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Amrita School of Engineering, Amaravati
B. Tech. CCE (I Semester)
End Sem Examination–December 2024
23MAT124-Engineering Mathematics-I

Maximum marks : 100

Duration :3 Hours

Course Outcomes(COs)

CO	Course Outcomes
01	Solve problems involving limits, derivatives and ODEs
02	Model and solve system of linear equations
03	Characterize systems using Eigen values and vectors
04	Apply the mathematical concepts learnt, to engineering problems

Answer ALL questions

1. (a) Define the following terms with one example each: (i) Positive Definite Matrix (ii) Negative Semi-Definite Matrix.

(b) Find the values of a in order that the rank of the matrix A is 3, where $A = \begin{pmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ a & 2 & 2 & 2 \\ 9 & 9 & a & 3 \end{pmatrix}$.

[4+6][CO02][BTL1]

2. (a) Given $f(x) = \int_1^{\sqrt{x} \tan x} \ln t \, dt$, compute $f'(x)$.

(b) Find the Rank of the matrix(using elementary row operation), where $A = \begin{pmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{pmatrix}$.

[3+7][CO01 & CO02][BTL2]

3. (a) Evaluate $\lim_{x \rightarrow \infty} \frac{x^2}{2x}$.

(b) Reduce the matrix to **normal form** and hence find the rank, where $A = \begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$.

[3+7][CO01 & CO02][BTL4]

4. (a) What is a Linear differential equation? Provide an example.

(b) Find the inverse of the matrix by using elementary transformations, where $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 7 \end{pmatrix}$.

[3+7][CO01 & CO02][BTL3]

5. (a) Examine the consistency and solve if possible, the following system of equations

$$x - 7y + 15z = -14; \quad 2x + 3y - 4z = 6; \quad 3x - 4y + 11z = -8; \quad 5x - y + 7z = -2.$$

(b) Find the values of λ and μ such that the following system of equation has (i) no solution (ii) unique solution (iii) many solutions

$$x + 2y + 3z = 5; \quad x + 3y - z = 4; \quad x + 4y + \lambda z = \mu.$$

[4+6][CO02][BTL4]

6. (a) Solve $x^2y \, dx - (x^3 + y^3)dy = 0$.

(b) Solve $x \frac{dy}{dx} - 2y = x^4 e^x$.

[5+5][CO01 & CO04][BTL3]

7. (a) Solve $x \frac{dy}{dx} + y = 3x^3y^2$.

(b) Solve $\frac{dy}{dx} = \frac{x^2 e^{\frac{y}{x}} + y^2}{xy}$.

[5+5][CO01 & CO04][BTL3]

8. (a) Evaluate: $\lim_{x \rightarrow 0} \frac{1}{x^3} \int_0^x \frac{t^3 - t^5}{1+t^2} dt$.

(b) Find the maximum (local maximum) and minimum (local minimum) points of the function

$$f(x, y) = x^4 - 2x^2 + y^4 - 2y^2 + 5.$$

[3+7][CO01][BTL4]

9. (a) Let the function $f(x, y)$ be defined as follows:

$$f(x, y) = \begin{cases} \frac{x^3 y}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0), \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$$

Determine whether the function $f(x, y)$ is continuous and differentiable at the point $(0, 0)$.

- (b) Let the function $q(x)$ be given as:

$$q(x) = \begin{cases} \frac{x^3 - x}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

Is $q(x)$ continuous and differentiable at $x = 0$?

[5+5][CO02][BTL3]

10. Check whether the matrix is diagonalizable $B = \begin{pmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ -3 & 4 & 6 \end{pmatrix}$.

[10][CO03][BTL4]

CO	Marks	BTL	Marks
CO01	30	BTL1	10
CO02	20	BTL2	20
CO03	30	BTL3	40
CO04	20	BTL4	30
		BTL5	0
		BTL6	0