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The Truth about Vagueness

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1. Introduction

What is the lesson of the Sorites Paradox? In this paper, we argue that it has largely been missed, and we return to a theorem we published in (2002), which we use to diagnose what has gone wrong in soritical reasoning, and to organize various things people have said about vagueness, such as Kit Fine's impossibility result (2008). The theorem, together with some *very* minimal assumptions, leads to the conclusion that no sentences containing vague predicates are either true or false—semantic nihilism. We offer a satisfying explanation of this result: vague predicates are simply not outfitted properly for the role in language that they are introduced to play. Together with the assumption that vague predicates must apply to extremal points, the theorem yields Fine's impossibility result. But we show that bringing on board this additional requirement on vagueness rests on a false assumption about the relation between having meaning intentions and executing them successfully. Sorites arguments are revealed to founder because they are unsound, but not because, for example, the step premise is false: it is rather that none of the premises or the conclusion are either true or false. We diagnose the struggle over understanding the Sorites Paradox as deriving largely from a tendency to tacitly acknowledge the defect in the practice with respect to the behavior of the predicate over the range of its application, while stubbornly refusing to acknowledge it for extremal cases or those nearby. This still leaves us with a puzzle, which is how to understand ordinary practice with vague terms if what we say using them is neither true nor false, and also how to understand the force of any theory we introduce, which must perforce employ vague terms itself in its expression. We have said some things about this in previous work, and here we take up a later suggestion made by Braun and Sider (2007), that to a first approximation appropriate assertibility of vague sentences tracks supertruth, which, though it seems to be on the right track, does not quite work, and we show how to improve upon it with a recursive account of what we call univocal supertruth.

2. A Theorem Based on Higher-Order Vagueness

We begin with a theorem that expresses a consequence of accepting higher-order vagueness—or boundarylessness, as Sainsbury has aptly put it (1991; 1999)—as an essential characteristic of semantic vagueness (see (Sorenson 2010) for a recent defense). Consider a sorites series with respect to 'is bald', b_0, b_1, b_2, ... b_k, that is, a series of items which call forth the kind of increasing hesitancy characteristic of vagueness as we are called on to say whether, as we move from b_0 to b_k, a certain b_i is such that 'is bald' applies or not. b_0 exemplifies a person who would ordinary be called "paradigmatically bald" and b_k someone who would ordinarily be said to be about as far on the opposite extreme as one could get. We can imagine k to be an especially large number, and b_1 to differ from b_0 in respect of having one more hair, and so on. We can state the form of a sorites argument with respect to this series as follows:

[1] b_0 is bald.
[2] For all n ($0 \leq n < k$), if b_n is bald, then b_{n+1} is bald.
[3] b_k is bald.

The argument form is valid. The premises seem compelling. For [1], b_0 is a paradigm for 'bald', and, for [2], how can very small differences, a single hair, ever make a difference to whether the predicate applies? Yet, everyone agrees that the argument is unsound. The question is what has gone wrong. Most commentators accept [1] and reject [3] and, hence, reject [2]. Epistemicists reject [2] on the grounds that there is specific number n which falsifies [2], though we can't in principle know what it is (Cargile 1969; Williamson 1994). Contextualists say that each given context determines some n or other for which [2] is false (Kamp 1981; Raffman 1994; Graff-Fara 2000; Shapiro 2006). A more traditional response is to argue that there are borderline cases, that for some range of n between 0 and k, b_n is neither bald nor not bald. Then [2] is not true for at least some n. But this runs into difficulty with higher-order vagueness—the denial that there are higher-order borderlines, between, for example, being bald, being neither bald nor not bald, and not being bald. For if there are no higher-order borderlines, then there is no precise delimiting of the borderline cases. Kit Fine has gone so far as to argue that, relative to accepting [1] and the negation of [3], this shows that vagueness is not possible (2008). Since evidently vagueness is possible, he suggests that we need to "modify the principles of classical logic" (p. 112) in order to accommodate it.

However, we can reach a more satisfying conclusion.

To do so, we must draw some distinctions. The first distinction is that between use and mention. Clearly, incautious operation with vague terms, as [1]-[3] show, can get us into trouble. Let's then first adopt the tactic of not using the vague terms we are examining but only mentioning them. This extends to using them after operators like 'definitely', except insofar as we interpret those as metalinguistic predicates of some sort (and even then the operator itself is vague). The second distinction is four-fold, that between anything applying to (being true of) something, its failing-to-apply to (being false of) something, its not applying to something, and its not failing-to-apply to something. 'Applying' and 'failing-to-apply' are semantic relations. 'Is even' applies to numbers divisible by two and fails-to-apply to numbers that are not divisible by two. In contrast, a horse does not apply to the number two, and it does not fail-to-apply to the number two. It neither applies nor fails-to-apply to anything. For a thing to apply to an object is for the object to meet its conditions of satisfaction; for a thing to fail to apply to an object is for the object to not meet its conditions of satisfaction. If something, like a horse, does not have conditions of satisfaction, it neither applies nor fails-to-apply to anything.

We make only one assumption about vagueness.

[HOV] There are no first or higher-order borderlines for vague predicates.

A first-order borderline between the application and failure of application of a predicate is pictured in (a). A second order borderline between application, neither application nor failure of application, and failure of application is pictured in (b). A third order borderline is pictured in (c)—label the categories how you will. [HOV] is incompatible with any higher-order borderlines. The absence of higher-order borderlines captures the idea that it is simply indeterminate where to draw the line between 'bald' applying to a member of a sorites series and that ceasing to be so. Without [HOV], the idea that vague predicates involve semantic indeterminacy disappears.

(a)

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applies                                                          fails-to-apply
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(b)

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applies                          doesn't apply                        fails-to-apply
                                 doesn't fail-to-apply
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(c)

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applies                                                               fails-to-apply

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We stipulate that the semantic predicates we use in the argument for our theorem obey the conditions [SP] (semantic precision) and [EXC] (exclusion).

[SP] 'applies to' and 'fails-to-apply to' are not vague.
[EXC] For any x, y, if x applies to y then x does not fail-to-apply to y, and for any x, y, if x fails-to-apply to y, then x does not apply to y.

We give the argument, again by stipulation, in a language with a bivalent first-order logic, and interpret the logical constants materially, that is to say, in terms of the usual truth functions in first-order logic.

On this basis, we can prove the following theorem [T].

If (i) for every n ($0 \leq n < k$), 'bald' applies to b_n iff 'bald' applies to b_{n+1} and
(ii) for every n ($0 \leq n < k$) , 'bald' fails-to-apply to b_n iff 'bald' fails-to-apply to b_{n+1}

then (iii) for every n ($0 \leq n \leq k$), 'bald' applies to b_n or

(iv) for every n ($0 \leq n \leq k$), 'bald' fails-to-apply to b_n or

(v) for every n ($0 \leq n \leq k$), 'bald' does not apply to b_n, and 'bald' does not fail-to-apply to b_n.

It is easy to see why [T] is true. Proof: Suppose that (i) and (ii) are true. If (i) is true, then if 'bald' applies to some b_i, it applies to every b_i, and, since no pair $<x, y>$ satisfies both 'applies' and 'fails-to-apply' (by [EXC]), 'bald' does not fail-to-apply to any b_i. Similarly, if (ii) is true, then if 'bald' fails-to-apply to some b_i, it fails-to-apply to every b_i, and, hence (by [EXC]), 'bald' does not apply to any b_i. Therefore, given (i) and (ii), there are three mutually exclusive options. Either 'bald' applies to every b_i, or 'bald' fails-to-apply to every b_i, or 'bald' does not apply to any b_i and 'bald' does not fail-to-apply to any b_i. Conditionalizing yields [T].

