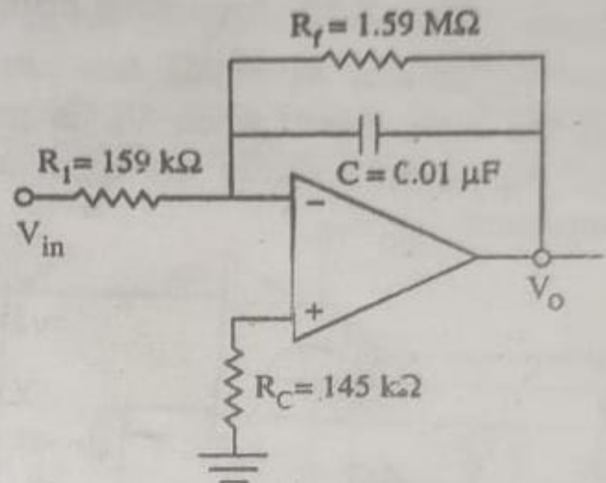


Fig. 7.27. Practical differentiator

When the input is 50 mV dc, the output is

(iii) $$V_o = \frac{-R_f}{R_1}\times 50\ mV = \frac{-1.59\times10^6}{159\times10^3}\times 50\ mV.$$

$$\therefore \quad V_o = -0.5\ V.$$

Example 7.11.

Find the peak value of the output of the ideal integrator where $R_1 = 100\ K\Omega$, $C = 0.01\ \mu F$, $Rc = 100\ K\Omega$ and $Vin = 0.5\sin(100t)$ V.

(ii) Repeat, when $V_i = 0.5\sin(10^3 t)$ V.

Solution:

The ideal integrator is shown in Figure 7.28.

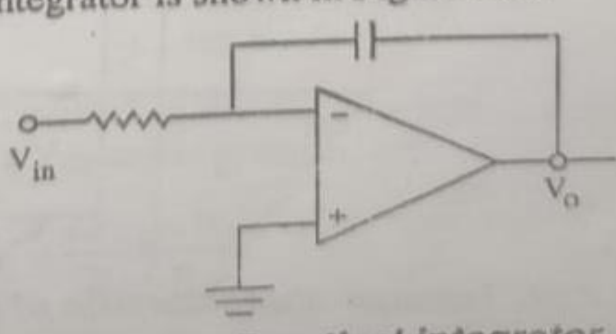


Fig. 7.28. Practical integrator

We know that $V_o = -\frac{1}{R_1C}\int(0.5\sin 100t)\,dt$

$$= \frac{-0.5}{R_1C\times 100}[-\cos 100t]_0^t$$

$$= \frac{+0.5}{100\times 10^3\times 0.01\times 10^{-6}\times 100}\cos 100t$$

$$= 5\cos 100\,t\ V.$$

$\therefore$ The peak voltage $= 5$ V.

(ii) If $V_i = 0.5\sin 10^3 t$,

then, $V_o = \frac{0.5}{10^3\times 100\times 10^3\times 0.01\times 10^{-6}}\cos 10^3 t$

or, $V_o = 0.5\cos 10^3 t.$

$\therefore$ The peak value $= 0.5$ V.

7.19. Multivibrators using Op-Amp

The multivibrators are very important regenerative circuits which are used commonly in timing applications.

The multivibrators are classified as:

(i) Monostable multivibrators, and,

(ii) Astable multivibrators.

(i) Monostable Multivibrator

The monostable multivibrator is also known as one-shot multivibrator. The circuit produces a single pulse of specified duration in response to each external trigger signal. For such a circuit, only one control-stable state exists. When an external trigger is voltage applied, the output changes its state. The new state is called the quasi-stable state. The circuit remains in this state for a fixed interval of time. After some time, it returns back to its original stable state.

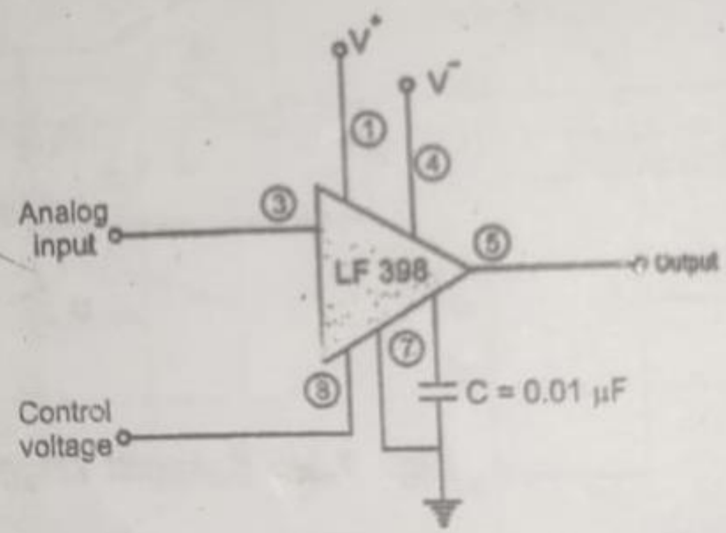


Fig. 7.38. Connection diagram of LF 398

(ii) Astable Muitivibrator

The astable mulivibrator is also called a free running multivibrator. It has two quasi-stable states, i.e., no stable state as such. The circuit's conditions oscillate between these two quasi-stable states. No external signal is required to produce changes in the state. The component values used decide the time for which the circuit remains in each state. Usually, since the astable multivibrator oscillates between the states, it is used to produce square wave.

