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Minimizing the Integer Ambiguity Search Space for RTK

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Abstract: Differential GPS carrier phase measurements have much lower noise and multipath error than that of pseudorange measurements. The result is centimeter accuracy for Real-Time Kinematic (RTK). However, the measurement of the carrier phase has a constant unknown integer ambiguity. Several technical issues are related to solving the integer ambiguity correctly. They are: proper stochastic model, search space definition and initialization, search space reduction, state and standard deviation calculation, validation and rejection criteria for the unique and correct candidate. Search space reduction is critically important. It not only affects the ambiguity resolution speed, but also defines the ambiguity resolution success rate. The smaller the search space, the easier it is to find the unique and correct candidate set. The paper analyzes the integer ambiguity search space in its residual domain. The search space is minimized by: Analyzing the maximum independent integer ambiguity measurement set theoretically; Selecting the best initial measurement set that minimizes the search range of each satellite in the set and reduces the error effects from noise that may cause the wrong integer ambiguity solution for the remaining satellites not contained in the initial measurement set.

Since the Residual Sensitive Matrix (S-Matrix) relates the integer ambiguity candidate set directly to post-fix residuals, it is not necessary to compute a fix for each candidate set thus making the integer ambiguity search process much more efficient. Also, minimizing the search space in the residual domain improves the search efficiency significantly and at the same time improves its reliability. Performance issues, such as recursive and weighted search techniques as well as methods for improving reliability, are also discussed in the article.

Key words: Ambiguity Resolution, Real Time Kinematic (RTK), Residual Sensitivity Matrix, and Singular Value Decomposition.

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0 Introduction

GPS integer ambiguity resolution is the key step to utilize carrier phase measurements for high accuracy navigation. The objective of the GPS integer ambiguity resolution is to solve for the unknown number of integer carrier cycles biasing the phase measurement, so that the low noise carrier phase measurement can be used as a range signal.

There are various categories of applications that need GPS integer ambiguity resolution:

1) Short range vs. long range;
2) Static position vs. dynamic/kinematic position;
3) Real-time processing vs. post-processing;
4) Known vs. unknown baseline.

Differential carrier phase can be written as

$$\nabla\phi\lambda = \begin{bmatrix} \boldsymbol{h} & \lambda \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \mathrm{N} \end{bmatrix} + \mathrm{n}_{\phi} \tag{1}$$

with $\nabla\phi$ being the differential carrier phase, $\boldsymbol{h}$ the vector between the antenna of GPS receiver and the satellites, $\boldsymbol{x}$ being the linearized position, N being the integer ambiguity, and n_{ϕ} being the differential phase noise plus the multipath error.

Eqn. (1) can not be solved directly since there are (4+n) unknown variables for n single difference GPS carrier phase measurements and (3+n) unknown variables for n double difference GPS carrier phase measurements. Theoretically, eqn. (1) cannot be used directly as a linear measurement model to estimate the real vector x and the integer ambiguity N by common

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methods, such as the Least Square and the Kalman filter, due to the fact that N is an integer. Hence, the estimation problem, based on eqn. (1), is nonlinear.

Therefore, in practical applications, GPS integer ambiguity resolution is accomplished by

1) solving the integer ambiguity with special search and hypothesis testing techniques, and

2) validating the result to make sure the integer ambiguity solution is unique and correct.

To solve and validate the integer ambiguity, there are three categories of methods. They are:

1) Long duration static observation [12]: This method is used in static mode where x is unchanged. The reasons for long term observation are that long time observation data can average the multipath error and GPS receiver noise and that due to h varying slowing (caused by GPS satellite motion) approximately 20 minutes are required for h to change enough for the set of equations to yield observability. The long convergence time is the major drawback of this approach.

2) GPS antenna special moving: GPS antennae swapping is described in [11]. Swapping location of two antennae causes the observability properties of the h matrix to change rapidly, but is rarely possible in real time kinematic and long baseline.

3) Searching methods: These methods need few assumptions and have attracted the attention of many researchers. Numerous methods have been reported. They are: Ambiguity Function Method (AFM) [2], Fast Ambiguity Resolution Approach (FARA) [4], Least Squares Ambiguity Search Technique [3, 7], Cholesky Decomposition [10], Fast Ambiguity Search Filter (FASF) [1], Least Square AMBiguity Decorrelation Adjustment (LAMBDA) [13], and Integrated Ambiguity Resolution Method [6].

The first two categories of methods are straightforward. The basic theory and steps of GPS integer ambiguity resolution search methods are (see [14] for details):

1) Linear stochastic model definition: Both differential pseudorange and carrier phase measurements have the linear equation as eqn. (1).

2) Ambiguity resolution initialization: This defines the initial integer ambiguity set and its search range.

3) Search space reduction: The initial integer ambiguity, initial real states, and their search ranges define the whole search space. For instance, n differential GPS measurements form a n-dimension integer search space.

