Computational Physics (22PHY106)

Unit 2

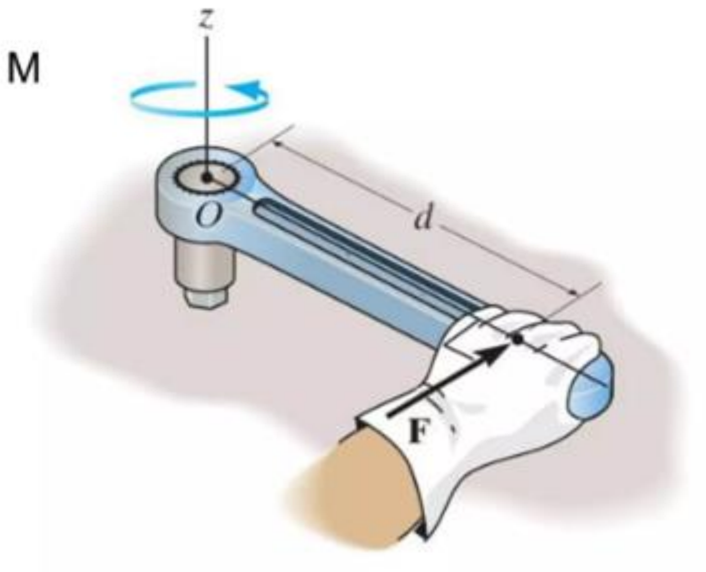
Equilibrium about a Point, Moment, Couple, Equivalent System, Equilibrium of Rigid Bodies, Degree-of-freedom and Constraints at Supports, Free Body Diagram.

Moment

A moment is a measure of rotation about a point.

or

Moment is defined as the turning effect produced by a force.



MOMENT OF A FORCE: It is the turning effect produced by a force, on the body, on which it acts. The moment of a force is equal to the product of the force and the perpendicular distance of the point, about which the moment is required and the line of action of the force.

Moment = Force × ⊥ Distance

$$M = F \times \perp d$$

where F = Force acting on the body, and

d = Perpendicular distance between the point, about which the moment is required and the line of action of the force.

A moment has both magnitude and direction.

THE UNIT OF A MOMENT

Since the moment of a force is the product of force and distance, therefore the units of the moment will depend upon the units of force and distance. Thus, if the force is in Newton and the distance is in meters, then the units of moment will be Newton-meter (briefly written as N-m).

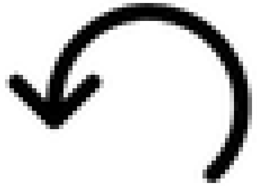
TYPES OF MOMENTS: Broadly speaking, the moments are of the following two types: 1. Clockwise moments. 2. Anticlockwise moments.

CLOCKWISE MOMENT

•Clockwise moment- A force causes an object or body to rotate in a clockwise direction. It is taken as positive.

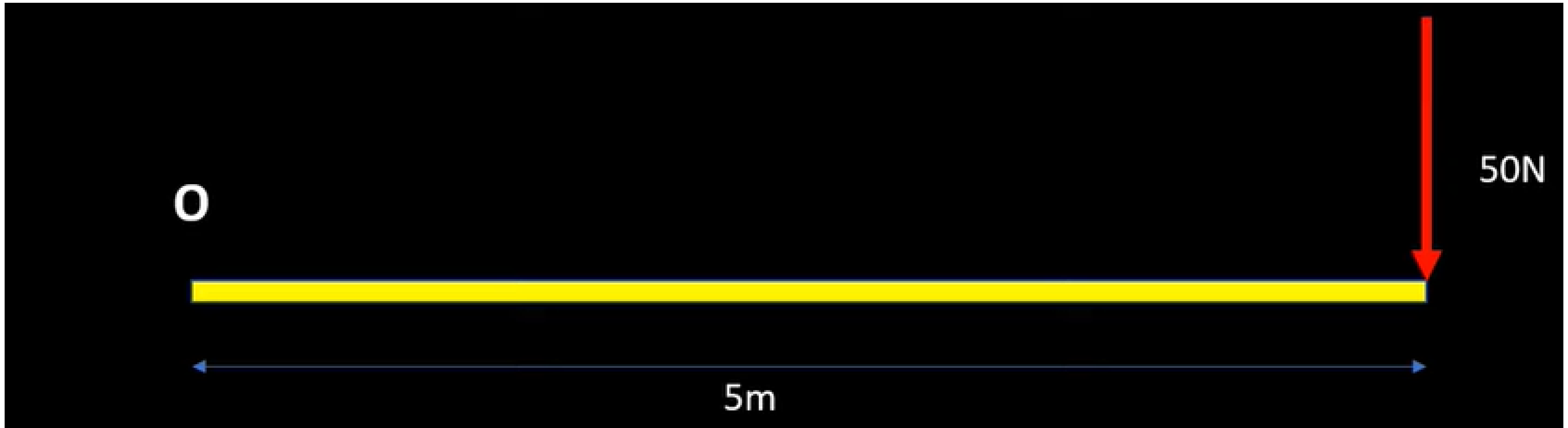
ANTICLOCKWISE MOMENT:

• Counter clockwise moment – A force causes an object or body to turn in an anti-clockwise direction. It is taken as negative.

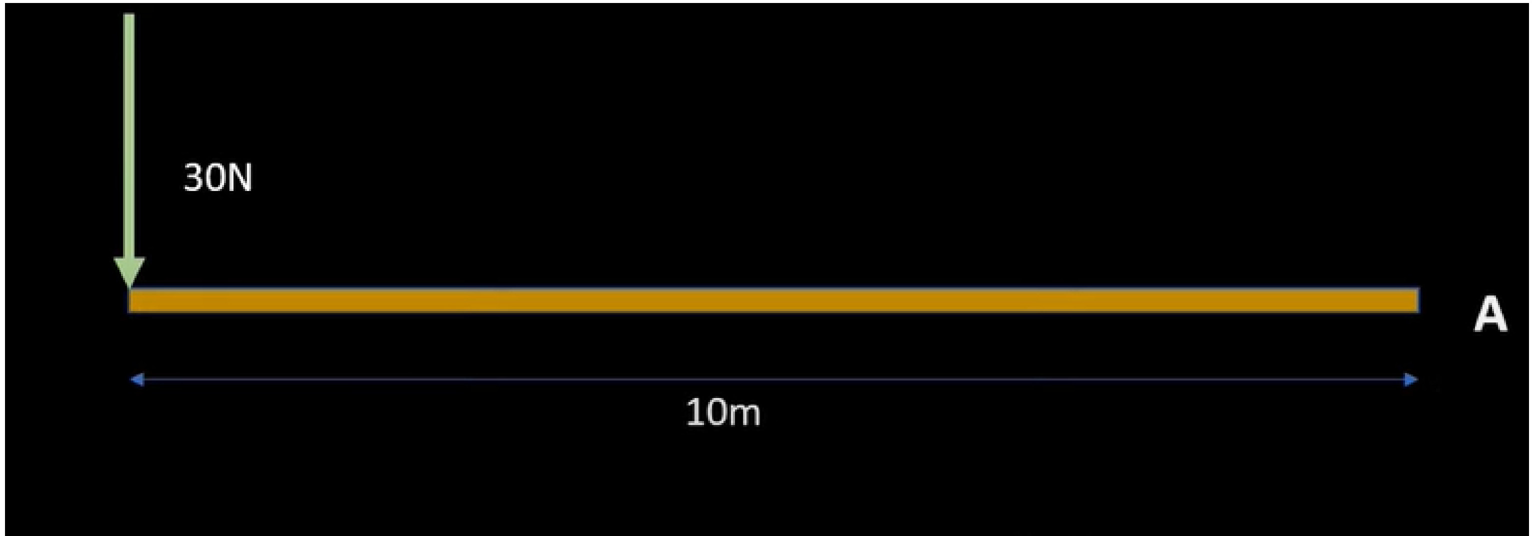


Counter clockwise direction is negative.

Note. The general convention is to take clockwise moment as positive and anticlockwise moment as negative.



Moment about the point O, $M_0 = F * x = 50 * 5 = 250$ Nm



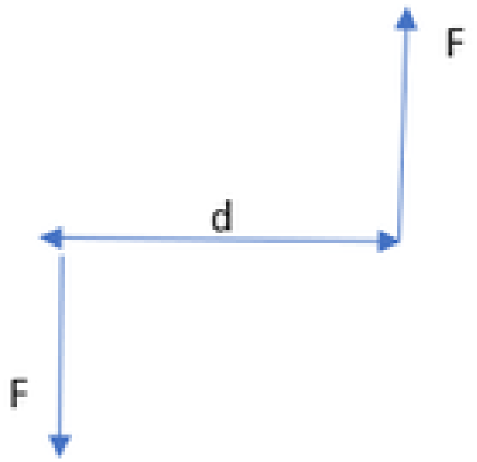
Moment about the point A, $M_A = F * x = 30 * 10 = 300$ Nm

COUPLE:

A pair of two equal and unlike parallel forces (*i.e*. forces equal in magnitude, with lines of action parallel to each other and acting in opposite directions) is known as a couple.

or

A couple is defined as a pair of two forces that are equal in magnitude but their direction are opposite to each other and the motion of lines do not coincide.



Here the forces are equal and opposite so we get the result of force is zero and there is no linear acceleration.

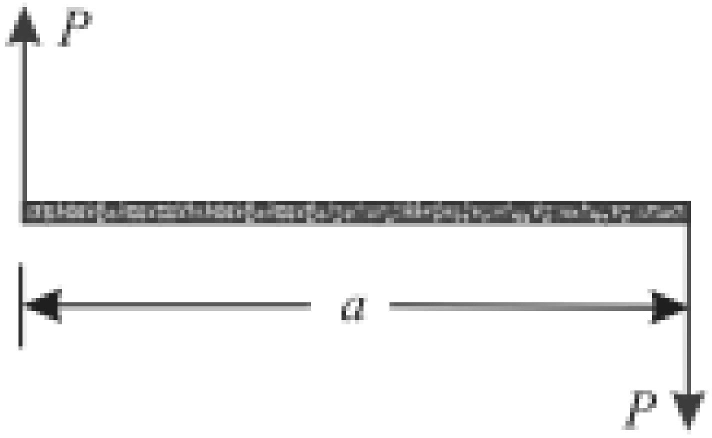
- As a matter of fact, a couple is unable to produce any translatory motion (*i.e.*, motion in a straight line). But it produces a motion of rotation in the body, on which it acts. The simplest example of a couple is the forces applied to the key of a lock, while locking or unlocking it.

Few Applications of the couple

- Opening and closing the cap of the bottle.
- Using the Key, unlock the locker.
- Turning the screwdriver.
- Helm wheel applied by the bus driver.

ARM OF A COUPLE:

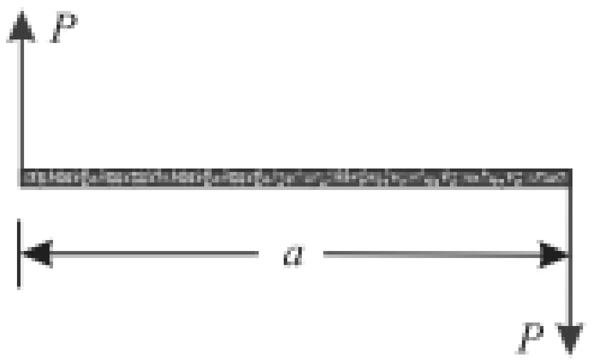
The perpendicular distance (a), between the lines of action of the two equal and opposite parallel forces, is known *as arm of the couple* as shown in Fig.



MOMENT OF A COUPLE: The moment of a couple is the product of the force (*i.e.*, one of the forces of the two equal and opposite parallel forces) and the arm of the couple. Mathematically:

Moment of a couple = $P \times a$

where P = Magnitude of the force, and a = Arm of the couple.



CLASSIFICATION OF COUPLES:

The couples may be, broadly, classified into the following two categories, depending upon their direction, in which the couple tends to rotate the body, on which it acts:

1. Clockwise couple, and 2. Anticlockwise couple

(*a*) Clockwise couple

(*b*) Anticlockwise couple

CLOCKWISE COUPLE:
A couple, whose tendency is to rotate the body, on which it acts, in a clockwise direction, is known as a clockwise couple as shown in Fig. (*a*). Such a couple is also called positive couple.

