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A sequential approach for the optimization of truck routes for solid waste collection

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ABSTRACT

The main objective of this paper is to present a sequential approach involving three phases for solving the optimization problem of truck routes for the collection of solid waste. The first phase executes the grouping of arcs based on an adapted model of the p-median problem, formulated as a problem of Binary Integer Linear Programming (BILP). The second phase refers to the development of a model for the solution to the Capacitated Arc Routing Problem (CARP), formulated as a Mixed Integer Linear Programming (MILP) problem. The third phase carries out the application of an adapted algorithm of Hierholzer for sequencing the arcs obtained in the preceding phase. The proposed methodology was tested using real data and efficiently solved the problem. The results led to a reduction in the distances traveled by trucks, which could promote money savings for the public coffers, as well as a reduction in carbon dioxide emissions.

Contents

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Nomenclature

ACS	Ant Colony System
b	truck capacity
BILP	binary integer linear programming
c_{ij}	cost of the arc (i, j)
CARP	Capacitated Arc Routing Problem
CCPP	Capacitated Chinese Postman Problem
CO_2	carbon dioxide
d_{ij}	product of the distance between vertices x_i and x_j and the weight w_i
FLP	Facility Location Problem
GAMS	General Algebraic Modeling System
K	number of vehicles
MCPP	Mixed Chinese Postman Problem
MILP	Mixed Integer Linear Programming
M-VRPTW	Multi-Depot Vehicle Routing Problem with Time Windows
n	number of vertices of the graph
p	number of medians to be installed
q_{ij}	demand of the arc (i, j)
SA	Sensitivity Analysis
TSP	Travelling Salesman Problem
VRP	Vehicle Routing Problem
w_j	demand of each vertex x_j

1. Introduction

Solid waste is the technical name given to garbage and can be regarded as any material that the owner or producer no longer considers valuable enough to keep. It is composed mainly of paper/cardboard, plastics, glass, metals, textile and food/garden waste. It is the result of human activity and its amount is directly proportional to industrial intensity and population increase. Solid waste is regarded as dangerous in relation to its physical, chemical, and infectious properties. Thus, the inadequate removal and collection of waste, as well as inadequate destination and final treatment, can cause a great impact on the environment (Buah et al., 2007).

The problem of solid waste in cities encompasses several factors, such as generation, collection, processing, and disposal. Collection is the most sensitive part in the eyes of the population and needs to be very well planned, since it represents about 50% of the operating costs of public sanitation, and not less important, the trucks that carry out this task emit about 1.24 kg of CO_2/km traveled and are considered large carbon dioxide emitters in the atmosphere (Detofeno and Steiner, 2010).

So, for this planning, route optimization of the trucks that collect waste is often treated as a Vehicle Routing Problem (VRP), which basically consists in establishing and organizing efficient routes so that vehicles can deliver and/or pickup merchandise, featuring a fleet of k vehicles which may or may not be identical, with the objective of serving a set of customers with a known demand. It is associated with special cases such as the Travelling Salesman Problem (TSP) and the Capacitated Chinese Postman Problem (CCPP) (Dror, 2001).

In this work, the subject under study is considered to be a combinatorial optimization problem, known in the literature as a Capacitated Arc Routing Problem (CARP), which was first proposed by Golden and Wong in 1981. The CARP, as proposed by these authors, considers a non-negative demand associated with each arc of the road network and a set of vehicles with known capacity that must traverse the arcs making collections or deliveries relating to the respective demands. Furthermore, the capacity of the trucks must not be exceeded. The goal is to search for a set of minimal cost routes that begin and end at a single point, often termed warehouse (Golden and Wong, 1981).

The CARP includes most real life applications in cases related to the collection or delivery of products and is classified as NP-Hard. Due to this fact, heuristics are frequently used to solve the problem more efficiently. Some papers from the literature highlight problem solution alternatives. Moghadam et al. (2014) proposed a simulated annealing algorithm and a hybrid metaheuristic algorithm combining Ant Colony System (ACS) and simulated annealing for a vehicle routing and scheduling problem in a network consisting of suppliers, customers, and a cross dock; Dondo and Cerdá (2009) presented a local search improvement algorithm to solve the multi-depot vehicle routing problem with time windows (M-VRPTW) that explores a large neighborhood of the current solution to discover a cheaper set of feasible routes; Belenguer et al. (2010) used a metaheuristic based on a cutting plane algorithm and on evolutionary local search for a split-delivery problem; Laporte et al. (2010) presented a neighborhood search heuristic for a problem with stochastic demands; Mourão and Almeida (2000) proposed a heuristic combined with a lower bounding method for a problem of waste collection; Mourão and Amado (2005) used heuristics such as Path-Scanning, Augment-Merge, and Ulusoy's algorithms, and Ghiani et al. (2005) developed an approach based on a well-known cluster-first, route-second heuristic, both for the problem of waste collection; Hertz et al. (2000) presented a method based on Tabu Search algorithm; Beullens et al. (2003) used a guide local search algorithm and Lacomme et al. (2001) used a Genetic Algorithm to solve the CARP; Shanmugasundaram et al. (2011) implemented the SAVGIS (free tool) in route optimization for collection and transportation of healthcare waste in Laos; Karadimas et al. (2007) implemented the Ant Colony System (ACS) algorithm for route optimization of solid waste trucks in Athens.

According to Usberti et al. (2011), there are two ways to solve the CARP from an exact algorithm: the first one is based on a branch and bound algorithm (Hirabayashi et al., 1992), while the second one transforms the CARP into a capacitated vehicle routing Problem (CVRP) and the problem is solved using a branch-and-cut-and-price algorithm (Longo et al., 2006). However, the authors caution that these approaches can only solve specific instances of relatively small size. Other authors, such as Eiselt et al. (1995), Dror (2001), Hertz (2005), Wøhlk (2008), and Corberán and Prins (2010) expanded the CARP study and presented exact and heuristic algorithms to improve its solution.

As noted by Dror (2001), there are two versions of the CARP regarding the number of vehicles to be included in the model. In the first version, this number is a fixed parameter. In the second one, it is considered to be a decision variable, which means that the algorithms can make use of an unlimited fleet of vehicles. Welz (1994) observed that determining the existence of a feasible solution for a given fixed number of vehicles is already an NP-hard problem and thus, for the second version of the CARP, this problem becomes even more difficult, which may explain the development of many heuristics for this purpose.

Several papers in the literature present a solution to the arc routing problem from different perspectives. Most authors use heuristic algorithms due to the complexity of using mathematical models in situations of medium to large scale. This difficulty is generally related to the formation of sub-routes, which increases with an increasing number of nodes and arcs. So, in the present paper, a sequential approach is presented, which allows a satisfactory solution to the problem without the formation of sub-routes.

The proposed methodology consists of three phases, in which two mathematical models and an exact algorithm are used. The first phase uses an adapted model of the p-median problem, also known as the facility location problem (FLP) (adapted because the model considers the demand of waste and the capacity of the trucks), to divide the area to be served by each vehicle, in other words, to make groupings of streets (edges/arcs) for each collection vehicle. The second phase uses a Mixed Integer Linear Programming (MILP) model, adapted from the CARP proposed by Dror (2001), which is applied to each group of arcs obtained from the previous step to minimize the total distance traveled by the truck. In the third phase, an adapted Hierholzer algorithm is applied for each graph from the previous step to obtain the route of each vehicle. The CARP and the Hierholzer algorithm were adapted due to the fact that the route begins and ends at different points, starting at the truck garage and ending at the landfill. For the CARP, the number of trucks is fixed at two, since, according to the company responsible for the waste collection in the region, this number is enough to perform the task.

In the p-median problem (first step), a deterministic model is used to group the vertices for each truck, considering the demand and the capacity of the vehicles. In the CARP (second step), another deterministic model is applied for each group obtained in the previous stage, for route optimization, considering only one truck per group, which avoids the formation of sub-routes in the solution. Because it is a solution obtained by a sequential method, the response obtained in the first step is used as the initial estimate in the second step, and that obtained in the second step is used in the third. As the first two models are deterministic, the solution found in each of these steps is unique.

