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lab - 2 ①

LAB -2

1) Eigen values and Eigen vectors

a) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ $\Rightarrow A-\lambda I \Rightarrow \begin{bmatrix} 0-\lambda & 1 & 0 \\ 1 & -1-\lambda & 1 \\ 0 & 1 & 0-\lambda \end{bmatrix}$

$S_1 = 0-1+0 = -1$

$S_2 = \begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix}$

$S_2 = -2$

$S_3 = 1(0)$
$= 0$

characteristic equation

$\lambda^3 - (-1)\lambda^2 + (-2)\lambda + 0 = 0$

$\lambda^3 + \lambda^2 - 2\lambda + 0 = 0$

$\lambda^3 + \lambda^2 - 2\lambda = 0$

$\lambda(\lambda^2 + \lambda - 2) = 0$

$\lambda = 0, \quad \lambda^2 + \lambda - 2 = 0$

$\lambda^2 + 2\lambda - \lambda - 2 = 0$

$\lambda(\lambda+2) - 1(\lambda+2) = 0$

$(\lambda+2)(\lambda-1) = 0$

-2
2 -1

$\lambda = -2, 0, 1$

if $\lambda = -2$

$\begin{bmatrix} +2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$y + 2z = 0 \qquad 2x + y = 0$

$y = -2z \qquad y = -2x$

$x_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

ii) $\lambda = 0$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\boxed{y = 0}$$

$x + z = 0$

$$\boxed{x = -z}$$

$$x_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

iii) $\lambda = 1$

$$\begin{bmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$y + (-z) = 0$

$$\boxed{y = z}$$

$-x + y = 0$

$$\boxed{y = x}$$

$$x_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\therefore$ solution

eigen values = $-2, 0, 1$

eigen vectors $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

checking othogonal of eigen vectors

$$\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = [1 + 0 - 1] = 0$$

$$\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 1 - 2 + 1 = 0$$

$$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 1 + 0 - 1 = 0$$

$\therefore$ proved othrogonal to each

3

b) $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

$[A-\lambda I]$

$\Rightarrow \begin{bmatrix} 2-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix}$

$\Rightarrow$ $S_1 = 2+2+2 = 6$

$S_2 = 3+4+3 = 10$

$S_3 \Rightarrow 4$

characterstic equation

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$\lambda^3 - 6\lambda^2 + 10\lambda - 4 = 0$

i) $\lambda = 2$ (checking condition)

$8 - 24 + 20 - 4 \Rightarrow$

$28 - 28$

$\Rightarrow 0$

$$\begin{array}{r|l} & \lambda^2 - 4\lambda + 2 \\ \lambda - 2 & \lambda^3 - 6\lambda^2 + 10\lambda - 4 \\ & \underline{\overset{(-)}{\cancel{\lambda^3}} \overset{(+)}{- 2\lambda^2}} \\ & -4\lambda^2 + 10\lambda \\ & \underline{\overset{(+)}{-\cancel{4\lambda^2}} \overset{(-)}{+ 8\lambda}} \\ & 2\lambda - 4 \\ & \underline{\overset{(-)}{\cancel{2\lambda}} \overset{(+)}{- 4}} \\ & 0 \end{array}$$

$\lambda^2 - 4\lambda + 2 = 0$

$\lambda = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{4 \pm \sqrt{8}}{2} \Rightarrow 2 \pm \sqrt{2}$

$\boxed{\lambda = 2, 2-\sqrt{2}, 2+\sqrt{2}}$

if $\lambda = 2$

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-y = 0$$

$$\boxed{y = 0}$$

$$-x - z = 0$$

$$\boxed{-x = z}$$

$$\therefore x_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

if $\lambda = 2 - \sqrt{2}$

$$\begin{bmatrix} \sqrt{2} & -1 & 0 \\ -1 & \sqrt{2} & -1 \\ 0 & -1 & \sqrt{2} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-y + \sqrt{2}z = 0$$

$$\boxed{y = \sqrt{2}z}$$

$$\sqrt{2}x - y = 0$$

$$\boxed{y = \sqrt{2}x}$$

$$\therefore x_2 = \begin{bmatrix} 1 \\ \sqrt{2} \\ 1 \end{bmatrix}$$

if $\lambda = 2 + \sqrt{2}$

$$\begin{bmatrix} -\sqrt{2} & -1 & 0 \\ -1 & -\sqrt{2} & -1 \\ 0 & -1 & -\sqrt{2} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-y - \sqrt{2}z = 0$