(*a*) Clockwise couple

ANTICLOCKWISE COUPLE:
A couple, whose tendency is to rotate the body, on which it acts, in an anticlockwise direction, is known as an anticlockwise couple as shown in Fig. (*b*). Such a couple is also called a negative couple.

(*b*) Anticlockwise couple

CHARACTERISTICS OF A COUPLE:

A couple (whether clockwise or anticlockwise) has the following characteristics:

1. The algebraic sum of the forces, constituting the couple, is zero.

2. The algebraic sum of the moments of the forces, constituting the couple, about any point is the same, and equal to the moment of the couple itself.

3. A couple cannot be balanced by a single force. But it can be balanced only by a couple of opposite sense.

4. Any no. of co-planer couples can be reduced to a single couple, whose magnitude will be equal to the algebraic sum of the moments of all the couples.

Questions

1. A boy is sitting on one side of a see-saw 3m away from a point. If the weight of the boy is 20 N then find the moment.

Given that the boy's weight caused an anticlockwise moment.

Moment = Force ×distance

$M = 20 \times 3$

$M = 60Nm.$

2. The moment of force is 20 N about a fixed point is 2Nm. Calculate the distance of a point from the line of action of force.

Given, Force = 20N and Moment = 2Nm.

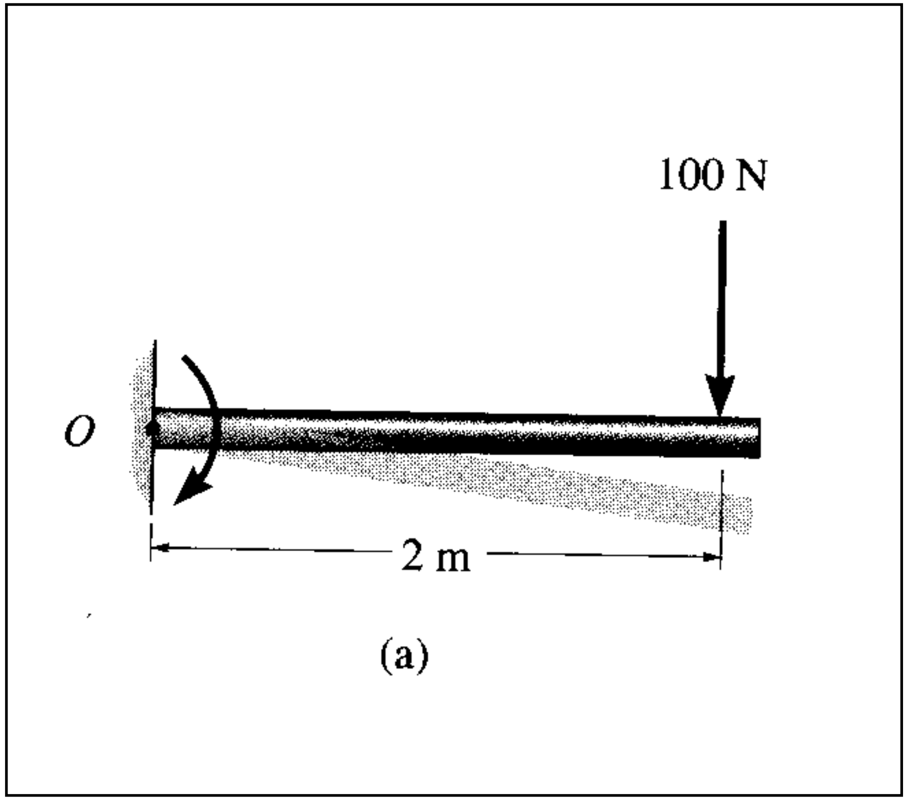
Using the formula of moment which is $M = F \times d$

$2 = 20 \times d$

$d = 2/20$

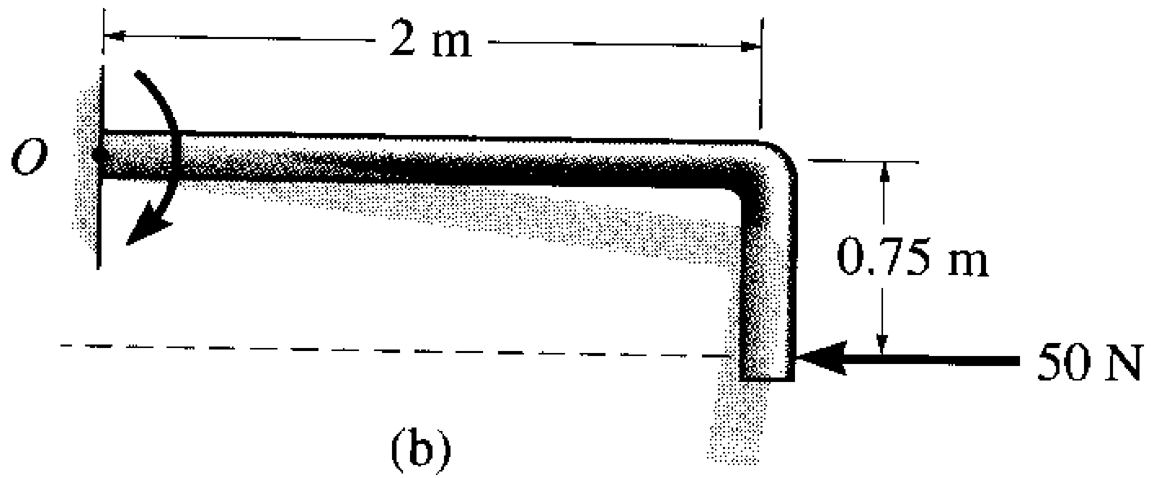
$d = 0.1m.$

3. Moment?



M_O = (100 N) (2 m) = +200 Nm

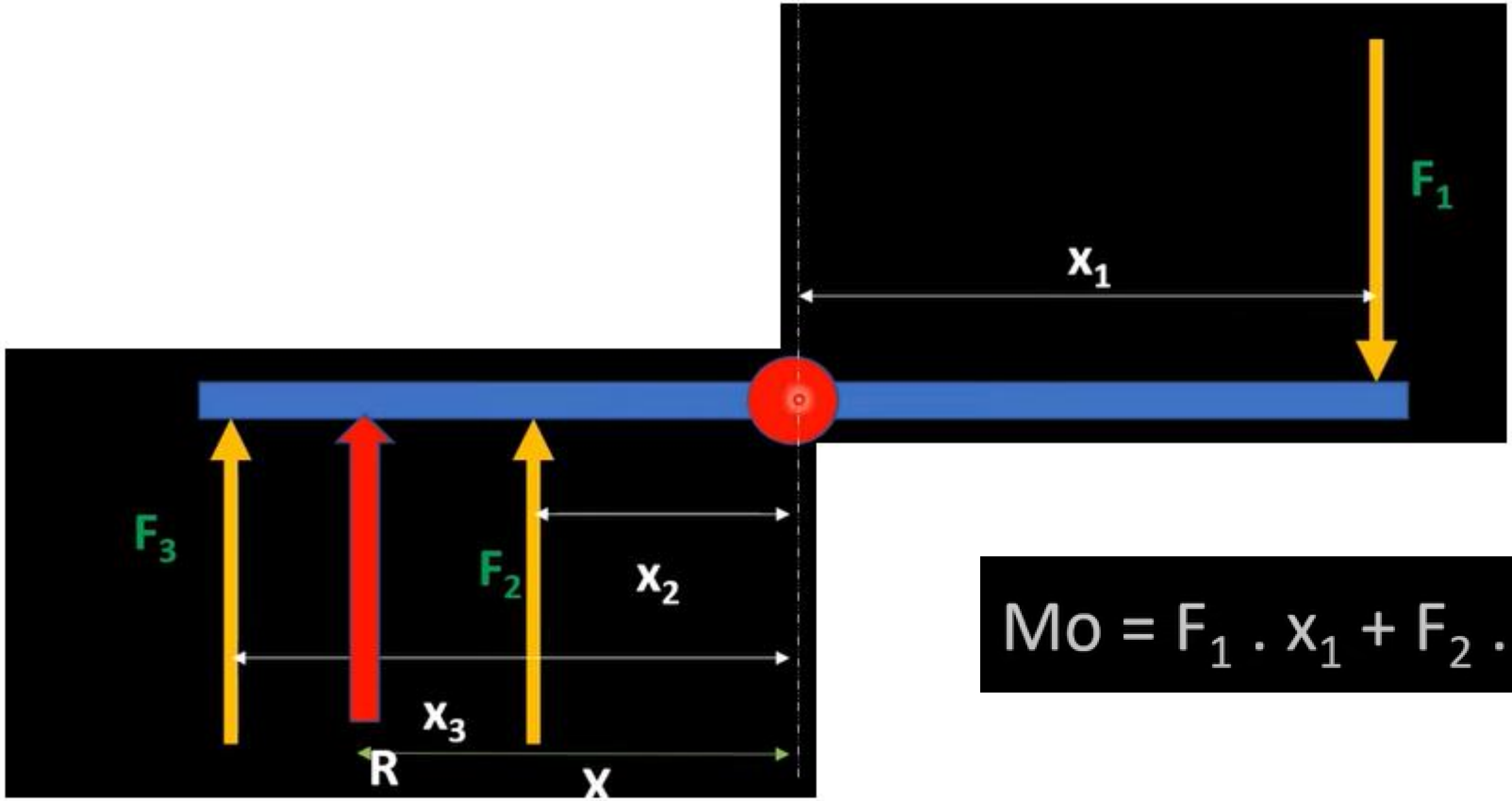
4.



$$M_O = (50\text{ N})\ (0.75\text{ m}) = 37.5\text{ Nm}$$

VARIGNON'S PRINCIPLE OR LAW OF MOMENTS:

It states, "If a number of coplanar forces are acting simultaneously on a particle, the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point."



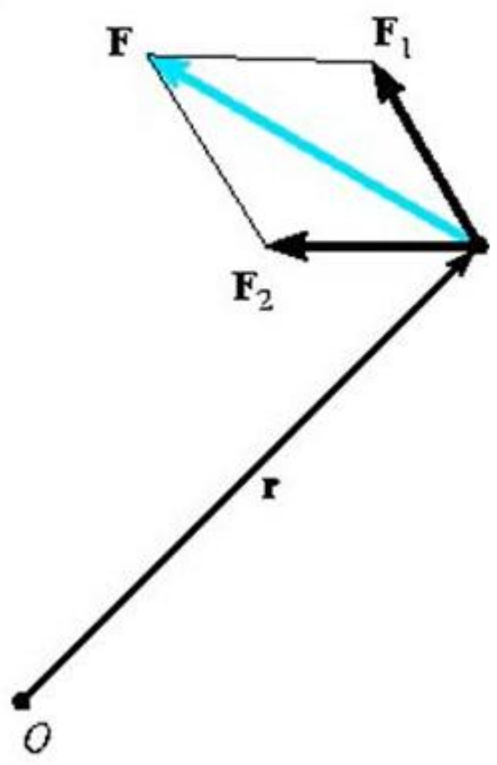
$$Mo = F_1 \,.\, x_1 + F_2 \,.\, x_2 + F_3 \,.\, x_3 = R.\, X$$

Varignon's principle

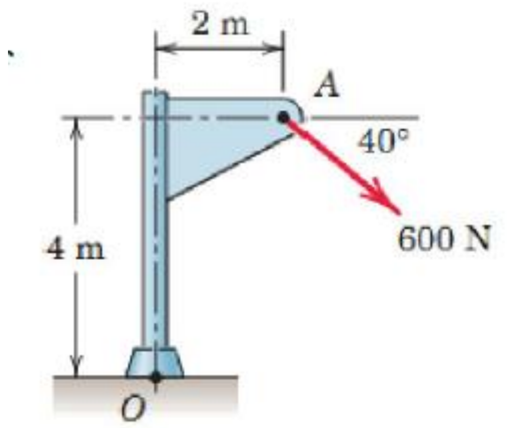
The moment of a force about any point is equal to the sum of the moments of the components of this force.

