Computational Physics (22PHY106)

Computational Physics

- **Physics** is a natural science that seeks to understand and explain the fundamental principles and laws governing the physical world.

- **Computational Physics** is a branch of physics that uses computers and numerical algorithms to solve complex physical problems. It combines computer science, physics, and applied mathematics to develop scientific solutions to these problems.

Course Objectives

- The course will lay down the basic concepts and techniques needed for verticals such as robotics.
- It will explore the concepts initially through computational experiments and then try to understand the concepts/theory behind them.
- It will help the students to perceive the engineering problems using the fundamental concepts in physics.
- Another goal of the course is to provide the connection between the concepts of physics, mathematics, and computational thinking.

Computational Physics course structure

- Credits are assigned to the courses based on the L-T-P pattern
- L-T-P-C: 2 0 3 3
- L- Lecture hours per week
- T- Tutorial hours per week
- P- Practical hours per week
- C- Credits earned for the course

- For Computational Physics, students attend 2 hours of lectures and 3 hours of practical work each week,

Syllabus

Unit 1

Newton’s Laws of Motion, Force as 3D Vector, Resolution of Forces, Resultant of Forces.

Course Outcomes

CO 1: Apply the principles of statics to solve elementary problems in physics.

Syllabus

Unit 2

Equilibrium about a Point, Moment, Couple, Equivalent System, Equilibrium of Rigid Bodies, Degree-of-freedom and Constraints at Supports, Free Body Diagram.

Course Outcomes

CO 2: Apply computational techniques to solve elementary problems in statics.

Syllabus

Unit 3

Kinematics of particles, assumptions, Cartesian, Cylindrical and Spherical frames, and motion of particles in them. Translation and rotation of rigid bodies in 2D – Translation and rotation of rigid bodies in 3D.

Course Outcomes

CO 3: Apply computational techniques to solve elementary problems in dynamics.

Syllabus

Unit 4

Kinematics of interconnected rigid bodies– Definition of a linkage – Definition of a mechanism –Four-bar mechanism.

Course Outcomes

CO 4: Analyze the motion of rigid bodies by applying fundamental principles of dynamics.

TEXTBOOKS

1. Merlam J.L and Kraige L.G., Engineering Mechanics, Volume I - statics, Volume 11- dynamics, John Wiley & Sons, New York, 2018.

2. Hibbeler R. C., Engineering Mechanics: Statics and Dynamics, 11th edition, Pearson Education India, 2017.

3. Elementary Mechanics Using Matlab – Malthe & Sorenssen – Undergraduate Lecture Notes in Physics, Springer International Publishing, 2015.

4. Elementary Mechanics Using Python – Malthe & Sorenssen – Undergraduate Lecture Notes in Physics, Springer International Publishing, 2015.

Reference

1. Beer F.P. and Johnston E.R., Vector Mechanics for Engineers - Volume I - Statics, Volume II - Dynamics, McGraw Hill, New York, 2004.

2. Shames I. H., Engineering Mechanics, Prentice Hall, New Delhi, 1996.

3. Statics with Matlab – Marghitu, Dupac& Madsen, Springer – Verlag London 2013.

4. Advanced Dynamics - Marghitu, Dupac& Madsen, Springer – Verlag London 2013.

5. Dukkipati R. V., MATLAB: An Introduction with Applications, New Age International; 2010.

EVALUATION PATTERN: 70 + 30

30- End semester Theory Examination (External)

70=50+20 (Internal)

20-Mid term examination

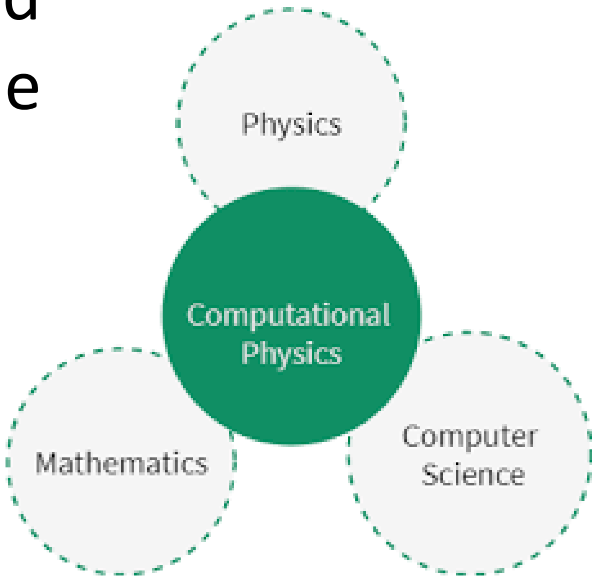
50= 25+25

25=Lab

25=15+15, 10=class test, 15=Assignment

Computational Physics

- **Physics** is a natural science that seeks to understand and explain the fundamental principles and laws governing the physical world.



- **Computational physics** is the study and implementation of numerical analysis to solve problems in physics for which a quantitative theory already exists; it combines computer science, physics and applied mathematics to develop scientific solutions to complex problems

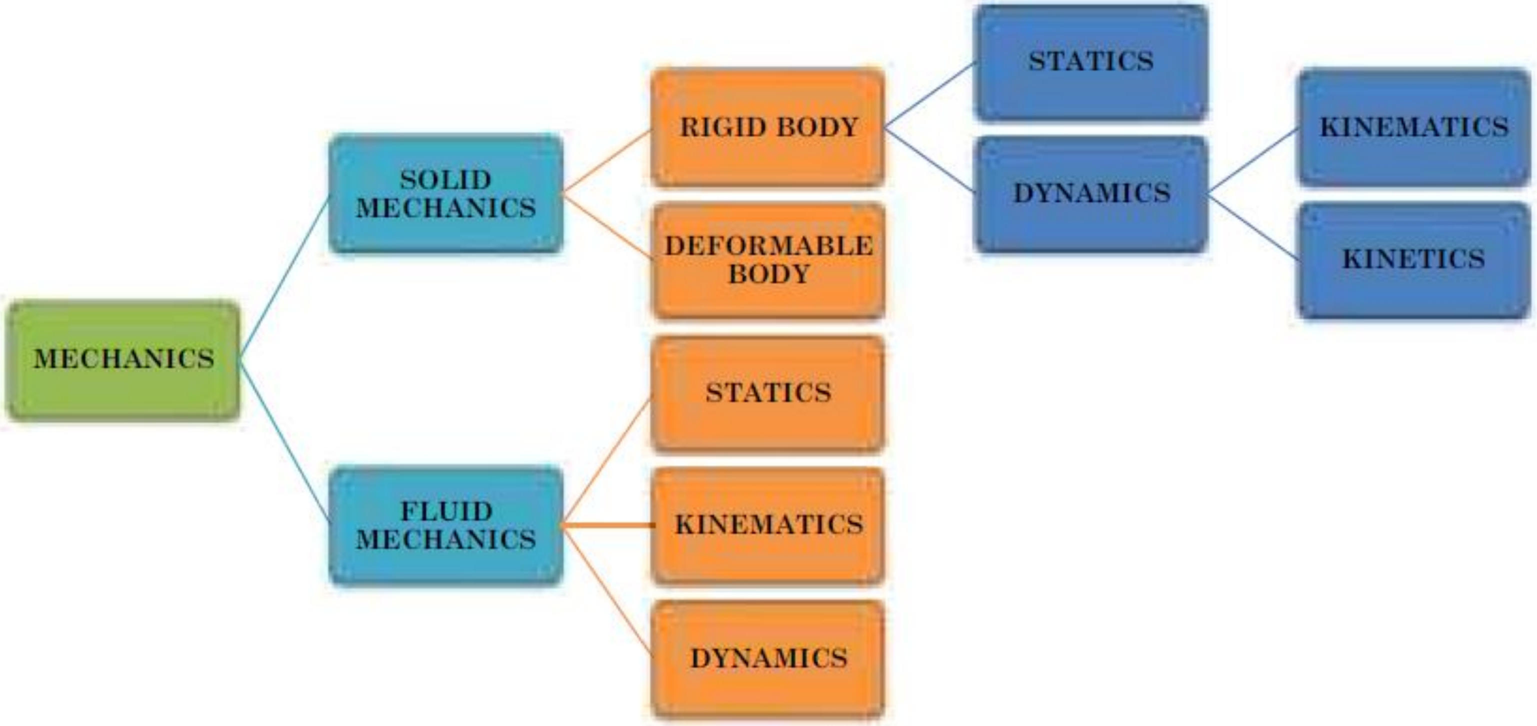
Syllabus

Unit 1

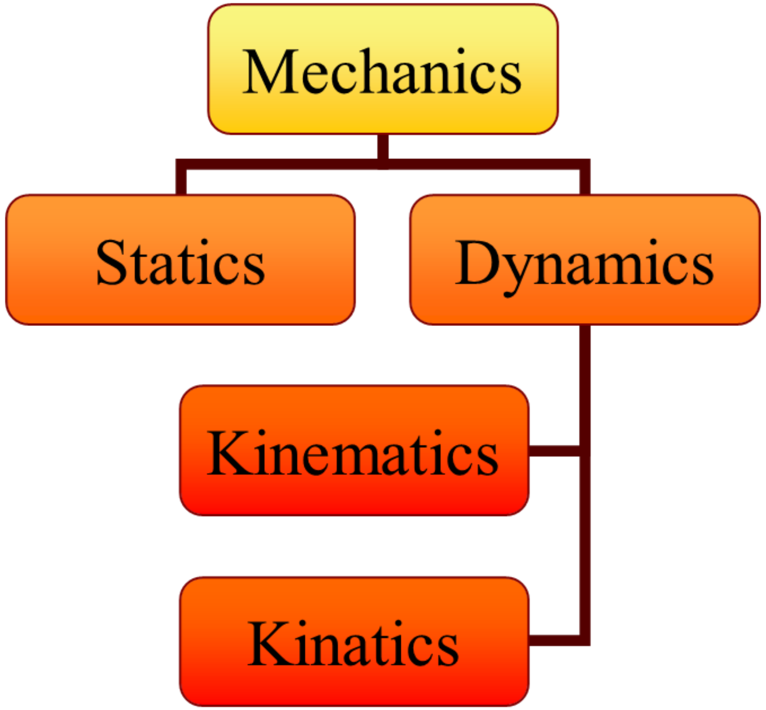
Newton's Laws of Motion, Force as 3D Vector, Resolution of Forces, Resultant of Forces.

Engineering Mechanics

- **Engineering mechanics** is a branch of science which deals with the effects of forces on objects.



Engineering Mechanics



Engineering Mechanics

Statics deals with objects and systems that are at rest or in a state of equilibrium, meaning they are not moving and have no acceleration.

Example

Book on a Table: A book resting on a table is an example of statics. In this scenario, the forces acting on the book (gravity pulling it downward and the table pushing upward) are balanced, and the book remains at rest.

Sitting in a chair

These examples involve everyday situations where objects are at rest or in equilibrium due to the balance of forces acting on them.

Engineering Mechanics

Dynamics deals with objects and systems that are in motion or experiencing acceleration.

Example

Car Acceleration-When a car accelerates on the highway, its an example of dynamics. The car is in motion, and forces are involved in changing its velocity.

Projectile Motion

Orbital Dynamics

Collisions

Engineering Mechanics

- Dynamics is divided into kinematics and kinetics

- Kinematics describes the motion of objects, while kinetics studies forces that cause changes of motion.

Basic Terms

- **Essential basic terms to be understood**
  - Space
  - Time
  - Mass
  - Force
  - Particle
  - Rigid Body

Space

Space is the geometric region occupied by bodies whose positions are described by linear and angular measurements relative to a coordinate system.

For three dimensional problems, three independent coordinates are needed. For two- dimensional problems, only two coordinates are required.

Time

- Time is the measure of the succession of events and is a basic quantity in dynamics.
- Time is not directly involved in the analysis of statics problems.

Mass

- Mass is a measure of the inertia of a body, which is its resistance to a change of velocity. The mass of a body affects the gravitational attraction force between it and other bodies.

Force

- Force is the action of one body on another. A force tends to move a body in the direction of its action. The action of a force is characterized by its magnitude, by the direction of its action, and by its point of application.

WHAT IS FORCE

- ✓ An action of one body on another.
- ✓ A push or pull upon an object resulting from the object's interaction with another object.
- ✓ Force is a vector.
- ✓ Unit is N (Newton)

Effects of force :-

i) Force can move a body at rest.

ii) Force can stop a moving body.

iii) Force can change the speed of a moving body.

iv) Force can change the direction of a moving body.

v) Force can change the shape and size of a body.

Balanced and unbalanced forces :-

i) Balanced forces :-

If two forces act on a body in opposite direction and if both the forces are equal, then the resultant force acting on the body is zero. Such forces are called balanced forces.

Balanced forces cannot change the state of rest or motion of a body.

$F_1 = F_2$

ii) Unbalanced forces :-

If two forces act on a body in opposite direction and if one force is greater than the other, then the resultant force is not equal to zero. Such forces are called unbalanced forces.

Unbalanced forces changes the state of rest or the motion of a body.

$F_1 > F_2$

Particle

- A particle is a body of negligible dimensions with respect to reference body.
- In the mathematical sense, a particle is a body whose dimensions are considered to be near zero so that we may analyze it as a mass concentrated at a point.

Rigid body

- A body is considered rigid when the change in distance between any two of its points is negligible for the purpose at hand.

or

A combination of large number of particles in which all particles remain at a fixed distance (practically) from one another before and after applying a load.

A rigid body does not deform under load

Particle: Body of negligible dimensions with respect to reference body

Rigid body: Body with <u>negligible deformations</u>

Non-rigid body / Elastic body: Body which can <u>deformable.</u>

Newton's Three Laws of Motion

First law (Law of Inertia): Every body continues in its state of rest or of uniform motion in a straight line unless compelled by an external unbalanced force to change that state.

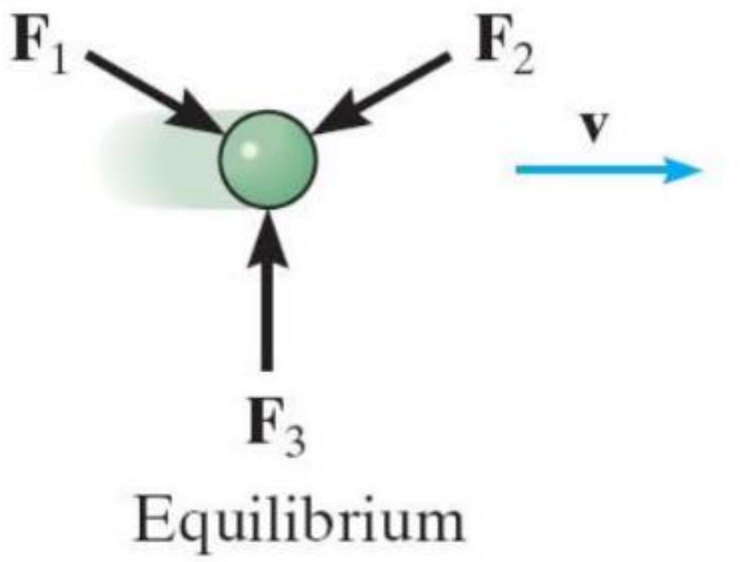
Second Law: The rate of change of momentum is directly proportional to the external unbalanced force and takes place in the direction of the force.

Third Law: To every action there is an equal and opposite reaction

Mechanics: Newton's Three Laws of Motion

The study of rigid body mechanics is formulated on the basis of Newton's laws of motion.

First Law: particle remains at rest or continues to move with uniform velocity (in a straight line with a constant speed) if there is no unbalanced force acting on it.



First law contains the principle of the equilibrium of forces - main topic of concern in Statics

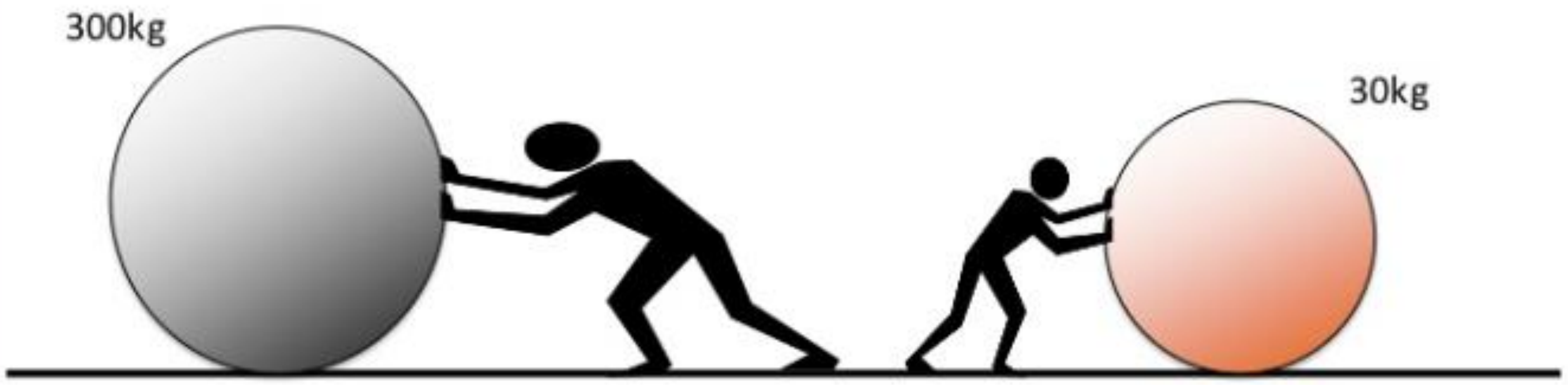
Law of inertia, states that an object at rest will remain at rest, and an object in motion will continue moving at a constant velocity in a straight line, unless acted upon by an external force.

