3. Lighter and smaller than a diesel engine of the same capacity.
4. The gas turbine is about one third the size of the turbine for a simple open gas turbine plant.
5. The free-piston is vibrationless.

Disadvantages :

1. Starting and control problems.
2. Synchronization problem not yet fully overcome.

5.22. RELATIVE THERMAL EFFICIENCIES OF DIFFERENT CYCLES

Fig. 5.47 shows the graphs between turbine inlet temperatures and thermal efficiency for different cycles and which are self explanatory.

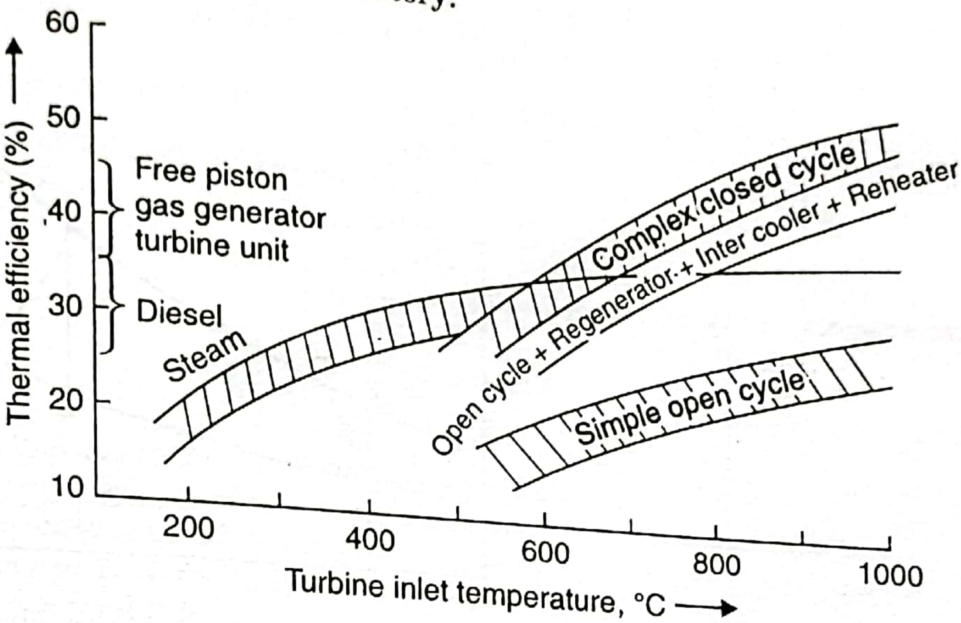


Fig. 5.47. Relative thermal efficiencies of different cycles.

WORKED EXAMPLES

Example 5.1. *The air enters the compressor of an open cycle constant pressure gas turbine at a pressure of 1 bar and temperature of 20°C. The pressure of the air after compression is 4 bar. The isentropic efficiencies of compressor and turbine are 80% and 85% respectively. The air-fuel ratio used is 90 : 1. If flow rate of air is 3.0 kg/s, find :*

(i) Power developed.

(ii) Thermal efficiency of the cycle.

Assume $c_p = 1.0$ kJ/kg K and $\gamma = 1.4$ for air and gases.

Calorific value of fuel = 41800 kJ/kg.

Solution. *Given :* $p_1 = 1$ bar ; $T_1 = 20 + 273 = 293$ K

$p_2 = 4$ bar ; $\eta_{compressor} = 80\%$; $\eta_{turbine} = 85\%$

Air-fuel ratio $= 90 : 1$; Air flow rate, $m_a = 3.0$ kg/s (*i*) **Power developed, P :**

Refer Fig. 5.48 (*b*)

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4}{1}\right)^{\frac{1.4-1}{1.4}} = 1.486$$

$$\therefore \quad T_2 = (20 + 273) \times 1.486 = 435.4 \text{ K}$$

$$\eta_{compressor} = \frac{T_2 - T_1}{T_2' - T_1}$$

$$0.8 = \frac{435.4 - 293}{T_2' - 293}$$

$$\therefore \quad T_2' = \frac{435.4 - 293}{0.8} + 293 = 471 \text{ K}$$

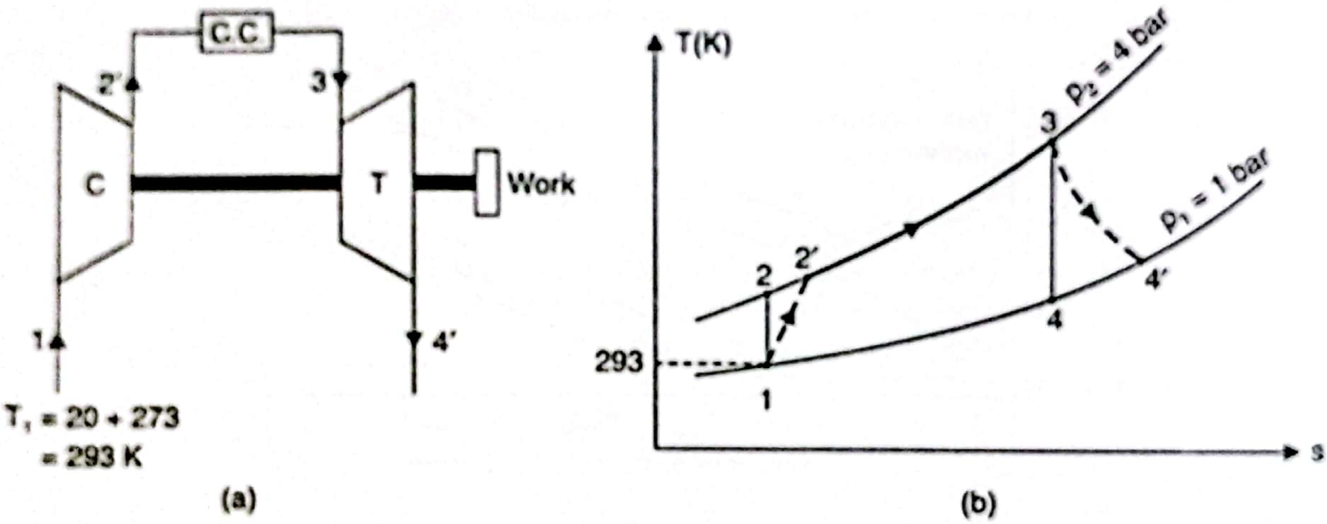


Fig. 5.48

Heat supplied by fuel = Heat taken by burning gases

$$m_f \times C = (m_a + m_f)\, c_p (T_3 - T_2')$$

(where m_a = mass of air, m_f = mass of fuel)

$$\therefore \quad C = \left(\frac{m_a}{m_f} + 1\right) c_p\, (T_3 - T_2')$$

$$\therefore \quad 41800 = (90 + 1) \times 1.0 \times (T_3 - 471)$$

i.e.,

$$T_3 = \frac{41800}{91} + 471 = 930 \text{ K}$$

Again,

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{4}\right)^{\frac{0.4}{0.4}} = 0.672$$

$$\therefore \quad T_4 = 930 \times 0.672 = 624.9 \text{ K}$$

$$\eta_{turbine} = \frac{T_3 - T_4'}{T_3 - T_4}$$

$$0.85 = \frac{930 - T_4'}{930 - 624.9}$$

$\therefore$ $T_4' = 930 - 0.85\ (930 - 624.9) = 670.6$ K

$$W_{turbine} = m_g \times c_p \times (T_3 - T_4')$$

(where m_g is the mass of hot gases formed per kg of air)

$\therefore$ $$W_{turbine} = \left(\frac{90+1}{90}\right) \times 1.0 \times (930 - 670.6)$$

$$= 262.28 \text{ kJ/kg of air.}$$

$$W_{compressor} = m_a \times c_p \times (T_2' - T_1) = 1 \times 1.0 \times (471 - 293)$$

$$= 178 \text{ kJ/kg of air}$$

$$W_{net} = W_{turbine} - W_{compressor}$$

$$= 262.28 - 178 = 84.28 \text{ kJ/kg of air.}$$

Hence *power developed*, $P = 84.28 \times 3 =$ **252.84 kW/kg of air.** **(Ans.)**

(*ii*) **Thermal efficiency of cycle, $\eta_{thermal}$:**

Heat supplied per kg of air passing through combustion chamber

$$= \frac{1}{90} \times 41800 = 464.44 \text{ kJ/kg of air}$$

$\therefore$ $$\eta_{thermal} = \frac{\text{Work output}}{\text{Heat supplied}} = \frac{84.28}{464.44} = 0.1814 \text{ or } \mathbf{18.14\%}. \quad \textbf{(Ans.)}$$

Example 5.2. *A gas turbine unit has a pressure ratio of 6 : 1 and maximum cycle temperature of 610°C. The isentropic efficiencies of the compressor and turbine are 0.80 and 0.82 respectively. Calculate the power output in kilowatts of an electric generator geared to the turbine when the air enters the compressor at 15°C at the rate of 16 kg/s.*

Take $c_p = 1.005$ kJ/kg K and $\gamma = 1.4$ for the compression process, and take $c_p = 1.11$ kJ/kg K and $\gamma = 1.333$ for the expansion process.

Solution. *Given* : $T_1 = 15 + 273 = 288$ K ; $T_3 = 610 + 273 = 883$ K ; $\frac{p_2}{p_1} = 6$,

$\eta_{compressor} = 0.80$; $\eta_{turbine} = 0.82$; Air flow rate = 16 kg/s

For compression process : $c_p = 1.005$ kJ/kg K, $\gamma = 1.4$

For expansion process : $c_p = 1.11$ kJ/kg K, $\gamma = 1.333$

In order to evaluate the net work output it is necessary to calculate temperatures T_2' and T_4'. To calculate these temperatures we must first calculate T_2 and then use the isentropic efficiency.

For an isentropic process, $$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (6)^{\frac{1.4-1}{1.4}} = 1.67$$

$\therefore$ $$T_2 = 288 \times 1.67 = 481 \text{ K}$$

Also, $$\eta_{compressor} = \frac{T_2 - T_1}{T_2' - T_1}$$

$$0.8 = \frac{481 - 288}{T_2' - T_1}$$

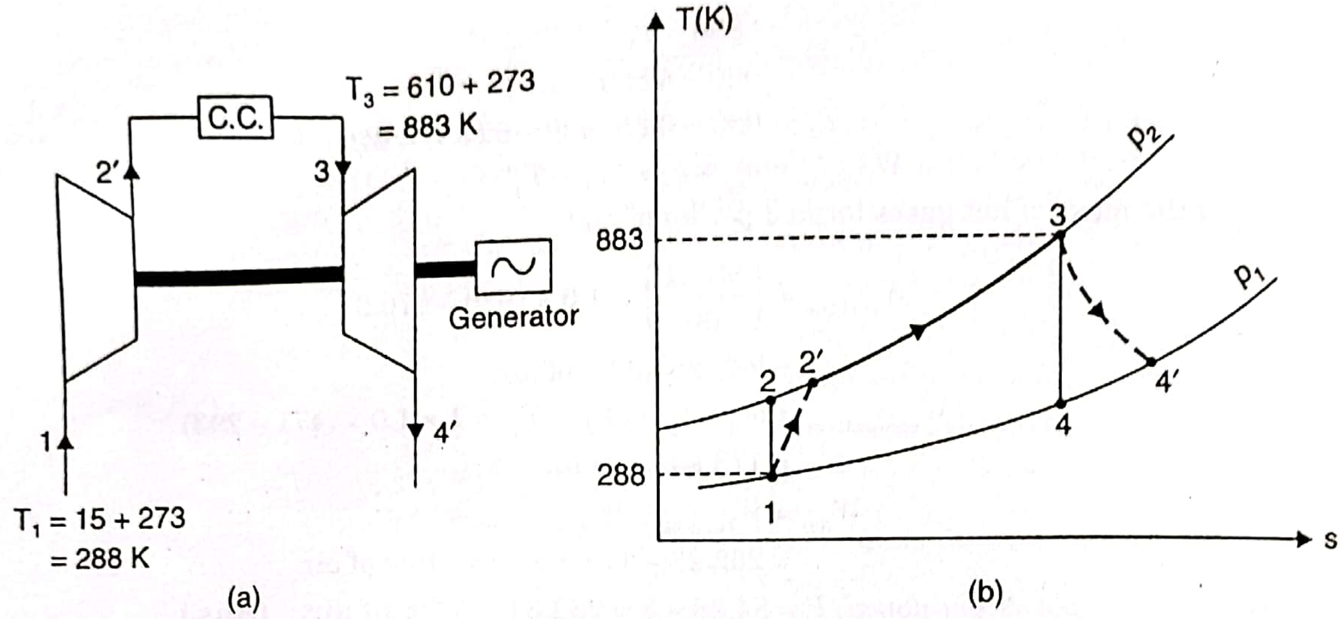


Fig. 5.49

$$\therefore \qquad T_2' = \frac{481 - 288}{0.8} + 288 = 529 \text{ K}$$

Similarly for the turbine, $\quad \dfrac{T_3}{T_4} = \left(\dfrac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (6)^{\frac{1.333-1}{1.333}} = 1.565$

$$\therefore \qquad T_4 = \frac{T_3}{1.565} = \frac{883}{1.565} = 564 \text{ K}$$

Also, $\quad \eta_{\text{turbine}} = \dfrac{T_3 - T_4'}{T_3 - T_4} = \dfrac{883 - T_4'}{883 - 564}$

$$\therefore \qquad 0.82 = \frac{883 - T_4'}{883 - 564}$$

$$\therefore \qquad T_4' = 883 - 0.82\,(883 - 564) = 621.4 \text{ K}$$

Hence,

Compressor work input, $W_{\text{compressor}} = c_p\,(T_2' - T_1)$
$= 1.005\,(529 - 288) = 242.2$ kJ/kg

Turbine work output, $W_{\text{turbine}} = c_p\,(T_3 - T_4')$
$= 1.11\,(883 - 621.4) = 290.4$ kJ/kg.

∴ Net work output, $W_{\text{net}} = W_{\text{turbine}} - W_{\text{compressor}}$
$= 290.4 - 242.2 = 48.2$ kJ/kg

Power in kilowatts $= 48.2 \times 16 =$ **771.2 kW. (Ans.)**

Example 5.3. *Calculate the thermal efficiency and work ratio of the plant is example 5.2, assuming that c_p for the combustion process is 1.11 kJ/kg K.*

Solution. Heat supplied $= c_p(T_3 - T_2')$
$= 1.11\,(883 - 529) = 392.9$ kJ/kg

$$\eta_{\text{thermal}} = \frac{\text{Net work output}}{\text{Heat supplied}} = \frac{48.2}{392.9} = 0.1226 \text{ or } \mathbf{12.26\%}. \quad \textbf{(Ans.)}$$

Now, Work ratio $= \dfrac{\text{Net work output}}{\text{Gross work output}} = \dfrac{48.2}{W_{turbine}} = \dfrac{48.2}{290.4} = \mathbf{0.166}$. **(Ans.)**

Example 5.4. *In a constant pressure open cycle gas turbine air enters at 1 bar and 20°C and leaves the compressor at 5 bar. Using the following data ; Temperature of gases entering the turbine = 680°C, pressure loss in the combustion chamber = 0.1 bar, $\eta_{compressor}$ = 85%, $\eta_{turbine}$ = 80%, $\eta_{combustion}$ = 85%, γ = 1.4 and c_p = 1.024 kJ/kg K for air and gas, find :*

(i) The quantity of air circulation if the plant develops 1065 kW.

(ii) Heat supplied per kg of air circulation.

(iii) The thermal efficiency of the cycle.

Mass of the fuel may be neglected.

Solution. *Given* : $p_1 = 1$ bar, $p_2 = 5$ bar, $p_3 = 5 - 0.1 = 4.9$ bar, $p_4 = 1$ bar,

$T_1 = 20 + 273 = 293$ K, $T_3 = 680 + 273 = 953$ K,

$\eta_{compressor} = 85\%$, $\eta_{turbine} = 80\%$, $\eta_{combustion} = 85\%$.

For air and gases : $c_p = 1.024$ kJ/kg K, $\gamma = 1.4$

Power developed by the plant, $P = 1065$ kW.

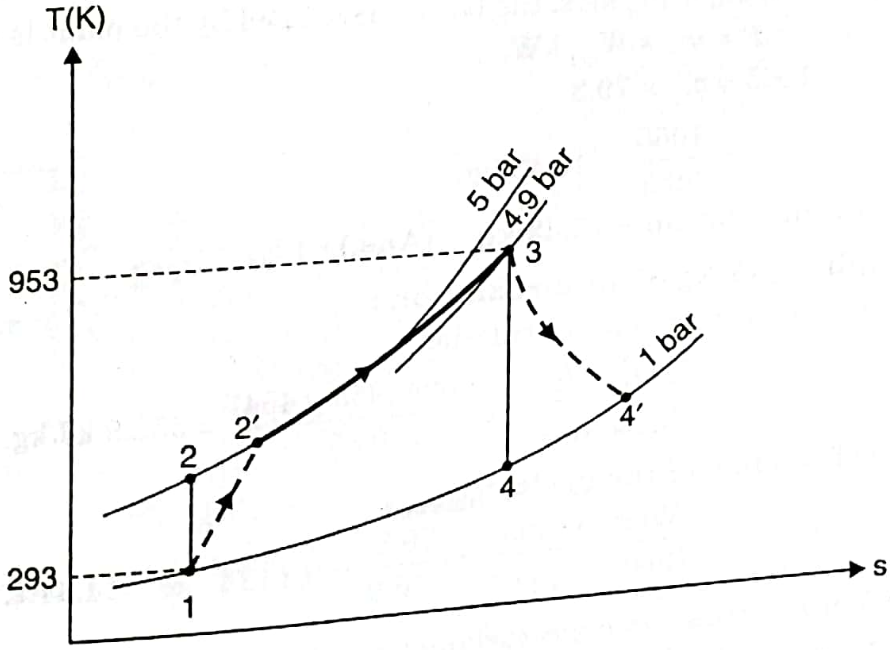


Fig. 5.50

(i) The quantity of air circulation, m_a :

For *isentropic compression 1-2,*

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{5}{1}\right)^{\frac{1.4-1}{1.4}} = 1.584$$

$\therefore$ $$T_2 = 293 \times 1.584 = 464 \text{ K}$$

Now, $$\eta_{compressor} = \frac{T_2 - T_1}{T_2' - T_1} \quad i.e. \quad 0.85 = \frac{464 - 293}{T_2' - 293}$$

$$\therefore \qquad T_2' = \frac{464 - 293}{0.85} + 293 = 494 \text{ K}$$

For isentropic expansion process 3-4,

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{4.9}\right)^{\frac{1.4-1}{1.4}} = 0.635$$

$$\therefore \qquad T_4 = 953 \times 0.635 = 605 \text{ K}$$

Now,

$$\eta_{turbine} = \frac{T_3 - T_4'}{T_3 - T_4}$$

$$0.8 = \frac{953 - T_4'}{953 - 605}$$

$$\therefore \qquad T_4' = 953 - 0.8(953 - 605) = 674.6 \text{ K}$$

$$W_{compressor} = c_p (T_2' - T_1) = 1.024\,(494 - 293) = 205.8 \text{ kJ/kg}$$

$$W_{turbine} = c_p (T_3 - T_4') = 1.024\,(953 - 674.6) = 285.1 \text{ kJ/kg.}$$

$$\therefore \qquad W_{net} = W_{turbine} - W_{compressor} = 285.1 - 205.8 = 79.3 \text{ kJ/kg of air}$$

If the mass of air flowing is m_a kg/s, the power developed by the plant is given by

$$P = m_a \times W_{net} \text{ kW}$$

$$1065 = m_a \times 79.3$$

$$\therefore \qquad m_a = \frac{1065}{79.3} = 13.43 \text{ kg.}$$

i.e., **Quantity of air circulation = 13.43 kg. (Ans.)**

(ii) Heat supplied per kg of air circulation :

Actual heat supplied per kg of air circulation

$$= \frac{c_p (T_3 - T_2')}{\eta_{combustion}} = \frac{1.024\,(953 - 494)}{0.85} = 552.9 \text{ kJ/kg.}$$

(iii) Thermal efficiency of the cycle, $\eta_{thermal}$:

$$\eta_{thermal} = \frac{\text{Work output}}{\text{Heat supplied}} = \frac{79.3}{552.9} = 0.1434 \quad \text{or} \quad 14.34\%. \quad \textbf{(Ans.)}$$

Example 5.5. *Air is drawn in a gas turbine unit at 15°C and 1.01 bar and pressure ratio is 7 : 1. The compressor is driven by the H.P. turbine and L.P. turbine drives a separate power shaft. The isentropic efficiencies of compressor, and the H.P. and L.P. turbines are 0.82, 0.85 and 0.85 respectively. If the maximum cycle temperature is 610°C, calculate :*

(i) *The pressure and temperature of the gases entering the power turbine.*

(ii) *The net power developed by the unit per kg/s mass flow.*

(iii) *The work ratio.*

(iv) *The thermal efficiency of the unit.*

Neglect the mass of fuel and assume the following :

For compression process $c_{pa} = 1.005$ *kJ/kg K and* $\gamma = 1.4$

For combustion and expansion processes : $c_{pg} = 1.15$ *kJ/kg K and* $\gamma = 1.333$.