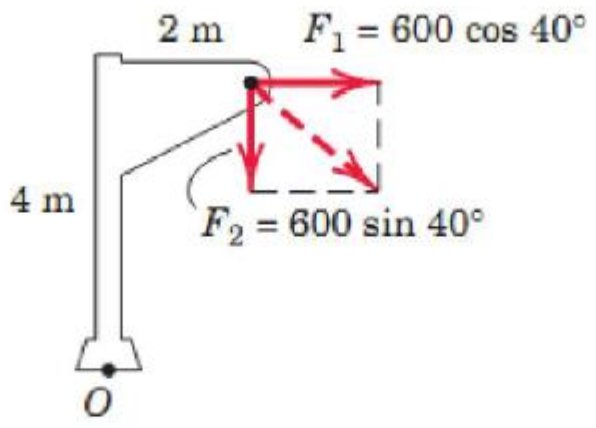
- For $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$,

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2$$
$$= \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2)$$
$$= \mathbf{r} \times \mathbf{F}$$



Calculate the magnitude of the moment about the base point O of the 600N force





By Varignon’s theorem,

F1 = 600 cos 40= 460 N,

F2 = 600 sin 40= 386 N

Mo= 460(4) +386(2) = 2610 N.m clockwise

Equilibrium:

- A body is said to be in equilibrium if it is at rest or moving with uniform velocity.

- When two or more forces act on a body such that their resultant or combining effect on the body is zero and the body retains its state of rest or of uniform motion then the body is said to be in equilibrium.

PRINCIPLES OF EQUILIBRIUM:

Though there are many principles of equilibrium, yet the following three are important from the subject point of view :

1. **Two force principle:** As per this principle, if a body in equilibrium is acted upon by two forces, then they must be equal, opposite and collinear.
2. **Three force principle:** As per this principle, if a body in equilibrium is acted upon by three forces, then the resultant of any two forces must be equal, opposite and collinear with the third force.
3. **Four force principle:** As per this principle, if a body in equilibrium is acted upon by four forces, then the resultant of any two forces must be equal, opposite and collinear with the resultant of the other two forces.

Types of Equilibrium

With respect to the state of a body, equilibrium may be divided into two categories:

1. Static equilibrium.
2. Dynamic equilibrium.

Static equilibrium

If the combined effect of all the forces acting on a body is zero and the body is in the state of rest then its equilibrium is termed as static equilibrium.

For example: All stationary bodies

Dynamic equilibrium

When a body is in state of uniform motion and the resultant of all the forces acting upon it is zero then it is said to be in dynamic equilibrium.

For example: Jump by using parachute

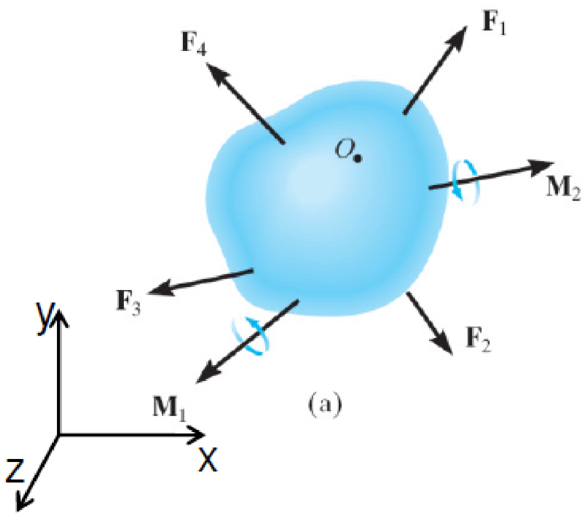
Rigid Body Equilibrium

A rigid body will remain in equilibrium provided

- sum of all the external forces acting on the body is equal to zero, and
- Sum of the moments of the external forces about a point is equal to zero

$$\begin{aligned} \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \\ \Sigma F_z &= 0 \end{aligned}$$

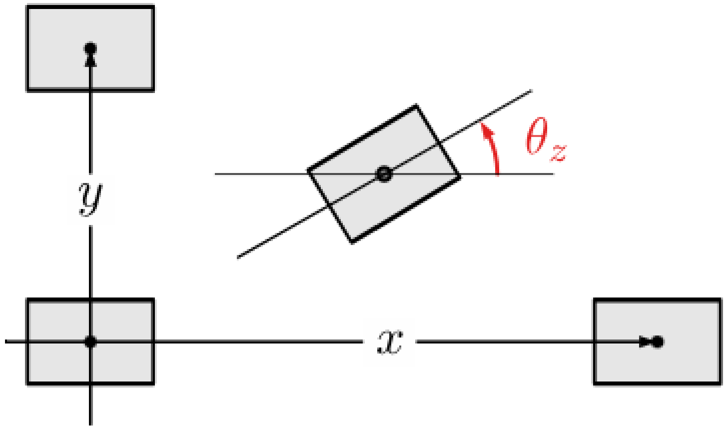
$$\begin{aligned} \Sigma M_x &= 0 \\ \Sigma M_y &= 0 \\ \Sigma M_z &= 0 \end{aligned}$$



Degree of Freedom

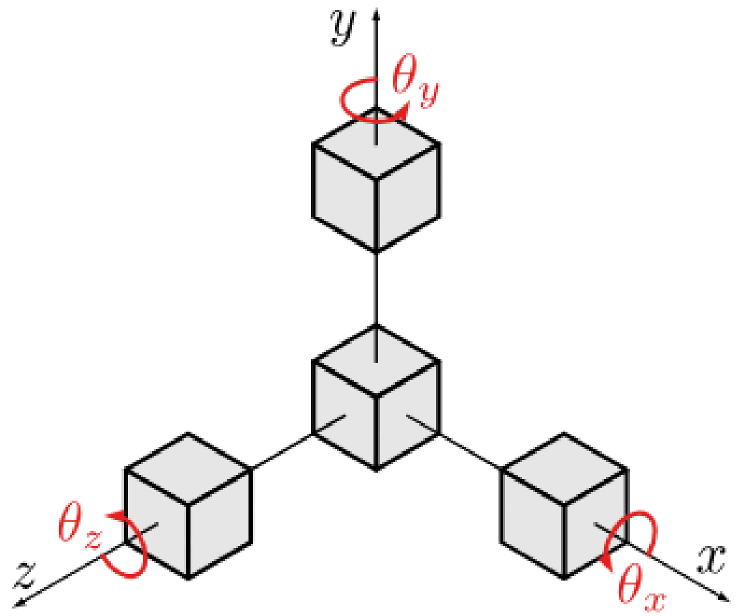
Degrees of freedom refers to the number of independent parameters or values required to specify the *state* of an object.

Two-dimensional rigid bodies in the xy plane have three degrees of freedom. Position can be characterized by the x and y coordinates of a point on the object, and orientation by angle θ_z about an axis perpendicular to the plane. The complete movement of the body can be defined by two linear displacements Δx and Δy, and one angular displacement $\Delta\theta_z$.



Two-dimensional rigid bodies have three degrees of freedom.

Three-dimensional rigid bodies have six degrees of freedom, which can be specified with three orthogonal coordinates x, y and z, and three angles of rotation, θ_x, θ_y and θ_z. Movement of the body is defined by three translations Δx, Δy and Δz, and three rotations $\Delta\theta_x$, $\Delta\theta_y$ and θ_z.



Three-dimensional rigid body have six degrees of freedom - three translations and three rotations.

For a rigid body in space, it has six degrees of freedom: three translational (along x, y, and z axis) and three rotational (about x, y, and z axes).

Constraints

- In engineering mechanics, **constraints** refer the conditions that limit or restrict the movement (translation and/or rotation) of a body in space. These constraints can come from physical supports, connections, or other surrounding structures.

A body can move in **six degrees of freedom (DOF)**:

- **Three translational DOF:** movement along the X, Y, and Z axes (linear motion).

- **Three rotational DOF:** rotation about the X, Y, and Z axes (angular motion).

Constraints reduce these degrees of freedom. Depending on the type of constraint, certain motions are blocked, limiting the body's freedom to move in one or more directions.

Types of Constraints

1.Fixed Constraints (also called built-in or clamped constraints):

- These constraints **completely restrict** both translational and rotational motion.
- The body is "fixed" in place, unable to move in any direction or rotate about any axis.

2. Pinned or Hinged Constraints:

•These allow **rotation** but prevent **translation** in any direction.

•The body can rotate freely about the pin or hinge but cannot move up, down, or sideways.

3. Roller Constraints:

•These allow movement (translation) in one direction (usually horizontal) but restrict motion in the perpendicular direction (usually vertical).

•They prevent motion in one axis while allowing free movement in another.

4. Slider Constraints:

- These constraints permit translational motion along a particular direction but prevent both rotational and translational motion in all other directions.

Supports

- A **support** is a type of constraint applied at specific points of a structure to prevent unwanted motion. Supports are used to stabilize structures by providing reactions to the forces and moments acting on them.

or

- Supports play a crucial role in defining constraints. They are points or elements in a structure that resist motion or rotation, providing stability.

Types of Supports:

- **Pinned support (Hinged)** allows **rotation** but restricts **translational motion**.
- **Roller support** allows **translational motion** along one direction (usually horizontal) while restricting motion in the perpendicular direction (usually vertical).
- **Fixed support** **completely restricts** both translational and rotational motion.
- **Sliding Support** allows **translation along a specific direction** (like a roller) but **prevents motion in all other directions and restricts rotation**.
- **Link Support** allows movement **along the direction of the link** while restricting motion perpendicular to it.

Free Body Diagram (FBD)

- A Free **Body Diagram (FBD)** is a powerful tool used in physics and engineering to analyze forces acting on an object.
- A **Free Body Diagram** is a **visual** representation of force and object interactions.
- Individual objects or members are **isolated** from their environment or system, illustrating all **external** forces acting upon them.

Free Body Diagram (FBD)

A **free body diagram (FBD)** is a graphical representation that shows all the external forces acting on a single object, isolated from its surroundings. In an FBD, the object is represented by a simple shape, such as a point or a box, and arrows are used to depict all the forces acting on it, indicating both the direction and magnitude of each force.

To draw a proper free body diagram, you must follow these steps:

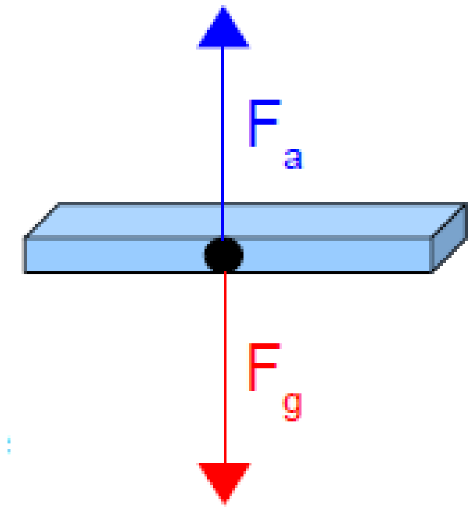
1. Draw a quick **sketch** of the object. Often a simple box will do.

2. Place a **dot** in the centre of the object. We basically treat this as the spot that all the forces are thought to act upon.

3. For every force acting on ***that*** object (we don't care about forces acting on any other objects), draw a **vector** that shows the size and direction of the force. Each vector must start from the dot and point outwards.

4. **Label** each vector based on the type of force it is. *Do **not** include numbers or calculations*

Example 1: **Sketch** a free body diagram for a book being held up by a person.



Free body diagram of forces on a book.

We draw a quick sketch of the book, and then put a dot in the centre.

Next, we identify that there are two forces acting on this book; the force of gravity pulling it down, and the applied force of a person pushing it up. Since the book is just being held up (not accelerating up or down), we can assume that the two forces are equal, so we will draw the vectors the same size.