4) State and standard deviation calculation in the reduced search space: For each integer ambiguity candidate set in the reduced space, estimate the real states and calculate the residual and standard deviation to identify the unique ambiguity candidate set.

5) Validation and rejection criteria for the unique and correct candidate: The statistic test is based on the statistical hypothesis testing theory.

Among these issues, search space reduction is critically important. It not only affects the ambiguity resolution speed, but also defines the ambiguity resolution success rate. The smaller the search space, the easier to find the unique and correct candidate set. The paper is focused on minimizing the search space to improve the calculation efficiency, to shorten the initial ambiguity resolution time, and to yield better reliability.

1 Search Space Reduction

One approach, to reduce the number of integer candidates without missing the correct candidate, is to decrease the diagonal element values of the covariance matrix that determine the search range. Two terms affect the covariance matrix. They are H, determined by GPS satellite geometry, and R, formed by the measurement residue errors. GPS users can do nothing about the satellite geometry. Hence, the existing methods to decrease the covariance matrix value are: improving the GPS receiver measurement performance; combining L1 measurement and L2 measurement to suppress the pseudorange measurement noise for dual frequency GPS receiver, or using phase smooth code (such as, Hatch filter) to reduce the measurement noise; and, applying the integer inverse matrix transformation to decorrelate the double differential measurement and to reduce the integer ambiguity covariance element value [12].

Second approach is to increase the search step length by using a longer wavelength (combine L1 phase and L2 phase). For the same search space defined in meters, the longer the wavelength, the smaller the number of integer search candidates. However, the phase estimate of the long wavelength usually also amplifies the measurement noise.

Also, the search space can be greatly reduced by cutting the search dimensions. In most cases, the number of satellite in view is more than six. Cutting the search dimensions can greatly reduce the number of searching candidates. For example, if each search range of four integer ambiguities is 10 cycles, the total search ambiguity candidate set size is 10^4, while for seven integer ambiguities, the total search ambiguity candidate set size is 10^7. The idea to partition the differential GPS measurements into a primary measurement set and a secondary measurement set was first presented in [7]. The phase measurements of the primary set define the reduced search space, while the phase measurements of the secondary set are used to validate the candidates.

1.1 Combing L1 and L2 Measurement

After differential calculation, the L1 and L2 pseudorange and carrier phase can be written as

$$\frac{\nabla\rho_1}{\lambda_1} = \frac{f_1}{c}r + \frac{f_2}{c}I_a + \frac{f_1}{c}(MP_1 + \eta_1) \qquad (2)$$

$$\frac{\nabla\rho_2}{\lambda_2} = \frac{f_2}{c}r + \frac{f_1}{c}I_a + \frac{f_2}{c}(MP_2 + \eta_2) \qquad (3)$$

$$(\nabla\phi_1 + N_1) = \frac{f_1}{c}r - \frac{f_2}{c}I_a + \frac{f_1}{c}n_1 \qquad (4)$$

$$(\nabla\phi_2 + N_2) = \frac{f_2}{c}r - \frac{f_1}{c}I_a + \frac{f_2}{c}n_2 \qquad (5)$$

As described in [9], adding eqn. (2) to eqn. (3) and re-arranging yields:

$$(\frac{\nabla\rho_1}{\lambda_1} + \frac{\nabla\rho_2}{\lambda_2})\lambda_n = r + I_a + \frac{\lambda_n}{\lambda_1}(MP_1 + \eta_1) + \frac{\lambda_n}{\lambda_2}(MP_2 + \eta_2) \qquad (6)$$

with $\lambda_n = \frac{c}{f_1 + f_2}$ called the narrow lane. subtracting eqn. (5) for eqn. (4) and re-arranging yields:

$$(\nabla\phi_1 - \nabla\phi_2)\lambda_w = r + I_a - (N_1 - N_2)\lambda_w + \frac{\lambda_w}{\lambda_1}n_1 - \frac{\lambda_w}{\lambda_2}n_2 \qquad (7)$$

with $\lambda_w = \frac{c}{f_1 - f_2}$ called the wide lane.

Since the right hand sides of eqn. (7) and eqn. (6) are directly comparable, the difference of the two equations,

$$(\frac{\nabla\rho_1}{\lambda_1} + \frac{\nabla\rho_2}{\lambda_2})\lambda_n - (\nabla\phi_1 - \nabla\phi_2)\lambda_w = (N_1 - N_2)\lambda$$
$$\frac{\lambda_n}{\lambda_1}(MP_1 + \eta_1) + \frac{\lambda_n}{\lambda_2}(MP_2 + \eta_2) + \frac{\lambda_w}{\lambda_1}n_1 - \frac{\lambda_w}{\lambda_2}n_2 \qquad (8)$$

provides a basis for estimating the wide lane integer (N_1-N_2) with the ionospheric delay being cancelled, the carrier noise n_1 and n_2 being small, and the coefficients of the code noise being significantly less than one. The standard deviations of terms of the multipath and noise on each estimate of (N_1-N_2) produced by eqn. (8) is approximately 0.7 times those of L_1 or L_2. The correct integer ambiguity of the wide-lane phase formed by eqn. (8) can be solved more easily than that of L_1 or L_2. Once (N_1 - N_2) is determined, eqn. (7) is available for accurate positioning with the wide-lane carrier phase and for aiding in direct estimation of N_1 and N_2.