This methodology was successfully applied using real data collected in the central region of Campo Mourão, Paraná, Brazil, aiming to plan the collection and transportation of waste in order to minimize the traveled distances as well as the costs of maintenance and refueling of the vehicles involved in this task and, consequently, to reduce the emissions of carbon dioxide into the atmosphere. This research covers only the routes included in the given area, which serves as an example, but it can be applied to any other area by simply changing the input data.

2. Methodology proposal

In the present work, a sequential methodology is developed to solve the route optimization problem of solid waste collection trucks. The problem is solved sequentially by using two different mathematical models and an algorithm interacting in three phases. Adjustments are made to the mathematical models and to the algorithm to allow viable and consistent results.

In the first phase, a BILP model based on the p-median problem is applied. The set of vertices is divided into two groups, one for each truck, considering the demands of the region and the capacity of the trucks.

In the second phase, a MILP model is proposed, adapted from the CARP proposed by Dror (2001), and applied to each group of arcs obtained from the previous phase.

In the third phase, an adapted Hierholzer algorithm is used to obtain the sequence of arcs that must be met (obtained in the previous phase), thus creating a route for each truck.

2.1. *p-median problem*

The p-median problem is a problem of location-allocation which aims to determine the configuration that minimizes installation costs of the facilities and to meet the demand of each client in a network connected by a finite number of paths. The mathematical model for the p-median problem with $p = 2$ considered in this paper was originally proposed by Christofides (1975) and adapted with the inclusion of the constraint (5). It is a model of a well-known BILP given by:

$$\min z = \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}x_{ij} \tag{1}$$

$$\sum_{i=1}^{n} x_{ij} = 1, \quad \forall j = 1, \ldots, n \tag{2}$$

$$\sum_{i=1} x_{ii} = p \tag{3}$$

$$x_{ij} \le x_{ii}, \quad \forall i, j = 1, \ldots, n \tag{4}$$

$$\sum_{j=1}^{n} w_j x_{ij} \le b.x_{ii}, \quad \forall i = 1, \ldots, n \tag{5}$$

$$x_{ij}\{0, 1\} \tag{6}$$

where

- n is the number of vertices of the graph;
- p is the number of medians to be installed (in this case, it is interpreted as the number of vehicles used);
- $[d_{ij}]$ is the matrix of weighed distances where d_{ij} is the product of the distance between vertices x_i and x_j and the weight w_j, which is the demand of each vertex x_j;
- b is truck capacity.

The objective function (1) is the minimization of the distances between vertices and median vertices. Constraint (2) ensures that each vertex x_j be allocated to one, and only one, median vertex x_i Constraint (3) ensures that there are only p median vertices. Constraint (4) ensures that allocations can only be made to median vertices. Constraint (5) is related to truck capacity b, which must not be exceeded. Furthermore, all w_j demands should be met. Constraint (6) imposes the condition that all variables must be binary.

This problem presents great prominence within the theory of location and its challenge is the installation of p facilities (medians) minimizing the sum of the distances of all vertices to the nearest facility. It grows in complexity as the size of the problem increases. In other words, it is an NP-hard problem, making the determination of the optimal solution within an acceptable time infeasible (Christofides and Beasley, 1982).

2.2. Capacitated Arc Routing Problem (CARP)

Initial studies on the Capacitated Arc Routing Problem were presented by Golden and Wong (1981). The authors presented the problem, its mathematical formulation, some relations with other problems from the literature, and its classification as NP-hard.

The basic mathematical model of the CARP involves simple periods of time (one day, for example) and considers the arcs of the network to be traversable in either direction (Lacomme et al., 2001). The model considered in the present work was adapted from that proposed by Dror (2001), which considers a graph $G(V,A)$ with V vertices and A arcs and also a set R of the required arcs. The adaptation was made with the inclusion of the Constraint (8.1) by considering the beginning and the end of the route to be at different points, beginning at vertex 1 (garage) and ending at vertex 141 (landfill). In the model, the following parameters are known:

- q_{ij} = demand of the arc $(i, j) \in R \subseteq A$;
- b = capacity of the vehicle;
- c_{ij} = cost of the arc $(i, j) \in A$;
- K = number of vehicles.

The variables of the model are:

- x_{ijk} = number of times a vehicle k passes through the arc $(i, j) \in A$;
- $y_{ijk} = \begin{cases} 1, \text{ if the vehicle k performs collection along the arc } (i, j) \in R; \\ 0, \text{ otherwise.} \end{cases}$

The mathematical model is given by:

$$\min z = \sum_{(i,j)\in A} \sum_{k=1}^{K} c_{ij} x_{ijk} \tag{7}$$

$$\sum_{p\in A} x_{pik} = \sum_{p\in A} x_{ipk}, \quad \forall k = 1, 2, \ldots, K \tag{8}$$

$$\sum_{i\in A} x_{1ik} = \sum_{i\in A} x_{i141k} = 1, \forall k = 1, 2, \ldots, K \tag{8.1}$$

$$\sum_{k=1}^{K} y_{ijk} = 1, \forall (i, j) \in R \tag{9}$$

$$x_{ijk} \geq y_{ijk}, \ \forall (i, j) \in R, k = 1, 2, \ldots, K \tag{10}$$

$$\sum_{(i,j)\in R} q_{ij} y_{ijk} \leq b, \forall k = 1, 2, \ldots, K \tag{11}$$

$$M \sum_{i\notin S, j\in S} x_{ijk} \geq \sum_{(j,p)\in A[S]\cap R} x_{ipk}, \ \forall S \subseteq N, \ 1 \notin S, \ A[S] \cap R \neq 0, \quad k = 1, 2, \ldots, K \tag{12}$$

$$y_{ijk} \in \{0; 1\}, \ \forall (i, j) \in R, k = 1, 2, \ldots, K \tag{13}$$

$$x_{ijk} \in Z^{+}, \ \forall (i, j) \in A, k = 1, 2, \ldots, K \tag{14}$$

The objective function (7) minimizes the cost represented by the total distance traveled by the vehicles. Constraint (8) assures the conservation of the flow of vehicles along the graph, that is, for each vehicle entering a vertex, there should be an equivalent vehicle coming out of that vertex. Constraint (8.1) was added to consider the beginning of the route at vertex 1 and the end of the route at vertex 141. Constraint (9) ensures that each arc be attended to only once during the entire path and by one single vehicle k. Constraint (10) ensures that a vehicle will only perform a delivery or collection in arc (i, j) if it passes through the named arc. Constraint (11) refers to vehicle capacity, which should not be exceeded. Constraint (12) prevents the formation of sub-routes, where M is a number as large as the maximum number of times that an arc can be crossed. Constraint (13) imposes that the variables y be binary and Constraint (14) represents the entirety of the variables x.

2.3. Hierholzer's method

Hierholzer's method is used to find an Euler circuit in a graph. Konowalenko et al. (2011) applied this method to a problem of obtaining routes of waste collection trucks using the solution found to the Mixed Chinese Postman Problem (MCPP). The method organizes the sequence of arcs that must be traversed (in this case determined by MCPP) and, in response, provides the route to be taken.

In the present work, Hierholzer's method considered certain arcs in the solution to the CARP (eulerian graph) to determine the sequence in which they should be crossed, thus creating the two routes that must be taken by the trucks that perform the collection of solid waste in the studied region.

As shown by Konowalenko et al. (2011), the exact Hierholzer's algorithm is originally given as follows: Let G (V, A) be an Eulerian graph with V vertices and A edges. Then:

(1) Hierholzer (G=(V, A)graph): circuit
(2) $G' := G \left\{ G' = (V', A') \right\}$
(3) v_0: = a vertex of G'
(4) C:= $[v_0]$ {Initially, the circuit contains only v_0}
(5) **While** A' is non-empty
(6) v_i: = a vertex C such that $d(vi) > 0$ in G'
(7) C': = Circuit in G' containing v_i
(8) $G' := G' - \{a|a$ is the edge contained in C'}
(9) In C, replace the vertex v_i by circuit C'
(10) **Return** C.

