

COMP 1805 Review Questions

From Rosen (6th ed.)

1.1: 33e) Construct a truth table for:

$$(p \leftrightarrow q) \vee (\neg q \leftrightarrow r)$$

1.2: 30) Show that the following is a tautology:

$$(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$$

1.4: 33) Rewrite each of these statements so that negations appear only within predicates:

- a) $\neg \forall x \forall y P(x, y)$
- b) $\neg \forall y \exists x P(x, y)$
- c) $\neg \forall y \forall x (P(x, y) \vee Q(x, y))$
- d) $\neg (\exists x \exists y \neg P(x, y) \wedge \forall x \forall y Q(x, y))$
- e) $\neg \forall x (\exists y \forall z P(x, y, z) \wedge \exists z \forall y P(x, y, z))$

1.6: 39) Prove that at least one of the real numbers a_1, a_2, \dots, a_n is greater than or equal to the average of these numbers.

1.6: 40) Use Exercise 39 to show that if the first 10 positive integers are placed around a circle, in any order, there exist three integers in consecutive locations around the circle that have a sum greater than or equal to 17.

1.7: 32) Prove that $\sqrt[3]{2}$ is irrational.

2.1: 19c) Find the power set of $\{\emptyset, \{\emptyset\}\}$

2.4: 31) Determine whether each of these sets is countable or uncountable. For those that are countable, exhibit a one-to-one correspondence between the set of natural numbers and that set.

- a) the negative integers
- b) the even integers
- c) the real numbers between 0 and $1/2$
- d) integers that are multiples of 7

3.2: 24)

- a) Show that $3x + 7$ is $\Theta(x)$
- b) Show that $2x^2 + x - 7$ is $\Theta(x^2)$
- c) Show that $\lfloor x + 1/2 \rfloor$ is $\Theta(x)$
- d) Show that $\log(x^2 + 1)$ is $\Theta(\log_2(x))$
- e) Show that $\log_{10} x$ is $\Theta(\log_2(x))$

4.1: 21) Prove that $2^n > n^2$ if n is an integer greater than 4

4.1: 31) Prove that 2 divides $n^2 + n$ whenever n is a positive integer

4.3: 13) Where f_n is the n th Fibonacci number, prove that

$$f_1 + f_3 + \dots + f_{2n-1} = f_{2n}$$

5.1: 31) How many strings of eight English letters are there

- a) that contain no vowels, if letters can be repeated?
- b) that contain no vowels, if letters cannot be repeated?
- c) that start with a vowel, if letters can be repeated?
- d) that start with a vowel, if letters cannot be repeated?
- e) that contain at least one vowel, if letters can be repeated?
- f) that contain exactly one vowel, if letters can be repeated?
- g) that start with X and contain at least one vowel, if letters can be repeated?
- h) that start and end with X and contain at least one vowel, if letters can be repeated?

5.2: 9(modified) what is the minimum number of students, each of whom comes from one of the 13 provinces and territories, who must be enrolled in a university to guarantee that there are at least 100 who come from the same province or territory?

5.2: 15) How many numbers must be selected from the set $\{1, 2, 3, 4, 5, 6\}$ to guarantee that at least one pair of these numbers add up to 7?

5.3: 38) How many ways are there to select 12 countries in the United Nations to serve on a council if 3 are selected from a block of 45, 4 are selected from a block of 57, and the others are selected from the remaining 69 countries?

5.4: 9) What is the coefficient of $x^{101}y^{99}$ in the expansion of $(2x - 3y)^{200}$?

5.5: 11) How many ways are there to choose eight coins from a piggy bank containing 100 identical pennies and 80 identical nickels?

8.1: 4) Determine whether the relation R on the set of all people is reflexive, symmetric, antisymmetric, and/or transitive, where $(a, b) \in R$ if and only if

- a) a is taller than b
- b) a and b were born on the same day
- c) a has the same first name as b
- d) a and b have a common grandparent

9.1: 12) Let G be an undirected graph with a loop at every vertex. Show that the relation R on the set of vertices of G such that uRv if and only if there is an edge associated to $\{u, v\}$ is a symmetric, reflexive relation on G .

9.2: 6) Show that the sum, over the set of people at a party, of the number of people a person has shaken hands with, is even. Assume that no one shakes their own hand.

9.7: 1) Can five houses be connected to two utilities without connections crossing?