ASSIGNMENT 2

1. Determine whether the following relations R on the set of all integers are reflexive, symmetric, antisymmetric, transitive, asymmetric, and/or irreflexive.
   (a) $R_1 = \{(x, y) : x + y = 0\}$
   (b) $R_2 = \{(x, y) : x = \pm y\}$
   (c) $R_3 = \{(x, y) : x = y + 1 \text{ or } x = y - 1\}$
   (d) $R_4 = \{(x, y) : x = 1\}$
   (e) $R_5 = \{(x, y) : x \text{ and } y \text{ are both negative or both nonnegative}\}$
2. Let $R = \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$ be a relation defined on $A = \{1, 2, 3, 4\}$. Find
   (a) Matrix representations of R, R^{-1}, $\bar{R}$, R^2.
   (b) Directed graph representations of R, R^{-1}, $\bar{R}$, R^2.
3. Find the reflexive closure, symmetric closure, and transitive closure of the following relations.
   (a) $R_1 = \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$ defined on $A = \{1, 2, 3, 4\}$.
   (b) $R_2 = \{(a, b) : a > b\}$ defined on $\mathbb{Z}$.
4. Show that the relation R on the set of all differentiable functions from $\mathbb{R}$ to $\mathbb{R}$ consisting of all pairs (f, g) such that $f'(x) = g'(x)$ for all real numbers x is an equivalence relation. Which functions are in the same equivalence class as the function $f(x) = x^2$?
5. Is (S, R) a poset if S is the set of all people in the world, and $(a, b) \in R$, where a and b are people, if
   (a) a is no shorter than b?
   (b) $a = b$ or a is a descendant of b?
   (c) a and b do not have a common friend?
6. Prove or disprove the following statements.
   (a) If a, b, c and d are integers such that a/b and c/d then $a + c/b + d$.
   (b) If a, b, c and d are integers such that a/b and c/d then ac/bd.
   (c) If a, b and c are integers such that $a \nmid b$ and $b \nmid c$ then $a \nmid c$.
7. (a) Do there exist integers x and y such that $x + y = 100$ and $\gcd(x, y) = 8$? Why or why not?
   (b) Prove that there exist infinitely many pairs of integers x and y such that $x + y = 87$ and $\gcd(x, y) = 3$.
8. (a) Prove that 7 has no expression as an integral linear combination of 18209 and 19043.
   (b) Find two rational numbers with denominators 11 and 13, respectively, and a sum of $\frac{7}{143}$.
9. Find all pairs of positive integers a and b such that $\gcd(a, b) = 12$ and $\operatorname{lcm}(a, b) = 360$.
10. Prove that the integer $53^{103} + 103^{53}$ is divisible by 39, and that $111^{333} + 333^{111}$ is divisible by 7.