4.4 Oscillator and [redacted]: Barkhausen Criteria

An oscillator is basically an amplifier with positive feedback where the feedback factor 'β' must be slightly greater than unity (Theoretically, the feedback factor or loop gain $A \cdot \beta = 1$)

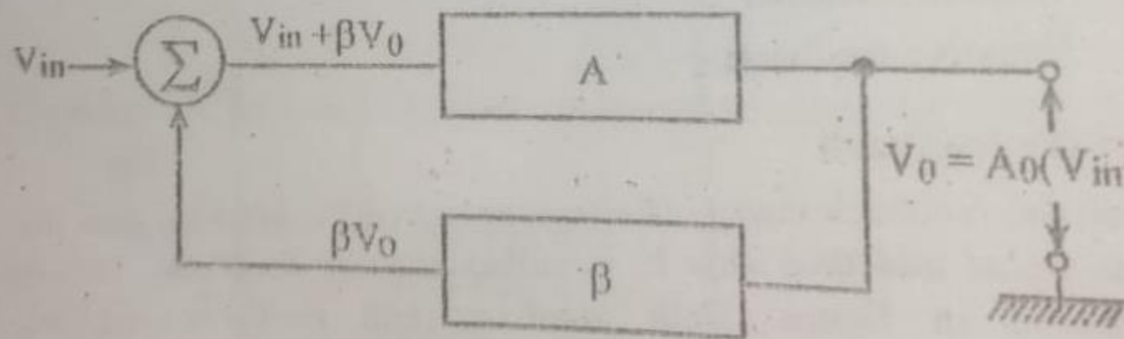


Fig.: Block diagram of oscillator.

From the circuit diagram, we have

$$V_0 = A_0 (V_{in} + \beta V_0)$$

or, $V_0 (1 - A_0\beta) = A_0 V_{in}$

or, $\dfrac{V_0}{V_{in}} = \dfrac{A_0}{1 - A_0\beta}$ = overall voltage gain of the system = A

$$\boxed{A = \frac{A_0}{1 - A_0\beta}} \quad \text{..............(1)}$$

$$\therefore \quad \boxed{V_0 = V_{in} A} \quad \text{..............(2)}$$

When $A_0\beta \rightarrow 1$, $A \rightarrow \infty$ and as a result $V_{in} \rightarrow 0$ for finite output voltage V_0. That means, output V_0 is produced. Without feeding any input, V_{in}. And it is called **Oscillator**. Thus to become an oscillator following two criteria, called **Barkhausen criteria** must be fulfilled.

The essential conditions for oscillations are:

- βA, i.e., loop gain, must be unity.
- The total phase shift (amplifier and feedback circuit) must be 0° or 360°. These conditions for oscillations are also called Barkhausen criteria for oscillation.

RC Phase Shift Oscillator:

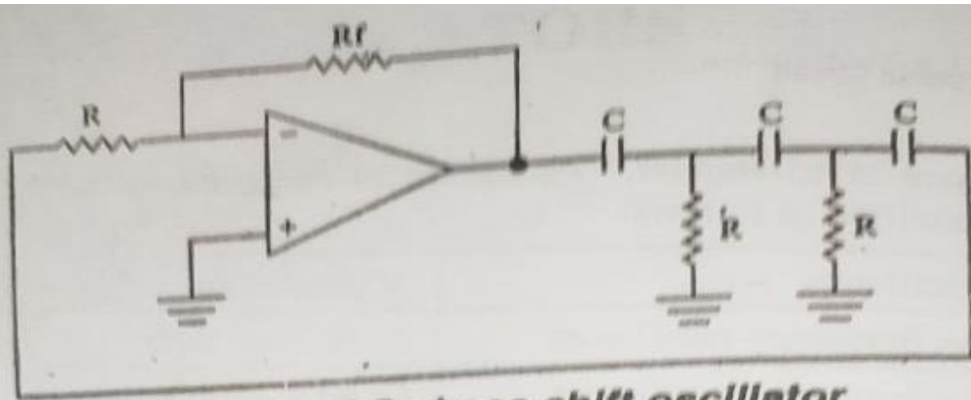


Fig. 5.4. ***RC phase shift oscillator***

The inverting operational amplifier provides a 180° phase shift in the signal passing through it.

When the signal passes through three RC sections, the cumulative phase shift is 180° at a certain frequency. Thus, the total phase shift becomes 360°, which is one condition of oscillation. Another condition of oscillation is that the loop gain must be unity.

Now, feedback ratio (β) can be expressed as

$$\beta = \frac{R^3}{(R^3 - 5RX_C^2) + j(X_C^3 - 6R^2X_C)}.$$

For oscillation, the angle of β must be 180°, i.e., β must be a pure real number;

i.e., $X_C^3 - 6R^2X_C = 0$

or, $$X_C^2 = 6R^2$$

or, $$\left(\frac{1}{\omega c}\right)^2 = 6R^2$$

or, $$(2\pi fc)^2 = 6R^2$$

or, $$f^2 = \frac{1}{(2\pi RC)^2 \times 6}.$$

$\therefore$ $$f = \frac{1}{2\pi RC \times \sqrt{6}}.$$

Now, $$\beta = \frac{R^3}{(R^3 - 5RX_C^2)} = \frac{R^3}{(R^3 - 5R \times 6R^2)}$$

i.e., $$\beta = -\frac{1}{29}.$$

Since $|\beta A|$ must be unity, A should be -29.

$$\text{But } A = \frac{V_o}{V_i} = -\frac{R_f}{R},$$

$$\text{i.e., } \frac{R_f}{R} = 29,$$

$$\text{i.e., } R_f = 29\,R.$$

Example 5.2

Design a RC phase shift oscillator that will oscillate at 100 Hz.

Solution:

$$\text{We know that } f = \frac{1}{2\pi RC\sqrt{6}}.$$

$$\text{Let } C = 0.5\ \mu F;$$

$$\text{then, } 100 = \frac{1}{2\pi R \times 0.5 \times 10^{-6} \times \sqrt{6}}.$$

$$\Rightarrow \quad R = 1300\ \Omega.$$

Again, to satisfy $|\beta A| = 1$, A must be 29,

$$\text{i.e., } \frac{R_f}{R} = 29.$$

$$\Rightarrow \quad R_f = 29\,R = 29 \times 1300.$$

$$\therefore \quad R_f = 37.7\ K\Omega.$$

Thus R-C phase shift oscillator is shown in the Figure 5.5.