An important observation is that, in line (7), the circuit is constructed arbitrarily, leaving v_i and selecting edges not used until it returns to v_i.

This algorithm was adapted to the problem in question with the goal of organizing the truck route traversing all arcs found in the solution obtained by the CARP, originating at vertex 1 (garage) and ending at vertex 141 (landfill). The adapted algorithm is:

- Step 1: Read the graph G (V, A) resulting from the CARP, the initial vertex (v_i), and the final vertex (v_f);
- Step 2: Search for a random route G' (set of arcs starting at the arc whose initial vertex is v_i and ending at the arc whose end vertex is v_f);
- Step 3: The new G will be: G => G − G';

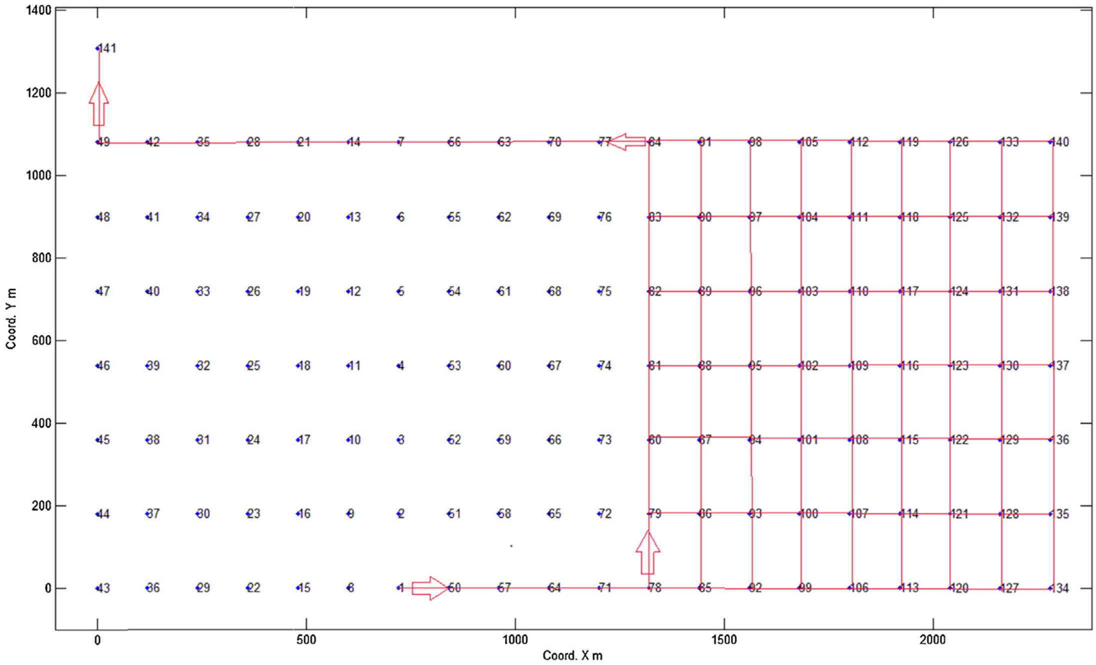

Fig. 1 – Current route taken by truck 1 in the central region of Campo Mourão.

- Step 4: Take the first arc of G (whose initial vertex is v_k) and form another route G'' until the final arc has the same final vertex v_k;
- Step 5: Insert route G'' in route G' (between two arcs whose initial and final vertex is v_k);
- Step 6: Return to step 3 until all arcs are covered, that is, $G = \{\}$.

3. Case Study

The developed sequential strategy was applied to a real problem in the city of Campo Mourão, Brazil. The population of the city is of approximately 90,000 inhabitants. The collection of solid waste in this city is performed by an outsourced company that provides undifferentiated and selective collection services. The truck routes for these two types of collection in the central region of the city were considered in the present work. The undifferentiated collection is held from Monday to Saturday during nighttime, the company using for that end two homogeneous compactor trucks, each with a capacity of 17 tons. The selective collection is held twice a week, on Wednesday and Saturday, by two 6-ton trucks, during daytime.

The average daily amount of solid waste collected by both undifferentiated collection trucks in this region is of 32 tons altogether. For the selective collection, the average daily amount of collected recyclable waste is of 9 tons. The company also informed that about US $ 10.00/km are spent with refueling and maintenance of the trucks.

The road network of the city can be treated as a graph $G(V, A)$, where V is the set of vertices (points or nodes located at the intersection of routes) and A is the set of arcs (lines connecting these points). For the undifferentiated collection, the central region of the city consists of 141 demand points (numbered 1–141) through which the collection trucks should traverse. "Vertex 1" is the garage where the route starts, and "vertex 141" is the landfill where it ends. The sum of the distances currently traveled by the two undifferentiated collection trucks in the studied region is 71,420 m.

Figs. 1 and 2 illustrate the current routes taken by the two undifferentiated collection trucks in the central region. It can be observed that the paths are presented in a simplified manner, without showing the arcs that are traversed more than once by each vehicle since it was not possible to obtain this information.

Truck 1 leaves the garage (vertex 1), performs the collection across the region marked with solid lines in Fig. 1, and goes to the landfill (vertex 141).

Truck 2 leaves the garage (vertex 1), performs the collection across the region marked with solid lines in Fig. 2, and goes to the landfill (vertex 141).

For the selective collection, the route taken by the two trucks is the same, except for the existence of two deposits for the final disposal of the collected material (each truck finishes the route in one of two deposits), which are represented by the vertices "141" and "142" (see Fig. 8).

The modified mathematical models for the FLP and the CARP (presented in Sections 2.1 and 2.2 respectively) were implemented in the software GAMS (General Algebraic Modeling System), version 24.2.3, using the solver CPLEX. The adapted Hierholzer algorithm (Section 2.3) was implemented in the software Matlab R2011. In both cases a computer with an Intel Core i5 processor and 4 GB of RAM was used.

4. Results and discussion

Initially, the CARP model proposed by Dror (2001), presented in Section 2.2, was directly used to optimize the routes of the two trucks that perform the undifferentiated collection in the studied area. However, Constraint (12), referring to the constraint that prevents the formation of sub-routes in the solution (routes that are not feasible due to the fact that they do not start at the garage and do not finish at the landfill), could not be successfully implemented due to the large number of combinations of the vertices of the problem (the number of constraints would be of the order of 2^{139}). This makes the Constraint (12) computationally infeasible, preventing its use.

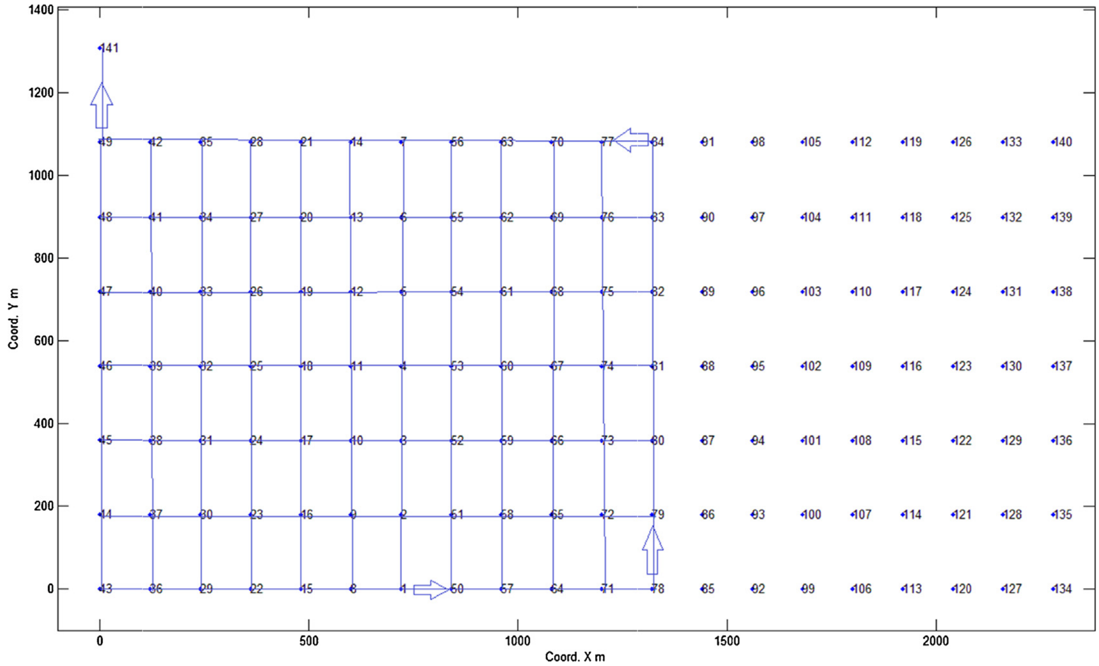

Fig. 2 – Current route taken by truck 2 in the central region of Campo Mourão.

