Cambridge Mathematics Education Project newsletter

Michaelmas term: November 2015

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Message from our Director - Martin Hyland

The Calculus is a major element of sixth form mathematics. One thing we use it for is finding maxima and minima. A compelling application is to derive the Law of Refraction from Fermat's Principle of Least Time. Suppose we teach that and some perceptive student observes that we are assuming that a straight line is the shortest distance between two points; and then asks why that is. Certainly we should think ourselves lucky because it is a very good question: but - as the ancient Greeks already appreciated - it does not have a straightforward answer. Sadly the internet will give an approach via the calculus of variations which is neither helpful nor to the point; I hope that CMEP will do better. Even so in the end this is one of those areas where teachers have to decide what it will be most useful to do with their students. Mathematicians like to say that mathematical thinking consists of making complex phenomena simple: but it is not that easy.

A basic element of the CMEP philosophy is that everyone has to understand mathematics for themselves. That is a challenge for all of us - to think about what we know and why.

Featured resource: Talking about Curves

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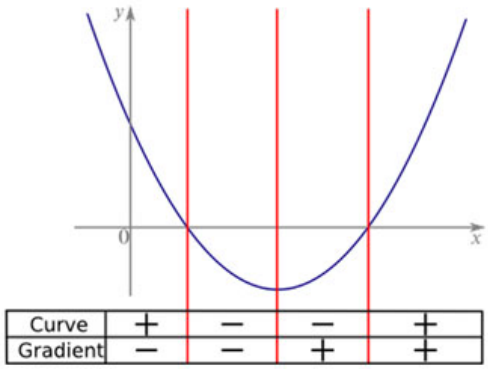
The following curve turns once. It changes sign twice and the sign of its gradient changes once, giving rise to four distinct regions, as shown in the image below:



What are the different possibilities of sign combination for a curve which turns exactly once?

How can you be sure that you have found them all?

A word from the authors: This investigation is an ideal task to introduce students to some of the concepts they will meet as they study calculus. It encourages students to really look at curves and to informally identify certain key features before introducing the formal language of maxima, minima, etc. Students should be encouraged to think about what a curve looks like, thinking primarily about sketches rather than algebraic definitions.

If you visit this problem on the site, you will also find an extension to this task: a carefully chosen selection of 'curves', provided as printable cards, which can be used to promote and broaden discussion in this investigation. Some of these curves may be less familiar to students e.g. $y=|x|$ and $x=y^2$ and these may help them to question their understanding of what a curve that turns 'exactly once' looks like.

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Teacher Perspective: Using 'A tangent is…' and 'Approaching asymptotes' in the classroom

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Michael Iyol, from Great Barr School, shares his experience of using 'A tangent is…' and 'Approaching asymptotes' with a group.

A tangent is….

Here are some suggestions of how we could define a tangent to a curve at a point.

1. "A tangent is a straight line which only meets the curve at that one point."
2. "A tangent is a straight line which touches the curve at that point only."
3. "A tangent is a straight line which meets the curve at that point, but the curve is all on one side of the line."
4. "A tangent is a straight line which meets the curve at that point, but near that point, the curve is all on one side of the line."

For each one, can you find or sketch an example to show that the proposed definition does not always work?

Can you come up with a better definition?

I teach at Great Barr School, a large comprehensive secondary in Birmingham, West Midlands. I heard a conversation between colleagues about a question posed by one A-level student who wanted to know more about what a tangent is. I remembered that CMEP has a brilliant resource that could help clarify the confusion.

On the day of the lesson I divided the group into pairs with clear instructions that each student must select the correct answer from four suggestions of how we could define tangent, then share their selection with a partner. After five minutes, the students were instructed to form groups of fours to discuss their answers. There were a few changes but they were fairly confident that the concept of tangent was now understood. The students were then asked if they could think of situations where their selected definition may fail. No counter examples were given.

I handled out the CMEP solution to challenge the first answer, "A tangent is a straight line which only meets the curve at that one point." At first a moment of disbelief, but gradually the students began to understand the generality of thinking about concepts in mathematics. I was asked to provide solutions to the rest of the options. These students were beginning to see the impact of counter examples as a way of deepening mathematical understanding. This went on for twenty minutes. One of the girls in class said "I think that this a good resource, since it has made me understand the true meaning of a tangent."

Approaching asymptotes

How would you describe what an asymptote is?

Here are some descriptions or statements about asymptotes.

1. "An asymptote is a line which a curve gets closer and closer to but doesn't meet."
2. "An asymptote is a line which a curve approaches as x tends to infinity."
3. "A curve can't cross an asymptote."
4. "Asymptotes are parallel to the coordinate axes."
5. "A graph can only have one asymptote parallel to each axis."
6. "Asymptotes occur when a function isn't defined for certain input values."
7. "A function tends to positive infinity on one side of an asymptote and tends to negative infinity on the other side."

The last half of the lesson I similarly introduced the "Approaching asymptotes" resource. The students remained in their groups to discuss asymptotes, especially how to describe them and identify them. Here the students had more confidence talking about asymptotes. There were 'light bulb' moments here as well because one students wrote, "These resources are very useful and I think that these kinds of resource … deepen my understanding … I think

that these worksheet will make maths lessons modernised … very positive way.