Newton’s First Law of Motion: Inertia

An object will not change its motion unless acted on by an unbalanced force.

- *if it is at rest, it will stay at rest*
- *if it is in motion, it will remain at the same velocity*

Objects with a **greater mass** have **more inertia**.
It takes **more force** to change their motion.



Mechanics: Newton’s Three Laws of Motion

- **Law II:** The acceleration of a particle is proportional to the vector sum of forces acting on it and is in the direction of this vector sum.

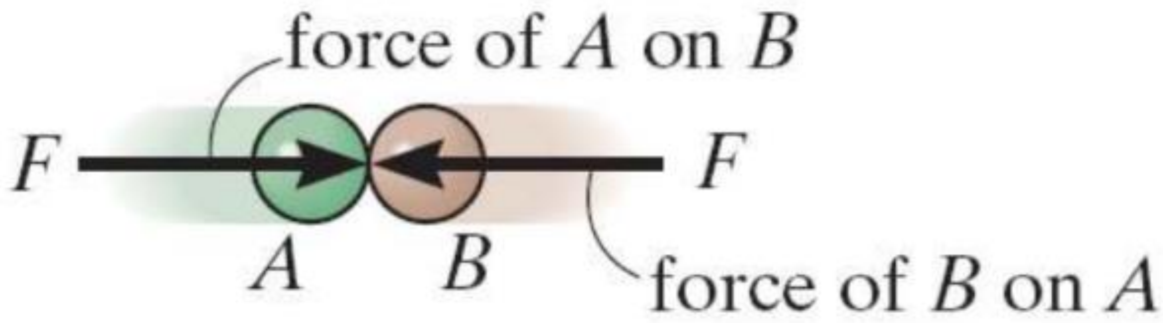
Accelerated motion

F = ma

Second Law forms the basis for most of the analysis in Dynamics

Mechanics: Newton's Three Laws of Motion

- **Law III:** The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear (they lie on the same line).

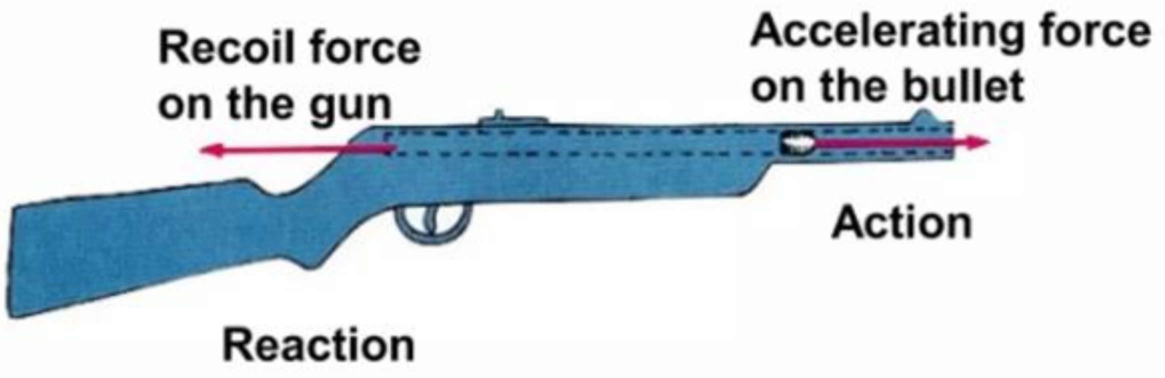


Action – reaction

Third law is basic to our understanding of Force → Forces always occur in pairs of equal and opposite forces.

Examples of action and reaction :-

i) When a bullet is fired from a gun, it exerts a forward force (action) on the bullet and the bullet exerts an equal and opposite force on the gun (reaction) and the gun recoils.



Examples

- A car with a mass of 1000 kg is accelerating at 2 m/s^2. What is the force acting on the car?

Solution: According to **Newton's Second Law**:

$$F = ma$$

Where:

$$m = 1000kg$$

$$a = 2\,m/s^2$$

Substituting the values:

$$F = 1000 \times 2 = 2000\ N$$

So, the force acting on the car is 2000 N

- A box with a mass of 50 kg is pushed with a force of 250 N. What is the acceleration of the box?

Solution: According to **Newton's Second Law**:

$$F = ma$$

$$a = \frac{F}{m}$$

$$a = \frac{250}{50} = 5\, m/s^2$$

So, the acceleration of the box is $5\, m/s^2$

- If a force of 10 N is applied to an object and it accelerates at 2 m/s^2, what is the mass of the object?

Solution: According to **Newton's Second Law**:

$$F = ma$$

$$m = \frac{F}{a}$$

$$m = \frac{10}{2} = 5\ kg$$

So, the mass of the object is 5 kg

- A car of mass 1500 kg accelerates from rest to 20 m/s in 10 seconds. What is the net force acting on the car?

  First, we calculate the acceleration using the formula:

$$a = \frac{v - u}{t}$$

Where:

$v = 20m/s$ *(final velocity)*

$u = 0\ m/s(initial\ velocity, since\ the\ car\ starts\ from\ rest)$
$t = 10\ seconds$

Substituting the values:

$$a = \frac{20 - 0}{10} = 2\, m/s^2$$

Now, using Newton's law, we can find the force.

$$F = ma = 1500 \times 2 = 3000\ N$$

So, the net force acting on the car is 3000 N

kinematic equations

$$v = u + at$$

$$S = ut + \frac{1}{2}at^2$$

$$v^2 - u^2 = 2\mathrm{aS}$$

- What force acting on a mass of 15kg for one minute can change its velocity from 10m/s to 50 m/s?

$$v = u + at; 50 = 10 + a\text{x}60$$

$$60a = 40$$

$$a = 40/60 = (2/3)ms^{-1}$$

$$F = ma = 15 \times (2/3) = 10\ N$$

Scalar quantities

Scalar quantities

These quantities have only magnitude and no direction. They obey the ordinary rules of algebra.

Examples: speed, distance, electric current, temperature, work etc.

Vector quantities

- These quantities possess both magnitude and direction.
- They are added and subtracted according to special laws such as parallelogram law of addition, triangle law of addition etc.
- Example: Force, velocity, acceleration, current density, intensity of electric field, angular velocity etc.

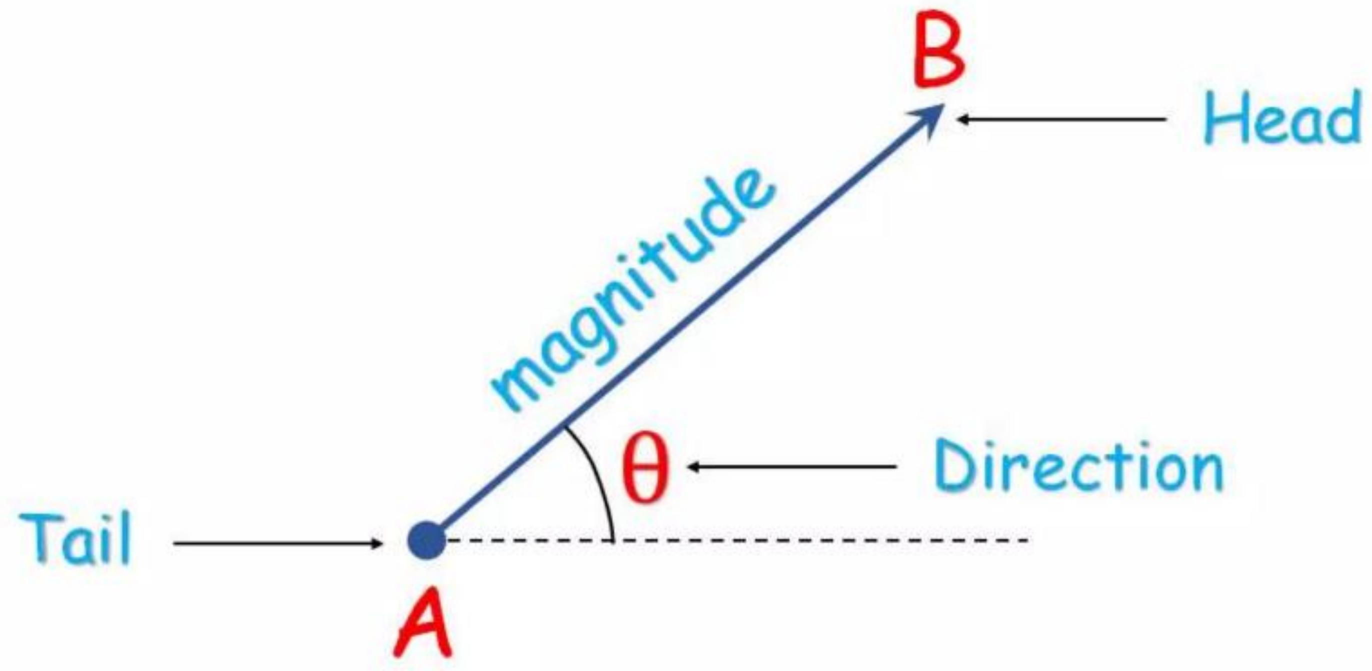
Vector quantities

Note

It is to be noted that all physical quantities having both magnitude and direction are not necessarily vectors. *All vectors obey the laws of vector algebra.*

For example, the electric current and time have both magnitude and direction; but they are scalars, because they do not obey the laws of vector algebra.

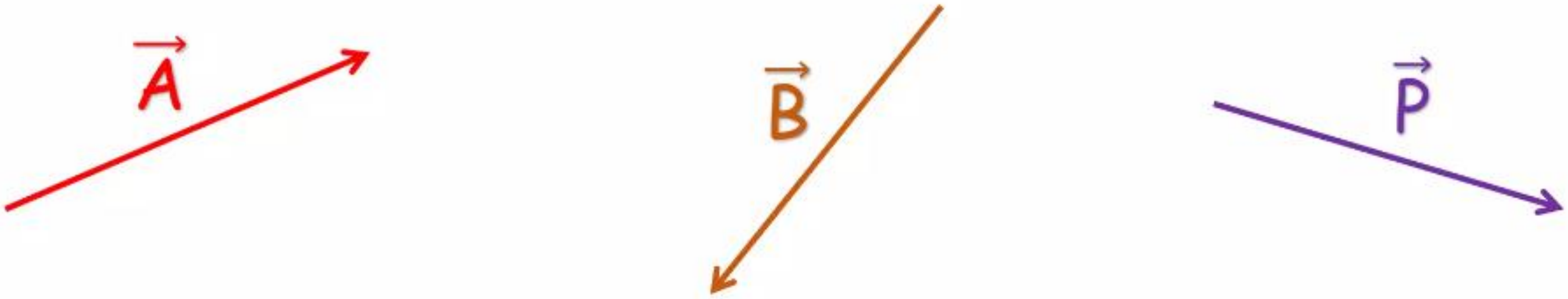
Representation of a vector



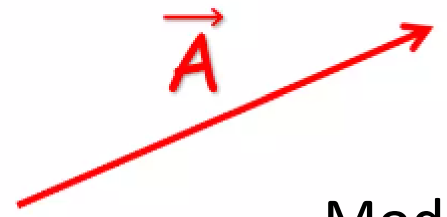
Symbolically it is represented as $\overrightarrow{AB}$

Representation of a vector

They are also represented by a single capital letter with an arrow above it.



Representation of a vector

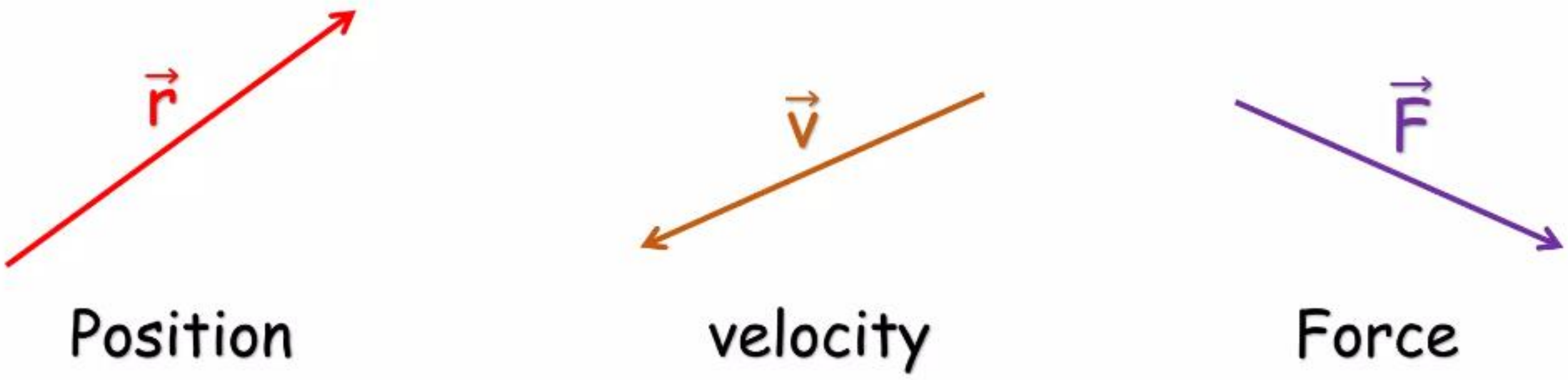


Modulus of a vector is the magnitude of the vector

The magnitude of a vector is always a positive quantity and is symbolized in *Itallic* type, written as *A* or |A|

Representation of a vector

Some vector quantities are represented by their respective symbols with an arrow above it.



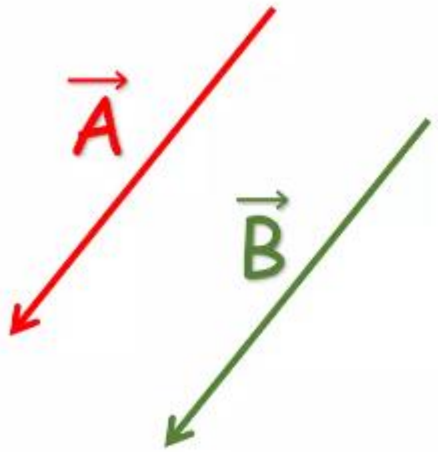
Types of vectors (on the basis of orientation)

Parallel Vectors

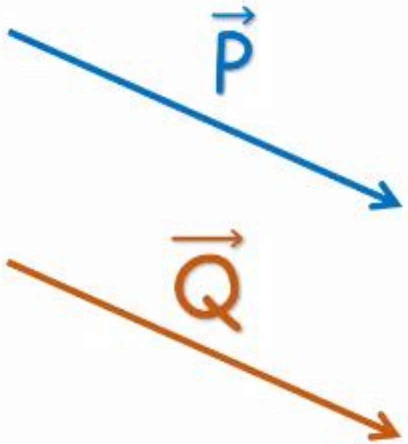
Two vectors are said to be parallel vectors, if they have same direction.

Equal Vectors

Two parallel vectors are said to be equal vectors, if they have same magnitude.



$\vec{A} = \vec{B}$



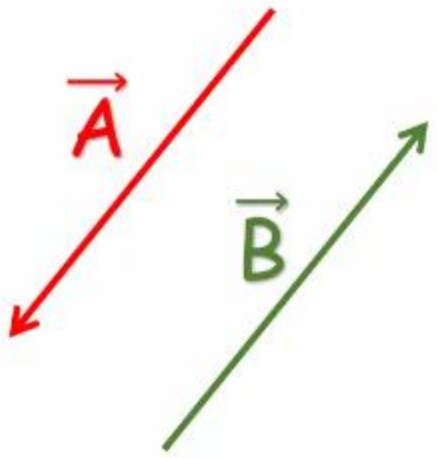
$\vec{P} = \vec{Q}$

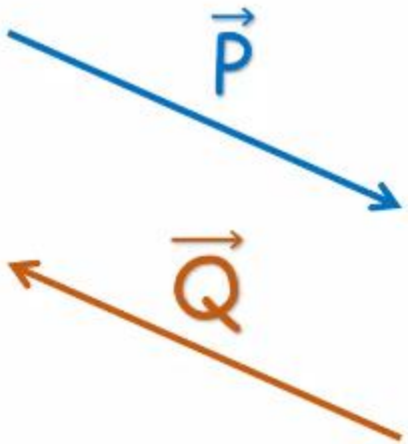
Anti-parallel Vectors

Two vectors are said to be anti-parallel vectors, if they are in opposite directions.

Negative Vectors

Two anti-parallel vectors are said to be negative vectors, if they have same magnitude.