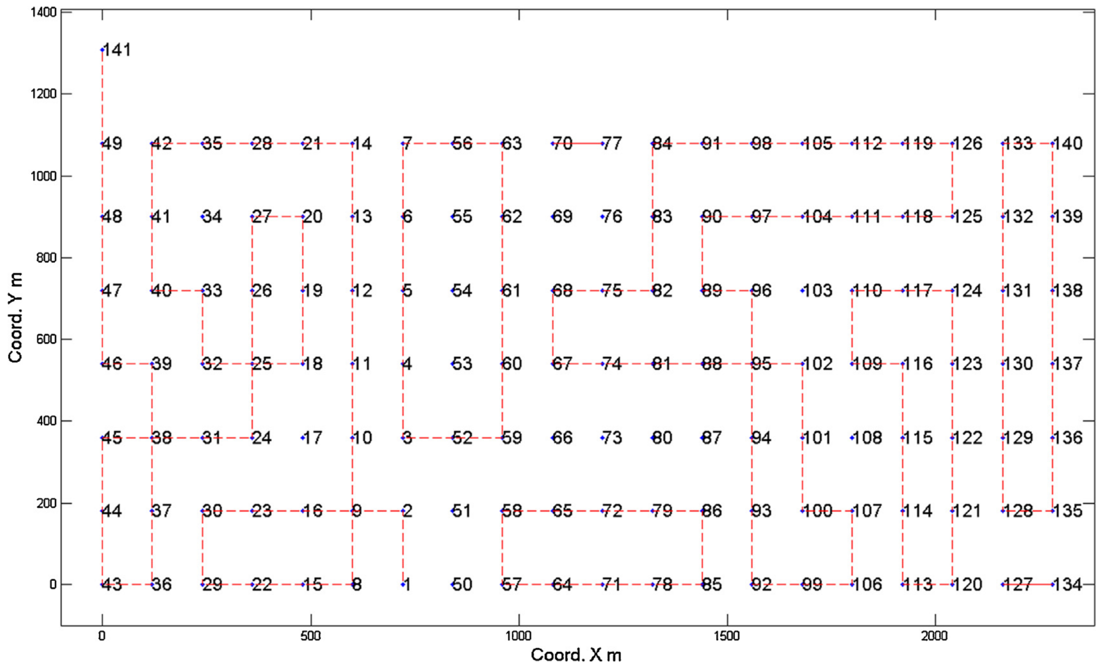

Fig. 3 – Solution to the particular CARP discussed for the first truck with the presence of sub-routes.

Thus, sub-routes were detected in the solution. Figs. 3 and 4 illustrate the results obtained for both trucks.

Therefore, as previously mentioned, it was decided to first group the points (two groups) and then to perform the routing in each group.

The results obtained with the use of the FLP model (Section 2.1) demonstrated the efficiency of the method in a very short computational time (10 s). This allows the conclusion that the number of vertices considered (141) did not represent an obstacle to the implementation of the model. Results are shown in Table 1, where it can be observed that the two determined medians are 18 and 102 and also that points 25, 26, . . ., 63, 65, 66, 68, 70 were designated to median 18 while points 64, 67, 69, 71, . . ., 140 were allocated to median 102. These results can be visualized in Fig. 5.

The two sets of points in Fig. 5 represent the sets of vertices allocated to the medians installed at vertices 18 and 102. The goal of minimizing the distances between the vertices by separating them into two groups, each one for a waste collection truck, was successfully achieved.

From the results obtained in the first phase, the mathematical model of the CARP (Section 2.2) was executed separately for each of the resulting groups from the FLP (Fig. 5) without using Constraint (12). The model proved to be very efficient, with a short execution time (15 s for the first group, median 18, and 17 s for the second group, median 102). Furthermore, the problem was solved without the formation of sub-routes in the solution, which is very desirable.

Table 2 shows the results obtained from the CARP model for integer variables x_{ijk}, taking into consideration the initial

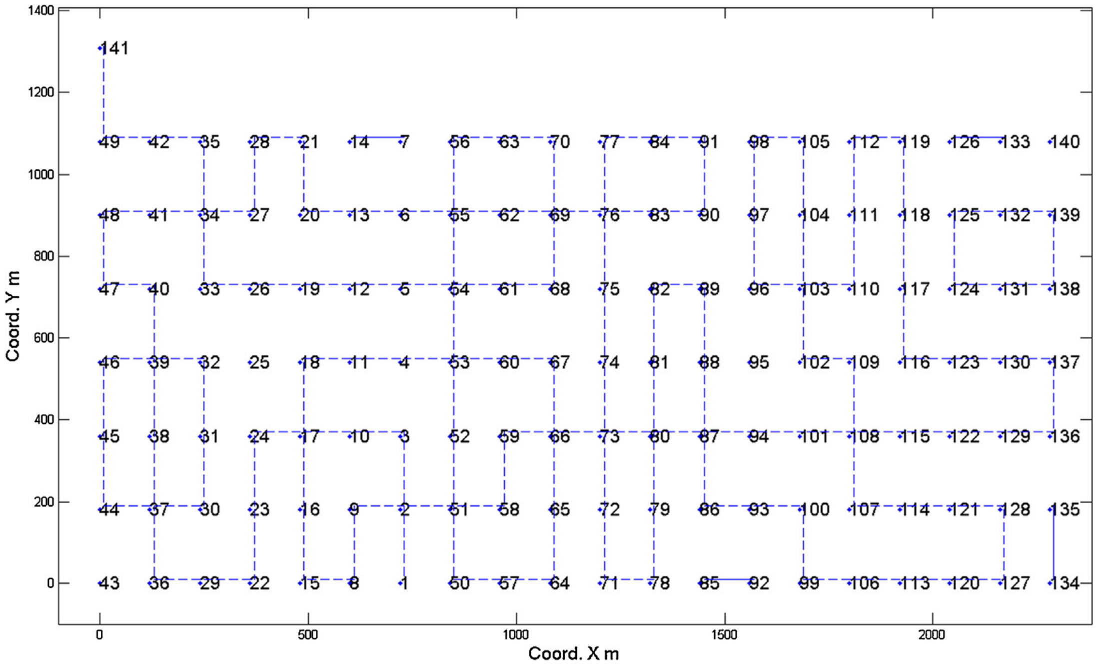

Fig. 4 – Solution to the particular CARP discussed for the second truck with the presence of sub-routes.

vertex (v_i) and the final vertex (v_f) of each arc that will be traversed by truck 1. The value of the integer variable x_{ijk} represents how many times the arc is traversed by the truck. In other words, considering as an example the first arc in the table, beginning at vertex 1 and ending at vertex 2, the value of the variable x_{ijk} is 1, which means that this arc is traversed only once. On the other hand, the arc beginning at vertex 46 and ending at vertex 45 is crossed twice by truck 1.

