π — what's in a number?

The number π is irrational: we cannot write it as a fraction; its decimal expansion is infinite and doesn't end in a repeating block of numbers. In fact, if we look at the first hundred digits of that expansion,

$$3.14159265358979323846264338327950288419716939937510$$
$$58209749445923078164062862089986280348253421170679,$$

then they look pretty random. Finding patterns in the expansion has been a favourite (and, if the movie Pi is to be believed, dangerous) occupation of numerologists and mystics, but they haven't been successful.

Many mathematicians believe that the digits of π really do have all the properties that a random sequence would have. But what does that mean? One prerequisite for randomness is that each of the digits from 0 to 9 should appear in equal proportion ($1/10$th of the time), each pair of digits should appear in equal proportion ($1/100$th of the time), each triple should appear in equal proportion ($1/1000$th of the time), and so on. That's what you would expect to happen if you cast a fair ten-sided die an infinite number of times and built a decimal from that. A sequence of numbers that has this property is called *normal*. Mathematicians believe that the sequence given by the decimal expansion of π is normal, but they haven't been able to prove it yet.

Normality has a curious consequence: it means that every sequence of digits would occur somewhere in the sequence. There are ways of translating text into numbers, for example one such is ASCII code in which a space is coded as 032, an A as 065 etc. This would mean that every name of every person who has ever lived or will live should be coded somewhere in the digits of π. For instance the name Jan has the ASCII code 074097110. There are 10^9 possible different sequences of 9 digits which theory suggests should all exist the same number of times somewhere in the digits of π; on average 1 in every 10^9 of these strings would encode the name Jan in ASCII. This also means that if we take long enough strings of digits every text that has ever been written, or will be written, would appear somewhere in the expansion of π (if it is indeed normal). The Bible, the complete works of Shakespeare, and the secret of the universe (if there is one) would all be encoded somewhere in π. No wonder so many people are fascinated by this number!