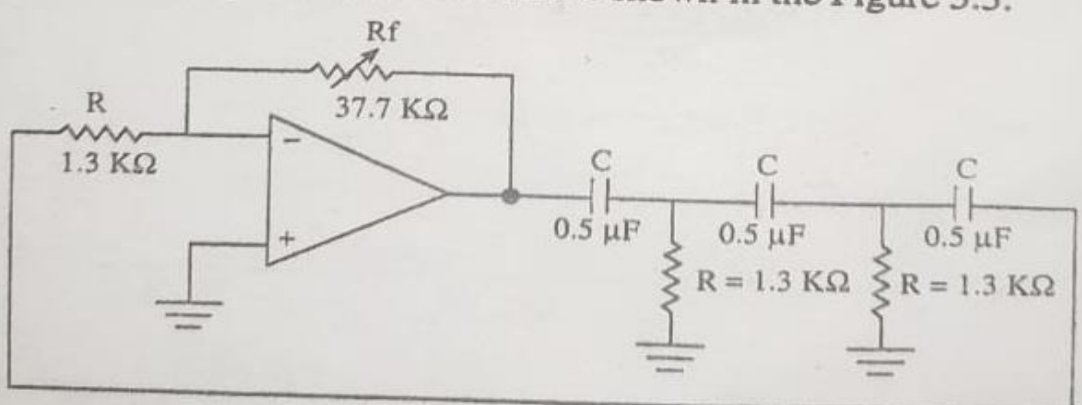


Fig. 5.5. R-C phase shift oscillator

R_f is an adjustable resistor so that the loop gain can be set precisely to 1 (unity).

5.7 Wien-Bridge Oscillator

The Wien bridge oscillator circuit is shown in Figure 5.6.

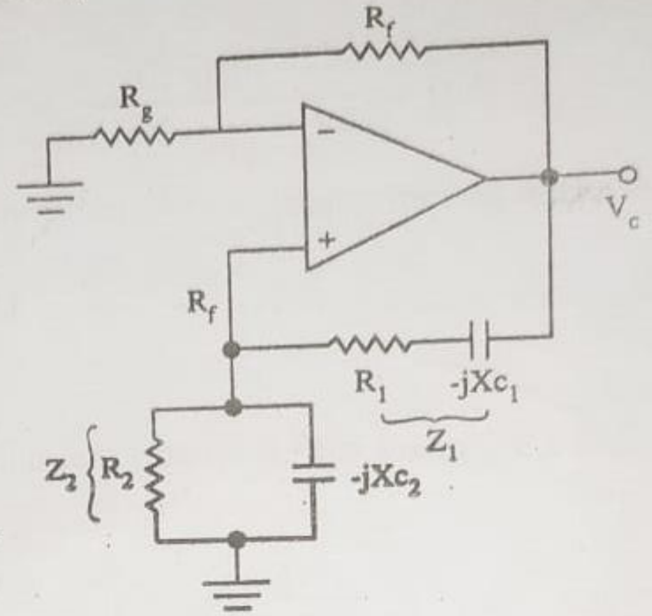


Fig. 5.6. Wien-bridge oscillator

Here, the op-Amp is used in the non-inverting configuration. So, it gives a 0° phase shift. If R_1, C_1, R_2 and C_2 are chosen properly, there will be no phase shift in the signal feedback to the amplifier at a certain frequency. So, the total phase shift around the loop will become zero. Similarly, the loop gain
$|\beta A| = 1$ for properly chosen impedances.

Now, $V_f = V+ = \beta V_o$.

Then $\beta = \dfrac{V_f}{V_o}$,

where $\beta = \dfrac{Z_2}{(Z_1 + Z_2)} = \dfrac{-jXc_2 \parallel R_2}{(R_1 - jXc_1) + (R_2 \parallel -jXc_2)}$,

or,
$$\beta = \frac{\dfrac{-jXc_2R_2}{R_2 - jXc_2}}{(R_1 - jXc_1) + \dfrac{(-jXc_2 \times R_2)}{(R_2 - jXc_2)}}$$

or,
$$\beta = \frac{-jR_2Xc_2}{(R_1 - jXc_1)(R_2 - jXc_2) - jR_2Xc_2}$$

or,
$$\beta = \frac{-jR_2Xc_2}{(R_1R_2 - jR_1Xc_1 - jR_2Xc_1 + j^2Xc_2Xc_2 - jR_2Xc_2)}.$$

Let $R_1 = R_2 = R$ and $C_1 = C_2 = C$.

Then, $\beta = \dfrac{-jRXc}{(R^2 - 3jRXc - Xc^2)}$ [Since $j^2 = -1$],

or, $$\beta = \frac{-jRXc}{j(R^2 - Xc^2) + 3RXc} = \frac{RXc}{3RXc + j(R^2 - Xc^2)}.$$

Here, β must be a pure real number for a zero phase shift,

i.e., $R^2 - Xc^2 = 0$

or, $$R^2 = Xc^2 \Rightarrow \frac{1}{\omega c} = R$$

or, $$\omega cR = 1$$

or, $$2\pi fRC = 1.$$

$\therefore$ $$f = \frac{1}{2\pi RC}.$$

$$\text{Now, } \beta = \frac{RXC}{3RXC} = \frac{1}{3}.$$

To satisfy $|\beta A| = 1$, A must be 3.

But the amplifier gain, $A = \left(1 + \dfrac{R_f}{R_g}\right)$.

$\therefore$ $$1 + \frac{R_f}{R_g} = 3.$$

$\therefore$ $$R_f = 2Rg.$$

Example 5.3

Design a Wein bridge oscillator that will oscillate at 25 KHz.

Solution:

We know that $f = \dfrac{1}{2\pi RC}$

or, $$25 \times 10^3 = \frac{1}{2\pi RC}.$$

Let $C = 0.001\ \mu F$.

