LAB-1

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lab-1

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1) Gaussian Elimination

a)
$$-7x - 3y + 3z = 12$$
$$2x + 2y + 2z = 0$$
$$-x - 4y + 3z = -9$$

Rearranging system of equation:

$$-x - 4y + 3z = -9 \rightarrow (1)$$
$$2x + 2y + 2z = 0 \rightarrow (2)$$
$$-7x - 3y + 3z = 12 \rightarrow (3)$$

$(1) \times 2 \rightarrow -2x - 8y + 6z = -18$

$(2) \rightarrow 2x + 2y + 2z = 0$

$$-6y + 8z = -18 \rightarrow (4)$$

$(1) \times 7 \rightarrow -7x - 28y + 21z = -63$

$(3) \rightarrow -7x - 3y + 3z = 12$ (signs: (+) (+) (−) (−))

$$-25y + 18z = -75 \rightarrow (5)$$

$(4) \times 25 \rightarrow -150y + 200z = -450$

$(5) \times 6 \rightarrow -150y + 108z = -450$ (signs: (+) (−) (+))

$$92z = 0$$

$$\boxed{z = 0}$$

Sub $z = 0$ in (4)

$$-6y + 0 = -18$$
$$-6y = -18$$
$$\boxed{y = 3}$$

Sub $z = 0$ and $y = 3$ in (1)

$$-x - 4(3) + 0 = -9$$
$$-x = +3$$
$$\boxed{x = -3}$$

$\therefore$ solution of the system of equation is $\begin{bmatrix} -3 \\ 3 \\ 0 \end{bmatrix}$

b) $x - y - z = 4 \rightarrow (1)$

$2x - 2y - 2z = 8 \rightarrow (2)$

$5x - 5y - 5z = 20 \rightarrow (3)$

$(1) \times 2 \rightarrow 2x - 2y - 2z = 8$

$(2) \rightarrow 2x - 2y - 2z = 8$

$0x + 0y + 0z = 0$

$(1) \times 5 \Rightarrow 5x - 5y - 5z = 20$

$(3) \rightarrow 5x - 5y - 5z = 20$

$0x + 0y + 0z = 0$

So, we should make two free variables

$y = s, z = t$

$x - s - t = 4$

$\boxed{x = s + t + 4}$

solution of system equation is $\begin{bmatrix} 4+s+t \\ s \\ t \end{bmatrix}$

independent solution

eg) $s = 0, t = 1$ $\quad$ $s = 1, t = 0$

$\begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$ $\quad$ $\begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}$

c) $x+y+z=0 \rightarrow (1)$

$-x-y+3z=3 \rightarrow (2)$

$-x-y-z=2 \rightarrow (3)$

(1) $\rightarrow x+y+z=0$

(2) $\rightarrow -x-y+3z=3$

$4z=3$

$z=\frac{3}{4}$

(1) $\rightarrow x+y+z=0$

(3) $\rightarrow -x-y-z=0$

$0x+0y+0z=2$

$\therefore$ which is not possible

$\therefore$ System of solution is no solution

d) $3x+4y-z=-6 \rightarrow (1)$

$-2y+10z=-8 \rightarrow (2)$

$4y-2z=-2 \rightarrow (3)$

(1) $\rightarrow 3x+4y-z=-6$

(2)$\times 2 \rightarrow 0x-4y+20z=-16$

$3x+19z=-22 \rightarrow (4)$

(1) $\rightarrow 3x+4y-z=-6$

(3) $\rightarrow 0x+4y-2z=-2$

(−) (+) (+)

$3x+z=-4 \rightarrow (5)$

④ → $3x + 19z = -22$

⑤ → $3x + z = -4$

(−) (−) (+)

$18z = -18$

$\boxed{z = -1}$

Sub $\boxed{z=-1}$ in ⑤ and ③

$3x - 1 = -4$

$3x = -3$

$\boxed{x = -1}$

$4y - 2(-1) = -2$

$4y + 2 = -2$

$4y = -4$

$\boxed{y = -1}$

∴ Solution of system of equation is $\begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$

