Electrostatics

Aims

- To reinforce some of the basic electrostatic principles using an electrometer
- To provide experimental verification of simple capacitor formulae
- To provide an exercise in careful experimental data acquisition and analysis (the determination of the permittivity of free space)

Objectives

After completing this experiment, you should:

- understand how an electrometer works
- have a practical understanding of the basics of electrostatics, e.g. conservation of charge in an isolated system, how charge distributes itself on a conducting surface and the principle of electrostatic screening
- fully understand the concept of capacitance, with emphasis on the parallel plate capacitor

Notes

It is a requirement of this lab to evidence your key results and observations using photos or screenshot. These should be included in your report.

The tasks required to complete the lab are highlighted in boxes, together with a rough idea of the time it should take you to complete.

The supplementary problems at the end are for reinforcing some of the ideas contained in this experiment. Please try these in your own time. You are not required to answer these as part of your write-up.

0. Introduction

The central piece of equipment used in this experiment is an **electrometer**. This is a high impedance voltage amplifier that measures small amounts of electric charge, inferred from the voltage developed across a fixed internal capacitor when the charge is deposited on it. Try to correlate its features with the schematic diagram of Fig. 1. Referring to Fig. 1, an internal capacitance of 10^{-7} F or 10^{-9} F can be selected in parallel with the input terminals AB by the position of the capacitance link ("C link"); here we will only use the 10^{-7} F internal capacitor.

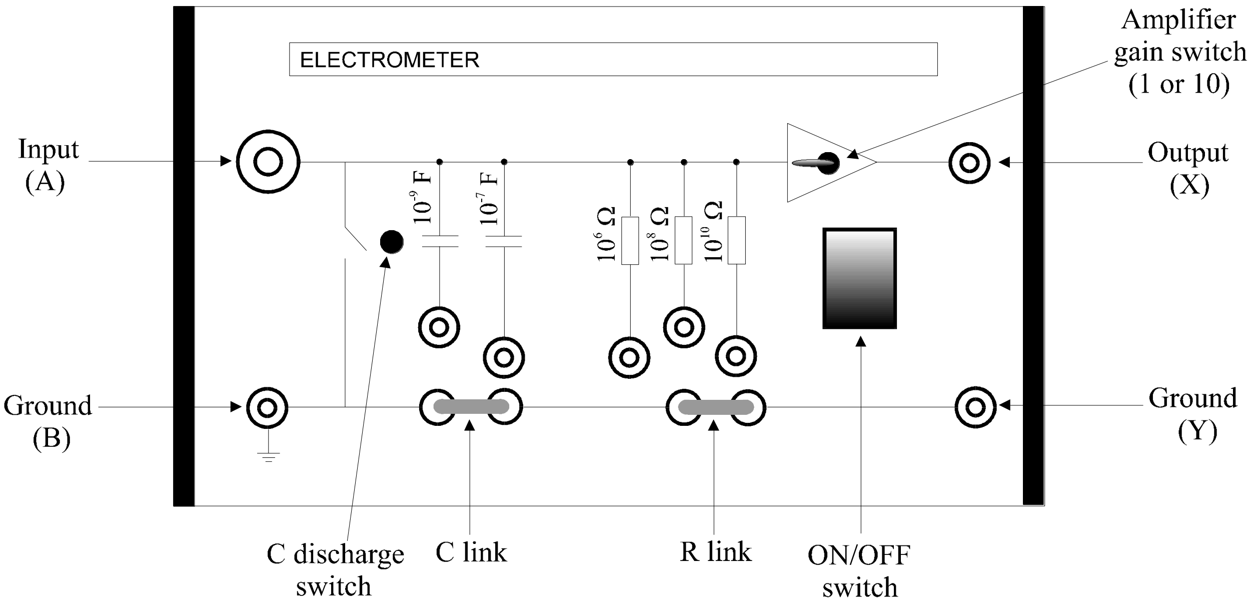


Figure 1: Schematic diagram of the electrometer

Any charge deposited on the internal capacitor (e.g. by applying a voltage across terminals AB and then disconnecting the voltage leads) will decay exponentially with time according to the value of the parallel resistance that can be selected using the resistance link ("R link") - this is dealt with in task 1. If no parallel resistance is selected (i.e. the R link just connects ground to ground), the internal capacitor can only discharge through the input circuit of the electrometer's voltage amplifier. Since this amplifier is close to ideal, its input impedance is very large (in excess of $10^{12}\,\Omega$) and the time constant of the parallel RC circuit thus formed is also very large. Hence, the charge remains on the internal capacitor and the voltage across it is maintained.