- These are the common forces acting on objects

F_g = force due to gravity

F_a = applied force

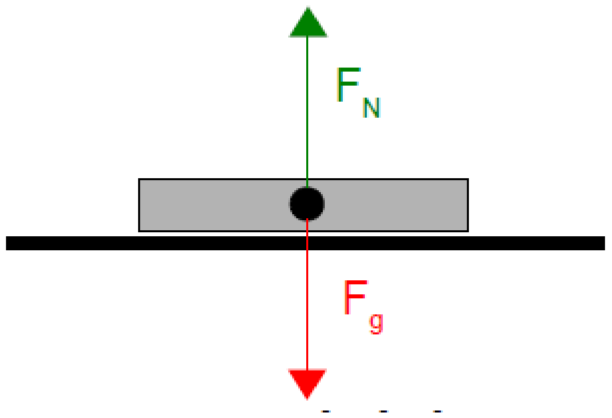
F_f = force due to friction

F_T = force of tension

F_N = normal force

F_{NET} = net force **Not drawn on free body diagrams**

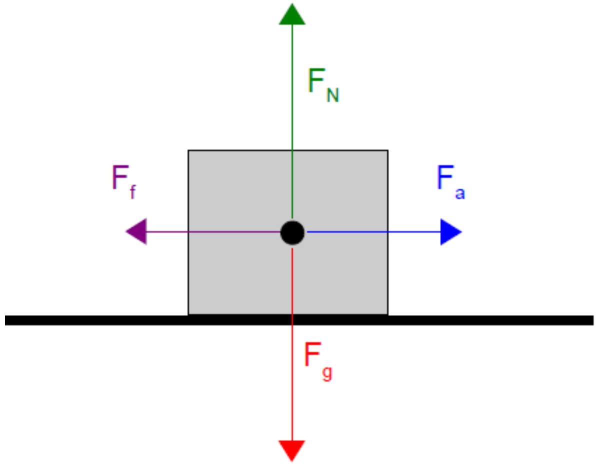
Example 2: **Sketch** a free body diagram of a laptop sitting on a table.



Free body diagram

- We know that there will be a force due to gravity (Fg) pulling the laptop down, but if that was the only force it should be dropping down towards the ground.
- There must be a force acting against the force of gravity that is holding the laptop up. This is happening because the table top is strong enough to hold up the laptop.
- We call this upwards force the **normal force**(F_N). *A normal force is exerted upwards by a surface (like a table or a floor) and is* ***perpendicular*** *to the surface*.

Example 3: You are trying to push a heavy box across the floor. It's causing you some trouble because the floor is not very smooth. **Sketch** a free body diagram of the box.



We still have force due to gravity and normal force, since it is still an object on a surface.

- You are trying to push it sideways, so that will be an applied force that you are exerting.
- It's tough to push because of friction between the box and the floor, so we'll also need to draw a force due to friction.
- **Force due to friction** (F_f) *always opposes the motion of an object. It is parallel to the surface the object is on.*
- Notice the normal force and force due to gravity are still equal. The applied force and force due to friction are also equal to each other.

Example 4:

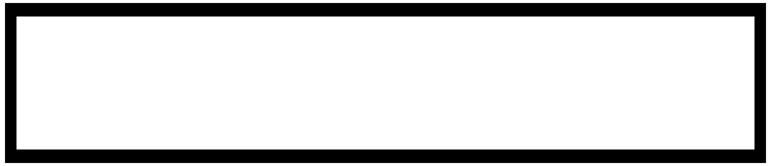
A stack of three books, each weighing 5 lb, is sitting on top of a table. Draw the Free Body Diagram (FBD) of the ***top*** *book*.

Free Body Diagram Procedure

1. **<u>Sketch</u>** the isolated object.

What is the isolated object?

<u>Top Book</u>



Free Body Diagram Procedure

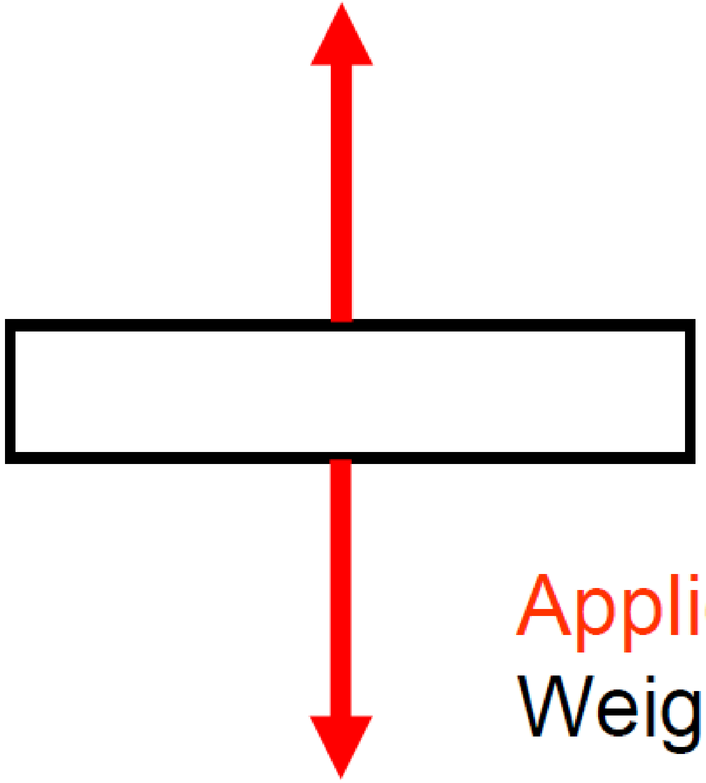
2. Sketch the **<u>applied</u>** and **<u>normal</u>** forces.

*When an object is in **contact with** and is **supported** by a second object, the second object can be replaced with a normal force which is perpendicular to the surface of the second object.*

Free Body Diagram Procedure

2. Sketch the **applied** and **normal** forces.

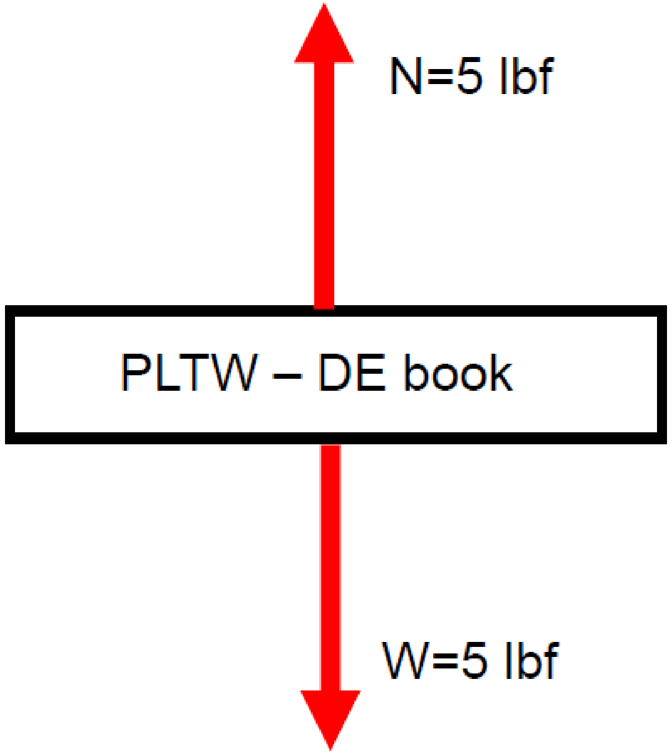
Normal Force:
Reaction force pushing up on the book, causing it not to fall.



Applied Force:
Weight of top book

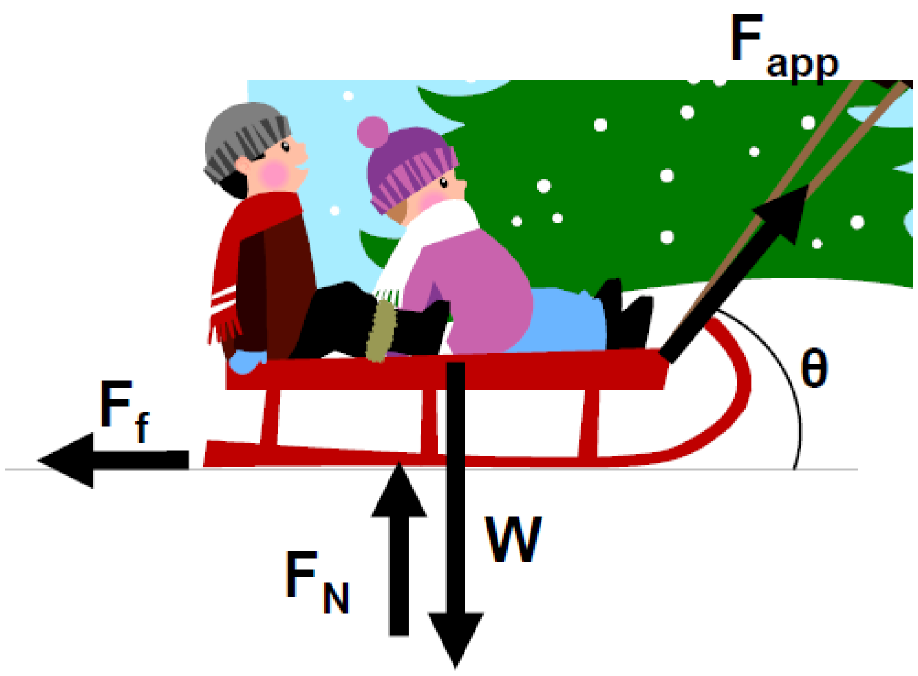
Free Body Diagram Procedure

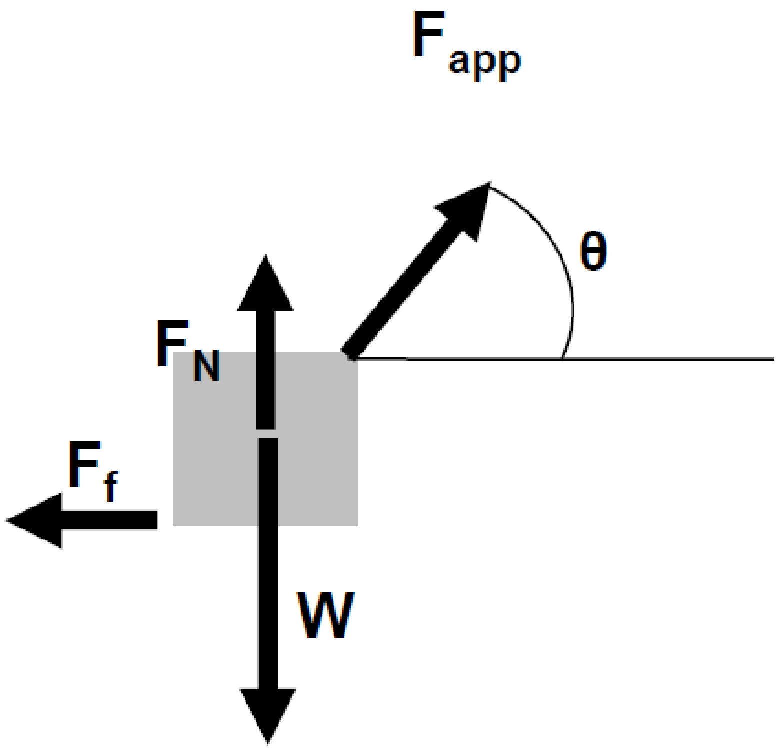
3. **Label** objects and forces.



