```
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```

```
%Find all the critical points
%310
%f(x,y) = 1 + x^2 + y^2
syms x y z
f= 1 + x^2 + y^2
```

f = $x^2 + y^2 + 1$

```
fx=diff(f,x)
```

fx = $2\,x$

```
fy=diff(f,y)
```

fy = $2\,y$

```
[xcr,ycr]=solve(fx,fy); [xcr,ycr]
```

ans = $\begin{pmatrix} 0 & 0 \end{pmatrix}$

```
fxx=diff(fx,x);
fxy=diff(fx,y);
fyy=diff(fy,y);
hessdetf=fxx*fyy-fxy^2;
xcr = xcr(1:1); ycr = ycr(1:1);
for k = 1:1
 [xcr(k), ycr(k), subs(hessdetf, [x,y], [xcr(k), ycr(k)]), subs(fxx, [x,y],
[xcr(k), ycr(k)])];
end
```

```
%3.11
%f(x,y) = (3*x - 2)^2 + (y - 4)^2
syms x y z
f= (3*x - 2)^2 + (y - 4)^2
```

f = $(y-4)^2 + (3\,x-2)^2$

```
fx=diff(f,x)
```

fx = $18\,x - 12$

```
fy=diff(f,y)
```

fy = $2\,y - 8$

```
[xcr,ycr]=solve(fx,fy); [xcr,ycr]
```

ans =

$$\begin{pmatrix} \frac{2}{3} & 4 \end{pmatrix}$$

```
fxx=diff(fx,x);
fxy=diff(fx,y);
fyy=diff(fy,y);
hessdetf=fxx*fyy-fxy^2;
xcr = xcr(1:1); ycr = ycr(1:1);
for k = 1:1
 [xcr(k), ycr(k), subs(hessdetf, [x,y], [xcr(k), ycr(k)]), subs(fxx, [x,y],
[xcr(k), ycr(k)])];
end
```

```
%312
%f(x,y) = x^4 + y^4 - 16*x*y
syms x y z
f= x^4 + y^4 - 16*x*y
```

f = $x^4 - 16\,x\,y + y^4$

```
fx=diff(f,x)
```

fx = $4\,x^3 - 16\,y$

```
fy=diff(f,y)
```

fy = $4\,y^3 - 16\,x$

```
[xcr,ycr]=solve(fx,fy); [xcr,ycr]
```

ans =

$$\begin{pmatrix} 0 & 0 \\ -2 & -2 \\ 2 & 2 \\ 2\,\mathrm{i} & -2\,\mathrm{i} \\ -2\,\mathrm{i} & 2\,\mathrm{i} \\ \sigma_3 & 2\,\sigma_1 \\ -\sigma_3 & -2\,\sigma_1 \\ \sigma_2 & -2\,\sigma_1\,\mathrm{i} \\ -\sigma_2 & 2\,\sigma_1\,\mathrm{i} \end{pmatrix}$$

where

$$\sigma_1 = (-1)^{1/4}$$

$$\sigma_2 = 2\ (-1)^{3/4}\,\mathrm{i}$$

$$\sigma_3 = 2\ (-1)^{3/4}$$

```
fxx=diff(fx,x);
fxy=diff(fx,y);
fyy=diff(fy,y);
hessdetf=fxx*fyy-fxy^2;
xcr = xcr(1:1); ycr = ycr(1:1);
for k = 1:1
 [xcr(k), ycr(k), subs(hessdetf, [x,y], [xcr(k), ycr(k)]), subs(fxx, [x,y],
[xcr(k), ycr(k)])];
end
```

```
%313
%f(x,y) = 15*x^3 - 3*x*y + 15*y^3
syms x y z
f= 15*x^3 - 3*x*y + 15*y^3
```

f = $15\,x^3 - 3\,x\,y + 15\,y^3$