Depressing the "push to zero" button inserts a $57\,\mathrm{k}\Omega$ resistor in parallel with the internal capacitor, which then discharges rapidly. This button is used for discharging the internal capacitor, or any external capacitor connected in parallel with it.

The amplifier has voltage gain 1 or 10 and its output is the amplified voltage across the internal capacitor, from which we can determine the charge on its plates using $Q = CV$. The ability of the electrometer to measure any charge transferred to the internal capacitor allows us to study many basic principles of electrostatics (task 2). By connecting a charged external capacitor in parallel with the uncharged internal capacitor and measuring the charge transferred to it, we can determine the value of the external capacitance (tasks 3-5).

Since the electrometer is battery operated, please switch it off at the end of the lab or when you take a break.

1. RC Time Constant

Suppose that a dc voltage V_0 is applied across the internal capacitor C_{int}. If the voltage leads are now removed and a resistance *R* is inserted in parallel with C_{int}, the charge *q* on C_{int} decays with time according to the equation

$$\frac{dq}{dt} + \frac{V}{R} = 0 \qquad (1)$$

where *V* is the voltage across the capacitor at some time *t* after disconnecting the voltage source. Assuming that $q = C_{\mathrm{int}} V$, solving eqn.(1) shows that *V* decreases exponentially with time

$$V(t) = V_0 e^{-t/\tau} \qquad (2)$$

where the time constant for the parallel RC circuit is $\tau = RC_{\mathrm{int}}$.

Task 1 - Test of eqn. (1) and determination of RC time constant (15 minutes)

Select the 10^{-7} F internal capacitor and connect the R link from ground to ground. Discharge the capacitor by pressing the "push to zero" button. Set the amplifier gain to 1, apply a dc voltage of 1 V to the input (across AB), and connect the output to VirtualBench. You only need to touch the terminals AB with the voltage leads to charge up the internal capacitor. Now disconnect the voltage leads and note that the charge remains on the internal capacitor.

Now study the effects of connecting the R link to each of the large parallel resistors indicated on the electrometer casing. Explain what happens to the charge on the internal capacitor in each case (don't forget to discharge and recharge the capacitor each time you change the parallel resistor).

Pre-lab
Read the lab sheet carefully. Understand the lab procedure. Calculate the time constants.

2. Basic Electrostatic Principles

Suppose that a metal sphere is plugged into terminal A of the electrometer input and the uncharged 10^{-7} F internal capacitor is selected, with the R link connected ground to ground. Referring to Fig. 2, if a positively charged rod is brought near the sphere, electrons from the top capacitor plate flow towards the sphere, leaving the sphere negatively charged and the top capacitor plate positively charged. Electrons are now attracted to the bottom capacitor plate and flow to it from earth, thus leaving it negatively charged, and a positive voltage appears on the electrometer output. This flow of charge occurs because 10^{-7} F is much larger than the capacitance of the sphere, so the internal capacitor acts as a charge reservoir. Therefore, any charge deposited on the sphere's surface is almost all transferred to the internal capacitor, and this charge can be determined by measuring the electrometer's output voltage. We will now use the electrometer in this and similar configurations for studying some basic electrostatic principles.

