To the limit

Teacher notes

Why use this resource?

This scaffolded task asks students to create a chain of reasoning to find the derivative of $f(x) = a^x$ from first principles, in the form of $f'(x) = a^x \times f'(0)$. Students are then asked to consider the special case when $f'(0) = 1$ to introduce them to the derivative of e^x. The resource allows students to review their understanding of differentiation from first principles, as well as giving them an algebraic and graphical view of why the derivative of e^x is e^x.

Preparation

The statements can be sorted interactively, or if you want to provide cards to cut out and sort, you can print them here.

There is a GeoGebra applet in the final section which you may want students to be able to manipulate themselves, but could also be looked at as a class.

Possible approaches

The warm-up offers a chance to review the ideas of differentiation from first principles.

The cards in the main problem can be used in different ways. You could ask students to sort the algebraic statements and write their own explanations for each step. Alternatively, some or all of the explanation cards could be used to support students' thinking. Once students have completed their argument they should think about the key questions below.

A special case looks at when $f'(0) = 1$ to introduce students to the derivative of e^x. This could be done individually, or used as a plenary to bring the whole group together.

Key questions

The gradient function of $f(x) = a^x$ is $f'(x) = a^x \times f'(0)$:

- What does the graph of $f'(x)$ look like?
- How will the gradient function change as a changes?
- There is a special case of the gradient function of a^x. What could it be?

Possible extension

What is $f'(0)$ for other values of a? The resource Differentiating exponentials takes a different approach to the problem and introduces students to the general result that $f'(x) = \ln a \times a^x$.