Q) Find all the critical points:

3.10. $f(x,y) = 1 + x^2 + y^2$

$$\frac{d(1+x^2+y^2)}{dx} = \frac{d(1)}{dx} + \frac{d(x^2)}{dx} + \frac{d(y^2)}{dx}$$

$$\Rightarrow 0 + 2x + 0$$

$$\Rightarrow 2x - ①$$

Equating ① to 'zero'

$$2x = 0$$

$$\boxed{x = 0}$$

$$\frac{d(1+x^2+y^2)}{dy} = \frac{d(1)}{dy} + \frac{d(x^2)}{dy} + \frac{dy^2}{dy}$$

$$\Rightarrow 0 + 0 + 2y$$

$$\Rightarrow 2y = 0$$

$$\boxed{y = 0}$$

∴ critical point are (0,0)

3.11) $f(x,y) = (3x-2)^2 + (y-4)^2$

$$\frac{d[(3x-2)^2 + (y-4)^2]}{dx} = 2(3x-2) \times 3$$

$$= (6x-4)3$$

$$= 18x - 12 \rightarrow ①$$

Equating ① to '0'

$$18x - 12 = 0$$

$$18x = 12$$

$$\boxed{x = \frac{2}{3}}$$

$$\frac{d[(3x-2)^2 + (y-4)^2]}{dy}$$

$$\Rightarrow 2(y-4)$$
$$\Rightarrow 2y - 4(2)$$
$$\Rightarrow 2y - 8$$

$$2y - 8 = 0$$
$$2y = 8$$
$$\boxed{y = 4}$$

$\therefore$ Critical point is $\left(\frac{2}{3}, 4\right)$

3.12) $f(x, y) = x^4 + y^4 - 16xy$

$$\frac{d(x^4 + y^4 - 16xy)}{dx}$$

$$\Rightarrow 4x^3 - 16y$$

Equating To Zero

$$4x^3 - 16y = 0$$
$$4x^3 = 16y$$
$$\boxed{x^3 = 4y}$$

$$\frac{d(x^4 + y^4 - 16xy)}{dy}$$

$$\Rightarrow 4y^3 - 16x$$

Equating to Zero
$$4y^3 - 16x = 0$$
$$4y^3 = 16x$$
$$\boxed{y^3 = 4x}$$

$$x^3 = 4y \qquad\qquad y^3 = 4x$$

$$\left(\frac{4^3}{4}\right)^3 = 4y$$

$$(2)^3 = 4x$$

$$(y^3)^3 = 4^4 \cdot y$$

$$x = \frac{8^2}{4}$$

$$y^9 = 2^8 \cdot y$$

$$\boxed{x = 2}$$

$$\frac{y^9}{y} = 2^8$$

$$y^8 = 2^8$$

$$\boxed{y = 2}$$

$$\therefore \text{ the critical point are } (2, 2)$$

2.13) $f(x, y) = 15x^3 - 3xy + 15y^3$

$\therefore$

$$\frac{d(15x^3 - 3xy + 15y^3)}{dx} = 3(15)x^2 - 3y$$

$$45x^2 - 3y = 0$$

$$45x^2 = 3y$$

$$\boxed{15x^2 = y}$$

$$\frac{d(15x^3 - 3xy + 15y^3)}{dy} = 45y^2 - 3x$$

$$45y^2 - 3x = 0$$

$$\boxed{15y^2 = x}$$

$$15(15y^2)^2 = y \qquad\qquad 18 \times \frac{1}{15^2} = x$$

$$15 \cdot 15^2 y^4 = y \qquad\qquad \frac{1}{15^2}$$

$$15^3 = \frac{1}{y^3} \qquad\qquad \boxed{x = \frac{1}{15}}$$

$$y^3 = \frac{1}{15^3}$$

$$\therefore \text{ the critical point are } \left(\frac{1}{15}, \frac{1}{15}\right)$$

$$\boxed{y = \frac{1}{15}}$$

find the critical points using algebraic techniques . verify
using partial derivaties ted

3.14) $f(x,y) = \sqrt{x^2 + y^2 + 1}$

$\quad f'(x) = \dfrac{x}{\sqrt{x^2 + y^2 + 1}}$ $\qquad\qquad f'(y) = \dfrac{y}{\sqrt{x^2 + y^2 + 1}}$

