

Why use this resource?

This Building blocks resource could be used to start thinking about integration by substitution. By exploring what happens to the areas of regions as functions are stretched and translated, students will think about ways of using existing knowledge which can later be connected to formal procedures.

The first problem can be tackled by thinking about transformations and relationships between the regions. Students are asked to explain how an answer can be deduced, which should help to support communication as well as reviewing more general points such as when the integrals of curves are positive or negative. Students can then tackle the second two problems by thinking about transformations and scaling, applying their knowledge to these more concrete cases. Thinking again about regions being made up of narrow strips may help to connect the algebraic techniques in integration with students' conceptual understanding of an integral.

Preparation

The GeoGebra files in the [Suggestion](#) should help students to think about what is happening so they may benefit from access to a computer or tablet to play with the files.

Possible approaches

There is quite a lot to take in when looking at the first diagram and it would be worth spending time encouraging students to look at this carefully. You could start by sharing just the diagram and inviting students to say what they see, or any questions they have. Students could record their thoughts individually, or discuss this in pairs. Depending on students' responses, this could be an opportunity for a plenary discussion, or you may prefer to move straight in to the first problem.

Although not the main focus of this resource, working on the first part of the problem may lead to discussion of the order of transformations of $y = f(x)$ that give $y = 3f(2x - 45)$. This can either be a stretch followed by a translation, or a translation followed by a stretch, and students may have noticed that the translations are not by the same amount. Discussion of this could be supported by the first GeoGebra file in the [Suggestion](#).

If students struggle to connect the first problem to the more concrete examples, you could offer them the GeoGebra files in the [Suggestion](#). These could also be used to support a class discussion after students have reported what they noticed when trying to compute the integrals.

Key questions

- What effect do different transformations have on the areas?
- Which direction is the function translated? How does this affect the x -intercepts?
- How did you make your decisions?
- Could you have tackled the problem a different way?

Possible support

Students often struggle to justify their reasoning and may need encouragement to think of simple ways of reporting their ideas such as

- Would labelling (other) areas help you to explain your reasoning?
- How much taller is the black curve and why?
- Which part of the equation is changing which feature of the curve?

The GeoGebra files in the [Suggestion](#) section may help students to visualize what is happening.

Possible extension

Students could generalise the concrete examples given, or try to combine stretches and transformation. They could also think about trying to justify algebraically what is happening and this could lead into thinking more formally about integration by substitution.