Solution. *Given* : $T_1 = 15 + 273 = 288$ K, $p_1 = 1.01$ bar, Pressure ratio $= \frac{p_2}{p_1} = 7$,

$\eta_{compressor} = 0.82$, $\eta_{turbine}$ (H.P.) = 0.85, $\eta_{turbine}$ (L.P.) = 0.85,

Maximum cycle temperature, $T_3 = 610 + 273 = 883$ K

(i) Pressure and temperature of the gases entering the power turbine, p_4' and T_4' :

Considering *isentropic compression 1-2*, we get

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (7)^{\frac{1.4-1}{1.4}} = 1.745$$

$\therefore$ $$T_2 = 288 \times 1.745 = 502.5 \text{ K}$$

Also $$\eta_{compressor} = \frac{T_2 - T_1}{T_2' - T_1}$$

$$0.82 = \frac{502.5 - 288}{T_2' - 288}$$

$\therefore$ $$T_2' = \frac{502.5 - 288}{0.82} + 288 = 549.6 \text{ K}$$

$$W_{compressor} = c_{pa}(T_2' - T_1) = 1.005 \times (549.6 - 288) = 262.9 \text{ kJ/kg}$$

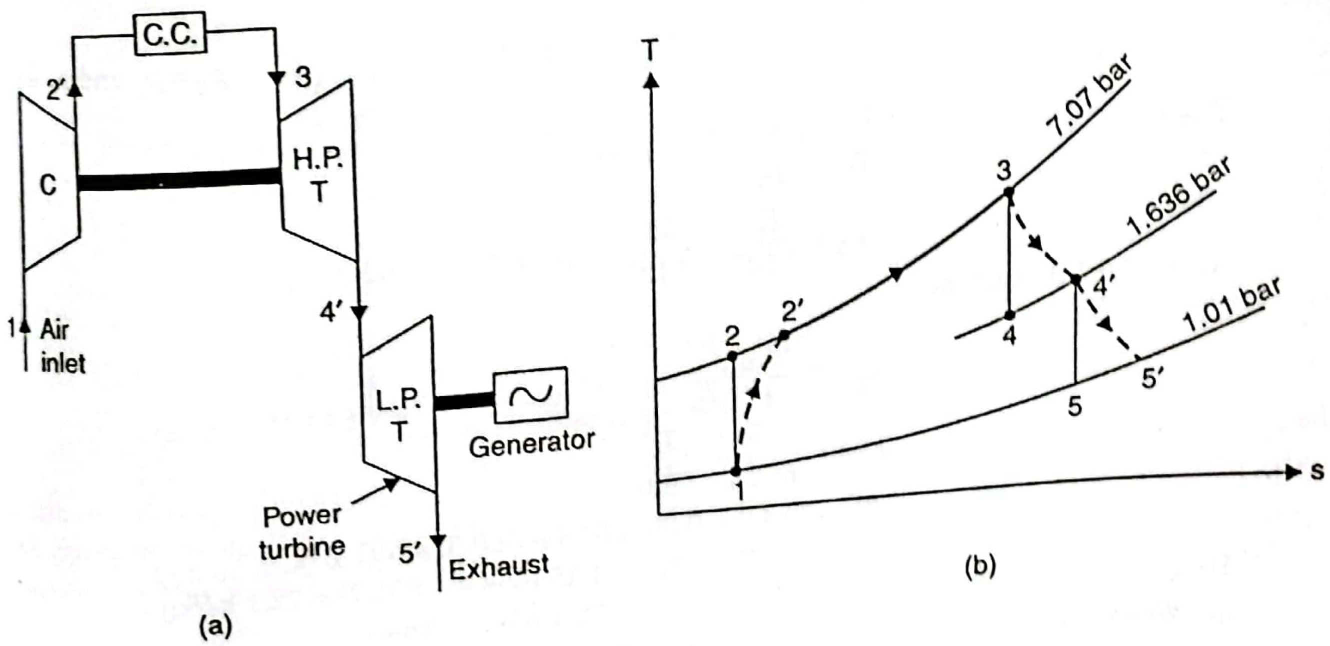


Fig. 5.51

Now, the *work output of H.P. turbine = Work input to compressor*

$\therefore$ $$c_{pg}(T_3 - T_4') = 262.9$$

i.e., $$1.15(883 - T_4') = 262.9$$

$\therefore$ $$T_4' = 883 - \frac{262.9}{1.15} = 654.4 \text{ K}$$

i.e., *Temperature of gases entering the power turbine* = **654.4 K. (Ans.)**

Again, for H.P. turbine :

$$\eta_{turbine} = \frac{T_3 - T_4'}{T_3 - T_4} \quad i.e. \;\; 0.85 = \frac{883 - 654.4}{883 - T_4}$$

$$\therefore \qquad T_4 = 883 - \left(\frac{883 - 654.4}{0.85}\right) = 614 \text{ K}$$

Now, considering *isentropic expansion process 3-4*, we get

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}}$$

or
$$\frac{p_3}{p_4} = \left(\frac{T_3}{T_4}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{883}{614}\right)^{\frac{1.33}{0.33}} = 4.32$$

i.e.,
$$p_4 = \frac{p_3}{4.32} = \frac{7.07}{4.32} = 1.636 \text{ bar}$$

i.e., *Pressure of gases entering the power turbine* = **1.636 bar. (Ans.)**

(*ii*) **Net power developed per kg/s mass flow, P :**

To find the power output it is now necessary to calculate T_5'.

The pressure ratio, $\frac{p_4}{p_5}$, is given by $\frac{p_4}{p_3} \times \frac{p_3}{p_5}$

i.e.,
$$\frac{p_4}{p_5} = \frac{p_4}{p_3} \times \frac{p_2}{p_1} = \frac{7}{4.32} = 1.62 \qquad (\because \;\; p_2 = p_3 \text{ and } p_5 = p_1)$$

Then,
$$\frac{T_4'}{T_5} = \left(\frac{p_4}{p_5}\right)^{\frac{\gamma-1}{\gamma}} = (1.62)^{\frac{0.33}{1.33}} = 1.27$$

$$\therefore \qquad T_5 = \frac{T_4'}{1.127} = \frac{654.4}{1.127} = 580.6 \text{ K.}$$

Again, for L.P. turbine

$$\eta_{turbine} = \frac{T_4' - T_5'}{T_4' - T_5}$$

i.e.,
$$0.85 = \frac{654.4 - T_5'}{654.4 - 580.6}$$

$$\therefore \qquad T_5' = 654.4 - 0.85\,(654.4 - 580.6) = 591.7 \text{ K}$$

$$W_{L.P.\ turbine} = c_{pg}\,(T_4' - T_5') = 1.15\,(654.4 - 591.7) = 72.1 \text{ kJ/kg}$$

Hence *net power output* (per kg/s mass flow) = **72.1 kW. (Ans.)**

(*iii*) **Work ratio :**

$$\textit{Work ratio} = \frac{\text{Net work output}}{\text{Gross work output}} = \frac{72.1}{72.1 + 262.9} = \mathbf{0.215.} \quad \textbf{(Ans.)}$$

(*iv*) **Thermal efficiency of the unit, $\eta_{thermal}$:**

Heat supplied $= c_{pg}(T_3 - T_2') = 1.15\,(883 - 549.6) = 383.4$ kJ/kg

$$\therefore \qquad \eta_{thermal} = \frac{\text{Net work output}}{\text{Heat supplied}} = \frac{72.1}{383.4} = 0.188 \text{ or } \mathbf{18.8\%}. \quad \textbf{(Ans.)}$$

Example 5.6. *In a gas turbine the compressor takes in air at a temperature of 15°C and compresses it to four times the initial pressure with an isentropic efficiency of 82%. The air is then passed through a heat exchanger heated by the turbine exhaust before reaching the combustion chamber. In the heat exchanger 78% of the available heat is given to the air. The maximum temperature after constant pressure combustion is 600°C, and the efficiency of the turbine is 70%. Neglecting all losses except those mentioned, and assuming the working fluid throughout the cycle to have the characteristic of air find the efficiency of the cycle.*

Assume R = 0.287 kJ/kg K and γ = 1.4 for air and constant specific heats throughout.

Solution. *Given* : $T_1 = 15 + 273 = 288$ K, Pressure ratio, $\frac{p_2}{p_1} = \frac{p_3}{p_4} = 4$, $\eta_{compressor} = 82\%$.

Effectiveness of the heat exchanger, $\varepsilon = 0.78$,

$\eta_{turbine} = 70\%$, Maximum temperature, $T_3' = 600 + 273 = 873$ K.

Efficiency of the cycle η_{cycle} :

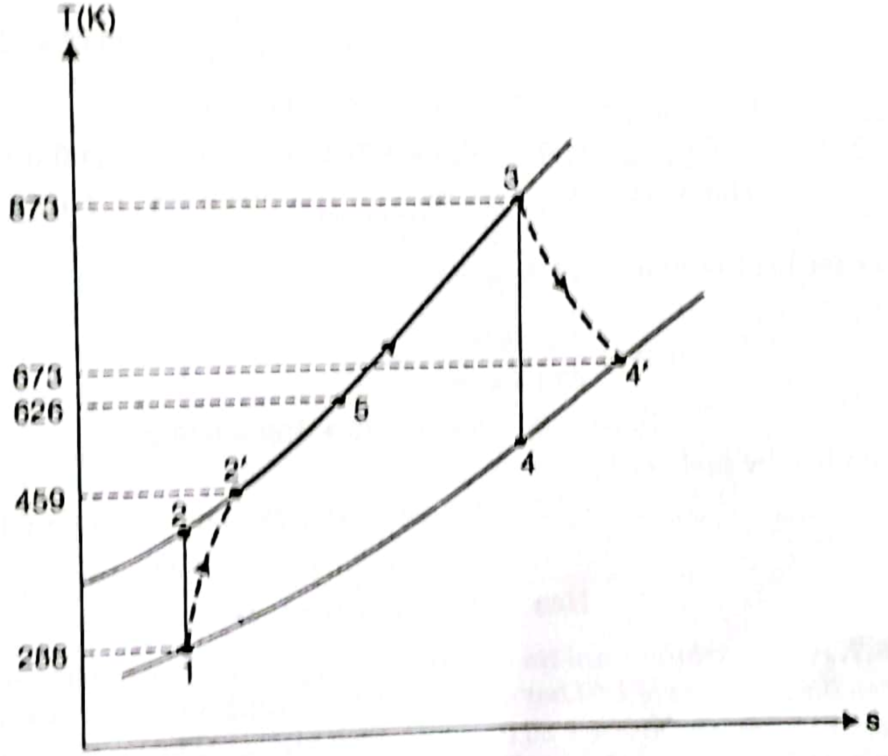


Fig. 5.52

Considering the *isentropic compression 1-2*, we get

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (4)^{\frac{1.4-1}{1.4}} = 1.486$$

∴ $$T_2 = 288 \times 1.486 = 428 \text{ K}$$

Now, $$\eta_{compressor} = \frac{T_2 - T_1}{T_2' - T_1}$$

$$0.82 = \frac{428 - 288}{T_2' - 288}$$

i.e., $$T_2' = \frac{428 - 288}{0.82} + 288 = 459 \text{ K}$$

∴

Considering the isentropic expansion process 3-4, we have

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = (4)^{\frac{1.4-1}{1.4}} = 1.486$$

$\therefore$
$$T_4 = \frac{T_3}{1.486} = \frac{873}{1.486} = 587.5 \text{ K.}$$

Again,
$$\eta_{\text{turbine}} = \frac{T_3 - T_4'}{T_3 - T_4} = \frac{873 - T_4'}{873 - 587.5}$$

i.e.,
$$0.70 = \frac{873 - T_4'}{873 - 587.5}$$

$\therefore$
$$T_4' = 873 - 0.7(873 - 587.5) = 673 \text{ K}$$

$$W_{\text{compressor}} = c_p(T_2' - T_1)$$

But
$$c_p = R \times \frac{\gamma}{\gamma - 1} = 0.287 \times \frac{1.4}{1.4 - 1} = 1.0045 \text{ kJ/kg K}$$

$\therefore$
$$W_{\text{compressor}} = 1.0045(459 - 288) = 171.7 \text{ kJ/kg}$$

$$W_{\text{turbine}} = c_p(T_3 - T_4') = 1.0045(873 - 673) = 200.9 \text{ kJ/kg}$$

$\therefore$
$$\text{Net work} = W_{\text{turbine}} - W_{\text{compressor}} = 200.9 - 171.7 = 29.2 \text{ kJ/kg.}$$

Effectiveness for heat exchanger, $\varepsilon = \dfrac{T_5 - T_2'}{T_4' - T_2'}$

i.e.,
$$0.78 = \frac{T_5 - 459}{673 - 459}$$

$\therefore$
$$T_5 = (673 - 459) \times 0.78 + 459 = 626 \text{ K}$$

$\therefore$ Heat supplied by fuel per kg

$$= c_p(T_3 - T_5) = 1.0045(873 - 626) = 248.1 \text{ kJ/kg}$$

$\therefore$
$$\eta_{\text{cycle}} = \frac{\text{Net work done}}{\text{Heat supplied by the fuel}} = \frac{29.2}{248.1} = 0.117 \text{ or } \mathbf{11.7\%}. \quad \textbf{(Ans.)}$$

Example 5.7. *A gas turbine employs a heat exchanger with a thermal ratio of 72%. The turbine operates between the pressures of 1.01 bar and 4.04 bar and ambient temperature is 20°C. Isentropic efficiencies of compressor and turbine are 80 and 85% respectively. The pressure drop on each side of the heat exchanger is 0.05 bar and in the combustion chamber 0.14 bar. Assume combustion efficiency to be unity and calorific value of the fuel to be 41800 kJ/kg calculate the increase in efficiency due to heat exchanger over that for simple cycle.*

Assume c_p is constant throughout and is equal to 1.024 kJ/kg K, and assume $\gamma = 1.4$.

For simple cycle the air-fuel ratio is 90 : 1, and for the heat exchange cycle the turbine entry temperature is the same as for a simple cycle.

Solution. Simple Cycle. Refer Fig. 5.53.

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4}{1}\right)^{\frac{1.4-1}{1.4}} = 1.486$$

$\therefore$
$$T_2 = 293 \times 1.486 = 435.4$$

Also,
$$\eta_{\text{compressor}} = \frac{T_2 - T_1}{T_2' - T_1}$$

$$0.8 = \frac{435.4 - 293}{T_2' - 293}$$

$\therefore$
$$T_2' = \frac{435.4 - 293}{0.8} + 293 = 471 \text{ K}$$

Now
$$m_f \times C = (m_a + m_f) \times c_p \times (T_3 - T_2')$$

[m_a = mass of air, m_f = mass of fuel]

$\therefore$
$$T_3 = \frac{m_f \times C}{c_p\,(m_a + m_f)} + T_2' = \frac{1 \times 41800}{1.024\,(90 + 1)} + 471 = 919.5 \text{ K}$$

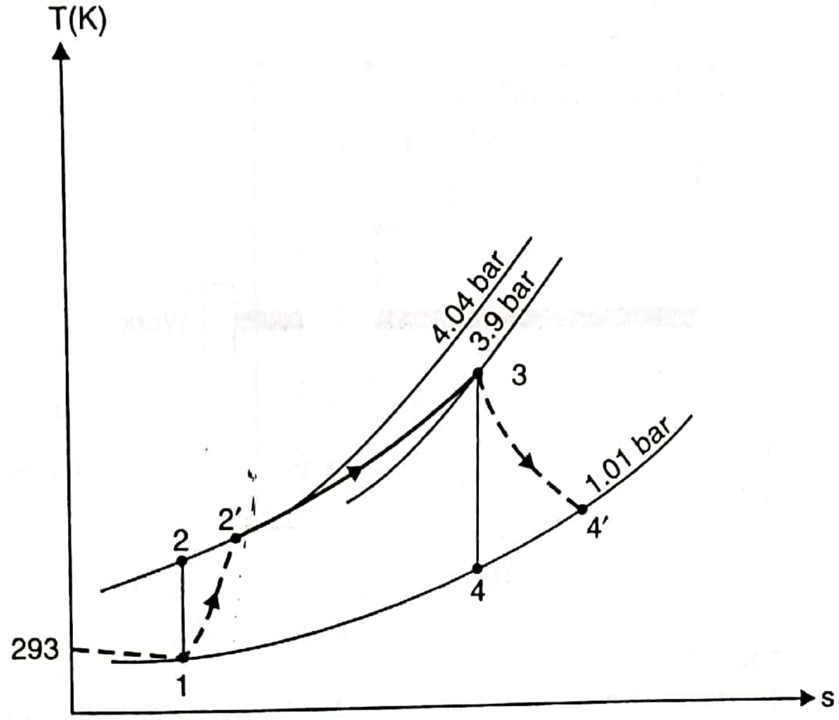


Fig. 5.53

Also,
$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma - 1}{\gamma}}$$

or
$$T_4 = T_3 \times \left(\frac{p_4}{p_3}\right)^{\frac{\gamma - 1}{\gamma}} = 919.5 \times \left(\frac{1.01}{3.9}\right)^{\frac{1.4 - 1}{1.4}} = 625 \text{ K}$$

Again,
$$\eta_{\text{turbine}} = \frac{T_3 - T_4'}{T_3 - T_4}$$

$\therefore$
$$0.85 = \frac{919.5 - T_4'}{919.5 - 625}$$

$\therefore$
$$T_4' = 919.5 - 0.85(919.5 - 625) = 669 \text{ K}$$

$$\eta_{\text{thermal}} = \frac{(T_3 - T_4') - (T_2' - T_1)}{(T_3 - T_2')}$$

$$= \frac{(919.5 - 669) - (471 - 293)}{(919.5 - 471)} = \frac{72.5}{448.5} = 0.1616 \text{ or } \mathbf{16.16\%}.\quad \textbf{(Ans.)}$$

Heat Exchanger Cycle. Refer Fig. 5.54 (*a*, *b*)

$$T_2' = 471 \text{ K (as for simple cycle)}$$
$$T_3 = 919.5 \text{ K (as for simple cycle)}$$

To find T_4' :

$$p_3 = 4.04 - 0.14 - 0.05 = 3.85 \text{ bar}$$
$$p_4 = 1.01 + 0.05 = 1.06 \text{ bar}$$

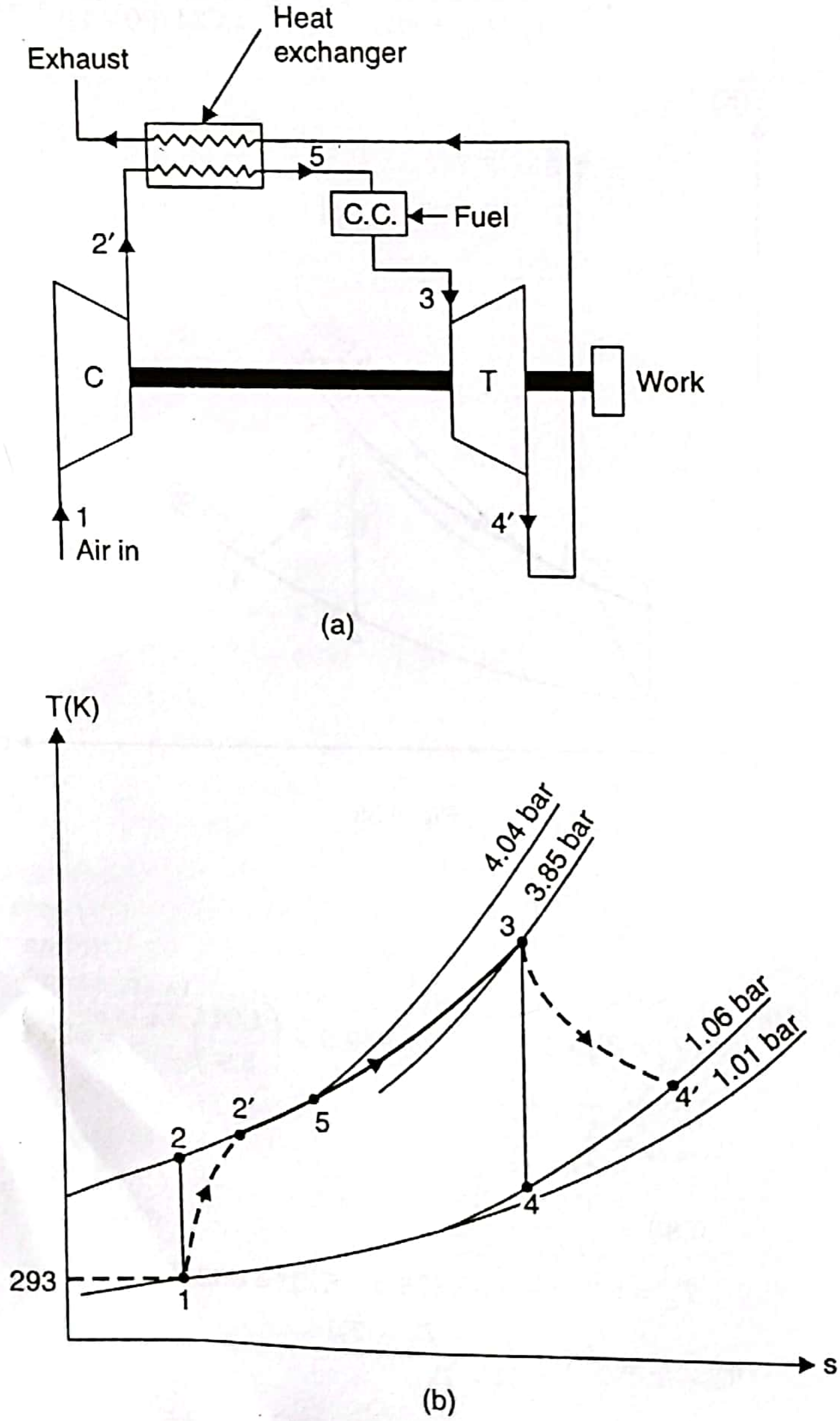


Fig. 5.54

$$\therefore \quad \frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1.06}{3.85}\right)^{\frac{1.4-1}{1.4}} = 0.69$$

i.e., $$T_4 = 919.5 \times 0.69 = 634 \text{ K}$$

$$\eta_{turbine} = \frac{T_3 - T_4'}{T_3 - T_4} \; ; \quad 0.85 = \frac{919.5 - T_4'}{919.5 - 634}$$

$$\therefore \quad T_4' = 919.5 - 0.85\,(919.5 - 634) = 677 \text{ K}$$

To find T_5 :

Thermal ratio (or effectiveness),

$$\varepsilon = \frac{T_5 - T_2'}{T_4' - T_2'} \qquad \therefore \quad 0.72 = \frac{T_5 - 471}{677 - 471}$$

$$\therefore \quad T_5 = 0.72(677 - 471) + 471 = 619 \text{ K}$$

$$\eta_{thermal} = \frac{(T_3 - T_4') - (T_2' - T_1)}{(T_3 - T_5)}$$

$$= \frac{(919.5 - 677) - (471 - 293)}{(919.5 - 619)} = \frac{64.5}{300.5} = 0.2146 \text{ or } \mathbf{21.46\%}$$

$\therefore$ **Increase in thermal efficiency** = 21.46 – 16.16 = **5.3%. (Ans.)**

Example 5.8. *A 5400 kW gas turbine generating set operates with two compressor stages ; the overall pressure ratio is 9 : 1. A high pressure turbine is used to drive the compressors, and a low-pressure turbine drives the generator. The temperature of the gases at entry to the high pressure turbine is 625°C and the gases are reheated to 625°C after expansion in the first turbine. The exhaust gases leaving the low-pressure turbine are passed through a heat exchanger to heat the air leaving the high pressure stage compressor. The compressors have equal pressure ratios and intercooling is complete between the stages. The air inlet temperature to the unit is 20°C. The isentropic efficiency of each compressor stage is 0.8, and the isentropic efficiency of each turbine stage is 0.85, the heat exchanger thermal ratio is 0.8. A mechanical efficiency of 95% can be assumed for both the power shaft and compressor turbine shaft. Neglecting all pressure losses and changes in kinetic energy calculate :*

(i) The thermal efficiency ; *(ii) Work ratio of the plant ;*

(iii) The mass flow in kg/s.