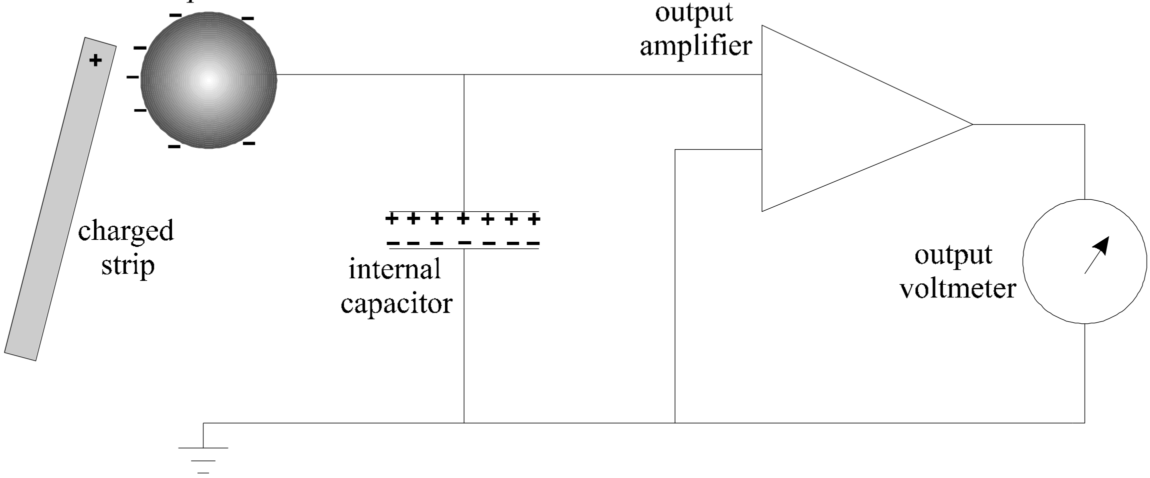
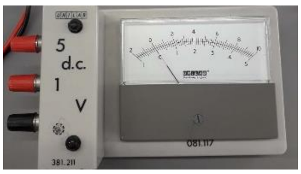


Figure 2: Induced charge distribution within the electrometer due to a + charged strip

Task 2: Illustrations of some basic electrostatic principles (40 minutes)
(The success of this depends on the moisture content of the air; it doesn't work too well on wet days, where the damp air allows a discharge path for any accumulated charge.)

This task could be fiddly. Make sure you do not unintentionally charge or discharge the objects!

Plug the hollow metal sphere into input A, connect the R link between ground to ground, <u>select an output voltage gain of 10</u> and set the output voltmeter to read 1 V full scale deflection. Discharge the internal capacitor. Suggest using the voltmeter to observe the output in this task.



The voltmeter

Electrostatic induction and charge conservation

a) Charge the red and grey strips by rubbing each of them with the cloth provided. Move the strip close to the hollow metal sphere (<u>without touching it</u>). Confirm that one charges negatively, the other positively. If you observe something unexpected, make sure you check and double-check your experiment.

b) Plug the two small metal spheres into their perspex supports and place them next to each other so that they touch. Bring a charged grey strip to about 1 mm away from one of the spheres (without touching it) and then separate the spheres (**whilst holding the rod still**) by removing the other sphere. Find the polarity of the charge on each of the small spheres using the electrometer. Draw simple sketches to explain why they become charged.

c) Repeat part b) above and this time discharge the positively charged, small sphere by bringing it into contact with the large hollow sphere. Measure the electrometer output voltage and determine the charge on the small sphere. Now touch the hollow sphere with the negatively charged small sphere. Measure the output voltage and comment on your result.

Electrostatic screening and charge distribution on a conducting surface

d) Now plug one of the small spheres into input A and plug the large sphere into a perspex support. Make sure everything is uncharged! Position the other small sphere so that it makes contact with the outside surface of the large hollow sphere and, whilst maintaining contact, bring a charged grey strip to about 1 mm from the outside surface of the hollow sphere. Keeping the grey strip fixed, break the contact and measure the charge on the small sphere.

e) Discharge everything and repeat part d) above, this time with the small sphere contacting the **inside** surface of the hollow sphere. Measure the charge on the small sphere and comment on your result in comparison with part d).

Pre-lab
Try to understand what each activity is for. What do you expect to observe from each activity (a to e) according to learned theory?