$\boxed{y = -\sqrt{2}z}$

$-\sqrt{2}x - y = 0$

$\boxed{y = -\sqrt{2}x}$

$\therefore x_3 = \begin{bmatrix} 1 \\ -\sqrt{2} \\ 1 \end{bmatrix}$

checking orthogonal of eigen vectors

$\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ \sqrt{2} \\ 1 \end{bmatrix} = -1 + 0 + 1 = 0$

$\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ -\sqrt{2} \\ 1 \end{bmatrix} = -1 + 0 + 1 = 0$

$\begin{bmatrix} 1 \\ \sqrt{2} \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ -\sqrt{2} \\ 1 \end{bmatrix} = 1 - 2 + 1 = 0$

$\therefore$ eigen values, 2, $2+\sqrt{2}$, $2-\sqrt{2}$

$\therefore$ eigenvectors are $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ \sqrt{2} \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -\sqrt{2} \\ 1 \end{bmatrix}$

2) a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$$A^T = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \qquad A^TA = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$\rightarrow$ i) $AA^T = \begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}$

$S_1 = 4$

$S_2 = 3$

characteristic equation

$\lambda^2 - S_1\lambda + S_2 = 0$

$\lambda^2 - 4\lambda + 3 = 0$

$\lambda^2 - 3\lambda - \lambda + 3 = 0$

$\lambda(\lambda-3) - 1(\lambda-3) = 0$

$(\lambda-1)(\lambda-3) = 0$

$\boxed{\lambda = 1, 3}$

3

-3 -1

-4

if $\lambda = 1$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x + y = 0$

$\boxed{x = -y}$

$x_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

if $\lambda = 3$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-x + y = 0$

$\boxed{+x = y}$

$x_2 = \begin{bmatrix} +1 \\ 1 \end{bmatrix}$

$\because$ eigen values 1, 3

$\because$ eigen vector $\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

ii) $A^T A$

$$\begin{bmatrix} 1-\lambda & 1 & 0 \\ 1 & 2-\lambda & 1 \\ 0 & 1 & 1-\lambda \end{bmatrix}$$

$S_1 = 1+2+1 = 4$

$S_2 = 3$

$S_3 = 0$

characteristic equation of the matrix

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$\lambda^3 - 4\lambda^2 + 3\lambda - 0 = 0$

$\lambda^3 - 4\lambda^2 + 3\lambda = 0$

$\lambda(\lambda^2 - 4\lambda + 3) = 0$

$\lambda = 0, \quad \lambda^2 - 4\lambda + 3 = 0$

$\lambda^2 - 3\lambda - \lambda + 3 = 0$

$\lambda(\lambda - 3) - 1(\lambda - 3) = 0$

$(\lambda - 3)(\lambda - 1) = 0$

$\boxed{\lambda = 0, 1, 3}$

if $\lambda = 0$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x + y = 0$

$\boxed{x = -y}$

$y + z = 0$

$\boxed{y = -z}$

$$x_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

if $\lambda = 1$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$y = 0$

$x + z = 0$

$\boxed{x = -z}$

$$x_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

if $\lambda = 3$

$$\begin{bmatrix} -2 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$y - 2z = 0$