Table 3 presents the results obtained for truck 2 in a similar fashion.

Figs. 6 and 7 illustrate the results shown in Tables 2 and 3 respectively. The minimized overall distance is 32,830 m for truck 1 and 37,570 m for truck 2. The total sum of these distances is 70,400 m, which is 1.020 m lower than that of the current route.

It can be observed that the distances traveled by the collection trucks in the current routes are close to the optimum value from this study. This is due to route planning using commercial software.

The appearance of duplicated arcs in Figs. 6 and 7 (a continuous segment and a dashed one) means that they will be traversed in both directions. The arcs represented by dashed segments will be traversed in only one direction.

Finally, the Hierholzer algorithm (Section 2.3), adapted to the situation to consider the results previously obtained from the CARP model, was applied.

The routes obtained by the algorithm are:

- Route for truck 1 (see Fig. 6): 1 2 3 10 11 4 3 52 53 60 59 66 59 60 61 54 55 56 63 56 55 62 63 62 61 68 61 54 53 4 5 6 13 14 7

Table 1 – Results obtained for the p-median problem.

V	M 18	M 102	V	M 18	M 102	V	M 18	M 102	V	M 18	M 102	V	M 18	M 102	V	M 18	M 102
2	1	0	25	1	0	48	1	0	71	0	1	94	0	1	117	0	1
3	1	0	26	1	0	49	1	0	72	0	1	95	0	1	118	0	1
4	1	0	27	1	0	50	1	0	73	0	1	96	0	1	119	0	1
5	1	0	28	1	0	51	1	0	74	0	1	97	0	1	120	0	1
6	1	0	29	1	0	52	1	0	75	0	1	98	0	1	122	0	1
7	1	0	30	1	0	53	1	0	76	0	1	99	0	1	123	0	1
8	1	0	31	1	0	54	1	0	77	0	1	100	0	1	124	0	1
9	1	0	32	1	0	55	1	0	78	0	1	101	0	1	125	0	1
10	1	0	33	1	0	56	1	0	79	0	1	102	0	1	126	0	1
11	1	0	34	1	0	57	1	0	80	0	1	103	0	1	127	0	1
12	1	0	35	1	0	58	1	0	81	0	1	104	0	1	128	0	1
13	1	0	36	1	0	59	1	0	82	0	1	105	0	1	129	0	1
14	1	0	37	1	0	60	1	0	83	0	1	106	0	1	130	0	1
15	1	0	38	1	0	61	1	0	84	0	1	107	0	1	131	0	1
16	1	0	39	1	0	62	1	0	85	0	1	108	0	1	132	0	1
17	1	0	40	1	0	63	1	0	86	0	1	109	0	1	133	0	1
18	1	0	41	1	0	64	0	1	87	0	1	110	0	1	134	0	1
19	1	0	42	1	0	65	1	0	88	0	1	111	0	1	135	0	1
20	1	0	43	1	0	66	1	0	89	0	1	112	0	1	136	0	1
21	1	0	44	1	0	67	0	1	90	0	1	113	0	1	137	0	1
22	1	0	45	1	0	68	1	0	91	0	1	114	0	1	138	0	1
23	1	0	46	1	0	69	0	1	92	0	1	115	0	1	139	0	1
24	1	0	47	1	0	70	1	0	93	0	1	116	0	1	140	0	1

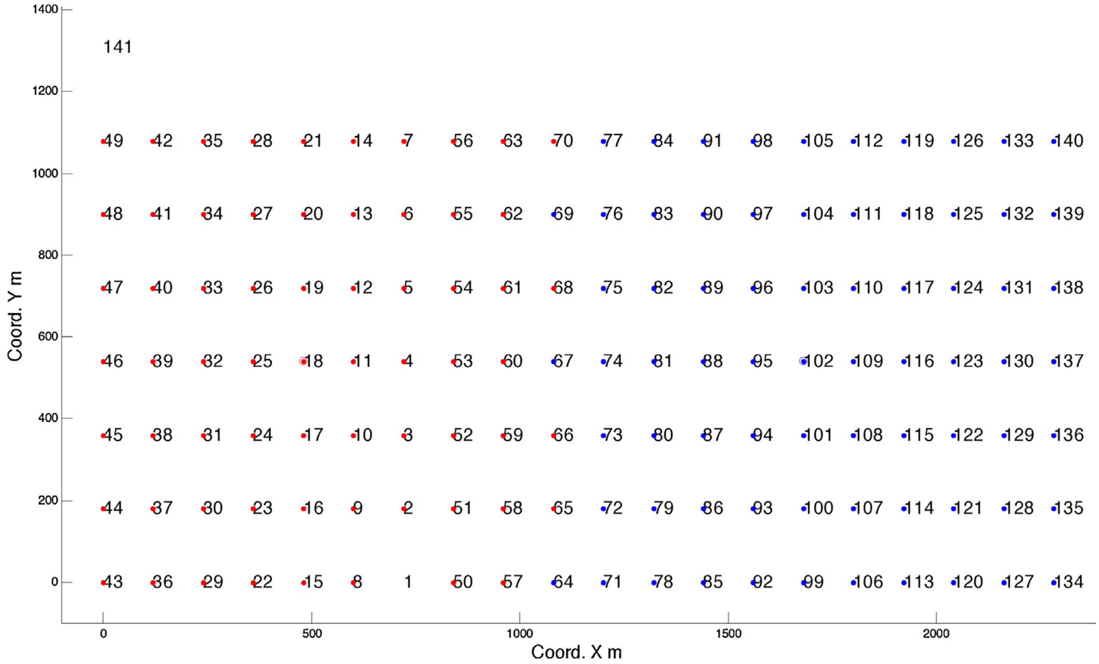

Fig. 5 – Solution to the p-median problem – located medians: 18, 102.

Table 2 – Results obtained from the CARP model for variables x_{ijk} for truck 1.

v_i	v_f	x_{ijk}	v_i	v_f	x_{ijk}	v_i	v_f	x_{ijk}	v_i	v_f	x_{ijk}	v_i	v_f	x_{ijk}
1	2	1	14	7	1	27	34	1	40	41	1	54	5	1
1	8	1	14	21	1	28	27	1	40	47	1	54	53	1
2	3	1	15	8	1	28	35	1	41	42	1	54	55	1
2	51	1	15	16	1	29	22	1	41	48	1	55	54	1
3	10	1	16	9	1	29	36	1	42	35	1	55	56	1
3	52	1	16	23	1	30	29	1	42	49	1	55	62	1
4	3	1	17	18	1	30	31	1	43	36	1	56	7	1
4	5	1	18	11	1	31	24	1	43	44	1	56	55	1
5	6	1	18	25	1	31	32	1	44	37	1	56	63	1
5	12	1	19	18	1	32	25	1	44	43	1	57	50	1
6	13	1	19	20	1	32	39	1	45	38	1	57	64	1
6	55	1	20	21	1	33	32	1	45	44	1	58	51	1
7	6	1	20	27	1	33	34	1	46	45	2	58	57	1
7	56	1	21	14	1	34	27	1	47	40	1	59	58	1
8	9	1	21	28	1	34	33	1	47	46	1	59	60	1
8	15	1	22	15	1	34	41	1	48	47	1	59	66	1
9	2	1	22	29	1	35	34	1	48	49	1	60	59	1
9	10	1	23	22	1	35	42	1	49	48	1	60	61	1
10	11	1	23	30	1	36	37	1	49	141	1	61	54	2
10	17	1	24	17	1	36	43	1	50	1	1	61	68	1
11	4	1	24	23	1	37	30	1	51	50	1	62	61	1
11	12	1	25	24	1	37	38	1	51	52	1	62	63	1
12	13	1	25	26	1	38	31	1	51	58	1	63	56	1
12	19	1	26	19	1	38	39	1	52	53	1	63	62	1
13	14	1	26	33	1	39	40	1	52	59	1	64	57	1
13	20	1	27	26	1	39	46	1	53	4	1	66	59	1
14	7	1	27	28	1	40	33	1	53	60	1	68	61	1

56 7 6 55 54 5 12 13 20 21 14 21 28 27 26 19 18 11 12 19 20 27 28 35 34 27 34 33 32 25 24 17 16 9 2 51 50 51 52 59 58 51 58 57 64 57 50 1 8 15 16 23 30 29 36 43 44 43 36 37 38 39 46 45 44 37 30 31 32 39 40 47 46 45 38 31 24 23 22 29 22 15 8 9 10 17 18 25 26 33 34 41 42 35 42 49 48 47 40 33 40 41 48 49 141.

