Logarithm laws

The *logarithm laws* are the rules by which logarithms may be combined, and are derived from the definition of the logarithm and the index laws.

The basic rules are:

$$\begin{aligned} \log_a a &= 1 \\ \log_a(xy) &= \log_a x + \log_a y \\ \log_a(x^n) &= n \log_a x \end{aligned}$$

From these we can derive other important rules:

$$\begin{aligned} \log_a 1 &= 0 \\ \log_a(x/y) &= \log_a x - \log_a y \\ \log_a(1/x) &= -\log_a x \\ \log_a \sqrt[n]{x} &= \tfrac{1}{n} \log_a x \end{aligned}$$

We can change the base of a logarithm:

$$\log_a x = \frac{\log_b x}{\log_b a}$$

And from the definition of logarithm, it is the inverse of exponentiation:

$$a^{\log_a b} = b \quad \text{and} \quad \log_a(a^b) = b.$$