Then, $R = 6366\ \Omega$.

Again, $\frac{R_f}{R_g} = 2$. Let $R_g = 10\ K\Omega$. Then, $R_f = 20\ K\Omega$.

The Wein bridge oscillator circuit is shown in figure 5.7.

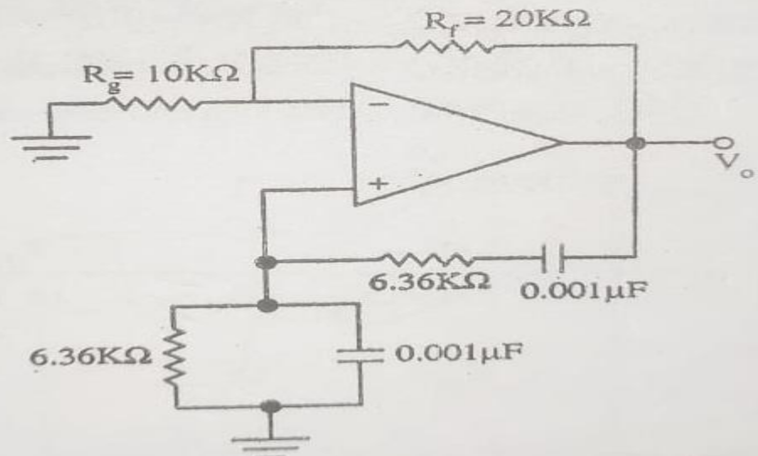


Fig. 5.7. Wein-bridge oscillator

5.8.3 *Crystal Oscillator*

In the crystal oscillators, the resonant circuit is replaced by mechanically vibrating crystals. The crystals (quartz) have a high degree of frequency stability.

A quartz crystal exhibits a very important property known as piezoelectric effect.

Piezoelectric effect

When a mechanical pressure is applied across a faces of the crystal, a voltage proportional to the mechanical pressure is appears across the crystal.

Conversely, when a voltage is applied across the crystal, the crystal is distorted by an amount proportional to the applied voltage. An alternating voltage applied to a crystal causes it to vibrate at its natural frequency.

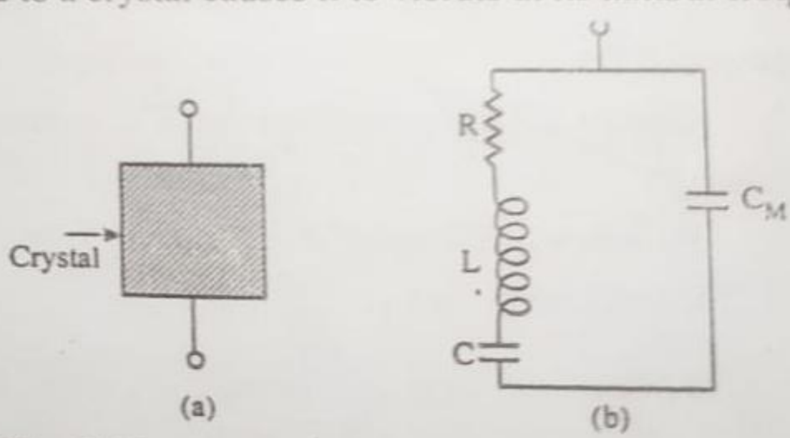


Fig. 5.12. (a) Symbol (b) Equivalent circuit

The two resonant frequencies are series resonance (f_s) and shunt resonance (f_p) frequencies.

$$f_s = \frac{1}{2\pi\sqrt{LC}} \text{ and}$$

$$f_p = \left(\frac{1}{2\pi\sqrt{\frac{LC}{1+\frac{C}{CM}}}} \right).$$

Now, for the series resonance (f_s)

$$X_L = X_C$$

$$\text{i.e. } \omega L = \frac{1}{\omega c}$$

or, $$\omega^2 LC = 1$$

or, $$\omega^2 = \frac{1}{LC}.$$

$\therefore$ $$\omega = \frac{1}{\sqrt{LC}},$$

or, $$f_s = \frac{1}{2\pi\sqrt{LC}}$$

For the shunt or parallel resonance frequency (f_p)

$$(X_L - X_C) = X_{CM}$$

or, $$\omega L - \frac{1}{\omega c} = \frac{1}{\omega c_M}$$

or, $$\frac{\omega^2 LC - 1}{\omega c} = \frac{1}{\omega c_M}$$

or, $$(\omega^2 LC - 1)C_M = C$$

or, $$\omega^2 LC.C_M = C + C_M$$

or, $$\omega^2 = \frac{C+C_M}{LC.C_M} = \frac{C}{LC.C_M} + \frac{C_M}{LC.C_M}$$

$$= \frac{1}{LC_M} + \frac{1}{LC} = \frac{C+C_M}{LC.C_M}$$

$$= \frac{1}{LC}\left[\frac{C+C_M}{C_M}\right]$$

$$= \frac{1}{LC}\left[1+\frac{C}{C_M}\right].$$

$$\therefore \qquad \omega_p = \sqrt{\frac{1}{LC}\left(1+\frac{C}{C_M}\right)}$$

or,

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{1+C/C_M}{LC}\right)}.$$

The impedance versus frequency plot is shown in Figure 5.13.

