

## Why use this resource?

This problem introduces concepts linked to integration as students are challenged to place four cards in order of the size of the given shaded area. The concepts of area under a curve, over and under-estimation, and the trapezium rule can be introduced as students grapple with this problem. They should be encouraged to carefully consider the layout of their work as they may find that they need to reflect on results and improve their estimates several times in order to solve it fully.

## Preparation

Cards can be printed off to give to students.

## Possible approach

It can be useful to begin by asking students to place their four cards in order of increasing area at first glance, before allowing any time for calculation. They can then be asked to compare to others nearby and then given more substantial time to attempt to confirm an order.

It is really important to encourage students to keep track of their calculations (not just their final values) as they are likely to need to revisit and improve on previous estimates in order to confidently place all four cards in order. For this reason it may be better to encourage work on paper (rather than on mini-whiteboards).

Allowing students to access spreadsheet software (if they think it useful) may help them to refine their estimations for the areas on cards (c) and (d) and could be a useful starting point for conversations about the trapezium rule.

## Key questions

- Which card might you choose to think about first? Why?
- Can you calculate all of the areas exactly?
- How could you estimate the area of (c)? Do you think this will be an over-estimate or an under-estimate?
- What is the same / different about the curves on cards (c) and (d)? How might this affect your estimations?

## Possible support

Students could be directed to cards (a) and (b) which can be calculated exactly. Encouraging them to identify and label values such as side lengths of rectangles / trapezia on their own copies of the diagrams may be helpful.

When considering cards (c) and (d) students could be encouraged to think about dividing the area into rectangles and / or trapezia.

Recognising that a very rough estimate is acceptable as a starting point is important.

Placing cards (a) to (c) in order is much easier than doing so for the complete set of four cards.

## Possible extension

Students could move on to consider [Problem areas](https://undergroundmathematics.org/introducing-calculus/approximating-areas)