Linear operator

A function f is called a *linear operator* if it has the two properties:

(i) $f(x+y) = f(x) + f(y)$ for all x and y;
(ii) $f(cx) = cf(x)$ for all x and all constants c.

It follows that $f(ax+by) = af(x) + bf(y)$ for all x and y and all constants a and b.

The most common examples of linear operators met during school mathematics are differentiation and integration, where the above rule looks like this:

$$\frac{d}{dx}(au+bv) = a\frac{du}{dx} + b\frac{dv}{dx}$$
$$\int_r^s (au+bv)\,dx = a\int_r^s u\,dx + b\int_r^s v\,dx,$$

where u and v are functions of x, a and b are constants, and r and s are the limits of integration.

See also linear function for a subtly different use of the term "linear".