Fundamental theorem of calculus

The *fundamental theorem of calculus* roughly says that integration and differentiation are inverses of each other. It has two parts.

First fundamental theorem of calculus

If we integrate and then differentiate, we get back to where we started.

More precisely, if $f(x)$ is a continuous function and a is a constant, then the function $F(x)$ defined by

$$F(x) = \int_a^x f(x)\,dx$$

is differentiable, with $F'(x) = f(x)$.

Second fundamental theorem of calculus

If we differentiate and then integrate, we get back to where we started, up to a constant.

More precisely, if $F(x)$ has a continuous derivative $f(x) = F'(x)$, then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

The fundamental theorem is regularly used in order to calculate integrals. For example, the derivative of $\frac{1}{3}x^3$ is x^2, so

$$\int_0^3 x^2\,dx = \left[\tfrac{1}{3}x^3\right]_0^3 = \tfrac{1}{3}(3^3) - \tfrac{1}{3}(0^3) = 9.$$

There are also stronger versions of these theorems.

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