# NUMERICAL METHODS
# FINAL EXAM

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## 1. Problem 1

(1) A. RAM
(2) D. All of the above.
(3) D. Both A and C.
(4) A. Linux kernel.
(5) C.

## 2. Problem 2

2.1. **(a).** We'll choose the origin to be the centre of the circle. The equation of the circle is then,

$$x^2 + y^2 = 100 \implies y(x) = \pm\sqrt{100 - x^2}. \tag{1}$$

The area of the circle is therefore

$$A = \int_{-10}^{10} 2\sqrt{100 - x^2}\, dx. \tag{2}$$

In the attached code I used the scipy.integrate.quad function to compute the integral. The numerical value I'm getting is 314.15926535897967. The analytic value is $100\pi \approx 314.1592653589793$. So our numerical algorithm is pretty accurate.

2.2. **(b).** I did not use any numerical integration for this. I directly plotted $r$ against $\pi r^2$. The plot is shown in Fig. 1.

2.3. **(c).** The interpolated value at $r = 13$ is 530.9291584566749. The actual value should be $169\pi \approx 530.929158456675$. The plot is shown in Fig. 2.

## 3. Problem 3

3.1. **(a).** The scatter plot of the data points given in "data.txt" is shown in Fig. 3.

3.2. **(b).** Using Cubic Spline Interpolation to fill in the gaps, the interpolated function is plotted in Fig. 4.

3.3. **(c).** The root of the interpolated function is at

$$x = 0.4515822420016775. \tag{3}$$

We obtain this by the scipy.optimize.bisect function applied to the interpolated function in the bracket $(-1, 1)$.

*Date*: February 10, 2022.

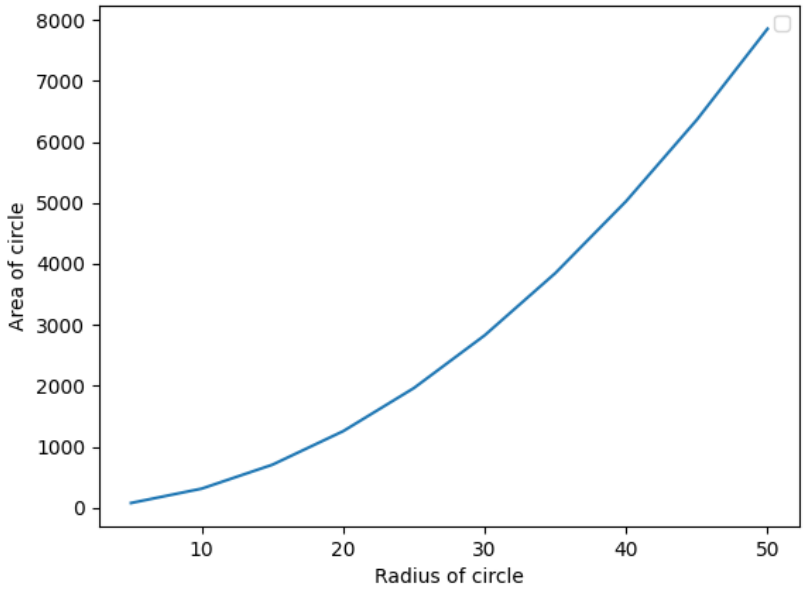


FIGURE 1. The area of a circle as function of its radius.

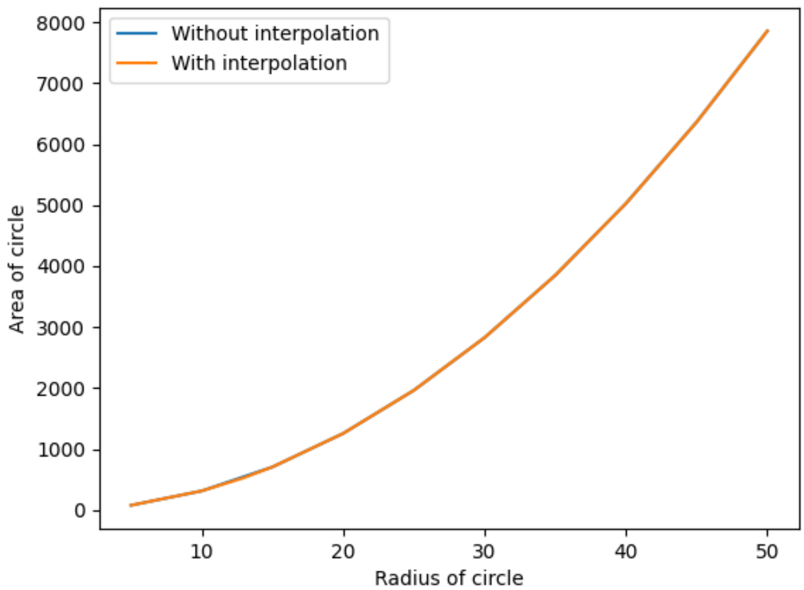


FIGURE 2. The area of a circle as function of its radius. The interpolated function and the actual function are almost the same except near $r = 13$ where we see a tiny difference (you might have to zoom in a little).