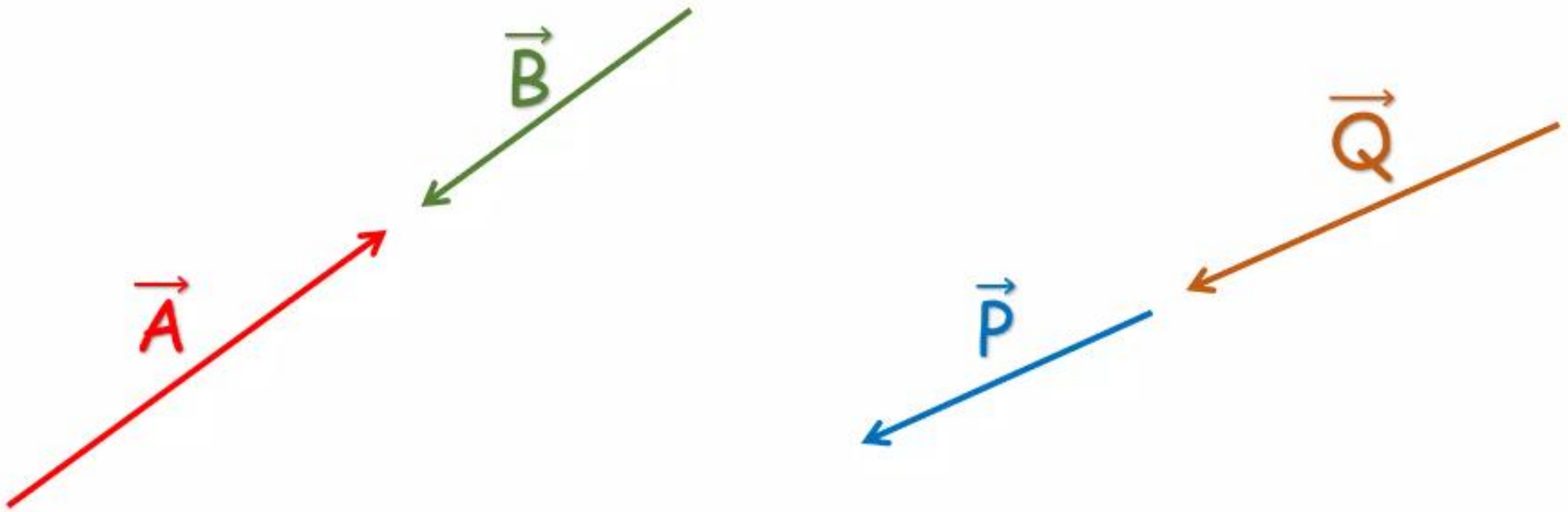


$$\vec{A} = -\vec{B}$$

$$\vec{P} = -\vec{Q}$$

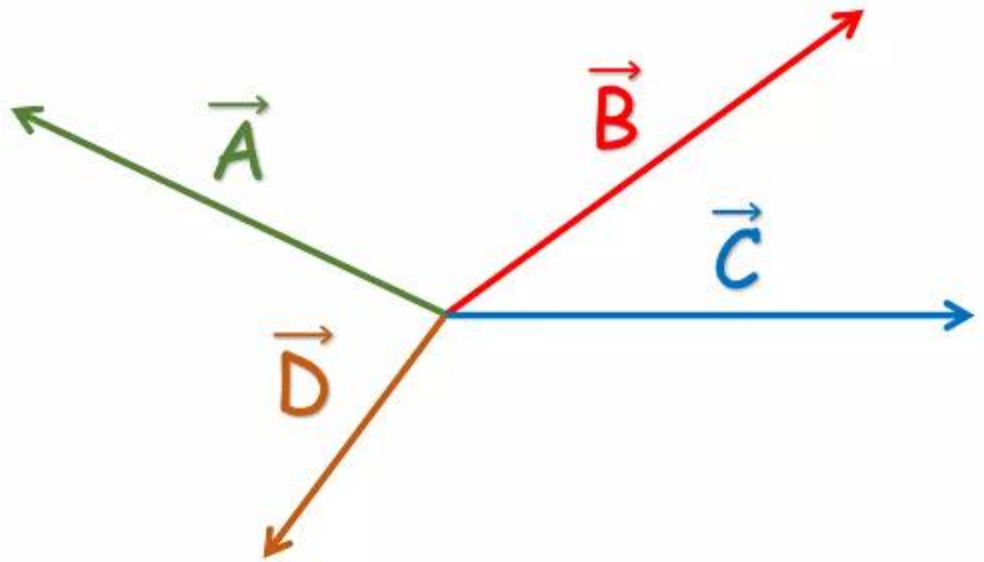
Collinear Vectors

Two vectors are said to be collinear vectors, if they act along a same line.



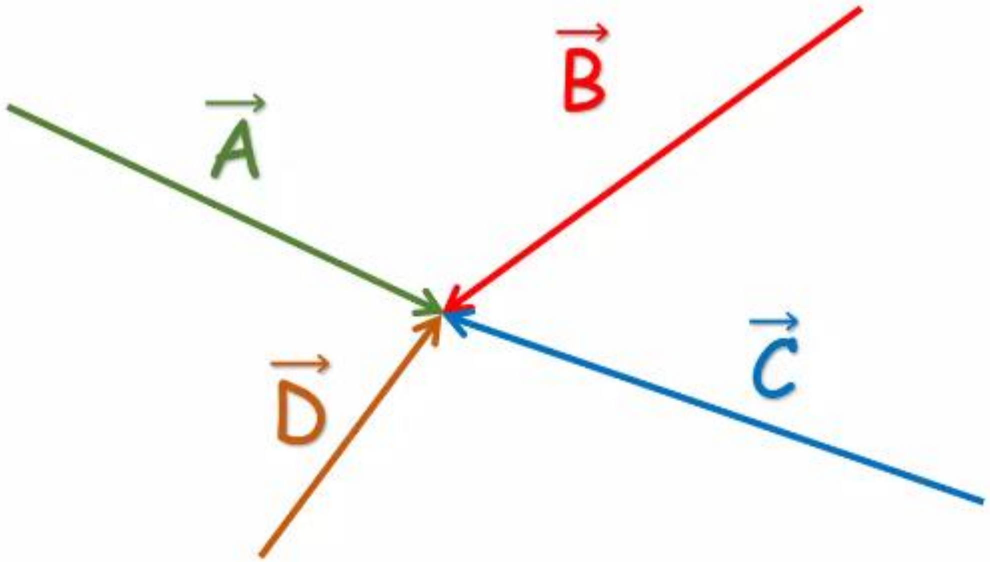
Co-initial Vectors

Two or more vectors are said to be co-initial vectors, if they have common initial point.



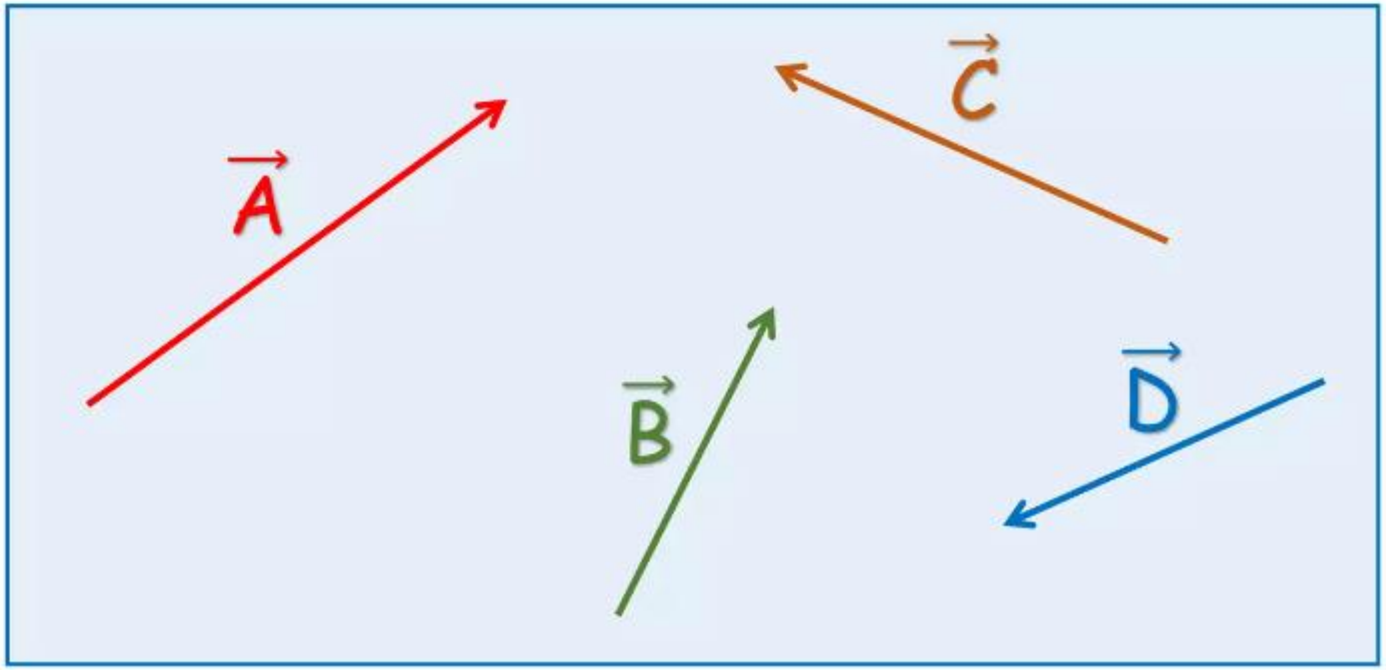
Co-terminus Vectors

Two or more vectors are said to be co-terminus vectors, if they have common terminal point.



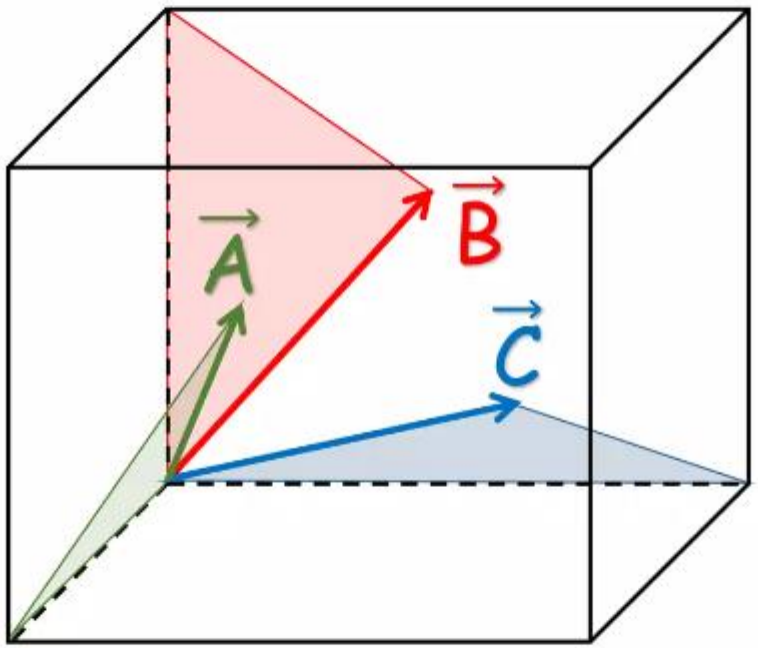
Coplanar Vectors

Three or more vectors are said to be coplanar vectors, if they lie in the same plane.



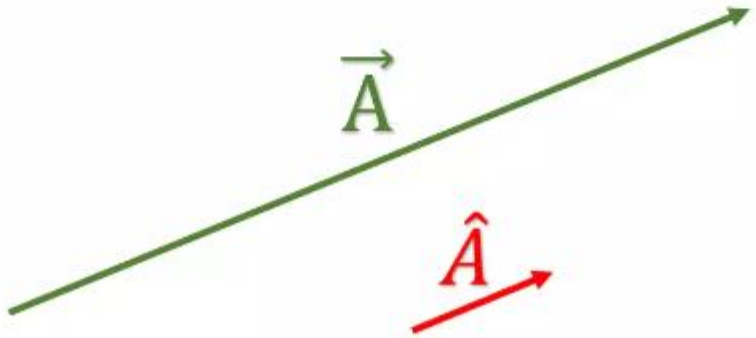
Non-coplanar Vectors

Three or more vectors are said to be non-coplanar vectors, if they are distributed in space.



Unit vectors

A unit vector is a vector that has a magnitude of exactly 1 and drawn in the direction of given vector.



Unit vectors

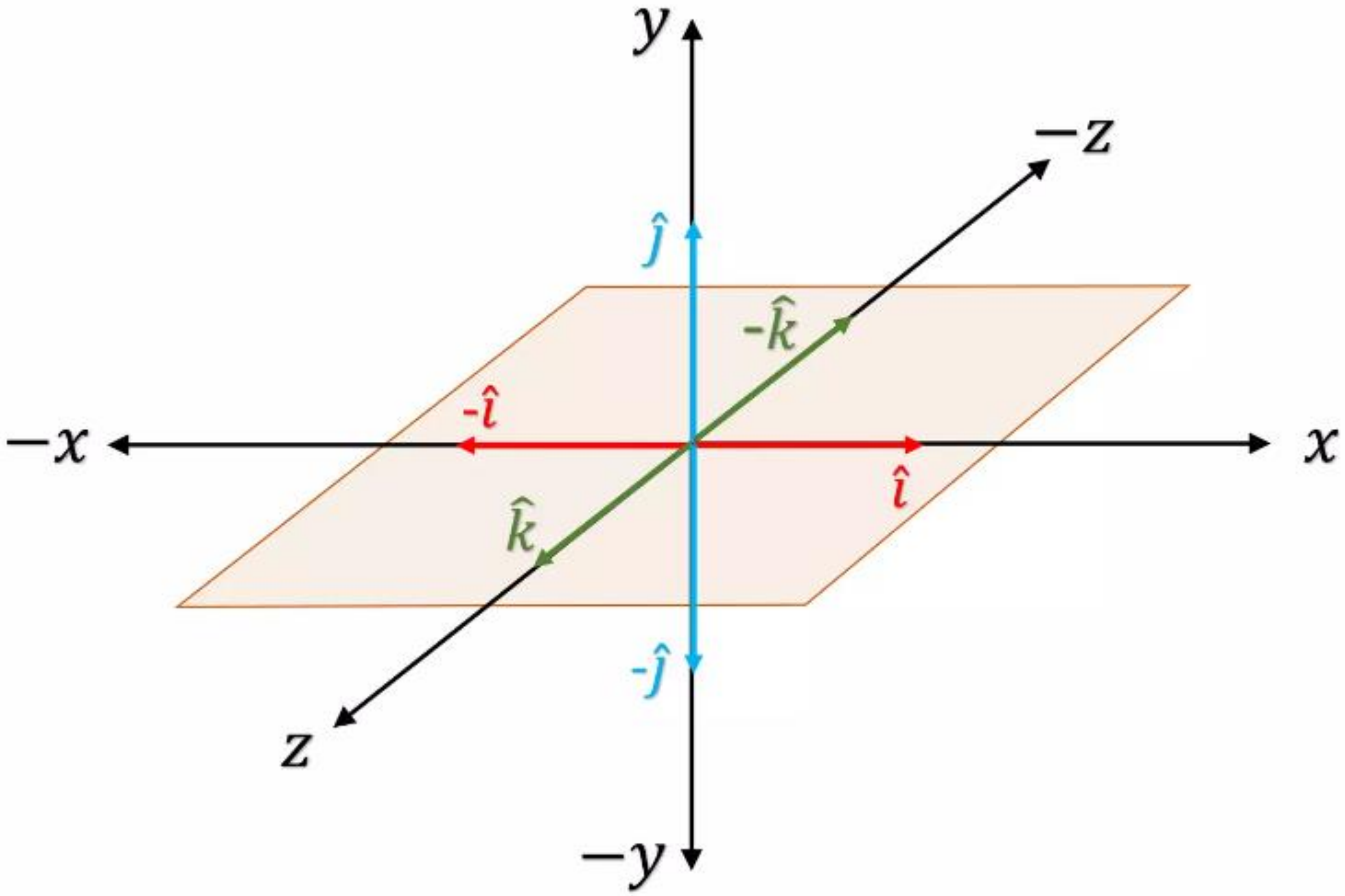
- A given vector can be expressed as a product of its magnitude and a unit vector.
- For example $\vec{A}$ may be represented as,

$$\boxed{\vec{A} = A\,\hat{A}}$$

A = magnitude of $\vec{A}$

$\hat{A}$ = unit vector along $\vec{A}$

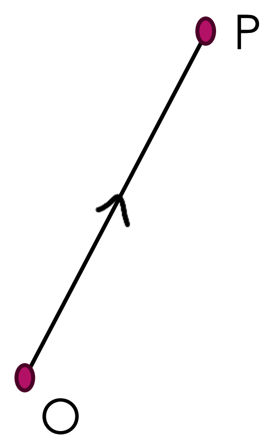
Cartesian unit vectors



Position vector

A vector representing the position of a point with respect to an arbitrary origin is called position vector.

Let ‘o’ be an arbitrary origin and P a point in space. The position vector of P with respect to ‘O’ is represented by $\overrightarrow{OP}$



Zero vector

It is that vector which has zero magnitude and an arbitrary direction. A zero vector is represented by $\vec{0}$. It is also called a null vector.

The main properties of a zero vector

- The result of adding a zero vector to any vector is the vector itself. $\vec{A} + \vec{0} = \vec{A}$
- The result of multiplication of a real number with zero vector is a zero vector itself and the result of multiplication of $\vec{0}$ and a vector $\vec{A}$ gives a zero vector. $n \times \vec{0} = \vec{0}$ and $\vec{0} \times \vec{A} = \vec{0}$
- The result of addition of a vector to its own negative vector is a zero vector.

$$\vec{A} + \left(-\vec{A}\right) = \vec{0}$$

Examples of zero vector

1. The velocity vector of a stationary object is a zero vector.
2. The acceleration of an object moving with uniform velocity is a zero vector.
3. The displacement of a stationary object is a zero vector
4. The position vector of the origin of co-ordinate axes is a zero vector.

Vectors

Vectors representing physical quantities can be classified as

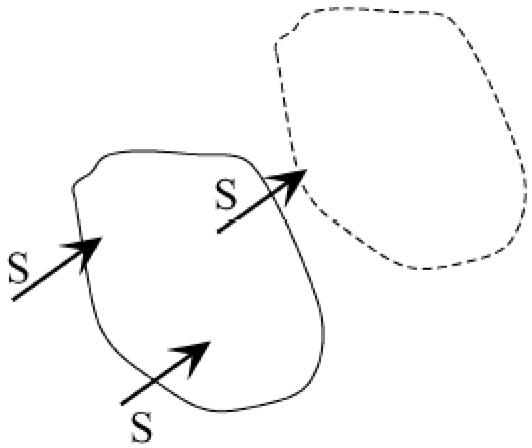
Free vectors

Sliding vectors

Fixed vectors

Free vector

A free vector is a vector that can move freely in space without being attached to any specific point. It is characterized by both magnitude and direction.

