17/4/2022 Assignment - 5 S. Praveenkumar

1st AIE

ch.en.u4aie22048

Q) Find all the critical points:

2.10. $f(x,y) = 1 + x^2 + y^2$

$$\frac{d(1+x^2+y^2)}{dx} = \frac{d(1)}{dx} + \frac{d(x^2)}{dx} + \frac{d(y^2)}{dx}$$

$$\Rightarrow 0 + 2x + 0$$

$$\Rightarrow 2x - ①$$

Equating ① to 'zero'

$$2x = 0$$

$$\boxed{x = 0}$$

$$\frac{d(1+x^2+y^2)}{dy} = \frac{d(1)}{dy} + \frac{d}{dy}(x^2) + \frac{dy^2}{dy}$$

$$\Rightarrow 0 + 0 + 2y$$

$$\Rightarrow 2y = 0$$

$$\boxed{y = 0}$$

$\therefore$ critical point are (0,0)

2.11) $f(x,y) = (3x-2)^2 + (y-4)^2$

$$\frac{d[(3x-2)^2 + (y-4)^2]}{dx} = 2(3x-2) \times 3$$

$$= (6x-4)3$$

$$= 18x - 12 \rightarrow (i)$$

equating (i) to '0'

$$18x - 12 = 0$$

$$18x = 12$$

$$\boxed{x = \frac{2}{3}}$$

$\frac{d}{dy}[(3x-2)^2+(y-4)^2]$

$\Rightarrow 2(y-4)$

$\Rightarrow 2y-4(2)$

$\Rightarrow 2y-8$

$2y-8=0$

$2y=8$

$\boxed{y=4}$

$\therefore$ Critical point is $\left(\frac{2}{3},4\right)$.

3.12) $f(x,y)=x^4+y^4-16xy$

$\frac{d}{dx}(x^4+y^4-16xy)$

$\Rightarrow 4x^3-16y$

Equating to zero

$4x^3-16y=0$

$4x^3=16y$

$\boxed{x^3=4y}$

$\frac{d}{dy}(x^4+y^4-16xy)$

$\Rightarrow 4y^3-16x$

Equating to zero

$4y^3-16x=0$

$4y^3=16x$

$\boxed{y^3=4x}$

$x^3 = 4y$ $\qquad\qquad\qquad$ $y^3 = 4x$

$\left(\frac{y^3}{4}\right)^3 = 4y$

$(2)^3 = 4x$

$(y^3)^3 = 4^4 . y$

$x = \frac{8^2}{4}$

$y^9 = 2^8 . y$

$\boxed{x = 2}$

$\frac{y^9}{\cancel{y}} = 2^8$

$y^8 = 2^8$

$\boxed{y = 2}$

$\therefore$ the critical point are $(2, 2)$

2.13) $f(x, y) = 15x^3 - 3xy + 15y^3$

$\frac{d(15x^3 - 3xy + 15y^3)}{dx} = 3(15)x^2 - 3y$

$45x^2 - 3y = 0$

$45x^2 = 3y$

$\boxed{15x^2 = y}$

$\frac{d(15x^3 - 3xy + 15y^3)}{dy} = 45y^2 - 3x$

$45y^2 - 3x = 0$

$\boxed{15y^2 = x}$

$15(15y^2)^2 = y$ $\qquad\qquad$ $\cancel{15} \times \frac{1}{15^2} = x$

$15 . 15^2 . y^4 = y$

$15^3 = \frac{1}{y^3}$ $\qquad\qquad$ $\boxed{x = \frac{1}{15}}$

$y^3 = \frac{1}{15^3}$

$\boxed{y = \frac{1}{15}}$ $\qquad$ $\therefore$ the critical point are $\left(\frac{1}{15}, \frac{1}{15}\right)$

find the critical points using algebraic techniques. Verify using partial derivaties test

3.14) $f(x,y) = \sqrt{x^2+y^2+1}$

$f'(x) = \frac{x}{\sqrt{x^2+y^2+1}}$ $f'(y) = \frac{y}{\sqrt{x^2+y^2+1}}$

$f''(x) = \frac{y^2+1}{(y^2+x^2+1)^{3/2}}$ $f''(y) = \frac{x^2+1}{(y^2+x^2+1)^{3/2}}$

Equating $f'(x)$ & $f'(y)$ to Zero.