$\quad f''(x) = \dfrac{y^2 + 1}{(y^2 + x^2 + 1)^{3/2}}$ $\qquad f''(y) = \dfrac{x^2 + 1}{(y^2 + x^2 + 1)^{3/2}}$

Equating $f'(x)$ & $f'(y)$ to zero,

$\dfrac{x}{\sqrt{x^2 + y^2 + 1}} = 0 \quad \Rightarrow \boxed{x = 0}$ $\qquad\qquad \dfrac{y}{\sqrt{x^2 + y^2 + 1}} = 0$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \boxed{y = 0}$

$\therefore$ critical points is (0,0)

mixed partial derivative

$\dfrac{d^2 f}{dx\,dy} = -3xy\,(x^2 + y^2 + 1)^{-5/2}$

$\qquad x = y = 0,$

$\qquad D = 0$

$\therefore D = 0,$ the test is inconclusive

3.15) $f(x,y) = -x^2 - 5y^2 + 8x - 10y - 13$

$\qquad \dfrac{d\,(-x^2 - 5y^2 + 8x - 10y - 13)}{dx} = -2x + 8$

$\qquad\qquad -2x + 8 = 0$

$\qquad\qquad 2x = 8$

$\qquad\qquad \boxed{x = 4}$

$$\frac{d(-x^2 - 5y^2 + 8x - 10y - 13)}{dy}$$

$$\Rightarrow -10y - 10$$

$$-10y - 10 = 0$$

$$10y = -10$$

$$\boxed{y = -1}$$

∴ critical point is $(4, -1)$

$$f''(x) = -2 \qquad\qquad f''(y) = -10$$
$$f''(x) < 0 \qquad\qquad f''(y) < 0$$

∴ it is maximum at $(4, -1)$

3.1b) $f(x,y) = x^2 + y^2 + 2x - 6y + 6$

$$f'(x) = 2x + 2 \qquad\qquad f'(y) = 2y - 6$$
$$f''(x) = 2 \qquad\qquad f''(y) = 2$$

$$f'(x) = 0 \qquad\qquad 2y - 6 = 0$$
$$2x = -2 \qquad\qquad 2y = 6$$
$$\boxed{x = -1} \qquad\qquad \boxed{y = 3}$$

$$f''(x) > 0 \qquad\qquad f''(y) > 0$$

∴ critical points is $(-1, 3)$

it is minimum at $(-1, 3)$

2.17) $f(x,y) = \sqrt{x^2+y^2} + 2$

$$f'(x) = \frac{x}{\sqrt{x^2+y^2}} \qquad\qquad f'(y) = \frac{y}{\sqrt{x^2+y^2}}$$

$$f''(x) = \frac{y^2}{(x^2+y^2)^{3/2}} \qquad\qquad f''(y) = \frac{x^2}{(x^2+y^2)^{3/2}}$$

equating $f'(x)$ and $f'(y)$ to zero

$$f'(x) = 0 \qquad\qquad f'(y) = 0$$
$$x = 0 \qquad\qquad y = 0$$

∴ critical points are $(0,0)$

the mixed partial derivative
$$\frac{d^2 f}{dx\,dy} = \frac{-3xy}{(x^2+y^2)^{5/2}}$$

since $x = y = 0$,
$$D = 0$$

∴ $D = 0$, the test is inconclusive

3.18) $f(x,y) = -x^3 + 4xy - 2y^2 + 1$

$$f'(x) = 4y - 3x^2 \qquad\qquad f'(y) = 4x - 4y$$
$$f''(x) = -6x \qquad\qquad f''(y) = -4$$

$$f''(x) = -6\left(\frac{4}{3}\right) = \frac{-24}{3} = -8$$

$$D = -8(-4) - 4(+4)$$
$$= 32 - 16$$
$$\boxed{D = 16} \qquad\qquad D > 0$$

$f\left(\frac{4}{3}, \frac{4}{3}\right)$ is a saddle point

2.19) $f(x,y) = x^2 y^2$

$f'(x) = 2y^2 x$ $f'(y) = 2x^2 y$

$f''(x) = 2y^2$ $f''(y) = 2x^2$

$= 4x^2 y^2 - 16x^2 y^2$

$= -12 x^2 y^2$

$f'(x) = 0$ $f'(y) = 0$

$2y^2 x = 0$ $2x^2 y = 0$

$\boxed{y = 0}$

$D = -12 x^2 y^2$

$D < 0$,

$f(0,0)$ is a saddle point