1)

$$\begin{bmatrix} 1 & 0.67 & 0.33 \\ 0.45 & 1 & 0.55 \\ 0.67 & 0.33 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

on a Decimal computer the ~~calculation is as~~ representation is

$$\begin{bmatrix} 0.1\times10^1 & 0.67\times10^0 & 0.33\times10^0 \\ 0.45\times10^0 & 0.1\times10^1 & 0.55\times10^0 \\ 0.67\times10^0 & 0.33\times10^0 & 0.1\times10^1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.2\times10^1 \\ 0.2\times10^1 \\ 0.2\times10^1 \end{bmatrix}$$

2nd row − 0.45 × 1st row

$$\begin{bmatrix} 0.1\times10^1 & 0.67\times10^0 & 0.33\times10^0 \\ 0 & 0.7\times10^0 & 0.4\times10^0 \\ 0.67\times10^0 & 0.33\times10^0 & 0.1\times10^1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.2\times10^1 \\ 0.11\times10^1 \\ 0.2\times10^1 \end{bmatrix}$$

$$\begin{bmatrix} 0.1\times10^1 & 0.67\times10^0 & 0.33\times10^0 \\ 0 & 0.7\times10^0 & 0.4\times10^0 \\ 0 & -0.12\times10^0 & 0.78\times10^0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.2\times10^1 \\ 0.11\times10^1 \\ 0.7\times10^0 \end{bmatrix}$$

$$\begin{bmatrix} 0.1\times10^1 & 0.67\times10^0 & 0.33\times10^0 \\ 0 & 0.1\times10^1 & 0.57\times10^0 \\ 0 & -0.12\times10^0 & 0.78\times10^0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.2\times10^1 \\ 0.16\times10^1 \\ 0.7\times10^0 \end{bmatrix}$$

$$\begin{bmatrix} 0.1\times10^1 & 0.67\times10^0 & 0.33\times10^0 \\ 0 & 0.1\times10^1 & 0.57\times10^0 \\ 0 & 0 & 0.85\times10^0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.2\times10^1 \\ 0.16\times10^1 \\ 0.89\times10^1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 0.1\times10^1 & 0.67\times10^0 & 0.33\times10^0 \\ 0 & 0.1\times10^1 & 0.57\times10^0 \\ 0 & 0 & 0.1\times10^1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.2\times10^1 \\ 0.16\times10^1 \\ 0.1\times10^1 \end{bmatrix}$$

$\therefore x_3 = 1$

on back substitution

$x_2 = 0.16 \times 10^1 - (0.57 \times 10^0 \times 0.1 \times 10^1)$

$= 0.16 \times 10^1 - 0.57 \times 10^0$

matching indices.

$0.16 \times 10^1 - 0.057 \times 10^1$

$= 0.103 \times 10^1$

round off

$= 0.1 \times 10^1$ or 1.

on back substitution

$x_1 = 0.2 \times 10^1 - (0.67 \times 10^0 + 0.33 \times 10^0)$

$= 0.2 \times 10^1 - (1.0 \times 10^0)$

$= 0.2 \times 10^1 - 0.1 \times 10^1$

$= 0.1 \times 10^1$

$\therefore$ Solution $= x_1 = x_2 = x_3 = 1$

Gauss - Sidel

$$\begin{bmatrix} 3 & -1 & 1 \\ 3 & 6 & 2 \\ 3 & 3 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$$

initial Guess

iter	x_1	x_2	x_3
0	0	0	0

$$x_1^1 = \frac{1-(0)}{3} = \frac{1}{3}$$

$$x_2^1 = \frac{0-(3\times\frac{1}{3})}{6} = \frac{-1}{6}$$

$$x_3^1 = \frac{4-\left[(3\times\frac{1}{3})+(3\times-\frac{1}{6})\right]}{7} = \frac{1}{2}$$

iter	x_1	x_2	x_3
0	0	0	0
1	$\frac{1}{3}$	$\frac{-1}{6}$	$\frac{1}{2}$

$$x_1^2 = \frac{1-\left[-1\left(\frac{-1}{6}\right)+1\left(\frac{1}{2}\right)\right]}{3} = \frac{1}{9}$$

$$x_2^2 = \frac{0-\left[(3\times\frac{1}{9})+2\left(\frac{1}{2}\right)\right]}{6} = \frac{-2}{9}$$

$$x_3^2 = \frac{4-\left[(3\times\frac{1}{9})+3\left(\frac{-2}{9}\right)\right]}{7} = \frac{13}{21}$$

iter	x_1	x_2	x_3
0	0	0	0
1	$\frac{1}{3}$	$-\frac{1}{6}$	$\frac{1}{2}$
2	$\frac{1}{9}$	$-\frac{2}{9}$	$\frac{13}{21}$

3

$$\begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 & -1 & 0 \\ 0 & -1 & 4 & 0 & 0 & -1 \\ -1 & 0 & 0 & 4 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 \\ 6 & 0 & -1 & 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 0 \\ 6 \\ -2 \\ 6 \end{bmatrix}$$

iter	x_1	x_2	x_3	x_4	x_5	x_6
0	0	0	0	0	0	0

$x_1^1 = \frac{0}{4} = 0$ | $x_2^1 = \frac{5-(0)}{4} = \frac{5}{4}$ | $x_3^1 = \frac{0-(0)}{4} = 0$