$\frac{x}{\sqrt{x^2+y^2+1}} = 0 \Rightarrow \boxed{x=0}$ $\frac{y}{\sqrt{x^2+y^2+1}} = 0$

$\boxed{y=0}$

$\therefore$ critical points is $(0,0)$

mixed partial derivative

$\frac{d^2 f}{dx dy} = -3xy(x^2+y^2+1)^{-3/2}$

$x=y=0$,

$D=0$

$\therefore D=0$, the test is inconclusive

3.15) $f(x,y) = -x^2 - 5y^2 + 8x - 10y - 13$

$\frac{d(-x^2-5y^2+8x-10y-13)}{dx} = -2x+8$

$-2x+8=0$

$2x=8$

$\boxed{x=4}$

$$\frac{d(-x^2-5y^2+8x-10y-13)}{dy}$$

$\Rightarrow -10y-10$

$-10y-10=0$

$10y=-10$

$\boxed{y=-1}$

$\therefore$ critical point is $(4,-1)$

$f''(x)=-2$ $\quad$ $f''(y)=-10$

$f''(x)<0$ $\quad$ $f''(y)<0$

$\therefore$ it is maximum at $(4,-1)$

2.1b) $f(x,y)=x^2+y^2+2x-6y+6$

$f'(x)=2x+2$ $\quad$ $f(y)=2y-6$

$f''(x)=2$ $\quad$ $y''(y)=2$

$f'(x)=0$ $\quad$ $2y-6=0$

$2x=-2$ $\quad$ $2y=6$

$\boxed{x=-1}$ $\quad$ $\boxed{y=3}$

$f''(x)>0$ $\quad$ $f''(y)>0$

$\therefore$ critical points is $(-1,3)$

it is minimum at $(-1,3)$

2.17) $f(x,y) = \sqrt{x^2+y^2} + 1$

$f'(x) = \dfrac{x}{\sqrt{x^2+y^2}}$ $\qquad$ $f'(y) = \dfrac{y}{\sqrt{x^2+y^2}}$

$f''(x) = \dfrac{y^2}{(x^2+y^2)^{3/2}}$ $\qquad$ $f''(y) = \dfrac{x^2}{(x^2+y^2)^{3/2}}$

equating $f'(x)$ and $f'(y)$ to zero

$f'(x) = 0$ $\qquad$ $f'(y) = 0$

$x = 0$ $\qquad$ $y = 0$

$\therefore$ critical points are $(0,0)$

the mixed partial derivative

$$\frac{d^2 f}{dx \cdot dy} = \frac{-3xy}{(x^2+y^2)^{5/2}}$$

since $x = y = 0$,

$D = 0$

$\because D = 0$, the test is inconclusive

3.18) $f(x,y) = -x^3 + 4xy - 2y^2 + 1$

$f'(x) = 4y - 3x^2$ $\qquad$ $f'(y) = 4x - 4y$

$f''(x) = -6x$ $\qquad$ $f''(y) = -4$

$f''(x) = -6\left(\frac{4}{3}\right) = -\frac{24}{3} = -8$

$D = -8(-4) - 4(+4)$

$= 32 - 16$

$\boxed{D = 16}$ $\qquad$ $D > 0$

$f\left(\frac{4}{3}, \frac{4}{3}\right)$ is a saddle point

Q.19) $f(x,y) = x^2 y^2$

$f'(x) = 2y^2 x$ $\qquad$ $f'(y) = 2x^2 y$

$f''(x) = 2y^2$ $\qquad$ $f''(y) = 2x^2$

$= 4x^2 y^2 - 16x^2 y^2$

$= -12x^2 y^2$

$f'(x) = 0$ $\qquad$ $f'(y) = 0$

$2y^2 x = 0$ $\qquad$ $2x^2 y = 0$

$\boxed{y = 0}$

$D = -12x^2 y^2$

$D < 0$,

$f(0,0)$ is a saddle point

```
%S.Praveen Kumar
%Ch.en.u4aie22048
%17-04-2023
```

```
%Find all the critical points
%310
%f(x,y) = 1 + x^2 + y^2
syms x y z
f= 1 + x^2 + y^2
```

