Sector spirals

Teacher notes

Why use this resource?

This resource provides an opportunity for students to practise calculating arc lengths and sector areas. Students who look at the problem as a whole before jumping in with calculations will notice patterns and relationships between the sectors that provide more efficient routes through the task.

Looking at the adapted diagrams encourages students to connect similar situations, thinking about what stays the same and what has changed. Hopefully they will notice the appearance of the special case where $r = 2$ for which arc length and sector area are equal for any angle.

Preparation

Students should know how to calculate the area and circumference of a circle.

It can be instructive for students to work in radians in order to reduce the formulae for arc length and sector area to $r\theta$ and $\frac{1}{2}r^2\theta$, but this is not assumed to be prior knowledge.

It might be useful to have one printout of the problem per pair of students. This could be folded so that only the first image and the instructions are visible.

Possible approach

Working in pairs, one student could be given a printed copy of the first image and instruction. Starting in this way will ensure that students have looked carefully at the image and begin to notice features that may help them to complete the rest of the problem.

Once this has been done, students should calculate the arc lengths and sector areas in the first image. Posing the question, "What do you notice?" once students have done this can prompt them to think more carefully about the calculations and values that they have just written down.

Only when students have had sufficient time to work with the first image should they move on to consider the second and third images.

The first image and the left-hand adapted image have many similar features so it is important that students meet all three images in order to understand how the calculations and properties associated with the sectors behave in all scenarios.

It is worth being aware that although it is less straight forward to interpret angles in the third diagram, there are still things that students can notice about the relationship between

arc length and sector area. There is no harm in students choosing to measure the angles in this diagram as a way to start noticing preserved properties.

Key questions

- How did you calculate the arc length and sector area? Can you think of another way of doing this?
- What stays the same and what is different when you compare each adapted image to the original?
- What is the ratio of arc length to sector area for each sector?
- Can you explain why the arc length and sector area are equal to each other for the sectors of radius 2?

Possible support

Ask students to draw each sector in isolation and to sketch the circle that it 'belongs to'. Encourage them to label the radius and sector angle and to think about how many of the sector will make a full circle.

Possible extension

Ask students to carefully consider the final questions posed in Things you might have noticed.