Function builder II

Teacher notes

Why use this resource?

This task is designed to build students' understanding of the properties of functions and their graphs particularly those of rational functions created by dividing one function by another.

Starting with functions in algebraic form, students have to find appropriate sketch graphs from a set of cards to fill each cell of a grid. This requires some algebraic manipulation and some graph sketching, or at least imagining what the graph might look like.

Preparation and possible approach

It would be a good idea to use Function builder I before this task, so students are familiar with the idea and with the starting functions.

Students are probably best working in pairs or threes. Each group will need a set of graph cards – these can be printed on card or laminated for reuse if appropriate. Each group will also need a grid (maybe in large format) on which to collect ideas and lay out the cards. You might want to use the printable of the problem page for each student to fill in and keep as a personal record.

The graphs are not all drawn to scale, so students should focus on the key features rather than estimating x and y values. Notice that graphs P and T differ only in horizontal scale, so could be interchanged (except that T has a hole in it – see below).

Students could be asked first if they can draw sketches of the $b(x)$ functions and their reciprocals, then use the cards to find the combined functions. Alternatively they could work with the cards throughout.

Note that there are more graphs than there are grid cells, so students attempting to use elimination will still need to think about the critical features and the behaviour of the graphs in different parts of the domain. A plenary could bring out different ways of approaching the problem.

Key questions

- Without doing the algebra, can you tell me what sort of function you expect to get with this combination?
- What do the functions in this row/column have in common and how do they differ?
- Which of the available cards are possible candidates for this cell?
- What do the functions represented by these graphs have in common and how do they differ? (E.g. Q and S or P and T)

- Describe what you see on the card. Can you label any features? Does that match any equation you have?

Possible support

There are different routes into the problem – row-by-row, column-by-column, etc – and students might sometimes benefit from changing tack. Encourage students to look for features algebraically ("How can we find where this function will have an x intercept?...a y intercept? ...an asymptote?"). Encourage students to think about multiplying the function $a(x)$ by the reciprocal function they found for the first column.

Possible extension

Students could be encouraged to use Desmos or a graphical calculator to create the three graphs that do not fit in the grid, exploring what aspects of an equation give the graph different features.

When cancelling down the fractions in the third row, note that the x values that are problematic in the uncancelled fraction should be excluded from the domain of the simplified function. This gives rise to holes in the graphs, which have been shown on the cards as open circles.

For further exercise at graph sketching, students could look at Worth 1000 words.