$\boxed{y = 2z}$

$$x_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$-2x + y = 0$ $\boxed{y = 2x}$

b) $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$

$AA^T = \begin{bmatrix} 5 & 15 \\ 15 & 45 \end{bmatrix}$ $\qquad A^TA = \begin{bmatrix} 10 & 20 \\ 20 & 40 \end{bmatrix}$

i) AA^T

$\begin{bmatrix} 5-\lambda & 15 \\ 15 & 45-\lambda \end{bmatrix}$

$S_1 = 50$

$S_2 = 0$

characteristic equation

$\lambda^2 - 50\lambda + 0 = 0$

$\lambda(\lambda - 50) = 0$

$\lambda = 0, \lambda = 50$

if $\lambda = 50$

$\begin{bmatrix} -45 & 15 \\ 15 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$-45x + 15y = 0$

$\overset{81}{\cancel{15}}y = \overset{93}{\cancel{45}}x$

$\boxed{x = \frac{y}{3}}$

$\therefore x_1 = \begin{bmatrix} \frac{1}{3} \\ 1 \end{bmatrix}$

if $\lambda = 0$

$$\begin{bmatrix} 5-\lambda & 15 \\ 15 & 45-\lambda \end{bmatrix} = \begin{bmatrix} 5 & 15 \\ 15 & 45 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 15 \\ 15 & 45 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$5x + 15y = 0$$

$$\cancel{5}x = -\cancel{15}^{3}y$$

$$\boxed{x = -3y}$$

$$x_2 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

ii) $A^T A$

$$\begin{bmatrix} 10-\lambda & 20 \\ 20 & 40-\lambda \end{bmatrix}$$

$S_1 = 50$

$S_2 = 0$

$$\lambda^2 - 50\lambda + 0 = 0$$

$$\lambda(\lambda - 50) = 0$$

$$\lambda = 0, \lambda = 50$$

if $\lambda = 50$

$$\begin{bmatrix} -40 & 20 \\ 20 & -10 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-40x + 20y = 0$

$40x = 20y$

$2x = y$

solution of $x_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

if $\lambda = 0$

$$\begin{bmatrix} 10 & 20 \\ 20 & 40 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$10x + 20y = 0$

$10x = -20y$

$\boxed{x = -2y}$

$x_2 = \begin{bmatrix} 1 \\ -\frac{1}{2} \end{bmatrix}$

c) $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

$A^T = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$AA^T = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ $A^T A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

→ AA^T and A^TA

$$\begin{bmatrix} 2-\lambda & 1 \\ 1 & 1-\lambda \end{bmatrix}$$

$S_1 = 3$

$S_2 = 1$

<u>charateristic equation</u>

$\lambda^2 - 3\lambda + 1 = 0$

$$\lambda = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$+3$

-3

$$= \frac{3 \pm \sqrt{9-4}}{2}$$

$$= \frac{3 \pm \sqrt{5}}{2}$$

$$\lambda = \frac{3+\sqrt{5}}{2} \qquad \lambda = \frac{3-\sqrt{5}}{2}$$

if $\lambda = \frac{3+\sqrt{5}}{2}$

$$\begin{bmatrix} 2-\frac{\sqrt{5}+3}{2} & 1 \\ 1 & 1-\frac{\sqrt{5}+3}{2} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\left(2 - \frac{\sqrt{5}+3}{2}\right)x + y = 0$$

$$2x - \left(\frac{\sqrt{5}+3}{2}\right)x + y = 0$$

$$y = -2x + \left(\frac{\sqrt{5}}{2} + \frac{3}{2}\right)x \qquad y = \left(-2 + \frac{\sqrt{5}+3}{2}\right)x$$

$$y = \left(\frac{\sqrt{5}+1}{2}\right)x$$

$$\therefore x_1 = \begin{bmatrix} 1 \\ \frac{1+\sqrt{5}}{2} \end{bmatrix}$$

if $\lambda = \frac{3-\sqrt{5}}{2}$

$$\begin{bmatrix} 2-\left(\frac{3-\sqrt{5}}{2}\right) & 1 \\ 1 & 1-\left(\frac{3-\sqrt{5}}{2}\right) \end{bmatrix}$$

$$\begin{bmatrix} \frac{1+\sqrt{5}}{2} & 1 \\ 1 & \frac{1+\sqrt{5}}{2} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\frac{1+\sqrt{5}}{2}x + y = 0$$