```
fx=diff(f,x)
```

fx = $45\,x^2 - 3\,y$

```
fy=diff(f,y)
```

fy = $45\,y^2 - 3\,x$

```
[xcr,ycr]=solve(fx,fy); [xcr,ycr]
```

ans =

$$\begin{pmatrix} 0 & 0 \\ \frac{1}{15} & \frac{1}{15} \\ -\frac{1}{30}+\frac{\sqrt{3}\,\mathrm{i}}{30} & -\frac{1}{30}-\frac{\sqrt{3}\,\mathrm{i}}{30} \\ -\frac{1}{30}-\frac{\sqrt{3}\,\mathrm{i}}{30} & -\frac{1}{30}+\frac{\sqrt{3}\,\mathrm{i}}{30} \end{pmatrix}$$

```
fxx=diff(fx,x);
fxy=diff(fx,y);
fyy=diff(fy,y);
hessdetf=fxx*fyy-fxy^2;
xcr = xcr(1:1); ycr = ycr(1:1);
for k = 1:1
 [xcr(k), ycr(k), subs(hessdetf, [x,y], [xcr(k), ycr(k)]), subs(fxx, [x,y],
[xcr(k), ycr(k)])];
end
```

```
%314
%f(x,y) = (x^2 + y^2 + 1)^1/2
syms x y z
f= (x^2 + y^2 + 1)^(1/2)
```

f = $\sqrt{x^2+y^2+1}$

```
fx=diff(f,x)
```

fx =

$$\frac{x}{\sqrt{x^2+y^2+1}}$$

```
fy=diff(f,y)
```

fy =

$$\frac{y}{\sqrt{x^2+y^2+1}}$$

```
[xcr,ycr]=solve(fx,fy); [xcr,ycr]
```

ans = $\begin{pmatrix} 0 & 0 \end{pmatrix}$

```
fxy=diff(fx,y)
```

fxy =

$$-\frac{x\,y}{(x^2+y^2+1)^{3/2}}$$

```
fyy=diff(fy,y)
```

fyy =

$$\frac{1}{\sqrt{x^2+y^2+1}}-\frac{y^2}{(x^2+y^2+1)^{3/2}}$$

```
hessdetf=fxx*fyy-fxy^2;
xcr = xcr(1:1); ycr = ycr(1:1);
for k = 1:1
 [xcr(k), ycr(k), subs(hessdetf, [x,y], [xcr(k), ycr(k)]), subs(fxx, [x,y],
[xcr(k), ycr(k)])];
end
%critical points are (0,0), the vaue of D = 0 which implies that
%the test results are inconclusive.
```

```
%315
%f(x,y) = -x^2 - 5*y^2 + 8*x - 10*y - 13
syms x y z
f= -x^2 - 5*y^2 + 8*x - 10*y - 13
```

f = $-x^2+8\,x-5\,y^2-10\,y-13$

```
fx=diff(f,x)
```

fx = $8-2\,x$

```
fy=diff(f,y)
```

fy = $-10\,y-10$

```
[xcr,ycr]=solve(fx,fy); [xcr,ycr]
```

ans = $\begin{pmatrix}4 & -1\end{pmatrix}$

```
fxy=diff(fx,y)
```

fxy = 0

```
fyy=diff(fy,y)
```

fyy = -10

```
hessdetf=fxx*fyy-fxy^2;
xcr = xcr(1:1); ycr = ycr(1:1);
for k = 1:1
 [xcr(k), ycr(k), subs(hessdetf, [x,y], [xcr(k), ycr(k)]), subs(fxx, [x,y],
[xcr(k), ycr(k)])];
end
%fxx and fyy < 0, hence it is maximum at (4,-1)
```

```
%316
```

```
%f(x,y) = x^2 + y^2 + 2*x - 6*y + 6
syms x y z
f= x^2 + y^2 + 2*x - 6*y + 6
```

f = $x^2 + 2\,x + y^2 - 6\,y + 6$

