Parametric preliminaries

Teacher notes

Why use this resource?

This resource introduces the idea of a parameter by looking at the locus of a point as its position changes according to some geometric constraints. Students are asked to visualise the path the point will trace out, and parametric equations are introduced by asking students to express the coordinates of the point in terms of a parameter (in this case an angle at the origin). Students are then given three more pairs of parametric equations and invited to sketch the curves they define.

Preparation

Mini whiteboards for sketching or flip-chart paper.

Possible approach

The resource lends itself to a think-pair-share model (flip-chart paper could be attached to the walls so that all students can see each others' sketches while ideas are shared).

Start by checking that students understand where the coordinates for the initial diagram come from.

Give students a minute or two of thinking time and then a few minutes of sharing (and sketching) in pairs to decide what the locus of P looks like in each case and what the coordinates of P are. They can then move into larger groups to compare their ideas. These could be presented by the groups to the class or, if time is an issue, relevant parts of the solution could be shown by the teacher. If the term 'parametric equations' has not already been used, it could be introduced at this point using the Parametric equations toggle in the problem page. Students can then return to working in groups to sketch curves from the three further pairs of parametric equations given.

When students present their ideas to each other, encourage them to trace out rather than just show the curves they've drawn to emphasise the direction in which the curve is traced out as the parameter increases.

Key questions

- Can you express length OQ in terms of $\angle QOP$? (What if you write $\angle QOP = t$?)
- Can you express OP in terms of t and the fixed length OQ?

For sketching the curves from parametric equations

- How is x varying as t varies? What about y?
- Are there specific points you can identify easily?
- Can you make the parameter the subject of one of your equations?
- Is another method for eliminating the parameter going to work better than substitution? Why might this be?

Possible support

For students struggling to get to grips with the initial two loci problems, some of the prompting questions above may help as well as encouraging students to sketch ideas. Students may find it helpful to sketch some points for given values of $\angle QOP$ and then think about how to "join the dots". Students may find it helpful to draw up a table of corresponding values of t, x and y. This strategy can also help students to get started on the second part of the problem, and in particular, they may start to be more strategic about the points they choose to find.

For the algebraic work to eliminate the parameter, students may need reminding about simultaneous equations and trigonometric identities.

Possible extension

Encourage students to experiment with changing one half of the pair of parametric equations and think about how that effects the final graph. Students could use a graphing calculator or Desmos to investigate. This is related to the resource Parametric paths.

Alternatively, in pairs, students should agree on a range of values for the parameter and then independently write one half each of a pair of parametric equations. They should then work together to sketch the curve and check their results with graphing software.