Powerful derivatives

Teacher notes



Why use this resource?

This short resource offers a chance for students to draw together what they have learnt about exponentials, logarithms and the chain rule to finally prove a very important result that they have been using since they first learnt calculus. It could well provoke interest in proof, while the brevity of the proof might raise questions of whether proofs need to be "complicated" (they don't!).

Preparation

Students will need some familiarity with the tools of this proof, namely, exponentials, logarithms and the chain rule. Students may benefit from having explored the earlier resource Binomials are the answer!, which looks at the derivative of x^n for positive integers n using the binomial theorem. A comparison between the arguments appears in the Solution section.

Possible approaches

Students can be presented with the problem and asked to search for a proof. They may well claim that they already "know" it, but the question of how can we *prove* it remains. If students are stuck, the question offered in the suggestion may well be the key to making further progress. The other way of rewriting x, namely $x = \ln(e^x)$, turns out not to be helpful; if students go down this route, they can be asked if they can think of any other way of rewriting x.

Alternatively, this problem could be used earlier as a key question to give a reason for learning the chain rule.

Key questions

• What does x^a mean?

Possible extension

If students have learnt about Taylor series, this could be used as a springboard for proving the general binomial theorem.