- Route for truck 2 (see Fig. 7): 1 71 72 73 74 67 65 72 79 78 71 78 85 86 87 80 79 86 93 94 87 88 89 96 97 98 105 98 91 90 89 82 75 74 73 80 81 74 75 76 83 82 83 84 91 84 77 76 69 67 74 81 82 81 88 95 94 101 100 99 106 107 114 115 122 123 130 131 138 137 130 129 122 121 114 113 120 121 128 127 128 129 136 137 136 135 128 135 134 127 120 113 106 99 92 85 92 93 100 107 108 101 102 103 96 95 102 109 108 115 116 117 124 123 116 109 110 111 118 125 124 131 132 139 138 139 140 133 132 125 126 133 126 119 118 117 110 103 104 111 112 119 112 105 104 97 90 83 76 77 70 69 70 141.

The numbers represent the vertices of the graph that form the arcs to be traversed by the trucks. The routes begin at vertex 1 (garage) and end at vertex 141 (landfill). All arcs are met and all waste is collected without exceeding the capacity of the trucks.

Table 3 – Results obtained from the CARP model for variables x_{ijk} for truck 2.

v_i	v_f	x_{ijk}	v_i	v_f	x_{ijk}	v_i	v_f	x_{ijk}	v_i	v_f	x_{ijk}	v_i	v_f	x_{ijk}
1	71	1	81	74	1	95	102	1	111	112	1	126	133	1
65	72	1	81	82	1	96	95	1	111	118	1	127	120	1
67	65	1	81	88	1	96	97	1	112	105	1	127	128	1
67	74	1	82	75	1	97	90	1	112	119	1	128	127	1
69	67	1	82	81	1	97	98	1	113	106	1	128	129	1
69	70	1	82	83	1	98	91	1	113	120	1	128	135	1
70	69	1	83	76	1	98	105	1	114	113	1	129	122	1
70	141	1	83	82	1	99	92	1	114	115	1	129	136	1
71	72	1	83	84	1	99	106	1	115	116	1	130	129	1
71	78	1	84	77	1	100	99	1	115	122	1	130	131	1
72	73	1	84	91	1	100	107	1	116	109	1	131	132	1
72	79	1	85	86	1	101	100	1	116	117	1	131	138	1
73	74	1	85	92	1	101	102	1	117	110	1	132	125	1
73	80	1	86	87	1	102	103	1	117	124	1	132	139	1
74	67	1	86	93	1	102	109	1	118	117	1	133	126	1
74	73	1	87	88	1	103	96	1	118	125	1	133	132	1
74	75	1	88	89	1	103	104	1	119	112	1	134	127	1
74	81	1	88	95	1	104	97	1	119	118	1	135	128	1
75	74	1	89	82	1	104	111	1	120	113	1	135	134	1
75	76	1	89	96	1	105	98	1	120	121	1	136	135	1
76	69	1	90	83	1	105	104	1	121	114	1	136	137	1
76	77	1	90	89	1	106	99	1	121	128	1	137	130	1
76	83	1	91	84	1	106	107	1	122	121	1	137	136	1
77	70	1	91	90	1	107	108	1	122	123	1	138	137	1
77	76	1	92	85	1	107	114	1	123	116	1	138	139	1
78	71	1	92	93	1	108	101	1	123	130	1	139	138	1
78	85	1	93	94	1	108	115	1	124	123	1	139	140	1
79	78	1	93	100	1	109	108	1	124	131	1	140	133	1
79	86	1	94	87	1	109	110	1	125	124	1			
80	79	1	94	101	1	110	103	1	125	126	1			
80	81	1	95	94	1	110	111	1	126	119	1			

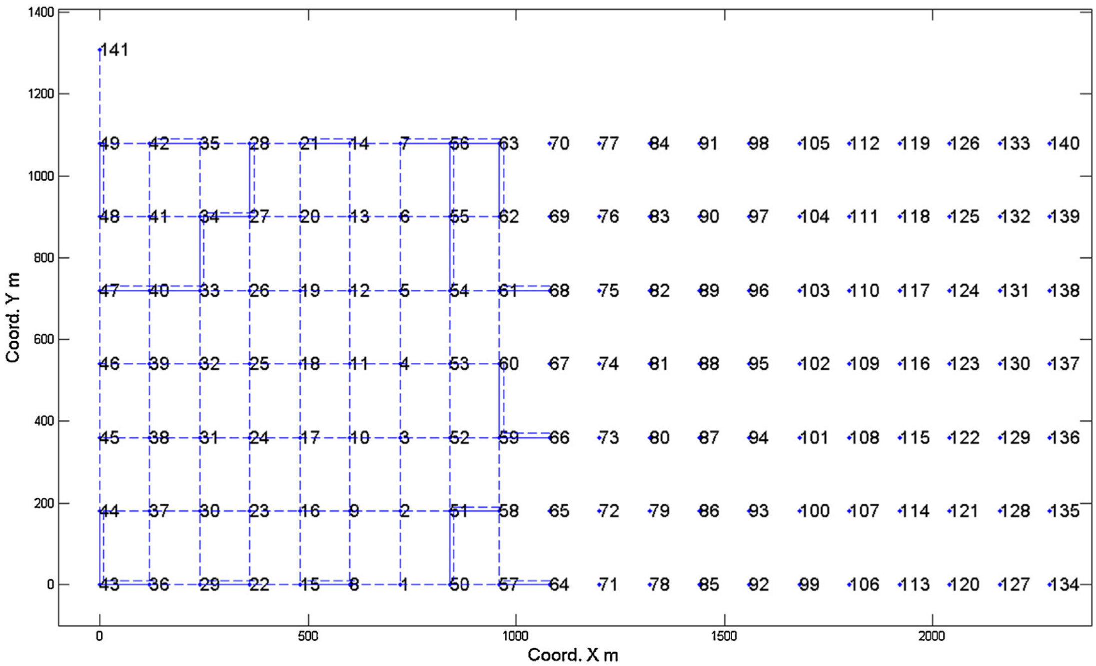

Fig. 6 – Solution obtained from the CARP model for truck 1.

The results for the selective collection, as well as the necessary changes in the previously presented mathematical models, will be presented in a simpler manner below.

The methodology used for the optimization of the routes of the selective collection trucks in the central region of the city was adapted from that used for the optimization of the routes of the undifferentiated collection trucks, in which case all collected waste was deposited in one location, the landfill of the city. In the case of selective collection, however, there are two deposits for waste disposal. These deposits are cooperatives that receive the waste and separate it for future use. On the map in Fig. 8, they are represented by vertices 141 and 142. The starting point for the trucks is the garage, represented by vertex 1.

