Average turning points

Teacher notes

Why use this resource?

This resource invites students to think about the shape of a cubic in a detailed way and to consider the symmetry or lack thereof in the cubic. It provides a good opportunity to think about what constitutes a proof. There are different entry points into this problem and it can be used to encourage multiple ways of representing cubics and flexibility of approaches.

Preparation

Students would likely benefit from having worked on Can you find... curvy cubics edition first, as this resource follows on from it, though it is not strictly necessary.

Possible approaches

- Students might explore the problem using graphing software such as Desmos or GeoGebra to develop a more nuanced understanding of the shape of a cubic, and to think about different algebraic representations of a cubic.
- Students might prefer to begin with an algebraic approach, or to use an algebraic approach to prove conjectures they have generated from using graphing software.
- Alternatively, they could use a graphical approach to develop an understanding of what is happening, and use this to convince themselves of the answer to this problem. This is unlikely to be an obvious approach to students, and may not feel like a "real" proof (partly due to its lack of algebra). It could be worth using prompting questions with the whole class to start them thinking about this approach, and then give further prompts to support them to progress further.

Key questions

- What exactly is student A's idea? Can you express it precisely?
- How are the locations of the stationary points and intersection points related to each other?
- How are the graphs of the cubic and its derivative related to each other?
- What must be true for a stationary point to be midway between two intersection points?

Possible support

Initial questions could include:

- Can you draw the cubic and locate the intersection and stationary points?
- Who is correct in this case, and how do you know?
- Can you draw another example?
- What patterns do you notice?

Students could be encouraged to explore numerically using graph-drawing software.

Possible extension

What would the case be for quartics or quintics (degree 4 and 5 polynomials)?