How not to solve a cubic...

Teacher notes

Why use this resource?

This resource provides an opportunity for students to demonstrate their understanding of the roots and coefficients of quadratic and cubics. Thinking about how quadratic and cubic roots can be related can be thought about in a variety of ways and so it is an ideal task for drawing out the connections between graphical and algebraic approaches.

Preparation

Students may benefit from having thought about writing a quadratic or cubic in terms of its roots, or from having thought about the factor theorem.

Possible approach

The warm-up section offers one entry route into the problem, by reinforcing the idea that the roots of a function correspond to where its graph meets the x-axis. Another route into it is to think about expressing a polynomial as a product of linear factors. These each provide a different perspective on the problem.

Key questions

- What does this problem mean graphically?
- In what different ways can you write a cubic equation?
- How can you generalise what you are doing?
- What conditions must the roots of the cubic satisfy?

Possible support

Some useful question may be:

- What do you know if, say, $x = 3$ is a common root?
- Is it easier to start from the cubic or from the quadratic?

Possible extension

- Can you find all cubics which have this property? Can you convince me that you have found them all?

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