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LAB-3

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a) Find the SVD of $A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}_{2\times 3}$

$A^T = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$

$AA^T = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ $A^TA = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

i) AA^T

$\begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix}$ $2-\lambda(2-\lambda)-1=0$

$\cancel{\lambda}^2 - 4\lambda + 3 = 0$

$\lambda = 1, \lambda = 3$

if $\lambda = 3$

$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$-x-y=0$

$-x=y$

$x_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

finding norm of x_1

$x_1 = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$

if $\lambda = 1$

$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

$x-y=0$

$x=y$

$x_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

finding norm of x_2

$x_2 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$

$U = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$

ii) $A^TA = \begin{bmatrix} 2-\lambda & -1 & 1 \\ -1 & 1-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{bmatrix}$

$S_1 = 4$

$S_2 = +3$

$S_3 = 0$

$\lambda^3 - 4t^2 + 3t + 0 = 0$

$\lambda(\lambda^2 - 4t + 3) = 0$

$\lambda = 0, 1, 3$

if $\lambda = 3$

$\begin{bmatrix} -1 & -1 & 1 \\ -1 & -2 & 0 \\ 1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$-x - y + z = 0$

$-x - 2y + 0z = 0$

$x_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

finding norm of x_1

$x_1 = \begin{bmatrix} \frac{2}{\sqrt6} \\ \frac{-1}{\sqrt6} \\ \frac{1}{\sqrt6} \end{bmatrix}$

if $\lambda = 1$

$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$x - y + z = 0$

$-x + 0y + 0z \quad 0$

$x_2 = \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}$

finding norm of x_2

$x_2 = \begin{bmatrix} 0 \\ -1/\sqrt2 \\ 1/\sqrt2 \end{bmatrix}$

if $\lambda = 0$

$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$2x - y + z = 0$

$-x + y + 0z = 0$

$x + 0y + z = 0$

$x_3 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

finding norm of x_3

$x_3 = \begin{bmatrix} -1/\sqrt3 \\ -1/\sqrt3 \\ 1/\sqrt3 \end{bmatrix}$

$$V = \begin{bmatrix} 2/\sqrt{6} & 0 & -1/\sqrt{3} \\ -1/\sqrt{6} & -1/\sqrt{2} & -1/\sqrt{3} \\ 1/\sqrt{6} & 1/\sqrt{2} & 1/\sqrt{3} \end{bmatrix} \qquad \Sigma = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{1} & 0 \end{bmatrix}$$

$A = U \Sigma V^T$

$$A = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{1} & 0 \end{bmatrix} \begin{bmatrix} 2/\sqrt{6} & -1/\sqrt{6} & 1/\sqrt{6} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} = A$$

2) find the SVD of A

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 5 & 4 & 3 \\ 4 & 5 & 3 \\ 3 & 3 & 2 \end{bmatrix} \qquad A^TA = \begin{bmatrix} 6 & 5 \\ 5 & 6 \end{bmatrix}$$

Eigenvalues of A^TA:

$$\begin{bmatrix} 6-\lambda & 5 \\ 5 & 6-\lambda \end{bmatrix} = 36 - 6\lambda - 6\lambda + \lambda^2 - 25$$

$$\lambda^2 - 12\lambda + 11 = 0$$

$$\boxed{\lambda = 11, 1}$$

if $\lambda = 11$

$$\begin{bmatrix} -5 & 5 \\ 5 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$-5x + 5y = 0$

$5x - 5y = 0$

$\lambda = 1$

$$\begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$5x + 5y = 0$

$5x + 5y = 0$

finding of norm of x_1 finding norm of x_2

$$x_1 = \begin{bmatrix} +1/\sqrt{2} \\ +1/\sqrt{2} \end{bmatrix} \qquad x_2 = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$V = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \qquad V^T = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$\sigma = \sqrt{11}$, $\sigma_2 = \sqrt{1} = 1$

$$\Sigma = \begin{bmatrix} \sqrt{11} & 0 \\ 0 & \sqrt{1} \end{bmatrix}$$

$$= \frac{1}{\sqrt{11}} \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \qquad u_2 = \frac{1}{\sqrt{1}} \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -1/\sqrt{2} \\ +1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 3/\sqrt{22} \\ 3/\sqrt{22} \\ 2/\sqrt{22} \end{bmatrix} \qquad = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{bmatrix}$$

$$U = \begin{bmatrix} 3/\sqrt{22} & 1/\sqrt{2} \\ 3/\sqrt{22} & -1/\sqrt{2} \\ 2/\sqrt{22} & 0 \end{bmatrix}$$

$$A = U \Sigma V^T = \begin{bmatrix} 3/\sqrt{22} & 1/\sqrt{2} \\ 3/\sqrt{22} & -1/\sqrt{2} \\ 2/\sqrt{22} & 0 \end{bmatrix} \begin{bmatrix} \sqrt{11} & 0 \\ 0 & \sqrt{1} \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 3/\sqrt{2} & 1/\sqrt{2} \\ 3/\sqrt{2} & 1/\sqrt{2} \\ 2/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{bmatrix}$$