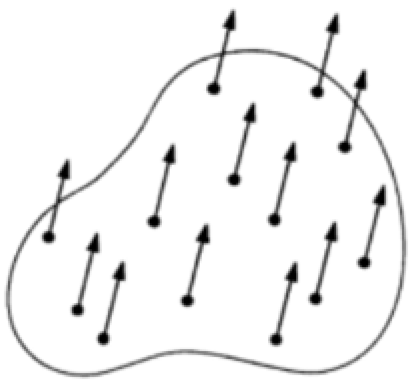


Free vector

Displacement of body moving without rotation

Free vector

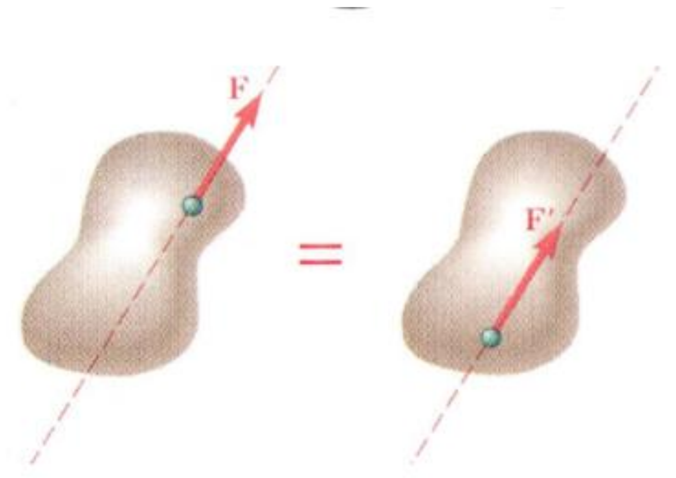
A ***free vector*** is one whose action is not confined to or associated with a unique line in space.



For example, if a body moves without rotation, then the movement or displacement of any point in the body may be taken as a vector. This vector describes equally well the direction and magnitude of the displacement of every point in the body. Thus, we may represent the displacement of such a body by a free vector.

Sliding vector

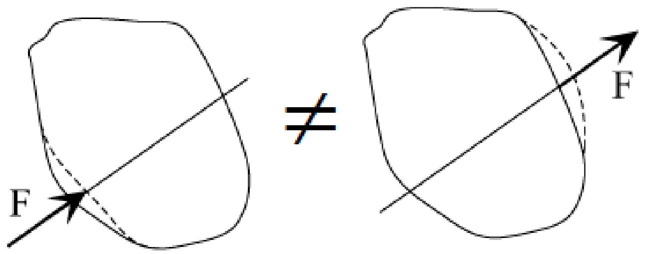
A ***sliding vector*** has a unique line of action in space but not a unique point of application.



For example, when an external force acts on a rigid body, the force can be applied at any point along its line of action without changing its effect on the body as a whole, and thus it is a sliding vector.

Fixed vector

A ***fixed vector*** is one for which a unique point of application is specified. It has a fixed magnitude and direction. The action of a force on a deformable or nonrigid body must be specified by a fixed vector at the point of application of the force. In this instance the forces and deformations within the body depend on the point of application of the force, as well as on its magnitude and line of action.



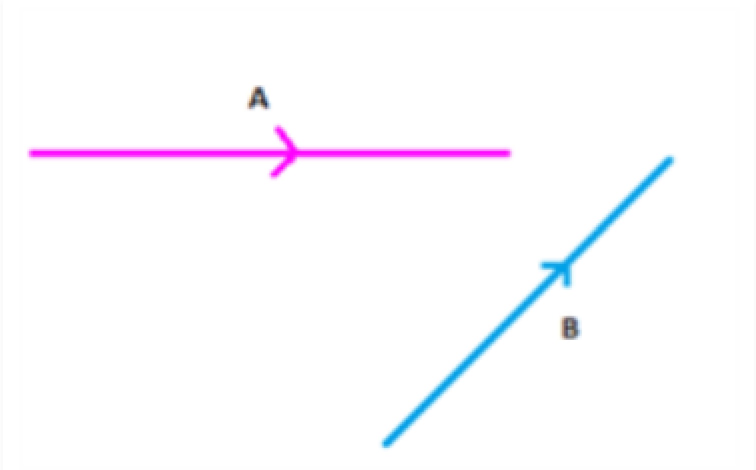
Deformable body

Fixed vector

The action of force on a deformable body

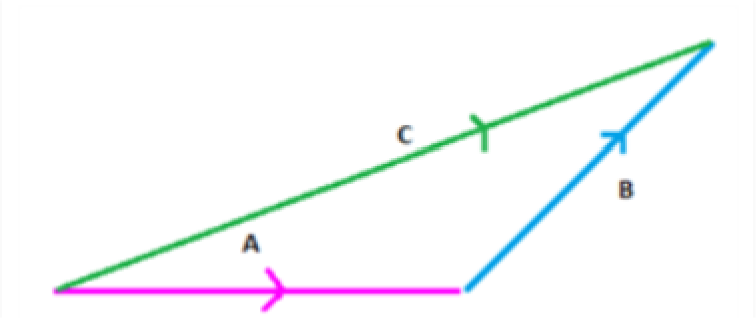
Triangle Law of Vector Addition:

Suppose, we have two vectors A and B as shown.



Now the method to add these is very simple, what we do is to simply place the tail of the second vector over the head of the first vector as shown below.

Now draw the resultant vector C from the tail of the first vector to the head of the second vector as shown in the below figure.



The resultant of the given vectors is given by the vector **C** which represents the sum of vectors **A** and **B.**

i.e. C = A + B

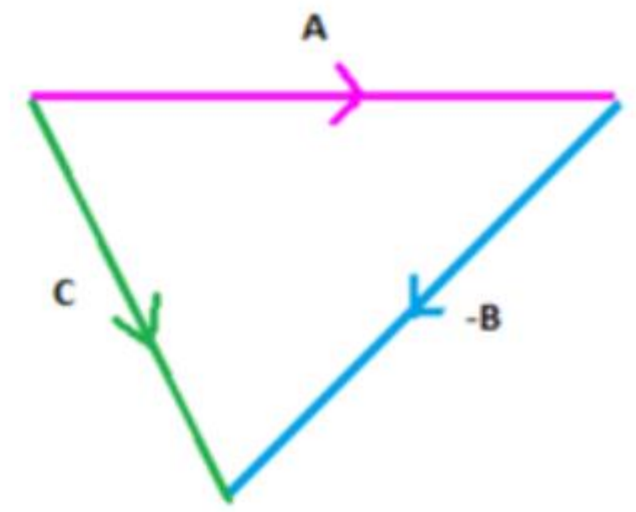
Vector addition is commutative in nature i.e.

if C = A + B; then C = B + A

Or

A + B = C = B + A

Similarly, if you want to subtract both the vectors using the triangle law then simply reverse the direction of any vector and add it to the other one as shown.

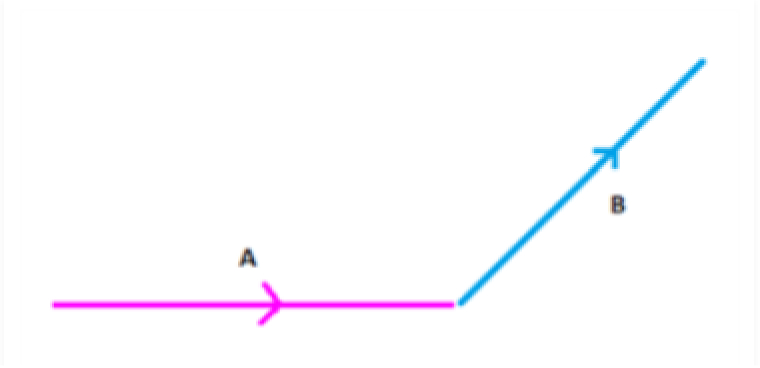


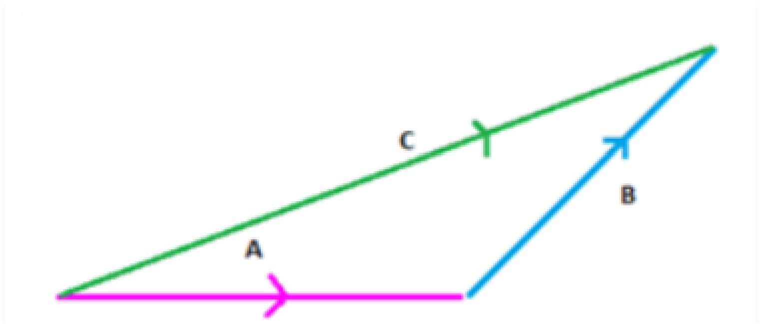
Now, this can be represented mathematically as:

C = A – B

Triangle law of vectors

If two sides of a triangle represent two given vectors in magnitude and direction and in the same order, then the third side of the triangle in the reverse order represents the vector sum of the vectors.

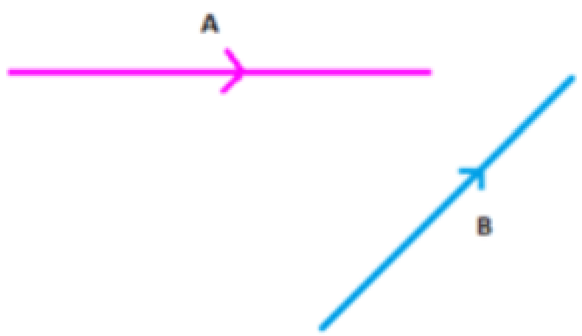




$C = A + B$

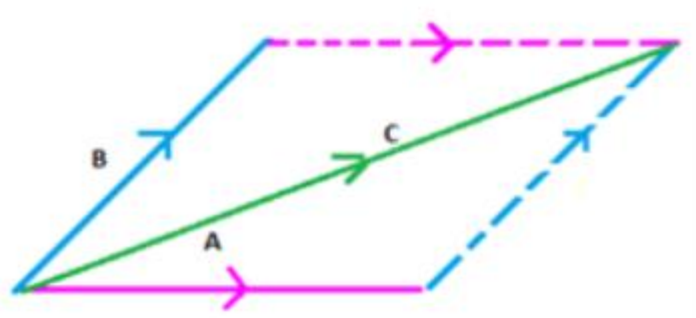
Parallelogram Law of Vector Addition:

This law is also very similar to the triangle law of vector addition. Consider the two vectors again.



Parallelogram Law of Vector Addition:

Now for using the parallelogram law, we represent both the vectors as adjacent sides of a parallelogram and then the diagonal emanating from the common point represents the sum or the resultant of the two vectors and the direction of the diagonal gives the direction of the resultant vector.



The resultant vector is shown by C. This is known as the parallelogram law of vector addition.

Commutative Property

$$\vec{C} = \vec{A} + \vec{B} = \vec{B} + \vec{A}$$

Therefore, addition of vectors obey commutative law.

Associative Property

$$\vec{D} = (\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$$

Therefore, addition of vectors obey associative law.

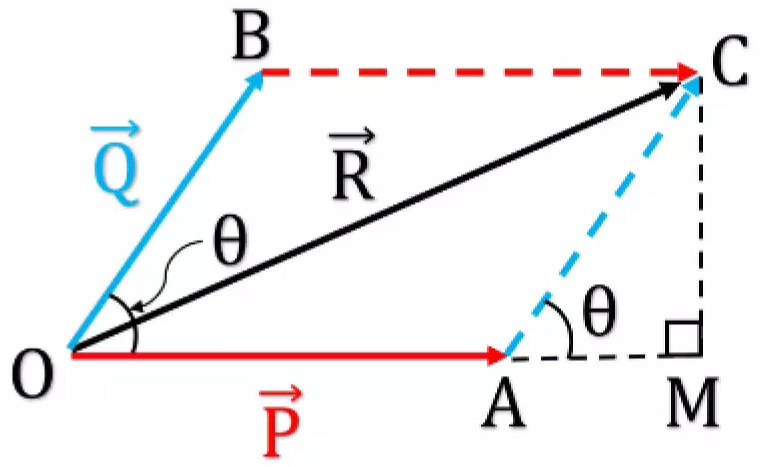
Subtraction of vectors

The subtraction of $\vec{B}$ from vector $\vec{A}$ is defined as the addition of vector $-\vec{B}$ to vector $\vec{A}$.

$$\boxed{\vec{A}-\vec{B}=\vec{A}+(-\vec{B})}$$

Analytical Method of Parallelogram Law of Vector Addition

Magnitude of Resultant



In ΔOCM,

$$OC^2 = OM^2 + CM^2$$

$$OC^2 = (OA + AM)^2 + CM^2$$

$$OC^2 = OA^2 + 2OA \times AM + AM^2 + CM^2$$

$$OC^2 = OA^2 + 2OA \times AM + AC^2$$

In ΔCAM,

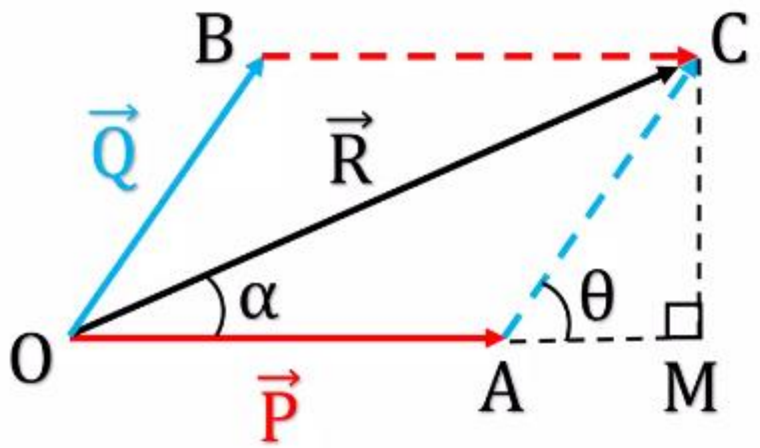
$$\cos\theta = \frac{AM}{AC} \quad \Rightarrow \quad AM = AC\cos\theta$$

$$OC^2 = OA^2 + 2OA \times AC\ \cos\theta + AC^2$$

$$R^2 = P^2 + 2P \times Q\cos\theta + Q^2$$

$$\boxed{R = \sqrt{P^2 + 2PQ\cos\theta + Q^2}}$$

Direction of Resultant



In ΔCAM,

$$\sin\theta = \frac{CM}{AC} \quad \Rightarrow \quad CM = AC\sin\theta$$

$$\cos\theta = \frac{AM}{AC} \quad \Rightarrow \quad AM = AC\cos\theta$$

In ΔOCM,

$$\tan\alpha = \frac{CM}{OM}$$

$$\tan\alpha = \frac{CM}{OA+AM}$$

$$\tan\alpha = \frac{AC\sin\theta}{OA+AC\cos\theta}$$

$$\boxed{\tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta}}$$

Case I - Vectors are parallel ($\theta = 0°$)

$$\vec{P} + \vec{Q} = \vec{R}$$

Magnitude:

$$R = \sqrt{P^2 + 2PQ\cos 0° + Q^2}$$

$$R = \sqrt{P^2 + 2PQ + Q^2}$$

$$R = \sqrt{(P+Q)^2}$$

$$\boxed{R = P + Q}$$

Direction:

$$\tan\alpha = \frac{Q\sin 0°}{P + Q\cos 0°}$$

$$\tan\alpha = \frac{0}{P+Q} = 0$$

$$\boxed{\alpha = 0°}$$

Case II - Vectors are perpendicular ($\theta = 90°$)

Magnitude:

$$R = \sqrt{P^2 + 2PQ\cos 90° + Q^2}$$

$$R = \sqrt{P^2 + 0 + Q^2}$$

$$\boxed{R = \sqrt{P^2 + Q^2}}$$

Direction:

$$\tan\alpha = \frac{Q\sin 90°}{P + Q\cos 90°} = \frac{Q}{P + 0}$$

$$\boxed{\alpha = \tan^{-1}\left(\frac{Q}{P}\right)}$$

Case III - Vectors are anti-parallel ($\theta = 180°$)

$$\vec{P} \quad - \quad \vec{Q} \quad = \quad \vec{R}$$

Magnitude:

$$R = \sqrt{P^2 + 2PQ\cos 180° + Q^2}$$

$$R = \sqrt{P^2 - 2PQ + Q^2}$$

$$R = \sqrt{(P - Q)^2}$$

$$\boxed{R = P - Q}$$

Direction:

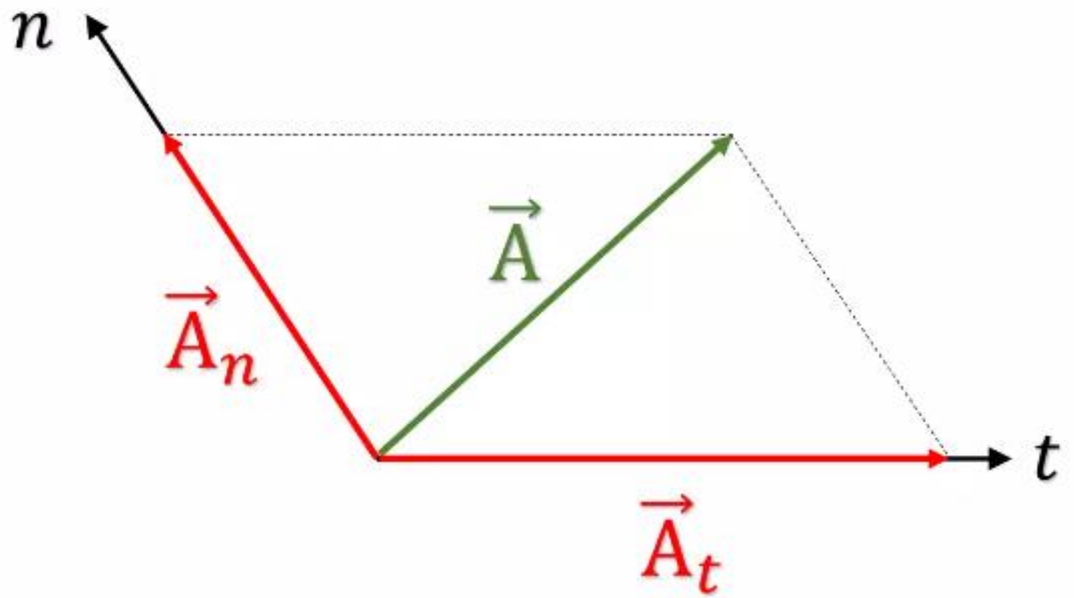
$$\tan\alpha = \frac{Q\sin 180°}{P + Q\cos 180°} = 0$$