3. Measurement of external capacitors

If an external capacitor C_{ext} is connected to a dc power supply providing a voltage V_{ext} then the charge stored on it is $Q = C_{\text{ext}} V_{\text{ext}}$. Suppose that the voltage leads are now removed and the charged external capacitor is connected **in parallel** with the uncharged internal capacitor C_{int}, as shown in Fig. 3. The charge Q now distributes itself between internal and external capacitors, and the voltage developed across C_{int} is given by

$$V_{\text{int}} = \frac{C_{\text{ext}}}{C_{\text{int}} + C_{\text{ext}}} V_{\text{ext}} \qquad (3)$$

Make sure that you can derive eqn.(3). The voltage V_{int} can be measured on the electrometer output, and since we know C_{int} (in this experiment we choose it to be 10^{-7} F) we can therefore work out the value of C_{ext}.

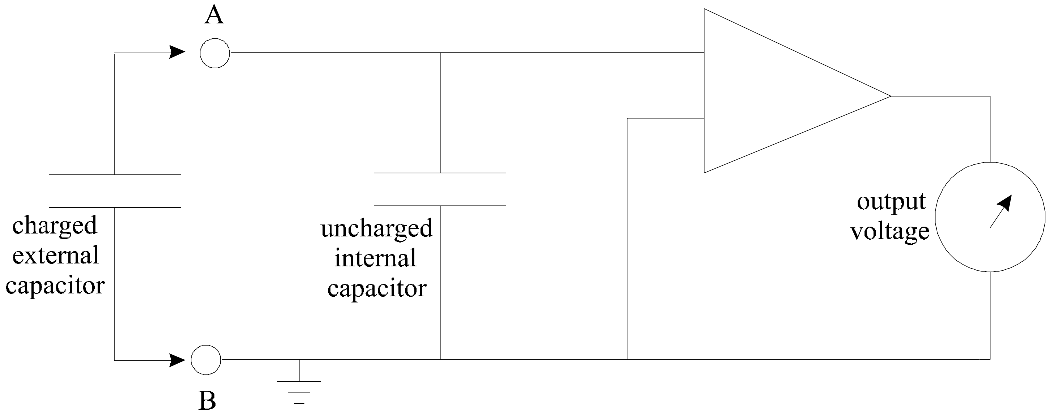
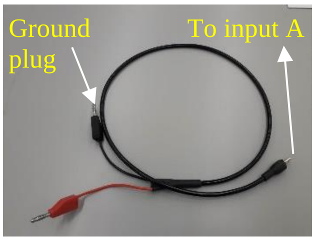


Figure 3: Determination of the capacitance of an external capacitor

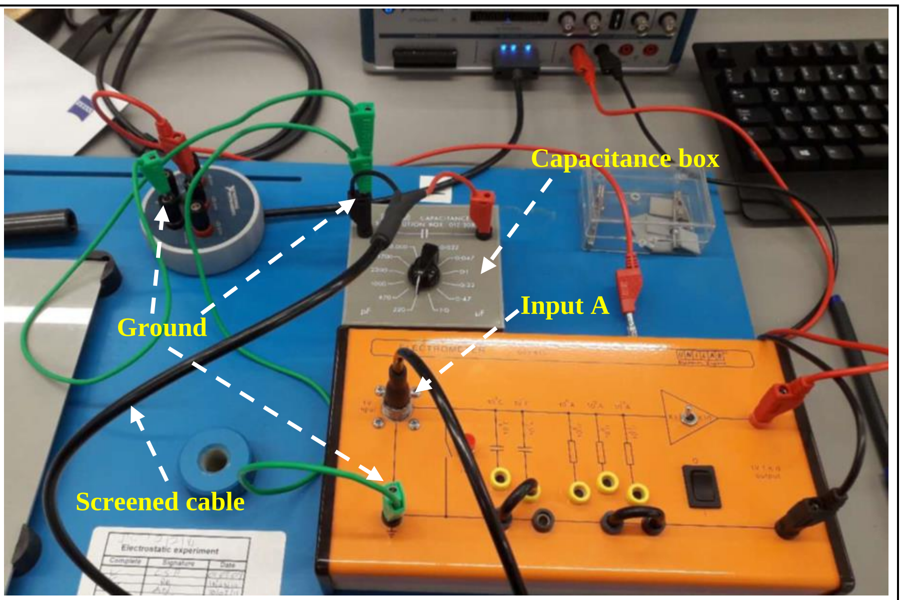
Eqn.(3) shows that in the limit when $C_{ext} << C_{int}$ (as is often the case)

$$V_{int} \approx \frac{C_{ext}}{C_{int}} V_{ext} \tag{4}$$

In this limit, almost all of the charge on the external capacitor is transferred to the internal capacitor (we remarked in section 2 that the internal capacitor acts as a "charge reservoir", and eqn.(4) quantifies this statement). We will use eqn.(4) extensively in what follows.