After the lesson I had a discussion with the two colleagues about the impact the resource had on the students. They were not surprised because the students had spread the word about counter examples and visualisation. I reflected on the lesson and came to the conclusion that spending time to understand the CMEP resources was a vital ingredient in employing the rich content of the materials.

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New and updated resources

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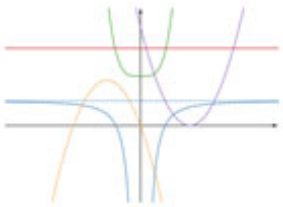
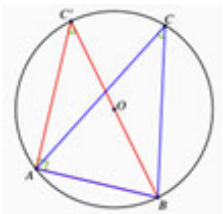
Here are some of our recent additions to the CMEP site. Some are brand new resources and some are resources that have been updated to improve them even further.

Odd or even or … (F2)



Students are asked to decide whether functions are odd or even or have some other kind of symmetry. As well as working with the graphs and equations of functions, there are opportunities to think about the effects of graph transformations. **Resource link**

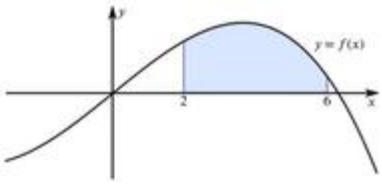
Sine-ing the way (G1)



This scaffolded task leads you through an interesting and unusual proof of the sine rule, as well as highlighting the relationship between a triangle and its circumcircle. **Resource link**

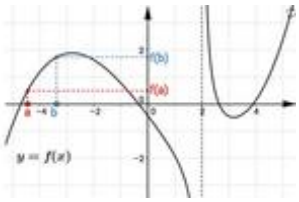
What else do you know? (C3)

Students are given the area under the curve in a specified region of an



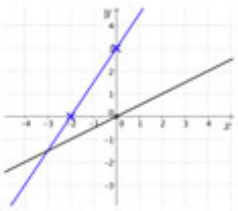
unknown function. They are then asked to consider what they can deduce about a selection of related integrals involving transformations of the original function. **Resource link**

Inequality flip-flop (F1)



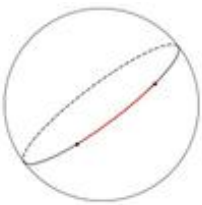
This resource is intended to support students' understanding of algebraic manipulation by encouraging them to think in terms of functions. In particular, it looks at how applying certain functions to an inequality affect the inequality sign. **Resource link**

Straight line pairs (E1)



A problem that asks students to construct two straight line graphs that satisfy certain constraints with regards to each other. Initially this is a simple problem that allows students to practise working with linear graphs. However there is scope to challenge students by asking them to think about more general solutions. **Resource link**

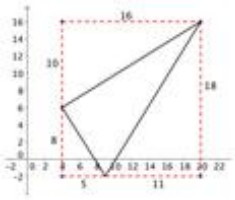
Haversine: Lost but lovely (G3)



This 'bigger picture' resource now has questions interspersed in the story of calculating distance on a sphere. With questions to think about and answer whilst reading, there is an opportunity for students to engage more deeply with the ideas presented. **Resource link**

What type of triangle? (G1)

This has been updated to include fewer questions than the original, but more variety in how you might



solve these problems. It now also offers an opportunity for students to consider the merits of different mathematical approaches. **Resource link**

Spread the word

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If you wish to demonstrate CMEP resources with colleagues or other schools, then please let us know. We can send you postcards and leaflets to hand out. The postcards show some of our resources and the leaflet explains the project's aims and beliefs as well as providing information about the website. You can also download the leaflet **here**.

If you are working with other schools then we can provide logins for them to access the site. We would be interested to hear from those spreading the word, and would love feedback from you about the site, the resources and how you have used them. For all of the above please contact us.

Meet the CMEP team - upcoming events

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Recent outings have included:

- 18th to 20th September 2015 - 5th International Realistic Mathematics Education Conference, where our writing team presented a session on our work. Read more
- 22nd to 25th September 2015 - The International Society for Design and Development in Education (ISDDE15), where our writing and evaluation team presented a poster.

And over the next few months:

- 7th November 2015 - BSRLM Conference. Our evaluation team will be presenting a session.
- 29th March to 1st April 2016 - ATM Conference. Our team will be presenting a session on CMEP.
- 1st to 3rd April 2016 - MA Conference. Our team will be presenting a session on CMEP.

- Our MEI-run CPD sessions are ongoing. More details can be found here.

Join the CMEP Community

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The Cambridge Mathematics Education Project (CMEP) is a new initiative, based in the University of Cambridge's Faculty of Mathematics and funded by a grant from the DfE, which aims to enhance post-16 mathematics education.

CMEP is developing innovative resources to help support and inspire teachers and students of A-level mathematics and similar qualifications. Throughout the project, we are carefully considering both the learning needs of the students and the associated issues facing teacher. We are working closely with teachers form a range of different schools throughout the development process to gain formative feedback and input.

Schools, college and academies across the UK are warmly invited to register as CMEP Affiliate Schools. Registration as an Affiliate School gives free access to the CMEP online resource. For more information, please click on the link below.

We would like to take this chance to thank our current Affiliate and Partner Schools for their feedback and comments left on the commenting system. We appreciate all feedback.