There are no vague terms in this theorem, nor in its canonical proof, as we have only mentioned the term 'bald', and no terms in use position in [T] are vague.

How is this related to the sorites paradox and vagueness? Answer: [HOV] is what underlies the attraction of [2] in the sorites argument and [HOV] entails (i) and (ii).

3. The Theorem [T], Borderline Cases, and the Non-application of Vague Predicates

There are no borderline cases of 'bald' if for every member b_i of any sorites series, 'bald' either applies to b_i or fails-to-apply to b_i. In this case, there is only a first-order borderline, and 'bald' is not vague. 'bald' is vague only if there are borderline cases for 'bald'. If there is a borderline case for 'bald', then for some sorites series there is a b_n such that 'bald' neither applies nor fails to apply to b_n. This suffices to establish that if 'bald' is vague, then

(vi) There is some b_n to which 'bald' does not apply;

(vii) There is some b_n to which 'bald' does not fail-to-apply.

There is universal agreement on (vi) and (vii). From them, together with [T] and [HOV], we can conclude that [N] (nihilism).

[N] 'bald' does not apply to any b_n and 'bald' does not fail-to-apply to any b_n.

This includes b_0 and b_k. For given [HOV] and [T], we know we must accept (iii), (iv) or (v). (vi) rules out (iii). (vii) rules out (iv). That leaves (v). We will return below to what this result tells us about what has gone wrong with vague predicates.

4. An Impossibility Result

Consider the three propositions [IND] (indeterminacy), [EXT] (extremal cases), and [E] (entailment):

[IND] Vague predicates exhibit indeterminacy across their range of application.

[EXT] Vague predicates apply and fail-to-apply to their positive and negative extremal cases, respectively. For example, 'bald' applies to the positive extremal case b_0 and fails-to-apply to the negative extremal case b_k.

[E] [IND] entails [HOV].

Suppose that one takes it to be *constitutive* of the phenomenon of vagueness that [IND] and [EXT] and [E] are true ([E] expresses how [IND] is understood). Then given theorem [T] we have the result that vagueness is impossible, that is, nothing satisfies both the requirements [IND] and [EXT]. This application of theorem [T] shows us the essential logical content of Kit Fine's (2008) argument that vagueness, characterized as including [IND], [EXT] and [E], is impossible (cf. pp. 114-5). Fine takes this to show that classical logic must be given up—we need a new logic to understand how vagueness is possible. We take this result just to show again that [EXT] is mistaken—a fact already made plain by [N]. Undoubtedly, the pull of [EXT] is strong. We have just seen a case of a philosopher and logician, who, in the face of a simple logical *reductio* is prepared to *give up logic* rather than even consider the possibility of its falsehood. So, what we need is to understand how [EXT] could be wrong given our clear intention that extremal cases should be paradigm instances of items in the extension and counterextension of vague predicates. Before we turn to this, however, we consider whether a *new* logic of vagueness or of natural language could be a solution to the problem.

5. Logic and Vagueness

The idea that we need a special logic for vagueness has been a longstanding theme in the literature on vagueness (Hallden 1949; Goguen 1969; Machina 1972; Wright 1976; Fine 2008).

What do we mean, however, by a logic of vagueness or of natural language? Here it is important to distinguish between two things. The first is an account of the semantic properties of a distinguished set of expressions that we call 'logical' on one or another ground, e.g., topic neutrality, membership in the set of minimally recursive devices of a language, and so on. The second is a formal model of the broadly semantic properties, or relevantly related properties (e.g., absence of meaning), of some set of expressions in a language. Max Black led the way in confusing these in (1937). Responding to Black in (1939), Carl Hempel was careful to draw the distinction, but this has not generally been thought to be as exciting as the conflation.

The first sort of logic is concerned with the semantic behavior, especially with respect to truth, of a set of expressions, such as 'and', 'or', 'not', 'if ... then ...', 'iff', 'for all *x*', 'for some *x*', 'few *x*', 'most *x*', etc. We'll call this the logic *of the language.*

The second sort involves terms that are not distinctively logical. For example, epistemic logic, the logic of belief, the logic of nonsense, deontic logic, and so on, are concerned not to characterize the semantic properties of *logical* terms (on any of the views of what singles these out), but to characterize some aspects of the semantic behavior, especially with respect to entailment relations, of sentences in relation to certain terms in them. These projects can be subdivided into those that focus on what are taken to be semantically complete expressions like 'believes' 'knows', 'ought', 'it is funny that', and the like, and those that focus on characterizing behavior of sentences containing terms that are defective in some way or other, as in the case of a logic of nonsense, which may, for example, aim to characterize safe inferential behavior in a language in which there are nonsense terms. On the assumption, which we make in this paper, that vagueness is a kind of semantic defect in a term, a logic of vagueness would be of this sort.

What are we to make, in light of this distinction, of the suggestion that we need a new logic in order to understand the possibility of vagueness?

First, consider the suggestion that we need a new (non-classical) logic of natural language. This can be taken as the suggestion that we need to rethink what the semantic properties of natural language logical constants actually are or that we should replace the actual natural language constants with their semantics with some distinct constants with theirs. But following the second suggestion could not help make sense of the possibility of vagueness in natural language as it is. So we put this aside. On the first suggestion, the idea must be that we have got the logic of natural languages wrong because if we have got it right, then vagueness is not possible. But how exactly is the semantics for the logical particles supposed to bear on the issue? Their semantics is evidently completely independent of the fact that there are any terms in the language that exhibit semantic vagueness, and it is clear that no matter what is the logic of natural language (in the sense at issue here), we could have semantically vague terms. And so we could have semantically vague terms in a language with a classical logic. Thus, the logic of natural language is not the right place to look for a solution to the problem.

It might be suggested, though, that given that there are vague terms in the language, the logic cannot be classical. This is apparently what Russell thought (1923). But this is a mistake. Suppose we have a language with a classical logic and no vague terms. To say that the logic is classical is to say that in the range of appropriate application for its logical terms, all sentences are either true or false. Suppose that we introduce a nonsense predicate into the language. Does the logic become non-classical? Does bivalence fail? How could the introduction of a predicate in a category change the meanings of any of the logical constants? They are defined for use over categories of terms. The meanings (or lack of meaning) of the items in the categories don't matter for characterizing the meanings of the logical terms. Therefore, the introduction of a new predicate in the language could not make any difference to their meanings. If the logic is classical, this doesn't change if we introduce a new predicate, even a new nonsense predicate.

But doesn't bivalence fail in our new language? No. Bivalence for a language L is properly expressed in [BV].

[BV] Every semantically complete sentence in L is either true or false in L.

A semantically complete sentence is one properly formed from components that are themselves semantically complete. A predicate is semantically complete only if its rules of application (positively) determine whether it is true of, false of, or neither true nor false of, any thing or n-tuple to which it is applied. When we introduce a nonsense predicate into a language and incorporate it into the recursive generation of complex expressions, we will get sentences that are not semantically complete. While these will not be true or false, they do not violate [BV] because they don't meet the condition for the claim of bivalence. *Pace* Dummett (1995, p. 211), being neither true or false is not *ipso facto* to have another *semantic* value akin to truth and falsity (Haack 1980). It is to not have any. It does not call for a reevaluation of the semantics of the logical constants that some things don't have semantic values. The planets are neither true nor false, but that does not mean that they are to be assigned some third semantic value—it just marks them as the sort of thing that is not up to having any sort of semantic value. The same point extends to the introduction of vague predicates into a language with a classical logic. Vague predicates are introduced without complete rules for their application. Thus, they are semantically incomplete. The presence of vague predicates in a language therefore does not show that the logic is not classical, and it does not show that [BV] fails.

