Delaunay Triangulation Spanner Notes

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Dobkin's Results

The Delaunay triangulation of a set of points in the plane is a spanner with spanning ratio $c \leq ((1+\sqrt{5})/2)\pi \approx 5.08$. This was proven in the paper "Delaunay Graphs Are Almost as Good as Complete Graphs" by Dobkin, Friedman, and Supowit [1] [2].

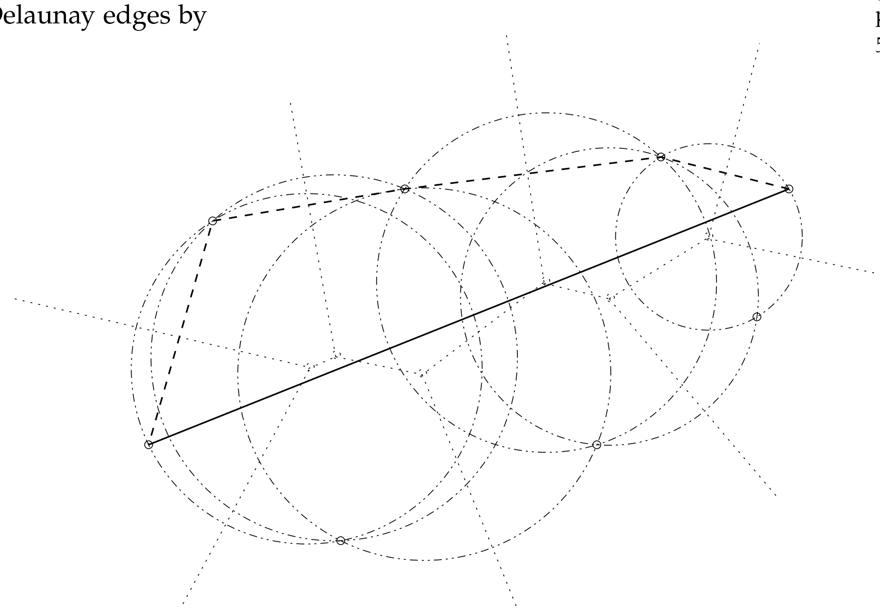
[1] David P. Dobkin, Steven J. Friedman, and Kenneth J. Supowit. Delaunay graphs are almost as good as complete graphs. In *Proceedings of the 28th Annual Symposium on Foundations of Computer Science*, SFCS '87, pages 20–26, Washington, DC, USA, 1987. IEEE Computer Society

[2] David P. Dobkin, Steven J. Friedman, and Kenneth J. Supowit. Delaunay graphs are almost as good as complete graphs. *Discrete Comput. Geom.*, 5(4):399–407, May 1990

Introduction

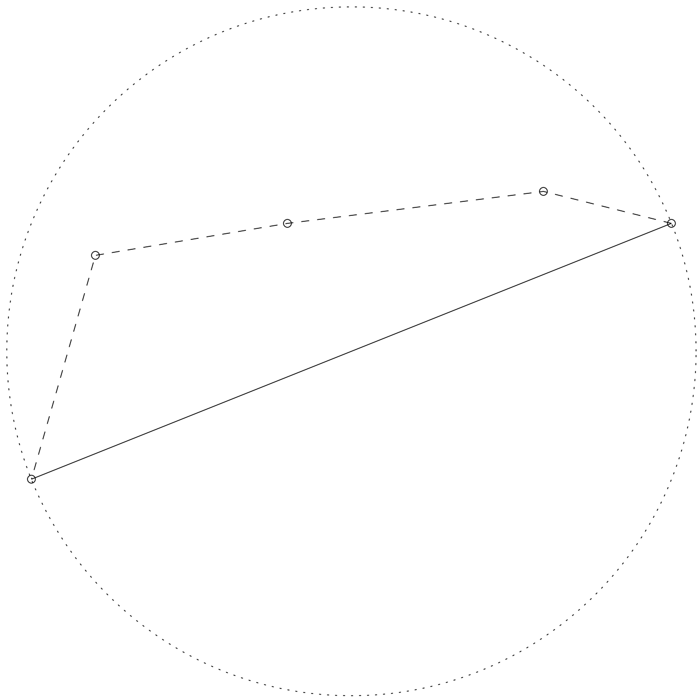
Let S be a set of points in the plane and $DT(S)$ be the edges of the Delaunay triangulation of S.

We consider the path between two arbitray points $a, b \in S$. Let the line connecting a, b be the *direct line*. We construct a path along the Delaunay edges by



One-Sided Path: The Easy Case

If all edges along the direct DT path between points $a, b \in S$ are either all above or all below the direct line, we say that this is a one-sided path.



Lemma 1. *Points along a direct DT path are monotonic in x.*
Lemma 2. *All points along the direct DT path from a to b are contained within or on the boundary of the circle with a and b diametrically opposed.*
Lemma 3. *The boundary of a connected union of circles has boundary at most $\pi \cdot (x_r - x_l)$ where x_r and x_l are the extreme x coordinates of any of the circles.*

From these lemmas, it follows that the one-sided path is at most $\pi/2$ times as long as the euclidean distance between the endpoints.

The Harder Case

The direct DT path may cross the direct line $\Omega(n)$ times, which can yield a much longer path.

Keil's Results

TODO

References

[1] David P. Dobkin, Steven J. Friedman, and Kenneth J. Supowit. Delaunay graphs are almost as good as complete graphs. In *Proceedings of the 28th Annual Symposium on Foundations of Computer Science*, SFCS '87, pages 20–26, Washington, DC, USA, 1987. IEEE Computer Society.

[2] David P. Dobkin, Steven J. Friedman, and Kenneth J. Supowit. Delaunay graphs are almost as good as complete graphs. *Discrete Comput. Geom.*, 5(4):399–407, May 1990.