Wave Optics Simulator

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Contents

Theoretical Approach

Scalar Wave Equation

Helmholtz equation: $\nabla^2 \psi + k^2 \psi = 0$ Source free

Where $\psi(\vec{r})$ is the Electro-Magnetic field

and we have the intensity: $I(\vec{r}) = |\psi(\vec{r})|^2$

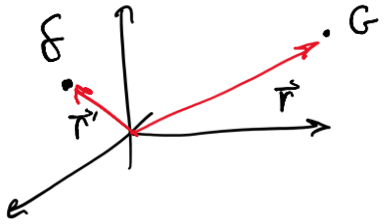
Plane waves: $\psi(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}$,

(a complete solution for the problem)

$\lambda = \frac{2\pi}{|k|}$

Green Function

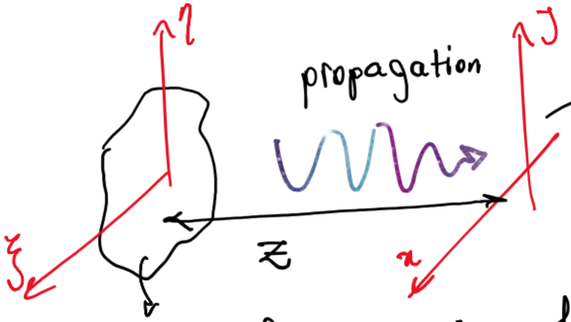
Green function: $\nabla^2 G + k^2 G = -\delta(\vec{r} - \vec{r}\,')$, $G = G(\vec{r}, \vec{r}\,')$



$$G(\vec{r}, \vec{r}\,') = G(|\vec{r} - \vec{r}\,'|) = \frac{e^{i\vec{k}\cdot(\vec{r} - \vec{r})}}{4\pi|\vec{r} - \vec{r}\,'|}$$

isotropic
homogeneous
space

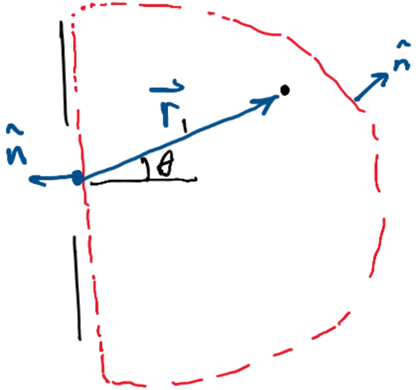
Planar POV



unknown field on the x-y plane after distance z

known field on this flat apperture

Huygens Principle



Green's 3rd theorem:

$$\int_{V} \left(U \nabla^2 V - V \nabla^2 U \right) dV = \oint_{S} \left(U \vec{\nabla} V - V \vec{\nabla} U \right) \cdot ds\,\hat{n}$$

if: $V = G(\vec{r}, \vec{r}_1)$ $\qquad \nabla^2 G = -\delta(\vec{r} - \vec{r}_0) - k^2 G$

$U = \psi(\vec{r}) \Rightarrow \nabla^2 \psi = -k^2 \psi$

$$\Rightarrow U(\vec{r}_1) = \frac{1}{j\lambda} \int_{\text{on the hole}} ds \, \cos\theta \, \frac{e^{jk|\vec{r}_1 - \vec{r}_0|}}{|\vec{r}_1 - \vec{r}_0|} \, U(\vec{r}_0)$$

$U(\vec{r}_0) \downarrow$ each point radiates in sphere

Near Field to Far Field

Antenna view :

near field

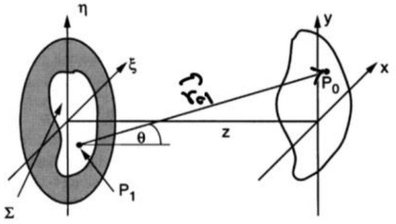
Fersnel Field

Far Field (Franhofer field)

$2D^2/\lambda$

distance

Paraxial Approximation



$\Longrightarrow$ با اعمال

$$U(P_0) = \frac{1}{j\lambda} \int_{\Sigma} U(P_1) \frac{e^{jkr_{01}}}{r_{01}} \cos\theta \, ds$$

[illegible] $\cos\theta = \dfrac{z}{r_{01}}$ ،

$$\Longrightarrow \quad U(x,y) = \frac{z}{j\lambda} \iint_{\Sigma} U(\xi,\eta) \frac{e^{jkR}}{R^2} d\xi \, d\eta \,, \quad R^2 = z^2 + (x-\xi)^2 + (y-\eta)^2$$

$$\sqrt{1+\alpha} \approx 1 + \frac{\alpha}{2} - \frac{\alpha^2}{8} + \cdots \qquad |\alpha| \ll 1 \Rightarrow R = z\left(1 + \left(\frac{x-\xi}{z}\right)^2 + \left(\frac{y-\eta}{z}\right)^2\right)^{\frac{1}{2}} \approx z\left[1 + \frac{1}{2}\,\frac{(x-\xi)^2 + (y-\eta)^2}{z^2}\right]$$

$$U(x,y) = \frac{e^{jkz}}{j\lambda z} \iint_{\Sigma} U(\xi,\eta) \exp\left[\frac{jk}{2z}\left((x-\xi)^2 + (y-\eta)^2\right)\right] d\xi \, d\eta \quad \Leftarrow \text{Fresnel Diffraction}$$

Free Space as a Transfer Function

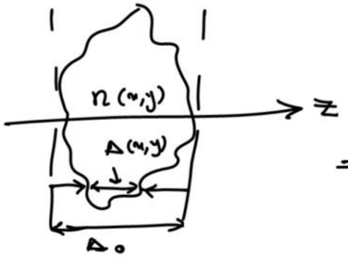
$$U(x,y) = \iint U(\xi,\eta)\, h(x-\xi, y-\eta)\, d\xi\, d\eta \leftarrow \text{Convolution!}$$

با یکم دقت متوجه می‌شویم که

$$h(x,y) = \frac{e^{jkz}}{j\lambda z} \exp\left(j\underbrace{\frac{k}{2z}}_{\frac{\pi}{\lambda z}} (x^2+y^2) \right) \longrightarrow \boxed{H(f_x,f_y) = e^{jkz} \exp\left(-j\pi\lambda z (f_x^2+f_y^2)\right)}$$

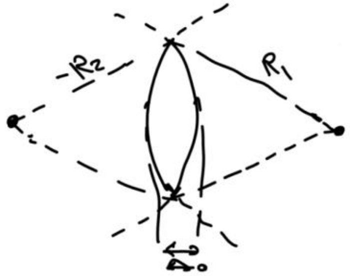
$$U(x,y) = F^{-1}\left\{ U(f_x,f_y)\, H(f_x,f_y) \right\}$$

