Can you find... curvy cubics edition

Teacher notes

Why use this resource?

This activity gets students thinking about how they can use their knowledge of derivatives and work "backwards", which is often a fruitful way of working. The problem could be used to check or consolidate understanding of stationary points or to motivate integration. It could also be used without any calculus as a sketching activity when exploring the nature of cubic curves.

This activity can be used as a warm-up or follow-up activity to Two-Way calculus

Possible approach

You might like to begin by considering turning points with students, perhaps using a mini-whiteboard activity in which they are asked to sketch specified examples. This can then lead into the main problem as students may find it helpful to sketch a cubic that has the specified conditions before trying to identify an equation and take an algebraic approach.

It can be useful for students to work in small groups, sharing and discussing their approaches in order to motivate questions such as: Do these conditions specify a unique cubic curve?

Students should be encouraged to justify why their cubics meet the specified conditions and explain their approach.

Sample student work is available and you might like to have a few copies of this on hand to give to groups of students at times while they are working on the problem. This may provoke discussion, give students a way into parts (c) and (d) of the problem, and encourage deeper thinking about the possible approaches.

Key questions

There are many suggested questions in the "Things you may have noticed" section, in particular you might like to consider the following:

- What does this mean for the derivative of the cubic function?
- What could $f'(x)$ look like if $f'(x) = 0$ has two distinct solutions?
- What shape did the curve you created in part (b) have? Could we use this to help create a cubic curve with these conditions?
- How could you convince someone else that your cubic meets these conditions?
- How could you find another example?
- Could you generalise your example?

Possible support

The related problem, Curvy cubics might be useful. Curvy cubics could be used before or after students have considered this problem as an opportunity to further discuss the properties of cubic curves and their derivatives.

You might like to offer students the opportunity to 'play' with cubic curves on Desmos. This may encourage them to find ways of manipulating cubics to create others, and should help to support their understanding of transformations.

Possible extension

- Can students find other cubics to match the criteria?
- Do all cubics satisfying one of the criteria have something in common that can be recognised without getting involved in calculus?

A version of this resource has been featured on the NRICH website.