Sine on the dotted line

Teacher notes

Why use this resource?

This resource encourages students to think about different ways of calculating the area under a curve. It may highlight various misconceptions and prompt students to think about the difference between an area and a definite integral.

Preparation

Mini white boards could be used by students to sketch diagrams. Sets of Expression cards and Student diagrams cards will probably be needed.

Possible approaches

Starting with the problem:

- Display or hand out just the problem diagram and ask students to *think* individually for 1-2 minutes about methods they could use to calculate the area (without actually calculating it).
- Students then *pair* up to discuss their methods and try to write expressions to match them.
- Give out the Expression cards to groups of 4 to 6 students. Ask them to *share* their findings so far and to match their own expressions with those on the cards.
    - Work out what is going on in any cards that don't match.
    - For each expression, is it correct? Will it evaluate to the shaded area or not?
    - If not, explain where it has gone wrong.
- As a plenary discussion (possibly using the interactive cards on a whiteboard), representatives from the groups take turns to explain why a chosen card is correct or incorrect.

Starting with the Expression cards:

- Show the problem to the class and give out sets of Expression cards to small groups of students (3s or 4s).
- Once students are underway give out the Student diagram cards to stimulate discussion. This might help flagging groups get their heads round some of the expressions, or encourage groups to consider other visualisations.
- As a plenary get each group to discuss one card, explaining why it does or does not give the correct value for the area.

Starting with Student diagram cards:

- Show the problem diagram and ask students to *think* about how they might calculate the area.
- Give out the Student diagram cards to *pairs* of students. For each card,
    - what method was this student trying to employ?
    - does it match their own way of thinking?
    - write an expression for the shaded area based on this method.
- Give out the Expression cards and ask students to match them up with their own expressions. Decide whether each one is correct or not and explain why.
- *Share* their findings as a plenary using the interactive cards.

Key questions

- Can you explain or draw a diagram of how the expression on the card is (trying to) calculate the shaded area?
- How would the thinking shown in this diagram help to find the shaded area?
- How can we calculate this (smaller) part of the area?
- Has some kind of transformation been applied here?
- Why do you think this one is correct but this one is not?
- Is this small area the same as any other area under the curve?
- What does it mean if a definite integral has a negative value?

Possible support

Use the Student diagram cards earlier with less confident students. Ask them to describe what each diagram shows and to look for an expression card that matches each one.

Possible extensions

- What other correct expressions can you find?
- What is the simplest way to correct each of the incorrect expressions?
- Can you use similar methods to find the area enclosed by $y = \cos x$ and $y = -\frac{1}{2}$ in the range $\frac{2\pi}{3} \leq x \leq \frac{4\pi}{3}$?