Meaningful areas

Teacher notes

Why use this resource?

The lovely enticing image is the gateway to lots of interesting thoughts about functions and integration.

Students are asked to find the areas of regions between the graphs of the functions in Curve match. In doing this, they may discover that they can use symmetry and averages to reduce the number of calculations they need to do and to explain why certain regions are the same size. In fact, all the areas between the curves can be found by finding only one integral.

Students can use integration techniques, function knowledge and symmetry to find out about the different areas and create more functions to divide the area into equal sized portions. They can come to the same result from more than one approach, (or different students using different techniques should find the same answers) which may help them to gain confidence moving fluidly between approaches or convince them of new ideas by reinforcing the validity using a technique they are more confident with. The link with means might not occur to many students but this and symmetry are tools we can draw on in many areas of maths (see pervasive ideas: averages and symmetry).

Preparation

If you have not previously used Curve match or it has been a while since the resource was used, students may need reassurance that they have correctly matched the graphs. This could be done in groups or as a whole class.

If students struggle to match the curves, or there is not time for this initial activity, students could be shown the solutions (near the top of the "Things you might have noticed" section) and asked to give a reason why these are the equations of the curves. (See Curve match for a full explanation.)

Possible approach

As is often the case, the "think-pair-share" model gives students time to ponder alone (probably with whiteboard or paper) then gain confidence by checking their understanding with a neighbour before sharing in a wider group.

Key questions

- How can symmetry help us to answer these questions about areas?
- Does it matter whether we combine our functions before or after integrating?
- What does the mean of two curves look like?
- How can we create a function symmetrical to another about the line $y = x$?
- How could we use integration with this new function?

Possible support

Students will probably find this activity easiest if they combine a variety of approaches. Prompting questions such as

- "Have you considered symmetry?"
- "What do you know about this larger region and how can you use it?"
- "What happens if you subtract functions from each other?"

may encourage different approaches.

Possible extension

The "Things you might have noticed" section suggests various options for reflection and extension. For example, students could try to find all the areas by computing only one integral. They could also try to split the region into sixths and then twelfths. Economics students might be interested in links with the Lorenz curve and Gini coefficient.