Apart from these considerations, the impossibility result can be given, as we have done, in a language that by stipulation has a classical logic and precise semantic predicates. Then whether the logic of natural languages is classical is neither here nor there. If certain premises can be shown to lead to a contradiction, it does not make it go away if you turn around and express them in a language without the resources to show that. What if someone argued that there could not in principle be a language with a classical logic? Good luck with that. Better: see §6 on the suggestion that it is impossible in principle for semantic predicates to be precise applied to vague expressions.

What about the suggestion that we need not a new logic for natural languages but a new logic of vagueness, in the spirit of the second sense of 'logic'? Is this what we need to resolve the impossibility result? How could it help? What such a logic would aim to do is simply to model something about the use of vague predicates, which, while not semantically complete, still have a practice associated with them. So it would attempt to model formally some features of that practice. But this is a descriptive project, and it has to accommodate whatever features we think attach to vagueness. If [IND] and [EXT] and [E] are constitutive of vagueness, that is part of the description. Add to this that there are vague terms, and then in light of [T], we get a contradiction. No, a "logic of vagueness" is a red herring—a misunderstanding of the relation of that sort of practice to the real work of understanding a phenomenon we are interested in. The understanding must precede the modeling, or, at least, it must be something that can be provided independently of it, as it provides the target of the modeling practice. We can't make something that is there go away with the right model of it.

Supervaluationism (Fine 1975), fuzzy logics and the idea of degrees of truth (Goguen 1969; Hajek 2009)—all of these are attempts to model aspects of the practice with vague predicates. In that sense they are theories of vagueness, but it is a mistake, if an exciting one, to think that we are discovering a new type of semantics for the language, as if this modeling of an incomplete practice were somehow the semantic framework for the language in the sense in which bivalence was thought to be.

6. A Dilemma

So, in the face of the result of §4, there are really only two choices. Both appear unpalatable. The first is to say that vagueness is *impossible*. This depends upon accepting as constitutive of vagueness not only that the application of vague predicates across their intended range is indeterminate as characterized by [HOV] but also that the positive and negative extremal points are fixed as in the extension and counterextension respectively of the predicate ([EXT]). The second choice is to hold onto [HOV], and reject the view that the extremal points are in the extension and counterextension of the vague predicates. Against the first is that it seems obvious that there are vague terms whose characteristic feature is captured by [HOV]. Against the second is the fact that the extremal points serve as semantic introductions to the terms—surely, if we say that heads with no hair at all on them are by *stipulation* in the extension of 'bald', we have determined beyond question that such heads are in its extension, even if we then fail to go on to give a precise cut off for withdrawing application as hairs are depleted in various configurations.

In this case, it is clear which one we have to choose. The challenge is to show how we can accept its consequences. The choice is clear because contradictions are not true. There *are* vague terms. Vagueness is essentially characterized by [HOV]. If it were characterized as well by [EXT], there would be no vague terms. Therefore, it is not characterized by [EXT], and, moreover, [EXT] is false.

7. How Could Extremal Points Fail to be in the Extension of Vague Predicates?

Suppose we introduce a new term 'spherish' by way of a positive exemplar, a perfect sphere, and a negative exemplar, a square. We then treat it as applying to the earth and the other planets, to bowling balls, balls of yarn, and the like, which are not perfect spheres, and withhold it from a range of other things such as galaxies, footballs, pinecones, tiddlywinks, and so on. That is the extent of what you can glean of the practice and indeed it is the extent of the practice altogether, for we do not have in mind any determinate line between things we use 'spherish' in connection with and things for which we say that it fails-to-apply.

Do you not at least know that 'spherish' *applies* to perfect spheres and *fails-to-apply* to squares? Indisputably you know that 'spherish' is intended to apply to perfect spheres and fail-to-apply to squares. If, therefore, the following principle [INT] (intention determination) is correct, you can infer that it in fact applies to perfect spheres and fails-to-apply to squares:

[INT] If anyone introduces a term intending that it have a certain item or type of item in its extension (counterextension), then the term has that item or items of that type in its extension (counterextension).

This is, no doubt, what people assume when they assume that extremal points must lie in the extension of vague predicates, for if this principle is true, it is also true that if a community's practices express the (shared) meaning intention of its members that an item fall in the extension of a predicate, then the item does so. Surely we are the masters of what goes in the extensions of our predicates!

However, [INT] is false. Putting things in a term's extension is not like putting apples in a basket. You have to secure that there is a basket in the first place. Suppose we introduce a term with two stipulations, that it is to apply to A and that it is to apply only to things that are F and not-G. Suppose it turns out that A is G. Does the predicate apply to A? The meaning intentions with which it is introduced require it to apply to A and require it not to apply to A. These intentions cannot be jointly satisfied. Therefore, our plan to introduce a predicate with a meaning that determines an extension for it by way of these specific rules cannot be carried out. In this case, we have not introduced a coherent set of rules for the use of the predicate. The predicate does not have a meaning, and it does not have an extension. We failed to fit it out to play the proper role of a predicate in a language. We might still have induced a practice with the term, for it might be a while before anyone notices that A is G, and so realizes that the intentions with which it was introduced cannot be carried out. But this just shows that a practice with a term may fail to be a meaning-inducing practice. (Plausibly this is what goes wrong with the semantic paradoxes—we impose conditions on the meaning of a predicate that, surprisingly, cannot all be satisfied, as is shown by our ability to derive a contradiction by following the rules we have introduced.)

We could try to repair [INT] by inserting 'coherently' in front of 'intending'. But a predicate can fail to be outfitted to play its role not only by its being introduced with intentions for its use which cannot be satisfied, but by its being introduced with intentions for its use which fail to determine an extension. For example, we may introduce a term indicating its extension by description: 'furgle' applies to whatever *chortelt* does in German. But *chortelt* is a complete nonsense word in German (introduced to translate 'chortle' in Lewis Carroll's *Jabberwocky* into German—unlike 'chortle' in English, 'chortelt' never came to have a meaning in German). It doesn't have an extension in German, so our intention that 'furgle' have the extension of *chortelt* in German fails to secure that it in turn has any extension. But suppose that we introduce 'furgle' with the intention that it apply to A and that its extension otherwise be the extension of *chortelt* in German. Does it not apply to A? But our intention was not just to introduce a term that has A in its extension. It was to introduce a term that had A and the members of the extension of *chortelt* in *its* extension. That was our extension introducing intention, and it fails because it presupposes falsely that *chortelt* has an extension in German. So in this case too we have failed to introduce a term with an extension even though it is perfectly clear that we intend that 'furgle' apply to A.

What these examples bring out is that in introducing a predicate with a meaning into the language has success conditions beyond intending to do so. Among the success conditions are that the rules given for a predicate's use, or for determining its rules of use, fix an extension for it. An extension is a set of things that the term applies to. In addition, we aim to specify a counter-extension, a set of things to which it fails-to-apply. If the complement of the extension is the counterextension, then the predicate divides things into those to which it applies and those to which it fails-to-apply. But minimally, it must determine an extension and a counter-extension.

Apply this to the case of 'spherish'. It is introduced with the intention that it apply to perfect spheres and that it not apply to squares, and certain other things. But in fact rules for determining an extension have not been given. The meaning introducing intention does not specify a plan for its use that suffices to fix an extension, and, hence, it does not suffice to fix a meaning (or a complete meaning, if you like). Still, it can induce a practice in the use of the expression. And we can, if we like, pretend that, or act as if, the practice has been filled in, because we have an idea about where safe areas of operation are because we know at least roughly what the intentions for use of the term leave unspecified. But the fact is that no extension has been determined, despite the intention that certain items clearly fall into the extension. But if the predicate has no extension (not even the empty set), there is no question of its applying, or failing-to-apply, to anything.