Neglect the mass of the fuel and assume the following :

For air : c_{pa} = 1.005 kJ/kg K and γ = 1.4

For gases in the combustion chamber and in turbines and heat exchanger, c_{pg} = 1.15 kJ/kg K and γ = 1.333.

Solution. Refer Fig. 5.55.

Given : $T_1 = 20 + 273 = 293$ K, $T_6 = T_8 = 625 + 273 = 898$ K

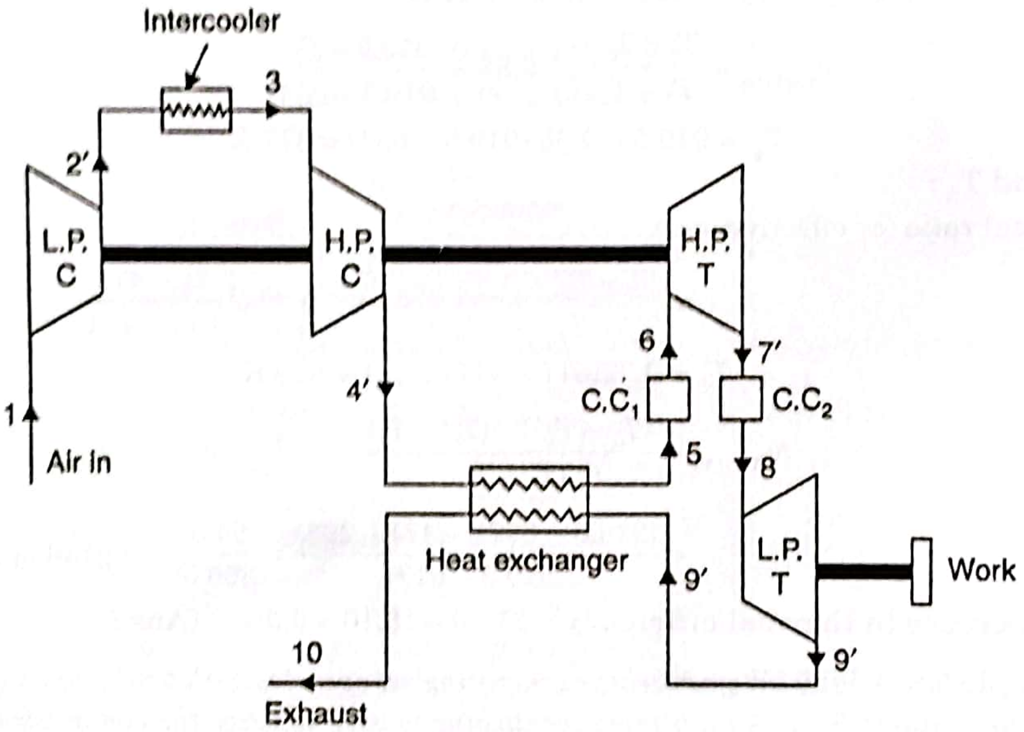
Efficiency of each compressor stage = 0.8

$\eta_{mech.} = 0.95$, $\varepsilon = 0.8$

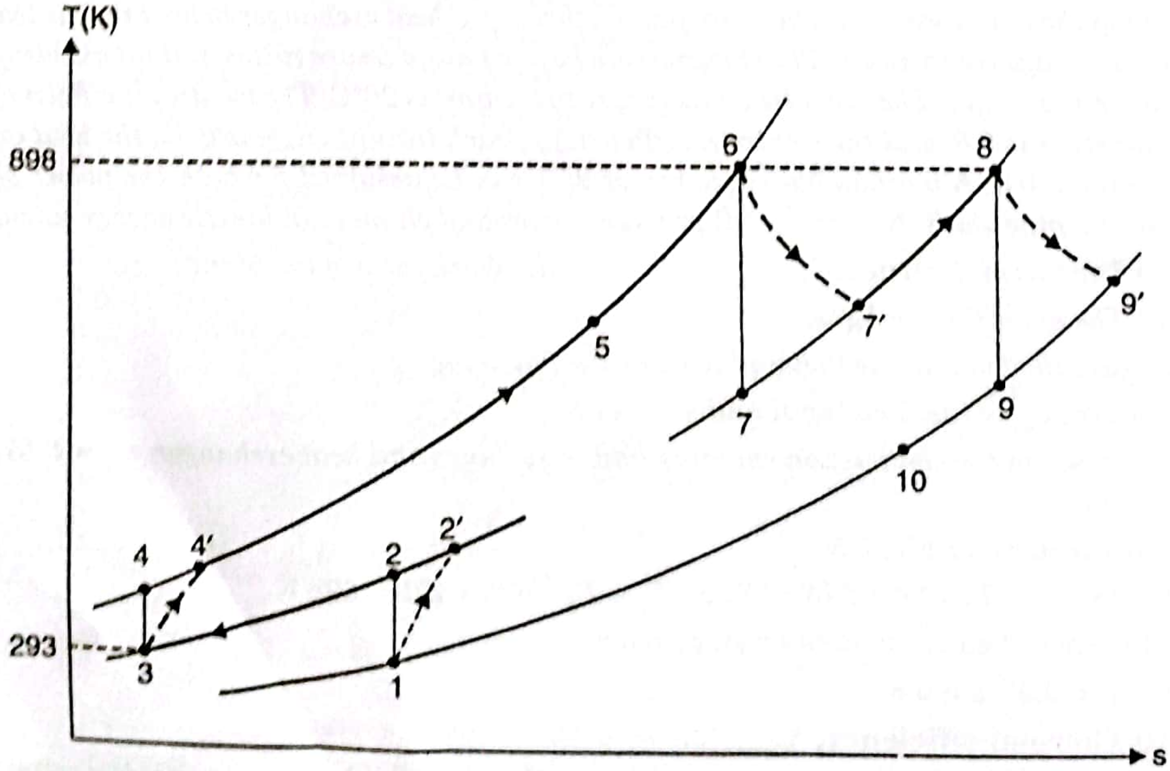
(i) Thermal efficiency, $\eta_{thermal}$:

Since the pressure ratio and the isentropic efficiency of each compressor is the same then the work input required for each compressor is the same since both compressor have the same air inlet temperature *i.e.,* $T_1 = T_3$ and $T_2' = T_4'$.

Also, $$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{and} \quad \frac{p_2}{p_1} = \sqrt{9} = 3$$



(a)



(b)

Fig. 5.55

$$\therefore \qquad T_2 = (20 + 273) \times (3)^{\frac{1.4-1}{1.4}} = 401 \text{ K}$$

Now, $$\eta_{compressor}\text{ (L.P.)} = \frac{T_2 - T_1}{T_2' - T_1}$$

$$0.8 = \frac{401 - 293}{T_2' - 293}$$

i.e., $$T_2' = \frac{401 - 298}{0.8} + 293 = 428 \text{ K}$$

Work input per compressor stage

$$= c_{pa}(T_2' - T_1) = 1.005\,(428 - 293) = 135.6 \text{ kJ/kg}$$

The H.P. turbine is required to drive both compressors and to overcome mechanical friction.

i.e., Work output of H.P. turbine $= \dfrac{2 \times 135.6}{0.95} = 285.5$ kJ/kg

$$\therefore \qquad c_{pg}\,(T_6 - T_7') = 285.5$$

i.e., $$1.15\,(898 - T_7') = 285.5$$

$$\therefore \qquad T_7' = 898 - \frac{285.5}{1.15} = 650 \text{ K}$$

Now, $$\eta_{turbine\text{ (H.P.)}} = \frac{T_6 - T_7'}{T_6 - T_7}\ ; \qquad 0.85 = \frac{898 - 650}{898 - T_7}$$

$$\therefore \qquad T_7 = 898 - \left(\frac{898 - 650}{0.85}\right) = 606 \text{ K}$$

Also, $$\frac{T_6}{T_7} = \left(\frac{p_6}{p_7}\right)^{\frac{\gamma-1}{\gamma}}$$

or $$\frac{p_6}{p_7} = \left(\frac{T_6}{T_7}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{898}{606}\right)^{\frac{1.333}{0.333}} = 4.82$$

Then, $$\frac{p_8}{p_9} = \frac{9}{4.82} = 1.86$$

Again, $$\frac{T_8}{T_9} = \left(\frac{p_8}{p_9}\right)^{\frac{\gamma-1}{\gamma}} = (1.86)^{\frac{1.333-1}{1.333}} = 1.16$$

$$\therefore \qquad T_9 = \frac{T_8}{1.16} = \frac{898}{1.16} = 774 \text{ K}$$

Also, $$\eta_{turbine\text{ (L.P.)}} = \frac{T_8 - T_9'}{T_8 - T_9}\ ; \quad 0.85 = \frac{898 - T_9'}{898 - 774}$$

$$\therefore \qquad T_9' = 898 - 0.85\,(898 - 774) = 792.6 \text{ K}$$

$\therefore$ Net work output $= c_{pg}(T_8 - T_9') \times 0.95$

$$= 1.15\,(898 - 792.6) \times 0.95 = 115.15 \text{ kJ/kg}$$

Thermal ratio or effectiveness of heat exchanger,

$$\varepsilon = \frac{T_5 - T_4'}{T_9' - T_4'} = \frac{T_5 - 428}{792.6 - 428}$$

i.e., $$0.8 = \frac{T_5 - 428}{792.6 - 428}$$

$\therefore$ $$T_5 = 0.8\,(792.6 - 428) + 428 = 719.7 \text{ K}$$

Now, $$\text{Heat supplied} = c_{pg}(T_6 - T_5) + c_{pg}(T_8 - T_7')$$
$$= 1.15\,(898 - 719.7) + 1.15\,(898 - 650) = 490.2 \text{ kJ/kg}$$

$\therefore$ $$\eta_{thermal} = \frac{\text{Net work output}}{\text{Heat supplied}} = \frac{115.15}{490.2}$$
$$= 0.235 \text{ or } 23.5\%. \quad \textbf{(Ans.)}$$

(*ii*) Work ratio :

Gross work of the plant $= W_{turbine\,(H.P.)} + W_{turbine\,(L.P.)}$

$$= 285.5 + \frac{115.15}{0.95} = 406.7 \text{ kJ/kg}$$

$\therefore$ **Work ratio** $$= \frac{\text{Net work output}}{\text{Gross work output}} = \frac{115.15}{406.7} = 0.283. \quad \textbf{(Ans.)}$$

(*iii*) Mass flow, $\dot{m}$:

Let the mass flow be $\dot{m}$, then

$$\dot{m} \times 115.15 = 4500$$

$\therefore$ $$\dot{m} = \frac{4500}{115.15} = 39.08 \text{ kg/s}$$

i.e., **Mass flow** $= 39.08$ **kg/s. (Ans.)**

Example 5.9. *In a closed cycle gas turbine there is two-stage compressor and a two-stage turbine. All the components are mounted on the same shaft. The pressure and temperature at the inlet of the first-stage compressor are 1.5 bar and 20°C. The maximum cycle temperature and pressure are limited to 750°C and 6 bar. A perfect intercooler is used between the two-stage compressors and a reheater is used between the two turbines. Gases are heated in the reheater to 750°C before entering into the L.P. turbine. Assuming the compressor and turbine efficiencies as 0.82, calculate :*

(*i*) *The efficiency of the cycle without regenerator.*

(*ii*) *The efficiency of the cycle with a regenerator whose effectiveness is 0.70.*

(*iii*) *The mass of the fluid circulated if the power developed by the plant is 350 kW.*

The working fluid used in the cycle is air. For air : $\gamma = 1.4$ and $c_p = 1.005$ kJ/kg K.

Solution. *Given :* $T_1 = 20 + 273 = 293$ K, $T_5 = T_7 = 750 + 273 = 1023$ K, $p_1 = 1.5$ bar, $p_2 = 6$ bar, $\eta_{compressor} = \eta_{turbine} = 0.82$,

Effectiveness of regenerator, $\varepsilon = 0.70$, Power developed, $P = 350$ kW.

For air : $c_p = 1.005$ kJ/kg, K, $\gamma = 1.4$

As per given conditions : $T_1 = T_3$, $T_2' = T_4'$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{and} \quad p_x = \sqrt{p_1 p_2} = \sqrt{1.5 \times 6} = 3 \text{ bar}$$

Now, $$T_2 = T_1 \times \left(\frac{p_2}{T_1}\right)^{\frac{\gamma-1}{\gamma}} = 293 \times \left(\frac{3}{1.5}\right)^{\frac{1.4-1}{1.4}} = 357 \text{ K}$$

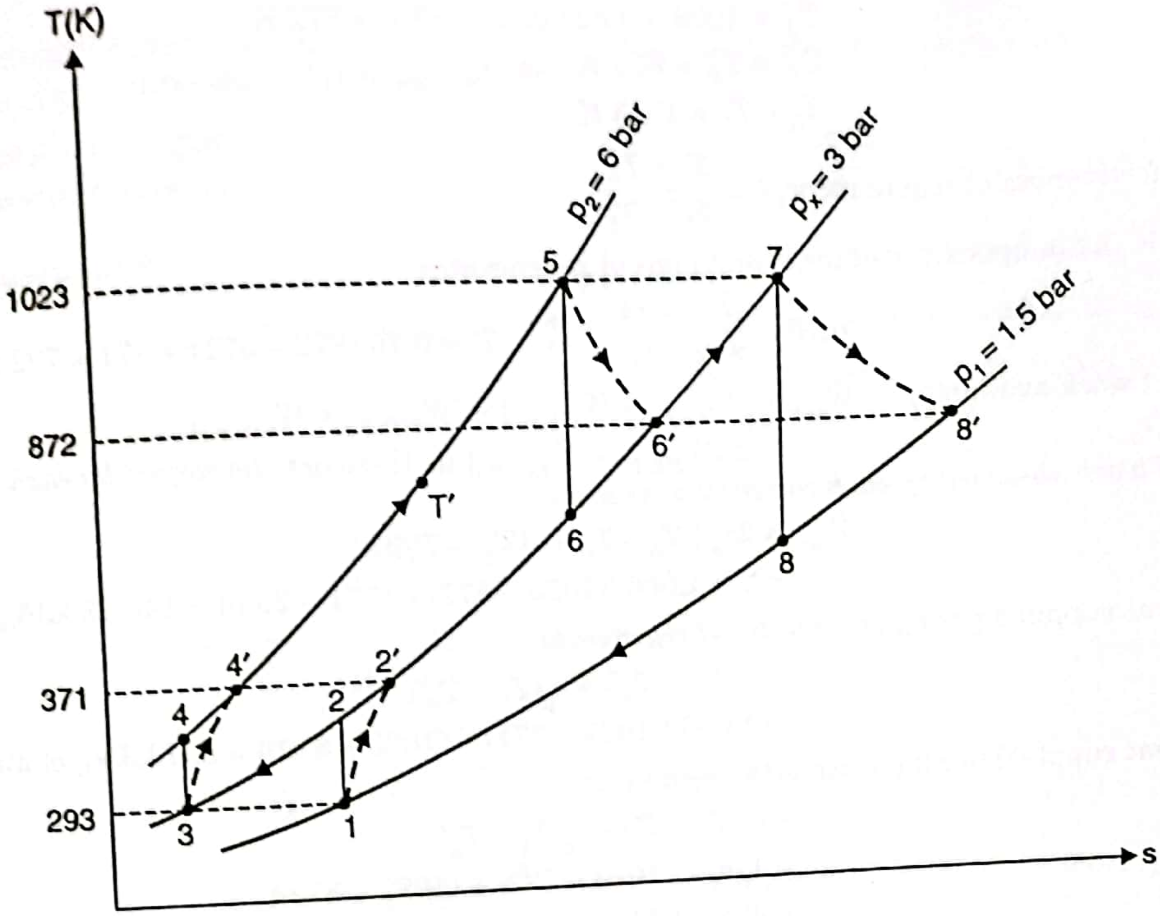


Fig. 5.56

$$\eta_{compressor\ (L.P.)} = \frac{T_2 - T_1}{T_2' - T_1}$$

$$0.82 = \frac{357 - 293}{T_2' - 293}$$

$$\therefore \quad T_2' = \frac{357 - 293}{0.82} + 293 = 371 \text{ K}$$

i.e.,
$$T_2' = T_4' = 371 \text{ K}$$

Now,
$$\frac{T_5}{T_6} = \left(\frac{p_5}{p_6}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_2}{p_x}\right)^{\frac{1.4-1}{1.4}} \quad \left[\because \begin{array}{l} p_5 = p_2 \\ p_6 = p_x \end{array}\right]$$

$$\frac{1023}{T_6} = \left(\frac{6}{3}\right)^{0.286} = 1.219$$

$$\therefore \quad T_6 = \frac{1023}{1.219} = 839 \text{ K}$$

$$\eta_{turbine\ (H.P.)} = \frac{T_5 - T_6'}{T_5 - T_6}$$

$$0.82 = \frac{1023 - T_6'}{1023 - 839}$$

$$\therefore \quad T_6' = 1023 - 0.82\,(1023 - 839) = 872 \text{ K}$$

$$T_8' = T_6' = 872 \text{ K} \quad \text{as} \quad \eta_{\text{turbine (H.P.)}} = \eta_{\text{turbine (L.P.)}}$$

and
$$T_7 = T_5 = 1023 \text{ K}$$

Effectiveness of regenerator, $\varepsilon = \dfrac{T' - T_4'}{T_8' - T_4'}$

where T' is the temperature of air coming out of regenerator

$$\therefore \quad 0.70 = \frac{T' - 371}{872 - 371} \quad i.e. \quad T' = 0.70\,(872 - 371) + 371 = 722 \text{ K}$$

Net work available, $W_{\text{net}} = [W_{T(\text{L.P.})} + W_{T(\text{L.P.})}] - [W_{C(\text{H.P.})} + W_{C(\text{L.P.})}]$

$= 2\,[W_{T(\text{L.P.})} - W_{C(\text{L.P.})}]$ as the *work developed by each turbine is same and work absorbed by each compressor is same.*

$$\therefore \quad W_{\text{net}} = 2c_p\,[(T_5 - T_6') - (T_2' - T_1)]$$

$$= 2 \times 1.005\,[(1023 - 872) - (371 - 293)] = 146.73 \text{ kJ/kg of air}$$

Heat supplied per kg of air *without regenerator*

$$= c_p(T_5 - T_4') + c_p(T_7 - T_6')$$

$$= 1.005\,[(1023 - 371) + (1023 - 872)] = 807 \text{ kJ/kg of air}$$

Heat supplied per kg of air *with regenerator*

$$= c_p(T_5 - T') + c_p(T_7 - T_6')$$

$$= 1.005\,[(1023 - 722) + (1023 - 872)]$$

$$= 454.3 \text{ kJ/kg}$$

(*i*) $\eta_{\text{thermal (without regenerator)}} = \dfrac{146.73}{807} = 0.182$ or **18.2%. (Ans.)**

(*ii*) $\eta_{\text{thermal (with regenerator)}} = \dfrac{146.73}{454.3} = 0.323$ or **32.3%. (Ans.)**

(*iii*) **Mass of fluid circulated, $\dot{m}$:**

Power developed, $P = 146.73 \times \dot{m}$ kW

$$\therefore \quad 350 = 146.73 \times \dot{m}$$

i.e.,
$$\dot{m} = \frac{350}{146.73} = 2.38 \text{ kg/s}$$

i.e., **Mass of fluid circulated = 2.38 kg/s. (Ans.)**

☞**Example 5.10.** *The air in a gas turbine plant is taken in L.P. compressor at 293 K and 1.05 bar and after compression it is passed through intercooler where its temperature is reduced to 300 K. The cooled air is further compressed in H.P. unit and then passed in the combustion chamber where its temperature is increased to 750°C by burning the fuel. The combustion products expand in H.P. turbine which runs the compressor and further expansion is continued in L.P. turbine which runs the alternator. The gases coming out from L.P. turbine are used for heating the incoming air from H.P. compressor and then expanded to atmosphere.*

Pressure ratio of each compressor = 2, isentropic efficiency of each compressor stage = 82%, isentropic efficiency of each turbine stage = 82%, effectiveness of heat exchanger = 0.72, air flow = 16 kg/s, calorific value of fuel = 42000 kJ/kg, c_v *(for gas) = 1.0 kJ/kg K,* c_p *(gas) = 1.15 kJ/kg K,* γ *(for air) = 1.4,* γ *(for gas) = 1.33.*

Neglecting the mechanical, pressure and heat losses of the system and fuel mass also, determine the following :

(i) The power output. *(ii) Thermal efficiency.*

(iii) Specific fuel consumption.

