22MAT122 Mathematics for Computing

Assignment - 2

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2-204
AIE

1) find linear and Quadratic approximation of function

$f(x,y) = \sin 2x + \cos y$

$f_x = \frac{d}{dx}(f(x,y)) = 2\cos 2x$

$f_{xx} = \frac{d}{dx}(f_x) = -4\sin 2x$

$f_y = \frac{d}{dy}(f(x,y)) = -\sin y$

$f_{yy} = \frac{d}{dy}(f_y) = -\cos y$

$f_{xy} = 0$

for near point $(0,0)$

$f_x = 2\cos 2(0) = 2$

$f_y = -\sin(0) = 0$

$L(x,y) = f(x,y) + f_x(x-a) + f_y(y-b)$

$= \sin 2(0) + \cos 0 + 2(x-0) + 0(y-0)$

$L(x,y) = 1 + 2x$

for near point $(0,0)$

$f_{xx} = -4\sin 2(0) = 0$

$f_{yy} = -\cos 0 = -1$

$f_{xy} = 0$

$Q(x,y) = L(x,y) + \frac{f_{xx}(0,0)}{2}(x-0)^2 + \frac{f_{yy}(0,0)}{2}(y-0)^2 + f_{xy}(0,0)(x-0)(y-0)$

$\Rightarrow 1 + 2x + 0(x-0)^2 + \frac{-y^2}{2} + 0$

$\Rightarrow 1 + 2x - \frac{y^2}{2}$

$Q(x,y) = 1 + 2x - \frac{y^2}{2}$

Q2) Minimise the function

$$f(x,y) = x^2 + y^2 - 2x - 4y + 5$$

$$\frac{\partial f}{\partial x} = 2x - 2 \qquad \frac{\partial f}{\partial y} = 2y - 4$$

$$\frac{\partial^2 f}{\partial x^2} = 2 \qquad \frac{\partial^2 f}{\partial y^2} = 2 \qquad \frac{\partial^2 f}{\partial x \partial y} = 0$$

$$H(x,y) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \qquad H^{-1} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$$

$$x_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \nabla f(x_0, y_0) = (2(1) - 2, 2(1) - 4)$$

$$= \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

$$x_{k+1} = x_k - H^{-1}(\Delta f(x_k))$$

first Horation:-

$$X_1 = X_0 - H^{-1}(\nabla f(x_0))$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$f(1,2)$

$$= 1^2 + 2^2 - 2(1) - 4(2) + 5$$

$$\Rightarrow 1 + 4 - 2 - 8 + 5$$

$$\Rightarrow -10 + 10$$

$$\Rightarrow 0$$

Second Iteration:

$$x_2 = x_1 - H^{-1}(\nabla f(x_1))$$

$$x_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\nabla f(x_1, y_1) = (0, 0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$H(x_1, y_1) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \qquad H^{-1} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$f(1,2) = 1^2 + 2^2 - 2(1) - 4(2) + 5$$

$$= 1 + 4 - 2 - 8 + 5$$

$$= -10 + 10$$

$$= 0$$

$$(x_0, y_0) = (1, 1)$$

$$(x_1, y_1) = (1, 2)$$

$$(x_2, y_2) = (1, 2)$$

Q3) Calculate the regression

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

x	y	x^2	y^2	xy
1	9	1	81	9
2	8	4	64	16
3	10	9	100	30
4	12	16	144	48
5	11	25	121	55
6	13	36	169	78
7	14	49	196	98

$\Sigma x = 28$ $\Sigma x^2 = 140$

$\Sigma y = 77$ $\Sigma y^2 = 875$

$\bar{x} = \frac{\Sigma x}{N} = \frac{28}{7} = 4$ $\bar{y} = \frac{\Sigma y}{N} = \frac{77}{7} = 11$

Regression x on y:

$x - \bar{x} = b_{xy}(y - \bar{y})$

$x - 4 = 0.929(y - 11)$

$x - 4 = 0.929y - 10.219$

$\boxed{x = 0.929y - 6.219}$

Regression coeffent of x on y

$$b_{xy} = \frac{N\Sigma xy - (\Sigma x)(\Sigma y)}{N\Sigma y^2 - (\Sigma y)^2}$$

$$= \frac{7(334) - 28(77)}{7(875) - 77^2}$$

$$= \frac{182}{196}$$

$b_{xy} = 0.929$

$$b_{yx} = \frac{N\Sigma xy - (\Sigma x)(\Sigma y)}{N\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{7(334) - 28(77)}{7(140) - 28^2}$$

$$= \frac{182}{196}$$

$b_{yx} = 0.929$

Regression of y on x

$y - \bar{y} = b_{yx}(x - \bar{x})$

$y - 11 = 0.929(x - 4)$

$\boxed{y = 0.929x + 7.284}$

4) Determine cumulative distribution function mean variance for,

$$f(x) = \frac{2x+1}{25}, \quad x = 0, 1, 2, 3, 4$$

$$f(0) = \frac{2(0)+1}{25} = \frac{1}{25} \qquad f(0) = 0.04$$

$$f(1) = \frac{2(1)+1}{25} = \frac{3}{25} \qquad f(1) = 0.12$$

$$f(2) = \frac{2(2)+1}{25} = \frac{5}{25} \qquad f(2) = 0.2$$

$$f(3) = \frac{2(3)+1}{25} = \frac{7}{25} \qquad f(3) = 0.28$$

$$f(4) = \frac{2(4)+1}{25} = \frac{9}{25} \qquad f(4) = 0.36$$

x	0	1	2	3	4
f(x)	1/25	3/25	5/25	7/25	9/25

Comulative distribution function,

$$f(x) = \sum f(x_i)$$

$$= \frac{1}{25} + \frac{3}{25} + \frac{5}{25} + \frac{7}{25} + \frac{9}{25}$$

$$\Rightarrow \frac{25}{25}$$

$$\Rightarrow 1$$

$$\text{mean}(\mu) = \sum_x x f(x)$$

$$\mu = 0\left(\frac{1}{25}\right) + 1\left(\frac{3}{25}\right) + 2\left(\frac{5}{25}\right) + 3\left(\frac{7}{25}\right) + 4\left(\frac{9}{25}\right)$$

$$= 0.12 + 0.4 + 0.89 + 1.44$$

$$\boxed{\mu = 2.8}$$

Variance, $\sigma^2 = V(x) = \sum_x f(x)(x-\mu)^2$

$$\Rightarrow \frac{1}{25}(0-2.8)^2 + \frac{3}{25}(1-2.8)^2 + \frac{5}{25}(2-2.8)^2 + \frac{7}{25}(3-2.8)^2 + \frac{9}{25}(4-2.8)^2$$

$$\Rightarrow 0.3136 + 0.3888 + 0.128 + 0.0112 + 0.5184$$

$$\sigma^2 = 1.36$$

Variance = 1.36