22MAT122 Mathematics for computing - 2

S. Praveenkumar
22048
AIE

Assignment - 1

01) Given:

Let, x = cashews

y = pistachios

z = almonds

$x + y + z = 900$ - (i)

30% almonds, 20% cashews, 10% pistachios were consumed

$1.0 - 0.3 = 0.7$

$1.0 - 0.2 = 0.8$

$1.0 - 0.1 = 0.9$

$0.8x + 0.9y + 0.7z = 770$ - (ii)

there are hundred more cashews than almond,

$x = 100 + z$ - (iii)

writing (i), (ii) and (iii) in Matrix form

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 900 \\ 0.8 & 0.9 & 0.7 & 700 \\ 1 & 0 & -1 & 100 \end{array}\right] \quad R_3 = R_3 + R_2, \quad R_2 \leftrightarrow 10R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 900 \\ 0 & 0.1 & -0.1 & 50 \\ 0 & -1 & -2 & -800 \end{array}\right] \quad R_3 = \frac{-1}{3}(R_3)$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 900 \\ 0 & 1 & -1 & 500 \\ 0 & 0 & -3 & -300 \end{array}\right]$$

using back substitution

$-3z = -300$

$\boxed{z = 100}$

$y - 1(100) = 500$

$\boxed{y = 600}$

$x + 600 + 100 = 900$

$\boxed{x = 200}$

$\therefore x = 200, y = 600, z = 100$

Q2) $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} + \begin{bmatrix} c & 1 \\ 1 & d \end{bmatrix} = \begin{bmatrix} a+c & 1 \\ 1 & b+d \end{bmatrix}$

$$k\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} = \begin{bmatrix} ka & 1 \\ 1 & kb \end{bmatrix}$$

i) $\vec{a}+\vec{b} = \vec{b}+\vec{a}$

$$\vec{a} = \begin{bmatrix} a_1 & 1 \\ 1 & b_1 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} a_2 & 1 \\ 1 & b_2 \end{bmatrix}$$

$$\vec{a}+\vec{b} = \begin{bmatrix} a_1+a_2 & 1 \\ 1 & b_1+b_2 \end{bmatrix} = \begin{bmatrix} a_2+a_1 & b_1 \\ 1 & b_2+b_1 \end{bmatrix}$$

$$\Rightarrow \vec{b}+\vec{a}$$

ii) $\vec{a}+(\vec{b}+\vec{c}) = (\vec{a}+\vec{b})+\vec{c}$

$$\vec{c} = \begin{bmatrix} a_3 & 1 \\ 1 & b_3 \end{bmatrix}$$

$$\begin{bmatrix} a_1 & 1 \\ 1 & b_1 \end{bmatrix} + \left[\begin{bmatrix} a_2 & 1 \\ 1 & b_2 \end{bmatrix} + \begin{bmatrix} a_3 & 1 \\ 1 & b_3 \end{bmatrix}\right] = \begin{bmatrix} a_1 & 1 \\ 1 & b_1 \end{bmatrix} + \begin{bmatrix} a_2+a_3 & 1 \\ 1 & b_2+b_3 \end{bmatrix}$$

$$= \begin{bmatrix} a_1+a_2+a_3 & 1 \\ 1 & b_1+b_2+b_3 \end{bmatrix} = \begin{bmatrix} (a_1+a_2)+a_3 & 1 \\ 1 & (b_1+b_2)+b_3 \end{bmatrix}$$

$$= (\vec{a}+\vec{b})+\vec{c}$$

iii) $\vec{a}+\vec{0} = \vec{a}$

$$\vec{0} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\vec{a}+\vec{0} = \begin{bmatrix} a_1+0 & 1 \\ 1 & b_1+0 \end{bmatrix} = \begin{bmatrix} a_1 & 1 \\ 1 & b_1 \end{bmatrix} = \vec{a}$$

$$\Rightarrow \vec{a}$$

iv) $1.\vec{a} = \vec{a}$

$$1.\begin{bmatrix} a_1 & , \\ , & b_1 \end{bmatrix} = \begin{bmatrix} a_1 . 1 & , \\ , & b_1 \end{bmatrix} = \begin{bmatrix} a_1 & , \\ , & b_1 \end{bmatrix} = \vec{a}$$