The sequential approach previously developed and used is composed of three phases. The first stage performs the

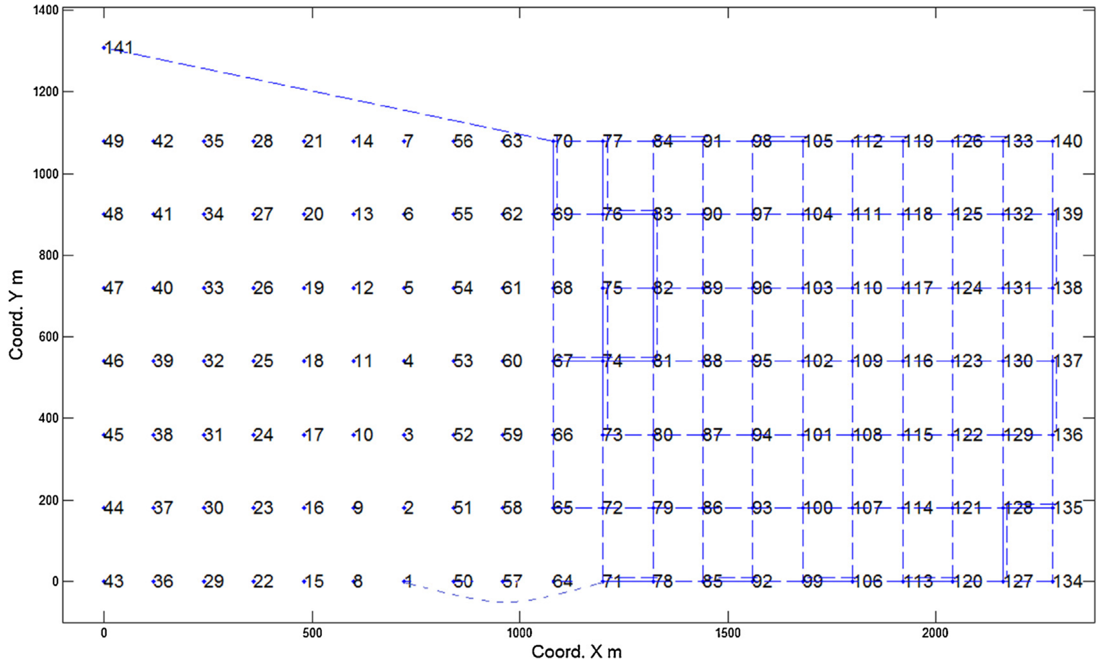

Fig. 7 – Solution obtained from the CARP model for truck 2.

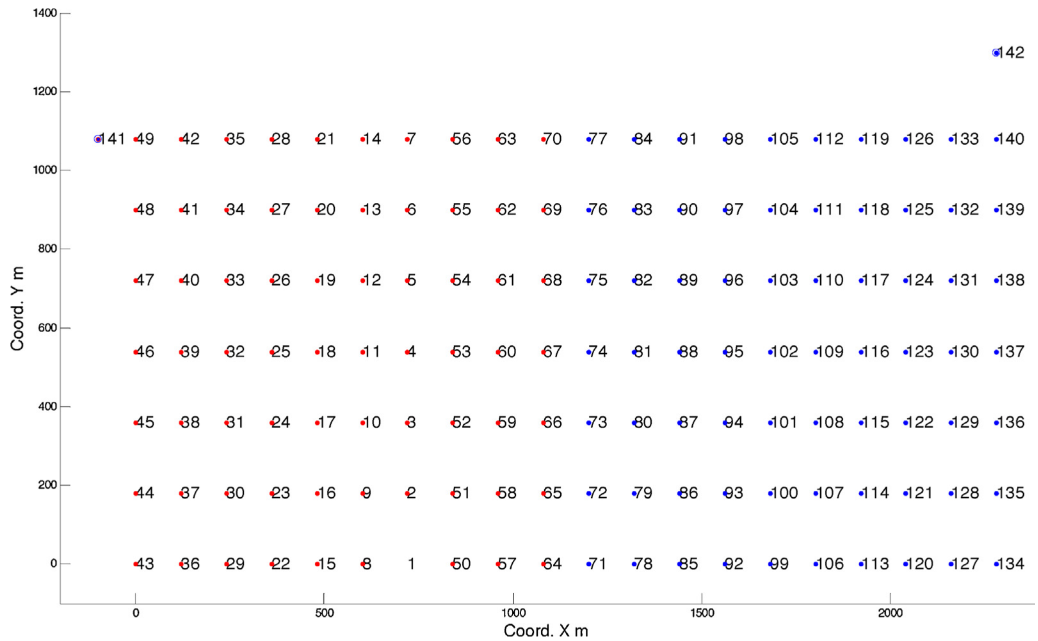

Fig. 8 – Solution to the p-median problem for the selective collection – located medians: 141, 142.

division of the vertices, from which the arcs that must be covered originate, in two groups, one for each truck, using the adapted p-median model. In this case, as there are two deposits for the disposal of collected waste, these deposits are predetermined as being the p-medians, and the other vertices are allocated to one of these medians from the model. The objective of this phase is to divide the set of vertices into two groups (one for each truck and, consequently, one for each deposit), minimizing the distance between the allocated vertices and the corresponding median (deposit) and taking into account the amount of waste to be collected and the capacity of the collection trucks.

The result obtained in this first step is presented in Fig. 8, where the medians are vertices 141 and 142 (deposits). The vertices in red were allocated to median 141, while those in blue were allocated to median 142.

The second phase uses the adapted CARP model, which is applied to each group of arcs obtained from the previous phase to minimize the total distance traveled by the selective collection trucks. Current routes followed by the two selective collection trucks cover a total distance of 49,780 m. The optimized routes totalize 46,080 m, which provides a reduction of approximately 7.5% in the total traveled distance.

Figs. 9 and 10 show the results of the CARP model for each selective collection truck.

In the third phase, the adapted Hierholzer algorithm is applied to each resulting graph of the previous stage to obtain the route of each vehicle. The result obtained for each graph is the following:

- ROUTE 1 (see Fig. 9): 1 2 3 4 5 12 13 6 5 54 53 4 11 12 19 20 13 14 7 56 55 6 7 56 63 62 55 54 61 62 63 70 69 62 69 68 67 60 53

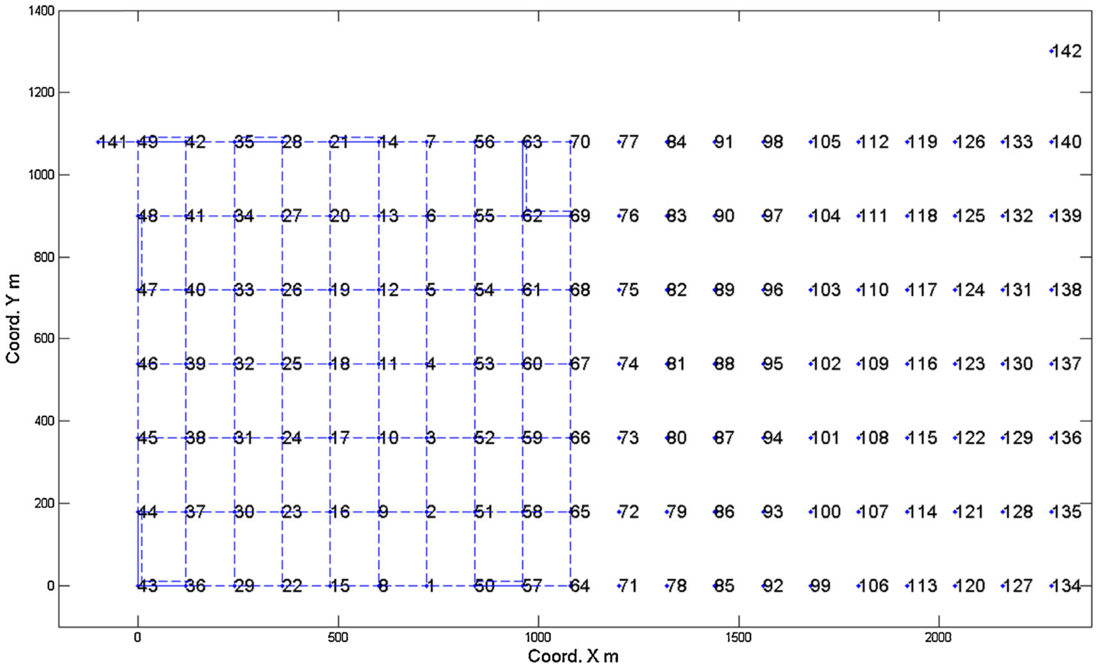

Fig. 9 – Solution obtained from the CARP model for selective collection truck 1.

