Arithmetic mean

The *arithmetic mean* is often known simply as the *mean*. It is an average, a measure of the centre of a set of data. The arithmetic mean is calculated by adding up all the values and dividing the sum by the total number of values.

For example, the mean of 7, 4, 5 and 8 is $\frac{7+4+5+8}{4} = 6$.

If the data values are x_1, x_2, ..., x_n, then we have $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, where $\bar{x}$ is a symbol representing the mean of the x_i values.

This rearranges to give the useful result

$$n\bar{x} = \sum_{i=1}^{n} x_i,$$

that is, the arithmetic mean is the number $\bar{x}$ for which having n copies of this number gives the same sum as the original data. So the sum of a set of numbers in some sense "averages" them.

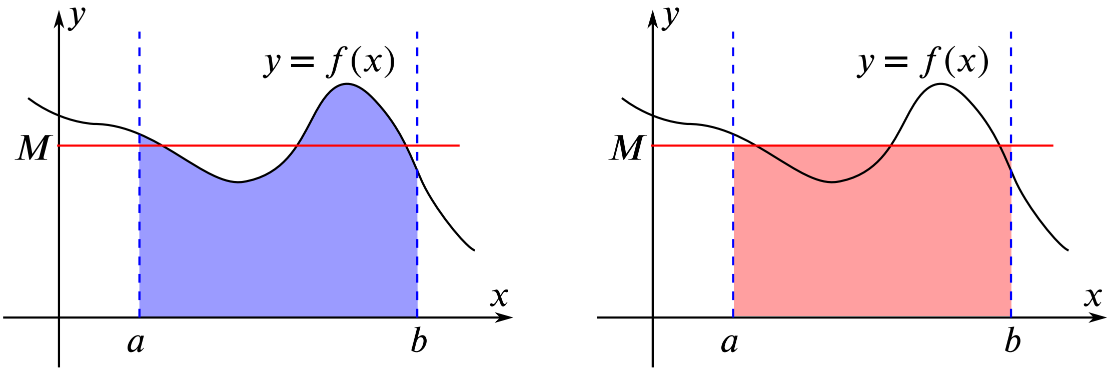
If the data are grouped, with f_i occurrences of the value x_i for $i = 1, 2, \ldots, n$, then their mean is given by

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i},$$

where the numerator is the sum of all of the x_i values and the denominator is the total number of values.

The arithmetic mean is sensitive to outlier values.

The mean value of a function $f(x)$ over the interval $a \le x \le b$ is likewise the value M for which the constant function $f(x) = M$ has the same "sum" as the original function. The "sum" of a function over an interval is the integral of the function, as shown in this sketch:



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Thus the mean M is given by $M(b-a) = \int_a^b f(x)\,dx$, so

$$M = \frac{\int_a^b f(x)\,dx}{b-a}.$$

The integral therefore "averages" the function.