Solution. *Given :* $T_1 = 293$ K, $T_3 = 300$ K, $\frac{p_2}{p_1} = \frac{p_4}{p_3} = 2$, $T_6 = 750 + 273 = 1023$ K,

$\eta_{compressor} = 82\%$, $\eta_{turbine} = 82\%$, $\varepsilon = 0.72$, $\dot{m}_a = 16$ kg/s, $C = 42000$ kJ/kg,

$c_{pa} = 1.0$ kJ/kg K, $c_{pg} = 1.15$ kJ/kg K, γ (for air) $= 1.4$, γ (for gas) $= 1.33$.

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (2)^{\frac{1.4-1}{1.4}} = 1.219$$

$\therefore$ $$T_2 = 293 \times 1.219 = 357 \text{ K}$$

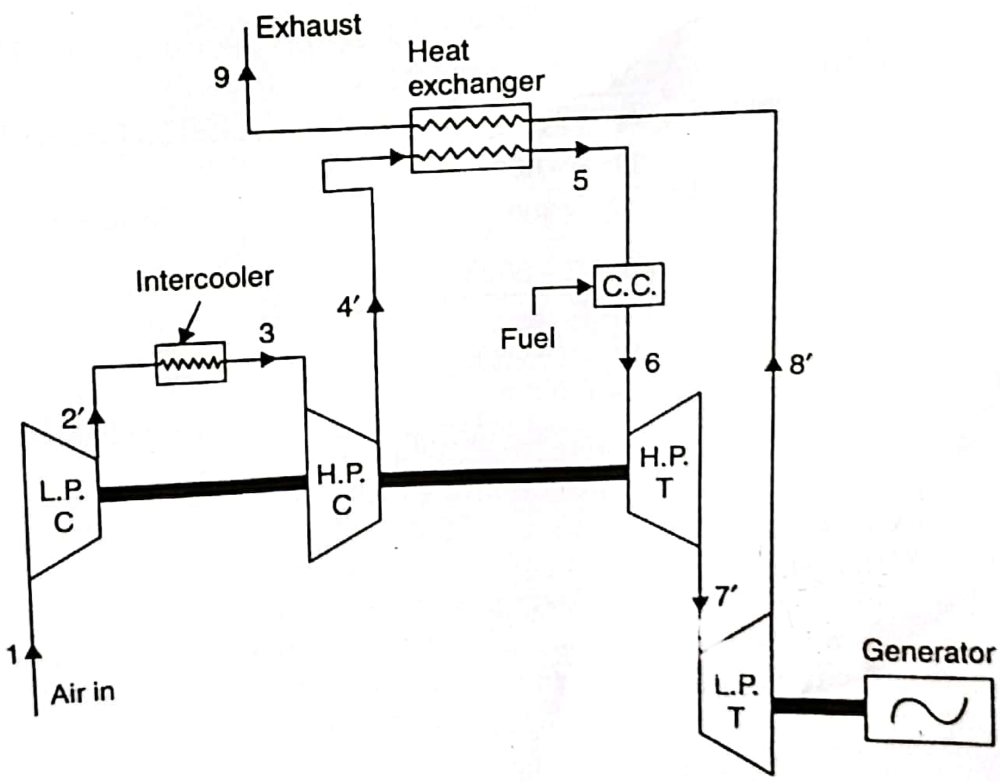
Also, $$\eta_{compressor} = \frac{T_2 - T_1}{T_2' - T_1}$$

$\therefore$ $$0.82 = \frac{357 - 293}{T_2' - 293}$$

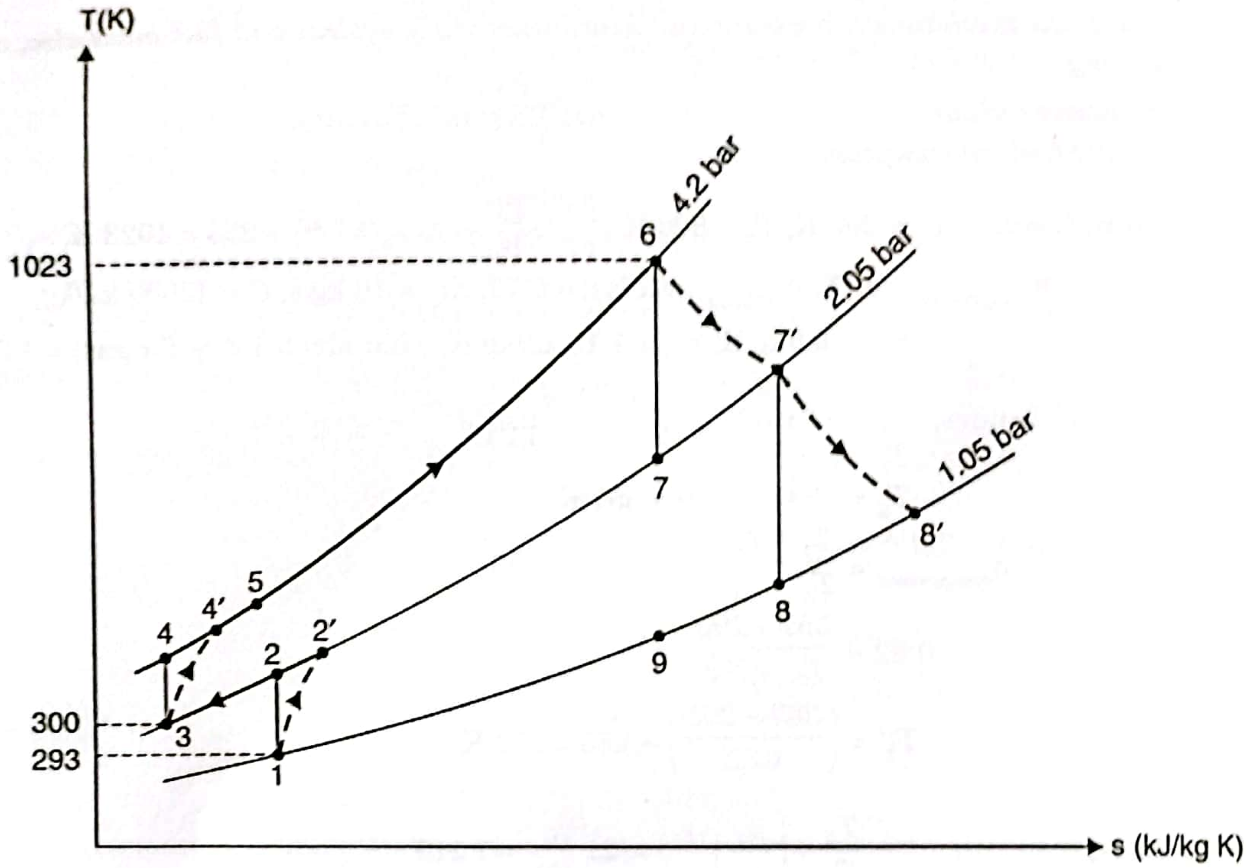
$\therefore$ $$T_2' = \left(\frac{357 - 293}{0.82}\right) + 293 = 371 \text{ K}$$

Similarly, $$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = (2)^{\frac{1.4-1}{1.4}} = 1.219$$

$\therefore$ $$T_4 = 300 \times 1.219 = 365.7 \text{ K}$$



(a)



(b)

Fig. 5.57

and
$$\eta_{compressor} = \frac{T_4 - T_3}{T_4' - T_3}$$

$\therefore$
$$0.82 = \frac{365.7 - 300}{T_4' - 300}$$

i.e.,
$$T_4' = \left(\frac{365.7 - 300}{0.82}\right) + 300 = 380 \text{ K}$$

Work output of H.P. turbine = Work input to compressor.

Neglecting mass of fuel we can write

$$c_{pg}\,(T_6 - T_7') = c_{pa}\,[(T_2' - T_1) + (T_4' - T_3)]$$

$$1.15(1023 - T_7') = 1.0[(371 - 293) + (380 - 300)]$$

or
$$1.15(1023 - T_7') = 158$$

$\therefore$
$$T_7' = 1023 - \frac{15.8}{1.15} = 886 \text{ K}$$

Also,
$$\eta_{turbine\,(H.P.)} = \frac{T_6 - T_7'}{T_6 - T_7}$$

i.e.,
$$0.82 = \frac{1023 - 886}{1023 - T_7}$$

$\therefore$
$$T_7 = 1023 - \left(\frac{1023 - 886}{0.82}\right) = 856 \text{ K}$$

Now, $$\frac{T_6}{T_7} = \left(\frac{p_6}{p_7}\right)^{\frac{\gamma-1}{\gamma}}$$

$\therefore$ $$\frac{p_6}{p_7} = \left(\frac{T_6}{T_7}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{1023}{856}\right)^{\frac{1.33}{1.33-1}} = 2.05$$

i.e., $$p_7 = \frac{p_6}{2.05} = \frac{4.2}{2.05} = 2.05 \text{ bar} \qquad [\because \; p_6 = 1.05 \times 4 = 4.2 \text{ bar}]$$

$$\frac{T_7'}{T_8} = \left(\frac{p_7}{p_8}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{2.05}{1.05}\right)^{\frac{1.33-1}{1.33}} = 1.18$$

$$T_8 = \frac{T_7'}{1.18} = \frac{886}{1.18} = 751 \text{ K}$$

Again, $$\eta_{\text{turbine (L.P.)}} = \frac{T_7' - T_8'}{T_7' - T_8}$$

$$0.82 = \frac{886 - T_8'}{886 - 751}$$

$\therefore$ $$T_8' = 886 - 0.82\,(886 - 751) = 775 \text{ K}$$

(*i*) **Power output :**

Net power output $= c_{pg}\,(T_7' - T_8')$

$= 1.15\,(886 - 775) = 127.6$ kJ/kg

$\therefore$ Net output per second $= \dot{m} \times 127.6$

$= 16 \times 127.6 = 2041.6$ kJ/s = **2041.6 kW. (Ans.)**

(*ii*) **Thermal efficiency :**

Effectiveness of heat exchanger, $\varepsilon = \dfrac{T_5 - T_4'}{T_8' - T_4'}$

i.e., $$0.72 = \frac{T_5 - 380}{775 - 380}$$

$\therefore$ $$T_5 = 0.72(775 - 380) + 380 = 664 \text{ K}$$

Heat supplied in combustion chamber per second

$= \dot{m}_a\, c_{pg}\,(T_6 - T_5)$

$= 16 \times 1.15(1023 - 664) = 6605.6$ kJ/s

$\therefore$ $$\eta_{\text{thermal}} = \frac{2041.6}{6605.6} = 0.309 \text{ or } \mathbf{30.9\%.}\ \textbf{(Ans.)}$$

(*iii*) **Specific fuel consumption :**

If m_f is the mass of fuel supplied per kg of air, then

$$m_f \times 42000 = 1.15(1023 - 664)$$

$\therefore$ $$\frac{1}{m_f} = \frac{42000}{1.15(1023 - 664)} = \frac{101.7}{1}$$

$\therefore$ Air-fuel ratio $= 101.7 : 1$

$\therefore$ **Fuel supplied per hour** $= \dfrac{16 \times 3600}{101.7} = 566.37$ kg/h

$\therefore$ **Specific fuel consumption** $= \dfrac{566.37}{2041.6} =$ **0.277 kg/kWh. (Ans.)**

Example 5.11. *Air is taken in a gas turbine plant at 1.1 bar 20°C. The plant comprises of L.P. and H.P. compressors and L.P. and H.P. turbines. The compression in L.P. stage is upto 3.3 bar followed by intercooling to 27°C. The pressure of air after H.P. compressor is 9.45 bar. Loss in pressure during intercooling is 0.15 bar. Air from H.P. compressor is transferred to heat exchanger of effectiveness 0.65 where it is heated by the gases from L.P. turbine. After heat exchanger the air passes through combustion chamber. The temperature of gases supplied to H.P. turbine is 700°C. The gases expand in H.P. turbine to 3.62 bar and air then reheated to 670°C before expanding in L.P. turbine. The loss of pressure in reheater is 0.12 bar. Determine :*

(i) The overall efficiency *(ii) The work ratio*

(iii) Mass flow rate when the power generated is 6000 kW.

Assume : Isentropic efficiency of compression in both stages = 0.82.

Isentropic efficiency of expansion in turbines = 0.85.

For air : $c_p = 1.005$ *kJ/kg K,* $\gamma = 1.4$.

For gases : $c_p = 1.15$ *kJ/kg K,* $\gamma = 1.33$.

Neglect the mass of fuel.

Solution. *Given :* $T_1 = 20 + 273 = 293$ K, $p_1 = 1.1$ bar, $p_2 = 3.3$ bar, $T_3 = 27 + 273 = 300$ K,

$p_3 = 3.3 - 0.15 = 3.15$ bar, $p_4 = p_6 = 9.45$ bar, $T_6 = 973$ K,

$T_8 = 670 + 273 = 943$ K, $p_8 = 3.5$ bar,

$\eta_{compressors} = 82\%$, $\eta_{turbines} = 85\%$, Power generated = 6000 kW,

Effectiveness, $\varepsilon = 0.65$, $c_{pa} = 1.005$ kJ/kg K, $\gamma_{air} = 1.44$, $c_{pg} = 1.15$ kJ/kg K and $\gamma_{gases} = 1.33$.

Refer Fig. 5.58

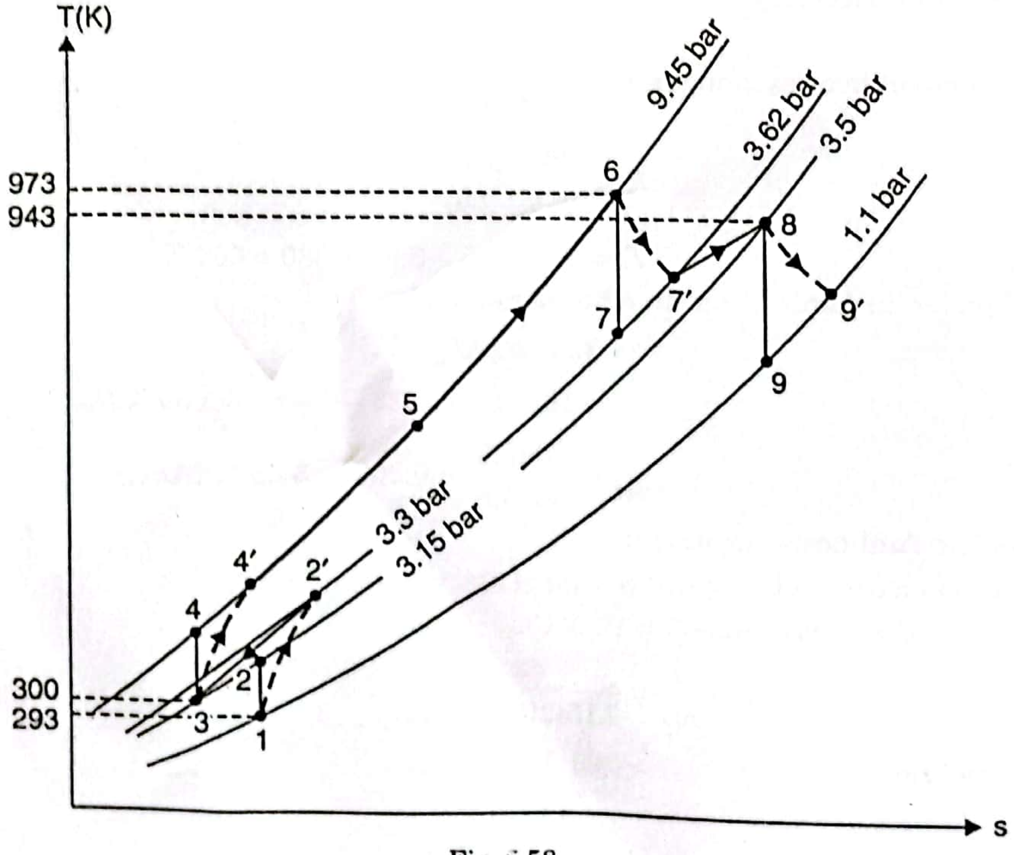


Fig. 5.58

Now, $$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{3.3}{1.1}\right)^{\frac{1.4-1}{1.4}} = 1.369$$

∴ $$T_2 = 293 \times 1.369 = 401 \text{ K}$$

$$\eta_{\text{compressor (L.P.)}} = 0.82 = \frac{T_2 - T_1}{T_2' - T_1} = \frac{401 - 293}{T_2' - 293}$$

∴ $$T_2' = \left(\frac{401 - 293}{0.82}\right) + 293 = 425 \text{ K}$$

Again, $$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{9.45}{3.15}\right)^{\frac{1.4-1}{1.4}} = 1.369$$

∴ $$T_4 = 300 \times 1.369 = 411 \text{ K}$$

Now, $$\eta_{\text{compressor (H.P.)}} = \frac{T_4 - T_3}{T_4' - T_3}$$

$$0.82 = \frac{411 - 300}{T_4' - 300}$$

∴ $$T_4' = \left(\frac{411 - 300}{0.82}\right) + 300 = 435 \text{ K}$$

Similarly, $$\frac{T_6}{T_7} = \left(\frac{p_6}{p_7}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{9.45}{3.62}\right)^{\frac{1.33-1}{1.33}} = 1.268$$

∴ $$T_7 = \frac{T_6}{1.268} = \frac{973}{1.268} = 767 \text{ K}$$

Also, $$\eta_{\text{turbine (H.P.)}} = \frac{T_6 - T_7'}{T_6 - T_7}$$

$$0.85 = \frac{973 - T_7'}{973 - 767}$$

∴ $$T_7' = 973 - 0.85(973 - 767) = 798 \text{ K}$$

Again, $$\frac{T_8}{T_9} = \left(\frac{p_8}{p_9}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{3.5}{1.1}\right)^{\frac{1.33-1}{1.33}} = 1.332$$

∴ $$T_9 = \frac{T_8}{1.332} = \frac{943}{1.332} = 708 \text{ K}$$

$$\eta_{\text{turbine (L.P.)}} = \frac{T_8 - T_9'}{T_8 - T_9}$$

$$0.85 = \frac{943 - T_9'}{943 - 708}$$

∴ $$T_9' = 943 - 0.85(943 - 708) = 743 \text{ K.}$$

Effectiveness of heat exchanger,

$$\varepsilon = 0.65 = \frac{T_5 - T_4'}{T_9' - T_4'}$$

i.e., $$0.65 = \frac{T_5 - 435}{743 - 435}$$

$\therefore$ $$T_5 = 0.65(743 - 435) + 435 = 635 \text{ K}$$

$$W_{\text{turbine (H.P.)}} = c_{pg}(T_6 - T_7') = 1.15(973 - 798) = 201.25 \text{ kJ/kg of gas}$$

$$W_{\text{turbine (L.P.)}} = c_{pg}(T_8 - T_9') = 1.15(943 - 743) = 230 \text{ kJ/kg of gas}$$

$$W_{\text{compressor (L.P.)}} = c_{pa}(T_2' - T_1) = 1.005(425 - 293) = 132.66 \text{ kJ/kg of air}$$

$$W_{\text{compressor (H.P.)}} = c_{pa}(T_4' - T_3) = 1.005(435 - 300) = 135.67 \text{ kJ/kg of air}$$

Heat supplied $$= c_{pg}(T_6 - T_5) + c_{pg}(T_8 - T_7') = 1.15(973 - 635) + 1.15(943 - 798) = 555.45 \text{ kJ/kg of gas}$$

(*i*) **Overall efficiency $\eta_{overall}$:**

$$\eta_{\text{overall}} = \frac{\text{Net work done}}{\text{Heat supplied}} = \frac{[W_{\text{turbine (H.P.)}} + W_{\text{turbine (L.P.)}}] - [W_{\text{comp. (L.P.)}} + W_{\text{comp. (H.P.)}}]}{\text{Heat supplied}}$$

$$= \frac{(201.25 + 230) - (132.66 + 135.67)}{555.45}$$

$$= \frac{162.92}{555.45} = 0.293 \text{ or } \mathbf{29.3\%}. \quad \textbf{(Ans.)}$$

(*ii*) **Work ratio :**

$$\textit{Work ratio} = \frac{\text{Net work done}}{\text{Turbine work}} = \frac{[W_{turbine\text{ (H.P.)}} + W_{turbine\text{ (L.P.)}}] - [W_{comp.\text{ (L.P.)}} + W_{comp.\text{ (H.P.)}}]}{[W_{turbine\text{ (H.P.)}} + W_{turbine\text{ (L.P.)}}]}$$

$$= \frac{(201.25 + 230) - (132.66 + 135.67)}{(201.25 + 230)} = \frac{162.92}{431.25} = 0.377.$$

i.e., *Work ratio* = **0.377. (Ans.)**

(*iii*) **Mass flow rate, $\dot{m}$:**

Net work done = 162.92 kJ/kg.

