Squaring the circle

"Squaring the circle"—it's an expression that is often used to describe something that is impossible, or very difficult to do. But why? The expression refers to the task of finding a square that has the same area as a given circle. That doesn't seem too difficult. First, measure the radius r of your circle and work out its area A using the formula

$$A = \pi r^2.$$

Then use a calculator to work out $\sqrt{A}$: since the area of a square is its side length squared, $\sqrt{A}$ is the side length of the square of area A you are looking for, which you can now draw—done!

That was easy, but now imagine you had been asked to construct the square geometrically, using only compasses and a straight edge (a ruler without markings), and without making any calculations in your head, on paper, or using a calculator. You would find this a lot harder to do and you would be in good company—the greatest mathematicians have puzzled over this problem since it was first posed by the ancient Greeks well over 2000 years ago. It wasn't until 1882 that the German mathematician Ferdinand von Lindemann explained why they were not getting anywhere: he showed that the task is actually impossible.

The reason is that π, the number that is so closely associated to circles, can never be the solution of a straightforward equation. This is in stark contrast to some other familiar irrational numbers, for example $\sqrt{2}$, which is a solution of the short equation

$$x^2 = 2.$$

The number π, on the other hand, is *transcendental*: it cannot be the solution of a polynomial with rational coefficients. It's a property that π shares with the number e and it's the reason why we can't square the circle: any geometric attempt to do this using a straightedge and compasses would translate into π being a solution of a polynomial equation.

Despite the incontrovertible proof of π's transcendentality, there are people who haven't given up. It's no rarity for an eminent mathematician to receive enthusiastic emails or letters containing purported circle-squaring methods. The stubbornness of two of such enthusiasts was illustrated beautifully by the logician Charles Dodgson, more famous as Lewis Carroll, author of *Alice in Wonderland*. In 1890, Dodgson wrote in the introduction to his book *A new theory of parallels*:

> The first of these two misguided visionaries filled me with a great ambition to do a feat I have never heard of as accomplished by man, namely to convince a circle squarer of his error! The value my friend selected for Pi was 3.2: the enormous error tempted me with the idea that it could be easily demonstrated to BE an

> error. More than a score of letters were interchanged before I became sadly convinced that I had no chance.

A few years after Dodgson's writing another hobby mathematician, Edwin J. Goodwin from Indiana, USA, also believed that he had found a method for squaring the circle which involved setting π equal to 3.2. He copyrighted his idea, but offered the Indiana government the opportunity to use the new "mathematical truth" in schools free of charge. A bill to that effect was submitted to the General Assembly in 1897, perhaps the only attempt ever made (at least in the modern era) to legislate mathematics—it took quite a lot of debating and the intervention of a maths professor to stop it from becoming law.