## 4. Problem 4

If we wish to evaluate an integral of the form:

$$I = \int_{-1}^{1} f(x)\, dx, \tag{4}$$

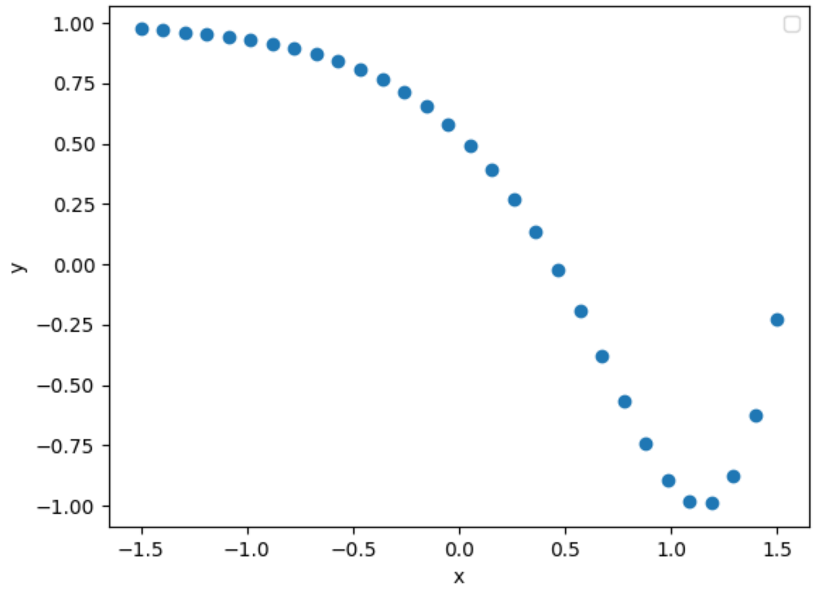


FIGURE 3. Scatter plot of data points in data.txt.

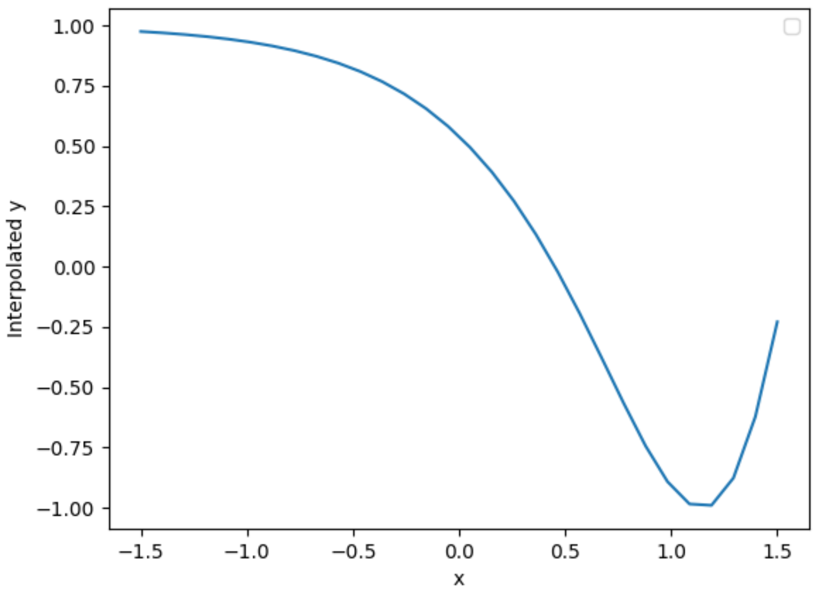


FIGURE 4. Interpolated function of the data points in data.txt.

using the Gaussian-Quadrature (2-point) method in the interval $(-1, 1)$, then we write

$$I = \int_{-1}^{1} f(x)\,dx = w_0 f(x_0) + w_1 f(x_1). \tag{5}$$

In class we solved for $x_0, x_1, w_0, w_1$ and obtained:

$$x_0 = -\sqrt{\frac{1}{3}}, \quad x_1 = \sqrt{\frac{1}{3}}, \quad w_0 = w_1 = 1. \tag{6}$$

Now in the problem, we have the integral

$$I = \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{1 - x^2 - y^2}\, dy\, dx = \int_{-1}^{1} g(x)\, dx, \tag{7}$$

where

$$g(x) = \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{1 - x^2 - y^2}\, dy. \tag{8}$$

By Gaussian-Quadrature (2-point) method, we have

$$I = \int_{-1}^{1} g(x)\, dx \approx g\left(\sqrt{\frac{1}{3}}\right) + g\left(-\sqrt{\frac{1}{3}}\right) = 2g\left(\sqrt{\frac{1}{3}}\right), \tag{9}$$

since $g(x) = g(-x)$ is even.

Now,

$$g\left(\sqrt{\frac{1}{3}}\right) = \int_{-\sqrt{2/3}}^{\sqrt{2/3}} \sqrt{\frac{2}{3} - y^2}\, dy. \tag{10}$$

In our code (prob4.py) we evaluate this integral numerically (using scipy.integrate.quad). This gives us

$$g\left(\sqrt{\frac{1}{3}}\right) = 1.0471975511965983. \tag{11}$$

Therefore,

$$I = 2g\left(\sqrt{\frac{1}{3}}\right) = 2.0943951023931966. \tag{12}$$

This is the volume of the half-sphere. To get the volume of the full sphere, we need to multiply this by 2. Therefore,

$$V_{sphere} = 2I = 4.188790204786393. \tag{13}$$

The analytic value is

$$V_{sphere} = \frac{4\pi}{3} \approx 4.1887902047863905. \tag{14}$$

So our numerical algorithm is pretty accurate.