$$\frac{1+\sqrt{5}}{2}x = -y$$

$$x_2 = \begin{bmatrix} 1 \\ -\left(\frac{1+\sqrt{5}}{2}\right) \end{bmatrix}$$

$\therefore$ eigen values are $\frac{3+\sqrt{5}}{2}$, $\frac{3-\sqrt{5}}{2}$

$\therefore$ eigen vectors $\begin{bmatrix} 1 \\ \frac{1+\sqrt{5}}{2} \end{bmatrix}$, $\begin{bmatrix} 1 \\ -\left(\frac{+1+\sqrt{5}}{2}\right) \end{bmatrix}$

3) <u>Diagonalize</u>

$$A = \begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$$

$S_1 = 2+3+2 = 7$

$S_2 = \begin{vmatrix} 3 & -1 \\ -2 & 2 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix}$

$= 6-2+4-1+6-2$

$= 11$

$S_3 = 5$

$\lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$

if $\lambda = 1$

$1 - 7 + 11 - 5 = 0$

$12 - 12$

$\Rightarrow 0$

$$\begin{array}{r|l} & \lambda^2 - 6\lambda + 5 \\ \lambda - 1 & \lambda^3 - 7\lambda^2 + 11\lambda - 5 \\ & \cancel{\lambda^3} - \lambda^2 \\ & (-) \quad (+) \\ \hline & -6\lambda^2 + 11\lambda \\ & \cancel{-6\lambda^2} + 6\lambda \\ & (+) \quad (-) \\ \hline & +5\lambda - 5 \\ & \cancel{5\lambda} - \cancel{5} \\ & (-) \quad (+) \\ \hline & 0 \end{array}$$

$\lambda^2 - 6\lambda + 5 = 0$

$\lambda^2 - 5\lambda - \lambda + 5 = 0$

$\lambda(\lambda - 5) - 1(\lambda - 5) = 0$

$(\lambda - 5)(\lambda - 1) = 0$

$\boxed{\lambda = 1, 5}$

5

-5 -1

-6

$\boxed{\lambda = 1, 1, 5}$

ii) $\lambda = 5$

$$\begin{bmatrix} -3 & 2 & -1 \\ -1 & -2 & -1 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-3x_1 + 2x_2 - x_3 = 0$$
$$x_1 - 2x_2 - x_3 = 0$$

$$\begin{array}{ccccccc} & x_1 & & x_2 & & x_3 & \\ 2 & & -1 & & -3 & & 2 \\ -2 & & -1 & & +1 & & 2 \end{array}$$

$$\frac{x_1}{-2-(2)} = \frac{x_2}{-1-3} = \frac{x_3}{6-(12)}$$

$$\frac{x_1}{-4} = \frac{x_2}{-4} = \frac{x_3}{4}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

if $\lambda = 1$

$$\begin{bmatrix} 1 & 2 & -1 \\ 1 & 2 & -1 \\ -1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 + 2x_2 - x_3 = 0$$

if $x_3 = 0$ (assume a free variable)

$$x_1 = -2x_2$$

$$x_2 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

for third eigen vector, we same eigen value 1

so, changing the free variable

$x_1 + 2x_2 - x_3 = 0$

$x_2 = 0$

$x_1 = x_3$

$\therefore x_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

forming $P = \begin{bmatrix} -1 & -2 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$P^{-1} = \begin{bmatrix} -0.25 & -0.5 & 0.25 \\ -0.25 & +0.5 & 0.25 \\ 0.25 & 0.5 & 0.75 \end{bmatrix}$

$A = \begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$

$D = P^{-1}AP$

$= \begin{bmatrix} -0.25 & -0.5 & 0.25 \\ -0.25 & +0.5 & 0.25 \\ 0.25 & 0.5 & 0.75 \end{bmatrix} \begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix} \begin{bmatrix} -1 & -2 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4) Orthogonally diagonalize

a) $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

$S_1 = 1+3+1 = 5$

$S_2 = -4$

$S_3 = -20$

Characteristic equation:

