Dimensional analysis

underground mathematics

Dimensional analysis is the analysis of the relationship between different physical quantities by tracking their physical 'dimensions' as calculations are performed.

Dimensions are usually expressed in terms of the SI base quantities such as mass ($[\mathrm{M}]$), length ($[\mathrm{L}]$) and time ($[\mathrm{T}]$). For example, if v represents a velocity, then we would write $[v] = [\mathrm{LT}^{-1}]$.

Dimensional analysis is commonly used to check the plausibility of equations and results. Any equation that models a physical system must have the same dimensions on both sides of it.

For example, ignoring air resistance, the period of a pendulum, T, with dimension $[\mathrm{T}]$, will be related to: the amplitude of its swing (the maximum angle from the vertical), $\theta_{\max}$, which is dimensionless; its length l, with dimension $[\mathrm{L}]$, and the acceleration due to gravity, g, with dimension $[\mathrm{LT}^{-2}]$. The only way to obtain $[\mathrm{T}]$ from $[\mathrm{L}]$ and $[\mathrm{LT}^{-2}]$ is $[\mathrm{T}] = \sqrt{[\mathrm{L}]/[\mathrm{LT}^{-2}]}$, so we obtain

$$T = f(\theta_{\max})\sqrt{\frac{l}{g}},$$

where $f(\theta_{\max})$ is some function.

Some authors write dimensions without square brackets.