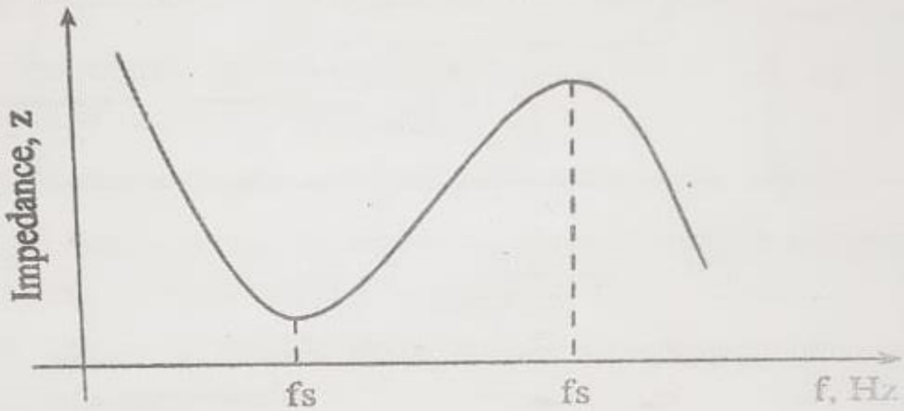


Fig. 5.13. Crystal impedance versus frequency plot

The oscillator with a crystal operating in series resonance and parallel resonance are shown in Figures 5.14 (a) and (b) respectively.

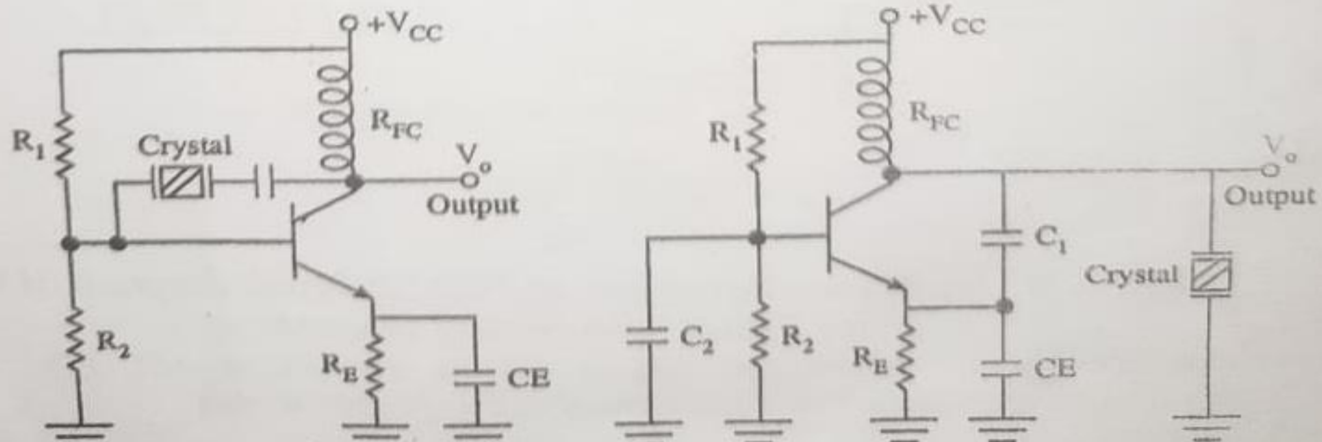


Fig. 5.14. (a) Series resonance (b) Parallel resonance

The parameters of a crystal oscillator equivalent circuit are $L_S = 0.8$ H, $C_S = 0.08$ pF, $R_S = 5$ KΩ, $C_P = 1$ pF. Calculate the series and the parallel resonance frequencies.

Solution:

For series resonance,

$$X_L - X_{CS} = 0$$

or, $$X_L = X_{CS}$$

or, $$\omega L = \frac{1}{\omega c_s}$$

or, $$\omega^2 = \frac{1}{\sqrt{LC_S}}$$

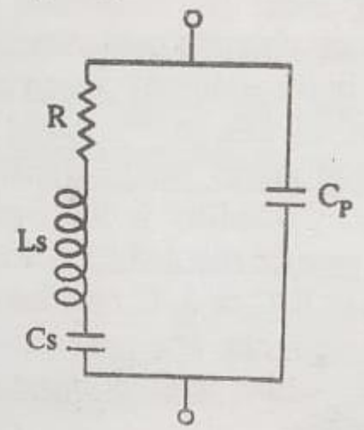


Fig. 5.15. Equivalent circuit of crystal oscillator

or, $$f_s = \frac{1}{2\pi\sqrt{LC_S}},$$

i.e., $$f_s = \frac{1}{2\pi\sqrt{0.8\times0.08\times10^{-12}}}.$$

∴ $$f_s = 629 \text{ KHz.}$$

For parallel resonance,

$$(X_L - X_{CS}) = X_{CP}$$

or, $$\omega L - \frac{1}{\omega c_s} = \frac{1}{\omega c_p}$$

or, $$\frac{\omega^2 LC_S - 1}{\omega c_S} = \frac{1}{\omega c_p}$$

or, $$\omega^2 LC_S = \frac{C_S}{C_P} + 1$$

or, $$\omega^2 = \frac{1}{LC_S}\left(1+\frac{C_S}{C_P}\right)$$

or, $$\omega = \sqrt{\frac{1+\frac{C_S}{C_P}}{LC_S}}.$$

∴ $$f_p = \frac{1}{2\pi}\frac{1+\frac{C_S}{C_P}}{LC_S} = \frac{1}{2\pi}\sqrt{\frac{1+\frac{0.08\times10^{-12}}{1\times10^{-12}}}{0.8\times0.08\times10^{-12}}},$$

or, $$f_p = 654 \text{ KHz}$$

Extra

7.20. Monostable Multiuibrator using an Op-Amp

The monostable multivibrator circuit using an op-amp is shown in Figure 7.39.

