Circular slice

Teacher notes

Why use this resource?

This resource asks students to make connections between the algebraic approach to an integral and the area it represents. The integral, which requires a trigonometric substitution, can also be found through geometrical reasoning, with students using inverse trigonometry and areas of sectors.

Possible approaches

There is a suggestion section which offers students some scaffolded questions to help them with their geometrical reasoning. One approach could be to use these questions as a warm-up before students tackle the main problem. This may be suitable if you feel that your students need support finding and using angles written as inverse trigonometric functions. (In this resource we have used the notation $\arccos$ rather than $\cos^{-1}$.)

Alternatively you could go straight to the main problem and use the scaffolded questions as and when they are appropriate. If you wish, you can print off a student worksheet which has the main problem on one side and the scaffolded questions on the other.

Key questions

- Why is it helpful to use radians to find the area of a sector?
- Are there other ways you could have calculated this integral?
- Can you think of other integrals that could be calculated using geometry?

Possible extension

The substitution $x = \cos\theta$ was chosen to make the geometry as straightforward as possible. What changes if $x = \sin\theta$ is used instead? Students could be asked to solve the integral with this substitution and see if they can also connect this answer to a geometrical approach to finding the area.