Free Body Diagram Practice

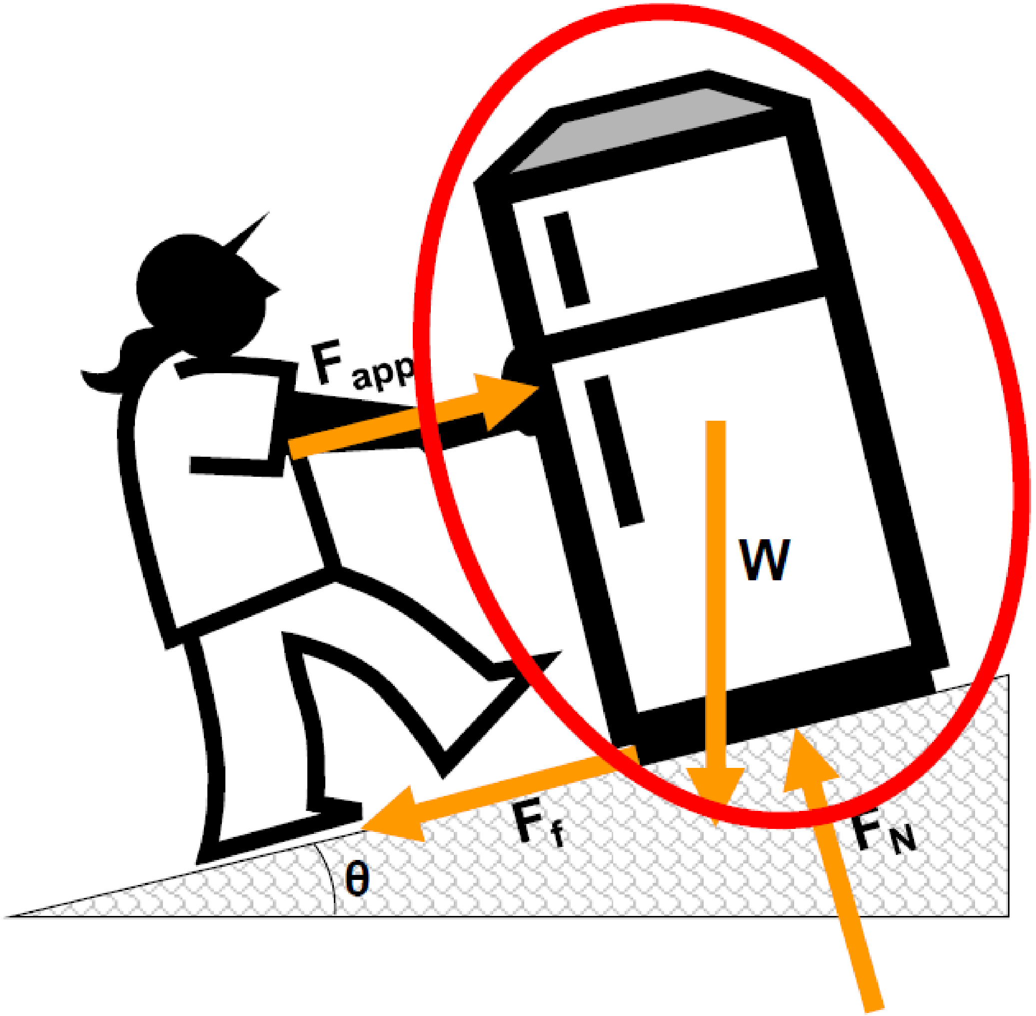
Create a FBD for the sled pictured below.

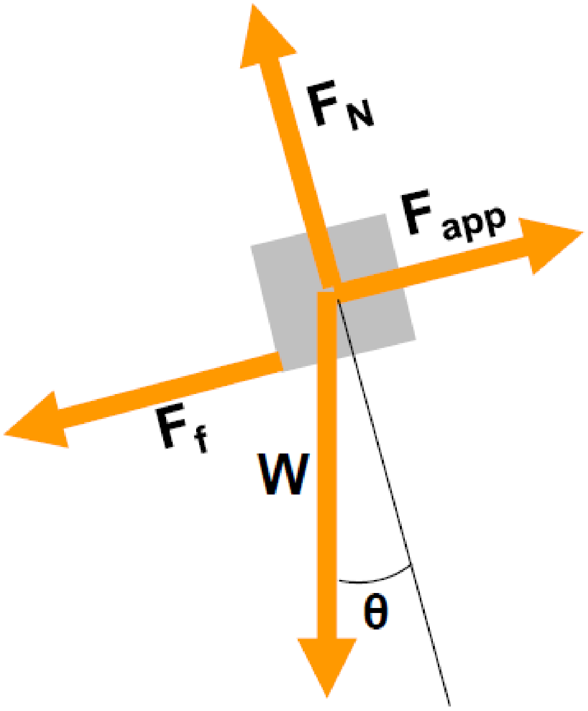




Free Body Diagram Practice

Create a FBD for the refrigerator pictured below.





Support reactions

- **Supports** play a crucial role in defining constraints. They are points or elements in a structure that resist motion or rotation, providing stability.

- **Support reactions** are the forces and moments exerted by a structure supports to maintain its equilibrium when subjected to external loads. These reactions counterbalance the applied loads, preventing the structure from translating (moving linearly) or rotating.

- The nature of these reactions depends on the type of support, such as roller, hinge (pin), or fixed support, each of which restricts certain degrees of freedom and provides corresponding reactions.

Free Body Diagram Reactions

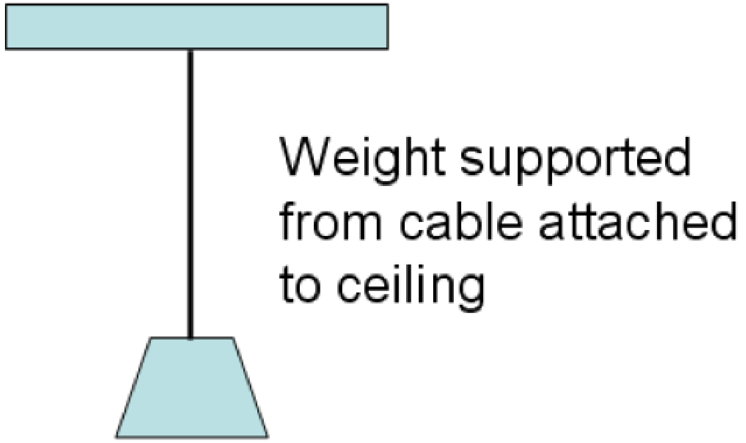
Different types of support reactions:

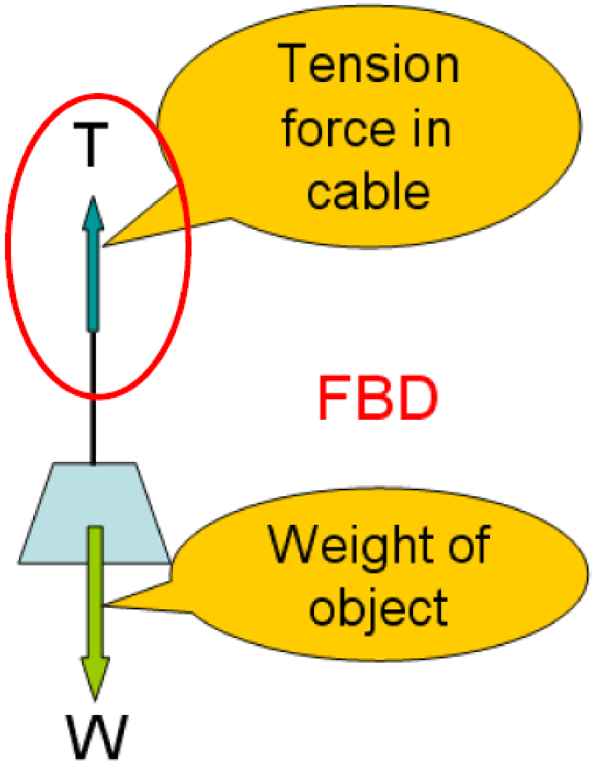
- Cable, rope, or chain
- **<u>Pin</u>**
- **<u>Roller</u>**
- Built-in end – Cantilever

To aid in completing free body diagrams, connections are often identified with letters.

Cable Support

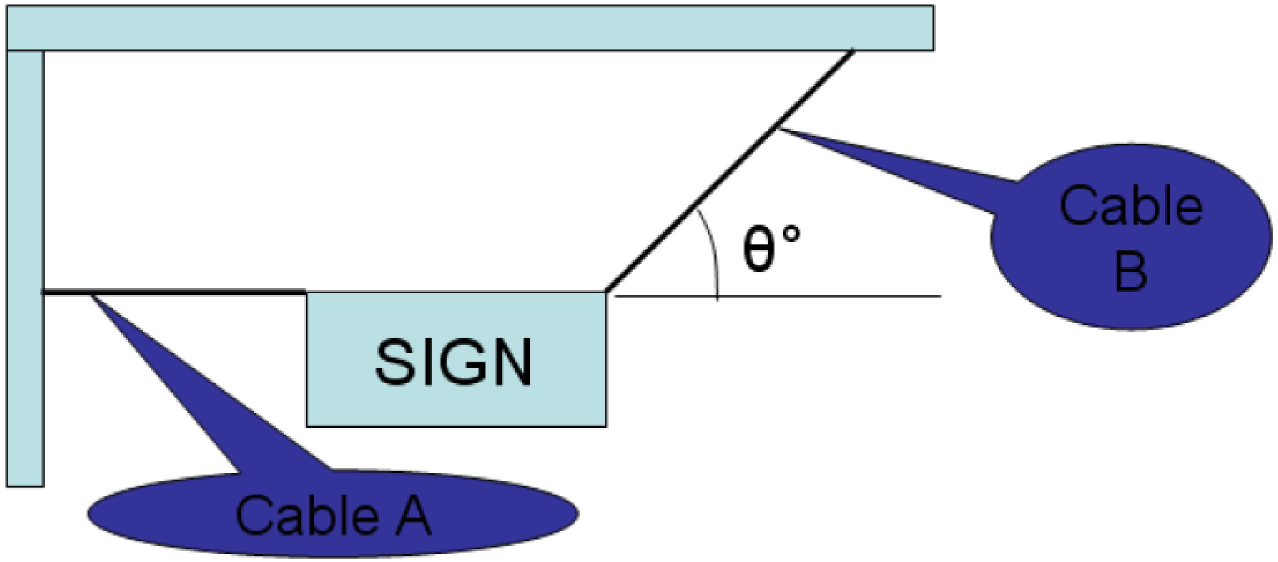
Cable, rope, chain – Replace with a **tension** force only.





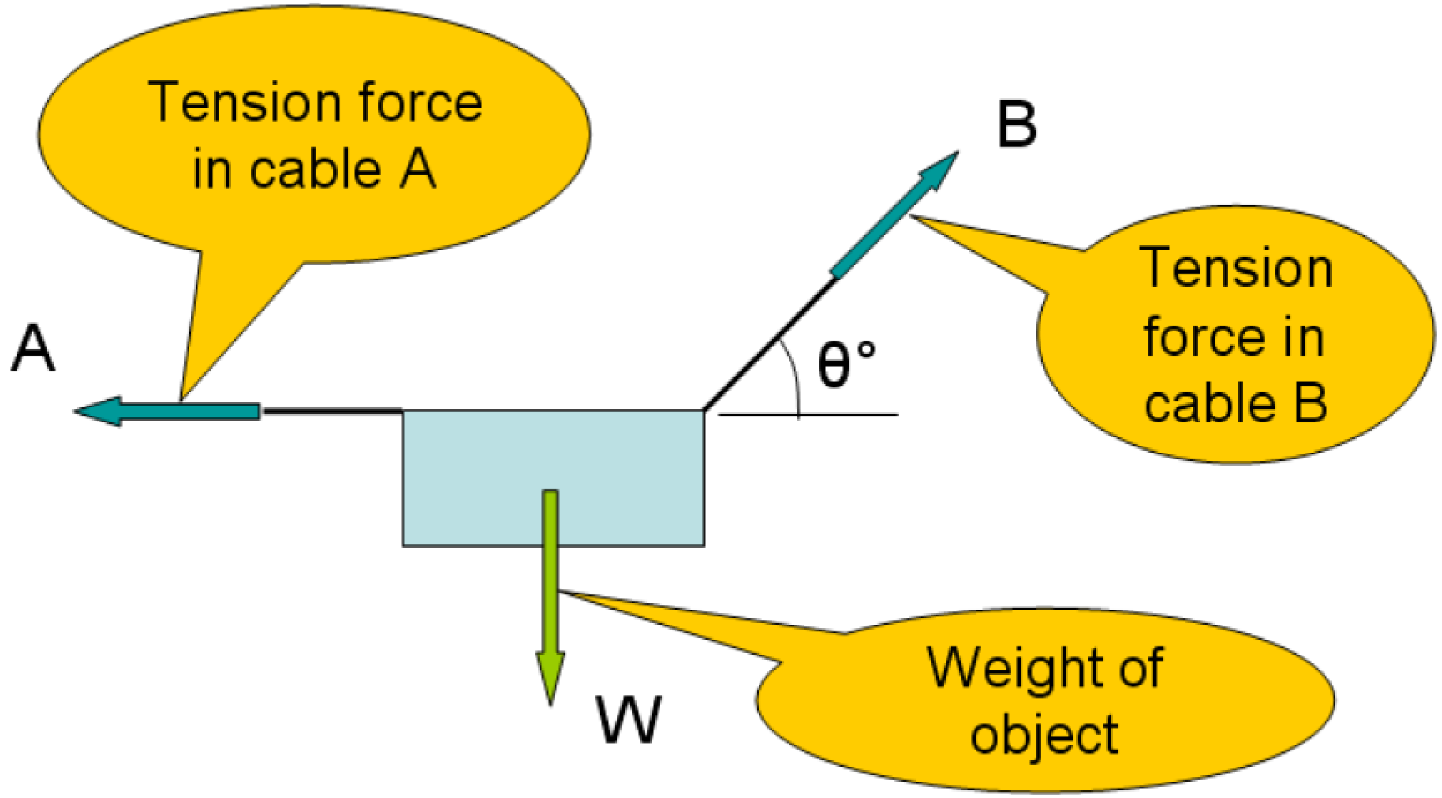
Cable Support

A sign with weight W is hung by two cables as shown. Draw the FBD of the sign and cables.



