Intro: Alice can have an entangled qubit to send 2 qubits of data at once?

But can only send 1.

Exercise 1. $|\phi^+\rangle_{AB} = \frac{|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B}{\sqrt{2}}$

1. $I_A |\phi^+\rangle_{AB} = \frac{|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B}{\sqrt{2}}$

2. $X_A |\phi^+\rangle_{AB} = \frac{|1\rangle_A|0\rangle_B + |0\rangle_A|1\rangle_B}{\sqrt{2}}$

3. $Z_A |\phi^+\rangle_{AB} = \frac{|0\rangle_A|0\rangle_B - |1\rangle_A|1\rangle_B}{\sqrt{2}}$

$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

4. $Z_A X_A |\phi^+\rangle_{AB} = Z_A \left(\frac{|1\rangle_A|0\rangle_B + |0\rangle_A|1\rangle_B}{\sqrt{2}} \right) = \frac{-|1\rangle_A|0\rangle_B + |0\rangle_A|1\rangle_B}{\sqrt{2}}$

Exercise 2.

1. $H_A C_{AB} I_A |\phi^+\rangle_{AB} = H_A C_{AB} \left(\frac{|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B}{\sqrt{2}} \right) = H_A \left(\frac{|0\rangle_A|0\rangle_B + |1\rangle_A|0\rangle_B}{\sqrt{2}} \right)$

$= \frac{-|1\rangle_A|0\rangle_B}{2} + \frac{|0\rangle_A|0\rangle_B}{\sqrt{2}} + \frac{|1\rangle_A|0\rangle_B}{\sqrt{2}} + \frac{|0\rangle_A|0\rangle_B}{\sqrt{2}} = |0\rangle_A|0\rangle_B$

2. $H_A C_{AB} X_A |\phi^+\rangle_{AB} = H_A C_{AB} \left(\frac{|1\rangle_A|0\rangle_B + |0\rangle_A|1\rangle_B}{\sqrt{2}} \right) = H_A \left(\frac{|1\rangle_A|1\rangle_B + |0\rangle_A|0\rangle_B}{\sqrt{2}} \right)$

$= \frac{|0\rangle_A|1\rangle_B + |1\rangle_A|1\rangle_B - |1\rangle_A|1\rangle_B + |0\rangle_A|1\rangle_B}{2} = |0\rangle_A|1\rangle_B$

3. $H_A C_{AB} Z_A |\phi^+\rangle_{AB} = H_A C_{AB} \left(\frac{|0\rangle_A|0\rangle_B - |1\rangle_A|1\rangle_B}{\sqrt{2}} \right) = H_A \left(\frac{|0\rangle_A|0\rangle_B - |1\rangle_A|0\rangle_B}{\sqrt{2}} \right)$

$\frac{-|1\rangle_A|0\rangle_B + |0\rangle_A|0\rangle_B - |1\rangle_A|0\rangle_B - |0\rangle_A|0\rangle_B}{2} = -|1\rangle_A|0\rangle_B$

4. $H_A C_{AB} Z_A X_A |\phi^+\rangle_{AB} = H_A C_{AB} \left(\frac{|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B}{\sqrt{2}} \right) = H_A \left(\frac{|0\rangle_A|1\rangle_B - |1\rangle_A|1\rangle_B}{\sqrt{2}} \right)$

$= \frac{-|1\rangle_A|1\rangle_B + |0\rangle_A|1\rangle_B - |1\rangle_A|1\rangle_B - |0\rangle_A|1\rangle_B}{2} = -|1\rangle_A|1\rangle_B$

Discussion:

Yes I am surprised

practical apps include better encryption algorithms

I'm not sure (superposition?)