If $P > Q$: $\boxed{\alpha = 0°}$

If $P < Q$: $\boxed{\alpha = 180°}$

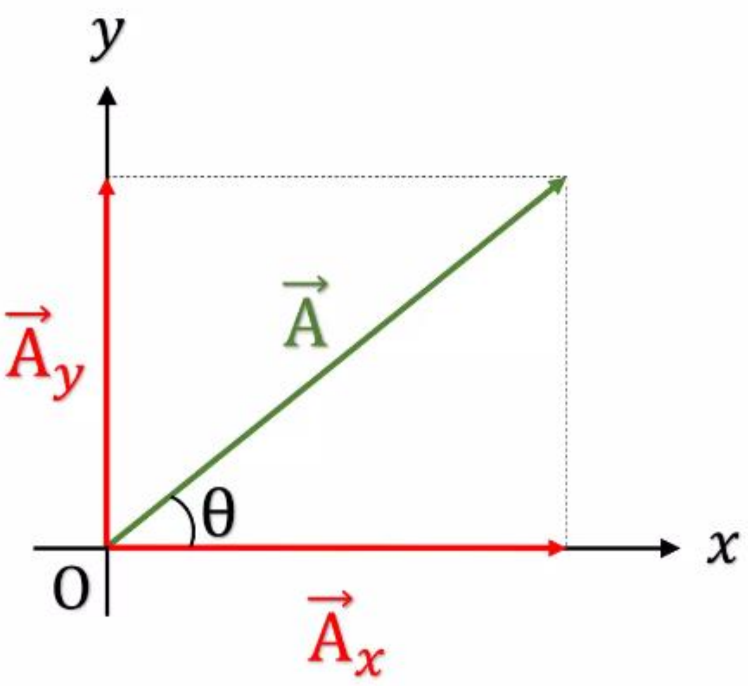
Resolution of a Vector

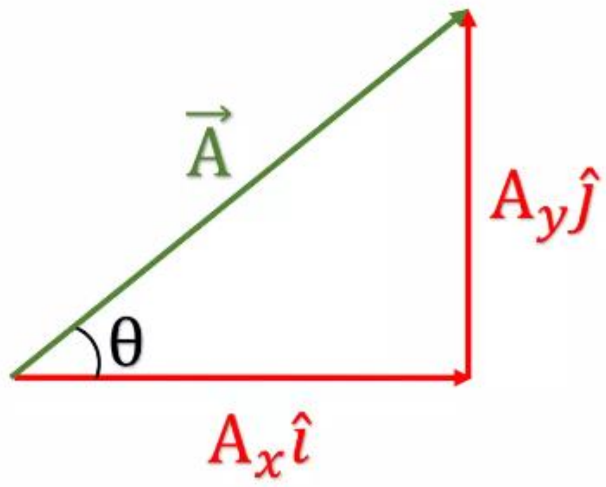
It is the process of splitting a vector into two or more vectors in such a way that their combined effect is same as that of the given vector.



- ➢ The process of splitting a vector into its components is called **resolution of the vector.**

Rectangular Components of 2D Vectors





$$\vec{A} = A_x\hat{\imath} + A_y\hat{j}$$

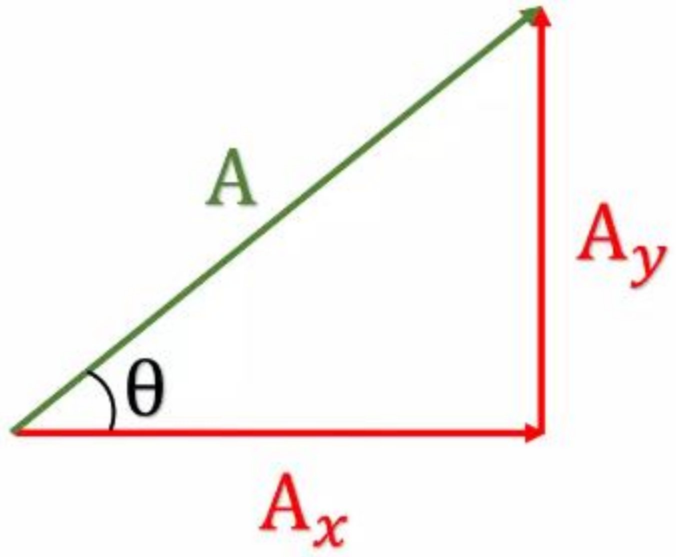
Given vector is split into two components along x axis and y axis

NOTE: In case of coplanar vector system ,given vector is resolved (split) into two Components one along x axis and other along y axis .such components are called Rectangular or Orthogonal vectors

Rectangular Components

when a vector is split into its mutually perpendicular components along the coordinate axis, these components are referred to as Rectangular components.

Rectangular Components of 2D Vectors

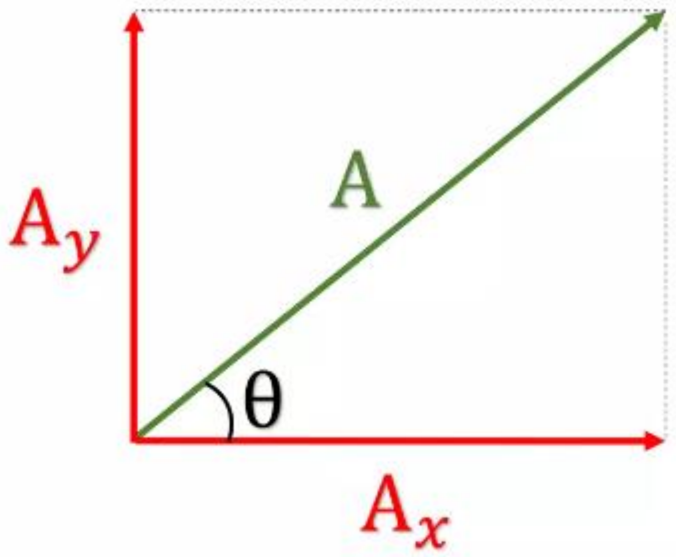


$$\sin\theta = \frac{A_y}{A} \Rightarrow \boxed{A_y = A\sin\theta}$$

$$\cos\theta = \frac{A_x}{A} \Rightarrow \boxed{A_x = A\cos\theta}$$

Magnitude & direction from components

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$$



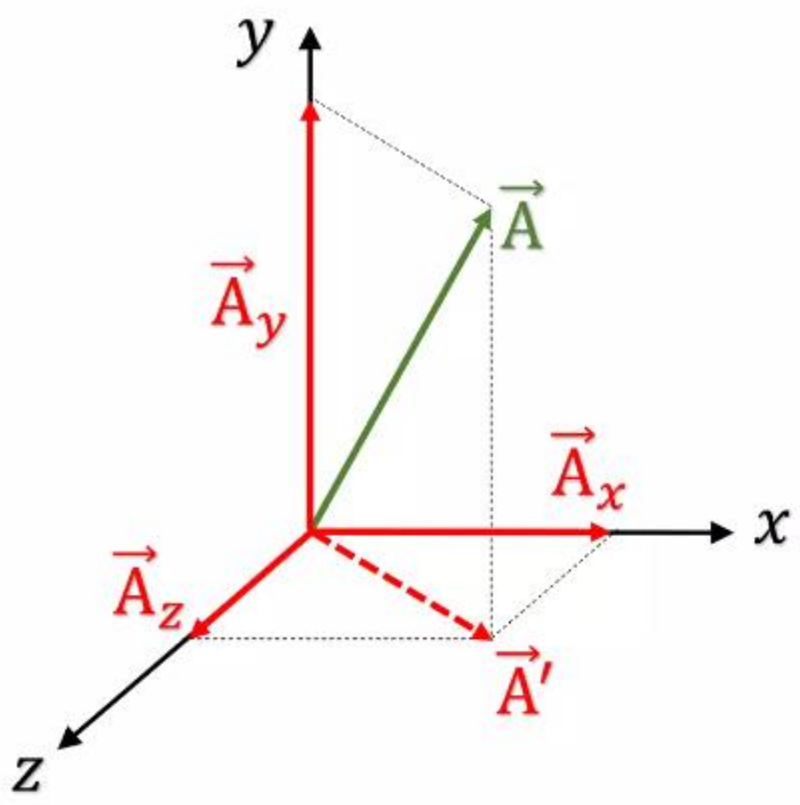
Magnitude:

$$A = \sqrt{A_x^2 + A_y^2}$$

Direction:

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

Rectangular Components of 3D Vectors



$$\vec{A} = \vec{A}' + \vec{A}_y$$

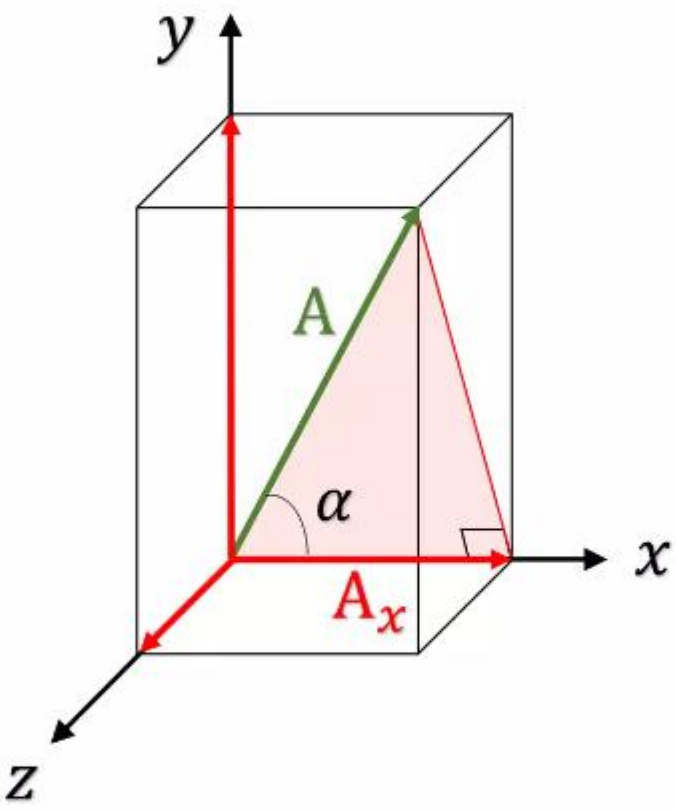
$$\vec{A} = \vec{A}_x + \vec{A}_z + \vec{A}_y$$

$$\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z$$

$$\boxed{\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}}$$

$\vec{A}'$ is the projection of $\vec{A}$ in x-z plane

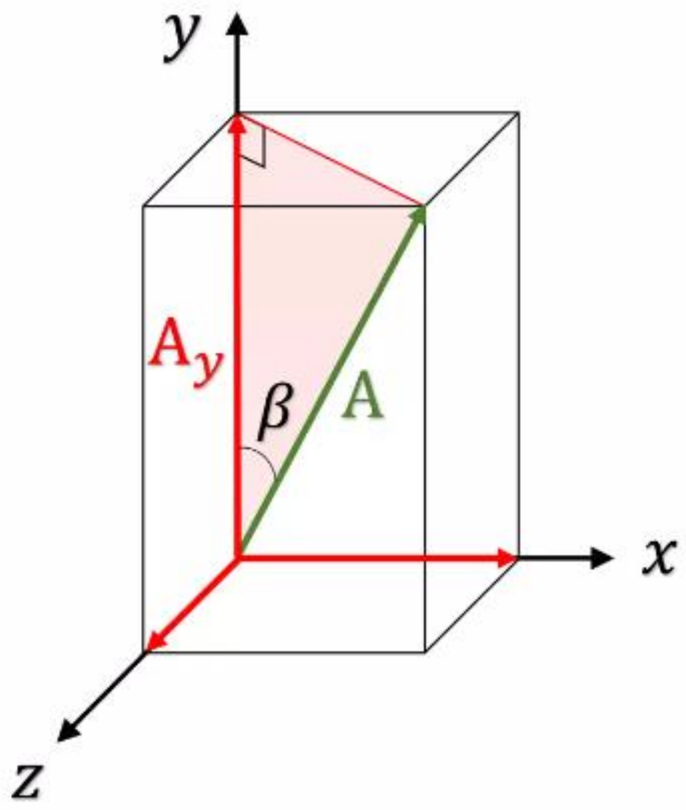
Rectangular Components of 3D Vectors



$$\cos\alpha = \frac{A_x}{A}$$

$$\boxed{A_x = A\cos\alpha}$$

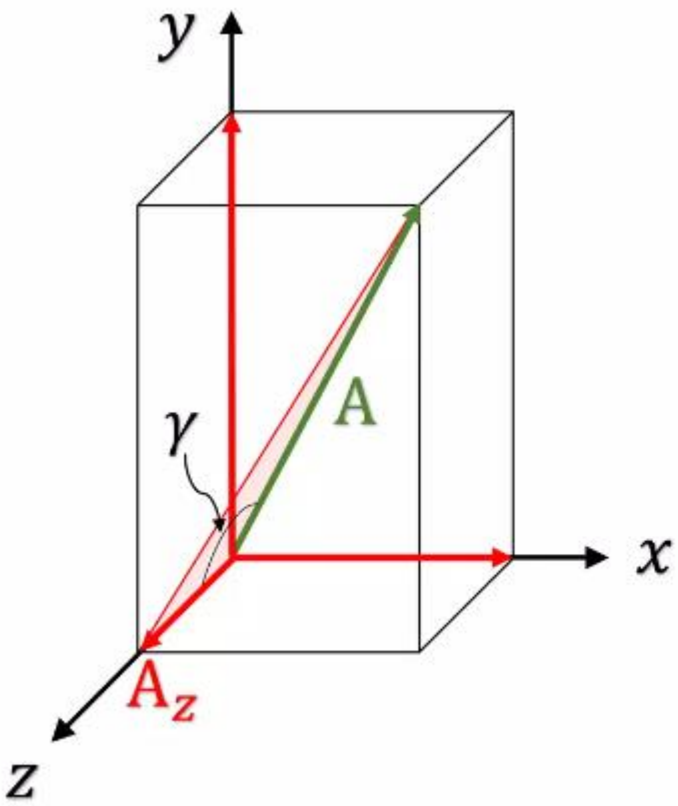
Rectangular Components of 3D Vectors



$$\cos\beta = \frac{A_y}{A}$$

$$\boxed{A_y = A\cos\beta}$$

Rectangular Components of 3D Vectors

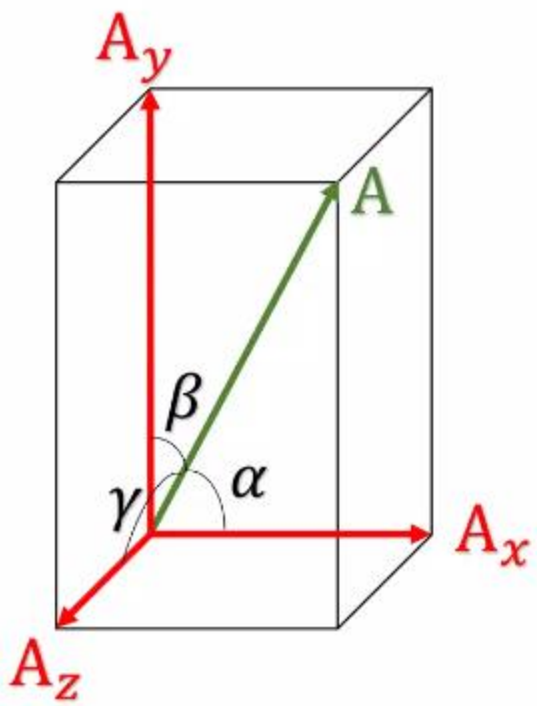


$$\cos\gamma = \frac{A_z}{A}$$

$$\boxed{A_z = A\cos\gamma}$$

Magnitude & direction from components

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$



Magnitude:

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Direction:

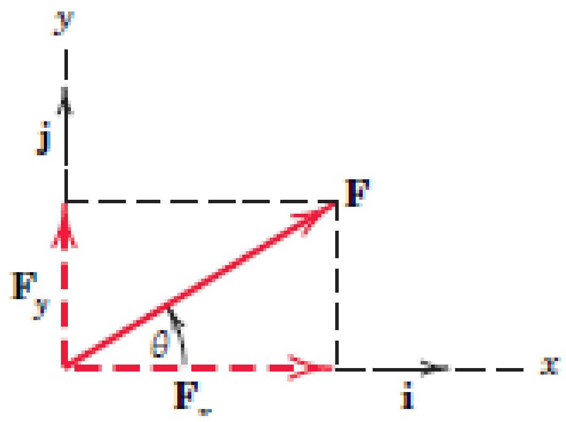
$$\alpha = \cos^{-1}\left(\frac{A_x}{A}\right) \qquad \beta = \cos^{-1}\left(\frac{A_y}{A}\right)$$

$$\gamma = \cos^{-1}\left(\frac{A_z}{A}\right)$$

Applying the Pythagorean Equation,

$$1 = \cos^2\alpha + \cos^2\beta + \cos^2\gamma$$

Two-Dimensional Force Systems



Rectangular Components

The most common two-dimensional resolution of a force vector is into rectangular components. It follows from the parallelogram rule that the vector **F** of **Fig.** may be written as

$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$$

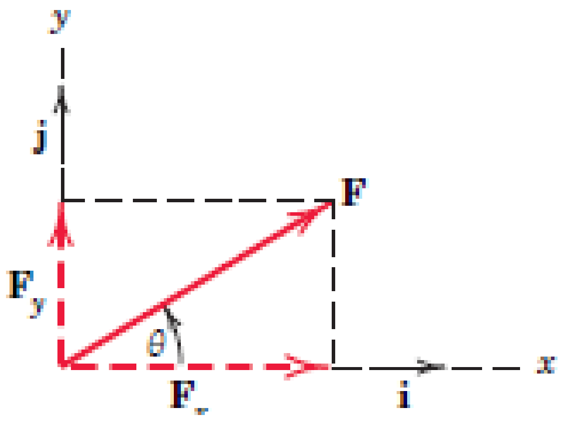
where $\mathbf{F}_x$ and $\mathbf{F}_y$ are *vector components* of $\mathbf{F}$ in the x- and y-directions. Each of the two vector components may be written as a scalar times the appropriate unit vector. In terms of the unit vectors $\mathbf{i}$ and $\mathbf{j}$ of Fig. $\mathbf{F}_x = F_x\mathbf{i}$ and $\mathbf{F}_y = F_y\mathbf{j}$, and thus we may write

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$$

where the scalars F_x and F_y are the x and y *scalar components* of the vector $\mathbf{F}$.