$\lambda^3 - 5\lambda^2 - 4\lambda + 20 = 0$

if $\lambda = 2$ (checking)

$8 - 5(4) - 4(2) + 20$

$\cancel{8} - \cancel{20} - \cancel{8} + \cancel{20}$

$\Rightarrow 0$

$$\begin{array}{r|l} & \lambda^2 - 3\lambda - 10 \\ \lambda - 2 & \lambda^3 - 5\lambda^2 - 4\lambda + 20 \\ & \cancel{\lambda^3} - 2\lambda^2 \\ & (-)\quad (+) \\ \hline & -3\lambda^2 - 4\lambda \\ & \cancel{-3\lambda^2} + 6\lambda \\ & (+)\quad (-) \\ \hline & -10\lambda + 20 \\ & \cancel{-10\lambda} + \cancel{20} \\ & (+)\quad (-) \\ \hline & 0 \end{array}$$

$\lambda^2 - 3\lambda - 10 = 0$

$\lambda^2 - 5\lambda + 2\lambda - 10 = 0$

$\lambda(\lambda + 2) - 5(\lambda + 2)$

$(\lambda + 2)(\lambda - 5) = 0$

$\lambda = 5, -2$

-10

-5 2

-3

$\lambda = 2, 5, -2$

if $\lambda = 5$

$$\begin{bmatrix} -4 & 1 & 3 \\ 1 & -2 & 1 \\ 3 & 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-4x_1 + x_2 + 3x_3 = 0$$

$$x_1 - 2x_2 + x_3 = 0$$

$$\frac{x_1}{\begin{vmatrix} 1 & 3 \\ -2 & 1 \end{vmatrix}} \quad \frac{x_2}{\begin{vmatrix} 3 & -1 \\ 1 & 1 \end{vmatrix}} \quad \frac{x_3}{\begin{vmatrix} -4 & 1 \\ 1 & -2 \end{vmatrix}}$$

$$\frac{x_1}{1-3(-2)} = \frac{x_2}{3-(-4)(1)} = \frac{x_3}{+8-1}$$

$$\Rightarrow \quad \frac{x_1}{7} = \frac{x_2}{7} = \frac{x_3}{7}$$

$$x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

if $\lambda = 2$

$$\begin{bmatrix} -1 & 1 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-x_1 + x_2 + 3x_3 = 0$$

$$x_1 + x_2 + x_3 = 0$$

$$\frac{x_1}{\begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix}} \quad \frac{x_2}{\begin{vmatrix} 3 & -1 \\ 1 & 1 \end{vmatrix}} \quad \frac{x_3}{\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}}$$

$$\frac{x_1}{-2} = \frac{x_2}{4} = \frac{x_3}{-2}$$

$$x_2 = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$$

if $\lambda = -2$

$$\begin{bmatrix} -3 & 1 & 3 \\ 1 & 5 & 1 \\ 6 & 1 & 3 \end{bmatrix}$$

$$3x_1 + x_2 + 3x_3 = 0$$
$$x_1 + 5x_2 + x_3 = 0$$

$$\begin{array}{cccc} & x_1 & x_2 & x_3 \\ 1 & 3 & 3 & 1 \\ 5 & 1 & 1 & 5 \end{array}$$

$$\frac{x_1}{1-15} = \frac{x_2}{3-3} = \frac{x_3}{15-1} \Rightarrow \frac{x_1}{-14} = \frac{x_2}{0} = \frac{x_3}{14}$$

$$x_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

find norms for eigenvectors

$$x_1 = \sqrt{1+1+1} = \sqrt{3}$$
$$x_2 = \sqrt{1+4+1} = \sqrt{6}$$
$$x_3 = \sqrt{1+0+1} = \sqrt{2}$$

$$\text{forming } P = \begin{bmatrix} +\frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$P^T = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{-1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & \frac{-1}{\sqrt{6}} \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$D = P^T A P$$

$$= \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{-1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & \frac{-1}{\sqrt{6}} \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 1 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

b) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{3}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{3}{2} \end{bmatrix}$

