Two for one

Teacher notes

Why use this resource?

What do we get when we integrate $\frac{1}{x}$? $\ln x$ or $\ln|x|$? This resource explores when and why we use the modulus sign.

The problem could be used as a way to introduce students to using the modulus sign, or as a way to think about why we use it. Solving the integrals in the first section is likely to either remind students that they need to use the modulus sign, or bring out the need for it. The second section asks students to sketch the gradient functions of $\ln x$ and $\ln(-x)$ to highlight that two different functions differentiate to $\frac{1}{x}$, but on different domains.

Preparation

As a minimum students must have met the result $\frac{d}{dx}(\ln x) = \frac{1}{x}$ or $\int_1^x \frac{1}{t}\,dt = \ln x$. The chain rule is not required, but means that students will have to work out the gradient of $\ln(-x)$ by sketching the graph and using their knowledge of the gradient function of $\ln x$.

Possible approach

The first section gives students an opportunity to get stuck, or to spot inconsistencies in their work. When calculating the second integral they might end up with $\ln(-2) - \ln(-3)$ and have to think about what is wrong with this statement. In any case, the integrals can be used to discuss questions such as 'Why can we use the modulus sign?' or 'How can we solve the integral with negative values of x?'. The second section can then be used to explore these questions and demonstrate the different antiderivatives that exist for $x > 0$ and $x < 0$.

Key questions

- What is the problem with this calculation?

$$\begin{aligned}\int_{-3}^{-2} \frac{1}{x}\,dx &= \left[\ln x\right]_{-3}^{-2} \\ &= \ln(-2) - \ln(-3) \\ &= \ln\frac{2}{3}\end{aligned}$$

- Why do we include a modulus sign in the logarithm when we integrate $\frac{1}{x}$?
- What do you notice about the gradient functions of $\ln x$ and $\ln(-x)$?

Possible support

The suggestion section contains a GeoGebra applet that allows students to check their sketching of $f(x)$, $g(x)$ and their gradient functions. This will suppor them in answering the final questions.

Possible extension

For students that have already met integration by substitution, they could use the substitution $u = -x$ to integrate $\displaystyle\int_{-3}^{-2} \frac{1}{x}\,dx$ and see if it matches their expectations.