Frightening function

Teacher notes

Why use this resource?

Students have to decide how to manipulate algebraic fractions in order to simplify a function. Familiar approaches, such as simplifying the numerator and finding a common denominator, are not as efficient as splitting the fraction into the sum of simpler fractions, and so offers a way to start thinking about partial fractions.

The 'taking it further' questions ask students to think more about the idea of partial fractions, but without formally introducing them to a given method.

You may also wish to use the function to ask students to think more generally about functions, graphs, and domains. What would you expect to see in the graph when thinking about the original algebraic form of $f(x)$? How does this compare with the simplified version?

Possible approach

Give students time to try their own approach before sharing ideas. You may wish to show students the questions in the Suggestion if they struggle to get started. Give them all the questions so they still have to decide which approach to take.

Encourage students to share approaches that didn't work, and to reflect on why they didn't work.

If you are using the task to begin thinking about partial fractions, highlight the different forms the simplified function might take, $f(x) = \dfrac{1}{x-5} - \dfrac{1}{x-1}$ and $f(x) = \dfrac{4}{(x-1)(x-5)}$, and ask students about how they could get from one form to the other.

Key questions

- What did you notice about the first approach you used to simplify $f(x)$? Did it work? If not, what did you do next?
- Can you add $\dfrac{1}{x-5} - \dfrac{1}{x-1}$ to make a single fraction? Can you split up $\dfrac{4}{(x-1)(x-5)}$ into the sum of two fractions? Which of these is easier to do?

Possible extension

The 'taking it further' questions can be used to see what students noticed when they simplified $f(x)$ and if they can apply it to rewriting these questions with partial fractions. They should compare the three examples and consider how are they similar and in what ways are they different.

If students think about whether all fractions in the form $\dfrac{p}{(x-q)(x-r)}$ can be rewritten as $\dfrac{A}{(x-q)}+\dfrac{B}{(x-r)}$, where p, q, r, A and B are constants, then they should also think what happens in the special case when $q = r$.