'Spherish' is a vague term. It is an artificial one. We introduced it solely for the purpose of illustration, but it has all the hallmarks of a vague term. We failed to give it an extension. Therefore, 'Spherish' does not apply to perfect spheres, though it is intended to. The same thing goes for natural language vague terms. They don't have extensions because the intentional practice they are embedded in is not up to the standards required. No extension, no application. In the case of a term like 'bald', while positive extremal cases are intended to fall in the extension, and a dimension of relevant variation gestured towards, the practice is silent on where to draw a line. This is what gives rise to [HOV]. This deprives the term of an extension. This is what the theorem [T] shows. A consequence is that despite the clear meaning intention that the extremal points fall into the extension, they don't, because the term has no extension.

This point obviously extends to the introduction of a predicate 'bald*' that by stipulation means 'is a man with no hair on his head and anyone who is bald' (supposing for the sake of argument that the first conjunct itself contains no vague terms). For this fails to secure an extension for the predicate 'bald*' and so it fails to apply even to any individual who has no hair on his head, though it is clear that the intent is that it does.

8. Are semantic terms precise?

From what has been said, it is clear that the assumptions of the argument for theorem [T] have as a consequence that 'bald' applies to anything only if it has an extension. And it can be seen that this is a consequence of assuming that the semantic predicates are precise [SP], for if 'applies' is precise, then any predicate stands in that relation to an object or tuple only if it has an extension. But are not semantic terms just as vague as the predicates to which we apply them? Is this not the problematic assumption of the argument?

Of course, one cannot reasonably object to [T] on the grounds that its semantic terms are imprecise without having any reason to think that that is so. Indeed, it would have to be a pretty powerful reason, for we simply stipulated that we were using precise semantic predicates. Any objection would have to maintain that this stipulation must in principle fail.

However, there is an argument that goes back to Russell's seminal 1923 paper on vagueness for the vagueness of semantic terms in *any* language which itself contains (any other) vague terms to which they are applied. (This seems to be what Braun and Sider have in mind in their rejection of our view in (2007, p. 154, n. 22)—we will show in §10 that this throws their own view into confusion given their official account of when one can assert claims in a vague language. See also their unargued assertion that 'expresses' is vague (p. 150)).

What is Russell's argument?

> Now 'true' and 'false' can only have a precise meaning when the symbols employed—words, perceptions, images, or what not—are themselves precise. We have seen that, in practice, this is not the case. (p. 64)
>
> ... it is possible to discover prehistoric specimens concerning which there is not, even in theory, a definite answer to the question, 'Is this a man?' As applied to such specimens, the proposition 'this is a man' is neither definitely true nor definitely false. Since all non-logical words have this kind of vagueness, it follows that the conceptions of truth and falsehood, as applied to propositions composed of or containing non-logical words, are themselves more or less vague. (p. 65)

Suppressing relativization to a language, the argument intended here seems to be the following (Ludwig and Ray, 2002, p. 455, n. 24).

> (i) There is a range of objects m_1, ... m_j such that it becomes more and more difficult to answer the question 'Is m_i a man?' as one moves from m_1 to m_j, (though the fault lies not in any failure in our knowledge of how the relevant portions of the world are or the meanings of the terms in the sentence—assume this qualification repeated below).
> (ii) To the degree that it is difficult to answer the question 'Is m_i a man?', it is to the same degree difficult to say whether 'm_i is a man' is true (or false).
> (iii) Therefore, there is a series of sentences, 'm_1 is a man', 'm_2 is a man', .. 'm_j is a man', to which the application of 'is true' is doubtful in the same way and to the same degree as is the application of 'is a man' to m_1 ... m_j. [(i), (ii)]
> (iv) Therefore, 'is true' ('is false') is vague in the same way and to the same degree as 'is a man'. [(iii)]

The trouble lies with premise (ii), which not true on any reading that will make the argument go through. Our semantic predicates are precise (by stipulation), and there are therefore certain conditions for their application. They apply or fail-to-apply to expressions of the right category which are fully meaningful. Vague predicates are semantically incomplete. They therefore do not come up to the standards required for our semantic predicates to apply or fail-to-apply to them. The same point holds for sentences containing vague predicates and their evaluations as true or false. It is therefore perfectly determinate whether 'this is a man' is true, false or neither true nor false: it is neither true nor false. Similarly for all other vague sentences, whether problematic or not. Thus, reading (ii) as meaning that the rules governing our semantic predicates don't give us guidance about whether 'm_i is a man' is true renders *it* false. This shows, incidentally, that the argument can give no reason to think that natural language semantic predicates are not precise either.

Why then do people hesitate over whether to say a sentence 'm_i is a man' is true or not when 'm_i' picks out a problematic case. For it is clearly this that Russell is taking as the evidence that the semantic predicates in a language with vague terms are vague. Answer: we ordinarily operate under the pretense that our terms are semantically complete, and since we are not unaware of where our practices give out, this usually gives us no trouble. If we deliberately turn our attention to a clearly problematic case for a predicate, but continue to act under the pretense that the predicate is fully meaningful (by using it rather than mentioning it, e.g.), we can ask whether the sentence we use is true or false, expecting some guidance from our practice. Our practice gives us no guidance, and so it seems to us as if we cannot answer the question. We are freed from this appearance of indeterminacy once we give up the pretense that the predicates in question are semantically complete.

Wait a minute! Aren't we just *begging the question* here in rejecting premise (ii) of the argument? But no, we are not. As we said, one cannot reasonably object to [T] on the grounds that its semantic terms are imprecise without having any reason to think that that is so (especially given the stipulation). The argument was supposed to give us a reason to think that the semantic predicates could not in principle be precise in application to vague terms. But on the assumption that the semantic terms are precise, the argument is unsound. Therefore, *it* cannot give us a *non-question begging* reason to accept its conclusion.

But still, might not, at least, natural language semantic terms be vague (for all you have said)? And shouldn't the argument be couched in terms of natural language semantic predicates? *What would make them vague?* Truth and falsity, applying and failing-to-apply, are outfitted for sentences expressing propositions, and predicates expressing concepts. For such objects, there is no imprecision in their application: if *s* expresses the proposition that *p* in L, and it is the case that *p*, then *s* is true in L; if *s* expresses the proposition that *p* in L, and it is not the case that *p*, then *s* is false in L. If *s* does not express any proposition in L, then it is neither true nor false in L. If *s* partially expresses a proposition in L, whatever that might be thought to come to, it does not express a proposition in L, and, hence, is neither true nor false in L. If *s* expresses, taken in one way, the proposition that p, and in another, the proposition that *q* in L, then relative to the one way it is true iff *p* and relative to the other it is true iff *q*, etc. That is how our ordinary semantic concepts operate. We have no trouble applying them to precise sentences. The problem with vague sentences lies in the vague predicates in them and not in the semantic predicates we apply to them.

But what about the relational verb "expresses" itself? Isn't that vague for any natural language? "We must … admit that the sentential truth predicate is vague, for a sentence is true (in English) iff the proposition it expresses (in English) is (propositionally) true, and 'expresses (in English)' is clearly vague" (Braun and Sider 2007, p. 139). And if that is true, isn't it also true that whether 'is true' applies to a sentence of English, e.g., is also vague? Is ‘expresses’ vague? Suppose for the moment that ‘English’ picks out a determinate recursive syntax and set of practices for the use of the expressions generated by it. Is it vague whether ‘John is bald’ expresses a proposition? No. No sentences that use vague terms express propositions! (Even Braun and Sider are committed to this.) That is why they are neither true nor false. There is nothing vague about it. But perhaps the problem lies with what language we are relativizing ‘expresses’ to, for, of course, ‘English’ intended as a designation of a language spoken by a linguistic community is vague. Thus, what set of practices we might relativize the semantics of expressions to has not been determined. But this doesn’t make ‘expresses’ vague. And even if ‘expresses’ were vague, that would not make ‘true’ and ‘false’ vague.