f = $x^2 + y^2 + 1$

```
fx=diff(f,x)
```

fx = $2\,x$

```
fy=diff(f,y)
```

fy = $2\,y$

```
[xcr,ycr]=solve(fx,fy); [xcr,ycr]
```

ans = $(0 \quad 0)$

```
fxx=diff(fx,x);
fxy=diff(fx,y);
fyy=diff(fy,y);
hessdetf=fxx*fyy-fxy^2;
xcr = xcr(1:1); ycr = ycr(1:1);
for k = 1:1
 [xcr(k), ycr(k), subs(hessdetf, [x,y], [xcr(k), ycr(k)]), subs(fxx, [x,y],
[xcr(k), ycr(k)])];
end
```

```
%3.11
%f(x,y) = (3*x - 2)^2 + (y - 4)^2
syms x y z
f= (3*x - 2)^2 + (y - 4)^2
```

f = $(y-4)^2 + (3\,x-2)^2$

```
fx=diff(f,x)
```

fx = $18\,x - 12$

```
fy=diff(f,y)
```

fy = $2\,y - 8$

```
[xcr,ycr]=solve(fx,fy); [xcr,ycr]
```

ans =

$$\begin{pmatrix} \frac{2}{3} & 4 \end{pmatrix}$$

```
fxx=diff(fx,x);
fxy=diff(fx,y);
fyy=diff(fy,y);
hessdetf=fxx*fyy-fxy^2;
xcr = xcr(1:1); ycr = ycr(1:1);
for k = 1:1
 [xcr(k), ycr(k), subs(hessdetf, [x,y], [xcr(k), ycr(k)]), subs(fxx, [x,y],
[xcr(k), ycr(k)])];
end
```

```
%312
%f(x,y) = x^4 + y^4 - 16*x*y
syms x y z
f= x^4 + y^4 - 16*x*y
```

f = $x^4 - 16\,x\,y + y^4$

```
fx=diff(f,x)
```

fx = $4\,x^3 - 16\,y$

```
fy=diff(f,y)
```

fy = $4\,y^3 - 16\,x$

```
[xcr,ycr]=solve(fx,fy); [xcr,ycr]
```

ans =

$$\begin{pmatrix} 0 & 0 \\ -2 & -2 \\ 2 & 2 \\ 2\,\mathrm{i} & -2\,\mathrm{i} \\ -2\,\mathrm{i} & 2\,\mathrm{i} \\ \sigma_3 & 2\,\sigma_1 \\ -\sigma_3 & -2\,\sigma_1 \\ \sigma_2 & -2\,\sigma_1\,\mathrm{i} \\ -\sigma_2 & 2\,\sigma_1\,\mathrm{i} \end{pmatrix}$$

where

$$\sigma_1 = (-1)^{1/4}$$

$$\sigma_2 = 2\ (-1)^{3/4}\,\mathrm{i}$$

$$\sigma_3 = 2\ (-1)^{3/4}$$

```
fxx=diff(fx,x);
fxy=diff(fx,y);
fyy=diff(fy,y);
hessdetf=fxx*fyy-fxy^2;
xcr = xcr(1:1); ycr = ycr(1:1);
for k = 1:1
 [xcr(k), ycr(k), subs(hessdetf, [x,y], [xcr(k), ycr(k)]), subs(fxx, [x,y],
[xcr(k), ycr(k)])];
end
```

```
%313
%f(x,y) = 15*x^3 - 3*x*y + 15*y^3
syms x y z
f= 15*x^3 - 3*x*y + 15*y^3
```

