Stretching an integral

Teacher notes

Why use this resource?

How do we integrate $\frac{1}{x}$? This resource offers an approach via transforming the graph of $y = \frac{1}{x}$ without needing calculus tools. It is a structured resource designed to lead students to the amazing result that the area under the graph behaves as a logarithm. Many students have been intrigued to see how seemingly disparate topics (transformations, integration and logs) combine to give this result.

Preparation

Students will need a copy of the diagrams page to write on. It may be helpful to print the problem page too so that students can easily work through the questions.

Possible approach

The Warm-up section reviews the effect on an equation of stretching its graph. It uses an example similar to $y = \frac{1}{x}$, but where the behaviours of stretches in the x and y directions differ from each other. This will help to clarify whether students understand the algebra involved.

Asking initial questions such as "What is the integral of x^n? Does this rule work for x^{-1}?" can help to set the scene for the main problem.

The idea of using a function to represent an integral – here, $I(a) = \int_1^a \frac{1}{x}\,dx$ – might be unfamiliar to students, and may need some further explanation.

Key questions

- How are $I(a)$ and $I(b)$ related?
- What does your answer suggest about the function $I(x)$?
- Once you know the value of a which gives $I(a) = 1$, what does this tell you?

Possible extension

The "Some further questions" part can be used for further discussion of closely related ideas.

Another interesting question to ask is "What is special about the function $y = \frac{1}{x}$ which makes this technique work?"