Mapping a function

Teacher notes

Why use this resource?

This is the first (and longest) of a series of three resources about mapping diagrams leading to a proof of the chain rule. (The second in the series is Mapping a derivative, which introduces differentiation through mapping diagrams, and the third is Chain mapping.) This resource explores mapping diagrams for linear functions. Mapping diagrams are likely to be familiar to students from their earlier years, but they frequently give way to graphs as a visual representation of functions. In this resource, though, we see that this alternative representation offers a very different perspective on linear functions, which will enrich students' understanding of them.

The approach taken in this resource is via mapping diagrams, a tool which is likely to be familiar to students from their earlier years.

In the Warm-up section, a variety of superficially identical mapping diagrams are presented, leading to the result that the "shape" of the mapping diagram of a linear function depends only upon the coefficient of x. This is then developed in the Problem section, where the precise relationship between this coefficient and the shape of the diagram is explored in depth. This is both interesting in its own right, showing that a linear function is essentially a scaling process, and provides the foundation upon which to understand differentiation through this approach. The "Taking it further" part of the Problem section is a digression which some students may find interesting, but it is not needed for the next resource in the series.

A fuller discussion of the potential power of this approach is discussed in the blog post Percolation, patience and the chain rule.

Preparation

Students will need to be able to draw several mapping diagrams, and blank ones can be downloaded and printed in advance. (Alternatively, students can draw their own axes, though this is likely to be more time-consuming and a lack of accuracy may obscure the interesting results.) There are also two interactive applets available for drawing mapping diagram for further exploration.

Possible approach

Students could initially be asked how many different ways they can think of to represent the function $f(x) = 3x$. Ideas might include a graph, a table of values, a function machine, a mapping diagram, a computer program and possibly others. This invites them to think about multiple representations of the function "object", before we look at one in detail.

These representations could later be compared in a plenary, once the mapping diagram representation has been explored in depth.

The Warm-up requires students to think carefully about how little effect changing scales on a mapping diagram may actually have. Students would benefit from attempting to argue why this is the case.

The Problem section develops the ideas further. The key idea required for the later resources in this series is that a linear function is a scaling, which can be seen by extending the arrows to meet in a focal point. Students might like to explore the relationship between the location of the focal point and the equation of the function, perhaps using the interactivity to gather some data.

A plenary could then be used to discuss either the closing questions or, more generally, to compare this representation to other representations discussed at the start of the lesson.

Key questions

- What does a linear function look like in this representation?
- How does the choice of a in $f(x) = ax + b$ change the diagram?
- What is the effect of centring our diagrams on the arrow from k to $f(k)$?
- How is this representation related to the normal graphical representation?

Possible extensions

- How is the location of the focal point related to the value of a in $f(x) = ax + b$?
- What would happen if we relaxed the convention on the central arrow being horizontal?