Inspecting identities

Teacher notes

Why use this resource?

Thinking about the behaviour of functions can help to make sense of identities. Similarly, by looking at identities satisfied by a function we can glean information about the behaviour of the function. In this resource, students are asked to use the half-angle formulae $\cos^2\frac{\theta}{2} \equiv \frac{1}{2}(1+\cos\theta)$ and $\sin^2\frac{\theta}{2} \equiv \frac{1}{2}(1-\cos\theta)$ to help them sketch graphs of $y=\cos^2\frac{\theta}{2}$ and $y=\sin^2\frac{\theta}{2}$.

Preparation

If printed graphs of $y=\cos\theta$ are available these could be used for students to sketch onto. Otherwise, $1\,\text{cm}$ square paper would be useful.

Possible approach

Students could be asked to start by sketching a $y=\cos\theta$ graph if pre-printed graphs are not available.

Ask students to think individually about what they would expect a $y=\cos^2\frac{\theta}{2}$ curve to look like using their cosine graph sketch as a starting point. After a minute or so they should add their $y=\cos^2\frac{\theta}{2}$ curve to their sketch then discuss their ideas with a partner. Pairs can then form small groups to share their ideas and agree on what they think the essential features of $y=\cos^2\frac{\theta}{2}$ and $y=\cos^2\frac{\theta}{2}$ are.

If graphing software is available students could check their graphs or this could be done as a whole class plenary, asking students to explain why the graphs have the features shown.

Key questions

- Are there any points on the graph we know straight away?
- What are the domain and range of the two functions?
- Can you describe $y=\cos^2\frac{\theta}{2}$ and $y=\sin^2\frac{\theta}{2}$ in terms of transformations of $y=\cos\theta$?

Possible support

To support students in thinking about functions and their graphs

- How could you decide whether $\cos^2\frac{\theta}{2}$ and $\cos\theta$ have the same period?
- What is the range of $\sin^2\frac{\theta}{2}$ and $\cos\theta$?

To support students in thinking about graph transformtions when looking at $\frac{1}{2}(1+\cos\theta)$ and $\frac{1}{2}(1-\cos\theta)$

- How does the $+1$ transform the graph?
- What effect does the minus sign before the cosine function have?
- What effect does the coefficient of $\frac{1}{2}$ have on the cosine graph?

Possible extension

- Can you transform a graph of $y=\sin\theta$ into either of these half-angle graphs?
- Can you use that to write down a new identity?

A version of this resource has been featured on the NRICH website.