**Harvard School of Engineering and Applied Sciences — CS 152: Programming Languages**

**Assignment 5**

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Name: Sharanya Pulapura
HUID: 50983277
Collaborators: None

*Make sure that the remaining pages of this assignment do not contain any identifying information.*

## 1 Monad Laws (15 points)

First we will show left unit:

```
bind (return x) f = f x
```

We know that

```
return x  = Writer(x, mempty)
```

Therefore, we simplify the left side to show it is equal to the right side:

```
bind Writer(x, mempty) f =
    let Writer(x1, w1) = Writer(x, mempty) in
    let Writer(y, w2) = f x1 in
    Writer(y, mappend w1 w2)
```

Since Writer($x_1$, $w_1$) = Writer($x$, mempty), then $x = x_1$ and $w_1$ = mempty, and $f\ x_1 = f\ x$. Therefore, our final output is Writer($y$, $w_2$) since $w_1$ is empty, and we see that Writer($y$, $w_2$) = $f\ x_1 = f\ x$, then our entire expression is equivalent to $f\ x$, as desired.

Next we will show right unit:

```
bind xM return = xM
```

We'll again simplify the left to show it is equal to the right.

```
bind xM return =
let Writer(x, w1) = xM in
let Writer(y, w2) = return x in
Writer(y, mappend w1 w2)
```

We saw before that return x is Writer($x$, mempty), so Writer($y$, $w_2$) = Writer($x$, mempty) and $y = x$ and $w_2$ = mempty. So our final output is equivalent to

```
Writer(y, mappend w1 w2)
= Writer(x, mappend w1 mempty)
= Writer(x, w1)
```

and Writer($x$, $w_1$) we originally defined as xM in our expansion of bind xM return above, so the two sides are equal.

Finally, we will show that

```
bind (bind xM f) g = bind xM (\ x -> bind (f x) g)
```

We start by expanding the left side:

```
bind (bind xM f) g =
let Writer(x1, w1) = (bind xM f) in
let Writer(x2, w2) = g x1 in
Writer(x2, mappend w1 w2)
```

Expanding the second bind, we get

```
bind xM f=
let Writer(x3, w3) = xM in
let Writer(x4, w4) = f x3 in
Writer(x4, mappend w3 w4)
```

Which tells us

```
Writer(x1, w1) = bind xM f = Writer(x4, mappend w3 w4)
x1 = x4
w1 = mappend w3 w4
mappend w1 w2 = mappend (mappend w3 w4) w2
Writer(x2, mappend w1 w2) = Writer(x2, mappend (mappend w3 w4) w2)
```

Where the output is ultimately

```
Writer(x2, mappend (mappend w3 w4) w2)
```

Then we expand the right side:

```
bind xM (\x -> bind (f x) g) =
let Writer(x3, w3) = xM in
let Writer(x5, w5) =  bind (f x3) g)  in
Writer(x5, mappend w3 w5)
```

We've maintained that Writer($x_3$, $w_3$) = xM from the left side. Now we expand the second bind, keeping in mind that Writer($x_4$, $w_4$) = f $x_3$, Writer($x_2$, $w_2$) = g $x_1$, and that $x_4 = x_1$, so Writer($x_1$, $w_4$) = f $x_3$, and Writer($x_2$, $w_2$) = g $x_4$

```
bind (f x3) g =
let Writer(x4, w4) = f x3 in
let Writer(x2, w2) = g x4 in
Writer(x2, mappend w4 w2)
```

Combining these, we can see that:

```
Writer(x5, w5) = Writer(x2, mappend w4 w2)
x5 = x2
w5 = mappend w4 w2
Writer(x5, mappend w3 w5) = Writer(x2, mappend w3 (mappend w4 w2))
```

where the final output is

```
Writer(x2, mappend w3 (mappend w4 w2))
```

According to the associativity of monoids,

```
Writer(x2, mappend w3 (mappend w4 w2)) =
Writer(x2, mappend (mappend w3 w4) w2)
```

So therefore the left and right sides of the equation for the associativity of the Writer monad are equal, as desired.

## 2 Lambda-Print (15 points)

$$e ::= x \mid \lambda x.\, e \mid e_1\ e_2 \mid s \mid \mathsf{concat}\ e_1\ e_2 \mid \mathsf{output}\ e \mid \mathsf{unit}$$
$$v ::= \lambda x.\, e \mid s \mid \mathsf{unit}$$

$$\textsc{Str}\ \frac{}{s \Downarrow \langle s, \text{``''} \rangle}$$

$$\textsc{Unit}\ \frac{}{\mathsf{unit} \Downarrow \langle \mathsf{unit}, \text{``''} \rangle}$$

$$\text{APP}\ \frac{e_1 \Downarrow \langle \lambda x.\, e, s \rangle \quad e_2 \Downarrow \langle v, s' \rangle \quad e\{v/x\} \Downarrow \langle v_1, s'' \rangle}{e_1\ e_2 \Downarrow \langle v_1, s + +s' + +s'' \rangle}$$

$$\textsc{Lam}\ \frac{}{\lambda x.\, e \Downarrow \langle \lambda x.\, e, \text{``''} \rangle}$$

$$\textsc{Concat}\ \frac{e_1 \Downarrow \langle s_1, s_1' \rangle \quad e_2 \Downarrow \langle s_2, s_2' \rangle}{\mathsf{concat}\ e_1 e_2 \Downarrow \langle s_1 + +s_2, s_1' + +s_2' \rangle}\ s1, s2 \text{ are strings}$$

$$\textsc{output}\ \frac{e \Downarrow \langle s_1, s_2 \rangle}{\mathsf{output}\ e \Downarrow \langle \mathsf{unit}, s_2 + +s_1 \rangle}\ \text{s1 is a string}$$

## 3 Monadic Interpreter (45 points)

*I followed instructions and submitted* `eval.hs` *on Gradescope!*