Bézout's Theorem

The simplest form of *Bézout's Theorem* states that if we have two simultaneous equations, each of which is a polynomial in x and y, then the number of solutions is at most the product of the degrees of the two polynomials (as long as the two polynomials do not have a non-constant factor in common).

For example, a circle (represented by $(x-a)^2+(y-b)^2=r^2$, a polynomial of degree 2) and a straight line (represented by $px+qy+r=0$, a polynomial of degree 1) intersect in at most $2\times 1=2$ points, while two conics (each represented by a polynomial of degree 2) intersect in at most $2\times 2=4$ points.

More sophisticated versions of Bézout's Theorem take into account further possibilities, for example there may be multiple roots (where a line is tangent to a circle, for instance) and there may be complex roots. Even with these included, the number of solutions is still at most the product of the degrees.

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