$s_1 = 1 + \frac{3}{2} + \frac{3}{2} \Rightarrow 4$

$s_2 = 5$

$s_3 = 2$

$\lambda^3 - 4\lambda^2 + 5\lambda - 2 = 0$

if $\lambda = 1$ Checking

$1 - 4 + 5 - 2$

$6 - 6$

$\Rightarrow 0$

$$\lambda^2 - 3\lambda + 2$$

$$\lambda - 1 \,\big)\, \lambda^3 - 4\lambda^2 + 5\lambda - 2$$

$$\cancel{\lambda^3} - \lambda^2$$

$$(-)\quad (+)$$

$$-3\lambda^2 + 5\lambda$$

$$-\cancel{3\lambda^2} + 3\lambda$$

$$(+)\quad (-)$$

$$2\lambda - 2$$

$$\cancel{2\lambda - 2}$$

$$(-)\quad (+)$$

$$0$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$\lambda^2 - 2\lambda - \lambda + 2 = 0$$

$$\lambda(\lambda - 2) - 1(\lambda - 2) = 0$$

$$(\lambda - 1)(\lambda - 2) = 0$$

+2

-2 -1

-3

$$\lambda = 1, 2$$

$$\lambda = 1, 1, 2$$

if $\lambda = 2$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-x_1 = 0$$

$$\boxed{x_1 = 0}$$

$$-\frac{x_2}{2} + \frac{x_3}{2} = 0$$

$$\cancel{+}\frac{x_2}{\cancel{2}} = +\frac{x_3}{2} \qquad \frac{x_2}{2} = \frac{x_3}{2}$$

$$\boxed{x_2 = x_3}$$

$$X_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$\lambda = 1$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\boxed{x_1 = 0}$

$$\frac{x_2}{2} = \frac{-x_3}{2}$$

$$x_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$\boxed{x_2 = 0}$

$$x_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

<u>finding norm</u>

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$x_1 = \sqrt{0+1+1} = \sqrt{2}$

$x_2 = \sqrt{0+1+1} = \sqrt{2}$

$x_3 = \sqrt{1+0+0} = \sqrt{1} = 1$

$$P = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \qquad P^T = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & 0 & 0 \end{bmatrix}$$