2) $y = ax^2 + bx + c$

$(-6, 8)$, $(-2, -12)$ & $(3, 8)$

$a(-6)^2 + b(-6) + c = 8$

$36a - 6b + c = 8 \rightarrow (1)$

$a(-2)^2 + b(-2) + c = -12$

$4a - 2b + c = -12 \rightarrow (2)$

$a(3)^2 + b(3) + c = 8$

$9a + 3b + c = 8 \rightarrow (3)$

$(1) \times 1 \rightarrow 36a - 6b + c = 8$

$(2) \rightarrow 4a - 2b + c = -12$ (−) (+) (−) (+)

$32a - 4b = 20 \rightarrow (4)$

$(1) \rightarrow 36a - 6b + c = 8$

$(3) \rightarrow 9a + 3b + c = 8$ (−) (−) (−) (−)

$27a - 9b = 0 \rightarrow (5)$

$32a - 4b = 20 \rightarrow (4)$

$27a - 9b = 0 \rightarrow (5)$

$(4) \times 9 \rightarrow 288a - 36b = 180$

$(5) \times 4 \rightarrow 108a - 36b = 0$ (−) (+) (−)

$180a = 180$

$\boxed{a = 1}$

Sub $\boxed{a = 1}$ in (4)

$32(1) - 4b = 20$

$32 - 4b = 20$

$-4b = -12$

$\boxed{b = 3}$

Sub $a=1$ $b=3$ in (2)

$4(1) - 2(3) + c = -12$

$4 - 6 + c = -12$

$-2 + c = -12$

$\boxed{c = -10}$

Solution of system of equation is $\begin{bmatrix} 1 \\ 3 \\ -10 \end{bmatrix}$

He wants to meet his friend at $p(7, 60)$

$ax^2 + bx + c = y$

$\Rightarrow (1)(7)^2 + 3(7) + (-10)$

$\Rightarrow 49 + 21 - 10$

$\Rightarrow 70 - 10$

$\Rightarrow 60$

$\therefore$ yes, he will meet his friend

3) LU Decomposition

a) $2x + 3y - z = 5$

$3x + 2y + z = 10$

$x - 5y + 3z = 0$

$$A \quad x \quad = \quad B$$

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 2 & 1 \\ 1 & -5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 0 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \qquad U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{32} \end{bmatrix}$$

$LU = A$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 2 & 1 \\ 1 & -5 & 3 \end{bmatrix}$$

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{33} \end{bmatrix} = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 2 & 1 \\ 1 & -5 & 3 \end{bmatrix}$$

$u_{11} = 2$, $u_{12} = 3$, $u_{13} = -1$

$l_{21} = \frac{3}{2}$, $u_{22} = -\frac{5}{2}$

$l_{31} = \frac{1}{2}$, $l_{32} = \frac{1}{2}$, $u_{23} = \frac{5}{2}$, $u_{33} = -3$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 3/2 & 1 & 0 \\ 1/2 & 13/5 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 2 & 3 & -1 \\ 0 & -5/2 & 5/2 \\ 0 & 0 & -3 \end{bmatrix}$$

$LUx = B$

$Ux = y$

$Ly = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 3/2 & 1 & 0 \\ 1/2 & 13/5 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 0 \end{bmatrix}$$

$y_1 = 5$

$\frac{3}{2} y_1 + y_2 = 10$

$\frac{15}{2} + y_2 = 10$

$y_2 = \frac{20 - 15}{2} = \frac{5}{2}$

$\frac{y_1}{2} + \frac{13}{6} y_2 + y_3 = 0$

$\frac{5}{2} + \frac{13}{2} + y_3 = 0$

$\boxed{y_3 = -9}$

$$y = \begin{bmatrix} 5 \\ 5/2 \\ -9 \end{bmatrix}$$

$Ux = y$

$$\begin{bmatrix} 2 & 3 & -1 \\ 0 & -\frac{5}{2} & \frac{5}{2} \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 5/2 \\ -9 \end{bmatrix}$$

