The Hitchikers guide to Kitaev's Periodic Table

Peadar Coyle

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Abstract

1 Introduction

Kitaev's goal is to classify topological phases, in his paper this is a table. He restricts to gapped systems exhibiting topological entanglement (for example the toric code). Both 2D and 3D systems are time reversal invariant insulators. More specifically, they consist of (almost) **noninteracting** fermions with a **gapped energy spectrum** and have both the time reversal symmetry (T) and a U(1) symmetry (Q). The latter is related to the particle number, which is conserved in insulators but not in superconductors or superfluids[1] There are some classical examples (classical meaning discovered when I was in nappies) of the applications of the first Chern number such as the TKNN invariant.[2] For integer quantum hall systems the invariant ν is related to the index theory and which can be expressed as the trace of a certain infinite operator, which represents the insertion of a magnetic flux quantum at an arbitrary point. [3] [4] His aim is to look for an enumeration of all possible phases.

Definition Two Hamiltonians belong to the same phase if they can be continuously transformed one to the other while maintaining the energy gap or localization.

The identity of a phase can be determined by some local probe[5] The table includes a general classification scheme for gapped free-fermion phases in all dimensions. The (mod 2) and (mod 8) patterns mentioned in the table are

[1] Felx: why is this so?

[2] As I understand this, this is mentioned in Barry Simons paper on the Geometric Phase, which I studied under Mark Dennis at Bristol, I must admit I didn't understand the paper until a year ago - because I didn't have the language of connections, principal bundles. I think the Geometric Phase exists on a Hermitian line bundle and can be explained by the first Chern class, I think Simon makes a reference to some connections - forgive the pun - between the Berry Phase and the TKNN invariant

[3] I think here in the paper, he makes reference to the exceedingly difficult paper by Kitaev on 'Anyons in an exactly solved model and beyond' cond-mat/0506438 - I was recommended by both Steve Simon and Joost Slingerland to read Parsa Bondersons thesis when trying to read Kitaev's language of superselectors, I have not at this moment looked at Kitaev

[4] I looked at Appendix C, it looks tricky and technical although I'm familiar with a lot of the mathematics, he uses a lot of functional analysis and homological algebra and even some K-theory

[5] Is this a reference to some experimental procedure

known as 'Bott Periodicity'; they are part of the mathematical theory called K-Theory. In particular the relation between the homotopy-theoretic and Clifford algebra versions of K-groups is important in this paper. A key idea in K-Theory is that of **stable equivalence**: when comparing two objects, X' and X", it is allowed to augment them by some object Y. The final twis is that K-theory deals with **difference** between objects rather than objects themselves. Thus, we consder one phase relative to another.

We now give exact definitions for d = 0 (meaning that the system is viewed as a single blob). The simplest case is where the particle number is conserved, but there are no other symmetries. A general free-fermion has this form

$$\sum_{jk} X_{jk}\hat{a}_j^\dagger \hat{a}_k$$

where $X = (X_{jk})$ is some Hermitian matrix representing electron hopping. Since we are interested in gapped sys- tems, let us require that the eigenvalues of X are bounded[6] from both sides, e.g., $\Delta \leq |\epsilon j| \leq E_max$. Furthermore he goes onto define homotopy. And describes the condition for when two matrices are homotopic, and then says This family of Hamiltonians is characterized by a nontrivial invariant, the first Chern number.

2 What is Bott Periodicity

[6]Felix: Does it mearly have to be bounded by definition of a gapped system? I'm having trouble translating from Physics to Math and vice versa here