Inverse trigonometric function

underground mathematics

The *inverse trigonometric functions* are the inverse functions of the trigonometric functions and are either written as $\arcsin x$, $\arccos x$ and so on, or as $\sin^{-1} x$, $\cos^{-1} x$ and so on.

As none of the trigonometric functions are one-to-one (injective), we cannot take their inverses without first restricting their domains. The domains that are normally chosen give rise to the following inverse trigonometric functions.

Function	**Domain**	**Range**
$y = \sin^{-1} x$	$\lvert x\rvert \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
$y = \cos^{-1} x$	$\lvert x\rvert \leq 1$	$0 \leq y \leq \pi$
$y = \tan^{-1} x$	$x \in \mathbb{R}$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
$y = \operatorname{cosec}^{-1} x$	$\lvert x\rvert \geq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2},\ y \neq 0$
$y = \sec^{-1} x$	$\lvert x\rvert \geq 1$	$0 \leq y \leq \pi,\ y \neq \frac{\pi}{2}$
$y = \cot^{-1} x$	$x \in \mathbb{R}$	$0 < y < \pi$

The range given in the above table is sometimes called the *"principal value range"* of the inverse trigonometric function.

So while an equation such as $\tan x = 1$ has infinitely many solutions ($\frac{\pi}{4}$, $\frac{5\pi}{4}$, $-\frac{3\pi}{4}$ and so on), when we write $\tan^{-1} 1$, we mean the single value which lies in the principal value range, which in this case is $\frac{\pi}{4}$.