Logarithm lattice

Teacher notes



Why use this resource?

This resource requires students to think about the relative values of logarithms and bring together various ideas such as the definition of a logarithm, logs as functions and the relationships between logs with different bases.

Preparation

The printable pages are ready to be used in class. There are also printable cards which can be cut up for students to sort and the extra *Problem table* consists of just the grid without any explanation.

A subset of these cards can also be used with Logarithm Lineup.

Note that not all the logarithms can be easily compared but that doesn't prevent students from completing the exercise. The second grid may look easier than the first but it requires some extra work because students can't use the extreme values of $\log_5 2$ and $\log_2 5$.

Possible approach

The main *Problem* has movable cards so the logarithms can be arranged on an interactive whiteboard or on portable devices if desired. Alternatively, students could use the cards or simply write their workings and solutions on the printable page.

One approach is to show students the logarithms or cards and ask them to group these or think about what they notice (such as reciprocal relationships, exact values, expressions that are less than or greater than 1). Can they find pairs of cards where they can explain why the value on one must be greater than the value on the other?

Then introduce the first grid and ask students to put the cards in appropriate places. Some students may notice that there are several ways to complete this grid, but they should be asked to think about which arrangements of cards they can completely justify, and which might be based on conjectures. If students are working in pairs, encourage them to seek justifications or explanations from each other. This will be especially important for getting the most out of working on the second grid.

The Taking it further section does not have cards as it is more open-ended.

Key questions

- Which of the logarithms can you find an exact value for?
- Comparing this pair of logarithms, what is the same and what is different?
- If we increase the base, how does that change the value of the logarithm?

Possible support

Students may need encouragement to break down the initial problem into bite-sized pieces, then put the pieces together in a logical manner.

For the *Taking it further* exercise students should be encouraged to think about generalising their solutions as far as possible, and to think about the possible values for the base and argument of a logarithm—including negatives, fractions and irrational numbers.

Possible extension

Students may have made several conjectures when trying to complete the first two grids. Ask them think about how they could prove these. Some may be quite difficult to prove, but thinking about this could be useful preparation for Logarithm Lineup.

A version of this resource has been featured on the NRICH website. You might like to look at some students' solutions that have been submitted there.