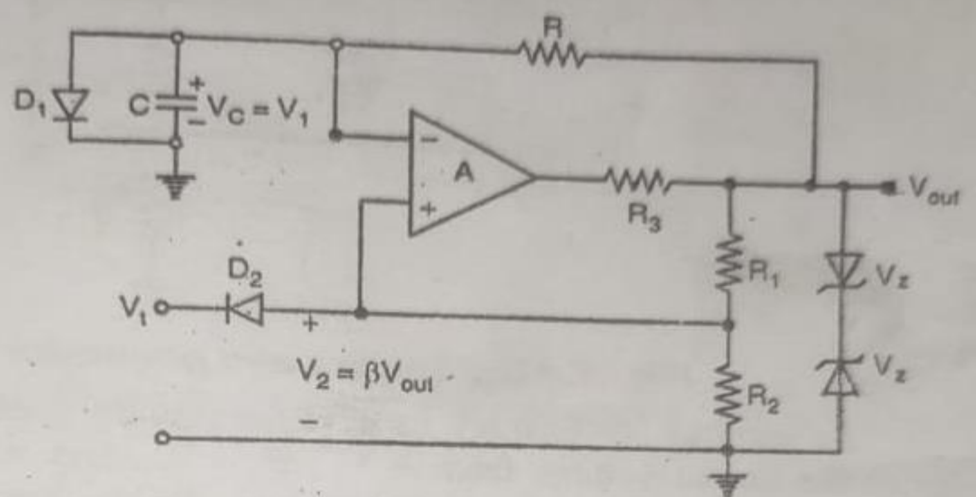


Fig. 7.39. Op-amp Monostable Multivibrator

The diode D_1 is a clamping diode connected across C. The diode clamps the capacitor voltage to 0.7 V when the output is at $+V_{sat}$. A narrow negative triggering pulse V_t is applied to the non-inverting input terminal through diode D_2.

To understand the operation of the circuit, let us assume that the output V_{out} is at $+ V_{sat}$, i.e., in its stable state. The diode D_1 conducts and the voltage across the capacitor C, i.e., V_C, gets clamped to 0.7 V. The voltage at the non-inverting input terminal is controlled by the potentiometric dividers of R_1 R_2 to βV_{out}, i.e., $+ \beta V_{sat}$, in the stable state.

Now, if V_t, a negative trigger of amplitude V_t, is applied to the non-inverting terminal so that the effective voltage at this terminal is less than 0.7 V $(+ (\beta V_{sat} + (-V_t))$, then the output of the op-amp changes its state from $+ V_{sat}$ to $-V_{sat}$.

The diode is now reverse-biased and the capacitor starts charging exponentially to $-V_{sat}$ through the resistance R. The time constant of this charging is $\tau = RC$.

The voltage at the non-inverting input terminal is now $-\beta V_{sat}$. When the capacitor voltage V_C becomes just slightly more negative than $-\beta V_{sat}$, the output of the op-amp changes its state back to $+V_{sat}$.

The capacitor now starts charging towards $+ V_{sat}$ through R until V_C reaches 0.7 V as the capacitor gets clamped to the voltage.

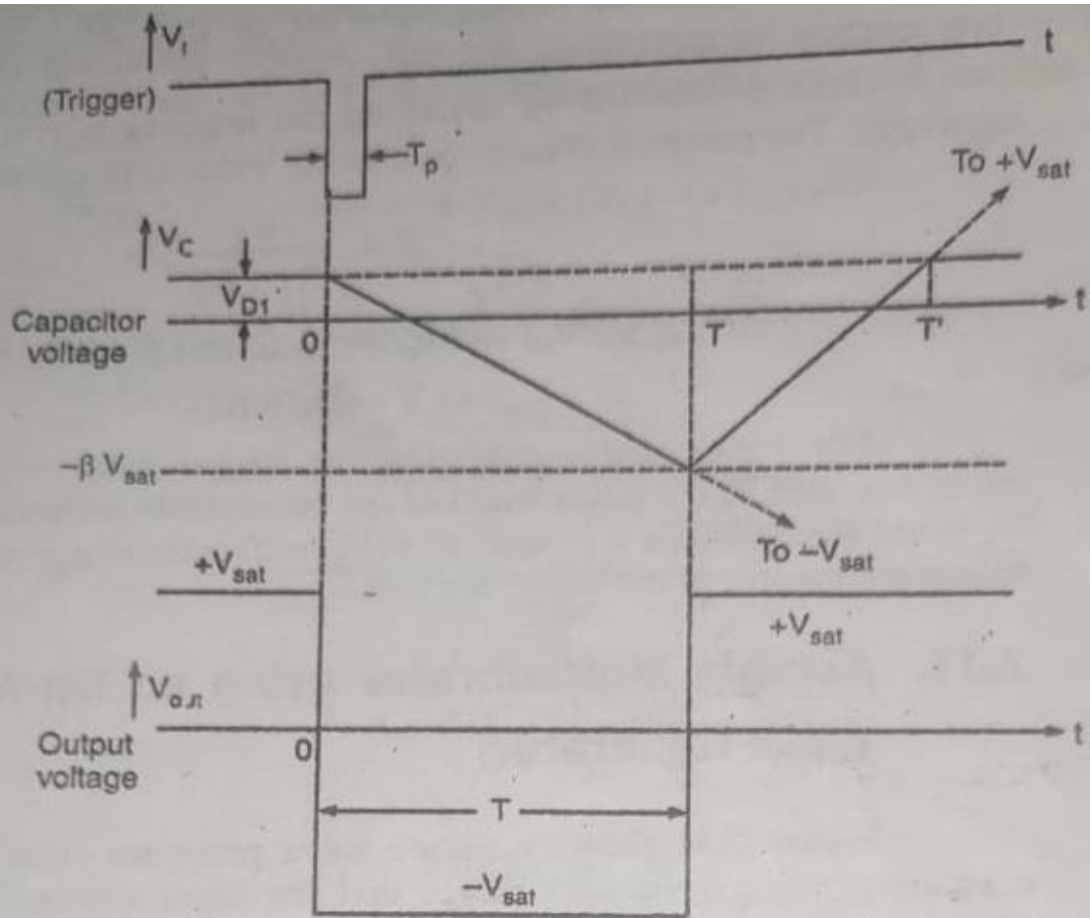


Fig. 7.40. Waveform of monostable multivibrator

7.20.1. Derivation of Expression for Pulse Width T

For a low pass RC circuit, let us assume

$$V_{in} = \text{initial voltage, and}$$

$$V_f = \text{final voltage.}$$

Then, the general solution is given by

$$V_{out} = V_f + (V_{in} - V_f)\, e^{-t/RC}.$$

Now, for the monostable multivibrator discussed above, the values of V_f and V_{in} are, $V_f = -V_{sat}$ and $V_{in} = V_{D1}$, (diode forward voltage).