Lens Transfer Fubction



$$\Longrightarrow \quad t_A(x,y) = \exp\left(jk_0\left[\underbrace{\Delta_0 - \Delta(x,y)}_{\text{طول [illegible] در فضای آزاد}} + n(x,y)\,\Delta(x,y)\right]\right)$$

$$= e^{jk_0\Delta_0}\, e^{jk_0(n(x,y)-1)\,\Delta(x,y)}$$



$R_1 > 0 \quad , R_2 < 0$

$$\rightarrow \quad \Delta(x,y) = \Delta_0 - R_1\left(1 - \sqrt{1 - \frac{x^2+y^2}{R_1^2}}\right) + R_2\left(1 - \sqrt{1 - \frac{x^2+y^2}{R_2^2}}\right)$$

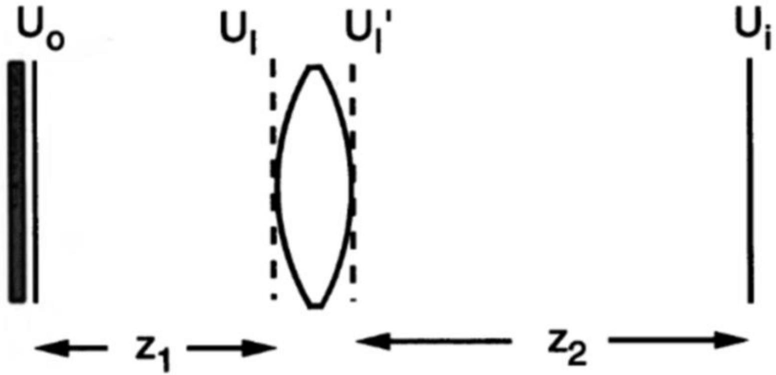
$$\sqrt{1 - \frac{x^2+y^2}{R_1^2}} \approx 1 - \frac{x^2+y^2}{2R_1^2} \quad : (\text{Paraxial})$$

Lens makers

$$\Delta(x,y) \approx \Delta_0 - \frac{x^2+y^2}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\text{if} \quad \frac{1}{f} := (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \Longrightarrow \quad \boxed{t_A(x,y) = e^{jk_0\Delta_0}\, e^{-j\frac{k}{2f}(x^2+y^2)}}$$

Imaging Condition



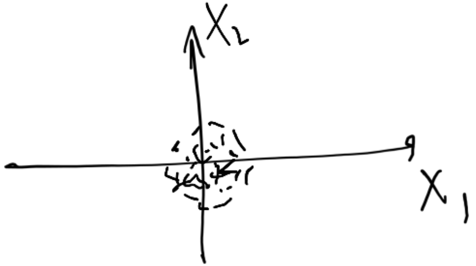
For having a point image of an initial point input we should satisfy the condition:

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$$

$$U_o = \delta(x-\xi)\,\delta(y-\eta)$$

Light as Quantum Photons

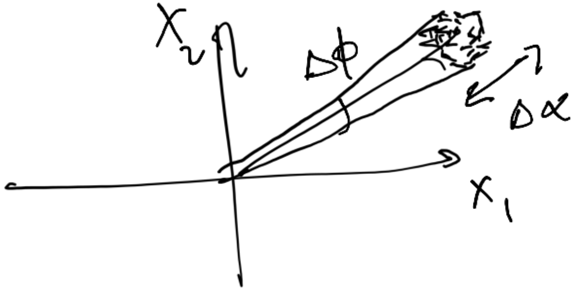
Instead of the classical coherent light we can shine the coherent state, although it has uncertaibty in both amplitude and phase.



$|0\rangle$ vacuum

Displacement

$\hat{D}(\alpha)$



coherent state: $|\alpha\rangle = \sum \frac{\alpha^n}{\sqrt{n!}} |n\rangle \cdot e^{-|\alpha|^2/2}$

Light as Quantum Photons

For different modes summing we will have the same wavefront for the coherent state which changes in the linear optical system.

set of $a^\dagger_{\vec{k}}$, $[a^\dagger_{\vec{k}}, a_{\vec{k}'}] = \delta_{\vec{k},\vec{k}'}$ in momentum

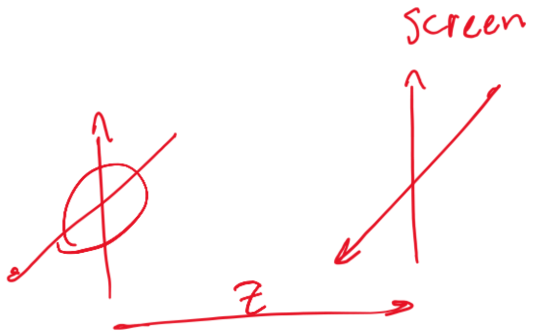
set of $a^\dagger_{\vec{x}}$, $[a^\dagger_{\vec{x}}, a_{\vec{x}'}] = \delta(\vec{x}-\vec{x}')$ in position

$$a^\dagger_U = \int dx\, dy\, \underbrace{U(x,y)}_{\downarrow} a^\dagger_{x,y}$$