v) $k(\vec{a}+\vec{B}) = k\vec{a} + k\vec{b}$

$$k\left[\left(\begin{matrix} a_1 & , \\ , & b_1 \end{matrix}\right) + \begin{bmatrix} a_2 & , \\ , & b_2 \end{bmatrix}\right] = k\begin{bmatrix} a_1+a_2 & , \\ , & b_1+b_2 \end{bmatrix}$$

$$\begin{bmatrix} ka_1+ka_2 & , \\ , & kb_1+kb_2 \end{bmatrix} = \begin{bmatrix} ka_1 & , \\ , & kb_1 \end{bmatrix} + \begin{bmatrix} ka_2 & , \\ , & kb_2 \end{bmatrix}$$

$$= k\begin{bmatrix} a_1 & , \\ , & b_1 \end{bmatrix} + k\begin{bmatrix} b_2 & , \\ , & b_2 \end{bmatrix}$$

$$\Rightarrow k\vec{a} + k\vec{b}$$

vi) $\vec{a} + (-\vec{a}) = 0$

$$-\vec{a} = \begin{bmatrix} -a & , \\ , & -b \end{bmatrix}$$

$$\vec{a} + (-\vec{a}) = \begin{bmatrix} a-a & , \\ , & b-b \end{bmatrix} = \begin{bmatrix} 0 & , \\ , & 0 \end{bmatrix} \Rightarrow \vec{0}$$

vii) $\vec{k}.(a+b) = a\vec{k} + b\vec{k}$

$$\vec{k} = \begin{bmatrix} k_1 & , \\ , & k_2 \end{bmatrix}$$

$$(a+b)\begin{bmatrix} k_1 & , \\ , & k_2 \end{bmatrix} = \begin{bmatrix} (a+b)k_1 & , \\ , & (a+b)k_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} ak_1+bk_1 & , \\ , & ak_2 \, bk_2 \end{bmatrix}$$

$$= \begin{bmatrix} ak_1 & , \\ 1 & ak_2 \end{bmatrix} + \begin{bmatrix} bk_1 & , \\ , & bk_2 \end{bmatrix} = a\vec{k} + b\vec{k}$$

viii) $\vec{K} \cdot (a+b) = a\vec{K} + b\vec{K}$

$$a+b\begin{bmatrix} k_1 & 1 \\ 1 & k_2 \end{bmatrix} = \begin{bmatrix} (a+b)k_1 & 1 \\ 1 & (a+b)k_2 \end{bmatrix}$$

$$= \begin{bmatrix} ak_1 + bk_1 & 1 \\ 1 & ak_2\ bk_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} ak_1 & 1 \\ 1 & ak_2 \end{bmatrix} + \begin{bmatrix} bk_1 & 1 \\ 1 & bk_2 \end{bmatrix} \Rightarrow a\vec{k} + b\vec{m}$$

∵ hence verified //

Q3) Determine $P^T A P$:

$$A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \quad A - \lambda I = \begin{bmatrix} 2-\lambda & -1 & -1 \\ -1 & 2-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{bmatrix}$$

$$\Rightarrow 2-\lambda^2 - 1 - 1 - (2-\lambda) - (2-\lambda) - (2-\lambda)$$

$$\Rightarrow (4+\lambda^2-4\lambda)(2-\lambda) \cdot 2 - 3(2-\lambda)$$

$$\Rightarrow 6\lambda^2 - \lambda^3 - 9\lambda = 0$$

$$\lambda = 0,\ \lambda = 3,\ \lambda = 3$$

$\lambda = 0$

$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \quad \begin{matrix} 2x - y - z = 0 \\ -x + 2y - z = 0 \\ -x + y + 2z = 0 \end{matrix} \quad x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\lambda = 3$

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix} \quad \begin{matrix} -x - y - z = 0 \\ -x - y - z = 0 \\ -x - y - z = 0 \end{matrix} \quad x_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \quad x_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\sqrt{(-1)^2 + (1)^2 + 0} = \sqrt{2}$$

$$P = \begin{bmatrix} 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{2} & 0 \\ 1/\sqrt{3} & 0 & 1/\sqrt{2} \end{bmatrix} \quad P^T = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix}$$

