Stretch the function

Teacher notes

Why use this resource?

This resource uses the idea of stretching (or vertically scaling) a function to allow students to practise evaluating definite integrals and to explore the difference between the definite integral and the area between the curve and the axis. The functions are carefully chosen to highlight particular issues.

Preparation

Initially students should sketch the functions by hand but access to graphing software could be useful.

Possible approaches

The Warm-up section gives a concrete example to work with which can be used to help define the main problem, ensure students are confident at integrating powers of x and appreciate why $\int af(x)\,dx = a\int f(x)\,dx$.

When attempting the main problem, students should be encouraged to sketch each of the curves to help them visualise the effect of scaling, and in some cases it will prove essential when students are thinking about areas.

The main problem raises the issue of integrating across a discontinuity, and this is explored further in the section titled A disagreement. In a light-hearted way, this challenges students to think more about why this kind of integral is or is not allowed.

Key questions

- Can you sketch that function?
- What are the significant features of that curve?
- Can you shade the area corresponding to that definite integral?
- What is it about that function that causes the problem here?
- Can you generalise about functions which cannot be scaled to give an integral of 1?

Possible extension

Students who have grasped the issues in the main problem could be encouraged to explore the question in A disagreement.

The mathematics required to do the integrals suggested goes beyond what is required for most students at this level. However 'A resolution' can be read and followed by an interested student, and it should give a taste of how this topic develops beyond school level.

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