f = $15\,x^3 - 3\,x\,y + 15\,y^3$

```
fx=diff(f,x)
```

fx = $45\,x^2 - 3\,y$

```
fy=diff(f,y)
```

fy = $45\,y^2 - 3\,x$

```
[xcr,ycr]=solve(fx,fy); [xcr,ycr]
```

ans =

$$\begin{pmatrix} 0 & 0 \\ \frac{1}{15} & \frac{1}{15} \\ -\frac{1}{30}+\frac{\sqrt{3}\,\mathrm{i}}{30} & -\frac{1}{30}-\frac{\sqrt{3}\,\mathrm{i}}{30} \\ -\frac{1}{30}-\frac{\sqrt{3}\,\mathrm{i}}{30} & -\frac{1}{30}+\frac{\sqrt{3}\,\mathrm{i}}{30} \end{pmatrix}$$

```
fxx=diff(fx,x);
fxy=diff(fx,y);
fyy=diff(fy,y);
hessdetf=fxx*fyy-fxy^2;
xcr = xcr(1:1); ycr = ycr(1:1);
for k = 1:1
 [xcr(k), ycr(k), subs(hessdetf, [x,y], [xcr(k), ycr(k)]), subs(fxx, [x,y],
[xcr(k), ycr(k)])];
end
```

```
%314
%f(x,y) = (x^2 + y^2 + 1)^1/2
syms x y z
f= (x^2 + y^2 + 1)^(1/2)
```

f = $\sqrt{x^2+y^2+1}$

```
fx=diff(f,x)
```

fx =

$$\frac{x}{\sqrt{x^2+y^2+1}}$$

```
fy=diff(f,y)
```

fy =

$$\frac{y}{\sqrt{x^2+y^2+1}}$$

```
[xcr,ycr]=solve(fx,fy); [xcr,ycr]
```

ans = $\begin{pmatrix} 0 & 0 \end{pmatrix}$

```
fxy=diff(fx,y)
```

fxy =

$$-\frac{x\,y}{(x^2+y^2+1)^{3/2}}$$

```
fyy=diff(fy,y)
```

fyy =

$$\frac{1}{\sqrt{x^2+y^2+1}} - \frac{y^2}{(x^2+y^2+1)^{3/2}}$$

```
hessdetf=fxx*fyy-fxy^2;
xcr = xcr(1:1); ycr = ycr(1:1);
for k = 1:1
 [xcr(k), ycr(k), subs(hessdetf, [x,y], [xcr(k), ycr(k)]), subs(fxx, [x,y],
[xcr(k), ycr(k)])];
end
%critical points are (0,0), the vaue of D = 0 which implies that
%the test results are inconclusive.
```

```
%315
%f(x,y) = -x^2 - 5*y^2 + 8*x - 10*y - 13
syms x y z
f= -x^2 - 5*y^2 + 8*x - 10*y - 13
```

f = $-x^2 + 8\,x - 5\,y^2 - 10\,y - 13$

```
fx=diff(f,x)
```

fx = $8 - 2\,x$

```
fy=diff(f,y)
```

fy = $-10\,y - 10$

```
[xcr,ycr]=solve(fx,fy); [xcr,ycr]
```

ans = $\begin{pmatrix} 4 & -1 \end{pmatrix}$

```
fxy=diff(fx,y)
```

fxy = 0

```
fyy=diff(fy,y)
```

fyy = -10

```
hessdetf=fxx*fyy-fxy^2;
xcr = xcr(1:1); ycr = ycr(1:1);
for k = 1:1
 [xcr(k), ycr(k), subs(hessdetf, [x,y], [xcr(k), ycr(k)]), subs(fxx, [x,y],
[xcr(k), ycr(k)])];
end
%fxx and fyy < 0, hence it is maximum at (4,-1)
```

```
%316
```

```
%f(x,y) = x^2 + y^2 + 2*x - 6*y + 6
syms x y z
f= x^2 + y^2 + 2*x - 6*y + 6
```