$D = P^T A P$

$$= \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1/\sqrt{3} & -1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{2} & 0 \\ 1/\sqrt{3} & 0 & 1/\sqrt{2} \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Hence P orthogonally diagonalizes A

4) find SVD.

$$A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 10 & 0 & 2 \\ 0 & 10 & 4 \\ 2 & 4 & 2 \end{bmatrix}$$

$\Rightarrow (10-\lambda^2)(2-\lambda) - 4(10-\lambda) = 160$

$-\lambda(\lambda-12)(\lambda-10)$

$\lambda_1 = 12, \lambda_2 = 10, \lambda_3 = 0$

$\lambda_1 = 12$

$$\begin{bmatrix} -2 & 0 & 2 \\ 0 & -2 & 4 \\ 2 & 4 & -10 \end{bmatrix}$$

$-2x + 0y + 2z = 0$

$0x - 2y + 4z = 0$

$2x + 4y - 10z = 0$

$$x_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$\sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$

$\lambda = 10$

$$\begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 4 \\ 2 & 4 & -8 \end{bmatrix}$$

$0x + 0y + 2z = 0$

$0x + 0y + 4z = 0$

$2x + 4y - 8z = 0$

$$x_2 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

$\sqrt{-2^2 + 1^2 + 0^2} = \sqrt{5}$

$\lambda = 0,$

$$\begin{bmatrix} 10 & 0 & 2 \\ 0 & 10 & 4 \\ 2 & 4 & 2 \end{bmatrix} \quad \begin{aligned} 10x + 0y + 2z &= 0 \\ 0 + 10y + 4z &= 0 \\ 2x + 4y + 2z &= 0 \end{aligned} \qquad x_3 = \begin{bmatrix} -1/5 \\ -2/5 \\ 1 \end{bmatrix}$$

$$\sqrt{(-1/5)^2 + (-2/5)^2 + 1^2} = \sqrt{5/6}$$

$\sigma_1 = \sqrt{12} = 2\sqrt{3}$

$\sigma_2 = \sqrt{10}$

$\sigma_3 = \sqrt{0} = 0$

$$\Sigma = \begin{bmatrix} 2\sqrt{3} & 0 & 0 \\ 0 & \sqrt{10} & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} \sqrt{6}/6 & -\frac{\cancel{2}\sqrt{5}}{\sqrt{5}} & -\frac{\sqrt{30}}{30} \\ \sqrt{6}/3 & \sqrt{5}/3 & -\frac{\sqrt{30}}{15} \\ \sqrt{6}/6 & 0 & \frac{\sqrt{30}}{6} \end{bmatrix}$$

$$u_i = \frac{1}{\sigma_i} \begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix} v_i$$

$$u_1 = \frac{1}{2\sqrt{3}} \begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{6}/6 \\ \sqrt{6}/3 \\ \sqrt{6}/6 \end{bmatrix} = \begin{bmatrix} \sqrt{2}/\sqrt{2} \\ \sqrt{2}/2 \end{bmatrix}$$

$$U_2 = \frac{1}{\sqrt{10}} \begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix} \begin{bmatrix} -2/\sqrt{5} \\ 5/3 \\ 0 \end{bmatrix} = \begin{bmatrix} -\sqrt{2}/2 \\ \sqrt{2}/2 \end{bmatrix}$$

$$U = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}_{2\times 2}$$

$A = U \Sigma V^T$

$$= \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \begin{bmatrix} 2/\sqrt{3} & 0 & 0 \\ 0 & \sqrt{10} & 0 \end{bmatrix} \begin{bmatrix} \sqrt{6}/6 & -\frac{2\sqrt{3}}{3} & -\frac{\sqrt{30}}{30} \\ \sqrt{6}/3 & \sqrt{3}/5 & -\sqrt{30}/5 \\ \sqrt{6}/6 & 0 & \sqrt{30}/6 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}_{2\times 3}$$

SVD was verified.