The sole reason to think otherwise comes from the kind of argument Russell originally sketched, an argument that commits what we might call the inverse fallacy of verbalism. The fallacy of verbalism, as Russell called it (1923, p. 147), consists in mistaking the properties of expressions for properties of things we apply them to. Russell described people who thought there were vague objects as committing this fallacy. The inverse fallacy of verbalism is mistaking the properties of things for the properties of the words we apply to them. In the argument above, this consists in taking the semantic indeterminacy of vague terms to apply *ipso facto* to terms which we apply to them. Absurd!

In any case, there is no barrier to introducing precise semantic terminology, and if there is any truth about vagueness to be had, it can and should be stated in a precise language. We do not want to couch our theoretical arguments in a vague language if we can avoid it! But the truth about vagueness in a precise language is that vague predicates neither apply nor fail-to-apply to anything. Thus, the only assumptions we need for the argument is that there can be precise semantic predicates and a bivalent classical language with terms that refer to vague expressions.

There remains the question of relativization of vague terms to a practice. To see that this is a red herring, consider a particular token use of ‘bald’ in your community. Run the argument on it. Generalize to every vague use of ‘bald’.

It can’t be emphasized enough that the knee-jerk reaction that our argument for [T] fails because the semantic predicates we used there must be vague is *completely bootless.*

9. Truth, Vagueness, and the Sorites

An upshot of the conclusion that vague predicates neither apply nor fail-to-apply to anything is that no sentence in which a vague predicate is used, as opposed to purely mentioned, is either true or false. For consider any sentence, S(v), containing a vague term v in a use position. This implies that in evaluating it for truth, at some point we must evaluate whether or not v applies or fails-to-apply to something. But being vague, v does not apply or fail-to-apply to anything, and so the evaluation of the sentence for truth must stop at that point. No truth-value can be assigned to the sentence. It is neither true nor false.

Thus, we have a simple, clear, true account of vagueness and what has gone wrong with the sorites arguments. Vague predicates are the offspring of an incomplete practice that does not fit them out for the semantic framework presupposed by the categories into which we intend to insert them. Presuppositions for their use with semantic vocabulary are not met. They neither apply nor fail-to-apply to anything. No sentences in which they appear are true or false. Consequently, the premises of the Sorites argument, as well as its conclusion, are neither true nor false. We may say the argument form is valid in the sense that any argument of that form is truth-preserving. But Sorites Arguments do not have true premises. Nor is the fault that one of the premises is false, for that too presupposes what is false, namely, that the practice is up to the presuppositions of the semantic framework.

The puzzle arises from our looking at it through two lenses at the same time. First, we look at the argument through the lens of everyday practice in which we idealize away from the vagueness of our terms to pursue the practical business of life, in which we adroitly avoid the use of terms where the practice fails us dramatically. From this standpoint, it seems okay to accept the first premise and reject the conclusion, and to take the argument as a material mode expression of the facts about the distribution of hair on heads. Second, though, we look at the puzzle from the standpoint of reflection on the inadequacies of our practice, which gives no guidance with respect to where to draw the line between applying 'bald' and withholding it, while clearly intending that decreasing/increasing head coverage by hair be relevant. Properly, what we recognize here should be expressed in the formal mode. With one eye still on everyday practice, it gets expressed instead as in the step premise (2), for semantic descent is okay relative to the idealization we bring to bear on accepting the first premise and rejecting the conclusion. The result is a massive confusion of levels of thought. This is precisely the mistake Peter Unger made in (1979a; 1979b). See (Ludwig & Ray, 2002, p. 443-4). If we carefully separate the theoretic position from the everyday practice, and stick to just mentioning the target predicates, what we get is the theorem [T], and then a satisfying diagnosis of the Sorites Paradox.

10. Vagueness and Ordinary Practice

It would be nice to stop there. Unfortunately, semantic nihilism seems to leave us with another puzzle, which is how to make sense of ordinary practice with vague terms. There are two faces to this puzzle. First, there is the task of explaining how we manage to communicate with terms which don't in fact apply or fail-to-apply to anything and when it is safe to ignore vagueness. Second, there is the task of making sense of the standpoint from which we theorize about this, since that standpoint itself inevitably must employ a language most of whose terms are themselves vague. Thus, it would seem that most of what we try to say about it will on the account itself not be true or false, just as much of what we have already said about it, if not all, must be counted as failing to be true or even false.

Faced with this, one might well start looking about for something else to say, to look for a lifeline from contextualism, which holds that there are rules that determine for any context a precise cutoff point, or from epistemicism, which holds that there are precise cutoffs but our knowledge of them is in principle inexact. These are different ways of denying that there is any semantic vagueness. But what are such theorists really thinking? They are making *empirical* claims about natural languages: that their semantics are precise for all terms. Surely that is massively implausible! And do they advance *empirical* arguments for their claims? No. The arguments are essentially predicated on there being unpalatable consequences of accepting semantic vagueness. This is like arguing that we have libertarian free will because the consequences of denying it are *too awful to contemplate*. But it is obvious that new terms enter natural languages all the time without there being any precise rules for their application across the entire intended range. We introduced 'spherish' above. Suppose it becomes widely used. The contextualist and epistemicist are each in their own way committed to denying that it is semantically vague. This is simply incredible. The right response, the only sensible response, is to deal with natural languages as they are, not try to make them out to be something else.

And so to business. Our first task is to explain how we communicate with words that neither apply nor fail-to-apply to anything. What we need to do is to describe the conditions by which we get on in the ordinary practice of using vague terms in a way that does not get us into trouble and which enables us to convey information. In earlier work, we invoked the term 'appropriate assertibility'. It is appropriate to assert '*x* is bald' of someone with no hair on his head. It is not appropriate to assert '*x* is bald' of someone in the middle of our sorites series $b_0 \ldots b_k$. So far so good. But what is the cut off? In the nature of the case, there is no general answer to this question. In earlier work, we said this (2002, p. 445):

> A sentence containing a vague predicate is appropriately assertible provided that the expectations that its sincere assertion, taken nonfiguratively, would thereby standardly generate, on the assumption that the speaker is using the terms in it in accordance with the practice, will not be frustrated.

This will make it highly pragmatic, but this seems, in fact, exactly right. We manage to communicate because there is a practice associated with vague terms that generates expectations (which can vary with context), which typically will have the shape of a probability profile over the range of application, and we avoid using vague expressions in sentences in contexts in which we think the expectations generated will be misleading. The expectations are set up in part by our conception of when a use is well within the standard practice, and that is in turn characterized in part by when it is not safe to use terms in application to objects or a range of objects because the practice doesn't give us much guidance in how to use them.

Can we do better? Braun and Sider (2007), who accept semantic nihilism, have introduced a theory of ignoring which is supposed to provide a more precise picture of when we can and cannot appropriately assert sentences containing vague predicates. They cite it as an improvement on what we said. The theory of ignoring makes use of a structure similar to that of supervaluation theories. We'll show that it does not give appropriate guidance, but suggest a refinement of the idea that has more promise.

Perhaps it is appropriate to add a note here about an important difference between Braun and Sider's take on vagueness and ours. They assimilate vagueness (following Lewis (1982)) to a species of ambiguity on the grounds that there are many candidate meanings for vague expressions just as there are multiple candidate meanings for ambiguous expressions. Thus, their reason for saying that vague sentences are not true is that ambiguous sentences quite generally are not true. 'John is bald' is not true because 'is bald' is ambiguous just as 'John went to the bank' is not true because 'bank' is ambiguous (ignoring its own vagueness here). To assign truth to a sentence, you have to first disambiguate it if it is ambiguous. But vagueness is not ambiguity! A term is ambiguous if it has more than one meaning associated with it in the language. The trouble with vague terms is not that they have many meanings associated with them (legitimate disambiguations, as Braun and Sider put it), but that they have, strictly speaking, none. In any case, if this were ambiguity, actual utterances of these sentences would after all be truth evaluable in context because speakers would typically use them with one of the meanings they have in the language; but that's not how it is. In the following, we'll ignore Braun and Sider's imputation of ambiguity and assume what they really have in mind is not an overabundance of meanings, but an underabundance, despite the talk of ambiguity and disambiguation in the paper. And we will use 'permissible precisification' in the following where they might use 'legitimate disambiguation'.