f = $x^2 + 2\,x + y^2 - 6\,y + 6$

```
fx=diff(f,x)
```

fx = $2\,x + 2$

```
fy=diff(f,y)
```

fy = $2\,y - 6$

```
hessdetf=fxx*fyy-fxy^2;
xcr = xcr(1:1); ycr = ycr(1:1);
for k = 1:1
 [xcr(k), ycr(k), subs(hessdetf, [x,y], [xcr(k), ycr(k)]), subs(fxx, [x,y],
[xcr(k), ycr(k)])];
end
%fxx and fyy > 0, hence it is minimum at (-1,3)
```

```
%317
%f(x,y) = (x^2 + y^2)^(1/2) + 1
syms x y z
f= (x^2 + y^2)^(1/2) + 1
```

f = $\sqrt{x^2 + y^2} + 1$

```
fx=diff(f,x)
```

fx =

$$\frac{x}{\sqrt{x^2 + y^2}}$$

```
fy=diff(f,y)
```

fy =

$$\frac{y}{\sqrt{x^2 + y^2}}$$

```
[xcr,ycr]=solve(fx,fy); [xcr,ycr]
```

ans = $\begin{pmatrix} 0 & 0 \end{pmatrix}$

```
fxy=diff(fx,y)
```

fxy =

$$-\frac{x\,y}{(x^2 + y^2)^{3/2}}$$

```
fyy=diff(fy,y)
```

fyy =

$$\frac{1}{\sqrt{x^2+y^2}} - \frac{y^2}{(x^2+y^2)^{3/2}}$$

```
%Since the critical points are (0,0), the vaue of D = 0 which implies that
%the test results are inconclusive.
```

```
%318
%f(x,y) = -x^3 + 4*x*y - 2*y^2 + 1
syms x y z
f= -x^3 + 4*x*y - 2*y^2 + 1
```

f = $-x^3 + 4\,x\,y - 2\,y^2 + 1$

```
fx=diff(f,x)
```

fx = $4\,y - 3\,x^2$

```
fy=diff(f,y)
```

fy = $4\,x - 4\,y$

```
[xcr,ycr]=solve(fx,fy); [xcr,ycr]
```

ans =

$$\begin{pmatrix} 0 & 0 \\ \frac{4}{3} & \frac{4}{3} \end{pmatrix}$$

```
fxy=diff(fx,y)
```

fxy = 4

```
fyy=diff(fy,y)
```

fyy = -4

```
hessdetf=fxx*fyy-fxy^2;
xcr = xcr(1:1); ycr = ycr(1:1);
for k = 1:1
 [xcr(k), ycr(k), subs(hessdetf, [x,y], [xcr(k), ycr(k)]), subs(fxx, [x,y],
[xcr(k), ycr(k)])];
end
D = -6*(4/3) * 4 -(4*4)
```

D = -48

```
%D is less than zero, f(4/3,4/3) is a saddle point
```

```
%319
%f(x,y) = x^2*y^2
syms x y z
f= x^2*y^2
```

f = $x^2\,y^2$

```
fx=diff(f,x)
```

fx = $2\,x\,y^2$

```
fy=diff(f,y)
```

fy = $2\,x^2\,y$

```
[xcr,ycr]=solve(fx,fy); [xcr,ycr]
```

ans = $\begin{pmatrix} 0 & 0 \end{pmatrix}$

```
fxy=diff(fx,y)
```

fxy = $4\,x\,y$

```
fyy=diff(fy,y)
```

fyy = $2\,x^2$

```
hessdetf=fxx*fyy-fxy^2;
xcr = xcr(1:1); ycr = ycr(1:1);
for k = 1:1
 [xcr(k), ycr(k), subs(hessdetf, [x,y], [xcr(k), ycr(k)]), subs(fxx, [x,y],
[xcr(k), ycr(k)])];
end
D = (2*y^2)*(2*x^2) - (16*x^2*y^2)
```

D = $-12\,x^2\,y^2$

```
%D is less than zero, f(0,0) is a saddle point
```