The screened cable



Experimental set-up for Task 3

Task 3: "Test" of $Q = CV$ by validation of eqn.(3) (40 minutes)

Turn the output voltage gain back to ×1.
Select the 10^{-7} F internal capacitor and connect the R link from ground to ground. Connect the screened cable into input A of the electrometer and insert the ground plug of the cable into the ground terminal B. Select one of the external capacitors C_{ext} on the external capacitance box

and discharge it by connecting it across AB and then pressing the "push to zero" button. Now disconnect the external capacitor and charge it by connecting it across the dc power supply. Use VirtualBench to set the power supply voltage to about 5 V. Ensure that a common ground point is used by connecting terminal B to the ground of the power supply. Disconnect the high voltage lead from the external capacitor C_{ext} and connect it in parallel with the internal capacitor C_{int} by touching the exposed external capacitor terminal with the centre conductor of the screened cable (connected to A). Measure the output voltage of the electrometer (which equals the voltage developed across the internal capacitor V_{int}) as a function of C_{ext}, varying C_{ext} from 220 pF up to 0.1 μF (*you may need to use appropriate combinations of amplifier gain and voltmeter sensitivity to obtain a stable voltage reading from VirtualBench*).

Plot the data for V_{int} as a function of C_{ext}. Also plot the results expected theoretically from eqn.(3), and the simple straight line approximation predicted by eqn.(4). Hence, determine the ratios $C_{\text{int}} / C_{\text{ext}}$ for which eqn.(4) is a reasonable approximation. (*Since the amplifier is battery-powered, it is not possible to obtain output voltages in excess of 4.5 V*).

Pre-lab
Calculate the expected V_{int} when C_{ext} changes from 220 pF to 0.1 μF.

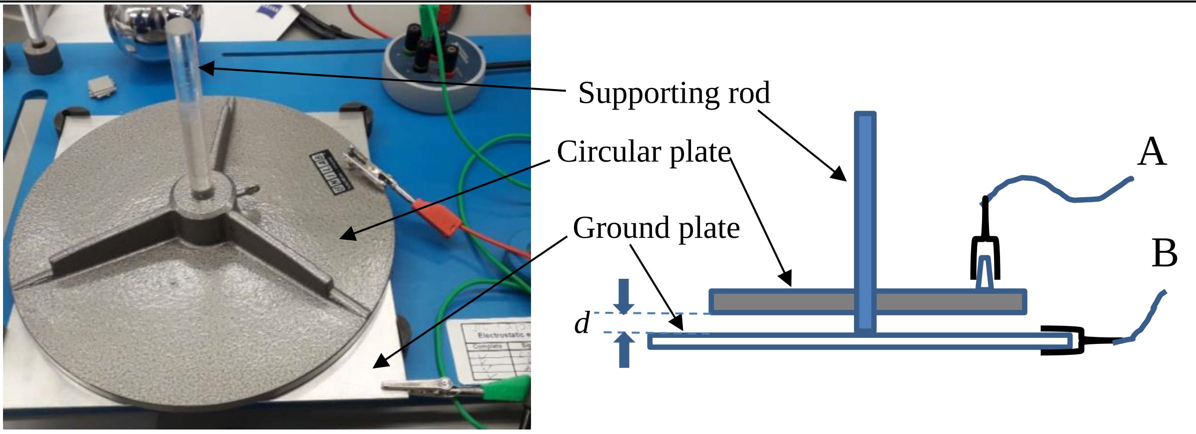
4. The parallel plate capacitor

Simple application of Gauss' law allows us to determine the capacitance of an air-spaced parallel plate capacitor