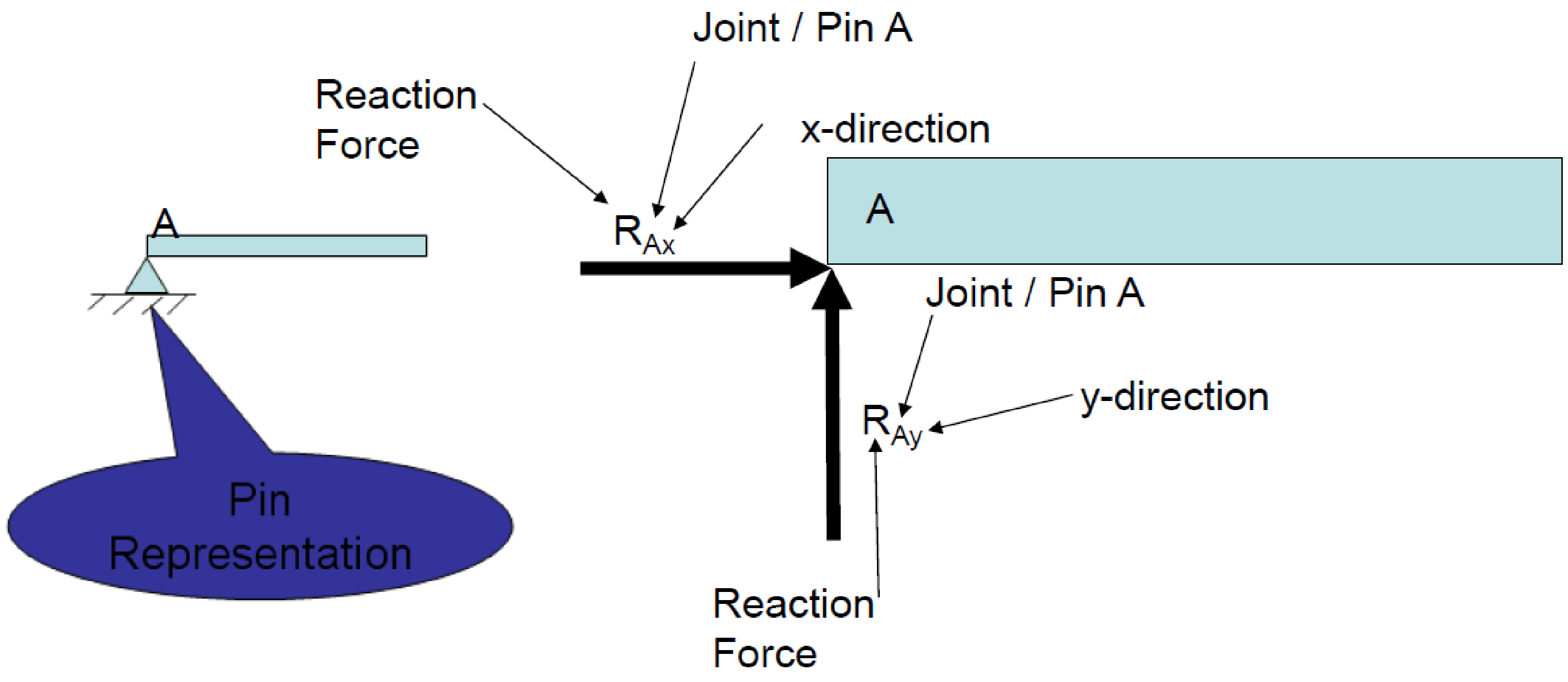
Cable Support

FBD of sign and cables:



Pin Support

Pin – Replaced with **TWO** reaction forces, one vertical (y) and one horizontal (x).

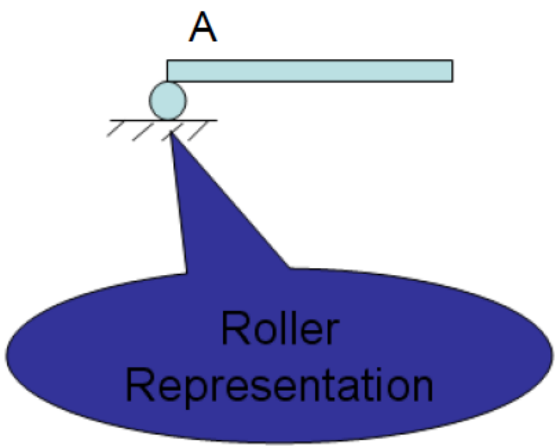


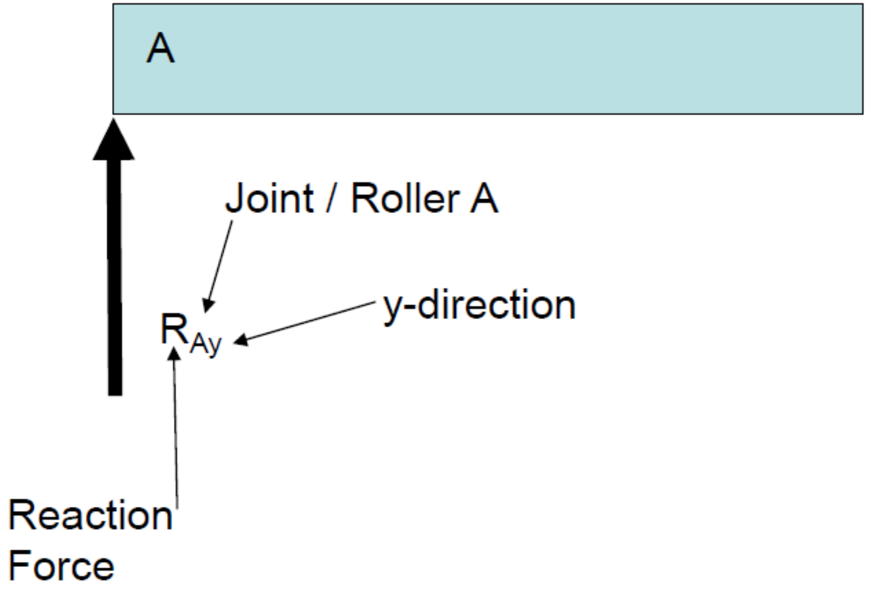
A pin support allows rotation but prevents translation in any direction. It provides two force reactions (horizontal and vertical).

Reactions: Horizontal and vertical forces

Roller Support

Roller – Replaced with <u>**ONE**</u> reaction force, perpendicular to surface.





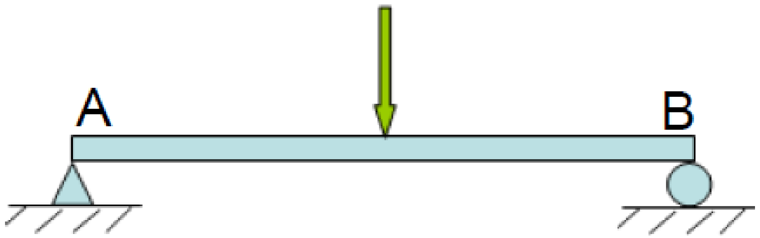
A roller support allows movement along one axis (usually horizontal) and prevents movement in the perpendicular direction (usually vertical). No moment reaction is provided.

Reaction: Vertical force

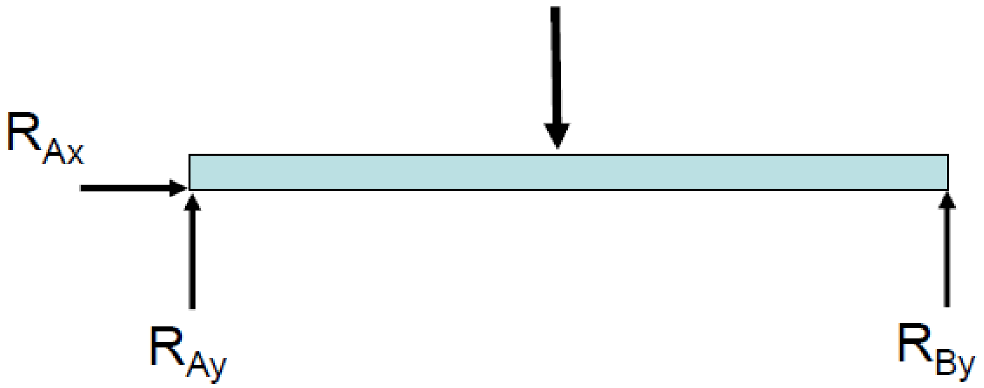
Common Support Reactions

Beams and truss bridges are usually supported with one pin support and one roller support. This is called a **simply supported** object.

Create a FBD for the simply supported beam.



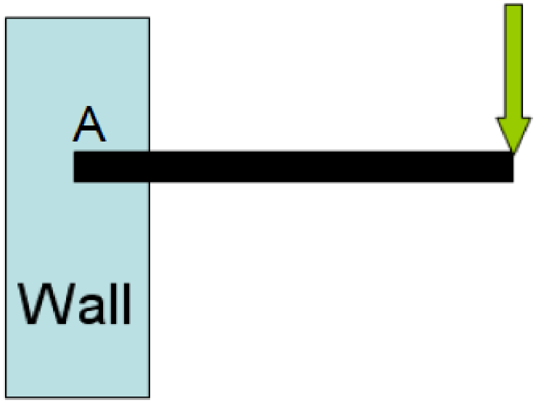
Roller at one end allows expansion/compression of beam or bridge.



Built-In-End Support

Built-in-end (cantilever) – Replaced with **TWO** forces: one horizontal and one vertical, and ONE moment

Create a FBD for the built-in-end cantilever.



A fixed support prevents both translation and rotation, providing both force and moment reactions. **Reactions**: Horizontal force, vertical force, and moment

Summary Support Reactions

Contact – Replace with a **normal** force.

Cable, rope, chain – Replace with **tension** force.

Pin – Replace with **two** reaction forces; one vertical and one horizontal.

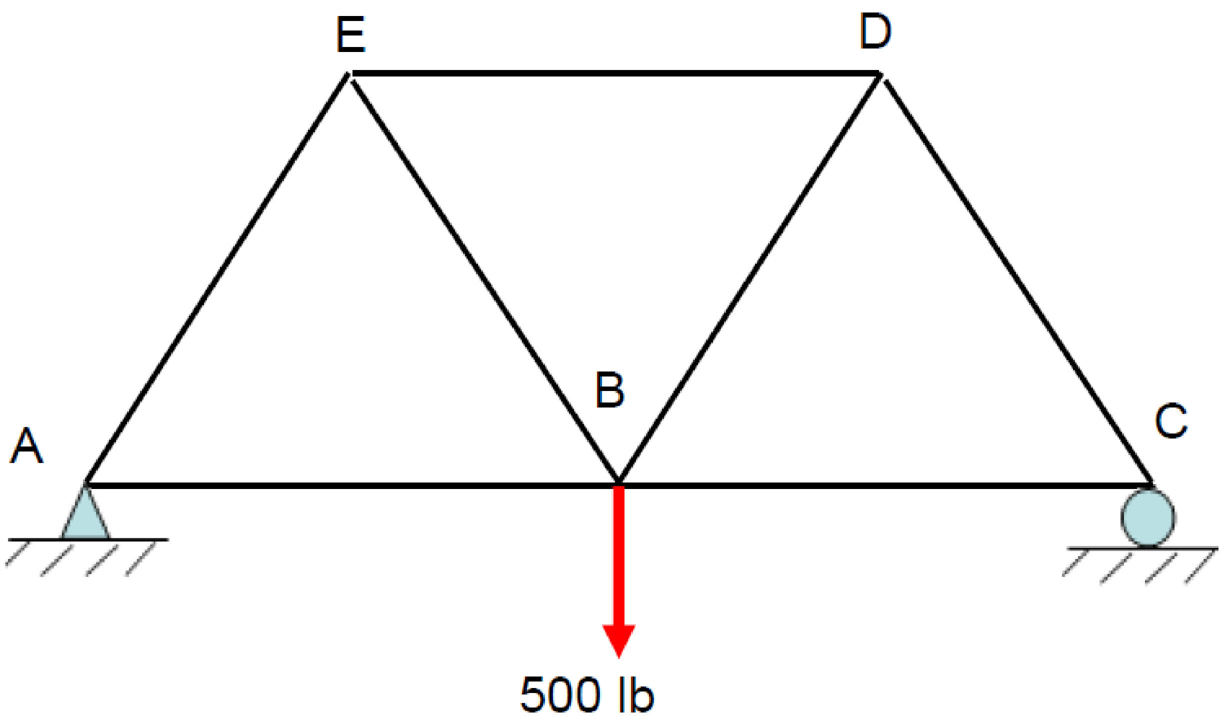
Roller – Replace with **one** reaction force perpendicular to surface.

