Designing a polynomial

Teacher notes

Why use this resource?

This resource approaches power series by asking students to find a polynomial that equals its own derivative. Students will probably initially respond that it can't be done. This resource explores the idea that trivial cases and infinite cases can be considered. It is an approach for getting the students to find the series expansion for e^x for themselves, without using any heavy machinery. It is an alternative to thinking about approximating a function at a particular point.

Preparation

Students may like to explore using Desmos, Geogebra or graphing calculators.

Possible approach

Students would benefit from a short time of thinking and pair discussion on just the question posed, before a whole class plenary unpacking the ideas in the prompting questions.

Key Questions

- What does differentiating do to the coefficent of x^n?
- Can we construct a polynomial $p(x)$ starting with a different constant term?

Possible extension

- Knowing that the exponential function e^x is its own derivative, the next natural question to ask is, *is this true for other exponential functions?* See resource Differentiating exponentials
- Find a polynomial $f(x)$ such that $f'(x) = -f(x)$ (as far as a certain term). How does this compare to the original problem?
- Can we design a polynomial such that $f''(x) = -f(x)$ (as far as a certain term). What function do we know that behaves like that? What happens if we need $f(0) = 1$?
- Ask students to find a pair of polynomials such that $f'(x) = g(x)$ and $g'(x) = -f(x)$. What functions do they know that behave like this?
- Set your own rules (students might at this stage investigate a pair where $f(x) = g'(x)$ and $g(x) = f'(x)$ thus finding the hyperbolic trig power series).

A version of this resource has been featured on the NRICH website. You might like to look at some students' solutions that have been submitted there.