Amrita Viswa Vidyapeetham

Amrita School of Engineering, Amaravati

B. Tech. CCE (I Semester)

Mid Term Examination–November 2024

23MAT124-Engineering Mathematics-I

Maximum marks : 50

Duration :2 Hours

Course Outcomes(COs)

CO	Course Outcomes
01	Solve problems involving limits, derivatives and ODEs
02	Model and solve system of linear equations
03	Characterize systems using Eigen values and vectors
04	Apply the mathematical concepts learnt, to engineering problems

Answer ALL questions

1. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$. [5][CO03][BTL2]

2. Let the function $f(x)$ be defined as follows:

$$f(x) = \begin{cases} \frac{x^2-1}{x-1}, & \text{if } x \neq 1 \\ 2, & \text{if } x = 1 \end{cases}$$

   Determine whether the function $f(x)$ is continuous and differentiable at the point $x = 1$. [5][CO01][BTL2]

3. (a) Verify if the function $f(x) = |x-1|$ is differentiable at $x = 1$?

   (b) Evaluate $\lim_{x \to 1^+} \frac{1-x^2}{|1-x|}$. [5][CO01][BTL3]

4. Prove that every differentiable function is continuous. Is the converse true? Explain with an example. [5][CO01][BTL2]

5. Find the maximum (local maximum) and minimum (local minimum) values of the function $f(x) = x^4 - 8x^3 + 18x^2$. [5][CO01][BTL3]

6. Find the maximum (local maximum) and minimum (local minimum) points of the function $f(x,y) = x^4 + y^4 - 4x^2 - 4y^2 + 16$. [5][CO01][BTL4]

7. Let the function $f(x, y)$ be defined as follows:

$$f(x, y) = \begin{cases} \dfrac{x^2 y}{\sqrt{x^2 + y^2}}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$$

Determine whether the function $f(x, y)$ is continuous and differentiable at the point $(0,0)$. [5][CO01][BTl

8. State and prove the Fundamental Theorem of Calculus (Part I). [5][CO01][BTL2]

9. (a) Given $f(x) = \int_1^{x^2} \sqrt{1 + t^3} dt$, compute $f'(x)$.

   (b) Evaluate $\lim_{x \to 0} \dfrac{1}{x^2} \int_0^x \dfrac{1 + t^2}{1 + \sin t} dt$ [5][CO01][BTL3]

10. (a) Let the function $f(t)$ be defined as follows:

$$f(t) = \begin{cases} t + 2, & \text{if } t < 0 \\ \sqrt{t}, & \text{if } \geq 0 \end{cases}.$$

   Evaluate $\lim_{t \to -1} f(t)$.

   (b) Given $f(x) = \int_{2x}^{x^4} (t^4 + 4)^6 dt$, find $f'(x)$. [5][CO01][BTL3]

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CO	Marks	BTL	Marks
CO01	45	BTL1	0
CO02	5	BTL2	20
CO03	0	BTL3	20
CO04	0	BTL4	10
		BTL5	0
		BTL6	0