Built-in-end (cantilever) – Replace with **one** horizontal force, **one** vertical force, and **one** moment.

Truss Bridge FBD

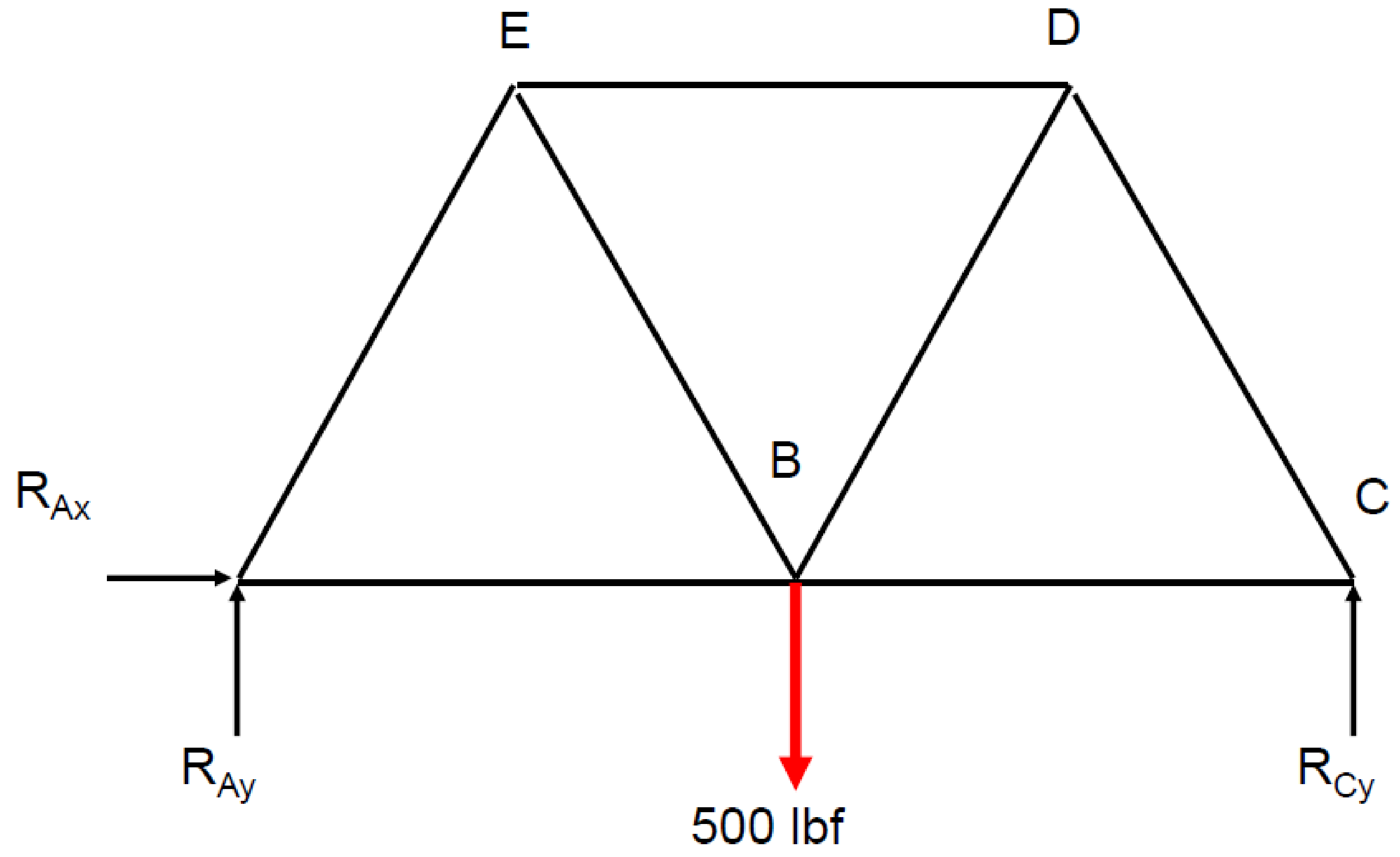
Supported with a **<u>pin</u>** at one end and a **<u>roller</u>** at the other.

Draw the FBD of the entire truss bridge.

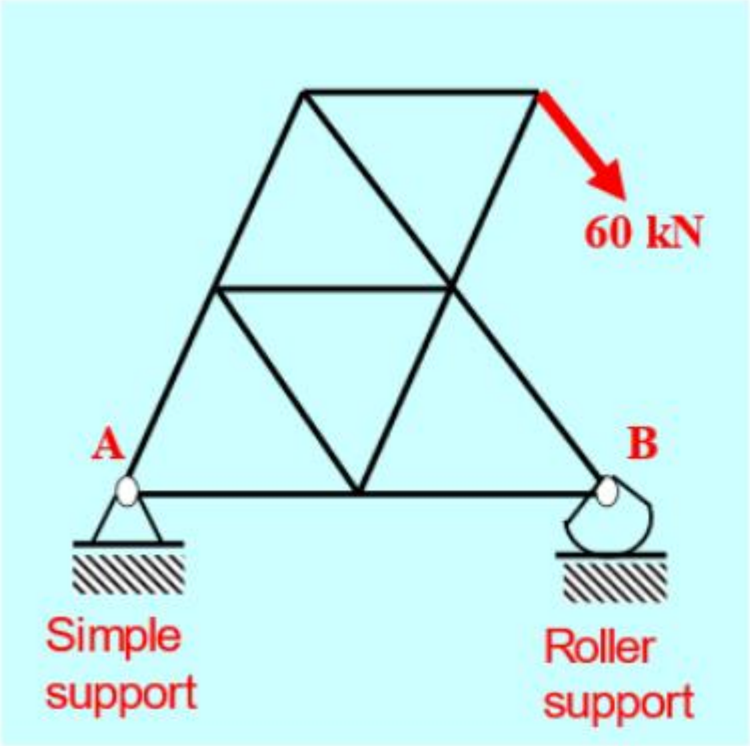


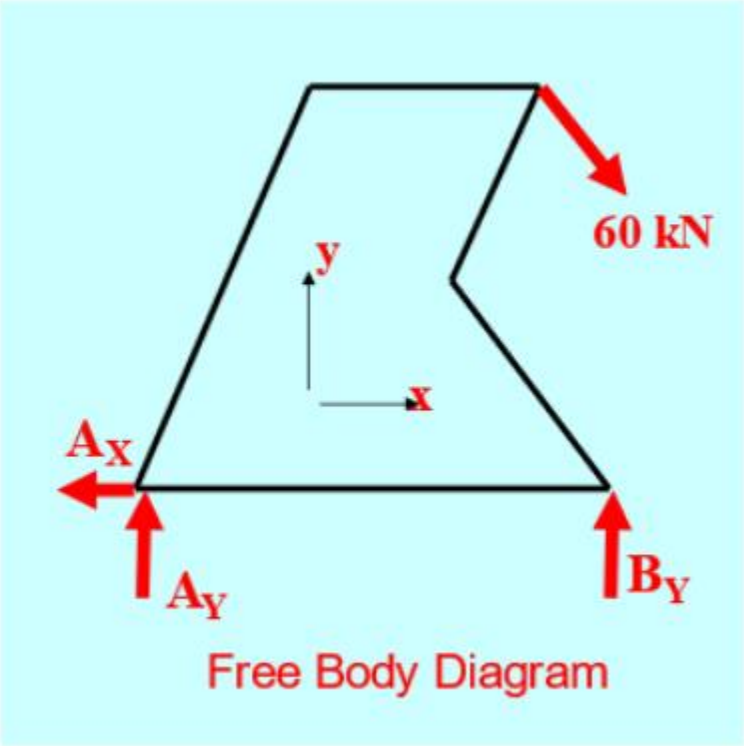
Truss Bridge FBD

FBD of the entire truss bridge:

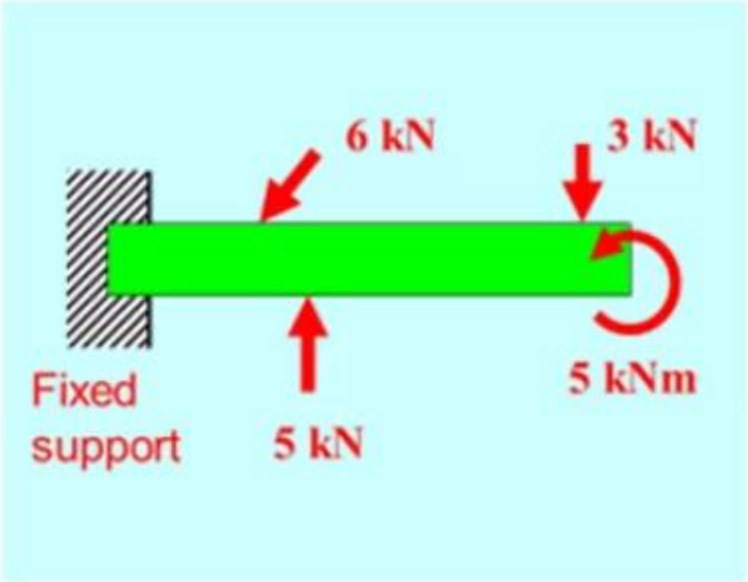


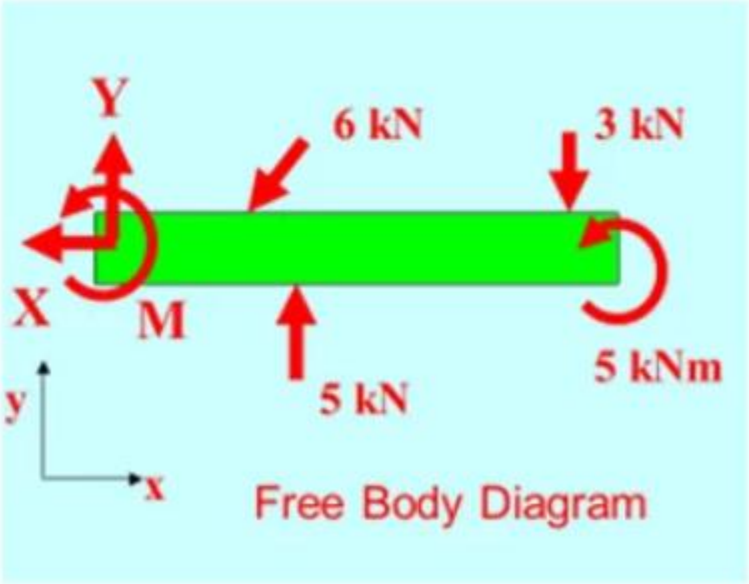
1. Plane Truss - Neglect the weight of the truss





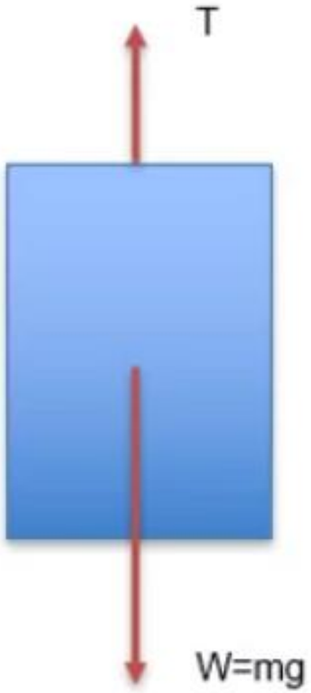
2. Cantilever beam (wing)