$x_1 = 1, x_2 = 2, x_3 = 3$

$$\therefore x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$x = 1, y = 2, z = 3$

b) $x + y + z = 3$

$2x - y - z = 3$

$x - y + z = 9$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & -1 \\ 1 & -1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 3 \\ 9 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$Ax = B$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_1 & 1 & 0 \\ l_2 & l_3 & 1 \end{bmatrix} \quad U = \begin{bmatrix} u_1 & u_2 & u_3 \\ 0 & u_4 & u_5 \\ 0 & 0 & u_6 \end{bmatrix}$$

$LU = A$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_1 & 1 & 0 \\ l_2 & l_3 & 1 \end{bmatrix} \begin{bmatrix} u_1 & u_2 & u_3 \\ 0 & u_4 & u_5 \\ 0 & 0 & u_6 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & -1 \\ 1 & -1 & 1 \end{bmatrix} \quad (LU = A)$$

$$\begin{bmatrix} u_1 & u_2 & u_3 \\ l_1 u_1 & l_1 u_2 + u_4 & l_1 u_3 + u_5 \\ l_2 u_1 & l_2 u_2 + l_3 u_4 & l_2 u_3 + l_3 u_5 + u_6 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

$u_1 = 1$, $u_2 = 1$, $u_3 = 1$

$l_1(1) = 2$ $\quad l_1 = 2$ $\quad 2(2) + u_4 = -1$

$2(1) + u_5 = -1$, $\quad l_2 = 1$

$u_5 = -3$

$l_3 = \frac{2}{3}$, $\quad u_6 = 2$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & \frac{2}{3} & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & -3 \\ 0 & 0 & 2 \end{bmatrix}$$

$$LUx = B$$

$$Ux = y \qquad Ly = B$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & \frac{2}{3} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 9 \end{bmatrix}$$

$y_1 = 3 \qquad y_2 = -3 \qquad y_3 = 8$

$$Ux = y$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & -3 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 8 \end{bmatrix}$$

$2x_3 = 8$

$\boxed{x_3 = 4}$

$-3x_2 - 3(4) = -3$

$-3x_2 - 12 = -3$

$-3x_2 = 9$

$\boxed{x_2 = -3}$

$x_1 - 3 + 4 = 3$

$x_1 + 1 = 3$

$\boxed{x_1 = 2}$

$\therefore$ solution is $\begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}$

```
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%Lab-1

%1)a)
% Ask user for matrix size
% Ask user for matrix size
n = input('Enter number of variables: ');

% Ask user for matrix coefficients
A = zeros(n);
for i = 1:n
    for j = 1:n
        A(i,j) = input(sprintf('Enter coefficient for A(%d,%d): ', i, j));
    end
end

% Ask user for right-hand side values
b = zeros(n,1);
for i = 1:n
    b(i) = input(sprintf('Enter value for b(%d): ', i));
end

% Gaussian elimination with partial pivoting
for k = 1:n-1
    % Partial pivoting
    [~,p] = max(abs(A(k:n,k)));
    p = p + k - 1;
    if p ~= k
        A([k,p],:) = A([p,k],:);
        b([k,p]) = b([p,k]);
    end

    % Elimination
    for i = k+1:n
        m = A(i,k) / A(k,k);
        A(i,k:n) = A(i,k:n) - m * A(k,k:n);
        b(i) = b(i) - m * b(k);
    end
end

% Back substitution
x = zeros(n,1);
x(n) = b(n) / A(n,n);
for i = n-1:-1:1
    x(i) = (b(i) - A(i,i+1:n) * x(i+1:n)) / A(i,i);
end

% Display solution
```

```
disp('Solution:');
```

```
Solution:
```

```
disp(x);
```

```
   -3.0000
    3.0000
    0.0000
```

```

