ASSIGNMENT 1

1. Express the following statements using logical expressions.
   (a) No number is both even and odd.
   (b) One more than any even number is an odd number.
   (c) There is prime number that is even.

2. Consider the statement "For all natural numbers n, if n is prime, then n is solitary."
   (a) Write the converse and the contrapositive of the statement
   (b) Write the negation of the original statement. What would you need to show to prove that the statement is false?
   (c) Is the statement "if 10 is prime, then 10 is solitary" true or false? Explain.
   (d) It turns out that 8 is solitary. Does this tell you anything about the truth or falsity of the original statement, its converse or its contrapositive? Explain.

   [Note: You don't need to know what is a solitary number to solve this problem.]

3. Without using the truth table prove the following equivalences.
   (a) $p \to (q \vee r) \equiv (p \to q) \vee (p \to r)$.
   (b) $(\neg p \to \neg q) \to (q \to p) \equiv T$

4. Prove that if x is irrational, then $1/x$ is irrational.

5. Use a proof by contradiction to show that there is no rational number r for which $r^3+r+1=0$. [Hint:Assume that $r = a/b$ is a root, where a and b are integers and a/b is in lowest terms. Obtain an equation involving integers by multiplying by b^3. Then look at whether a and b are each odd or even.]

6. Prove that $\displaystyle\sum_{i=0}^{n}\left(\frac{-1}{2}\right)^i = \frac{2^{n+1}+(-1)^n}{3.2^n}$ for every non negative integers by using mathematical induction.

7. Express the statement "Not everyone is perfect" using predicates and quantifiers and then find the negation of the statement.

8. Let $P(x)$ and $Q(x)$ be the statements "x is a superhero" and "x has a good heart" respectively. Express each of the following quantification's in English.
   (a) $\exists x\ \neg Q(x)$
   (b) $\neg\ \exists x\ Q(x)$
   (c) $\forall x\ (P(x) \to Q(x))$
   (d) $\forall x\ (P(x) \wedge Q(x))$
   (e) $\forall x\ P(x) \to \forall x\ Q(x)$

9. Show that the premises "If Ranjith is able and willing to pass all students he would do so", "If Ranjith is unable to pass all students then he is impotent", "If he is unwilling to pass students he is evil", "Ranjith does not pass all students", "If Ranjith is a teacher then he is neither impotent not evil" leads to the conclusion "Ranjith is not a teacher".

10. Use rules of inference to show that if $\forall x\ (P(x) \to Q(x) \wedge S(x))$ and $\forall x\ (P(x) \wedge R(x))$ are true then $\forall x\ (R(x) \wedge S(x))$ is true.