Let's turn to Braun and Sider's theory of ignoring vagueness. Begin with supervaluationism. Supervaluation theories introduce the concept of supertruth. A sentence is supertrue iff it is true under all permissible precisfications of it. Some who invoke supertruth go on to say that vague sentences that are supertrue are true. We agree with Braun and Sider that this is a mistake. The vague sentence is vague, hence, not true, though a precisification of it can be true because it is meets the presupposition of the application of truth to it.

The idea that Braun and Sider have is to use supertruth as a guide to appropriate assertibility, or, in their terms, as a guide to when you can afford to ignore vagueness. There seems to be something to this idea because the idea of a permissible precisification is intended to capture constraints laid down by the practice, a kind of schema for filling in what is left out. A precisification is not permissible if it violates something that the practice clearly lays down—like the treatment of extremal points.

Braun and Sider put their idea initially like this:

> On our view, ordinary speakers typically and harmlessly ignore vagueness. And when doing so, it is reasonable to speak, in a sense to be defined, the *approximate truth*.
>
> When all the legitimate disambiguations of a sentence are true, call that sentence *approximately true*. (p. 135)

Here the notion of a legitimate disambiguation plays the same role as the notion of a precisification in supervaluationism. Of course, "permissible precisification" is itself vague.

If it were not, [HOV] would be false. This is a place where the theory itself employs vague expressions. Let us put this aside for now, however. Later they say explicitly:

> … the concept of approximate truth … is essentially the concept of supertruth. (p. 146)

The rough idea is that we often ignore vagueness in practice and when we do so, approximate truths (as well as truths) are appropriately assertible. The suggestion looks promising initially. Thus, for example, 'If John is bald, then someone with one hair less than John is bald' comes out as true on all permissible precisifications of 'is bald', and so, being supertrue, it is safe to ignore vagueness with respect to it, and it is, thereby, approximately true. Similarly, we might say that it is safe to ignore vagueness in 'John is bald' provided that John has no hair on his head, since 'John is bald' is supertrue. Likewise, if something is superfalse, it is safe to ignore vagueness in asserting its negation. If this works, modulo the worry about 'precisification' being vague, it gives us a systematic account of when it is okay to assert a sentence containing a vague expression. The trouble is that this rough idea does not work, as Braun and Sider well know. Some claims that are approximately true are nonetheless not assertible, (and some claims that are assertible are not approximately true). So, Braun and Sider's view is outfitted with a special escape clause: we often ignore vagueness in practice and when we do, truths and approximate truths are what is appropriately assertible, *unless it is not appropriate to ignore vagueness in that case*. Their strategy for handling the sorts of cases just mentioned, then, is to declare that these trouble cases are cases where it is not proper to ignore vagueness, and hence those mere approximate truths are not appropriate to assert.

But do Braun and Sider offer us any substantial account of when it is proper to ignore vagueness that amounts to more than the advice to do it when it won't get you into trouble and don't do it when it will? That, we submit, would be *less* informative than the account we gave earlier. The problem Braun and Sider are facing is this. They want to turn the idea of supervaluations to a new end—assertibility rather than truth—but they have chosen to simply piggyback on the standard supervaluation mechanism of supertruth *unamended*. As a result, they inherit a classic set of supervaluationist problems in a slightly new guise. Supervaluationists identify supertruth (aka approximate truth) with truth, but, for example, some sentences that come out as supertrue no one would accept as true. So, of course, these same kinds of cases haunt Braun and Sider as things that come out approximately true but are not assertible. For example (these parallel examples Braun and Sider give and we use their labels), (∃) is supertrue, but not appropriately assertible.

(∃) For some n, b_n is bald and b_{n+1} is not bald

Similarly, if b_i is a "borderline case", (∨) is not appropriately assertible,

(∨) b_i is bald or b_i is not bald

but (∨) comes out supertrue. This is a result of the fact that precisifications force sharp borderlines, so any sentences that will come out true on the assumption of sharp borderlines and uniform interpretation count as supertrue. But some of these statements we don't want to assert because to do so is to adopt a commitment with respect to the actual sentences that they are not vague in cases in which ignoring vagueness gets us into trouble.

A consequence of this is that the theory of ignoring vagueness—when it is safe to ignore vagueness in asserting a sentence—cannot be given simply in terms of "approximate truth." What they can say is this: if a sentence is neither supertrue nor superfalse, then it is not safe to ignore vagueness with respect to it. If a sentence is supertrue, then it is appropriately assertible unless it is not appropriate to ignore vagueness in that case. If a sentence is superfalse, then its negation is assertible unless it is not appro priate to ignore vagueness in that case.

What other clues do Braun and Sider gives us for when we can ignore vagueness? Their remarks on this are relatively sparse: you cannot ignore vagueness when it is "hard to ignore" or has been explicitly put forward (p. 138); when sentences "manifestly concern borderline cases" or "draw attention to their vague nature" (p. 146).

When is it hard to ignore vagueness? If you are thinking about vagueness, it is hard to ignore, but it doesn't seem to be a problem if one is talking about practical matters, or even talking about vagueness using admittedly vague terms, as Braun and Sider do. Why can we not ignore vagueness in the case of borderline cases? Because it gets us into trouble. And it gets us into trouble because the practice gives us little guidance in these cases and so to use 'is bald' with respect to a midway case will set up expectations in our audience that we know will be frustrated. What about sentences that "draw attention to their vague nature"? The same thing goes, given the examples that Braun and Sider provide, namely, sentences like (∃) and (∨). What is meant by 'draws attention to their vague nature' is that we recognize that our pretense that terms are not vague gets us into trouble in these cases. To sum up, we can safely ignore vagueness when it doesn't get us into trouble! This advice seems appropriate, but doesn't amount to a theory.

In §11, we will show how the mechanism of supertruth can be used to better model appropriate assertibility. This improved account would help Braun and Sider avoid the trouble cases they inherit from supervaluationism. And insofar as we are successful, it also obviates their (unsatisfying) treatment of these cases by appeal to a loose notion of the propriety/impropriety of ignoring.

There is a final difficulty for Braun and Sider, given (i) their view that something is appropriately assertible only if it is approximately true, and, hence, only if it is supertrue, even if supertruth is not sufficient for appropriate assertibility, and (ii) their denial that semantic predicates applied to vague expressions are precise. It is their view that, e.g., [BNT] is appropriately assertible. It expresses (approximately) the application of their central thesis to a particular vague sentence.

[BNT] 'A man with no hair on his head is bald' is not true.

[BNT] is not true, of course, on the account itself, if 'true' is vague, but they are committed to its being approximately true, at least. But if 'true' is vague, [BNT] *is not approximately true, but instead it is superfalse*, and, hence, only its negation could be appropriately assertible *on their own view.* For, each precisification of the language that Braun & Sider can admit will precisify 'true' and 'bald' both, but then each will make the sentence 'A man with no hair on his head is bald' true, and, hence, relative to the correlatively precisified truth predicate, [BNT] will be false on every precisification. Thus, on their own view, it is not appropriately assertible. Worse, since [BNT] is superfalse, its negation is supertrue. Hence, not only is [BNT] not assertible, if supertruth is taken as a guide to appropriate assertibility, its negatum is appropriately assertible.