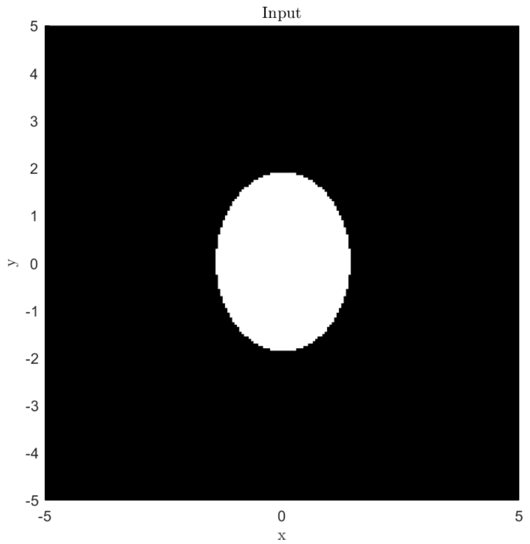
linearly changes

No Lens Test

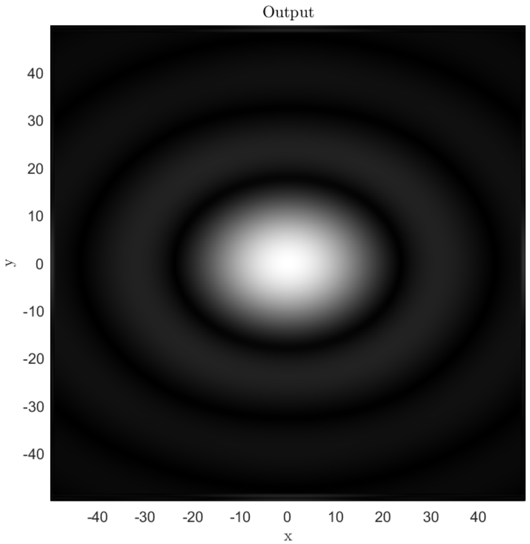
Circle Hole



Screen
z

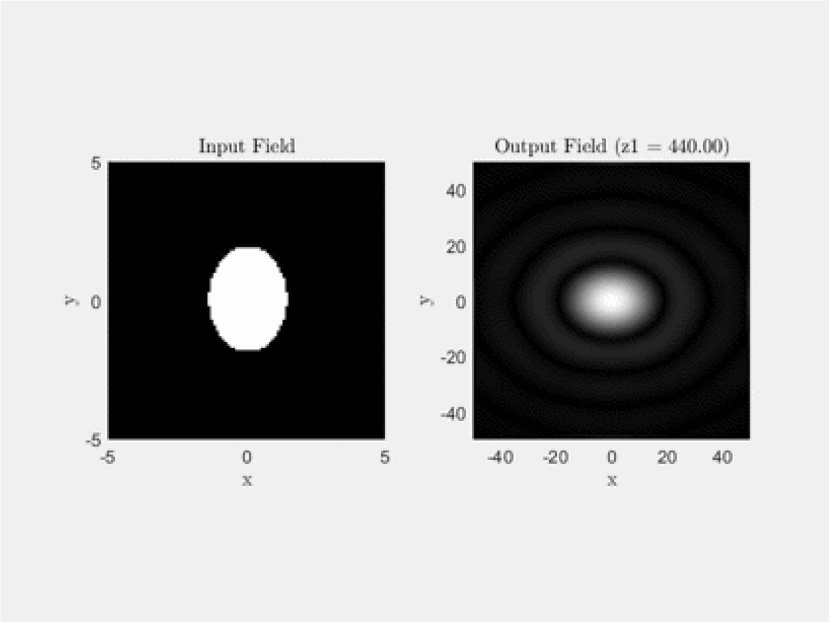


Input
5
4
3
2
1
0
-1
-2
-3
-4
-5
y
-5
0
5
x

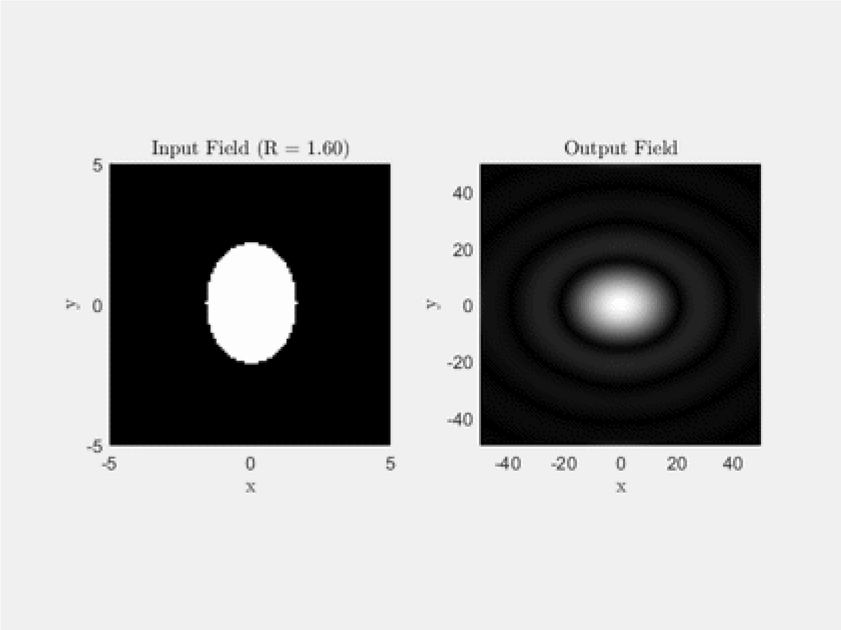


Output
40
30
20
10
0
-10
-20
-30
-40
y
-40
-30
-20
-10
0
10
20
30
40
x

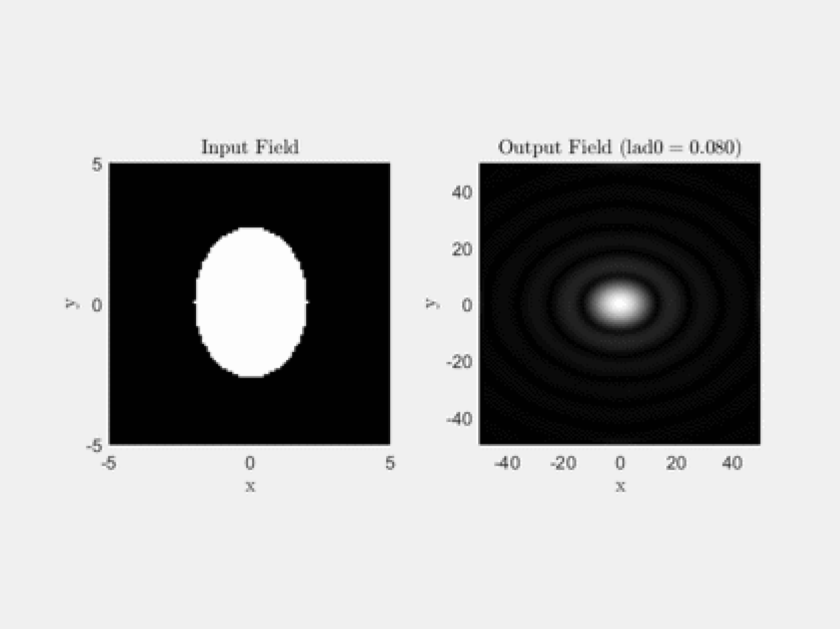
Circle Hole



Circle Hole



Circle Hole

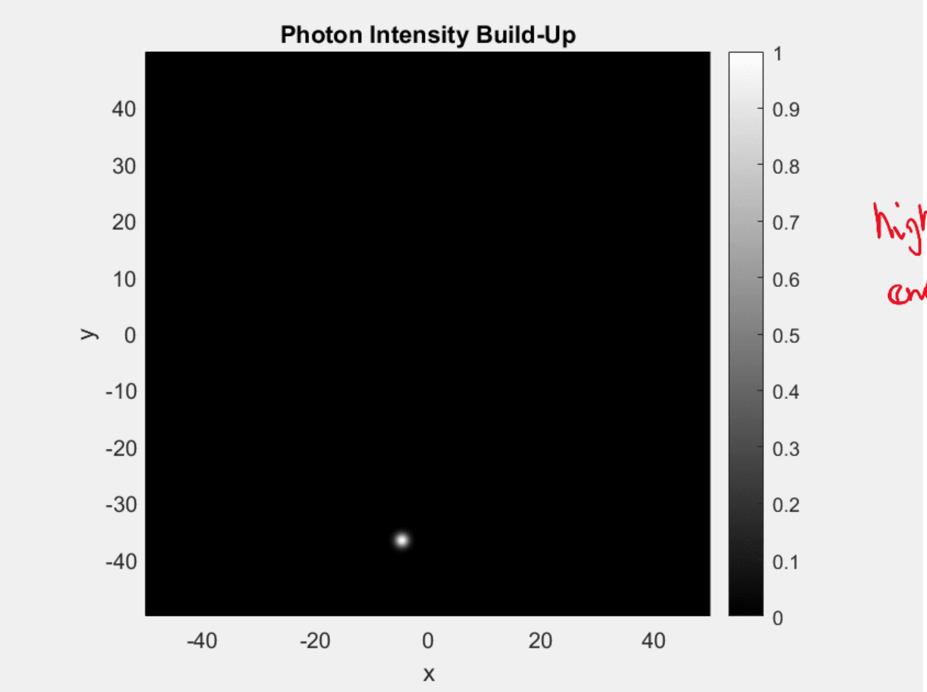


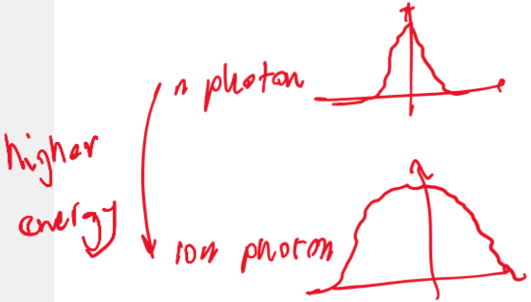
Input Field
Output Field (lad0 = 0.080)
5
0
-5
-5
0
5
x
y
40
20
0
-20
-40
-40
-20
0
20
40
x
y