Since mass of fuel is neglected, for 6000 kW, mass flow rate,

$$\dot{m} = \frac{6000}{162.92} = 36.83 \text{ kg/s}$$

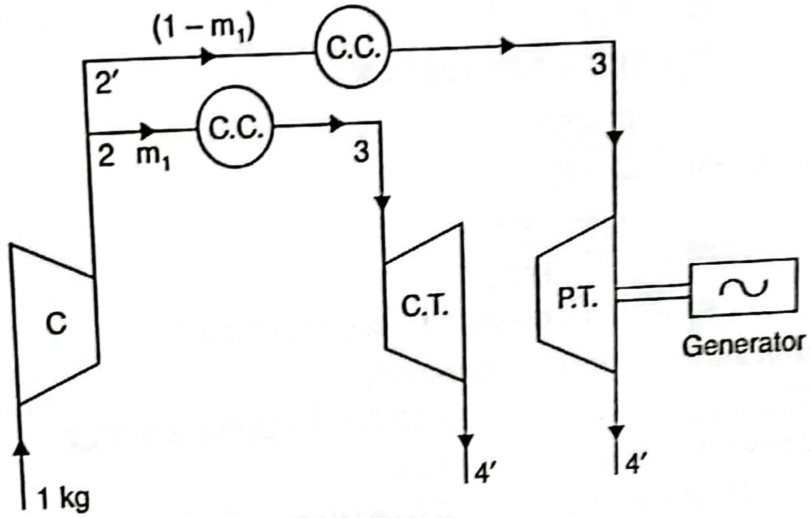
i.e., *Mass flow rate* = **36.83 kg/s. (Ans.)**

ADDITIONAL / TYPICAL EXAMPLES

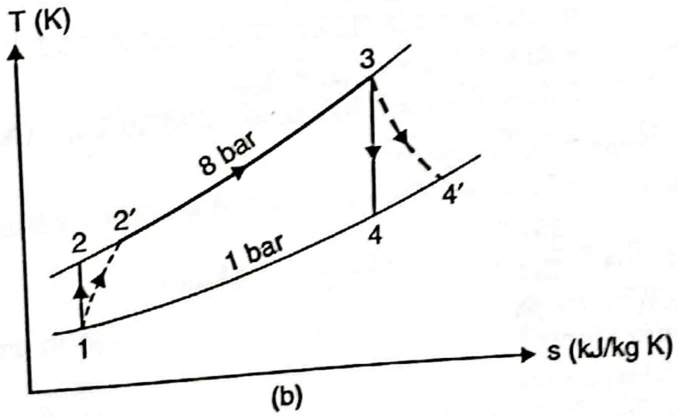
Example 5.12. *A gas turbine plant consists of two turbines. One compressor turbine to drive compressor and other power turbine to develop power output and both are having their own combustion chambers which are served by air directly from the compressor. Air enters the compressor* *at 1 bar and 288 K and is compressed to 8 bar with an isentropic efficiency of 76%. Due to heat added in the combustion chamber, the inlet temperature of gas to both turbines is 900°C. The isentropic efficiency of turbines is 86%. The mass flow rate of air at the compressor is 23 kg/s. The calorific value of fuel is 4200 kJ/kg. Calculate the output of the plant and the thermal efficiency if mechanical efficiency is 95% and generator efficiency is 96%. Take c_p = 1.005 kJ/kg K and γ = 1.4 for air and c_{pg} = 1.128 kJ/kg K and γ = 1.34 for gases.* **(M.U.)**

Solution. *Given :* $p_1 = 1$ bar ; $T_1 = 288$ K ; $p_2 = 8$ bar, $\eta_{(isen)} = 76\%$; $T_3 = 900°C$ or 1173 K, $\eta_{t(isen)} = 86\%$, $m_a = 23$ kg/s ; C.V. = 4200 kJ/kg ; $\eta_{mech.} = 95\%$; $\eta_{gen.} = 96\%$; $c_p = 1.005$ kJ/kg ; $\gamma_a = 1.4$; $c_{pg} = 1.128$ kJ/kg K ; $\gamma_g = 1.34$.

The arrangement of the plant and the cycle are shown in Fig. 5.59 (*a*), (*b*) respectively.



(a)



(b)

Fig. 5.59

Considering *isentropic compression process 1-2*, we have

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{8}{1}\right)^{\frac{1.4-1}{1.4}} = 1.811$$

$$\therefore \qquad T_2 = 288 \times 1.811 = 521.6 \text{ K}$$

Also, $$\eta_{C(isen.)} = \frac{T_2 - T_1}{T_2' - T_1}$$

or $$0.76 = \frac{521.6 - 288}{T_2' - 288}$$

or $$T_2' = \frac{521.6 - 288}{0.76} + 288 = 595.4 \text{ K}$$

Considering *isentropic expansion process 3-4*, we have

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{8}\right)^{\frac{1.34-1}{1.34}} = 0.59$$

$$\therefore \qquad T_4 = 1173 \times 0.59 = 692.1 \text{ K}$$

Also, $$\eta_{T(isen.)} = \frac{T_3 - T_4'}{T_3 - T_4}$$

or $$0.86 = \frac{1173 - T_4'}{1173 - 692.1}$$

$$\therefore \qquad T_4' = 1173 - 0.86(1173 - 692.1) = 759.4 \text{ K}$$

Consider 1 kg of air flow through compressor.

$$W_{compressor} = c_p\,(T_2' - T_1) = 1.005(595.4 - 288) = 308.9 \text{ kJ}$$

This is equal to work of compressor turbine.

$$\therefore \qquad 308.9 = m_1 \times c_{pg}\,(T_3 - T_4'), \text{ neglecting fuel mass}$$

or $$m_1 = \frac{308.9}{1.128\,(1173 - 759.4)} = 0.662 \text{ kg}$$

and flow through the power turbine $= 1 - m_1 = 1 - 0.662 = 0.338$ kg

$$\therefore \qquad W_{PT} = (1 - m_1) \times c_{pg}(T_3 - T_4')$$
$$= 0.338 \times 1.128\,(1173 - 759.4) = 157.7 \text{ kJ}$$

$\therefore$ **Power output** $= 23 \times 157.7 \times \eta_{mech.} \times \eta_{gen.}$

$= 23 \times 157.7 \times 0.95 \times 0.96 =$ **3307.9 kJ. (Ans.)**

$$Q_{input} = c_{pg}T_3 - c_{pa}T_2'$$
$$= 1.128 \times 1173 - 1.005 \times 595.4 = 724.7 \text{ kJ/kg of air}$$

Thermal efficiency, $\eta_{th} = \dfrac{157.7}{724.7} \times 100 =$ **21.76%. (Ans.)**

Example 5.13. *(a) Why are the back work ratios relatively high in gas turbine plants compared to those of steam power plants ?*

(b) In a gas turbine plant compression is carried out in two stages with perfect intercooling and expansion in one stage turbine. If the maximum temperature (T_{max} K) and minimum temperature (T_{min}K) in the cycle remain constant, show that for maximum specific output of the plant, the optimum overall pressure ratio is given by

$$r_{opt.} = \left(\eta_T \cdot \eta_C \cdot \frac{T_{max}}{T_{min}}\right)^{\frac{2\gamma}{3(\gamma-1)}}$$

where γ *= Adiabatic index ;* η_T *= Isentropic efficiency of the turbine.*

η_C *= Isentropic efficiency of compressor.* **(N.U.)**

Solution. (*a*) **Back work ratio** may be defined as *the ratio of negative work to the turbine work in a power plant*. In gas turbine plants, air is compressed from the turbine exhaust pressure to the combustion chamber pressure. This work is given by $-\int vdp$. As the specific volume of air is very high (even in closed cycle gas turbine plants), the compressor work required is very high, and also *bulky compressor is required*. In steam power plants, the turbine exhaust is changed to liquid phase in the condenser. The pressure of condensate is raised to boiler pressure by condensate extraction pump and boiler feed pump in series since the specific volume of water is very small as compared to that of air, the pump work $(-\int vdp)$, is also very small. From the above reasons, the back work ratio

$$= \frac{-\int vdp}{\text{Turbine work}}$$

for gas *turbine plants is relatively high compared to that for steam power plants.*

(*b*) Refer Fig. 5.60.

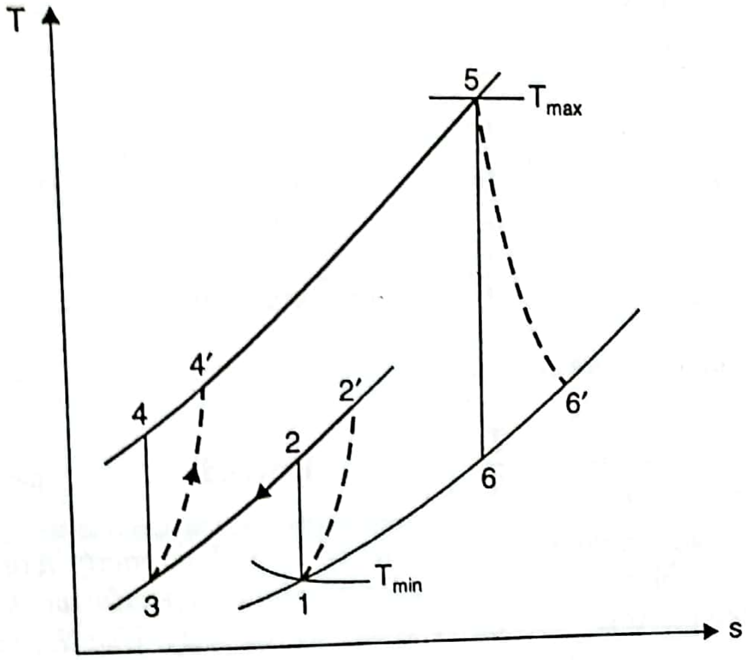


Fig. 5.60

Assuming optimum pressure ratio in each stage of the compressors as $\sqrt{r}$, we have

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

or

$$T_2 = T_{min} \times (r)^{\frac{\gamma-1}{2\gamma}}$$

$$W_{compressor} = 2[c_p\,(T_2' - T_1)] \text{ for both compressors}$$

$$= 2c_p\,\frac{T_2 - T_1}{\eta_C} = \frac{2c_p}{\eta_C}\,T_{min}\left[(r)^{\frac{\gamma-1}{2\gamma}} - 1\right], \text{ as } T_1 = T_{min}$$

Also,

$$\frac{T_5}{T_6} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (r)^{\frac{\gamma-1}{2\gamma}}$$

$$\therefore \qquad T_2' = 298 + \frac{452 - 298}{0.82} = 485.8 \text{ K}$$

Turbine :
$$\frac{T_4}{T_5} = \left(\frac{p_4}{p_5}\right)^{\frac{\gamma-1}{\gamma}} = (4.3)^{\frac{1.4-1}{1.4}} = 1.517$$

$$\therefore \qquad T_5 = \frac{T_4}{1.57} = \frac{950}{1.517} = 626.2 \text{ K}$$

$$\eta_T = 0.88 = \frac{T_4 - T_5'}{T_4 - T_5} = \frac{950 - T_5'}{950 - 626.2}$$

$$\therefore \qquad T_{5'} = 950 - 0.88\,(950 - 626.2) = 665 \text{ K}$$

Work developed per hour (neglecting fuel mass)

$$= m_a \times c_{pa}\,[(T_4 - T_{5'}) - (T_{2'} - T_1)] \times \eta_{mech.} \times \eta_{gen.} = 12 \times 10^3 \text{ kW}$$

where m_a is the mass of air passed per second.

$$m_a \times 1.005\,[(950 - 665) - (485.8 - 298)] \times 0.94 \times 0.94 = 12 \times 10^3$$

$$86.31\, m_a = 12 \times 10^3$$

$$\therefore \qquad m_a = 139.03 \text{ kg/s}$$

Heat exchanger :

Effectiveness of heat exchanger,

$$\epsilon = \frac{T_3 - T_{2'}}{T_{5'} - T_{2'}} \quad \text{(neglecting fuel mass)}$$

$$\therefore \qquad 0.68 = \frac{T_3 - 485.8}{665 - 485.8}$$

or
$$T_3 = 485.8 + 0.68\,(665 - 485.8) = 607.6 \text{ K}$$

Combustion chamber :

Considering the combustion process in combustion chamber, we have :

Now, $\qquad m_f \times C \times \eta_{comb.} \times 0.9 = m_a\, c_{pa}\,(T_4 - T_3)$

where m_f is the mass of fuel burned per sec.

or
$$m_f \times 41000 \times 0.95 \times 0.9 = 139.03 \times 1.005\,(950 - 607.6)$$

$$\therefore \qquad m_f = 1.365 \text{ kg/s}$$

$$\text{Cost of fuel} = \frac{1.365 \times 3600}{1000} \times 4500 = \text{Rs. } 17690.4\text{/h}$$

Total cost per hour = Cost of fuel / hour + All other charges including profit per hour
= 17690.4 + 3500 = Rs. 21190.4/h

$$\therefore \quad \textit{Cost of energy generated} = \frac{21190.4}{12 \times 10^3} = \textbf{Rs. 1.76/kWh.} \quad \textbf{(Ans.)}$$

Example 5.15. *A gas turbine power plant works on constant pressure open cycle. It consists of compressor, generator, combustion chamber and turbine (the compressor, turbine and generator mounted on the same shaft). The following data is given for this plant :*

The pressure and temperature of air entering into the compressor	*= 1 bar, 25°C*
The pressure of air leaving the compressor	*= 4 bar*
Isentropic efficiency of the compressor	*= 82 per cent*
Isentropic efficiency of the turbine	*= 86 per cent*

Effectiveness of the regenerator	= 72 per cent
Pressure loss in regenerator along air side	= 0.08 bar
Pressure loss in regenerator along gas side	= 0.08 bar
Pressure loss in the combustion chamber	= 0.04 bar
Combustion efficiency	= 92 per cent
Mechanical efficiency	= 94 per cent
Generation efficiency	= 94 per cent
Calorific value of fuel used	= 40000 kJ/kg
Flow of air	= 24 kg/s
Atmospheric pressure	= 1.03 bar
The maximum temperature of the cycle	= 690°C

Determine the following :

(i) The power available at the generator terminals,

(ii) The overall efficiency of the plant, and

(iii) The specific fuel consumption.

Take $\gamma = 1.4$ *for air and gases.*

$c_{pa} = 1$ kJ/kg K ; $c_{pg} = 1.1$ kJ/kg K.

Solution. *Given :* $p_1 = 1$ bar ; $T_1 = 25 + 273 = 298$ K ; $p_2 = p_{2'} = 4$ bar ; $\eta_c = 82\%$; $\eta_T = 86\%$; $\epsilon = 0.72$; $\eta_{comb.} = 92\%$; $\eta_{mech.} = 94\%$; $\eta_{gen.} = 94\%$; $C = 40000$ kJ/kg ; $m_a = 24$ kg/s ; $p_{atm.} = 1.03$ bar ; $T_4 = 690 + 273 = 963$ K.

The schematic arrangement of the plant and its corresponding *T-s* diagram are shown in Fig. 5.62 (*a*) and (*b*) respectively.

Pressure at the inlet to the turbine, $p_4 = 4 - (0.08 + 0.04) = 3.88$ bar

Pressure at exit of the turbine, $p_5 = 1.03 + 0.08 = 1.11$ bar

Compressor :

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4}{1}\right)^{\frac{1.4-1}{1.4}} = 1.486$$

$\therefore$
$$T_2 = T_1 \times 1.486 = 298 \times 1.486 = 442.8 \text{ K}$$

$$\eta_{comp.} = 0.82 = \frac{T_2 - T_1}{T_{2'} - T_1} = \frac{442.8 - 298}{T_{2'} - 298}$$

$\therefore$
$$T_{2'} = 298 + \frac{442.8 - 298}{0.82} = 474.6 \text{ K}$$

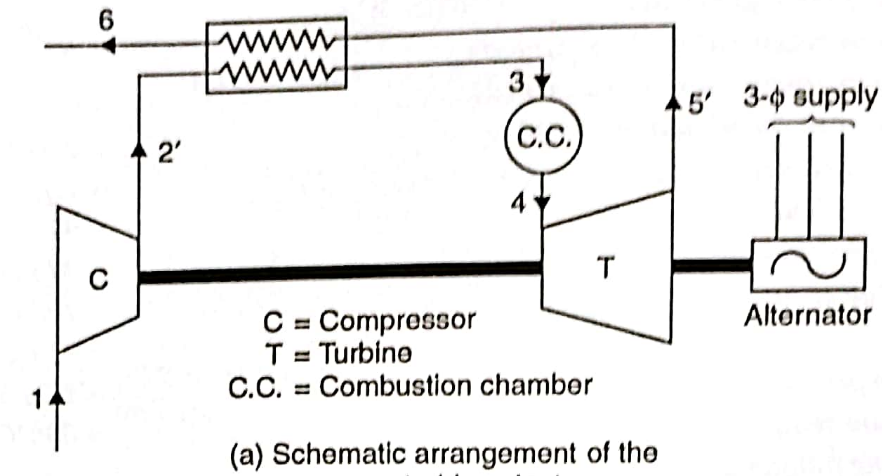
Turbine :

$$\frac{T_4}{T_5} = \left(\frac{p_4}{p_5}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{3.88}{1.11}\right)^{\frac{1.4-1}{1.4}} = 1.43$$

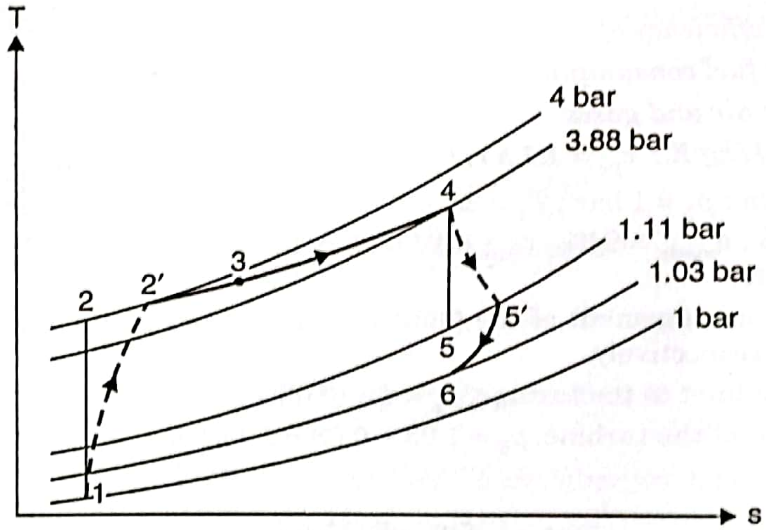
$\therefore$
$$T_5 = \frac{T_4}{1.43} = \frac{963}{1.43} = 673.4 \text{ K}$$

$$\eta_{turbine} = 0.86 = \frac{T_4 - T_{5'}}{T_4 - T_5} = \frac{963 - T_{5'}}{963 - 673.4}$$

$\therefore$
$$T_{5'} = 963 - 0.86\,(963 - 673.4) = 713.9 \text{ K}$$



(a) Schematic arrangement of the gas turbine plant



(b) T-s diagram

Fig. 5.62

Combustion Chamber :

Considering the combustion process in combustion chamber, we have

$$m_f \times C \times \eta_{comb.} = (m_a + m_f)\, c_{pg}\,(T_4 - T_3)$$

$$\therefore \qquad C \times \eta_{comb.} = \left(\frac{m_a}{m_f} + 1\right) c_{pg}\,(T_4 - T_3)$$

or
$$40000 \times 0.92 = \left(\frac{m_a}{m_f} + 1\right) \times 1.1\,(963 - T_3)$$

or
$$\frac{m_a}{m_f} = \frac{40000 \times 0.92}{1.1(963 - T_3)} - 1 = \frac{33455}{(963 - T_3)} - 1 \qquad \text{...(1)}$$

Also, effectiveness of the regenerator,

$$\epsilon = \frac{m_a\, c_{pa}\,(T_3 - T_{2'})}{(m_a + m_f)\, c_{pg}\,(T_{5'} - T_{2'})}$$

or $$0.72 = \frac{1(T_3 - 474.6)}{\left(1+\frac{m_f}{m_a}\right) \times 1.11(713.9 - 474.6)}$$

or $$\frac{m_f}{m_a} = \frac{(T_3 - 474.6)}{0.72 \times 1.11(713.9 - 474.6)} - 1 = \frac{(T_3 - 474.6)}{191.2} - 1 = \frac{T_3 - 665.8}{191.2}$$

or $$\frac{m_a}{m_f} = \frac{191.2}{T_3 - 665.8} \quad ...(2)$$

From eqns. (1) and (2), we get

$$\frac{33455}{963 - T_3} - 1 = \frac{191.2}{T_3 - 665.8} \quad \text{or} \quad \frac{33455 - (963 - T_3)}{963 - T_3} = \frac{191.2}{T_3 - 665.8}$$

$$\frac{32492 + T_3}{963 - T_3} = \frac{191.2}{T_3 - 665.8}$$

$$(32492 + T_3)(T_3 - 665.8) = 191.2(963 - T_3)$$

$$32492\, T_3 - 21633174 + T_3^2 - 665.8\, T_3 = 184126 - 191.2\, T_3$$

$$T_3^2 + 32017\, T_3 - 21817300 = 0$$

$\therefore$ $$T_3 = \frac{-32017 \pm \sqrt{(32017)^2 + 4 \times 21817300}}{2}$$

$$= \frac{-32017 \pm 33352}{2} = 667.5 \text{ K}$$

i.e., $$T_3 = 667.5 \text{ K}$$

and $$\frac{m_a}{m_f} = 112 : 1.$$

(i) The power available at generator terminals :

$$W_{comp.} = 1 \times c_{pa}(T_{2'} - T_1) = 1(474.6 - 298) = 176.6 \text{ kJ/kg or air}$$

$$W_{turbine} = (1 + m_f) \times c_{pg} \times (T_4 - T_{5'})$$

$$= \left(1 + \frac{1}{112}\right) \times 1.1(963 - 713.9) = 276.4 \text{ kJ/kg of air}$$

$$W_{net} = W_{turbine} - W_{comp.}$$
$$= 276.4 - 176.6 = 99.8 \text{ kJ/kg of air}$$

Work available per kg of air at the terminals of generator

$$= 99.8 \times \eta_{mech.} \times \eta_{gen.} = 99.8 \times 0.94 \times 0.94 = 88.18 \text{ kJ/kg}$$

Power available at the generator terminal

$$= \frac{24 \times 88.18}{1000} = \mathbf{2.116\ MW.} \quad \textbf{(Ans.)}$$

(ii) Overall efficiency of the plant, $\eta_{overall}$:

$$\eta_{overall} = \frac{88.18}{\frac{1}{112} \times 40000} \times 100 = \mathbf{24.69\%.} \quad \textbf{(Ans.)}$$

(iii) Specific fuel consumption :

Fuel required per hour $= (24 \times 3600) \times \dfrac{1}{112} = 771.43$ kg/h

$\therefore$ Specific fuel consumption $= \dfrac{771.43}{2.116 \times 1000}$ = **0.364 kg/kWh. (Ans.)**

Example 5.16. *The following data relate to a gas turbine plant :*

Power developed	= 5 MW
Inlet pressure and temperature of air to the compressor	= 1 bar, 30°C
Pressure ratio of the cycle	= 5
Isentropic efficiency of the compressor	= 80 per cent
Isentropic efficiency of both turbines	= 85 per cent
Maximum temperature in both turbines	= 550°C

$c_{pa} = 1.0\ kJ/kg\ K$; $c_{pg} = 1.15\ kJ/kg\ K$; $\gamma\,(air) = 1.4$; $\gamma\,(gases) = 1.33$.