$x_4^1 = \frac{6-(0)}{4} = \frac{3}{2}$ | $x_5^1 = \frac{-2-(0)}{4} = -\frac{1}{2}$ | $x_6^1 = \frac{6-(0)}{4} = \frac{3}{2}$

iter	x_1	x_2	x_3	x_4	x_5	x_6
0	0	0	0	0	0	0
1	0	5/4	0	3/2	$-\frac{1}{2}$	$\frac{3}{2}$

$$x_1^2 = \frac{0-[-1\times 5/4 \quad -1\times 3/2]}{4} = \frac{11}{16}$$

$$x_2^2 = \frac{5-[(-1\times 0)+(-1\times 0)+(-1\times -\frac{1}{2})]}{4} = \frac{9}{8}$$

$$x_3^2 = \frac{0-[(-1\times \frac{5}{4}) \quad (1\times(3/2))]}{4} = \frac{11}{16}$$

$$x_4^2 = \frac{6-[(-1\times 0)+(-1\times -\frac{1}{2})]}{4} = \frac{13}{8}$$

$$x_5^2 = \frac{-2-[(-1\times \frac{5}{4})+(-1\times \frac{1}{2})+(-1\times \frac{3}{2})]}{4} = \frac{9}{16}$$

$$x_6^2 = \frac{6-[(-1\times 0)+(-1\times -\frac{1}{2})]}{4} = \frac{13}{8}$$

$\therefore$

iter	x_1	x_2	x_3	x_4	x_5	x_6
0	0	0	0	0	0	0
1	0	$5/4$	0	$3/2$	$-\frac{1}{2}$	$3/2$
2	$\frac{11}{16}$	$\frac{9}{8}$	$\frac{11}{16}$	$\frac{13}{8}$	$\frac{9}{16}$	$\frac{13}{8}$

Gauss-Sidel method

$$\begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 & -1 & 0 \\ 0 & -1 & 4 & 0 & 0 & -1 \\ -1 & 0 & 0 & 4 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 0 \\ 6 \\ -2 \\ 6 \end{bmatrix}$$

iter	x_1	x_2	x_3	x_4	x_5	x_6
0	0	0	0	0	0	0

$$x_1' = \frac{0-(0)}{4} = 0$$

$$x_2' = \frac{5-(0)}{4} = \frac{5}{4}$$

$$x_3' = \frac{0-[(-1\times\frac{5}{4})+0]}{4} = \frac{5}{16}$$

$$x_4' = \frac{6-[(-1\times 0)+(-1\times 0)]}{4} = \frac{3}{2}$$

$$x_5' = \frac{-2-[(-1\times\frac{5}{4})+(-1\times\frac{3}{2})+(-1\times 0)]}{4} = \frac{3}{16}$$

$$x_6' = \frac{6-[(-1\times\frac{5}{16})+(-1\times\frac{3}{16})]}{4} = \frac{13}{8}$$

iter	x_1	x_2	x_3	x_4	x_5	x_6
0	0	0	0	0	0	0
1	0	$\frac{5}{4}$	$\frac{5}{16}$	$3/2$	$\frac{3}{16}$	$\frac{13}{8}$

$$x_1^2 = \frac{0 - \left[\left(-1 \times \frac{5}{4}\right) + \left(-1 \times \frac{3}{2}\right)\right]}{4} = \frac{11}{16}$$

$$x_2^2 = \frac{5 - \left[\left(-1 \times \frac{11}{16}\right) + \left(-1 \times \frac{5}{16}\right) + \left(-1 \times \frac{3}{16}\right)\right]}{4} = \frac{99}{64}$$

$$x_3^2 = \frac{0 - \left[\left(-1 \times \frac{99}{64}\right) + \left(-1 \times \frac{13}{8}\right)\right]}{4} = \frac{203}{256}$$

$$x_4^2 = \frac{6 - \left[\left(-1 \times \frac{11}{16}\right) + \left(-1 \times \frac{3}{16}\right)\right]}{4} = \frac{55}{32}$$

$$x_5^2 = \frac{-2 - \left[\left(-1 \times \frac{99}{64}\right) + \left(-1 \times \frac{55}{32}\right) + \left(-1 \times \frac{13}{8}\right)\right]}{4} = \frac{185}{256}$$

$$x_6^2 = \frac{6 - \left[\left(-1 \times \frac{203}{256}\right) + \left(-1 \times \frac{185}{256}\right)\right]}{4} = \frac{481}{256}$$

iter	x_1	x_2	x_3	x_4	x_5	x_6
0	0	0	0	0	0	0
1	0	$\frac{5}{4}$	$\frac{5}{16}$	$3/2$	$3/16$	$\frac{13}{8}$
2	$\frac{11}{16}$	$\frac{99}{64}$	$\frac{203}{256}$	$\frac{55}{32}$	$\frac{185}{256}$	$\frac{481}{256}$

3) Assume The complex Matrix to be

$$X = A + iB$$

let $X^{-1} = C + iD$

$\therefore XX^{-1} = (A+iB)(C+iD) = I$

$\therefore AC - BD + i(AD + BC) = I$

$\therefore AC - BD = I \quad -①$

$AD + BC = 0 \quad - ②$

$\Rightarrow AD = -BC$

$D = -A^{-1}BC$ - substituting in ①

$AC + BA^{-1}BC = I$ (multiply both sides with C^{-1})

$\therefore (A + BA^{-1}B) = C^{-1}$

$\therefore C = (A + BA^{-1}B)^{-1}$

$\therefore D = -A^{-1}B\,(A + BA^{-1}B)^{-1}$

$= -(A B^{-1} A + B)^{-1}$

$= -(B + AB^{-1}A)^{-1}$

$\therefore X^{-1} = (A + BA^{-1}B)^{-1} - i(B + AB^{-1}A)^{-1}$