$D = P^T A P$

$$= \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{3}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{3}{2} \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

```
% S.PRAVEEN KUMAR
% AIE   % CH.EN.U4AIE22048
%LAB-2

%1
A = [0 1 0; 1, -1, 1; 0, 1, 0]
```

```
A = 3×3
     0     1     0
     1    -1     1
     0     1     0
```

```
s = size(A);
if s(1)~=s(2)
  error('Error: Input must be square.')
end
I = eye(length(A));
%
syms x
eq1 = det(A-I*x) == 0 ;
eigVal = double(solve(eq1,x));
%
eigVec = zeros(s);
for i = 1:length(A)
  syms y
  eq2 = (A-eigVal(i)*I)*y == 0;
  eigVec(:,i) = double(solve(eq2,y));
end
[V,D] = eig(A)
```

```
V = 3×3
   -0.4082   -0.7071    0.5774
    0.8165    0.0000    0.5774
   -0.4082    0.7071    0.5774
D = 3×3
   -2.0000         0         0
         0   -0.0000         0
         0         0    1.0000
```

```
%2
A = [1 1 0; 0, 1, 1]
```

```
A = 2×3
     1     1     0
     0     1     1
```

```
B = transpose(A)
```

```
B = 3×2
     1     0
     1     1
     0     1
```

```
C=A*B;
```

```
D=B*A;
s = size(C);
if s(1)~=s(2)
  error('Error: Input must be square.')
end
I = eye(length(C));
%
syms x
eq1 = det(C-I*x) == 0 ;
eigVal = double(solve(eq1,x));
%
eigVec = zeros(s);
for i = 1:length(C)
  syms y
  eq2 = (C-eigVal(i)*I)*y == 0;
  eigVec(:,i) = double(solve(eq2,y));
end
[V,D] = eig(C)
```

```
V = 2×2
   -0.7071    0.7071
    0.7071    0.7071
D = 2×2
     1     0
     0     3
```

```
%%
syms x

eq1 = det(D-I*x) == 0 ;
eigVal = double(solve(eq1,x));
%
eigVec = zeros(s);
for i = 1:length(D)
  syms y
  eq2 = (D-eigVal(i)*I)*y == 0;
  eigVec(:,i) = double(solve(eq2,y));
end
[V1,E] = eig(D)
```

```
V1 = 2×2
     1     0
     0     1
E = 2×2
     1     0
     0     3
```

```
%3
A = [2 2 -1;1 3 -1;-1 -2 2];

% Find the eigenvalues and eigenvectors of A
[V,D] = eig(A);
```

```
% Rearrange the eigenvalues and eigenvectors in ascending order
[d,ind] = sort(diag(D));
Ds = D(ind,ind);
Vs = V(:,ind);

% Check if the eigenvectors form a basis for R^2
if rank(Vs) == 2
    disp('The eigenvectors form a basis for R^2')
else
    disp('The eigenvectors do not form a basis for R^2')
end
```

```
The eigenvectors do not form a basis for R^2
```

```
% Compute the diagonal inverse of D
D_inv = zeros(size(Ds));
for i = 1:size(Ds,1)
    if Ds(i,i) ~= 0
        D_inv(i,i) = 1/Ds(i,i);
    end
end

% Compute the diagonalized matrix
A_diag = Vs*D_inv*inv(Vs);

% Display the results
disp('Original matrix A:');
```

```
Original matrix A:
```

```
disp(A);
```

```
     2     2    -1
     1     3    -1
    -1    -2     2
```

```
disp('Diagonalized matrix A_diag:');
```

```
Diagonalized matrix A_diag:
```

```
disp(A_diag);
```

```
    0.8000   -0.4000    0.2000
   -0.2000    0.6000    0.2000
    0.2000    0.4000    0.8000
```

```
disp('Diagonal matrix D:');
```

```
Diagonal matrix D:
```

```
disp(Ds);
```

```
    1.0000         0         0
         0    1.0000         0
         0         0    5.0000
```

```
disp('Eigenvector matrix V:');
```

```
Eigenvector matrix V:
```

```
disp(Vs)
```

```
   -0.0409   -0.9045    0.5774
    0.4631    0.3015    0.5774
    0.8853   -0.3015   -0.5774
```

```
%4
A = [1 1 3;1 3 1;3 1 1];

% Initialize the diagonal matrix D
D = zeros(size(A));

% Initialize the eigenvector matrix V
V = eye(size(A));

% Set a tolerance value for the maximum number of iterations
tol = 1e-10;

% Perform iterations until the off-diagonal elements are less than the
tolerance
while max(max(abs(triu(A,1)))) > tol
    % Find the index of the largest off-diagonal element
    [row, col] = find(abs(triu(A,1)) == max(max(abs(triu(A,1)))));

    % Compute the rotation angle theta
    if A(row,row) == A(col,col)
        theta = pi/4;
    else
        theta = 0.5*atan(2*A(row,col)/(A(row,row)-A(col,col)));
    end

    % Compute the rotation matrix G
    G = eye(size(A));
    G(row,row) = cos(theta);
    G(col,col) = cos(theta);
    G(row,col) = -sin(theta);
    G(col,row) = sin(theta);

    % Update the matrices A and V
    A = G'*A*G;
    V = V*G;
end
```

```
% Extract the diagonal elements of A into D
D = diag(diag(A));

% Display the diagonalized matrix D and the eigenvector matrix V
disp('Diagonalized matrix D:')
```

```
Diagonalized matrix D:
```

```
disp(D)
```

```
     5     0     0
     0     2     0
     0     0    -2
```

```
disp('Eigenvector matrix V:')
```

```
Eigenvector matrix V:
```

```
disp(V)
```

```
    0.5774   -0.4082   -0.7071
    0.5774    0.8165         0
    0.5774   -0.4082    0.7071
```