Photons Sampling

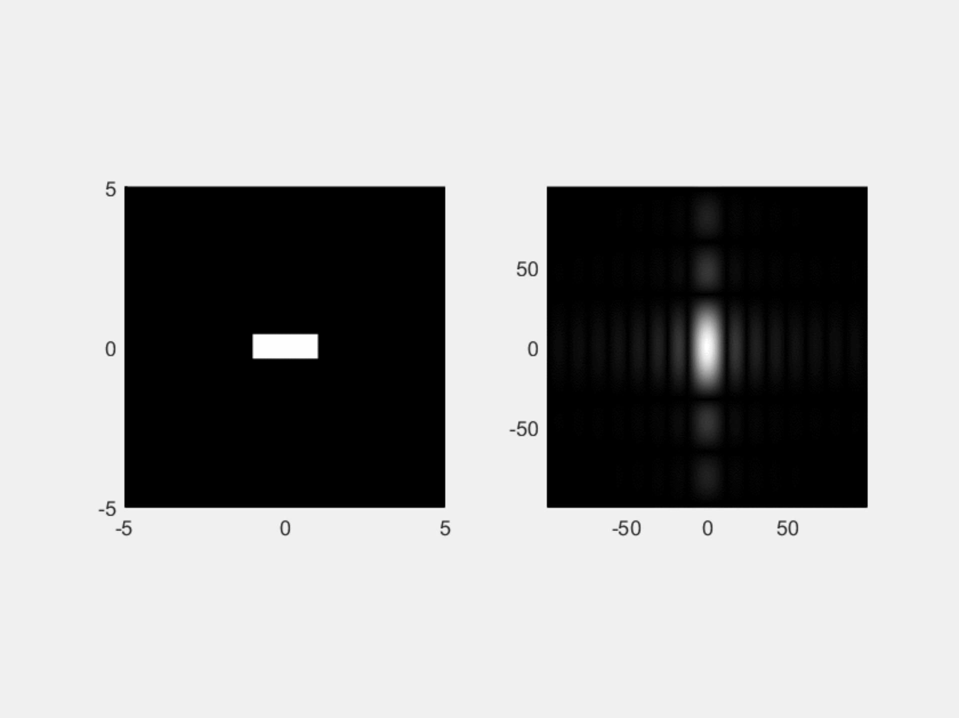
For coherent state we have

$$P(n) = \frac{e^{-\bar{n}}}{n!}\bar{n}^{n}$$

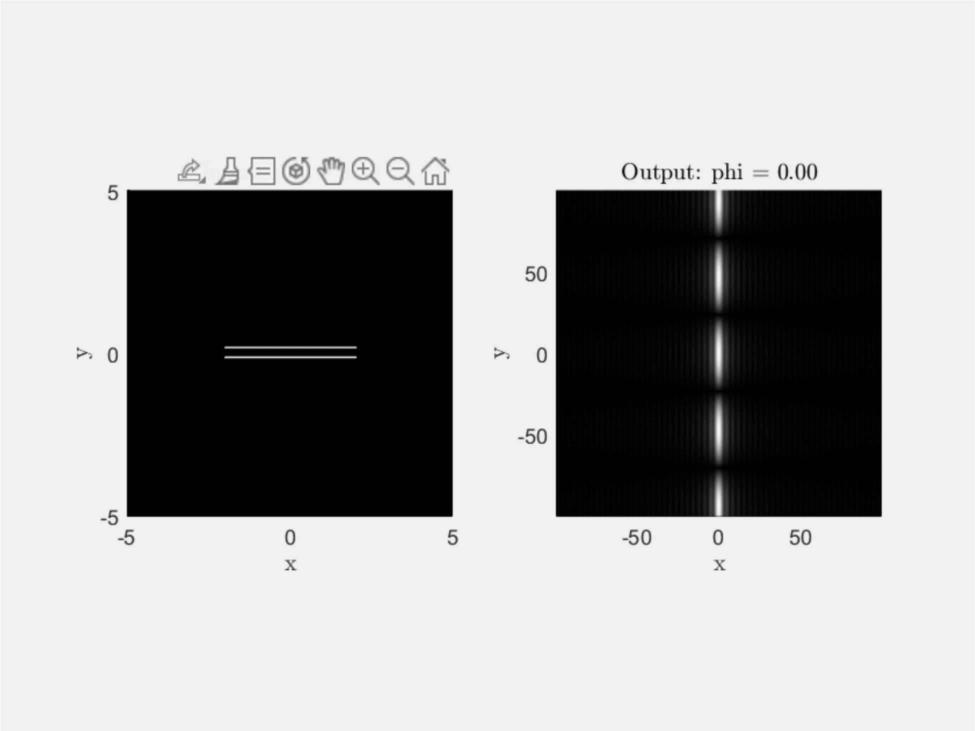




Single Slit

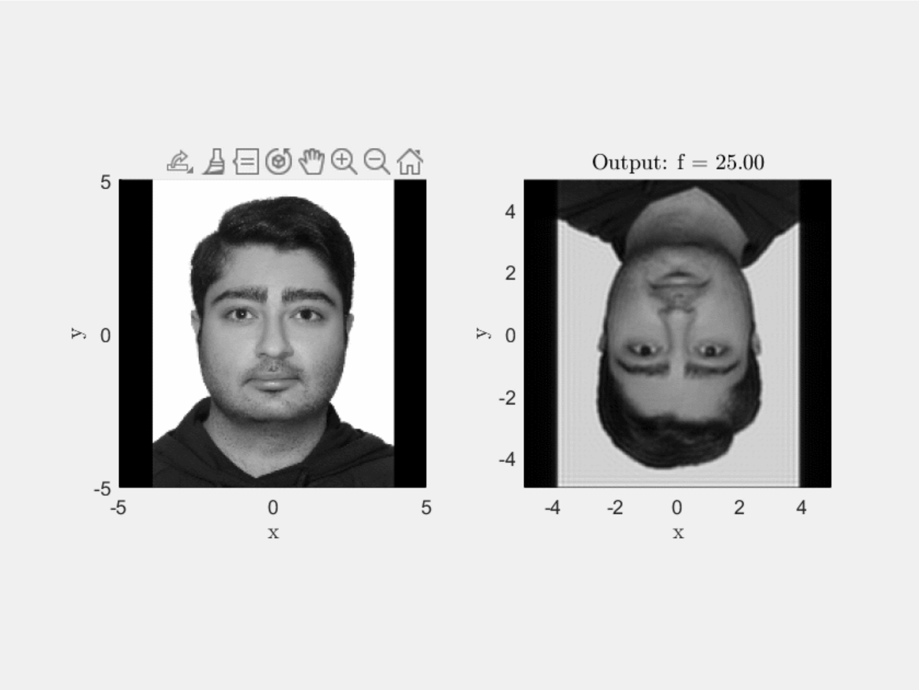


Double Slit

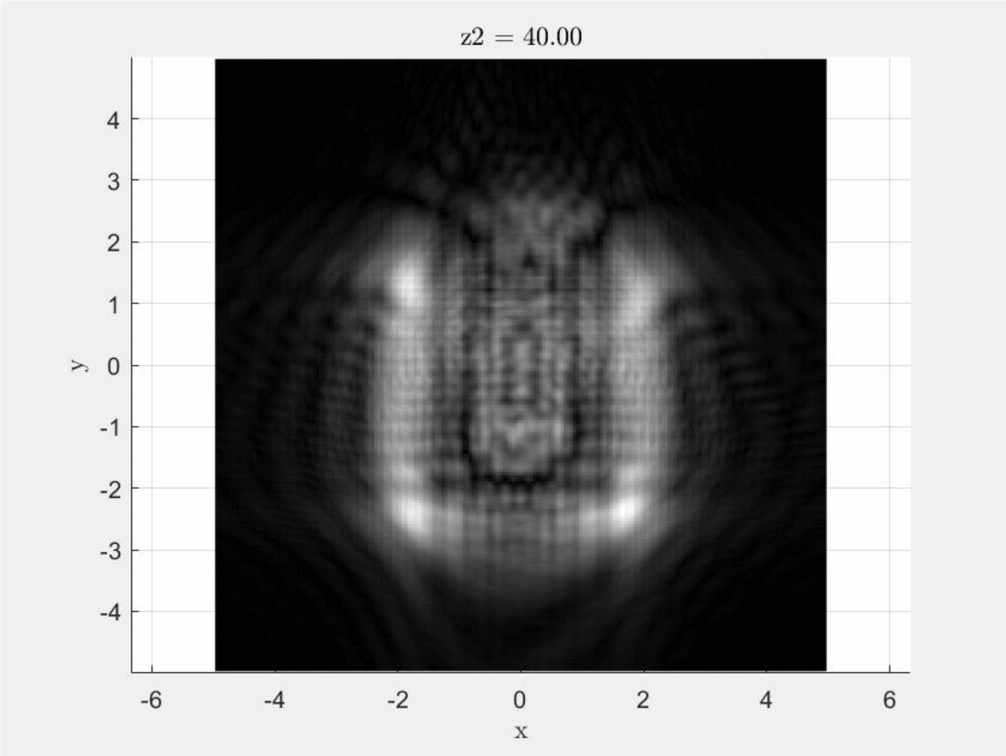


