$f'(x) = \lim_{h\to 0}\left(a^x\left(\dfrac{a^h-1}{h}\right)\right)$	The expression is factorised as a^x is a common factor.
$f'(x) = a^x \lim_{h\to 0}\left(\dfrac{a^h-a^0}{h}\right)$	As $h \to 0$ the limit will give the gradient function at $x = 0$.
$f'(x) = \lim_{h\to 0}\left(\dfrac{a^{x+h}-a^x}{h}\right)$	The expression in the limit gives the gradient of a chord between $x = 0$ and $x = h$ on $f(x) = a^x$.
$f'(x) = a^x \lim_{h\to 0}\left(\dfrac{a^h-1}{h}\right)$	This is the general formula for differentiating from first principles.
$f'(x) = a^x \times f'(0)$	This is an expression for the gradient of $f(x) = a^x$.
$f'(x) = \lim_{h\to 0}\left(\dfrac{f(x+h)-f(x)}{h}\right)$	As a^x is not affected by h it can be taken outside of the limit.