while $V_{out} =$ output = capacitor voltage V_C.

$$\therefore \quad V_C = -V_{sat} + (V_{D1} - [-V_{sat}])\, e^{-t/RC}.$$

At $t = T$,

$$V_C = -\beta V_{sat}.$$

$$\therefore \quad -\beta V_{sat} = V_{sat} + (V_{DI} + V_{sat})\, e^{-T/RC}.$$

$$\therefore \quad (V_{DI} + V_{sat})e^{-T/RC} = -V_{sat}(1 - \beta).$$

or, $$e^{-T/RC} = \frac{V_{sat}(1-\beta)}{(V_{D1}+V_{sat})}.$$

$$\text{Then, } T = RC \ln\left[\frac{1+V_{D1}/V_{sat}}{1-\beta}\right]$$

This is obtained by absorbing the negative sign inside the natural logarithm. The potential divider decides the value of β, given by,

$$\beta = \frac{R_2}{R_1+R_2}.$$

If $V_{sat} >> V_{D1}$ and $R_1 = R_2$ so that $\beta = 0.5$, then

$$T = 0.69\ RC.$$

For a monostable operation, the trigger pulse width T_p should be much less than T, the pulse width of the monostable multivibrator. The diode D_2 is not essential but it is used to void malfunctioning if any positive noise spikes are present in the triggering line.

7.21. Astable Multivibrator using an Op-Amp (Square Wave Generator)

Figure 7.41 shows a square wave generator circuit. It looks like a comparator with hysteresis except that the input voltage is replaced by a capacitor. The circuit has time dependent elements such as resistance and capacitor to set the frequency of oscillation.

As shown in Figure 7.41, the comparator and positive feedback resistors R_1 and R_2 form an inverting Schmitt trigger. When V_{out} is at $+V_{sat}$, the feedback voltage is called the upper threshold voltage V_{UT} and is given as

$$V_{UT} = \frac{R_1(+V_{sat})}{R_1+R_2}.$$

When V_{out} is at $(+V_{sat})$, the feedback voltage is called the lower-threshold voltage V_{UT} and is given as

$$V_{LT} = \frac{R_1(-V_{sat})}{R_1+R_2}.$$

to the inverting (–) input. This is illustrated in Figure 7.41. As long as the capacitor voltage VC is less than V_{UT}, the output voltage remains at $+V_{sat}$.

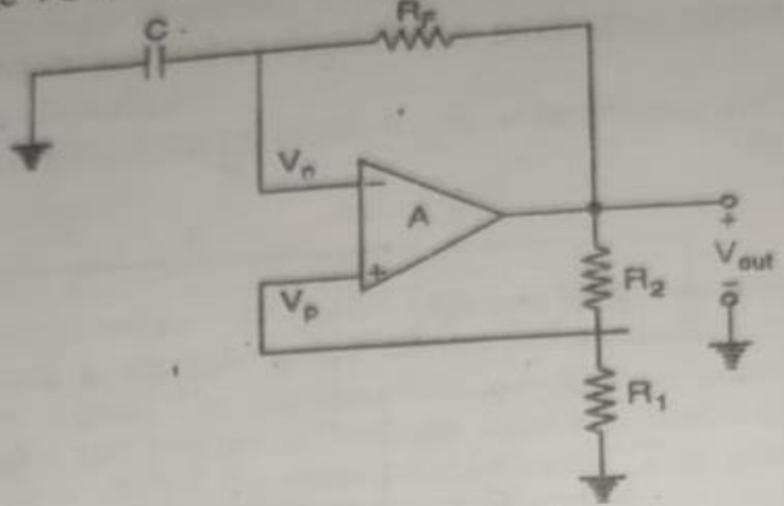


Fig. 7.41. Square wave generator

As soon as V_C charges to a value slightly greater than V_{UT}, this switches the output voltage from $+V_{sat}$ to $-V_{sat}$ and we have $VP = V_{LT}$, which is negative with respect to the ground. As V_{out} switches to $-V_{sat}$, the capacitor starts discharging via R_F, as shown in the figure 7.41. The current I_2 discharges the capacitor to 0 V and then recharges the capacitor to V_{LT}. When V_C becomes slightly more negative than the feedback voltage V_{LT}, the output voltage V_{out} switches back to $+V_{sat}$. As a result, the condition in Figure 7.41 is reestablished except that the capacitor now has a initial charge equal to V_{LT}. The capacitor will discharge from V_{LT} to 0 V and then recharge to V_{UT}, and the process is a repeating one. Once the initial cycle is completed, the waveforms become periodic, as shown in Figure 7.42.