[BT] 'A man with no hair on his head is bald' is true.

But this is what their theory denies. Of course, Braun and Sider can beg off asserting the negation of [BNT] because it gets them into trouble! But they can't get [BNT] to come out assertible on their own account, even in the context of their theorizing about vagueness. Nor can they get the right result *by precisifying 'true' without precisifying 'bald'* (perhaps taking the mentioned sentence to be in an object language relative to the truth predicate), because they are committed to the impossibility of precise semantic predicates in application to vague sentences (p. 154, n. 22). This is not a difficulty for our view, for on our view [BNT] is not a vague sentence at all, and we have shown that 'true' does not apply to any vague sentences. The treatment of appropriate assertibility that we propose in §11 will not alleviate this problem for Braun and Sider. This problem can only be solved by accepting that semantic terms for a vague language need not themselves be vague (cf. §8). As a definite bonus, they would also then be able (must) accept theorem (T)—which is the foundation of the strongest argument for that core view which the four of us share, semantic nihilism.

11. A New Proposal: Univocal Supertruth Models Appropriate Assertibility

Still, as we noted, there seems to be something right in thinking that if we can make use of the notion of permissible precisifications, we can make at least a little headway on saying when vague sentences are appropriately assertible. The mistake was to try to let the notion of supertruth alone do the job. We need instead a refinement of the idea.

Supertruth applied to safe cases or to sentences witnessed by safe cases gets the right result. For example, it gets the right result for 'That is red or it is not red' asserted of a clear case, and likewise for 'A man with no hair is bald', and 'If a man is bald, a man with one less hair is bald'. Asserting these is all safe, and well within the practice. What goes wrong in the case of sentences like ($\exists$) and ($\vee$) is that these come out as supertrue not because they are about (relatively) safe cases, that is, not because of anything to do with how the practice constrains the set of permissible precisifications, but merely because every precisification yields precise predicates. What we want, then, is to weed out sentences that are supertrue independently of facts about the actual practice with a term that constrains the set of permissible precisifications.

It is clear that supertruth gives the right results when applied to atomic sentences. For these will turn out to be supertrue only when all terms in them refer to safe cases for their predicates. Here supertruth relies only on the guidance that the practice gives. Our suggestion will be to start with the assignment of supertruth to atomic sentences, and then characterize as univocally supertrue molecular and quantified sentences whose truth conditions can be cashed out in terms of supertruth (or superfalsity, in the case of one use of negation—see below) for atomic sentences. In this way, we characterize which sentences can be used safely, and eliminate the problem cases that result from the fact that plain supertruth applies to sentences directly, rather than being calculated in a recursive manner.

Before we develop our proposal, we note that the force of negation in ordinary speech can be taken in two ways. First, it can be taken as conveying that the negated sentence is not safe to assert: and this can be either because it is simply false (or superfalse) or because it lies outside the area of safe use. Second it can be taken as conveying that the negated sentence is not safe to assert specifically because it is false (or superfalse). We call negation taken with the first sort of force *weak negation* and the second *strong negation*. Weak negation is typically intended when we are asserting negations of non-atomic sentences and strong negation when we assert negations of atomic sentences. But, as we will show below, there are exceptions. In consequence, when we develop our proposal, we will give separate clauses for weak and strong negation, and the application to ordinary language sentences will depend on how the force of negations are to be interpreted in context.

With this as background, we propose:

[A] A vague sentence is appropriately assertible iff it is univocally supertrue.

The set of permissible precisifications M of L is a set of classical models. All the models of that set have the same domain of discourse D and agree on the reference assignments of all names. Univocal supertruth is then defined as follows:

[UST] A sentence S is *univocally supertrue* in L iff$_{df}$ for the set M of permissible precisifications of L, S is univocally supertrue relative to M.

To define the right hand side, we introduce some terminology.

M* is a β-variant of M for S iff$_{df}$ for some $x \in$ D, M* is the result of reassigning the first fresh constant β not in S to x in every model $m \in$ M.

S[α/β] = the result of substituting β for each free appearance of α in S (if any).

We now define univocal supertruth relative to a set of models M for L as follows.

[USTM] S *is univocally supertrue relative to* a set of models M of L [i.e., S is *ustrue in* M] iff_{df}

(a) if S is atomic, then S is true in every model $m \in$ M (i.e., S is supertrue with respect to M);
(b) if S is the conjunction of S_1 and S_2, then S_1 is ustrue in M and S_2 is ustrue in M;
(c) if S is the disjunction of S_1 and S_2, then S_1 is ustrue in M or S_2 is ustrue in M;
(d) if S is the conditionalization of S_2 on S_1, then if S_1 is ustrue in M, then S_2 is ustrue in M
(e) if S the weak negation of S_1, then it is not the case that S_1 is ustrue in M;
(f) if S is the strong negation of S_1, then S_1 is superfalse in M.
(g) if S is the universal quantification of S_1 with respect to a variable α, then for all β-variants M* of M for S_1, $S_1[\alpha/\beta]$ is ustrue in M;
(h) if S is the existential quantification of S_1 with respect a variable α, then for some β-variant M* of M for S_1, $S_1[\alpha/\beta]$ is ustrue in M*.

We will assume in the following discussion, unless otherwise noted, that the negations of atomic sentences employ strong negation and those of molecular and quantified sentences employ weak negation. This typically gets the right results, and so serves as a good rule of thumb for figuring out when a vague sentence is assertible.

Consider [USTM] first in application to ($\vee$), repeated here.

($\vee$) b_i is bald or b_i is not bald

b_i is a borderline case but ($\vee$) comes out supertrue because every precisification of it puts b_i either under 'bald' or 'not bald'. But ($\vee$) is not univocally supertrue because that would require one of the disjuncts to be supertrue, and neither is. This correlates, as desired, with the non-assertibility of ($\vee$). The same result obtains for any borderline case between any two predicates. Now consider the application to

($\exists$) For some n, b_n is bald and b_{n+1} is not bald

($\exists$) is supertrue because every precisification draws a precise borderline. But ($\exists$) is not univocally supertrue because *there is no choice of n* relative to which 'b_n is bald and b_{n+1} is not bald' is univocally supertrue, which is what ($\exists$)'s *univocal* supertruth would require. This correctly correlates with the non-assertibility of ($\exists$). Now consider

There is a set of all and only bald men.

This is supertrue, but it is not appropriately assertible. Is it univocally supertrue? No, because there is no set that will "supersatisfy" '*x* is a set of all and only bald men'. For no matter what is in a given set, on some precisification either something in it is not in the extension of 'bald' or something not in it is.

Now consider a case involving the idea of permissible precisifications itself, when a term for that is a predicate in the object language, and where 'red' is in *its* object language.

There is a set of all and only the permissible precisifications of 'red'.

This is supertrue, but not appropriately assertible! But happily it is not univocally supertrue. For there is no set which is in all the permissible precisifications of 'permissible precisification' in the object language. Now suppose 'red' is in the very language of the sentence itself. Then when we precisify the language, precisifying 'red' at the same time, it turns out that

There is but one permissible precisification of 'red'.

is true no matter the precisification, and so supertrue! But is certainly not appropriately assertible! Accordingly, though, it is not *univocally* supertrue. For it to be univocally supertrue there has to be a model *m* that satisfies 'is the only permissible precisification of "red"' on every precisification of that, and there is no such model. We also get the right result for

There is a permissible precisification of 'red',

of course, because there is a model *m* (one can think of it as the most conservative one—an extremal case for 'permissible precisification of "red"') that satisfies 'is a permissible precisification of "red"'. The successful application of the account to a language containing the theoretical apparatus used for defining univocal super truth is especially gratifying.

