Can you find... series edition

Teacher notes

Why use this resource?

This resource gets students thinking about how they can construct examples of infinite series that meet certain constraints (all of which can be met either by geometric or arithmetic series). The problem could be used to check or consolidate understanding of convergence of geometric series, or it could be used to introduce series following work on sequences. Working on this problem may bring out the distinction between a sequence converging and a series converging.

Preparation

Copies of the Student work may be helpful for some students as they start to work on part (c). You might want to slice the print-out in two so students can think about the ideas in the first part before seeing the sketch graph.

Possible approach

Present the first three parts of the problem and ask students to spend a few minutes working on these individually. Mini-whiteboards may help to encourage students to try out examples of sequences and also to use sketches as part of their reasoning.

Then ask students to share their ideas in pairs or small groups. As well as discussing their reasoning, this may encourage students to generalise their ideas and think about modifying their series.

At this point the Now try this toggle could be revealed. Some students' series may already meet one or more of these extra conditions, in which case they could be asked to think about further constraints, or further examples of series that meet the original and extra constraints.

Key questions

- What does it mean to say that a series converges?
- Could you draw a sketch to represent your series?
- Could you modify a series you already know about?
- If the terms in a series get closer to zero as n increases, does this mean the series converges?
- Does increasing always mean getting further from zero?

Possible support

Students may find it helpful to start by thinking about sequences and then move on to series. Ask for any examples of sequences they can give. What about a sequence whose terms are all positive? Or all negative? Or a sequence where the terms alternate sign? Can they suggest a sequence whose terms are increasing but getting closer together? What happens if the terms in these sequences are added up? Access to a spreadsheet may be helpful to support exploration.

Some of the sequences and series in Square spirals could be used to remind students that infinite series can converge.

Possible extension

There are many ways to adapt the terms in a series, but how does this affect whether the series converges? Students could start to explore this using calculators, spreadsheets, or Desmos.