

4) Stiff differential equations is a differential equation for which certain numerical methods of solving become the equations are numerically unstable, unless the step-size is taken to be extremely small. A linear system, a system of differential equations is termed stiff if the ratio between the largest and the smallest eigen values is large.

- Decay processes generally give stiff differential equations. we use Backward integration methods to solve stiff differential equations.

- we can use `scipy.integrate.solve_ivp` - with method set to 'BDF' for solving stiff Differential equations. ~~backward~~ 'LASODA', 'Radau', ~~can~~ methods can also be used.

14)

1) `gsl-odeiv2-step-rk2` - Explicit Runge-kutta(2,3) method -

2) `gsl-odeiv2-step-rk4` - 4th order classic Runge-kutta method with Error estimation carried with step doubling method

3) gsl-odeiv2-step-rkck - Explicit Runge-Kutta (Cash-Karp (4/5) method).

- The Functions are used by first defining ODE system.

`gsl-odeiv2-system - sys = f,`

`int (*function) (double t, const double y[], double dydt[], void *params)`

as we can solve multiple coupled ODE's using `y[]`

`, int (*jacobian) (double t, const double y[], double dfdy, void *params)`

`= size-t dimension`

Dimension of system of equations - or number of equations

`void *params`

< Then we need to apply a driver to the which tells the method of stepping.

`gsl-odeiv-driver id = gsl-odeiv2-driver-alloc-new`

`(id const gsl-odeiv2-system *sys,`

stem type to `← const, gsl-odeiv2-step type *T, const double`

be used defined `hstart, const double epsabs, const double epsrel`

above `ex gsl-odeiv2-step-rkck`

`hstart`

(as adaptive step size)

absolute and relative errors

Then we have to apply the driver

int get-odeivz-driver-~~also~~ apply-kutta

(get-odeivz-driver *d, double *t, const double t~~0~~,
 double y[])

double y[]

Define ranges of t
 [a, b]

(stores the solved values ~~and~~
 of y - ~~should~~ contain y0 initially)

12) General Runge-Kutta method of order s can be written as

$$y_{t+h} = y_t + h \sum_{i=1}^s a_i k_i + O(h^{s+1})$$

where $k_i = \cancel{y_t} h \sum_{j=1}^s B_{ij} f(k_j, t_n + \alpha_j h)$

~~we want to~~

from above we can see for sth order f has to be evaluated s times. ~~taking~~

$$y_{n+1} = y_n + \{a_1 k_1 + a_2 k_2 + a_3 k_3 + a_4 k_4 + a_5 k_5\} / h$$

$$k_1 = h f(x_n, y_n)$$

$$k_2 = h f(x_n + v_2 h, y_n + v_2 k_1)$$

$$k_3 = h f(x_n + v_3 h, y_n + \{ (u_3 v_1 - 1) k_1 + k_2 \} / (8 v_1))$$

$$k_4 = h f(x_n + \frac{2}{3} h, y_n + \{ (10 v_2 - 2) k_1 + 2 k_2 + 8 v_3 k_3 \} / (27 v_1))$$

$$k_5 = h f(x_n + (7 + \sqrt{21}) h / 14, y_n + \{ (E 77 v - 56) + (17 v - 8) (\sqrt{21}) \} k_1 - 8 (7 + \sqrt{21}) k_2 + u_5 (7 + \sqrt{21}) v k_3 - 3 (21 + \sqrt{21}) v k_4 \} / 343 v)$$

1960s,

(189)

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~~1/4 as~~ K as 0 (a) $y y' = 0$)