The scalar components can be positive or negative, depending on the quadrant into which $\mathbf{F}$ points. For the force vector of Fig. the x and y scalar components are both positive and are related to the magnitude and direction of $\mathbf{F}$ by

$F_x = F\cos\theta$	$F = \sqrt{F_x^{\,2} + F_y^{\,2}}$
$F_y = F\sin\theta$	$\theta = \tan^{-1}\dfrac{F_y}{F_x}$



In adding the force vectors $\mathbf{F}_1$ and $\mathbf{F}_2$, we may write

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = (F_{1_x}\mathbf{i} + F_{1_y}\mathbf{j}) + (F_{2_x}\mathbf{i} + F_{2_y}\mathbf{j})$$

or

$$R_x\mathbf{i} + R_y\mathbf{j} = (F_{1_x} + F_{2_x})\mathbf{i} + (F_{1_y} + F_{2_y})\,\mathbf{j}$$

from which we conclude that

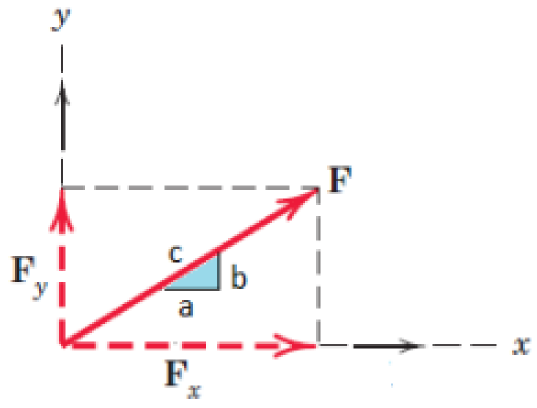
$$R_x = F_{1_x} + F_{2_x} = \Sigma F_x$$

$$R_y = F_{1_y} + F_{2_y} = \Sigma F_y$$

The term ΣF_x means "the algebraic sum of the x scalar components".

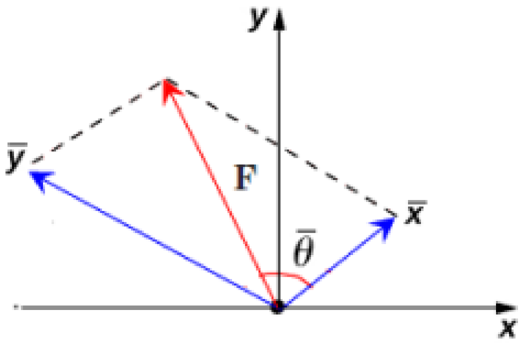
The direction of ***F*** can also be defined using a small "slope" triangle. Given the slope of the line of action of the force as

$$c = \sqrt{a^2 + b^2}$$



$$Fx = F\cos\theta \quad \rightarrow \quad Fx = F \cdot \frac{a}{c} \quad \rightarrow$$

$$Fy = F\sin\theta \quad \rightarrow \quad Fy = F \cdot \frac{b}{c} \quad \uparrow$$



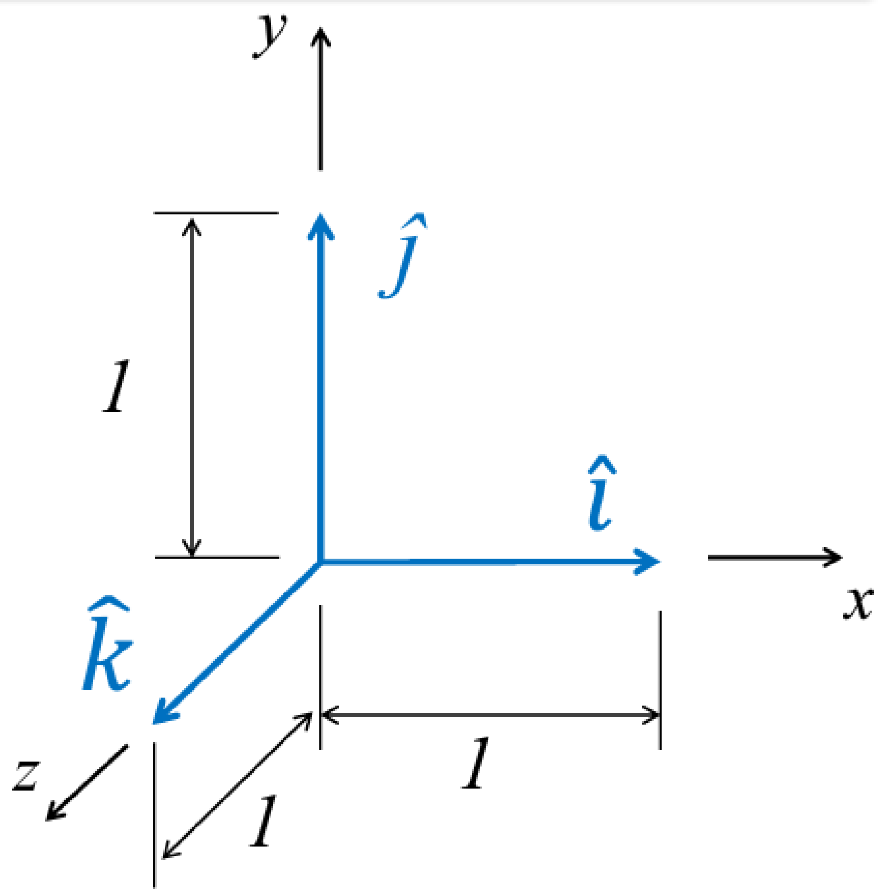
$$F\bar{x} = F\cos\bar{\theta} \quad \nearrow$$

$$F\bar{y} = F\sin\bar{\theta} \quad \nwarrow$$

Forces in Three-Dimensional Space

Rectangular Components of a Force in Three-Dimensional Space

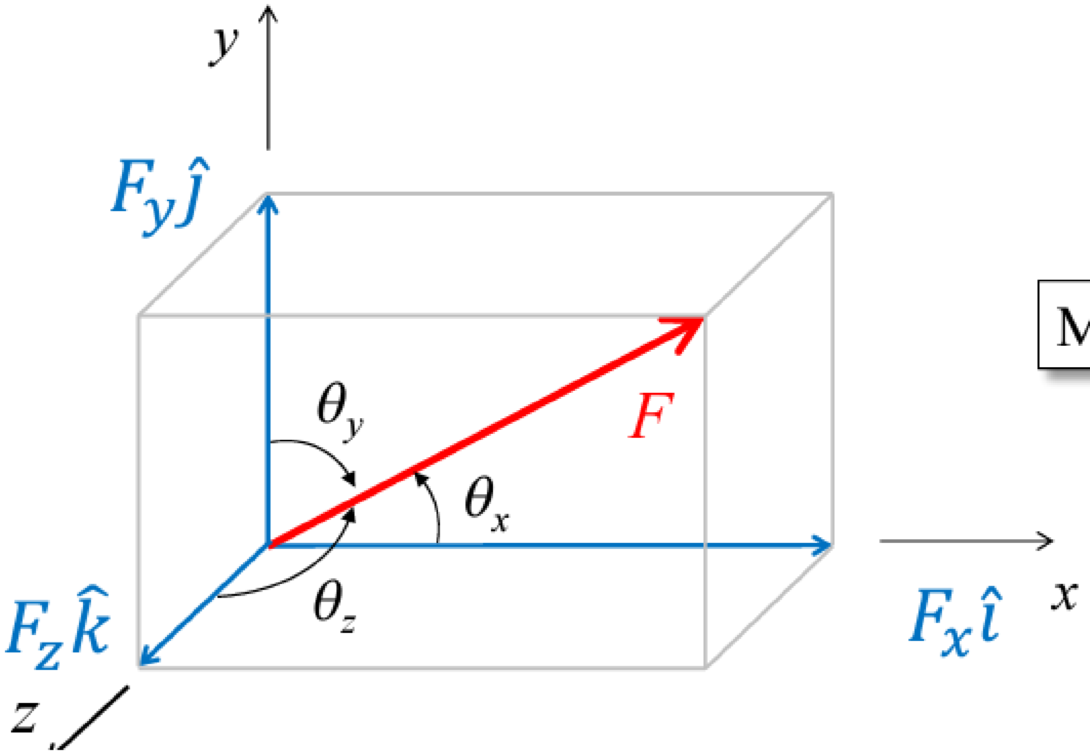
Define unit vectors in the *x*, *y* and *z* directions



Rectangular Components of a Force in Three-Dimensional Space

Cartesian Vector Form of $\boldsymbol{F}$

$$\boldsymbol{F} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}$$



Scalar components of $\boldsymbol{F}$

$$F_x = F\cos\theta_x$$

$$F_y = F\cos\theta_y$$

$$F_z = F\cos\theta_z$$

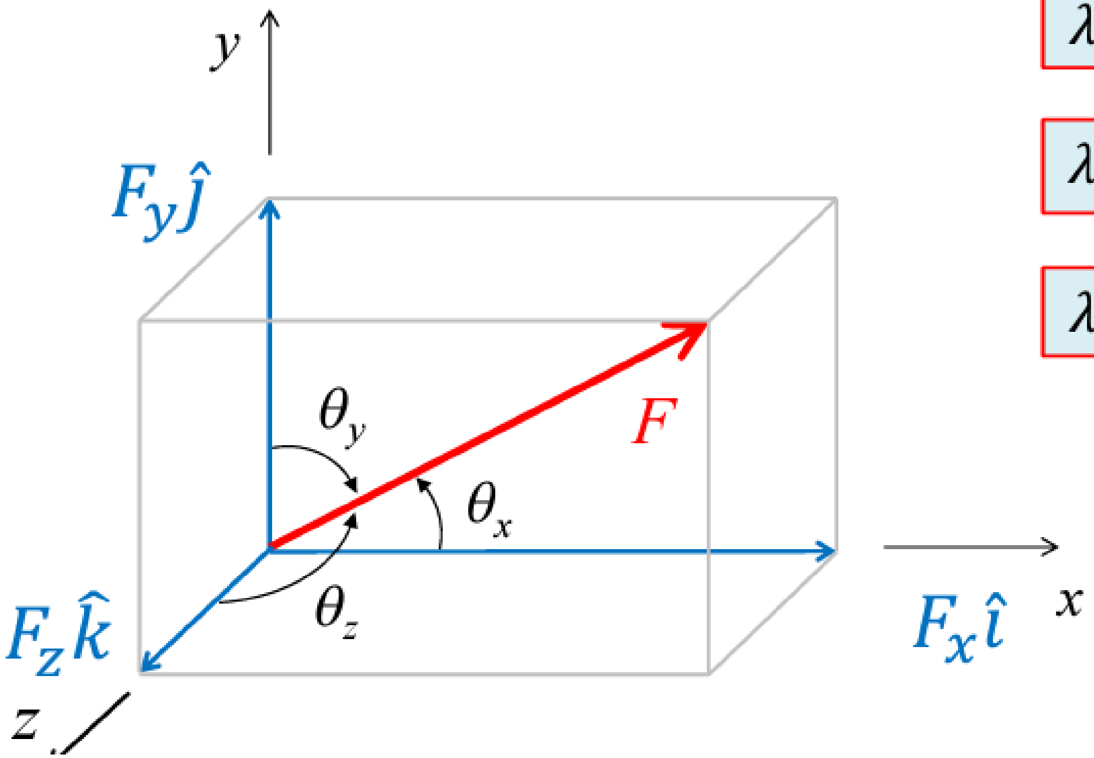
Magnitude of $\boldsymbol{F}$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

Rectangular Components of a Force in Three-Dimensional Space

$$\boldsymbol{F} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}$$

Direction of $\boldsymbol{F}$ is defined by direction cosines



$$\lambda_x = \cos\theta_x$$

$$\lambda_y = \cos\theta_y$$

$$\lambda_z = \cos\theta_z$$

Rectangular Components of a Force in Three-Dimensional Space

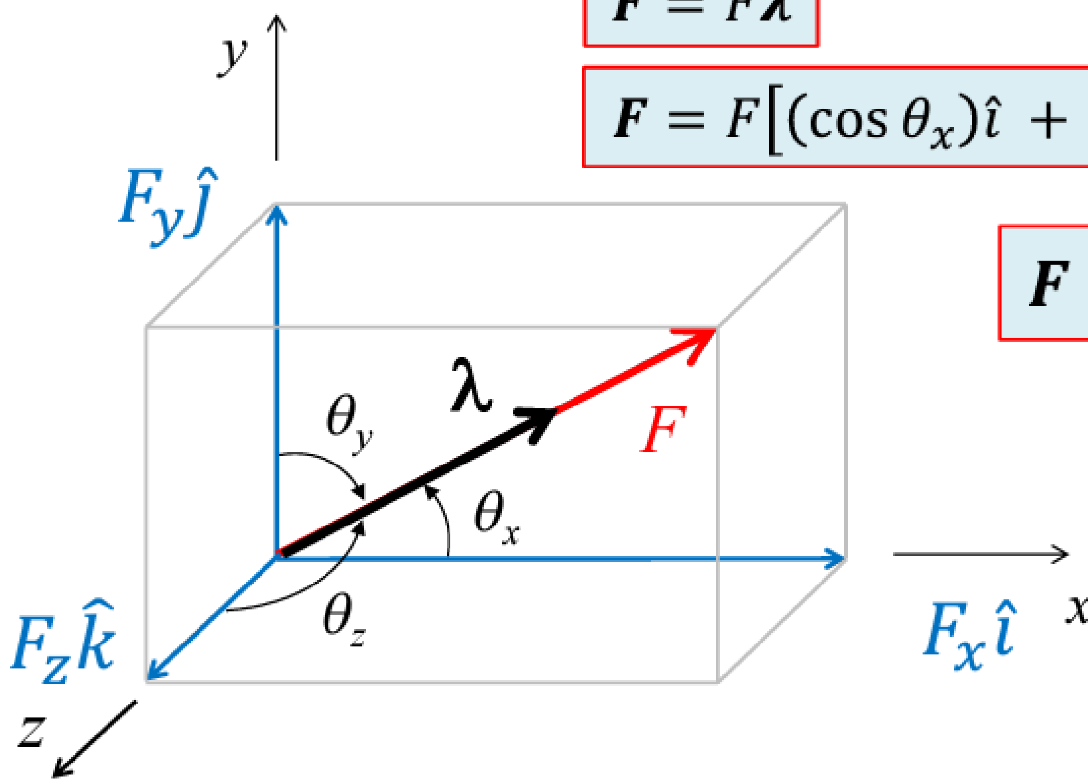
unit vector in the direction of ***F***

$$\boldsymbol{\lambda} = (\cos\theta_x)\hat{\imath} + (\cos\theta_y)\hat{\jmath} + (\cos\theta_z)\hat{k}$$

$$\boldsymbol{F} = F\boldsymbol{\lambda}$$

$$\boldsymbol{F} = F\left[(\cos\theta_x)\hat{\imath} + (\cos\theta_y)\hat{\jmath} + (\cos\theta_z)\hat{k}\right]$$

The force magnitude F times a unit vector λ which characterizes the direction of **F**



$$\boldsymbol{F} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}$$

$$\lambda = \lambda_x i + \lambda_y j + \lambda_z k$$

$$F_x = F\cos\theta_x$$

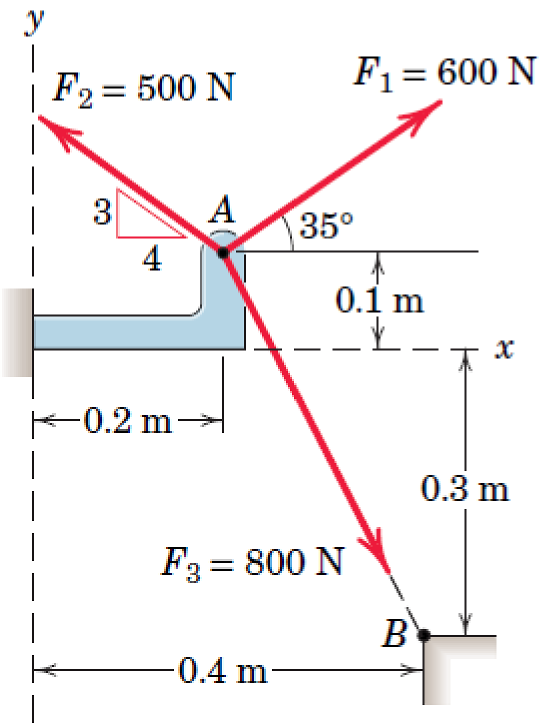
$$F_y = F\cos\theta_y$$

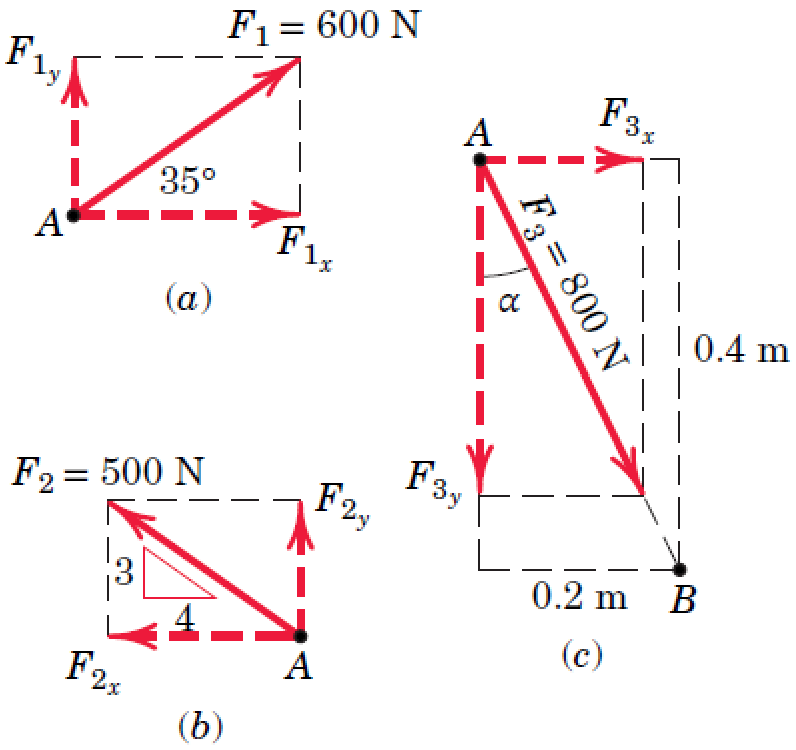
$$F_z = F\cos\theta_z$$

The scalar components of the unit vector λ are the direction cosine of the line of action of F

Problems

The forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$, all of which act on point A of the bracket, are specified in three different ways. Determine the x and y scalar components of each of the three forces.