3) find the SVD of A

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{2\times 4}$$

$$A^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}_{4\times 2}$$

$$AA^T = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}_{2\times 2} \qquad A^TA = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

AA^T

$$\begin{bmatrix} 2-t & 0 \\ 0 & 2-t \end{bmatrix} \qquad 2-t\,(2-t) \Rightarrow t^2-4t+4=0$$

$$t=2,\ t=2$$

if $t=2$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$0x+0y=0$

assign ~~one~~ free variable

$$x_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad x_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$U = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1-t & 0 & 1 & 0 \\ 0 & 1-t & 0 & 1 \\ 1 & 0 & 1-t & 0 \\ 0 & 1 & 0 & 1-t \end{bmatrix}$$

$$\Rightarrow t^4-4t^3+4t^2 = t^2(t^2-4t+4)$$

$$\Rightarrow t^2(t-2)(t-2)$$

$$t = 0, 2, 2, 0$$

if $t=2$

$$\begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = 0$$

$$a + 0b + c + 0d = 0$$

$$0a + b + 0c + d = 0$$

$$x_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad x_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

if $t=0$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = 0$$

$$a + 0b + c + 0d = 0$$

$$0a + b + 0c + d = 0$$

$$x_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad x_4 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$V = \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix} \qquad V^T = \begin{bmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix}$$

$$A = U \Sigma V^T$$

$$U = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} \sqrt{2} & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 \end{bmatrix}$$

$$A \Rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A //$$

4) Find the SVD of A

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix} \qquad A^T = \begin{bmatrix} 3 & 2 \\ 2 & 3 \\ 2 & -2 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 17 & 8 \\ 8 & 17 \end{bmatrix}_{2\times 2} \qquad A^TA = \begin{bmatrix} 13 & 12 & 2 \\ 12 & 13 & -2 \\ 2 & -2 & 8 \end{bmatrix}$$

$$AA^T \begin{bmatrix} 17-\lambda & 8 \\ 8 & 17-\lambda \end{bmatrix} \qquad \lambda^2 - 34\lambda + 225 = 0$$

$$(\lambda - 25)(\lambda - 9) = 0$$

$$\boxed{\lambda = 9, 25}$$

if $\lambda = 25$

$$\begin{bmatrix} -8 & 8 \\ 8 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-8x + 8y = 0$

$$x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

finding norm x_1

$$x_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

if $\lambda = 9$

$$\begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$8x + 8y = 0$

$$x_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

finding norm x_2

$$x_2 = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$$

$$U = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

i) $\underline{A^T A}$

$$\begin{bmatrix} 13-\lambda & 12 & 2 \\ 12 & 13-\lambda & -2 \\ 2 & -2 & 8-\lambda \end{bmatrix}$$

$$\lambda^3 - 34\lambda^2 + 225\lambda = 0$$

$$(\lambda - 25)(\lambda - 9)(\lambda - 0) = 0$$

$$\boxed{\lambda = 25, \lambda = 9, \lambda = 0}$$

if $\lambda = 25$

$$\begin{bmatrix} -12 & 12 & 2 \\ 12 & -12 & -2 \\ 2 & -2 & -17 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

find norm of x_1

$$x_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$$

if $\lambda = 9$

$$\begin{bmatrix} 4 & 12 & 2 \\ 12 & 4 & -2 \\ 2 & -2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$$

finding norm x_2

$$x_2 = \begin{bmatrix} 1/\sqrt{18} \\ -1/\sqrt{18} \\ 4/\sqrt{18} \end{bmatrix}$$

if) $\lambda = 0$

$$\begin{bmatrix} 13 & 12 & 2 \\ 12 & 13 & -2 \\ 2 & -2 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_3 = \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$$

find norm x_3

$$x_3 = \begin{bmatrix} \frac{2}{3} \\ \frac{-2}{3} \\ \frac{-1}{3} \end{bmatrix}$$

$$V_B = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{18} & 2/3 \\ 1/\sqrt{2} & -1/\sqrt{18} & -2/3 \\ 0 & 4/\sqrt{18} & -1/3 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} \sqrt{\lambda} & 0 & 0 \\ 0 & \sqrt{\lambda} & 0 \\ 0 & 0 & \sqrt{\lambda} \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \end{bmatrix}$$

$$A = \left[U \Sigma V^T \right]$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{18} & -1/\sqrt{18} & 4/\sqrt{18} \\ 2/3 & -2/3 & -1/3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$$

$$\Rightarrow \underline{\underline{A}}$$