Single Lens Simulations

Single Lens

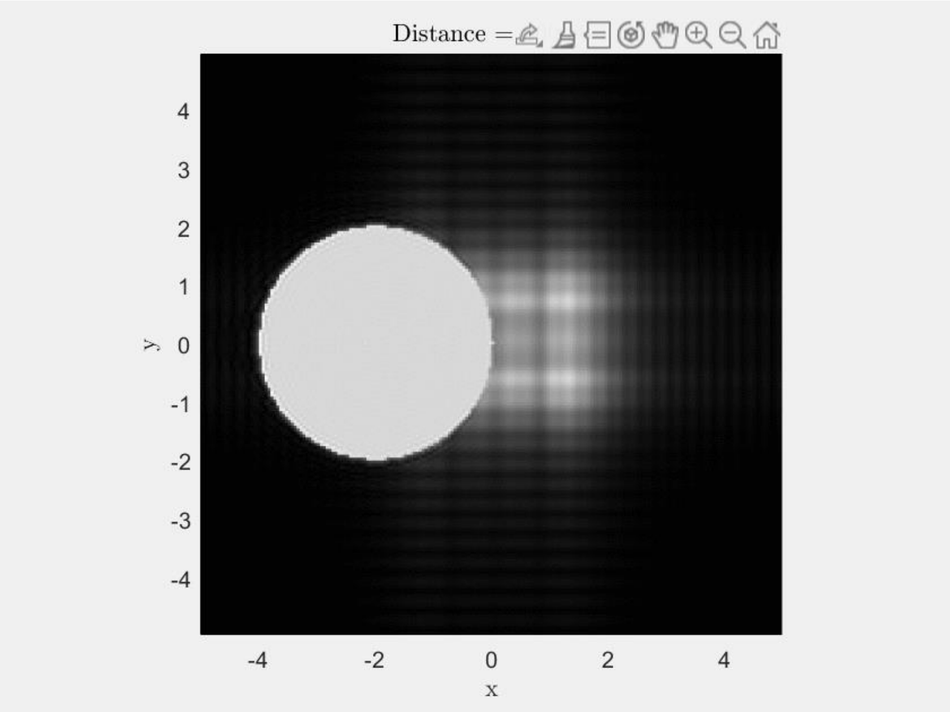


Single Lens

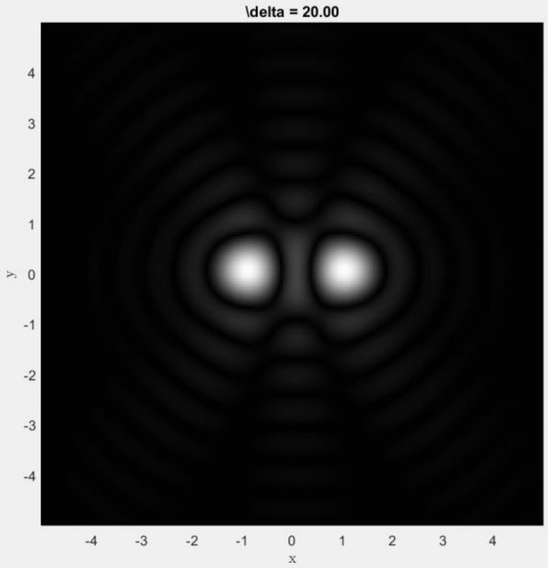


2f System

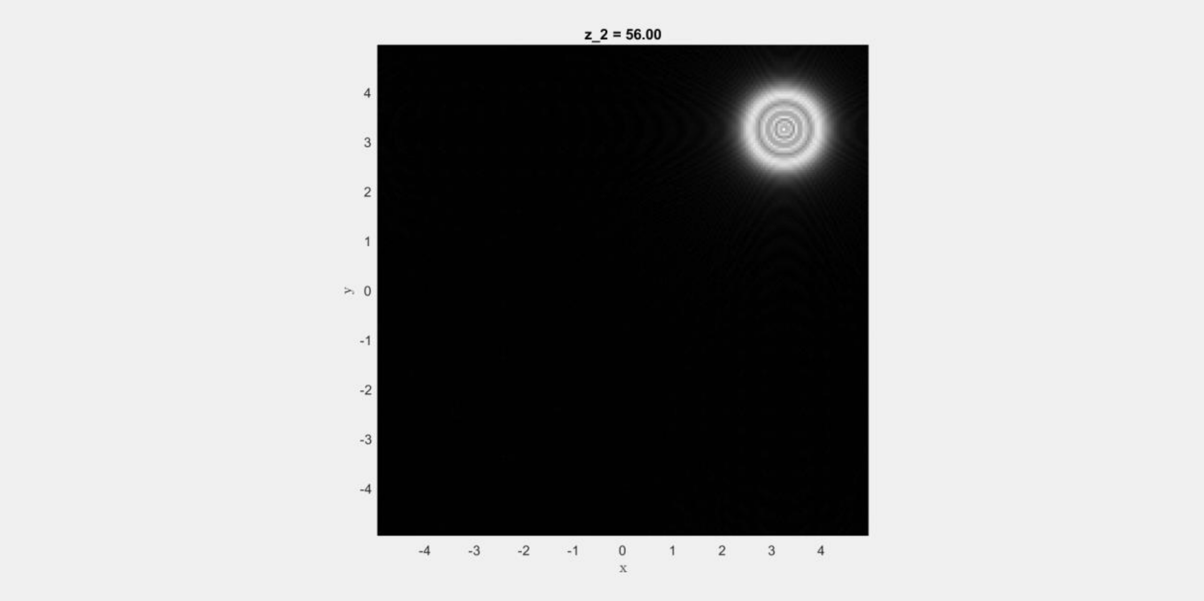
Focusing



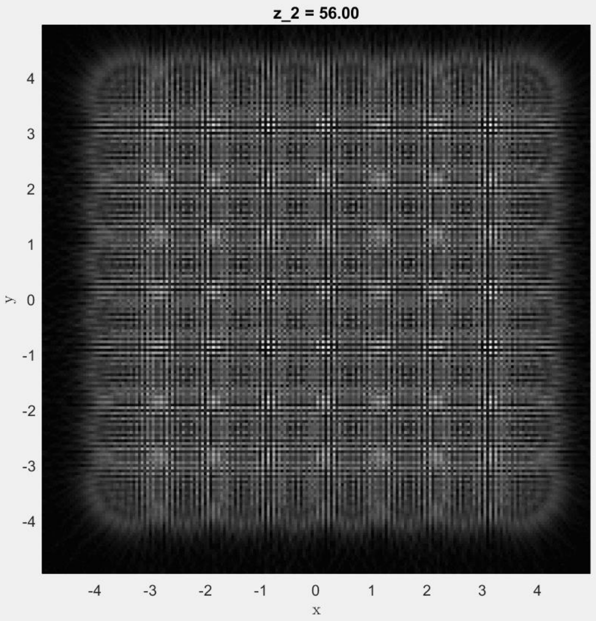
Two Stars Reileigh Criterion



Comma Abboration