Solution The scalar components of $\mathbf{F}_1$, from Fig. *a*, are

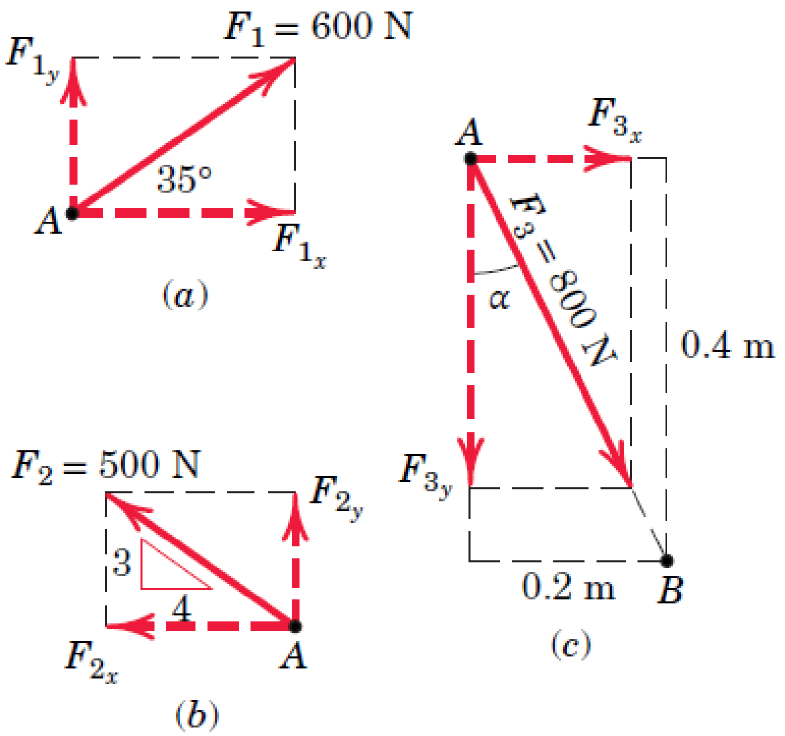
$$F_{1_x} = 600 \cos 35° = 491 \text{ N}$$

$$F_{1_y} = 600 \sin 35° = 344 \text{ N}$$

The scalar components of $\mathbf{F}_2$, from Fig. *b*, are

$$F_{2_x} = -500(\tfrac{4}{5}) = -400 \text{ N}$$

$$F_{2_y} = 500(\tfrac{3}{5}) = 300 \text{ N}$$



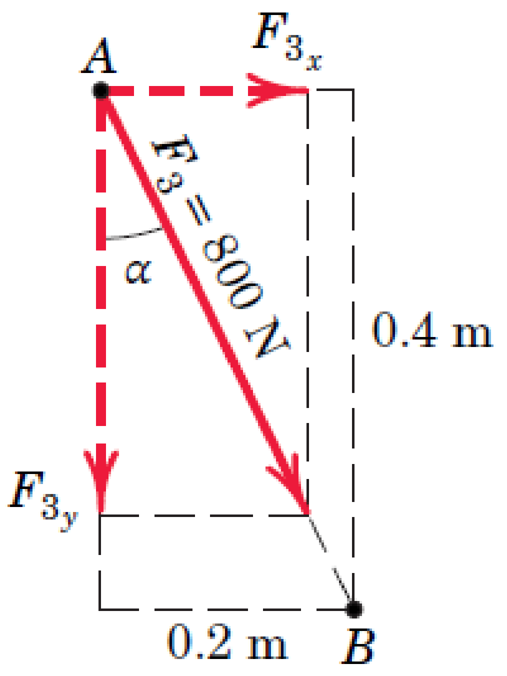
Note that the angle which orients $\mathbf{F}_2$ to the x-axis is never calculated. The cosine and sine of the angle are available by inspection of the 3-4-5 triangle. Also note that the x scalar component of $\mathbf{F}_2$ is negative by inspection.

The scalar components of $\mathbf{F}_3$ can be obtained by first computing the angle α of Fig. c.

$$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6°$$

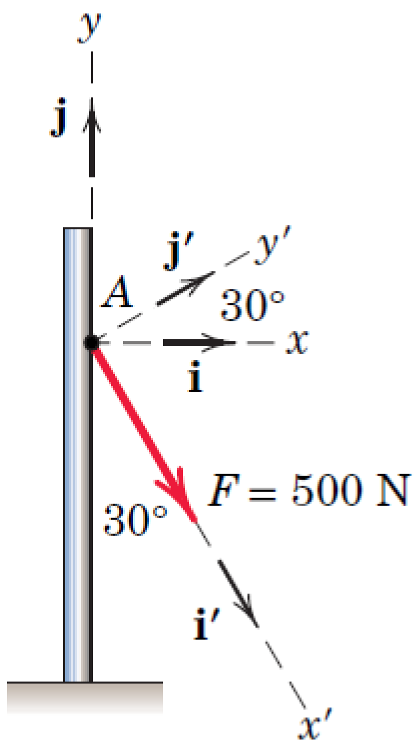
Then, $$F_{3_x} = F_3 \sin\alpha = 800 \sin 26.6° = 358 \text{ N}$$ ①

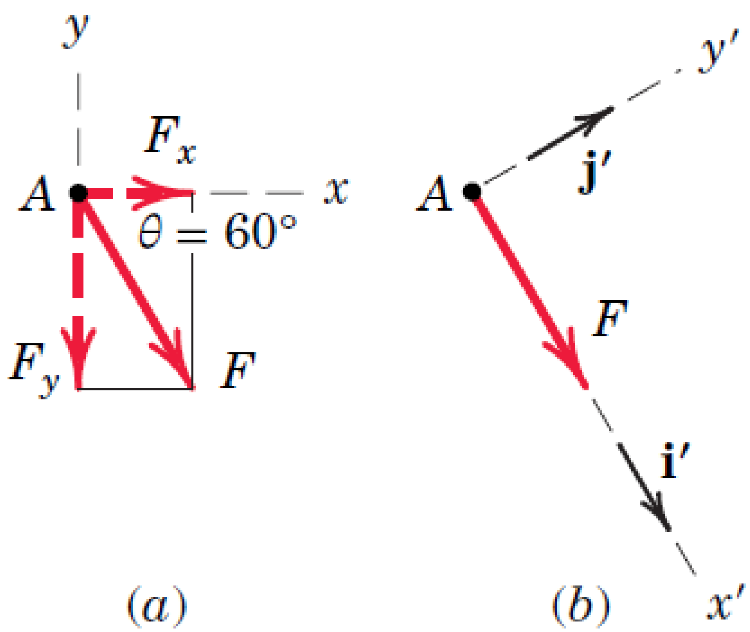
$$F_{3_y} = -F_3 \cos\alpha = -800 \cos 26.6° = -716 \text{ N}$$



(c)

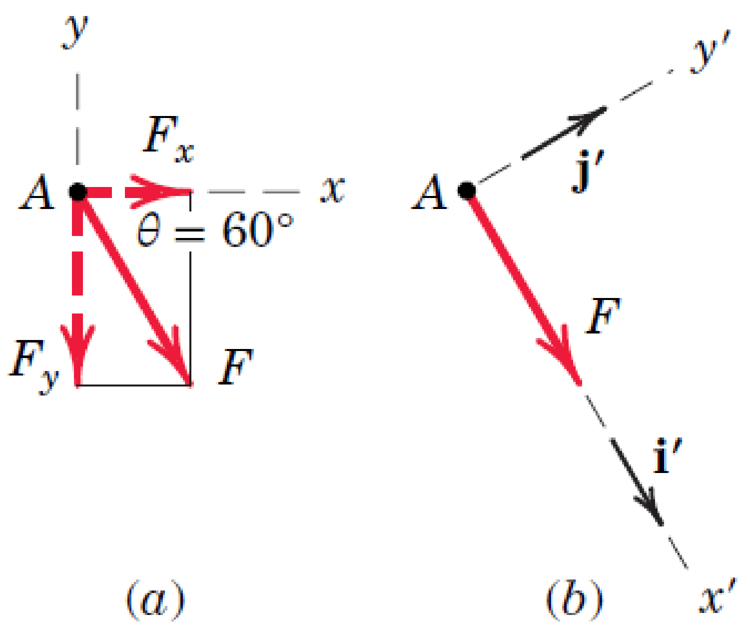
The 500-N force **F** is applied to the vertical pole as shown. (1) Write **F** in terms of the unit vectors **i** and **j** and identify both its vector and scalar components. (2) Determine the scalar components of the force vector **F** along the x'- and y'-axes.





Solution ***Part (1).*** From Fig. a we may write $\mathbf{F}$ as

$$\mathbf{F} = (F\cos\theta)\mathbf{i} - (F\sin\theta)\mathbf{j}$$
$$= (500\cos 60^\circ)\mathbf{i} - (500\sin 60^\circ)\mathbf{j}$$
$$= (250\mathbf{i} - 433\mathbf{j})\ \text{N}$$



The scalar components are $F_x = 250$ N and $F_y = -433$ N. The vector components are $\mathbf{F}_x = 250\mathbf{i}$ N and $\mathbf{F}_y = -433\mathbf{j}$ N.

Part (2). From Fig. b we may write $\mathbf{F}$ as $\mathbf{F} = 500\mathbf{i}'$ N, so that the required scalar components are

$$F_{x'} = 500\ \text{N} \qquad F_{y'} = 0$$

SYSTEM OF FORCES: When two or more forces act on a body, they are called to form a system of forces. Following systems of forces are important from the subject point of view;

1. **Coplanar forces**: The forces, whose lines of action lie on the same plane, are known as coplanar forces.
2. **Collinear forces**: The forces, whose lines of action lie on the same line, are known as collinear forces
3. **Concurrent forces:** The forces, which meet at one point, are known as concurrent forces. The concurrent forces may or may not be collinear.
4. **Coplanar concurrent forces**: The forces, which meet at one point and their lines of action also lie on the same plane, are known as coplanar concurrent forces.
5. **Coplanar non-concurrent forces:** The forces, which do not meet at one point, but their lines of action lie on the same plane, are known as coplanar non-concurrent forces.
6. **Non-coplanar concurrent forces**: The forces, which meet at one point, but their lines of action do not lie on the same plane, are known as non-coplanar concurrent forces.
7. **Non-coplanar non-concurrent forces**: The forces, which do not meet at one point and their lines of action do not lie on the same plane, are called non-coplanar non-concurrent forces.

We will find out the resultant force for many forces acting on a rigid body by using the following equations:-

$$R_X = \sum F_X$$

$$R_Y = \sum F_Y$$

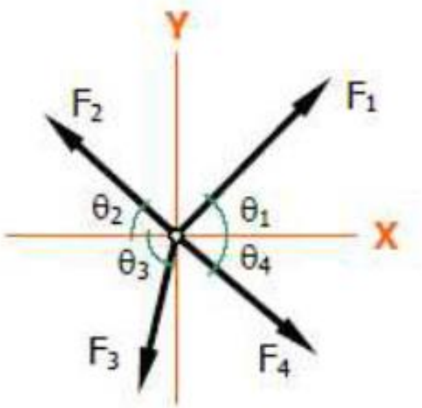
$$R = \sqrt{{R_X}^2 + {R_Y}^2}$$

The direction of resultant force may be determined as:

$$\theta = \tan^{-1}(\frac{R_Y}{R_X})$$

Where θ is the angle between the resultant and the x axis. The resultant passes through the point of concurrence of the forces of the system, and its sense can be detetmined from the components R_X and R_Y.

In x-y plane, the resultant of coplanar concurrent force system where the lines of action of all forces pass through a common point can be found by the following formulas:

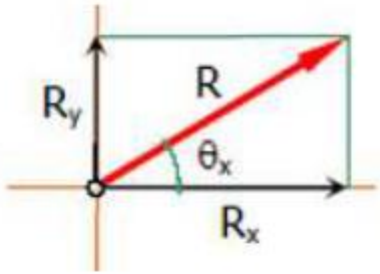


$$R_x = \sum F_x \quad \rightarrow^{+}$$

$$R_x = F_{1x} - F_{2x} - F_{3x} + F_{4x}$$

$$R_y = \sum F_y \quad \uparrow^{+}$$

$$R_y = F_{1y} + F_{2y} - F_{3y} - F_{4y}$$

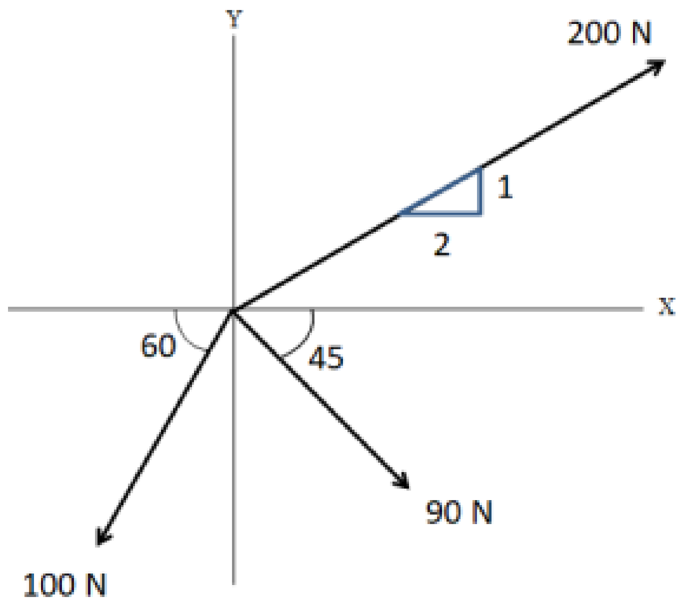


$$R = \sqrt{{R_x}^2 + {R_y}^2}$$

$$\theta_x = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

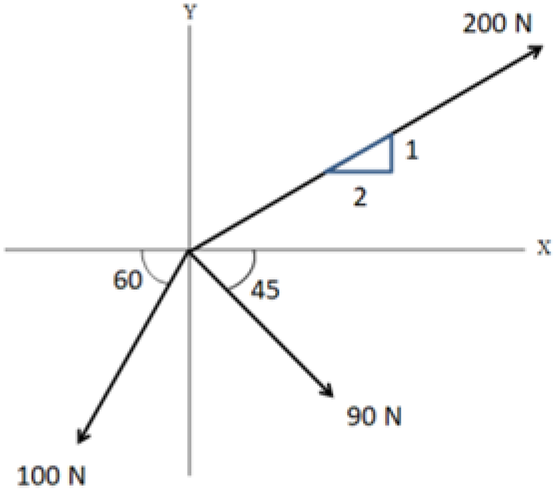
Examples

Find the resultant force for the concurrent coplanar force system, shown in figure.



Example

Find the resultant force for the concurrent coplanar force system, shown in figure.