$$C_{\text{pp}} \approx \frac{\varepsilon_0 A}{d} \tag{5}$$

where A is the plate area and d is the plate separation. By charging C_{pp} to some known voltage V_{ext}, then disconnecting the voltage leads and connecting it in parallel with the large uncharged internal capacitor C_{int}, we can use eqn.(4) to determine C_{pp} (provided that it is much smaller than C_{int}) using the electrometer to measure the voltage developed across the internal capacitor V_{int} (just like task 3). However, since C_{pp} is usually very small, we have to include the finite capacitance C_{cable} of the screened cable that we connect across terminals AB, which appears in **parallel** with C_{pp} (can you see why?). Thus, using eqns. (4) and (5)

$$C_{\text{pp}} + C_{\text{cable}} \approx \frac{V_{\text{int}}}{V_{\text{ext}}} C_{\text{int}} \approx \frac{\varepsilon_0 A}{d} + C_{\text{cable}} \tag{6}$$



Experimental set-up for Task 4

Task 4: Validation of eqn.(6) and estimation of ε_0 (50 minutes)

Use exactly the same method as that for task 3 to test eqn.(6) using the large, circular parallel plate capacitor as the external capacitor. Apply a constant supply voltage of about 20 V, and use the cable with the croc clip to ground one of the capacitor plates. Vary the spacing between the plates *d* using the 0.8 mm thick dielectric spacers provided. *You have to be very careful to discharge all capacitors to get good results here*. Measure the voltage developed across the internal capacitor V_{int} when connected in parallel with the uncharged internal capacitor C_{int}. Plot V_{int} as a function of 1/*d* (with at least three different *d* values) and hence find the value of ε_0. Include an estimate of the random error in your value, and comment on any systematic errors in the measurement. Estimate C_{cable} from your graph and comment on how this value affects the results of task 3?

Pre-lab
Assume the outer radius of the circular plate is 11 cm and the inner radius of the plate is 0.7 cm (where the supporting rod goes). Calculate the capacitance of the plates when *d* is 0.8 mm, 1.6 mm and 2.4 mm respectively.

Task 5: Evaluation of relative permittivity (10 minutes)

Measure the capacitance of the large circular capacitor with a 0.8 mm plate spacing. Now measure the capacitance when the plates are filled completely with the 1 mm thick dielectric sheet. What is the relative permittivity of the sheet?

Pre-lab
Work out a simple strategy to extract the relative permittivity of the sheet.

*****Don't forget to switch the electrometer off when you finish the experiment.**

5. Supplementary Problems

(Try these in your own time.)

a) If a parallel plate capacitor is filled with a dielectric of resistivity $10^{14}\,\Omega\,\mathrm{m}$ and relative permittivity 2.7, how long does it take for any charge deposited on its plates to decay to half its initial value via electrical conduction through the dielectric? Does this answer depend on the shape of the capacitor plates?

b) Estimate the capacitances of the two sorts of conducting spheres used in task 2. If each sphere is charged and then connected in parallel with the 10^{-7} F internal capacitor, what proportion of charge is transferred to the internal capacitor in each case? What radius must a conducting sphere have if its capacitance equals that of the 10^{-7} F internal capacitor?

c) Use Gauss' law to show that, for any static electric charge distribution, the electric field inside a conductor is always zero. If a solid metal wire carries an electric current, is the electric field still zero inside the wire? What is the electric field outside of the current-carrying wire?

d) A charged capacitor is connected in parallel with an identical uncharged capacitor. What fraction of the initial stored electrostatic energy is lost on connection? What happens to this "lost energy"? What factors determine the time taken for charge to redistribute between the two capacitors?

e) Find the magnitude of $\underline{E}$ and the free charge density on the plates of the charged air-spaced capacitor of task 4. What happens to these values as the plate separation increases? If the spacers used in the experiment have a relative permittivity of 2.3, what contribution do they make to the overall capacitance of the structure?

f) From the dimensions of the screened coaxial cable used in this experiment, calculate C_{cable} assuming that the dielectric inside the cable has a relative permittivity of 2.0.

g) Find the magnitudes of $\underline{E}$, $\underline{P}$ and $\underline{D}$ within the dielectric for the charged capacitor of task 5. What is the free surface charge density on each capacitor plate, and what is the density of the polarisation charges developed on the surfaces of the dielectric sheet?