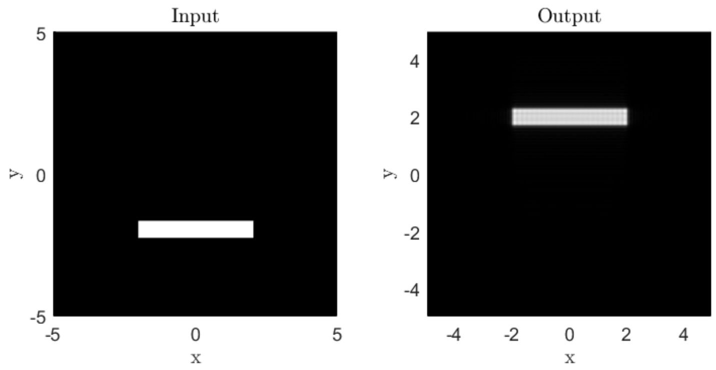


Point Spread Function

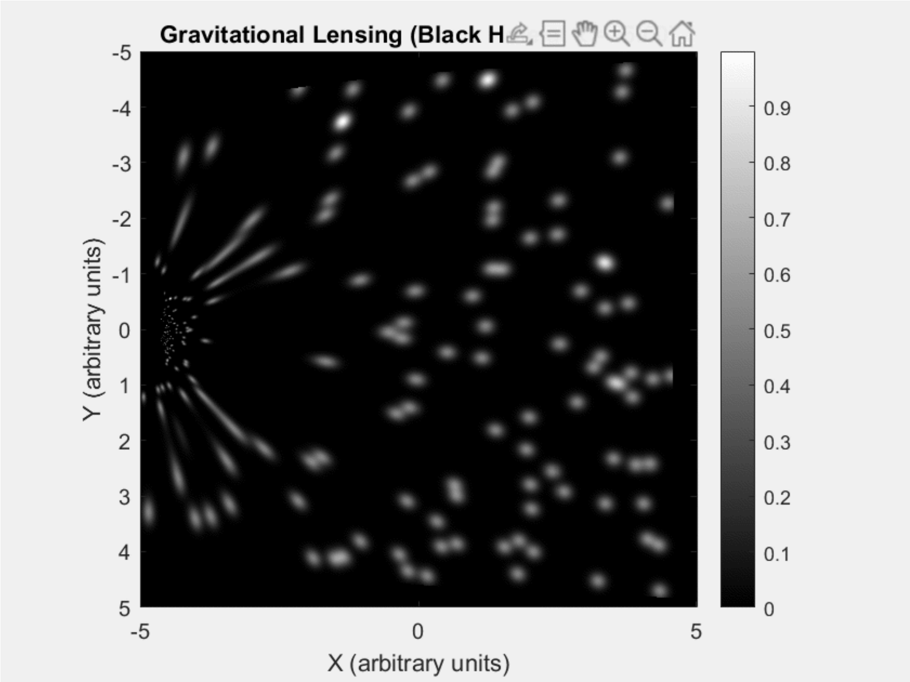


Perfect Lens

Perfect Lens

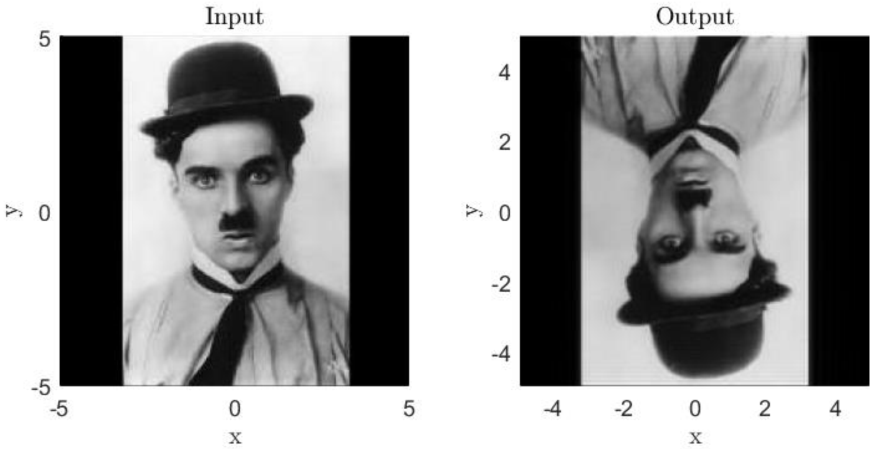


Black Hole as a Lens

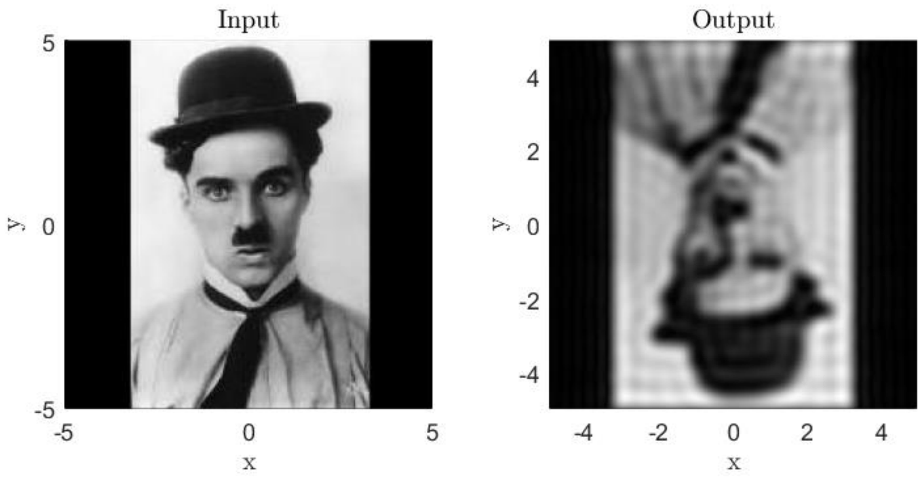


Multi Lens Simulations

4f System