Solution:-

$$\rightarrow R_X = \sum F_X$$

$$= 200\,\frac{2}{\sqrt{5}} - 100\cos 60 + 90\cos 45 = +192.4\text{ N} \rightarrow$$

$$\uparrow R_Y = \sum F_Y$$

$$= 200\,\frac{1}{\sqrt{5}} - 100\sin 60 - 90\sin 45 = -60.8\text{ N}$$

$$= 60.8\text{N} \downarrow$$

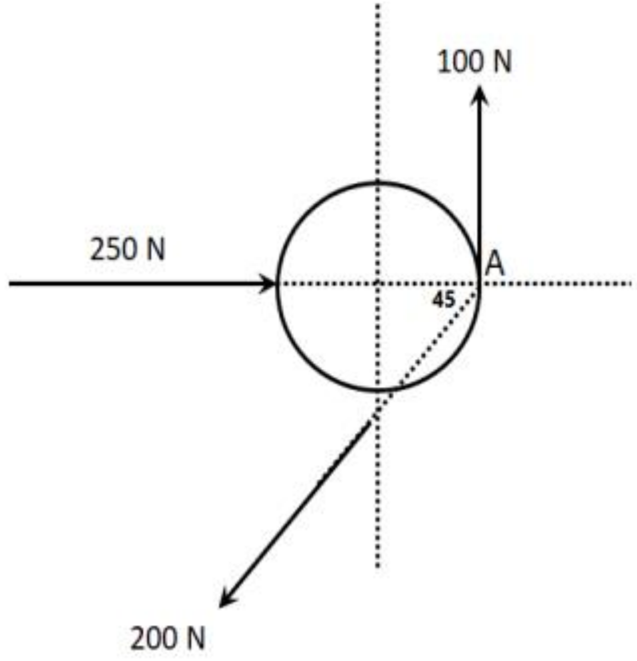
$$R = \sqrt{(R_X)^2 + (R_Y)^2} = \sqrt{(192.4)^2 + (60.8)^2} = 202\text{ N}$$

$$\theta = \tan^{-1}\left(\frac{R_Y}{R_X}\right)$$

$$\theta = \tan^{-1}\left(\frac{60.8}{192.4}\right) = -17.5$$

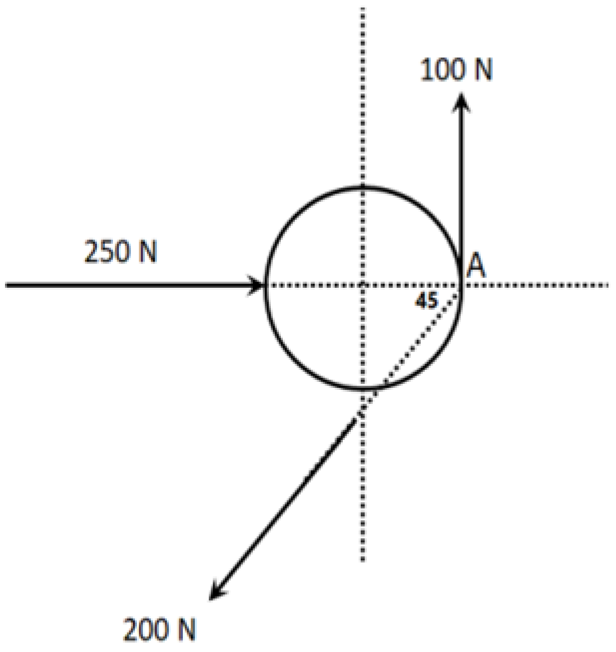
Example

Determine the resultant force for the forces system shown in figure.



Example

Determine the resultant force for the forces system shown in figure.



Solution:-

$\rightarrow R_X = \sum F_X$

$=250 - 200\cos 45 = 108.6N\rightarrow$

$\uparrow R_Y = \sum F_Y$

$=100 - 200 \sin 45 = -41.4N = 41.4N \downarrow$

$R = \sqrt{(R_X)^2 + (R_Y)^2}$

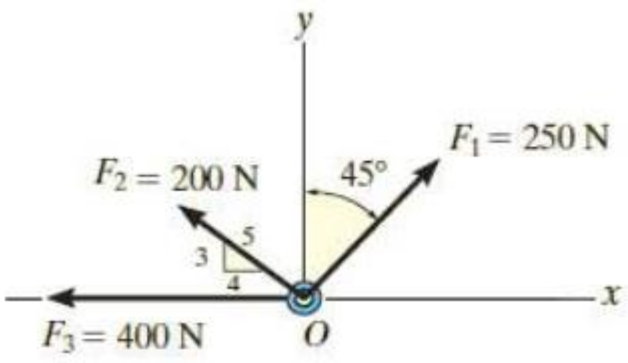
$= \sqrt{(108.6)^2 + (41.4)^2}$

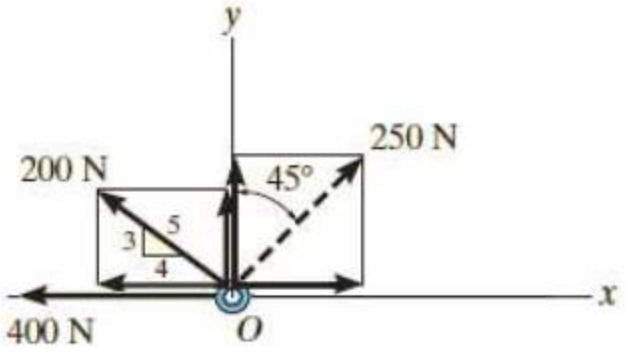
$=116.2 \text{ N}$

$\theta = \tan^{-1}(\frac{R_Y}{R_X})$

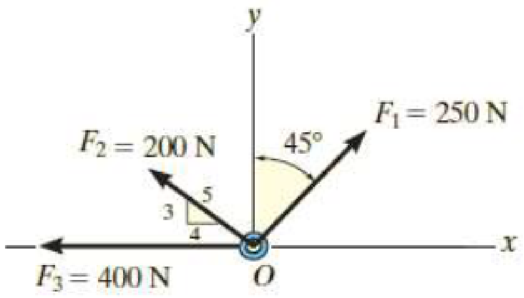
$\theta = \tan^{-1}(\frac{41.4}{108.6}) = -20.8$

Example Determine the magnitude and direction of the resultant forces system shown in Figure.

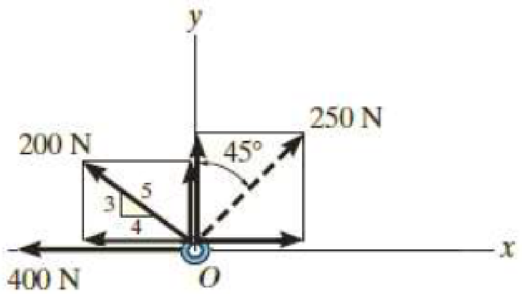




Example Determine the magnitude and direction of the resultant forces system shown in Figure.



Solution:



$$F_{1x} = 250 \times \sin 45 = 176.8\ N \ \rightarrow$$

$$F_{1y} = 250 \times \cos 45 = 176.8\ N \ \uparrow$$

$$F_{2x} = 200 \times \frac{4}{5} = 160\ N \ \leftarrow$$

$$F_{2y} = 200 \times \frac{3}{5} = 120\ N \ \uparrow$$

$$F_{3x} = 400\ N \ \leftarrow$$

$$F_{3y} = 0$$

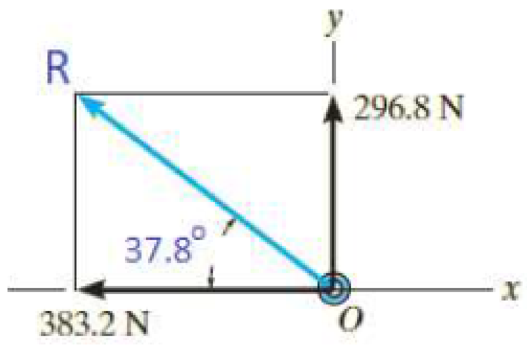
$$\xrightarrow{+} R_x = \sum F_x = 176.8 - 160 - 400$$

$$R_x = -383.2\ N = \ 383.2\ N \leftarrow$$

$$\uparrow^{+} R_y = \sum F_y = 176.8 + 120 + 0 = 296.8\ N \quad \uparrow$$

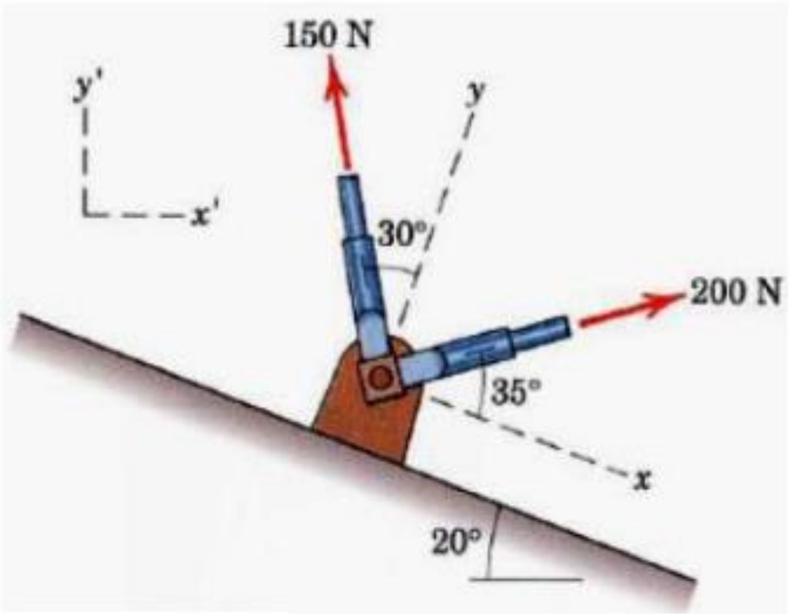
$$R = \sqrt{{R_x}^2 + {R_y}^2} = \sqrt{(383.2\ )^2 + (296.8)^2} = 484.7\ N$$

$$\theta_x = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{296.8}{383.2}\right) = 37.8^{\circ}$$



Example

Find the resultant of the two force shown in figure.



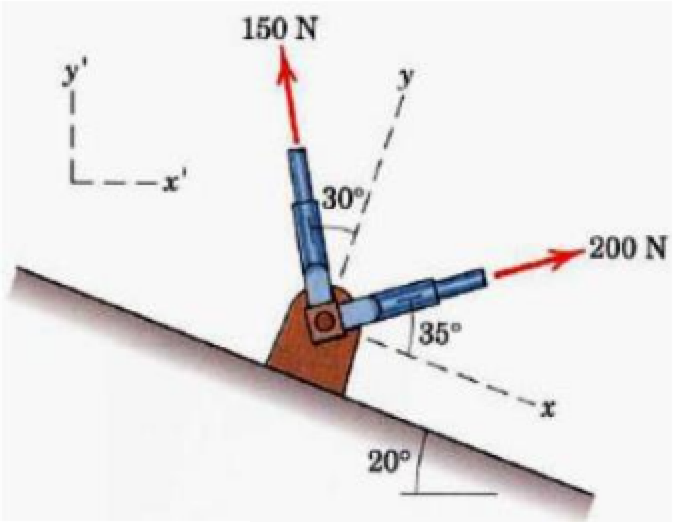
Example

Find the resultant of the two force shown in figure.

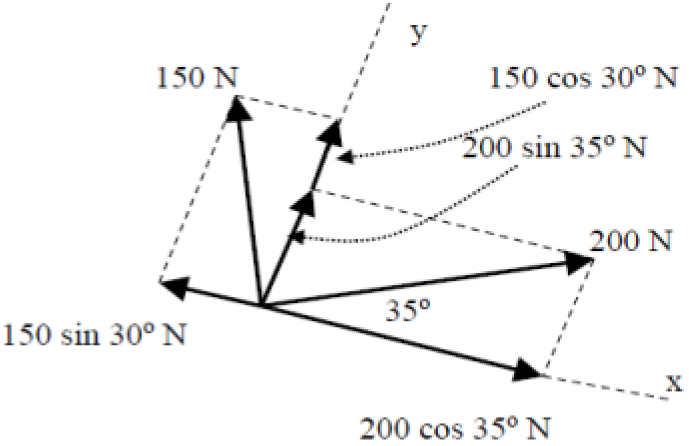
Solution:-

$\rightarrow R_x = \sum F_x = 200\cos 35 - 150\sin 30 =$
88.8 N

$\uparrow R_y = \sum F_y = 200\sin 35 + 150\cos 30 =$
245 N

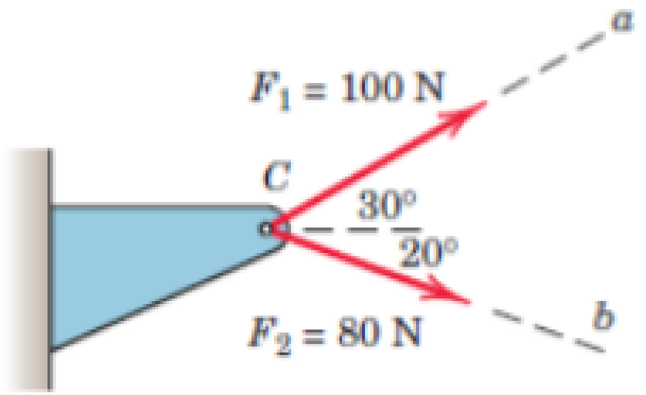


$$R = \sqrt{(R_X)^2 + (R_Y)^2} = \sqrt{88.8^2 + 245^2}$$
$$= 260.6N$$



Example

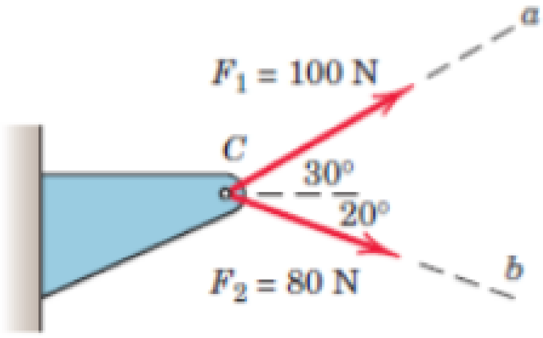
Forces F1 and F2 acts on the bracket as shown in figure. Determine their resultant R.



Example

Forces F1 and F2 acts on the bracket as shown in figure. Determine their resultant R.

Solution:-



$$\rightarrow R_X = \sum F_X$$

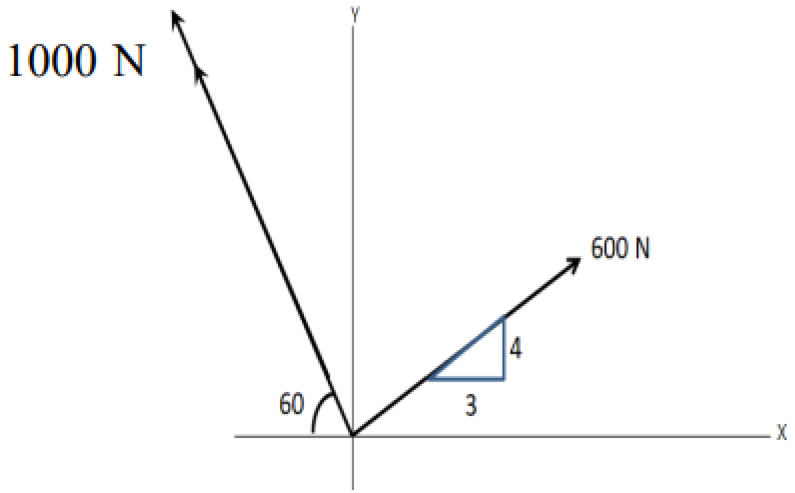
$$= 100\cos 30 + 80\cos 20 = 161.77\text{N} \rightarrow$$

$$\uparrow R_Y = \sum F_Y = 100\sin 30 - 80\sin 20 = 22.63\text{N} \uparrow$$

$$R = \sqrt{(R_X)^2 + (R_Y)^2} = \sqrt{161.77^2 + 22.63^2} = 163.4\text{N}$$

Example

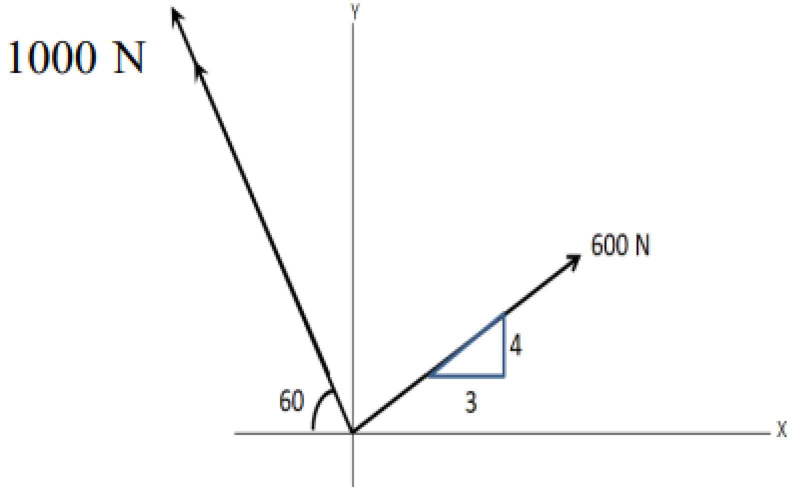
The 1000 N force is a resultant of two forces, one of which is 600 N, Determine the other force.



Example (3):-

The 1000 N force is a resultant of two forces, one of which is 600 N, Determine the other force.

Solution:-



$\rightarrow R_X = \sum F_X$

$-1000 \cos 60 = 600\frac{3}{5} + F2_X$

$-1000 \times 0.5 = 360 + F2_X$

$F2_X = -860\ N = 860N \leftarrow$

$\uparrow R_Y = \sum F_Y$

$1000 \sin 60 = 600\frac{4}{5} + F2_Y$

$F2_Y = 386.02\ N \uparrow$

$F = \sqrt{(860)^2 + (386.02)^2}$

$=942.62$ N

$= \tan^{-1}(\frac{R_Y}{R_X})$

$= \tan^{-1}(\frac{386}{860}) = 24.17$

Thank You…