```

```
%1)b)

% Ask user for matrix size
n = input('Enter number of variables: ');

% Ask user for matrix coefficients
A = zeros(n);
for i = 1:n
    for j = 1:n
        A(i,j) = input(sprintf('Enter coefficient for A(%d,%d): ', i, j));
    end
end

% Ask user for right-hand side values
b = zeros(n,1);
for i = 1:n
    b(i) = input(sprintf('Enter value for b(%d): ', i));
end

% Gaussian elimination with partial pivoting
for k = 1:n-1
    % Partial pivoting
    [~,p] = max(abs(A(k:n,k)));
    p = p + k - 1;
    if p ~= k
        A([k,p],:) = A([p,k],:);
        b([k,p]) = b([p,k]);
    end

    % Elimination
    for i = k+1:n
        m = A(i,k) / A(k,k);
        A(i,k:n) = A(i,k:n) - m * A(k,k:n);
        b(i) = b(i) - m * b(k);
    end
end

% Back substitution
x = zeros(n,1);
```

```
x(n) = b(n) / A(n,n);
for i = n-1:-1:1
    x(i) = (b(i) - A(i,i+1:n) * x(i+1:n)) / A(i,i);
end

% Display solution
disp('Solution:');
```

```
Solution:
```

```
disp(x);
```

```
   NaN
   NaN
   NaN
```

```
%1)c

% Ask user for matrix size
n = input('Enter number of variables: ');

% Ask user for matrix coefficients
A = zeros(n);
for i = 1:n
    for j = 1:n
        A(i,j) = input(sprintf('Enter coefficient for A(%d,%d): ', i, j));
    end
end

% Ask user for right-hand side values
b = zeros(n,1);
for i = 1:n
    b(i) = input(sprintf('Enter value for b(%d): ', i));
end

% Gaussian elimination with partial pivoting
for k = 1:n-1
    % Partial pivoting
    [~,p] = max(abs(A(k:n,k)));
    p = p + k - 1;
    if p ~= k
        A([k,p],:) = A([p,k],:);
        b([k,p]) = b([p,k]);
    end

    % Elimination
    for i = k+1:n
        m = A(i,k) / A(k,k);
        A(i,k:n) = A(i,k:n) - m * A(k,k:n);
        b(i) = b(i) - m * b(k);
    end
```

```
end

% Back substitution
x = zeros(n,1);
x(n) = b(n) / A(n,n);
for i = n-1:-1:1
    x(i) = (b(i) - A(i,i+1:n) * x(i+1:n)) / A(i,i);
end

% Display solution
disp('Solution:');
```

```
Solution:
```

```
disp(x);
```

```
   NaN
   NaN
   NaN
```

```
%1)d

% Ask user for matrix size
n = input('Enter number of variables: ');

% Ask user for matrix coefficients
A = zeros(n);
for i = 1:n
    for j = 1:n
        A(i,j) = input(sprintf('Enter coefficient for A(%d,%d): ', i, j));
    end
end

% Ask user for right-hand side values
b = zeros(n,1);
for i = 1:n
    b(i) = input(sprintf('Enter value for b(%d): ', i));
end

% Gaussian elimination with partial pivoting
for k = 1:n-1
    % Partial pivoting
    [~,p] = max(abs(A(k:n,k)));
    p = p + k - 1;
    if p ~= k
        A([k,p],:) = A([p,k],:);
        b([k,p]) = b([p,k]);
    end

    % Elimination
    for i = k+1:n
```

```
        m = A(i,k) / A(k,k);
        A(i,k:n) = A(i,k:n) - m * A(k,k:n);
        b(i) = b(i) - m * b(k);
