Proving half-angle formulae

Teacher notes

Why use this resource?

This resource provides a collection of diagrams that students can use to help them give a geometric proof of the formula $\cos^2\frac{\theta}{2} = \frac{1}{2}(1+\cos\theta)$. The same ideas can be adapted to give geometric proofs of results for $\sin^2\frac{\theta}{2}$ and $\tan^2\frac{\theta}{2}$.

Preparation

The diagrams for the first problem are available to print as cards. You might like to laminate them for re-use so that students can add any other labels they might need.

Alternatively, the problem page has an interactive set of cards, which students could use if tablets are available.

Mini-white boards (one per card, if available) or spare sheets of paper might be useful for students to combine their ideas about each card. Then the cards (with their accompanying notes) can be rearranged easily as students work towards a proof.

If the further problems in the Looking again section are to be used then print-outs of these diagrams would be helpful.

Possible approach

Students might spend a few minutes looking at the diagrams alone before discussing in pairs or threes how these could be used to prove the result. Encourage students to write down any relationships they see as equations using the labels on the cards.

Working in their small groups students can try rearranging the order of their ideas to create a coherent argument to prove (one of) the identities. The suggested proof uses two sequences of diagrams to get to different expressions for the length of the same line segment. This may be different from other proof-sorts students have used, where one clear sequence of cards may have been the aim.

Key questions

- What result are you aiming for? How could this be proved?
- What can you write down about this diagram?
- Do you know a relationship between these angles?
- Which sides do you think will be useful? Can you express them in terms of a marked angle?
- Do you know a relationship between these sides?

Possible support

Note that certain lines have been drawn as solid rather than dashed lines, and some angles have been marked in bold. It may be helpful to draw attention to these, as well as asking some of the following questions.

- What formulae or relationships about triangles do you know that might be useful?
- What formulae or relationships about circles do you know that might be useful?
- Can you write them down algebraically? - Can you apply them to any angles/ sides in these diagrams?

Possible extension

Can you think of a different way to use these diagrams to prove the same result?

A version of this resource has been featured on the NRICH website. You might like to look at some students' solutions that have been submitted there.