Tangent or normal

Teacher notes

Why use this resource?

This resource approaches tangents and normals in a slightly unusual way. The core activity of this resource is to match gradient functions (derivatives) with tangent/normal pairs, and from there to deduce the original functions. Along the way, students will have to think carefully about the meaning of a gradient function, and how it relates to the equations of tangents, normals and their points of intersection.

Students start in the Warm-up section by pairing up the equations of tangent and normal lines (though they are not introduced as such) and finding their points of intersection, which offers an opportunity to practise the coordinate geometry of straight lines.

Then in the Tangent or normal section, given a set of gradient functions, students must work backwards to determine which gradient function corresponds to which pair of lines, and further determine which line is the tangent and which is the normal. This encourages deep thinking about the relationships between these things.

Finally, in the Original curves section, the students are asked to recover the equations of the original curves, a task that requires the reverse of differentiation, and draws attention to the significance of the arbitrary constant.

Preparation

Cards can be used showing the equations of the tangent and normal lines to be matched. Single and double sets of cards are available.

Possible approaches

One way to use this resource is in the way presented: asking students to do the Warm-up task first, and then explaining what these lines are, continuing with the remaining sections.

Another approach would be to give the problem all in one go, giving the students the task of figuring out what to do. This will obviously be more challenging, and require more sophisticated problem-solving skills. In this guise, the problem might look something like this:

- Here are the equations of eight straight lines and the gradient functions of four curves. The eight straight lines are the tangent and normal at one point on each of the four curves. What are the equations of the four curves?

Another approach would be to give the equations of the eight straight lines, explaining that these are the equations of the tangents and normals to one point on each of four curves, and then ask the students what they might do with this information. This could lead to discussions about what other information they might want, the need to pair them up and value of finding the points of intersection.

Key questions

- What do you know about the gradients of a tangent and the corresponding normal?
- Can you sketch the lines and what the curve might look like at that point?
- If we had the equation of the curve, how would we find its gradient function? So what would the equation have to look like to give a gradient function like this?

Possible support

Students might need help to see the connection between the intersections of the lines and the possible curves.

Possible extension

Students could be asked to sketch the original curves and their tangent and normal lines.

They could be asked to design similar problems for one another, starting with the equation of a curve.