Connect three?

Teacher notes

Why use this resource?

It can be easy to assume that any three numbers can always belong to an arithmetic progression, in some order, because they can always lie on some straight line. In this problem students are challenged to think deeply about the properties of an AP and are prompted to consider sets of three numbers that contain integers, rational and irrational numbers. In these examples students are likely to grapple with ideas such as HCF, common differences, properties of irrational numbers and, in some cases, proof by contradiction.

An important idea to emerge is that the difference between any two terms (not necessarily consecutive) in an AP must be an integer multiple of the common difference.

This resource could be used to introduce the language and properties of APs and could be used or revisited as an opportunity to make connections to the proof of why the square root of 2 is irrational and then as a further context for proof by contradiction.

Preparation

It might be helpful to have printed sets of the cards in the Suggestion section. This will encourage students to sort the cards into categories and to notice properties of these numbers in these examples.

At the end of the Things you might have noticed section of the resource you can find a summary showing for which cards an AP can or cannot be found.

Possible approach

You might like to begin by posing the problem to the whole class and inviting suggestions of three numbers to try. It could be useful to take this opportunity to discuss whether the numbers suggested need to be consecutive terms in the AP and whether they need to be the first terms in the AP.

If using this problem before students have encountered the language of arithmetic progressions you might like to ask students to find 'linear sequences'. Terminology such as AP and common difference could then be naturally introduced throughout the lesson or at the end as part of a plenary session.

The cards in the Suggestion section of the resource could then be given to students in pairs or small groups.

Key questions

- Before considering any of the cards in the Suggestion section, do you think it is always possible to find an AP containing any three numbers chosen? Why?
- What is the same and what is different between the sets of numbers on each card?
- Which card looks like it will be the easiest to think about? What about the hardest? Why?
- You have found an AP containing the numbers on card C (for example). Is there another AP that would also contain these numbers in some order?
- If you decide that it is not possible to find an AP containing all three numbers on a card, can you change one value so that it is now possible?

Possible support

Use the cards in the Suggestion section of the resource to give students something concrete to work with. Encourage them to work in pairs to first categorise the sets of numbers (in whatever way they wish), before drawing their attention to cards C, D, E and I as a potential entry point (if they have not already done so). In particular, card C is a good starting point.

Possible extension

Asking students to think more deeply about the cards containing irrational numbers can be interesting. You might like to challenge them to construct a proof of why the numbers on one of cards A, B or G cannot belong to an AP, and then to modify this proof for the other two cards. You might like to encourage students to think about the proof of why the square root of 2 is irrational to support this challenge.