It is easy to verify that our proposal makes appropriately assertible various sentences we want to count.

0 is a small number.
A man with zero hairs on his head is bald.
If a man is bald, then a man with one less hair is bald.
John is bald or not bald. [where John is *not* a borderline case]

What about a disjunction in which one disjunct is about a borderline case and another about a safe case?

Mr. Nohair is bald or Mr. Borderline is bald

This is univocally supertrue. It seems to us that it is also appropriately assertible, and so we believe that univocal supertruth gives the right result for this case. But it would be easy to modify the account if one thought that it should not count as appropriate assertible by requiring for the univocal supertruth of disjunctions that each disjunct be univocally supertrue, or one be univocally supertrue and the other univocally superfalse.

It is clear that on this account, just as we want, logically equivalent sentences are not generally equally assertible. A sentence, e.g., 'Mr. Borderline is bald or Mr. Borderline is not bald', may not be assertible, so that its negation is assertible when interpreted weakly, 'It is not the case that: Mr. Borderline is bald or Mr. Borderline is not bald', though the logically equivalent 'Mr. Borderline is bald and Mr. Borderline is not bald' is not assertible. This is why we treat conditionals separately. 'If Mr. Borderline is bald, then Mr. Borderline is bald' is assertible, but the logically equivalent 'Mr. Borderline is not bald or Mr. Borderline is bald' is not. It might be objected to the treatment of conditionals that any conditional whose antecedent is not univocally supertrue will be true, e.g., any conditional of the form

If Mr. Borderline is bald, then p.

For example: If Mr. Borderline is bald, then I am bald. But this can be handled by an extension of your favorite story about the oddness of asserting a conditional on the basis of its having a false antecedent. Ours is that the oddness arises from a generalized conversational implicature that the conditional is true (or assertible) on non-truth functional grounds.

The point about logical equivalence extends to definitional equivalence more generally. Suppose we define '*x* is schmald' as '*x* is bald or *x* is not bald'. Let '*b*' be a borderline case of baldness. Then neither '*b* is schmald' nor '*b* is bald or *b* is not bald' are assertible (the former is not even supertrue, while the latter, though supertrue, is not univocally supertrue). But while 'it is not the case that *b* is schmald' is not assertible either (strong negation), in contrast 'It is not the case that: *b* is bald or *b* is not bald' is assertible (weak negation). This is the result we want.

What about this case (proposed by Vann McGee): 'Mr. B is bald or nearly bald', where Mr. B is close to, but on the far side of, the borderline of applicability for 'bald' (i.e., in the borderlands but near the border). Then it seems assertible. Is it univocally supertrue? Why not? Plausibly the effect of modifying 'bald' with 'nearly' is to produce a complex predicate whose application borderline is closer to the middle of the borderline range for 'bald'—speaking under an idealization, of course—which would allow the second disjunct for some borderline cases for 'bald' to be univocally supertrue while the first disjunct is not.

Let's return to the two forces that negation can have. Consider the negation of an atomic sentence: 'Mr. Borderline is not bald'. What do we intend to convey by this? In standard cases, to assert this would imply that Mr. Borderline was on the far side of the borderlands, that is, definitely in the complement of 'is bald'. Thus, we standardly interpret the negation of an atomic sentence as strong negation. Suppose, however, that someone asserts, 'Mr. Borderline is bald,' and someone else, apprised of the fact that Mr. Borderline is a borderline case, denies this by saying 'No, he isn't' or 'No, he isn't bald'. In this case, it is clear that weak negation is intended. There are also cases of negations of molecular sentences which we will be inclined to interpret as strong negation, e.g., 'It is not the case that Mr. Borderline is bald and Mr. Borderline is bald'. This is an odd entry into a conversation, of course, but it would usually be taken to be a roundabout way of saying what one could say by saying 'It is not the case that Mr. Borderline is bald', and if the context is one in which that negation would be interpreted as strong, then so will the negation on the conjunction. If invariably the negation of an atomic sentence were interpreted as strong negation and the negation of a non-atomic sentence were interpreted as weak negation, then we could revise clauses (e) and (f) so that the first was restricted to non-atomics and the second to atomics. However, given that there is no syntactic criterion for interpreting the intended force of negation, we must instead disambiguate in context and then apply the test for assertibility. This should not be unexpected given that we are interested in tracking a pragmatic response to the implicit knowledge of the incomplete practice associated with most of our words.

Finally, return to [BNT] and [BT], repeated here.

[BNT] 'A man with no hair on his head is bald' is not true.
[BT] 'A man with no hair on his head is bald' is true.

On our view, [BNT] is true, and its negation, [BT] is false. That's fine. Neither sentence is vague. But doesn't this leave us with a puzzle about ordinary practice? For [BNT] does not seem to be appropriately assertible, while [BT] does seem to be appropriately assertible, in everyday life, outside the context of theorizing. Even we would do it. But we have just given an account of when a *vague* sentence is appropriately assertible. We did not give, of course, a complete account of when some sentence in the ordinary practice is appropriately assertible. In theorizing, we assert [BNT] because it is true; we do not assert [BT] because it is false. But in the ordinary practice, where we operate in safe areas as if our terms were precise, it is clear that we want 'S is true' to be appropriately assertible just when S is appropriately assertible, that is to say, we allow semantic ascent for appropriately assertible vague sentences when we are operating under the idealization that they are precise and we are playing it safe. (On this last point, we take it that this same understanding is part of what underlies Braun & Sider's treatment of a closely related question (pp. 149-52).)

There is one last meta-issue to take up, namely, that we use vague terms inevitably in giving the account itself, and, in particular, 'permissible precisification' is used under an idealization that it is precise. Does this not undercut the whole enterprise? Not if we are right—in what we intend to convey. You do see, we maintain, what it is that we intend to convey in using what vague terms we did, just as you do in everyday life when someone tells you that his uncle has "gone bald." Given that, you can now apply the account to itself to explain how by giving it we managed to stay safe and convey the gist of it to you. And if you have an objection to *that*—that we can actually convey things to you in that way—we invite you to give it. You will perforce use some vague terms. If you are successful in conveying your objection to us, then it will be a bad objection, since if it were correct, you could not successfully convey it. And if you are not successful, you have not raised an objection. Then we will invite you to continue to try until you are successful, and thereby demonstrate that your objection is a bad objection. In short, if you cannot convey an objection, you don't have a good one, and if you convey an objection, you don't have a good one. Therefore, you do not have a good objection.

12. Conclusion

We have reviewed our result that shows that all vague sentences fail to be evaluable for truth or falsity. We have shown that this result also yields as an easy corollary Kit Fine's impossibility result; for, since Fine takes it to be definitional of vague terms that they are true of their positive extremal points and false of their negative extremal points, it is an immediate result of theorem [T] that vagueness is impossible. But we have shown that Fine's assumption cannot be maintained, and that there is no prospect for resolving the difficulty with a new logic, as he suggests. We have shown that the attraction of taking positive and negative extremal points to fall in the extension and counterextension of vague predicates rests on a mistaken assumption about the relation between having a meaning intention and its success conditions. We have defended the result from the charge that semantic predicates cannot in principle be precise in application to a vague language. We have offered a clear and satisfying diagnosis of what goes wrong in sorites arguments. Finally, we have taken up a suggestion by Braun and Sider for how to characterize when assertoric use of vague sentences is safe, namely, in the initial incarnation, when they are supertrue. This suggestion is inadequate, as Braun and Sider recognize. We have now shown how to refine the idea to give a satisfying account of when vague sentences are appropriately assertible by giving a recursive definition of *univocal supertruth*. There is more to be said, of course, about the view we are promoting—some of which we *have* said and will not repeat here. The interested reader should consult (Ludwig & Ray, 2002) for more information.

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