Cambridge Mathematics Education Project newsletter

Summer term: May 2015

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Welcome from Professor Martin Hyland, CMEP Project Director

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This is the first of what I hope will be a series of messages from the CMEP project directors, who are Lynne McClure and myself.

I have taught maths at Cambridge for almost 40 years so naturally am interested in specific issues of mathematical content. But in my research work I deal in particular with the conceptual underpinnings of mathematics; so I am concerned that the project draw attention to the range of significant abstract ideas which connect areas and so support mathematical understanding.

One leading example is the idea of the average. That might be introduced for the first time in Statistics; and the mean or expected value is a basic notion in Probability. But the concept is found all over mathematics. At a simple level, it explains why summing an arithmetic series and finding the area of a trapezium are essentially the same problem. One of mankind's great discoveries, that of the Centre of Mass by Archimedes, is a sophisticated average. Indeed that average and others run through Mechanics; and as a result there are (often overlooked) parallels between ideas in Mechanics and Statistics.

By highlighting Pervasive Ideas like the average, we aim to encourage students' conceptual understanding and so promote their development as versatile mathematicians.

Martin Hyland

Join the CMEP community

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The Cambridge Mathematics Education Project (CMEP) is a new initiative, based in the University of Cambridge's Faculty of Mathematics and funded by a grant from the DfE, which aims to enhance advanced post-16 mathematics education.

CMEP is developing innovative resources to help support and inspire teachers and students of A-level mathematics and similar qualifications. Throughout the

project, we are carefully considering both the learning needs of the students and the associated issues facing teachers. We are working closely with teachers from a range of different schools throughout the development process to gain formative feedback and input.

Schools, colleges and academies across the UK are warmly invited to register as CMEP Affiliate Schools. Registration as an Affiliate School gives free access to the CMEP online resources. For more information, please click on the link below.

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Featured resource: 't for tan'

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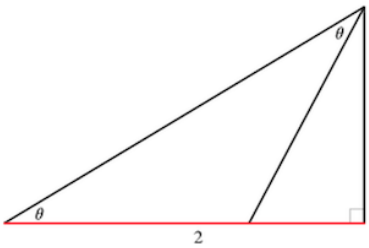
If we write $t = \tan\theta$, then the following equations are true.

$$\tan(2\theta) = \frac{2t}{1-t^2},$$
$$\sin(2\theta) = \frac{2t}{1+t^2},$$
$$\cos(2\theta) = \frac{1-t^2}{1+t^2}.$$

Can you use this diagram to obtain these formulae?



A word from the authors:

This resource shows the power of working with a helpful diagram, and encourages students to continue to take a geometrical approach to trigonometric formulae. Students can find various important lengths and angles, using straightforward techniques that they will be familiar with. However, they may find it challenging to find a useful approach to deduce the formulae, and this is where being willing to try different methods and collaborate with their peers can lead to more success in the task.

To access the full resource on the CMEP site, please click on the link below. (Note: you will need to have registered for the CMEP site to log in - please see above to details of how to request access as an Affiliate School, if not already registered.)

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Featured resource: 'Integral Chasing'

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A word from the authors:

Below is a selection of integrals, some of which you can not do without performing others first and some that require the use of other mathematical skills.

Can you find the missing positive numbers a to d?

$$(1)\ \int_{a}^{5} 10x + 3\ \mathrm{d}x = 114$$

$$(2)\ \int_{2a}^{9} b\sqrt{x} + \frac{a}{\sqrt{x}}\ \mathrm{d}x = 42$$

$$(3)\ \int_{\frac{1}{2}}^{1} \frac{1}{x^5} - \frac{1}{x^2}\ \mathrm{d}x = \frac{c+1}{4}$$

$$(4)\ \int_{6}^{c+2} x^{\frac{b}{a}} \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)\ \mathrm{d}x = ab^a d^a$$

Test your solution

Once you have found the missing positive numbers a to d, can you use them in the statement below to test your values?

The area formed between the x-axis and the lines $x = b$ and $x = d$, and the curve $y = (x - 2a)(x + 1)$ is $\frac{cd}{a(a+b)}$.

Remember, when we are looking at *area* what must we check about the curve between the lines $x = b$ and $x = d$ when it is plotted?

The idea behind this resource is for students to get further practice of integrating powers of x, whilst exposing some of the common misconceptions in order for them to be addressed. For the students who have tried this there were a range of outcomes, from strengthening their skills in manipulating powers of x, all the way through to greater awareness of how the evaluation of integrals links to finding areas between a curve and the x-axis.

To access the full resource on the CMEP site, please click on the link below. For other calculus resources you might also like to try, have a look at 'Can you find …curvy cubics edition' or for a longer task, 'Two way calculus'.

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Teacher perspective: using 'Two Way Calculus' in the classroom

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Katie Binks, from Long Road Sixth Form College, shares her experience of

	The curve is . . .creasing for $x > 1$	Has a local . . .imum with y-coordinate 1	
Has a stationary point at $(1, 1)$	$y = \frac{1}{4}(x^3 + 3x^2 - 9x + 9)$		$y = \frac{1}{12}(3x^4 - 4x^3 + 13)$
		$y = \frac{1}{6}(2x^3 + 9x^2 + 6)$	$y = 1 - \frac{1}{4}x^4 - x^3$
Has an . . . number of stationary points			$y = x^5 - x^3 + 5$

using Two Way Calculus with a group:

"I displayed the grid on the board from the beginning and asked some questions about the wording to make sure they understood, e.g. a student sketched what they thought it meant to be increasing for x>1. Students suggested some techniques they might try in order to discover things. They then worked for 10 minutes and then I had students feed back what they had found out. At this point we had a suggestion of an even (!) number of stationary points, but got the increasing and minimum filled in and a suggested function for the top row, which was the translated standard quadratic in completed square form. From then on the students went up and filled in things if they thought they had something that worked, and worked in twos or fours, as they liked consulting freely between groups.

The result was quite lively but very productive.

1. One pair of students ended up using Autograph with the constant controller to investigate the effect of removing different bits of a quintic after realizing that there was something which was having the effect of "collapsing" turning points together to create a stationary point of inflection and thinking about both the differential and second differential having a factor of x.

2. There was lots of discussion between different students about the number of roots and how they could prove how many there were. This included a lot of work on comparing algebraic attempts to using their calculator functionality and understanding how they could use the two together to feel more confident in their solutions.

3. One pair of students fairly quickly realized they could reuse functions and filled in the table quickly. They were then challenged to come up with a different row title for the middle row.

4. One group found a link between the two functions on the middle row concerning the form of the differential (which actually was linked to having x=-3 as a stationary point, but they did not recognize that so had a complicated row title) so formed their function by integrating the differential that they felt fitted the pattern.

5. At the end we discussed what they had learned, which ranged from students finally cracking factorizing, to understanding more about shapes of polynomials, links between functions and their differentials and between similar functions. Students were very positive about the confidence they had gained."

To find the 'Two Way Calculus' resource on the CMEP site, please click on the link below.

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Meet the CMEP team - upcoming events

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The CMEP team will be presenting sessions at the following events over the next few months:

18th June 2015 – CMEP session at the Leicestershire Heads of Maths meeting

25th-27th June 2015 – MEI Conference

3rd July 2015 - FMSP KS5 London Network Day, Tower Hamlets

5th September 2015 – Mathematical Association – CMEP session at One-day Conference for Teachers of Secondary Mathematics