If a reheater is used between two turbines at a pressure 2.24 bar, calculate the following :

(*i*) *the mass flow rate of air, and* (*ii*) *the overall efficiency.*

Neglect the mass of the fuel. **(P.U. Winter, 1999)**

Solution. *Given :* Power developed = 5 MW ; $p_1 = 1$ bar ; $T_1 = 30 + 273 = 303$ K ;

$r_p = \dfrac{p_3}{p_1} = 5$ or $p_3 = 5 \times 1 = 5$ bar ; $p_2 = 2.24$ bar ; $T_3 = T_5 = 550 + 273 = 823$ K ; $\eta_{comp.} = 80\%$,

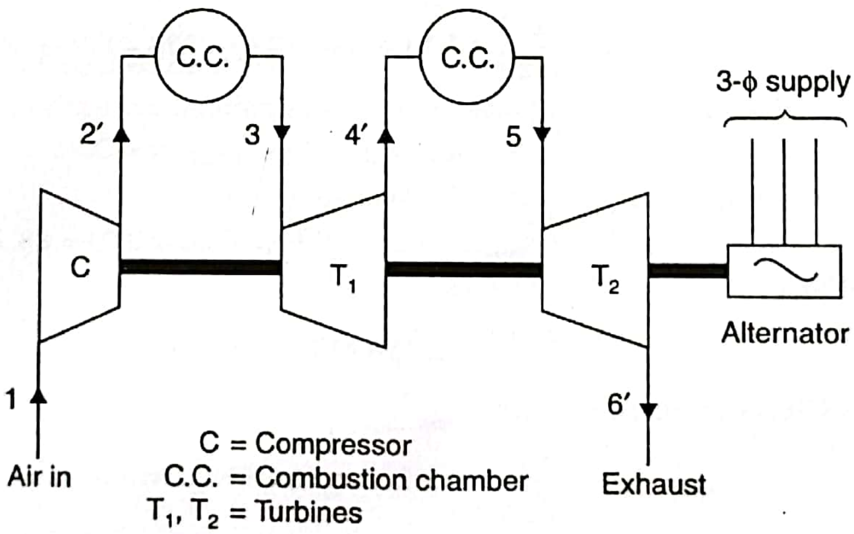
$\eta_{T_1} = \eta_{T_2} = 85\%$; $c_{pa} = 1$ kJ/kg K ; $c_{pg} = 1.15$ kJ/kg K ; γ (air) = 1.4 ; γ (gases) = 1.33.

The schematic arrangement and its corresponding *T-s* diagram are shown in Fig. 5.63 (*a*) and (*b*) respectively.

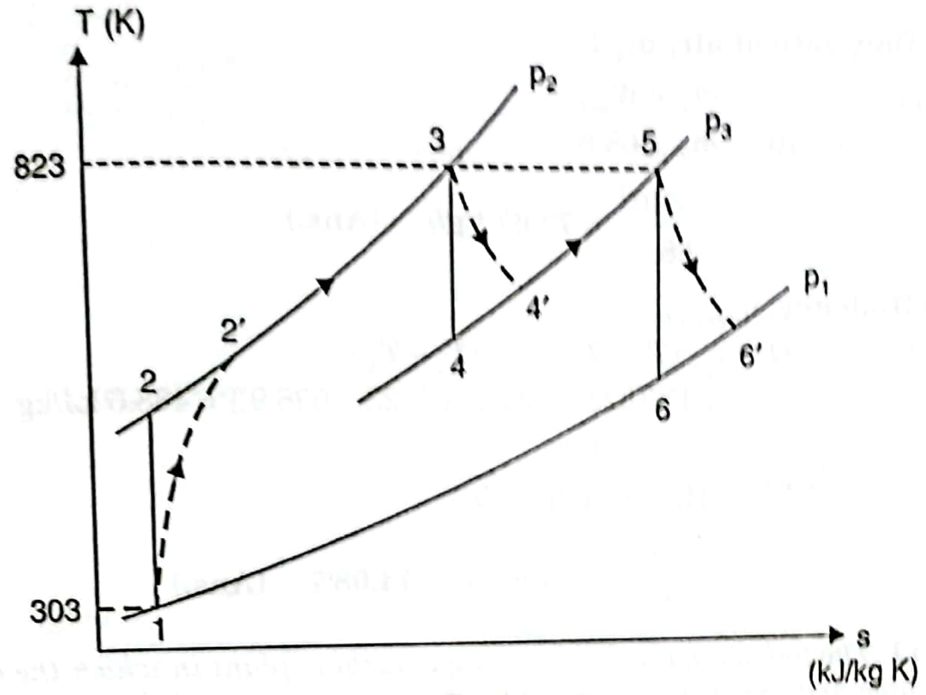
Compressor :
$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (5)^{\frac{1.4-1}{1.4}} = (5)^{0.2857} = 1.584$$

$\therefore$
$$T_2 = T_1 \times 1.584 = 303 \times 1.584 = 480 \text{ K}$$

$$\eta_{comp.} = 0.8 = \frac{T_2 - T_1}{T_{2'} - T_1} = \frac{480 - 303}{T_{2'} - 303}$$



(a) Flow diagram for the plant



(b) T-s diagram

Fig. 5.63

$$\therefore \quad T_{2'} = 303 + \frac{(480 - 303)}{0.8} = 524.2 \text{ K}$$

$$W_{comp.} = c_{pa}(T_{2'} - T_1) = 1.0(524.2 - 303) = 221.2 \text{ kJ/kg}$$

Turbine, T_1:

$$\frac{T_3}{T_4} = \left(\frac{p_2}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{5}{2.24}\right)^{\frac{1.33-1}{1.33}} = \left(\frac{5}{2.24}\right)^{0.2481} = 1.22$$

$$\therefore \quad T_4 = \frac{T_3}{1.22} = \frac{823}{1.22} = 674.6 \text{ K}$$

$$\eta_{T_1} = 0.85 = \frac{T_3 - T_{4'}}{T_3 - T_4} = \frac{823 - T_{4'}}{823 - 674.6}$$

$$\therefore \quad T_{4'} = 823 - 0.85(823 - 674.6) = 696.9 \text{ K}$$

Turbine, T_2:

$$\frac{T_5}{T_6} = \left(\frac{p_3}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{2.24}{1}\right)^{\frac{1.33-1}{1.33}} = (2.24)^{0.2481} = 1.22$$

$$\therefore \quad T_6 = \frac{T_5}{1.22} = \frac{823}{1.22} = 674.6 \text{ K}$$

$$\eta_{T_2} = 0.85 = \frac{T_5 - T_{6'}}{T_5 - T_6} = \frac{823 - T_{6'}}{823 - 674.6}$$

$$\therefore \quad T_{6'} = 823 - 0.85\,(823 - 674.6) = 696.9 \text{ K}$$

$$\therefore \quad (W_{turbine})_{total} = 2\,c_{pg}\,(T_3 - T_{4'}) \quad \text{as} \quad T_3 = T_5 \text{ and } T_{4'} = T_{6'}$$

$$= 2 \times 1.15\,(823 - 696.9) = 290 \text{ kJ/kg}$$

$$\therefore \quad W_{net} = (W_{turbine})_{total} - W_{comp.}$$

$$= 290 - 221.2 = 68.8 \text{ kJ/kg}$$

(i) The mass flow rate of air, $\dot{m}_a$:

Power developed $= \dot{m}_a \times W_{net}$

$$5 \times 10^3 = \dot{m}_a \times 68.8$$

$$\therefore \quad \dot{m}_a = \frac{5 \times 10^3}{68.8} = \mathbf{72.67\ kg/s.} \quad \textbf{(Ans.)}$$

(ii) Overall efficiency, $\eta_{overall}$:

Heat supplied, $\quad Q_s = c_{pg}(T_3 - T_{2'}) + c_{pg}(T_5 - T_{4'})$

$$= 1.15\,[(823 - 524.2) + (823 - 696.9)] = 488.6 \text{ kJ/kg}$$

$$\therefore \quad \eta_{overall} = \frac{W_{net}}{\text{Heat supplied } (Q_s)}$$

$$= \frac{68.8}{488.6} = 0.1408 \text{ or } \mathbf{14.08\%.} \quad \textbf{(Ans.)}$$

Example 5.17. *The following data refer to a gas turbine plant in which the compression is carried out in one stage and expansion is carried out in two stages with reheating to the original temperature.*

Capacity of the gas turbine plant	*= 6 MW*
Temperature at which air is supplied	*= 20°C*
Suction and exhaust pressure	*= 1 bar*
Pressure ratio	*= 6*
Maximum temperature limit	*= 750°C*
Isentropic efficiency of compressor	*= 80 per cent*
Isentropic efficiency of each turbine	*= 84 per cent*
Effectiveness of heat exchanger	*= 0.72*
Calorific value of fuel	*= 18500 kJ/kg*

$c_{pa} = 1$ *kJ/kgK* ; $c_{pg} = 1.15$ *kJ/kg K* ; γ *(air) = 1.4,* γ*(gas) = 1.33.*

Determine the following :

(i) *A/F ratio entering in the first turbine, and*

(ii) *Thermal efficiency of the cycle,*

(iii) *Air supplied to the plant,*

(iv) *Fuel consumption of the plant per hour.*

Solution. *Given :* Capacity of the plant = 6 MW ; $T_1 = 20 + 273 = 293$ K ; $p_1 = 1$ bar ;

$$r_p = \frac{p_3}{p_1} = 6 \text{ or } p_3 = 6 \text{ bar} ; T_4 = T_5 = 750 + 273 = 1023 \text{ K} ;$$

$$\eta_{comp.} = 80\%,\ \eta_{T_1} = \eta_{T_2} = 84\% ;\ \epsilon = 0.72,\ C = 18500 \text{ kJ/kg} ;$$
$$c_{pa} = 1 \text{ kJ/kg K} ;\ c_{pg} = 1.15 \text{ kJ/kg K} ;\ \gamma(\text{air}) = 1.4 ;\ \gamma(\text{gas}) = 1.33.$$

The schematic arrangement of the gas turbine plant and its corresponding T-s diagram are shown in Fig. 5.64 (a) and (b) respectively.

$$p_2 = \sqrt{p_3 \times p_1} = \sqrt{6 \times 1} = 2.45 \text{ bar}$$

Compressor :

$$\frac{T_2}{T_1} = \left(\frac{p_3}{p_1}\right)^{\frac{\gamma - 1}{\gamma}} = (6)^{\frac{1.4-1}{1.4}} = (6)^{0.2857} = 1.668$$

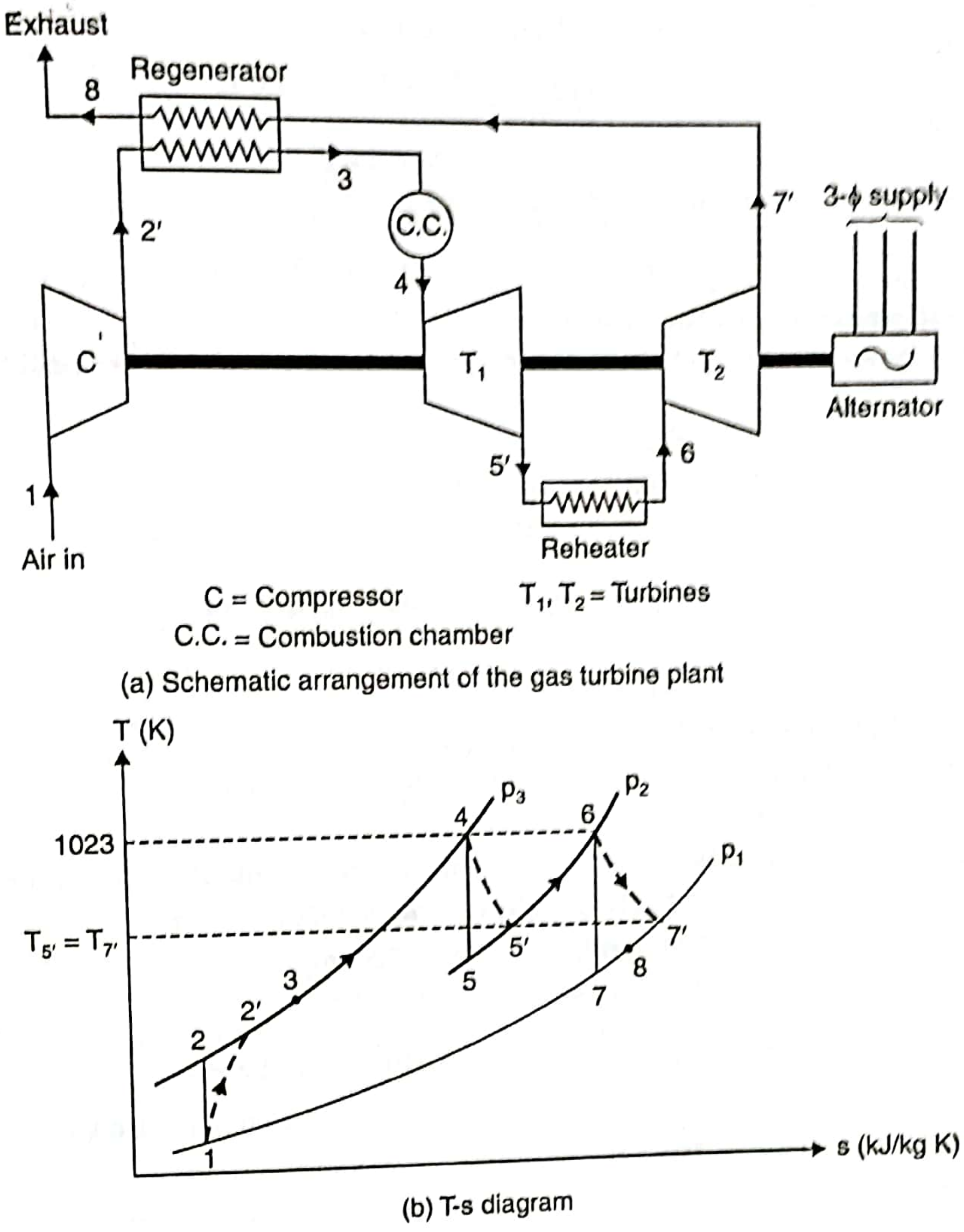


Fig. 5.64

$$\therefore \quad T_2 = T_1 \times 1.668 = 293 \times 1.668 = 488.7 \text{ K}$$

$$\eta_{comp.} = 0.8 = \frac{T_2 - T_1}{T_{2'} - T_1} = \frac{488.7 - 293}{T_{2'} - 293}$$

$$\therefore \quad T_{2'} = 293 + \frac{488.7 - 293}{0.8} = 537.6 \text{ K}$$

Turbine-1

$$\frac{T_4}{T_5} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{6}{2.45}\right)^{\frac{1.33-1}{1.33}} = (2.45)^{0.2481} = 1.249$$

$$\therefore \quad T_5 = \frac{T_4}{1.249} = \frac{1023}{1.249} = 819 \text{ K}$$

$$\eta_{T_1} = 0.84 = \frac{T_4 - T_{5'}}{T_4 - T_5} = \frac{1023 - T_{5'}}{1023 - 819}$$

$\therefore \quad T_{5'} = 1023 - 0.84\,(1023 - 819) = 851.6\ \text{K}$

$\eta_{T_1} = \eta_{T_2}$ (Given), $\therefore \quad T_{5'} = T_{7'} = 851.6$ K

Regenerator :

Effectiveness, $$\epsilon = 0.72 = \frac{T_3 - T_{2'}}{T_{7'} - T_{2'}} = \frac{T_3 - 537.6}{851.6 - 537.6}$$

$\therefore \quad T_3 = 537.6 + 0.72\,(851.6 - 537.6) = 763.7$ K

(*i*) A/F ratio entering in the first turbine :

Let m_{f_1} = Mass of fuel supplied per kg of air to the combustion chamber (and hence to the first turbine)

$\therefore \quad (1 + m_{f_1})\, c_{pg}\,(T_4 - T_3) = m_{f_1} \times C$

or $\quad (1 + m_{f_1}) \times 1.15\,(1023 - 763.7) = m_{f_1} \times 18500$

or $\quad 298.2\,(1 + m_{f_1}) = 18500\, m_{f_1}$

$$\therefore \quad m_{f_1} = \frac{298.2}{(18500 - 298.2)} = 0.01638 \text{ kg/kg of air}$$

$$\therefore \quad A/F\ ratio = \frac{1}{0.01638} = \mathbf{61 : 1.} \quad \textbf{(Ans.)}$$

(*ii*) Thermal efficiency of the cycle, $\eta_{thermal}$:

Considering the flow with burning through the reheater, we have

$$(1 + m_{f_1} + m_{f_2})\, c_{pg}\,(T_6 - T_{5'}) = m_{f_2} \times C$$

where m_{f_2} is the fuel supplied in the reheater per kg of air entering into the compressor.

$\therefore \quad (1 + 0.01638 + m_{f_2}) \times 1.15\,(1023 - 851.6) = m_{f_2} \times 18500$

or $\quad 197.11(1.01638 + m_{f_2}) = 18500\, m_{f_2}$

or $\quad 200.34 + 197.11\, m_{f_2} = 18500\, m_{f_2}$

$$\therefore \quad m_{f_2} = \frac{200.34}{(18500 - 197.11)} = 0.0109 \text{ kg/kg of air}$$

$\therefore \quad W_{comp.} = 1 \times c_{pa}(T_{2'} - T_1) = 1 \times 1 \times (537.6 - 293) = 244.6$ kJ/kg of air

$$\begin{aligned}
W_{\text{Turbine (total)}} &= W_{T_1} + W_{T_2} \\
&= (1 + m_{f_1})\, c_{pg}\,(T_4 - T_{5'}) + (1 + m_{f_1} + m_{f_2})\, c_{pg}\,(T_6 - T_{7'}) \\
&= (1 + 0.01638) \times 1.15(1023 - 851.6) + (1 + 0.01638 + 0.0109) \\
&\qquad \times 1.15(1023 - 851.6) \\
&= 1.15(1023 - 851.6)\,[(1 + 0.01638) + (1 + 0.01638 + 0.0109)] \\
&= 402.8 \text{ kJ/kg of air.}
\end{aligned}$$

$$\begin{aligned}
W_{net} &= W_{\text{turbine (total)}} - W_{comp.} \\
&= 402.8 - 244.6 = 158.2 \text{ kJ/kg of air}
\end{aligned}$$

$$\therefore \quad \eta_{thermal} = \frac{W_{net}}{\text{Heat supplied } (Q_s)} = \frac{W_{net}}{(m_{f_1} + m_{f_2}) \times C}$$

$$= \frac{158.2}{(0.01638 + 0.0109) \times 18500} = 0.3135 \quad \text{or} \quad \mathbf{31.35\%.} \quad \textbf{(Ans.)}$$

(*iii*) Air supplied to the plant, m_a :

$$m_a \times W_{net} = 6 \times 1000$$

$$m_a = \frac{6000}{W_{net}} = \frac{6000}{158.2} = \mathbf{37.93\ kg/s.} \quad \textbf{(Ans.)}$$

(iv) Fuel consumption of the plant per hour :

Fuel consumption of the plant per hour

$$= m_a(m_{f_1} + m_{f_2}) \times 3600$$
$$= 37.93(0.01638 + 0.0109) \times 3600 = \mathbf{3725\ kg/h.} \quad \textbf{(Ans.)}$$

Example 5.18. *An open cycle constant pressure gas turbine power plant of 1500 kW capacity comprises a single stage compressor and two turbines with regenerator. One turbine is used to run the compressor and other one runs the generator. Separate combustion chamber is used for each turbine. Air coming out from the regenerator is divided into two streams, one goes to compressor turbine and other to power turbine. The pressure and temperature of air entering the compressor are 1 bar and 25°C. The maximum temperature in the compressor turbine is 750°C and in power turbine is 800°C. The maximum pressure in the stream is 5 bar. The exhaust pressure of both turbine is 1 bar and both exhausts pass through the regenerator. The temperature of exhaust entering the regenerator is 475°C.*

Use the following data :

Isentropic efficiency of compressor ($\eta_{comp.}$)	= *82 per cent*
Isentropic efficiency of compressor turbine (η_{T_1})	= *84 per cent*
Isentropic efficiency of power turbine (η_{T_2})	= *89 per cent*
Calorific value of fuel	= *40500 kJ/kg*
Combustion efficiency ($\eta_{comb.}$) *in both the combustion chambers*	= *94 per cent*
Mechanical efficiency ($\eta_{mech.}$) *for both the turbines*	= *89 per cent*
Effectiveness of regenerator ($\in$)	= *0.72*

$c_{pa} = 1.005$ *kJ/kg K* ; $c_{pg} = 1.1$ *kJ/kg K;* γ*(for air)* = *1.4* ; γ*(for gases)* = *1.35*

Neglecting pressure losses, heat losses and mass of fuel, determine :

(i) Plant efficiency, *(ii) Specific fuel consumption, and*

(iii) Air fuel ratio.

Solution. *Given* : Capacity of the plant = 1500 kW ; $p_1 = 1$ bar ; $T_1 = 25 + 273 = 298$ K ;
$T_4 = 750 + 273 = 1023$ K ; $T_6 = 800 + 273 = 1073$ K ; $p_1 = 1$ bar ; $p_2 = 5$ bar ;
$T_8 = 475 + 273 = 748$ K ; $\eta_{comp.} = 82\%$; $\eta_{T_1} = 84\ \%$; $\eta_{T_2} = 89\%$;
$C = 40500$ kJ/kg ; $\eta_{comb.} = 94\%$; $\eta_{mech} = 89\%$, $\in = 0.72$ $c_{pa} = 1.005$ kJ/kg K ;
$c_{pg} = 1.1$ kJ/kg K ; γ (for air) = 1.4 ; γ (for gases) = 1.35.

