Root mean square

The *root mean square* is a type of mean.

Given real numbers a_1, a_2, ..., a_n, the root mean square (often abbreviated to RMS) is obtained by calculating the arithmetic mean of the squares of a_1, ..., a_n, and then taking the square root of this:

$$\sqrt{\frac{a_1^2 + a_2^2 + \cdots + a_n^2}{n}}$$

It is useful when trying to measure the average "size" of numbers, where their sign is unimportant, as the squaring makes all of the numbers non-negative.

The most common case of using the root mean square is when calculating the standard deviation of a set of numbers x_1, ..., x_n. The standard deviation is the root mean square of the deviations of these numbers from the mean, that is, the root mean square of $(x_1 - \bar{x})$, ..., $(x_n - \bar{x})$, where $\bar{x}$ is the mean of x_1, ..., x_n, so

$$\text{standard deviation} = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}}.$$

The root mean square can also be used for continuous functions, with integration replacing summation. If the function $f(x)$ is defined for $a \le x \le b$, then the root mean square value of $f(x)$ over this interval is

$$\sqrt{\frac{1}{b-a}\int_a^b (f(x))^2\,dx}.$$

The root mean square is an example of a power mean.