Parametric paths

Teacher notes

Why use this resource?

Students are given a set of curves and a table of parametric equations. Students have to match each curve with its parametric equations and sketch the two missing curves. Many of the curves can be placed in the table by noticing how key features eliminate other possibilities, but students should also be encouraged to convince themselves about why each pair of equations describes that particular curve. The layout of the table allows students to explore what happens when one of the parametric equations changes while the other is fixed. This may help to reinforce understanding of the role the parameter plays in determining the shape of the curve.

Preparation

Each pair or group of students will need copies of the curves and the table or access to devices for using the interactive card sort. (If students are using the interactive, they may like to export or print an image of their partially completed table so that they can sketch in the missing curves.) If students will be working in groups, an A3 copy of the table and larger cards would be useful.

Students may find mini-whiteboards helpful for sketching ideas.

Possible approach

To help students notice features of the curves, you could ask them to group the curves before introducing the table (space is available above the interactive table for this). Then ask students to group the cards in a different way. Ask them to make a note of how they have grouped them and what features they notice. Based on these features, students may be able to suggest possible parametric equations for either the x- or y-coordinates (see Possible support below). Alternatively, ask students to select two curves and describe one feature they share and one way in which they differ.

Students will probably get most out of this task if they work in pairs or threes. Joining up pairs into larger groups after an initial exploration period might be helpful to promote discussion and comparison of ideas. Encourage students to justify why they have placed cards in particular positions. The interactive table (displayed in full screen mode) could be used by students to share their ideas and reasoning in a full class discussion.

Key questions

Looking at the curves

- What do you notice about this curve?
- What can you conjecture about the parametric equations of this curve?

Looking at the table

- What can you say about the shape of the curve from this pair of parametric equations?
- What will be similar about all the curves in this row/column?
- How do x and y change as t changes?
- What is the range of x-values as t varies across its domain? What about the y-values?

Possible support

Some students may need support to see how they can use what they already know to get started.

Ask the students to group the curves.

- What criteria are they using?
- Can they identify a feature of the curves in the group and say something about their parametric equations (or x- and y-coordinates)?
- Does one of their groups look as if all the y-values fit $y = t^2$, or $y = 2\sin t$, or x-values fit with $x = \cos t$?

Ask the students to look at the middle column of the table. Can they find Cartesian equations for the curves defined by these parametric equations instead?

Possible extension

Ask students to experiment with combining parametric equations in different ways.

- What other curves can they come up with?
- Can they find anything surprising?
- Can they design a different table?