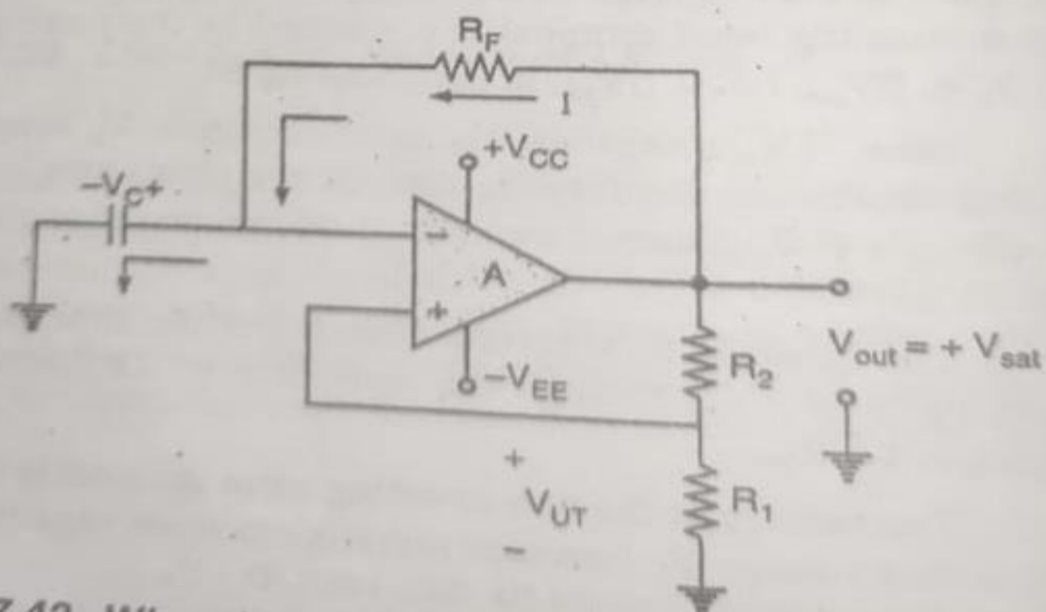
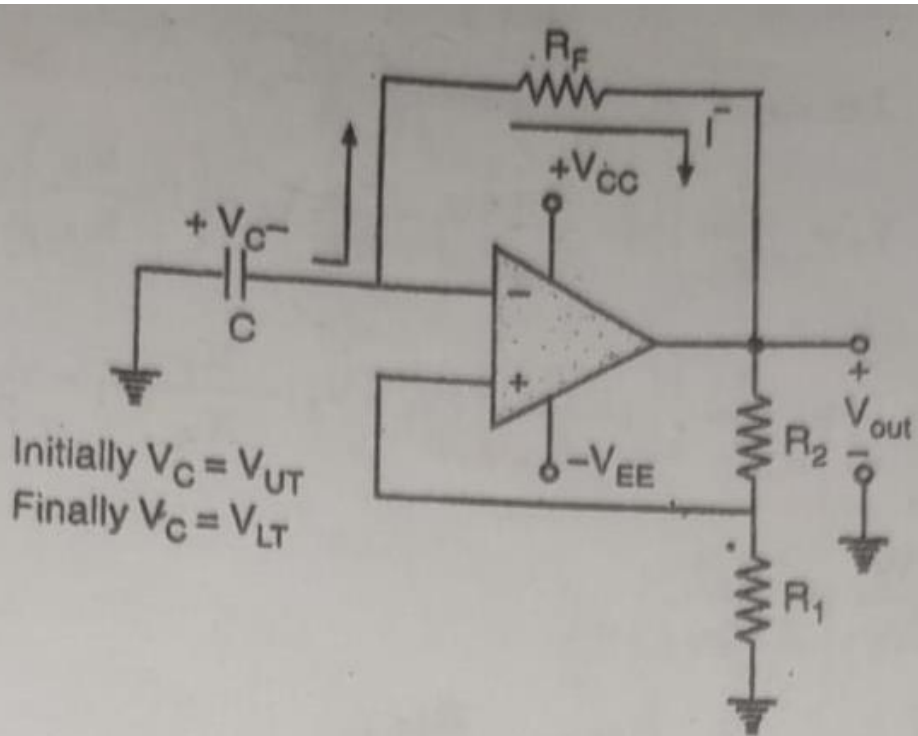


Fig. 7.42. When $V_0 = +V_{sat}$, the capacitor charges towards V_{UT}



(a) When $V_{out} = -V_{sat}$, the capacitor charges towards V_{LT}

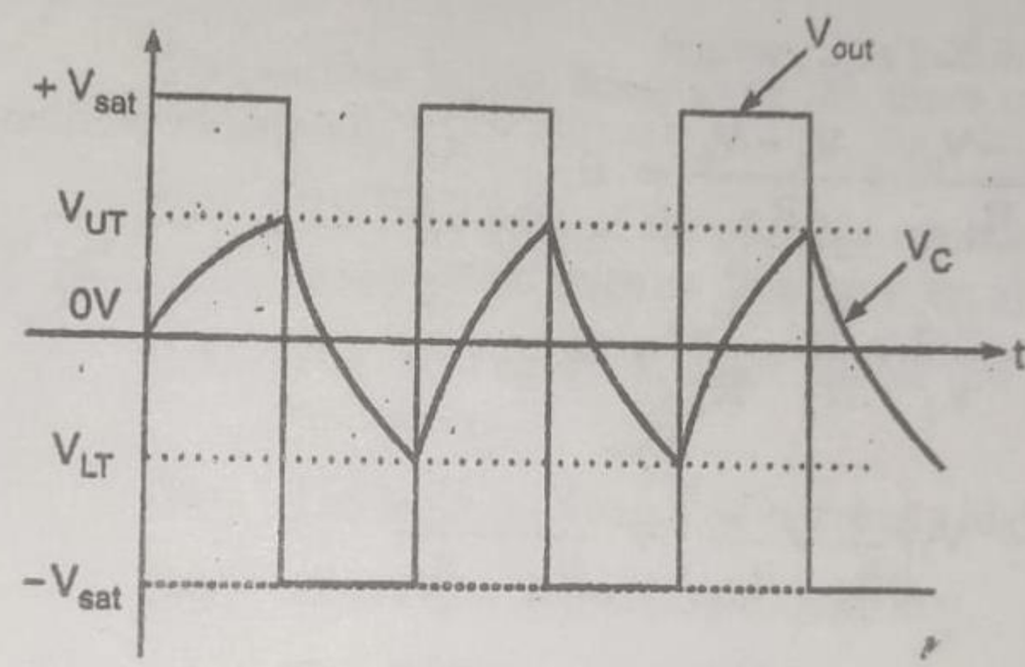


Fig. 7.43. Waveforms