

Intermediary Report
A New Approach to the Maximum Flow Problem

Praveen Kumar Shanmugam and Shridharan Chandramouli

University of Utah

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Problem

Problem Definition

The problem of finding the maximum flow in a given graph is solved using preflow concept of Karzanov. By incorporating the dynamic tree data structure [1] of Sleator and Tarjan the new approach achieves a running time of $\mathcal{O}(nm \log(n^2/m))$ on an n -vertex, m -edge graph.

A minimum cut is a cut of minimum capacity. The max-flow, min-cut theorem of Ford and Fulkerson states that the value of a maximum flow is equal to the capacity of a minimum cut.

Graph $G = (V, E)$ is a directed graph with vertex set V and edge set E . Size of V is denoted by n and size of E by m . G is a network if it has two distinct distinguished vertices, a *source* s and a *sink* t , and a positive capacity $c(v, w)$ on each directed edge (v, w) . A flow f on G is a real-valued function on vertex pairs satisfying. The value of a flow f is the net flow into the sink,

$$|f| = \sum_{v \in V} f(v, t) \quad (1)$$

A maximum flow is a flow of maximum value. A *cut* S, \bar{S} is a partition of the vertex set ($S \cup \bar{S} = V, S \cap \bar{S} = \emptyset$) with $s \in S$ and $t \in \bar{S}$. The capacity of the cut is

$$c(S, \bar{S}) = \sum_{v \in S, w \in \bar{S}} c(v, w) \quad (2)$$

The flow across the cut is

$$f(S, \bar{S}) = \sum_{v \in S, w \in \bar{S}} f(v, w) = |f| \quad (3)$$

Main Result

The new approach makes the algorithm as fast as any known method for any graph density and faster on graphs of moderate density. And the algorithm is efficient in distributed and parallel implementations. The parallel implementation running in $\mathcal{O}(n^2 \log n)$ time and uses only $\mathcal{O}(m)$ space with a time bound $\mathcal{O}(n^3)$.

Importance of Result

TODO: Explain why this result is important (provide some background with references)

Impact

TODO: Explain what impact (if any) this result has had (or might have)

Outline

Outline, in a page, your best understanding of how the paper does what it does, focusing on key techniques. For example, it might be something like

- **previous papers did X**
- **they hit a bottleneck Y**
- **This paper uses key idea Z to get improvement A1, and then key idea Z' to get improvement A2, and so on.**

References

- [1] SLEATOR, D. D., AND TARJAN, R. E. A data structure for dynamic trees. *J. Comput. Syst. Sci.* 26, 3 (June 1983), 362–391.