The schematic arrangement of the plant and its corresponding *T-s* diagram are shown in Fig. 5.65 (*a*) and (*b*) respectively.

Compressor :

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{5}{1}\right)^{\frac{1.4-1}{1.4}} = (5)^{0.2857} = 1.584$$

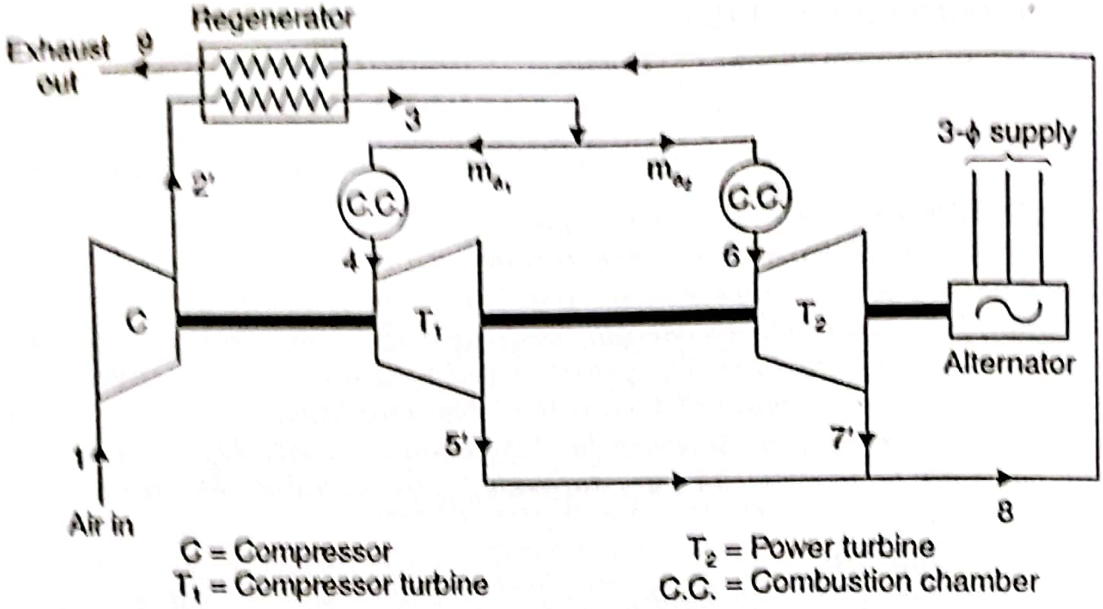
$$\therefore \quad T_2 = T_1 \times 1.584 = 298 \times 1.584 = 472 \text{ K}$$

$$\eta_{comp.} = 0.82 = \frac{T_2 - T_1}{T_{2'} - T_1} = \frac{472 - 298}{T_{2'} - 298}$$

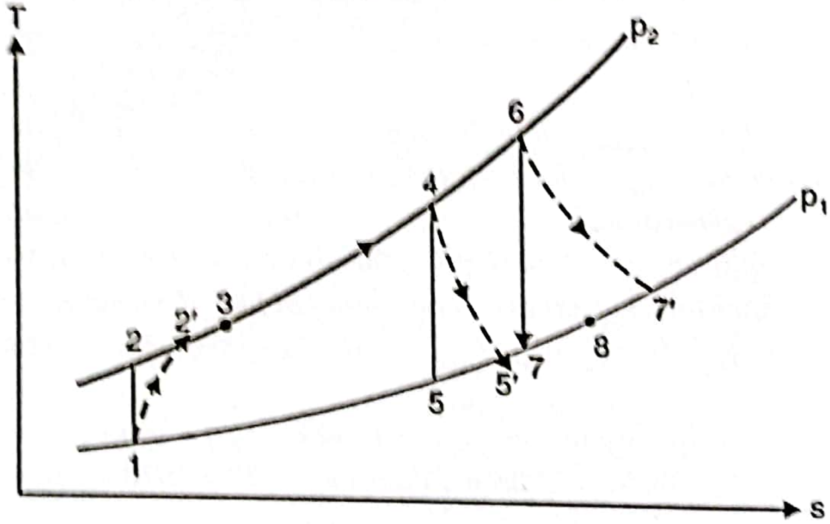
$$\therefore \quad T_{2'} = 298 + \frac{472 - 298}{0.82} = 510.2 \text{ K}$$

Compressor turbine :

$$\frac{T_4}{T_5} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (5)^{\frac{1.35-1}{1.35}} = (5)^{0.2592} = 1.518$$



(a) Schematic arrangement of the gas turbine plant



(b) T-s diagram

Fig. 5.65

$$\therefore \quad T_5 = \frac{T_4}{1.518} = \frac{1023}{1.518} = 673.9 \text{ K}$$

$$\eta_{T_1} = 0.84 = \frac{T_4 - T_{5'}}{T_4 - T_5} = \frac{1023 - T_{5'}}{1023 - 673.9}$$

$$\therefore \quad T_{5'} = 1023 - 0.84(1023 - 673.9) = 729.7 \text{ K}$$

Power Turbine :

$$\frac{T_6}{T_7} = \left(\frac{p_2}{p_1}\right)^{\frac{1.35-1}{1.35}} = (5)^{0.2592} = 1.518$$

$$\therefore \quad T_7 = \frac{T_6}{1.518} = \frac{1073}{1.518} = 706.8 \text{ K}$$

$$\eta_{T_2} = 0.89 = \frac{T_6 - T_{7'}}{T_6 - T_7} = \frac{1073 - T_{7'}}{1073 - 706.8}$$

$$T_{7'} = 1073 - 0.89(1073 - 706.8) = 747.1 \text{ K}$$

∴

Let, m_{a_1} = Mass of air passing through the compressor turbine, and

m_{a_2} = Mass of air passing through the power turbine.

Then, power output of the power turbine is given by :

$$m_{a_2} \times c_{pg} (T_6 - T_{7'}) \times \eta_{mech.} \times \eta_{gen.} = 1500$$

$$m_{a_2} \times 1.1 (1073 - 747.1) \times 0.89 \times 1 = 1500$$

or

$$m_{a_2} = 4.7 \text{ kg/s}$$

∴

Power developed by the compressor turbine = Power absorbed by the compressor

∴

$$m_{a_1} \times c_{pg}(T_4 - T_5') \times \eta_{mech.} = (m_{a_1} + m_{a_2}) \times c_{pa}(T_2' - T_1)$$

or

$$m_{a_1} \times 1.1(1023 - 729.7) \times 0.89 = (m_{a_1} + 4.7) \times 1.005(510.2 - 298)$$

$$287.1\ m_{a_1} = (m_{a_1} + 4.7) \times 213.26$$

∴

$$m_{a_1} = \frac{4.7 \times 213.26}{(287.1 - 213.26)} = 13.57 \text{ kg/s}$$

The exhaust gases of both turbines are mixed before entering into the regenerator. Therefore, the temperature of the gases entering into the regenerator is designated by the point 8 on the T-s diagram and it is given by

$$m_{a_1} \times c_{pg} \times T_5' + m_{a_2} \times c_{pg} \times T_{7'} = (m_{a_1} + m_{a_2}) \times c_{pg} \times T_8$$

where, T_8 is the temperature after mixing.

∴

$$T_8 = \left(\frac{m_{a_1}}{m_{a_1} + m_{a_2}}\right) \times T_{5'} + \left(\frac{m_{a_2}}{m_{a_1} + m_{a_2}}\right) \times T_{7'}$$

$$= \left(\frac{13.57}{13.57 + 4.7}\right) \times 729.7 + \left(\frac{4.7}{13.57 + 4.7}\right) \times 747.1$$

$$= 541.98 + 192.19 = 734.2 \text{ K}$$

Regenerator :

Effectiveness,

$$\epsilon = 0.72 = \frac{m_a\ c_{pa}\ (T_3 - T_{2'})}{m_g\ c_{pg}\ (T_8 - T_{2'})}$$

or

$$0.72 = \frac{c_{pa}\ (T_3 - T_{2'})}{c_{pg}\ (T_8 - T_{2'})} \qquad (\because\ m_a = m_g, \text{ as fuel mass is neglected})$$

$$= \frac{1.005\ (T_3 - 510.2)}{1.1\ (748 - 510.2)}$$

or

$$T_3 = 510.2 + \frac{0.72 \times 1.1\ (748 - 510.2)}{1.005} = 697.6 \text{ K}$$

Combustion chambers :

Total heat supplied in both combustion chambers

$$= c_{pg}\ m_{a_1}(T_4 - T_3) + c_{pg}\ m_{a_2}(T_6 - T_3) = m_f \times C \times \eta_{comb.}$$

or

$$1.1[13.57(1023 - 697.6) + 4.7(1073 - 697.6)] = m_f \times 40500 \times 0.94$$

$$1.1(4415.68 + 1764.38) = 38070 \times m_f$$

∴

$$m_f = 0.178 \text{ kg/s}$$

(i) Plant efficiency :

$$\text{Plant efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{1500}{0.178 \times 40500} = 0.208 \text{ or } \mathbf{20.8\%.} \quad \textbf{(Ans.)}$$

(*ii*) **Specific fuel consumption :**

$$\text{Specific fuel consumption} = \frac{0.178 \times 3600}{1500} = \mathbf{0.427\ kg/kWh.} \quad \textbf{(Ans.)}$$

(*iii*) **Air-fuel (A/F) ratio :**

$$\text{A/F ratio} = \frac{m_{a_1} + m_{a_2}}{m_f} = \frac{13.57 + 4.7}{0.178} = \mathbf{102.64.} \quad \textbf{(Ans.)}$$

Example 5.19. *The air supplied to a gas turbine plant is 10 kg/s. The pressure ratio is 6 and pressure at the inlet of the compressor is 1 bar. The compressor is two-stage and provided with perfect intercooler. The inlet temperature is 300 K and maximum temperature is limited to 1073 K.*

Take the following data :

Isentropic efficiency of compressor each stage ($\eta_{comp.}$) = 80%

Isentropic efficiency of turbine ($\eta_{turbine}$) = 85%

A regenerator is included in plant whose effectiveness is 0.7. Neglecting the mass of fuel, determine the thermal efficiency of the plant.

Take c_p *for air = 1.005 kJ/kg K.* **(P.U. Summer, 1997)**

Solution. *Given :* $\dot{m}_a = 10$ kg/s ; $r_p = 6$, $p_1 = 1$ bar ; $T_1 = 300$ K ; $T_6 = 1073$ K ;

$$\eta_{comp.} = 80\% \ ; \ \eta_{turbine} = 85\% \ ; \ \epsilon = 0.7.$$

The schematic arrangement of the gas turbine plant and its corresponding *T-s* diagram are shown in Fig. 5.66(*a*) and (*b*) respectively. As the cooling is perfect,

$$p_2 = \sqrt{p_1 p_3} = \sqrt{1 \times 6} = 2.45 \text{ bar} \qquad \left[\because\ r_p = 6 = \frac{p_3}{p_1} = \frac{p_3}{1} \text{ or } p_3 = 6 \text{ bar}\right]$$

Considering *isentropic compression in L.P. compressor*, we have :

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}} = \left(\frac{2.45}{1}\right)^{\frac{1.4 - 1}{1.4}} = 1.29$$

$$\therefore \quad T_2 = T_1 \times 1.29 = 300 \times 1.29 = 387 \text{ K}$$

Also,
$$\eta_{comp.(L.P.)} = 0.8 = \frac{T_2 - T_1}{T_2' - T_1} = \frac{387 - 300}{T_2' - 300}$$

$$\therefore \quad T_{2'} = 300 + \frac{387 - 300}{0.8} = 408.7 \text{ K}$$

$$\therefore \quad W_{comp.(L.P.)} = 1 \times c_{pa}(T_2' - T_1) \text{ per kg of air}$$

$$= 1 \times 1.005(408.7 - 300) = 109.2 \text{ kJ/kg}$$

Since *intercooling is perfect*, therefore,

$$W_{comp.(L.P.)} = W_{comp.\ (H.P.)}$$

$\therefore$ Total compressor work,

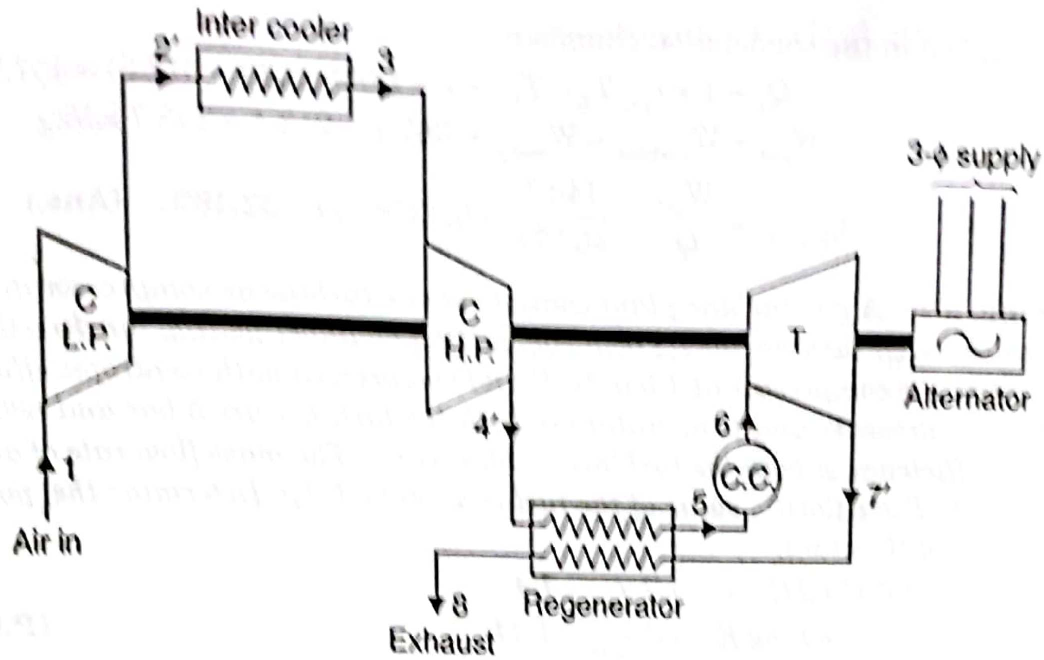
$$W_{comp.\ (total)} = 2 \times 109.2 = 218.4 \text{ kJ/kg}$$

As
$$T_3 = T_1 \text{ and } r_p = \frac{p_2}{p_1} = \frac{p_3}{p_2} \quad \text{(for perfect intercooling)}$$

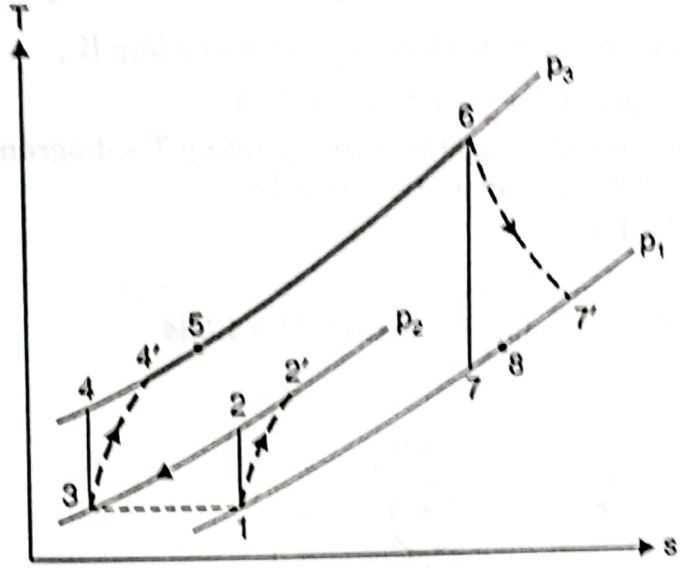
$$\therefore \quad T_{4'} = T_{2'} = 408.7 \text{ K}$$

Considering *isentropic expansion in the turbine*, we have

$$\frac{T_6}{T_7} = \left(\frac{p_3}{p_1}\right)^{\frac{\gamma - 1}{\gamma}} = (6)^{\frac{1.4 - 1}{1.4}} = 1.668$$



(a) Schematic arrangement of the gas turbine plant



(b) T-s diagram

Fig. 5.66

$$\therefore \qquad T_7 = \frac{T_6}{1.668} = \frac{1073}{1.668} = 643.3 \text{ K}$$

Also, $$\eta_{turbine} = 0.85 = \frac{T_6 - T_{7'}}{T_6 - T_7} = \frac{1073 - T_{7'}}{1073 - 643.3}$$

or $$T_{7'} = 1073 - 0.85(1073 - 643.3) = 707.7 \text{ K}$$

$$\therefore \qquad W_{turbine} = c_{pa}(T_6 - T_{7'}) = 1.005(1073 - 707.7) = 367.1 \text{ kJ/kg}$$

The effectiveness (ε) of *regenerator* is given by,

$$\varepsilon = 0.7 = \frac{T_5 - T_{4'}}{T_{7'} - T_{4'}} = \frac{T_5 - 408.7}{707 - 408.7}$$

$$T_5 = 408.7 + 0.7(707 - 408.7) = 617.5 \text{ K}$$

Heat supplied in the combustion chamber,

$$Q_s = 1 \times c_{pa}(T_6 - T_5) = 1 \times 1.005(1073 - 617.5) = 457.77 \text{ kJ/kg}$$

$$W_{net} = W_{turbine} - W_{comp.} = 367.1 - 218.4 = 148.7 \text{ kJ/kg}$$

$$\therefore \quad \eta_{thermal} = \frac{W_{net}}{Q_s} = \frac{148.7}{457.77} = 0.3248 \text{ or } \mathbf{32.48\%.} \quad \textbf{(Ans.)}$$

Example 5.20. *A gas turbine plant consists of one turbine as compressor drive and other to drive a generator. Each turbine has its own combustion chamber getting air directly from the compressor. Air enters the compressor at 1 bar 15°C and compressed with isentropic efficiency of 76 percent. The gas inlet pressure and temperature in both the turbines are 5 bar and 680°C respectively. Take isentropic efficiency of both the turbines as 86 percent. The mass flow rate of air entering compressor is 23 kg/s. The calorific value of the fuel is 42000 kJ/kg. Determine the, power output and thermal efficiency of the plant.*

Take, $c_{pa} = 1.005$ *kJ/kg K, and* $\gamma_{air} = 1.4$;

$c_{pg} = 1.128$ *kJ/kg K, and* $\gamma_{gas} = 1.34$ **(P.U. June, 1998]**

Solution. *Given* : $p_1 = 1$ bar, $T_1 = 15 + 273 = 288$ K ; $p_2 = 5$ bar,

$$T_3 = T_5 = 680 + 273 = 953 \text{ K} ; \ \eta_{comp.} = 76\% ; \ \eta_{T_1} = \eta_{T_2} = 86\% ;$$

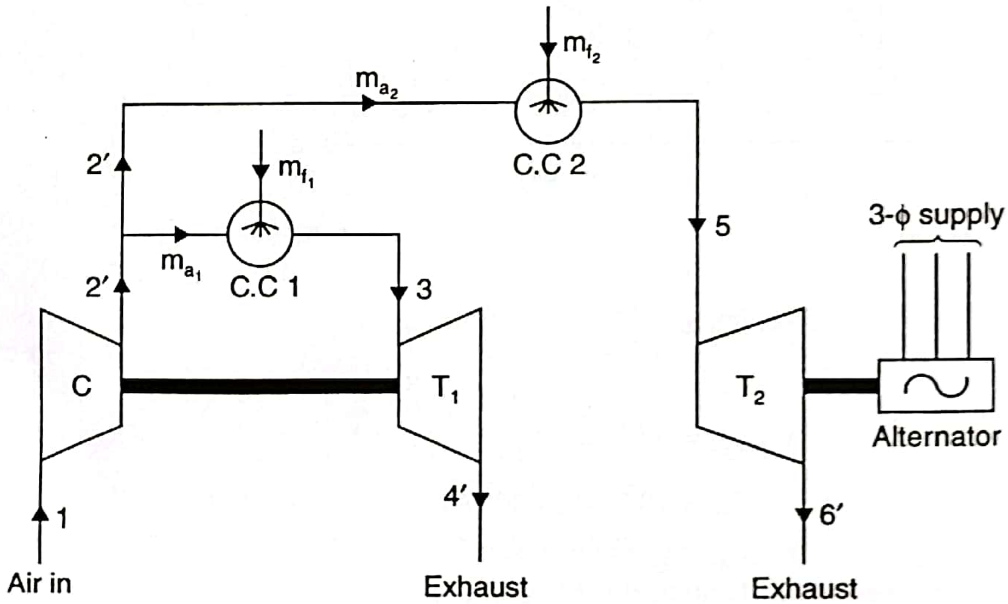
$$m_a (= m_{a_1} + m_{a_2}) = 23 \text{ kg/s} ; C = 42000 \text{ kJ/kg} ; c_{pa} = 1.005 \text{ kJ/kg K} ;$$

$$c_{pg} = 1.128 \text{ kJ/kg K} ; \gamma_{air} = 1.4 ; \gamma_{gas} = 1.34.$$

The schematic arrangement of the plant and its corresponding T-s diagrams (separately for $C - T_1$ and $C - T_2$) are shown in Fig. 5.67 (*a*) and (*b*) respectively.