52 3 10 11 18 19 26 27 20 21 14 21 28 27 34 33 32 31 24 23 16 15 8 1 50 57 58 59 66 65 64 57 50 51 52 59 60 61 68 67 66 65 58 51 2 9 10 17 18 25 26 33 40 41 42 35 28 35 34 41 48 47 46 45 38 37 30 29 22 23 30 31 38 39 46 45 44 37 36 43 44 43 36 29 22 15 8 9 16 17 24 25 32 39 40 47 48 49 42 49 141.

- ROUTE 2 (see Fig. 10): 1 71 78 79 72 71 78 85 86 79 80 73 72 73 74 75 74 81 80 87 86 87 88 87 94 93 86 93 92 85 92 93 100 99 92 99 106 107 100 101 94 95 96 89 82 75 76 77 84 83 76 83 82 81 88 89 88 95 102 103 104 97 96 103 110 109 108 101 102 109 116 115 108 107 114 115 122 121 120 113 106 113 114 121 128 129 122 123 116 117 110 111 112 105 98 91 84 91 90 83 90 89 90 97 98 105 104 111 118 117 124 123 130 129 136 135 134 127 120 127 128 135 136 137 138 131 124 125 126 119 112 119 118 125 132 131 130 137 138 139 132 133 126 133 140 139 140 142.

It can be observed that the two routes start at vertex 1, which is the garage, and end at the deposits, which are vertex 141 for the first route and vertex 142 for the second one. All arcs were covered and all recyclable waste was collected without exceeding the capacity of the trucks.

In situations where the number of deposits is greater than two, the same methodology can be applied, considering the deposits as the p-medians of the first step and then executing the last two steps.

And yet, considering that the trucks involved in the collection task are large carbon dioxide emitters in the atmosphere, the results for the optimized routes in both cases studied, undifferentiated collection and selective collection, allow a reduction of approximately 914 kg/year of CO_2, which

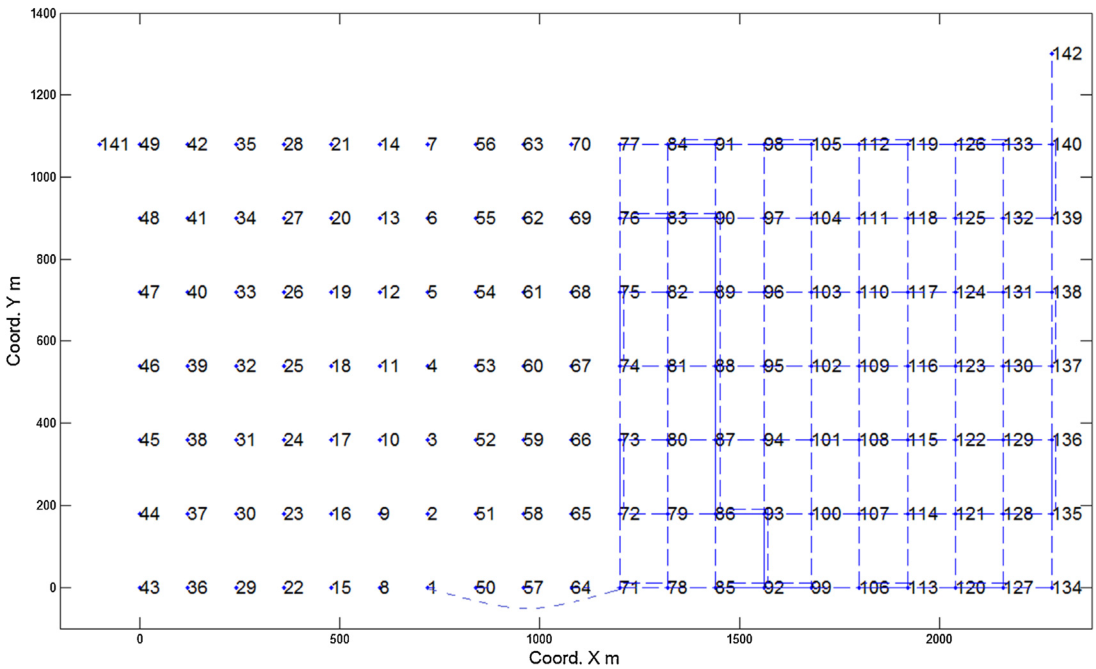

Fig. 10 – Solution obtained from the CARP model for selective collection truck 2.

Table 4 – SA for the number of trucks (fixed capacity of 6 tons for each truck).

Number of trucks	Situation relative to the best response
1	Infeasible
2	100% (best response)
3	Increase of 17.5% in the total traveled distance

Table 5 – SA for the capacity of trucks (number of trucks fixed as 2).

Truck capacity	Situation relative to the best response
3 ton	Infeasible
6 ton	100% (best response)
9 ton	Even with increased truck capacity, solution remains unaltered

promotes benefits to the environment and to the health of population.

4.1. *Sensitivity analysis*

The sensitivity analysis (SA) aims to check if a model produces logical results when undergoing changes in the input parameters and the need for accuracy of these parameters (Gowda et al., 1999). The stability of the solution is evaluated under a *ceteris paribus* condition ("other things held constant"), where the effects of changing the value of a single input parameter or variable are considered while keeping all others constant (Hazell and Norton, 1986).

Thus, the analyzed input parameters were number of trucks and truck capacity. The results are based on the selective collection, which provided the best results (values in bold in the following tables) and are presented in Tables 4 and 5.

From these results it can be seen that, by fixing the truck capacity at 6 tons, a single truck cannot perform the collection because its capacity is lower than the demand. For three trucks, the result would be feasible, but the cost would be 17.5% higher.

Regarding the capacity of the trucks, it is clear that two 3-ton trucks cannot accomplish the task, because the demand is greater than the sum of their capacities. On the other hand, two 9-ton trucks show no alteration in the viability of the result compared with 6-ton trucks.

Therefore, the number of trucks is an important factor for the viability of the solution. The lower the number of trucks, the greater the amount of saved money, as long as their combined capacity is sufficient to carry out the task. Otherwise, the problem becomes infeasible.

5. Conclusions

The development of this study has shown that methods involving exact algorithms can be used for determining vehicle routes in a medium-sized region and that the proposed sequential approach is efficient for this purpose. The developed methodology made it possible to change a problem initially classified as NP-hard into an easily solvable one.

The proposed procedure uses three sequential steps. The first step uses the p-median model. This model only performs the grouping of arcs, one for each truck, considering the demand and the capacity of the vehicles. Its standalone application does not optimize the routes of the vehicle. In the second step, the CARP model is used for route optimization for each set obtained in the previous phase, considering only one truck per set and without taking into account restriction 12 of the model. This restriction would generate a very large number of inequalities (in situations of medium and large sizes) due to its combinatorial nature, leading to infeasible solutions. In the third step, the Hierholzer algorithm is applied to determine the sequence of the arcs obtained in the previous step, generating the route for each vehicle. This sequential approach avoids the formation of sub-routes in the solution.

Two applications were performed using real data. The first one involved the undifferentiated collection in the central region of the city and provided a reduction of 1.5% in the total traveled distance, which can result in saving US $ 3825/year, related to expenses with refueling and maintenance of the trucks. The other one considered the selective collection in the same region and allowed a reduction of 7.5% in the total traveled distance, which can result in saving US $ 4146/year considering that this collection is performed only twice a week. The optimized routes, in both situations, also allow a reduction in carbon dioxide emissions in the atmosphere of approximately 914 kg/year.

Based on the sensitivity analysis results, considering variations in the input parameters "number of trucks" and "capacity of trucks," it can be concluded that the number of trucks is inversely proportional to the savings obtained with the optimal results. However, their capacity has to be sufficient to avoid making the problem infeasible.

Finally, results that provide benefits to the environment as well as savings to the public coffers are always valid, justifying the importance of the developed methodology.

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