This shows that the original equation is equivalent to

$$\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}+\frac{c}{a}=0.$$

Since $a \neq 0$, we can divide by a to get

$$x^2+\frac{b}{a}x+\frac{c}{a}=0.$$

We complete the square.

We can rewrite the right-hand side by putting it over a common denominator:

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}.$$

Consider $ax^2+bx+c=0$, where $a \neq 0$.

Get the squared term on one side of the equation:

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}.$$

Subtracting $\frac{b}{2a}$ from both sides and putting the right-hand side over a common denominator gives

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

We can take the square root of both sides.

Since x appears only once in the equation, we can rearrange this to solve for x.

Taking account of the possibility of positive and negative square roots, we see

$$x+\frac{b}{2a}=\frac{\pm\sqrt{b^2-4ac}}{2a}.$$