Implicit circles

Teacher notes

Why use this resource?

This resource presents opportunities for students to see implicit differentiation as a useful or necessary technique for finding the gradient function of more complex curves. The questions involving the circle can all be solved using geometric reasoning, although students may well attempt to differentiate here. However the gradient of the ellipse would be more challenging to solve geometrically and is likely to encourage students to think about differentiating.

Preparation

Students need to be familiar with the chain rule.

Possible approach

This is an ideal starting point for thinking about implicit differentiation. It is likely some students would naturally use geometry to find the gradients and some would try differentiation. If students attempt to differentiate to find the gradient of the circle they may succeed by treating the positive and negative case differently, but they may run into difficulty. Getting stuck is likely to be very productive, as it creates the need for implicit differentiation.

It may be beneficial for students to share the approaches they took to find the gradient of a general point on the circle, before attempting to find the gradient of the ellipse.

Key questions

- What geometrical properties could you use to solve this?
- Could you use calculus to find the gradient?
- What problems arise if we try to differentiate $y = \pm\sqrt{4 - x^2}$?
- How can we differentiate y^2 with respect to x?

There is an opportunity to think about notation within this resource and check that students understand $\frac{d}{dx}(\cdots)$.