Interval notation

Mathematicians frequently want to talk about *intervals* of real numbers such as "all real numbers between 1 and 2", without mentioning a variable. As an example, "The range of the function $f : x \mapsto \sin x$ is all real numbers between -1 and 1".

A compact notation often used for these intervals of real numbers is as follows:

- $(1,2)$ means all real numbers between 1 and 2, excluding the endpoints
- $[1,2]$ means all real numbers between 1 and 2, including the endpoints

We can also write these intervals using set notation as $\{x : 1 < x < 2\}$ and $\{x : 1 \leq x \leq 2\}$ respectively.

If needed, we can also mix the two types of bracket, so $(1,2]$ means the interval $\{x : 1 < x \leq 2\}$ and $[1,2)$ means $\{x : 1 \leq x < 2\}$.

The interval "all real numbers greater than -5" is written as $(-5,\infty)$, and "all real numbers less than or equal to 7" is written as $(-\infty,7]$. This does not mean that ∞ is a number; it is just a convenient shorthand.

Although the notation $(1,2)$ is exactly the same as the notation for coordinates, the two are rarely confused because the context will make it clear which is meant.