

Completing the square

Completing the square is a method for rewriting a quadratic expression in a variable such as x using only one occurrence of the variable, by combining the x^2 and x terms into a single square.

In general, the expression $x^2 + bx + c$ can be rewritten as $(x + \frac{1}{2}b)^2 - \frac{1}{4}b^2 + c$, where the $(x + \frac{1}{2}b)^2$ term expands to give both the x^2 and bx terms.

The expression $ax^2 + bx + c$ can be rewritten similarly by first taking a out as a factor, giving

$$ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right),$$

and then rewriting the quadratic expression within the parentheses as above.

This technique can be used to solve quadratic equations and to derive the quadratic formula.