General solutions

Teacher notes

Why use this resource?

By asking students what they can say about A and B if $\tan A = \tan B$, this resource introduces the general solutions of trigonometric equations such as $\tan x = \sqrt{3}$. The interactive graphs could be used to explore these and similar equations, revealing how the symmetry and periodicity of the functions comes into play. Students may also connect the graphs and solutions with the unit circle.

Possible approach

Students could work in pairs or small groups. Suggest that they try to sketch the graph of $\tan x$ or $\cos x$ first and think about solving an equation such as $\tan x = 1$ or $\cos x = \frac{1}{2}$. Alternatively, students could use the GeoGebra apps to explore the problem. Expressing the general solutions algebraically is part of the challenge and provides an opportunity to encourage students to express themselves clearly and precisely.

Key questions

- Why do you need to specify that n is an integer in expressions such as $\frac{\pi}{4} + n\pi$?
- How is solving $\cos x = \frac{1}{2}$ different from solving $\sin x = \frac{1}{2}$ or $\tan x = \frac{1}{2}$?
- What can you say about $\tan A$ and $\tan B$ if $\sin A = \sin B$ and $\cos A = \cos B$?

Possible support

Students could first try to solve an equation such as $\sin x = \frac{1}{2}$ for x in a restricted interval, such as 0 to π and think about what would happen if they extend the graph beyond this interval. This could help to bring out the symmetry of $\sin x$ about $\frac{\pi}{2}$.

When working in the restricted interval 0 to π, students may be able to link this back to the ambiguous case when solving triangles.

Possible extension

What can you say about A and B if $\sin A = -\sin B$ and $\cos A = -\cos B$? What can you then say about $\tan A$ and $\tan B$? When exploring this, students may start to connect the graphical approach with the unit circle.

Students could also think about how they could use the ideas in this resoure to solve other trigonometric equations such as $\tan 2A = \tan 2B$ or $\tan(2A + \frac{\pi}{3}) = 1$.