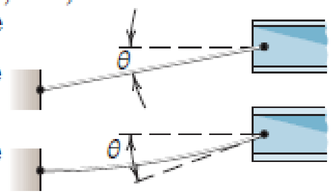
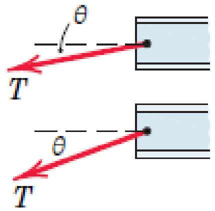
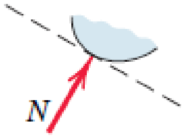
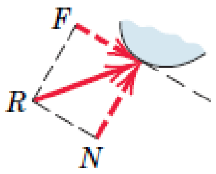
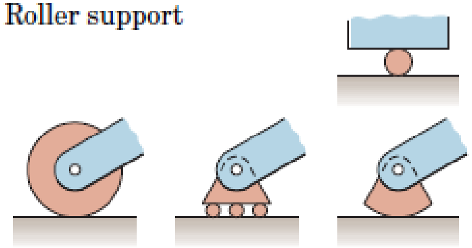
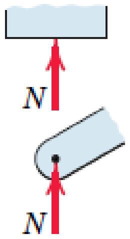
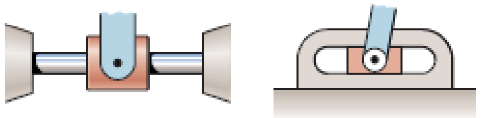
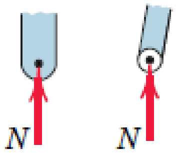


Example Free Body Diagram of Person in a Lift

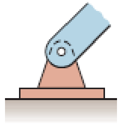
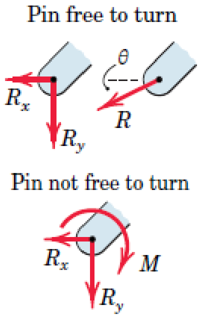
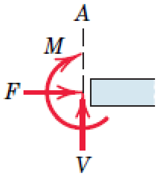
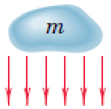
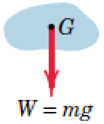
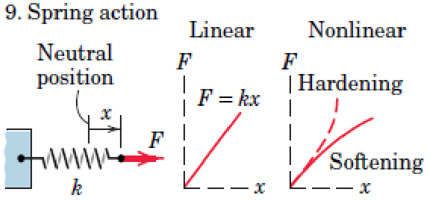
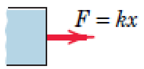
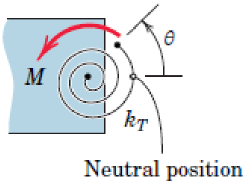
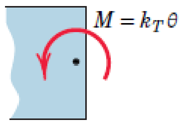
The lift is supported by a cable shown on the top of the diagram.



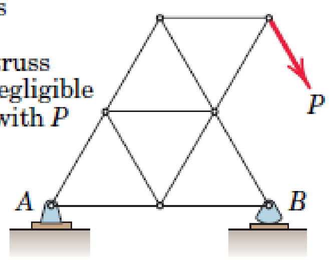
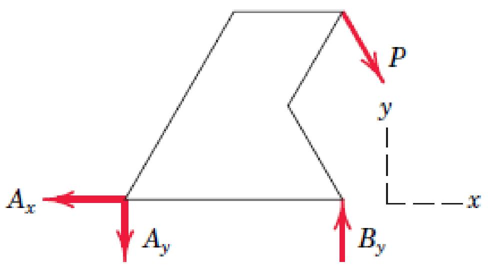
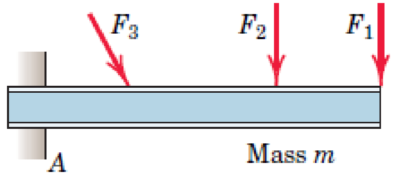
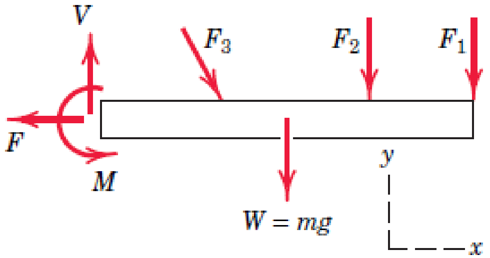
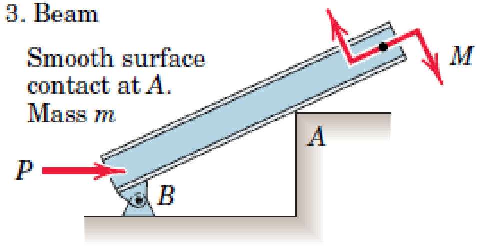
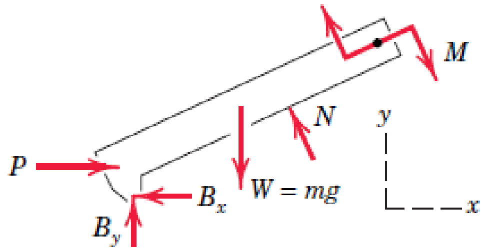
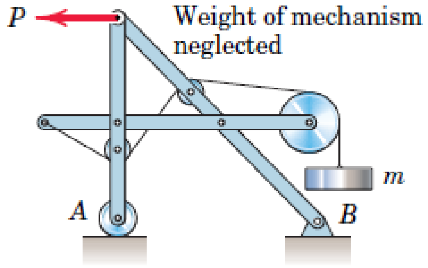
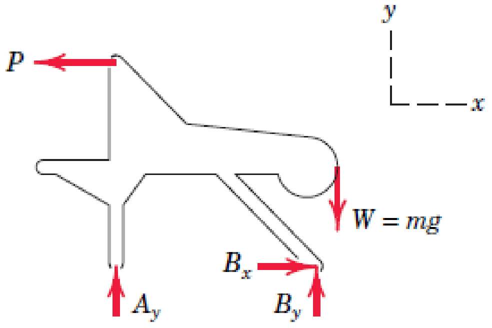
MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS

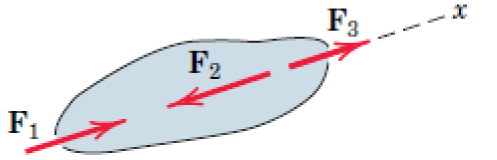
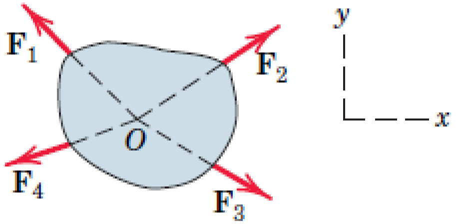
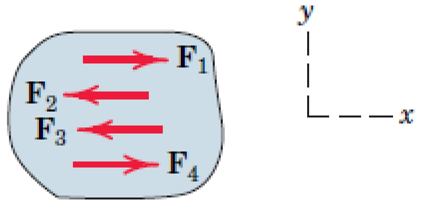
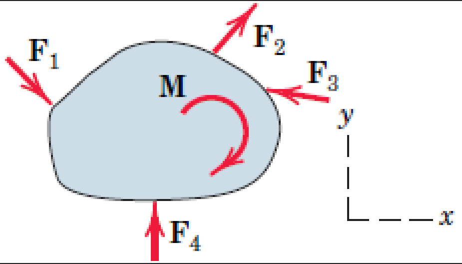
Type of Contact and Force Origin	Action on Body to Be Isolated
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible 	 Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces 	 Contact force is compressive and is normal to the surface.
3. Rough surfaces 	 Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant contact force R.
4. Roller support 	 Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide 	 Collar or slider free to move along smooth guides; can support force normal to guide only.

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (*cont.*)

Type of Contact and Force Origin	Action on Body to Be Isolated
6. Pin connection 	 A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.
7. Built-in or fixed support 	 A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction 	 The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center of gravity G.
9. Spring action 	 Spring force is tensile if the spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.
10. Torsional spring action 	 For a linear torsional spring, the applied moment M is proportional to the angular deflection θ from the neutral position. The stiffness k_T is the moment required to deform the spring one radian.

SAMPLE FREE-BODY DIAGRAMS

Mechanical System	Free-Body Diagram of Isolated Body
1. Plane truss Weight of truss assumed negligible compared with P 	
2. Cantilever beam 	
3. Beam Smooth surface contact at A. Mass m 	
4. Rigid system of interconnected bodies analyzed as a single unit Weight of mechanism neglected 	

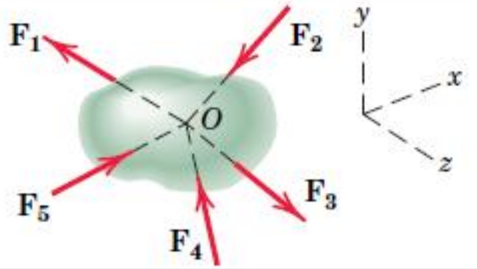
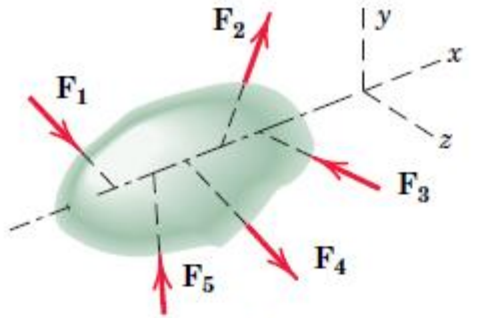
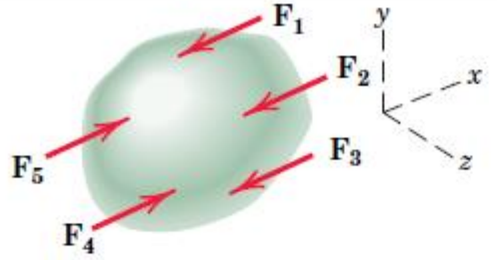
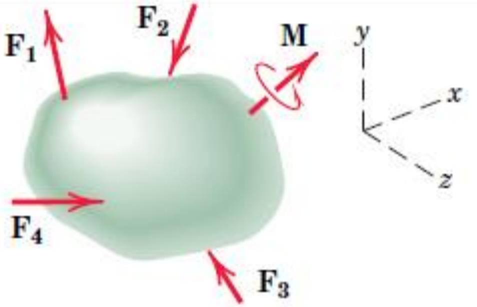
CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0 \quad \Sigma M_z = 0$
4. General		$\Sigma F_x = 0 \quad \Sigma M_z = 0$ $\Sigma F_y = 0$

Category 1, equilibrium of collinear forces, clearly requires only the one force equation in the direction of the forces (x-direction), since all other equations are automatically satisfied.

Category 2, equilibrium of forces which lie in a plane (x-y plane) and are concurrent at a point O, requires the two force equations only, since the moment sum about O, that is, about a z-axis through O, is necessarily zero. Included in this category is the case of the equilibrium of a particle.

Category 3, equilibrium of parallel forces in a plane, requires the one force equation in the direction of the forces (x-direction) and one moment equation about an axis (z-axis) normal to the plane of the forces.

Category 4, equilibrium of a general system of forces in a plane (x-y), requires the two force equations in the plane and one moment equation about an axis (z-axis) normal to the plane.

CATEGORIES OF EQUILIBRIUM IN THREE DIMENSIONS			
Force System	Free-Body Diagram	Independent Equations	
1. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$	
2. Concurrent with a line		$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$	$\Sigma M_y = 0$ $\Sigma M_z = 0$
3. Parallel		$\Sigma F_x = 0$	$\Sigma M_y = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$	$\Sigma M_x = 0$ $\Sigma M_y = 0$ $\Sigma M_z = 0$

Categories of Equilibrium

Category 1, equilibrium of forces all concurrent at point O, requires all three force equations, but no moment equations because the moment of the forces about any axis through O is zero.

Category 2, equilibrium of forces which are concurrent with a line, requires all equations except the moment equation about that line, which is automatically satisfied.

Category 3, equilibrium of parallel forces, requires only one force equation, the one in the direction of the forces (x-direction as shown), and two moment equations about the axes (y and z) which are normal to the direction of the forces.

Category 4, equilibrium of a general system of forces, requires all three force equations and all three moment equations.

Thank You…