```
fx=diff(f,x)
```

fx = $2\,x + 2$

```
fy=diff(f,y)
```

fy = $2\,y - 6$

```
hessdetf=fxx*fyy-fxy^2;
xcr = xcr(1:1); ycr = ycr(1:1);
for k = 1:1
 [xcr(k), ycr(k), subs(hessdetf, [x,y], [xcr(k), ycr(k)]), subs(fxx, [x,y],
[xcr(k), ycr(k)])];
end
%fxx and fyy > 0, hence it is minimum at (-1,3)
```

```
%317
%f(x,y) = (x^2 + y^2)^(1/2) + 1
syms x y z
f= (x^2 + y^2)^(1/2) + 1
```

f = $\sqrt{x^2 + y^2} + 1$

```
fx=diff(f,x)
```

fx =

$$\frac{x}{\sqrt{x^2 + y^2}}$$

```
fy=diff(f,y)
```

fy =

$$\frac{y}{\sqrt{x^2 + y^2}}$$

```
[xcr,ycr]=solve(fx,fy); [xcr,ycr]
```

ans = $\begin{pmatrix} 0 & 0 \end{pmatrix}$

```
fxy=diff(fx,y)
```

fxy =

$$-\frac{x\,y}{(x^2 + y^2)^{3/2}}$$

```
fyy=diff(fy,y)
```

fyy =

$$\frac{1}{\sqrt{x^2+y^2}}-\frac{y^2}{(x^2+y^2)^{3/2}}$$

```
%Since the critical points are (0,0), the vaue of D = 0 which implies that
%the test results are inconclusive.
```

```
%318
%f(x,y) = -x^3 + 4*x*y - 2*y^2 + 1
syms x y z
f= -x^3 + 4*x*y - 2*y^2 + 1
```

f = $-x^3+4\,x\,y-2\,y^2+1$

```
fx=diff(f,x)
```

fx = $4\,y-3\,x^2$

```
fy=diff(f,y)
```

fy = $4\,x-4\,y$

```
[xcr,ycr]=solve(fx,fy); [xcr,ycr]
```

ans =

$$\begin{pmatrix} 0 & 0 \\ \frac{4}{3} & \frac{4}{3} \end{pmatrix}$$

```
fxy=diff(fx,y)
```

fxy = 4

```
fyy=diff(fy,y)
```

fyy = -4

```
hessdetf=fxx*fyy-fxy^2;
xcr = xcr(1:1); ycr = ycr(1:1);
for k = 1:1
 [xcr(k), ycr(k), subs(hessdetf, [x,y], [xcr(k), ycr(k)]), subs(fxx, [x,y],
[xcr(k), ycr(k)])];
end
D = -6*(4/3) * 4 -(4*4)
```

D = -48

```
%D is less than zero, f(4/3,4/3) is a saddle point
```

```
%319
%f(x,y) = x^2*y^2
syms x y z
f= x^2*y^2
```

f = $x^2\, y^2$

```
fx=diff(f,x)
```

fx = $2\, x\, y^2$

```
fy=diff(f,y)
```

fy = $2\, x^2\, y$

```
[xcr,ycr]=solve(fx,fy); [xcr,ycr]
```

ans = $\begin{pmatrix} 0 & 0 \end{pmatrix}$

```
fxy=diff(fx,y)
```

fxy = $4\, x\, y$

```
fyy=diff(fy,y)
```

fyy = $2\, x^2$

```
hessdetf=fxx*fyy-fxy^2;
xcr = xcr(1:1); ycr = ycr(1:1);
for k = 1:1
 [xcr(k), ycr(k), subs(hessdetf, [x,y], [xcr(k), ycr(k)]), subs(fxx, [x,y],
[xcr(k), ycr(k)])];
end
D = (2*y^2)*(2*x^2) - (16*x^2*y^2)
```

D = $-12\, x^2\, y^2$

```
%D is less than zero, f(0,0) is a saddle point
```