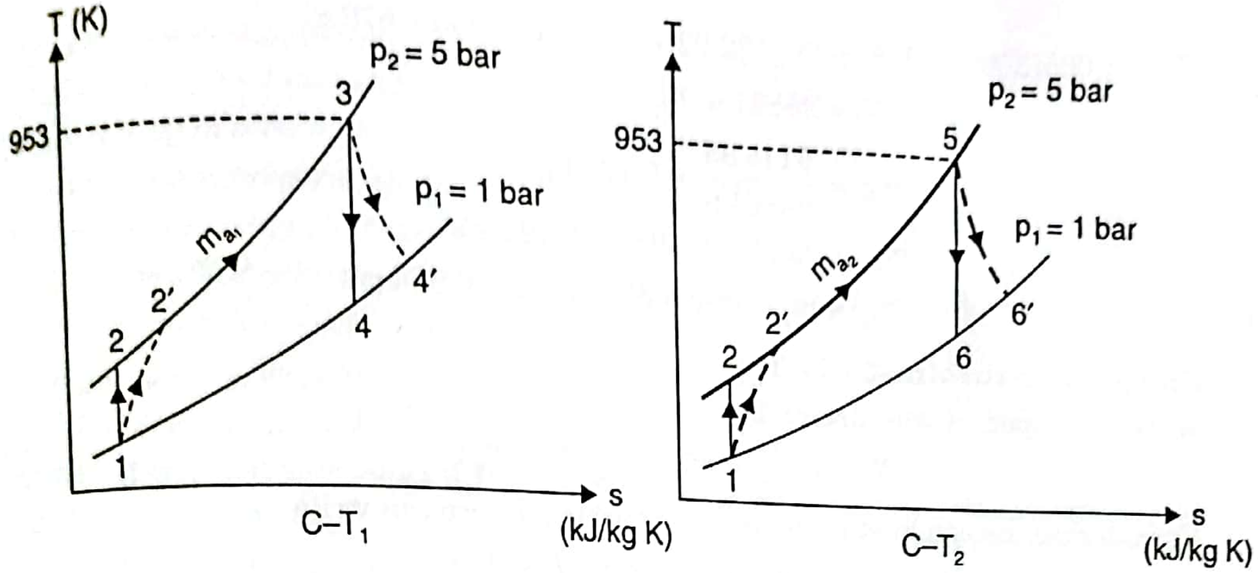
Compressor-Turbine-1 ($\mathbf{C - T_1}$) :

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (5)^{\frac{1.4-1}{1.4}} = (5)^{0.2857} = 1.584$$



C = Compressor
T_1, T_2 = Turbines
C.C – 1, C.C – 2 = Combustion chambers

(a) Schematic arrangement of the plant



(b) T-s diagrams

Fig. 5.67

$$\therefore \quad T_2 = T_1 \times 1.584 = 288 \times 1.584 = 456.2 \text{ K}$$

$$\eta_{\text{comp.}} = 0.76 = \frac{T_2 - T_1}{T_{2'} - T_1} = \frac{456.2 - 288}{T_{2'} - 288}$$

$$\therefore \quad T_2' = 288 + \frac{456.2 - 288}{0.76} = 509.3 \text{ K}$$

Let $\quad m_{f_1}$ = Fuel supplied in $C.\ C - 1$

Then, $\quad m_{f_1} \times C = (m_{a_1} + m_{f_1}) \times c_{pg}(T_3 - T_{2'})$

or
$$C = \left(\frac{m_{a_1}}{m_{f_1}} + 1\right) \times c_{pg}(T_3 - T_{2'})$$

or
$$42000 = \left(\frac{m_{a_1}}{m_{f_1}} + 1\right) \times 1.128(953 - 509.3)$$

$$\therefore \quad \frac{m_{a_1}}{m_{f_1}} = \frac{42000}{1.128\,(953 - 509.3)} - 1 = 82.92$$

Also,
$$\frac{T_3}{T_4} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}} = (5)^{\frac{1.34 - 1}{1.34}} = (5)^{0.254} = 1.5$$

$$\therefore \quad T_4 = \frac{T_3}{1.5} = \frac{953}{1.5} = 635.3 \text{ K}$$

$$\eta_{T_1} = 0.86 = \frac{T_3 - T_{4'}}{T_3 - T_4} = \frac{953 - T_{4'}}{953 - 635.3}$$

$$\therefore \quad T_{4'} = 953 - 0.86(953 - 635.3) = 679.8 \text{ K}$$

Now, the work required to run the compressor must be *equal to* the work developed by T_1.

$$\therefore \quad m_a \times c_{pa}(T_{2'} - T_1) = (m_{a_1} + m_{f_1}) \times c_{pg}(T_3 - T_{4'})$$

$$= m_{f_1}\left(\frac{m_{a_1}}{m_{f_1}} + 1\right) \times c_{pg}(T_3 - T_{4'})$$

or $$23 \times 1.005(509.3 - 288) = m_{f_1}\,(82.92 + 1) \times 1.128(953 - 679.8)$$

or $$5115.35 = 25861.6\, m_{f_1}$$

$$\therefore \quad m_{f_1} = \frac{5115.35}{25861.6} = 0.198 \text{ kg/s}$$

$$\therefore \quad m_{a_1} = 82.92 \times 0.198 = 16.42 \text{ kg/s}$$

and $$m_{a_2} = m_a - m_{a_1} = 23 - 16.42 = 6.58 \text{ kg/s}$$

Compressor-turbine-2 (C – T_2) :

Work developed by the turbine T_2,

$$W_{T_2} = (m_{a_2} + m_{f_2}) \times c_{pg}(T_5 - T_{6'}) \quad \text{...}(i)$$

Considering the combustion chamber of turbine T_2, we can write

$$m_{f_2} \times C = (m_{a_2} + m_{f_2}) \times c_{pg}(T_5 - T_{2'})$$

or $$m_{f_2} \times 42000 = (6.58 + m_{f_2}) \times 1.128(953 - 509.3)$$

$$= 500.49(6.58 + m_{f_2}) = 3293.22 + 500.49\, m_{f_2}$$

$$\therefore \quad m_{f_2} = \frac{3293.22}{(42000 - 500.49)} = 0.079 \text{ kg/s} \qquad \left(\text{or } \frac{m_{a_2}}{m_{f_2}} = \frac{6.58}{0.079} = 83.3\right)$$

Substituting the values in eqn. (*i*), we get

$$W_{T_2} = (6.58 + 0.079) \times 1.128(953 - 679.8)$$

($\because$ $T_{6'} = T_{4'}$ as per given conditions)

$$= \mathbf{2052.1 \text{ kJ/s.}} \quad \textbf{(Ans.)}$$

The capacity of turbine T_2 to run the compressor,

$$W_{T_1} = m_{a_1} \times c_{pa}(T_{2'} - T_1)$$

$$= 16.42 \times 1.005(509.3 - 288) = 3651.9 \text{ kJ/s}$$

Total fuel consumed, $m_f = m_{f_1} + m_{f_2}$

$$= 0.198 + 0.079 = 0.277 \text{ kg/s}$$

$\therefore$ Thermal efficiency of the plant,

$$\eta_{\text{thermal}} = \frac{W_{T_2}}{m_f \times C}$$

$$= \frac{2052.1}{0.277 \times 42000} = 0.1764 \text{ or } \mathbf{17.64\%.} \quad \textbf{(Ans.)}$$

Example 5.21. *A gas turbine plant consists of two-stage compressor with intercooler and it is driven by a separate turbine. The gases coming out from first turbine are passed to the power turbine after reheating to the temperature which is equal to the temperature at the inlet of the compressor turbine. The power turbine generates the electrical energy. A regenerator is used for heating the air before entering into the combustion chamber by using the exhaust gases coming out of power turbine.*

Use the following data :

Ambient air temperature and pressure	*= 15°C, 1 bar*
Maximum cycle temperature	*= 1000 K*
Air mass flow	*= 20 kg/s*
Isentropic efficiency of each compressor	*= 80 per cent*
Insentropic efficiencies of turbines :	$\eta_{T_1} = 87\%$; $\eta_{T_2} = 80\%$
Pressure drop in intercooler	*= 0.07 bar*

Pressure drop in regenerator (H.E.) = *0.1 bar in each side*
Effectiveness of heat exchanger = *0.75*
Pressure drop in combustion chamber = *0.15 bar*
Pressure drop in reheater = *0.1 bar*
Mechanical efficiency for compressor-turbine = *99%*
Combustion efficiency (in combustion chamber and reheater) = *98%*
Compression ratio of each stage of compressor = *2 : 1*
Calorific value of fuel used = *43500 kJ/kg*
$c_{pa} = 1$ *kJ/kg K* ; $c_{pg} = 1.1$ *kJ/kg K*, $\gamma_{air} = 1.4$; $\gamma_{gas} = 1.33$.
Assuming perfect intercooling, determine :
(i) Net output of plant, *(ii) Specific fuel consumption of the plant, and*
(iii) Overall efficiency of the plant. **(K.U., 1999)**

Solution. *Given :* $T_1 = 15 + 273 = 288$ K ; $p_1 = 1$ bar ; $T_6 = T_8 = 1000$ K ; $m_a = 20$ kg/s ;

$$\eta_{C_1} = \eta_{C_2} = 80\% \ ; \ \eta_{T_1} = 87\% \ ; \ \eta_{T_2} = 80\% \ ;$$

$$(\Delta p)_{intercooler} = 0.07 \text{ bar} \ ; \ (\Delta p)_{regenerator} = 0.1 \text{ bar in each side} \ ; \ \epsilon = 0.75 \ ;$$

$$(\Delta p)_{C.C.} = 0.15 \text{ bar} \ ; \ (\Delta p)_{reheater} = 0.1 \text{ bar} \ ; \ \eta_{mech(T_1)} = 99\% \ ;$$

$$\eta_{comb.\ (C.C.\ and\ reheater)} = 98\% \ ; \ (r_p)_{C_1, C_2} = 2 : 1 \ ; \ C = 43500 \text{ kJ/kg} \ ;$$

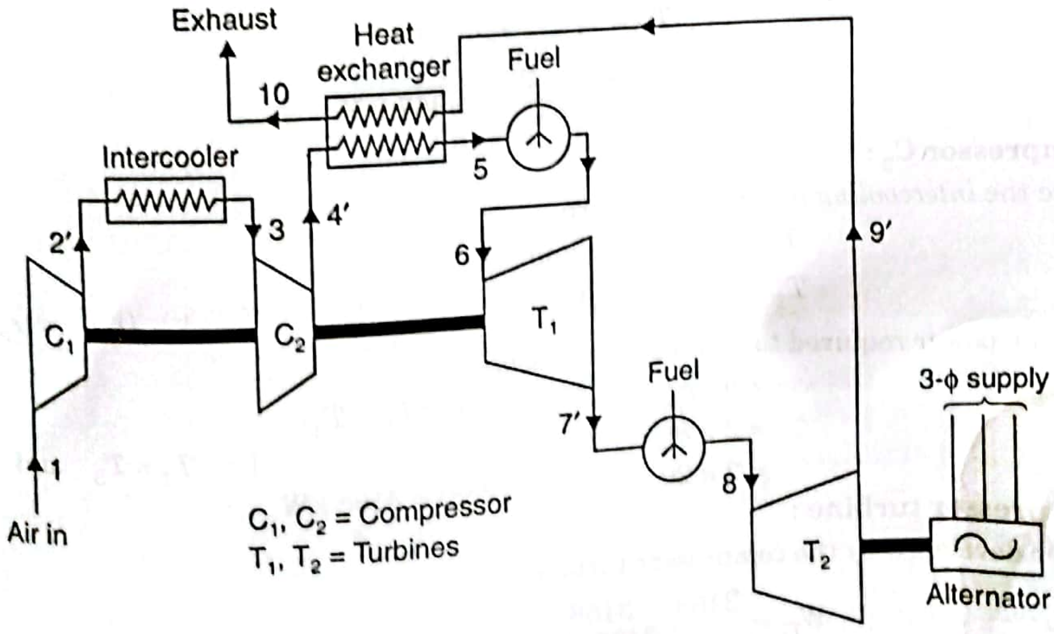
$$c_{pa} = 1 \text{ kJ/kg K} \ ; \ c_{pg} = 1.1 \text{ kJ/kg K} \ ; \ \gamma_{air} = 1.4 \ ; \ \gamma_{gas} = 1.33$$

The schematic arrangement of the gas turbine plant and its corresponding T-s diagram are shown in Fig. 5.68 (a) and (b) respectively.

As for given data, we have :

$$p_1 = 1 \text{ bar}, \qquad p_2 = 2 \text{ bar as } \frac{p_2}{p_1} = 2 \text{ (Given)}$$

$$p_3 = p_2 - (\Delta p)_{intercooler} = 2 - 0.07 = 1.93 \text{ bar}$$



(a) Schematic arrangement of the plant

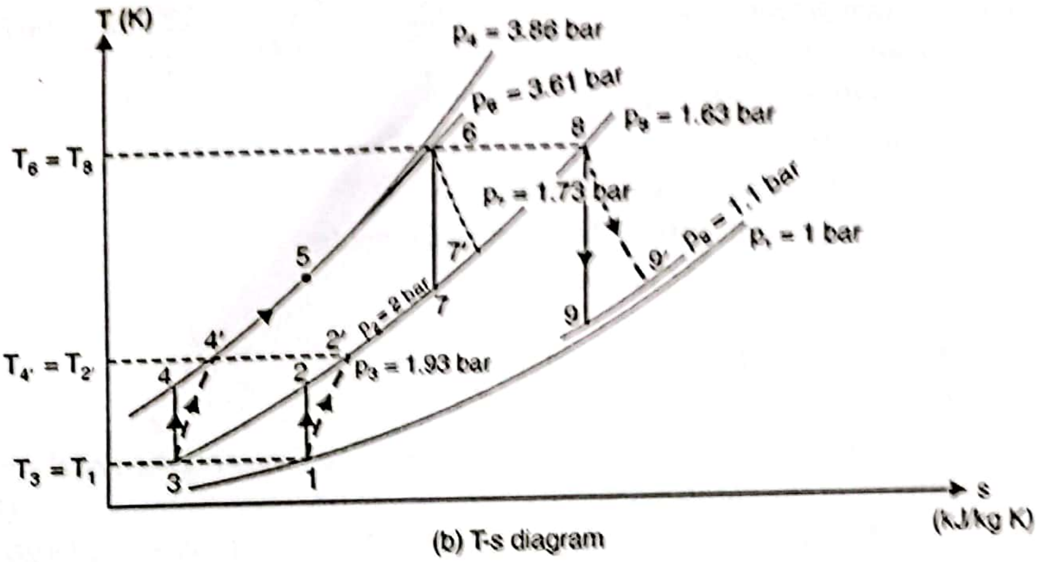


Fig. 5.68

$$p_4 = 2p_3 = 2 \times 1.93 = 3.86 \text{ bar} \qquad [\because (r_p)_{C_1, C_2} = 2:1]$$

$$p_6 = p_4 - (\Delta_p)_{\text{H.E. air side}} - (\Delta p)_{C.C} = 3.86 - 0.1 - 0.15 = 3.61 \text{ bar}$$

p_7 is to be calculated

$$p_8 = p_7 - (\Delta p)_{\text{reheater}}$$

$$p_9 = 1 + (\Delta p)_{\text{H.E. gas side}} = 1 + 0.1 = 1.1 \text{ bar}$$

Compressor C_1 :

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left[(r_p)_{C_1}\right]^{\frac{\gamma-1}{\gamma}} = (2)^{\frac{1.4-1}{1.4}} = (2)^{0.2857} = 1.22$$

$\therefore$ $$T_2 = T_1 \times 1.22 = 288 \times 1.22 = 351.36 \text{ K}$$

$$\eta_{C_1} = 0.8 = \frac{T_2 - T_1}{T_{2'} - T_1} = \frac{351.36 - 288}{T_{2'} - 288}$$

$\therefore$ $$T_{2'} = 288 + \frac{351.36 - 288}{0.8} = 367.2 \text{ K}$$

Compressor C_2 :

Since the *intercooling is perfect* therefore,

$$T_1 = T_3 = 288 \text{ K, and}$$

$$T_{2'} = T_{4'} = 367.2 \text{ K} \qquad [\because (r_p)_{C_1} = (r_p)_{C_2} = 2:1]$$

$\therefore$ The power required to run the compressors

$$= m_a c_{pa}(T_{2'} - T_1) + m_a c_{pa}(T_{4'} - T_3)$$

$$= 2 \times m_a c_{pa}(T_{2'} - T_1) \qquad [\because T_1 = T_3 \text{ and } T_{2'} = T_{4'}]$$

$$= 2 \times 20 \times 1(367.2 - 288) = 3168 \text{ kW}$$

Compressor turbine :

Power developed by the compressor turbine,

$$W_{T_1} = \frac{3168}{\eta_{mech}} = \frac{3168}{0.99} = 3200 \text{ kW}$$

$\therefore$ Work developed by the compressor turbine per kg of air

$$= \frac{3200}{20} = 160 \text{ kJ/kg}$$

The work developed by the compressor turbine per kg of air is given by

$$c_{pg}(T_6 - T_{7'}) = 160$$

$$\therefore \quad T_6 - T_{7'} = \frac{160}{1.1} = 145.45 \text{ K}$$

$$\frac{T_6}{T_7} = [(r_p)_{T_1}]^{\frac{\gamma-1}{\gamma}}$$

$$\eta_{T_1} = \frac{T_6 - T_{7'}}{T_6 - T_7} \quad \text{or} \quad T_6 - T_{7'} = \eta_{T_1}(T_6 - T_7)$$

or $$T_6 - T_{7'} = \eta_{T_1} \times T_6\left(1 - \frac{T_7}{T_6}\right) = \eta_{T_1}\, T_6\left[1 - \frac{1}{\{(r_p)_{T_1}\}^{\frac{\gamma-1}{\gamma}}}\right]$$

or $$T_6 - T_{7'} = 0.87 \times 1000\left[1 - \frac{1}{\{(r_p)_{T_1}\}^{\frac{1.33-1}{1.33}}}\right]$$

or $$145.45 = 870\left[1 - \frac{1}{\{(r_p)_{T_1}\}^{0.2481}}\right]$$

or $$0.167 = 1 - \frac{1}{\{(r_p)_{T_1}\}^{0.2481}}$$

$$\therefore \quad (r_p)_{T_1} = \left(\frac{1}{1-0.167}\right)^{\frac{1}{0.2481}} = 2.09$$

Also, $$(r_p)_{T_1} = 2.09 = \frac{p_6}{p_7} \quad \therefore \quad p_7 = \frac{p_6}{2.09} = \frac{3.61}{2.09} = 1.73 \text{ bar}$$

$$p_8 = p_7 - (\Delta p)_{\text{reheater}} = 1.73 - 0.1 = 1.63 \text{ bar}$$

$$(r_p)_{T_2} = \frac{p_8}{p_9} = \frac{1.63}{1.1} = 1.48$$

where $(r_p)_{T_1}$ and $(r_p)_{T_2}$ are the pressure ratios of the turbines T_1 and T_2.

Turbine T_2 :

$$\frac{T_8}{T_9} = \left(\frac{p_8}{p_9}\right)^{\frac{\gamma-1}{\gamma}} = (1.48)^{\frac{1.33-1}{1.33}} = (1.48)^{0.2481} = 1.1$$

$$\therefore \quad T_9 = \frac{T_8}{1.1} = \frac{1000}{1.1} = 909.1 \text{ K}$$

$$\eta_{T_2} = 0.8 = \frac{T_8 - T_{9'}}{T_8 - T_9} = \frac{1000 - T_{9'}}{1000 - 909.1}$$

$$\therefore \quad T_{9'} = 1000 - 0.8(1000 - 909.1) = 927.3 \text{ K}$$

(i) Net output of the plant :

The power output of the plant is only from turbine T_2.

$\therefore$ Net-output of the plant $= m_a\, c_{pa}\, (T_8 - T_{9'})$

$= 20 \times 1.0(1000 - 927.3) = $ **1454 kW. (Ans.)**

(ii) Specific fuel consumption of the plant :

The effectiveness of heat exchanger

$$\epsilon = 0.75 = \frac{T_5 - T_{4'}}{T_{9'} - T_{4'}} = \frac{T_5 - 367.2}{927.3 - 367.2}$$

$$\therefore \quad T_5 = 367.2 + 0.75(927.3 - 367.2) = 787.3 \text{ K}$$

The total heat supplied in the plant per kg of air,

$$Q_s = c_{pg}\,(T_6 - T_5) + c_{pg}\,(T_8 - T_{7'}) = c_{pg}\,[(T_6 - T_{7'}) + (T_8 - T_5)]$$

$$= 1.1(145.45 + 1000 - 787.3) = 394 \text{ kJ/kg)}$$

Assuming m_f is mass of fuel supplied per second in combustion chamber and reheater, then,

Heat developed = Heat-gained by air

$$m_f \times C \times \eta_{comb.} = 20 \times 394$$

or $$m_f \times 43500 \times 0.98 = 20 \times 394$$

$$\therefore \quad m_f = \frac{20 \times 394}{43500 \times 0.98} = 0.185 \text{ kg/s} = 666 \text{ kg/h}$$

$\therefore$ Specific fuel consumption (s.f.c.) $= \dfrac{666}{1454} =$ **0.458 kg/kWh. (Ans.)**

(iii) Overall efficiency of the plant, $\eta_{overall}$:

$$\eta_{overall} = \frac{\text{Output}}{\text{Input}}$$

$$= \frac{1454}{20 \times 394} = 0.1845 \quad \text{or} \quad \mathbf{18.45\%}. \quad \textbf{(Ans.)}$$

HIGHLIGHTS

1. The major fields of application of gas turbines are :
   (i) Aviation (ii) Power generation
   (iii) Oil and gas industry (iv) Marine propulsion.
2. A gas turbine plant may be defined as one "in which the principal prime-mover is of the turbine type and the working medium is a permanent gas".
3. A simple gas turbine plant consists of the following :
   (i) Turbine (ii) Compressor
   (iii) Combustor (iv) Auxiliaries.
   A modified plant may have in addition an *intercooler, a regenerator, a reheater etc.*
4. Methods for improvement of thermal efficiency of open cycle gas turbine plant are :
   (i) Intercooling (ii) Reheating (iii) Regeneration.
5. *Free-piston engine plants* are the conventional gas turbine plants with the difference that the air compressor and combustion chamber are replaced by a free piston engine.