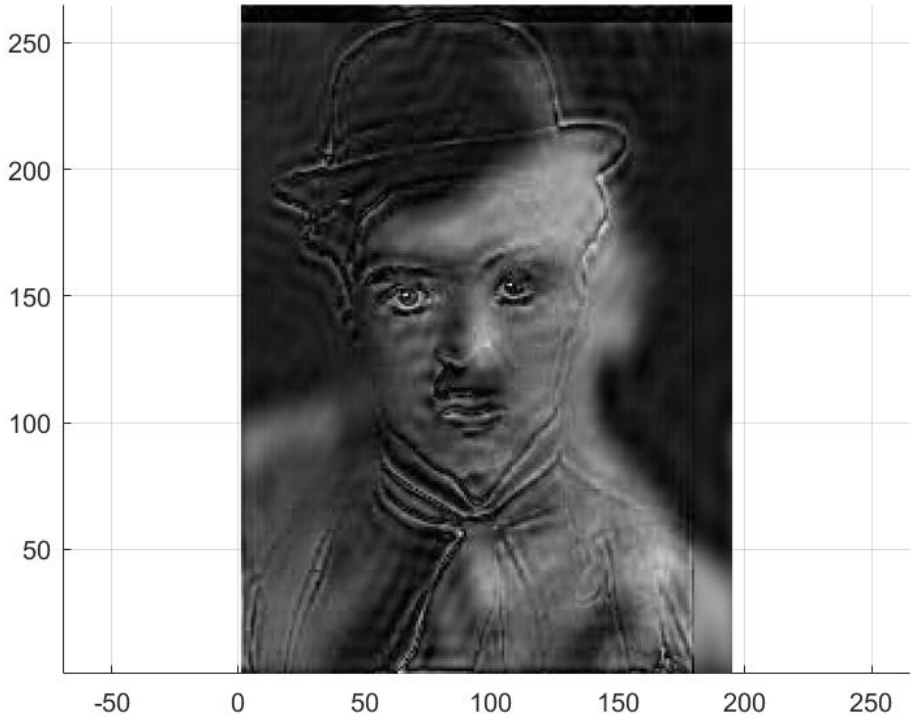


Low Pass Filter

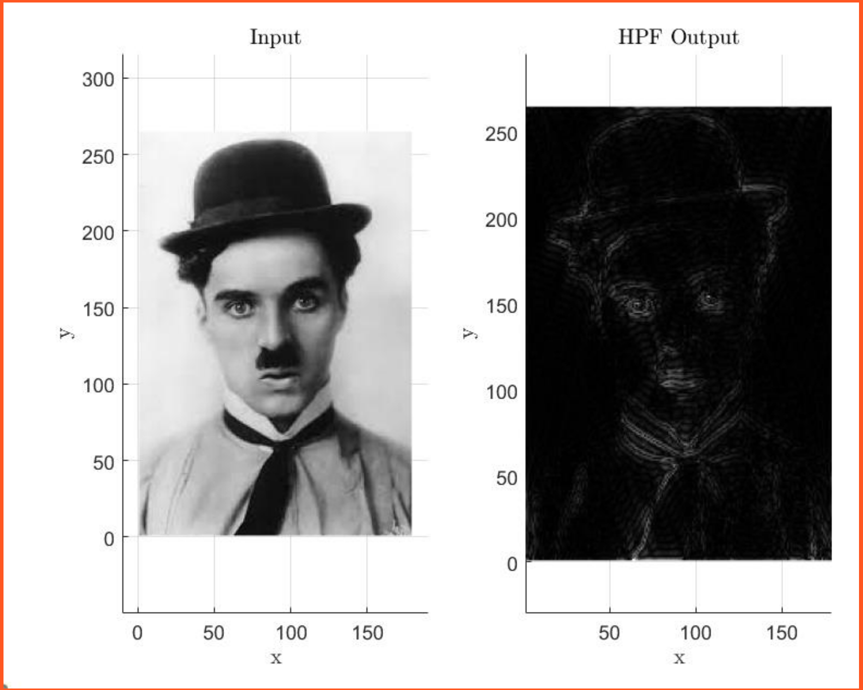


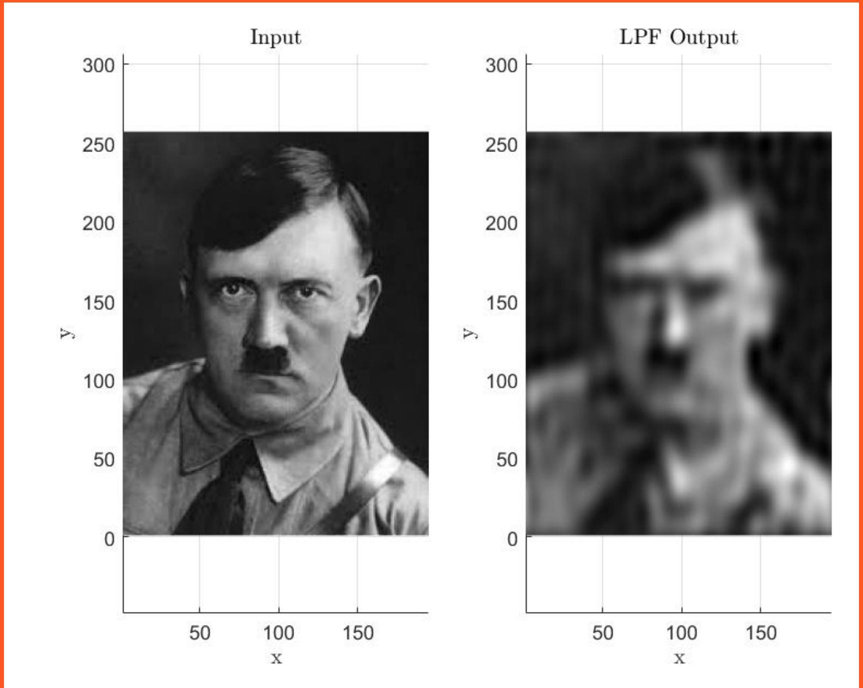
High Pass Filter

Hybrid Image



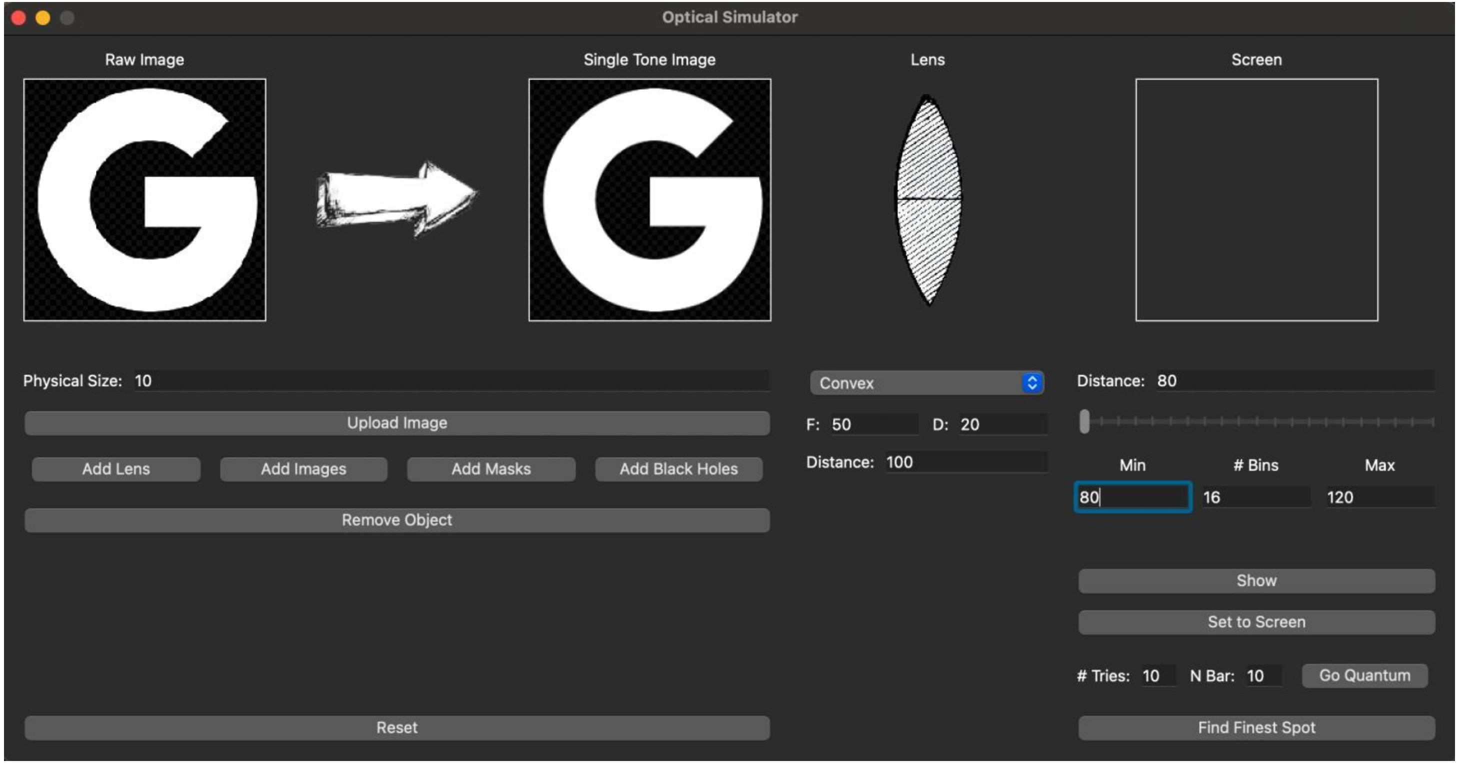
Hybrid Image





Say Hello to the Application

User Interface



Appendix

Any question?

Refrences

- ***J. Goodman. Introduction to Fourier optics, 3rd ed., by JW Goodman. Englewood, CO: Roberts & Co. Publishers, 2005, (2005)***
- M. Rezai and J. A. Salehi, “Quantum fourier optics (QFO),” 2023

Materials

Found around the house!

- 2 drinking glasses
- Table salt
- 2 eggs
- Water