    end
end

% Back substitution
x = zeros(n,1);
x(n) = b(n) / A(n,n);
for i = n-1:-1:1
    x(i) = (b(i) - A(i,i+1:n) * x(i+1:n)) / A(i,i);
end

% Display solution
disp('Solution:');
```

```
Solution:
```

```
disp(x);
```

```
   -1
   -1
   -1
```

```
%2)

% Ask user for matrix size
n = input('Enter number of variables: ');

% Ask user for matrix coefficients
A = zeros(n);
for i = 1:n
    for j = 1:n
        A(i,j) = input(sprintf('Enter coefficient for A(%d,%d): ', i, j));
    end
end

% Ask user for right-hand side values
b = zeros(n,1);
for i = 1:n
    b(i) = input(sprintf('Enter value for b(%d): ', i));
end

% Gaussian elimination with partial pivoting
for k = 1:n-1
    % Partial pivoting
    [~,p] = max(abs(A(k:n,k)));
    p = p + k - 1;
    if p ~= k
        A([k,p],:) = A([p,k],:);
        b([k,p]) = b([p,k]);
```

```
    end

    % Elimination
    for i = k+1:n
        m = A(i,k) / A(k,k);
        A(i,k:n) = A(i,k:n) - m * A(k,k:n);
        b(i) = b(i) - m * b(k);
    end
end

% Back substitution
x = zeros(n,1);
x(n) = b(n) / A(n,n);
for i = n-1:-1:1
    x(i) = (b(i) - A(i,i+1:n) * x(i+1:n)) / A(i,i);
end

% Display solution
disp('Solution:');
```

```
Solution:
```

```
disp(x);
```

```
    1
    3
  -10
```

```
%3)a

A = [2 3 -1 ; 3 2 1 ; 1 -5 3]
b = [5 ; 10 ; 0]
n = size(A,1);
L = eye(n);
U = A;
for k = 1:n-1
 for i = k+1:n
 L(i,k) = U(i,k)/U(k,k);
 U(i,k:n) = U(i,k:n) - L(i,k)*U(k,k:n);
 end
end
disp('L = ');
disp(L);
disp('U = ');
disp(U);
y = zeros(n,1);
y(1) = b(1);
for i = 2:n
 y(i) = b(i) - L(i,1:i-1)*y(1:i-1);
end
x = zeros(n,1);
```

```
x(n) = y(n)/U(n,n);
for i = n-1:-1:1
 x(i) = (y(i) - U(i,i+1:n)*x(i+1:n))/U(i,i);
end
disp('X = ');
disp(x);
```

```
L = 3×3
     1     0     0
     0     1     0
     0     0     1
U = 3×3
     2     3    -1
     3     2     1
     1    -5     3
Solution:
     1
     2
     3
```

```
%3)b

A = [1 1 1;2 -1 -1;1 -1 1]
```

```
A = 3×3
     1     1     1
     2    -1    -1
     1    -1     1
```

```
b = [3 ; 3 ; 9]
```

```
b = 3×1
     3
     3
     9
```

```
n = size(A,1);
L = eye(n);
U = A;
for k = 1:n-1
 for i = k+1:n
 L(i,k) = U(i,k)/U(k,k);
 U(i,k:n) = U(i,k:n) - L(i,k)*U(k,k:n);
 end
end
disp('L = ');
```

```
L =
```

```
disp(L);
```

```
    1.0000         0         0
    2.0000    1.0000         0
    1.0000    0.6667    1.0000
```

```
disp('U = ');
```

```
U =
```

```
disp(U);
```

```
     1     1     1
     0    -3    -3
     0     0     2
```

```
y = zeros(n,1);
y(1) = b(1);
for i = 2:n
 y(i) = b(i) - L(i,1:i-1)*y(1:i-1);
end
x = zeros(n,1);
x(n) = y(n)/U(n,n);
for i = n-1:-1:1
 x(i) = (y(i) - U(i,i+1:n)*x(i+1:n))/U(i,i);
end
disp('X = ');
```

```
X =
```

```
disp(x);
```

```
     2
    -3
     4
```