1.2 Phase Smoothed Code across Measurements' Samples

Given the GPS code and carrier phase measurements, it is natural to consider the methods of combining two measurements to achieve the higher-accuracy range information for the integer ambiguity round off process. One method is to regard the integer ambiguity as a real number, then try to solve the ambiguity as a part of the stochastic estimation process, which is summarized in Section 7.4.1 and Section of 7.4.2 of [3] and needs more computation for both the dynamic updating and covariance calculation. Another method is a decoupled implementation, which is called Hatch filter [9] and more efficient. Both of methods can be referred to as carrier-smoothed-code.

For the RTK, the decoupled phase-smoothed-code can be implemented by averaging the measurements based on the first row of eqn. [8], which can recursively reduce the multipath and noise errors across the measurement samples.

2 Search Space Reduction Based on the Residual Sensitive Matrix

The residual Sensitive Matrix (S-Matrix), defined and derived in [8] and re-written in Section 2.1, relates the integer ambiguity set to the quadratic measurement residual vector directly for the unique correct integer ambiguity set validation and detection. It is computationally efficient, in that a simple process is used to compute the residual vector without computing a position solution or the related middle adjustments. Position solutions are computed only when the residual vector indicates the particular integer ambiguity permutation to be the unique correct set.

This article details an improvement based on the Singular Value Decomposition (SVD) of the Residual Sensitivity Matrix to find the minimum search space. The technique not only improves the calculation efficiency and ambiguity resolution time, but also improves the reliability.

2.1 Ambiguity Search Space Definition and Validation Criteria

For each satellite, the differential measurement eqn.(1) can be re-written as:

$$(\nabla\phi_i + N_i)\lambda = \boldsymbol{h_i x} + n_{\phi i} \qquad (9)$$

If there are n satellites in the view, all the measurements can be written in array format as:

$$(\nabla\boldsymbol{\Phi}+\boldsymbol{N})\lambda=\boldsymbol{Hx}+\boldsymbol{n}_{\varphi} \tag{10}$$

where $\nabla\boldsymbol{\Phi}=[\nabla\phi_1,\nabla\phi_2,\cdots,\nabla\phi_n]^T$ is the differential carrier phase measurement vector formed by each satellite, $\boldsymbol{N}=[N_1,N_2,\cdots,N_n]^T$ is integer ambiguity vector formed by each satellite, $\boldsymbol{H}=[h_1,h_2,\cdots,h_n]^T$ is the measurement vector matrix from user to satellites, and $\boldsymbol{n}_{\varphi}=[n_{\phi 1},n_{\phi 2},\cdots,n_{\phi n}]^T$ is the carrier phase measurement noise vector formed by each satellite.

Using the pseudorange or carrier phase smoothed pseudorange, the initial ambiguity N_0 can be estimated by rounding off. If the search width of each satellite is δN, the total candidate set number is δN^{n-1}, with n being the number of satellites used. For instance, if $\delta N=4$ and n=7, the total search number is 4096. And for each candidate set, the real state is

$$\hat{\boldsymbol{x}}=[\boldsymbol{H}^T\boldsymbol{R}^{-1}\boldsymbol{H}]^{-1}\boldsymbol{H}^T\boldsymbol{R}^{-1}(\nabla\boldsymbol{\Phi}+\hat{\boldsymbol{N}}_{\boldsymbol{0}}+\boldsymbol{\Delta N})\lambda \tag{11}$$

where $\boldsymbol{R}=\begin{bmatrix}\sigma_1^2 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \sigma_n^2\end{bmatrix}$ is the measurement covariance matrix formed by the differential carrier phase noise with σ_i being the standard deviation of satellite i differential carrier phase noise, $\boldsymbol{\Delta N}=[\delta N_1,\delta N_2\cdots,\delta N_n]^T$ is the integer ambiguity vector formed from search width δN for each satellite.

The calculated phase range residual vector is:

$$\boldsymbol{\Delta_\Phi}=(\nabla\boldsymbol{\Phi}+\hat{\boldsymbol{N}}_0+\boldsymbol{\Delta N})\lambda-\boldsymbol{H}\hat{\boldsymbol{X}} \tag{12}$$

$$=(\boldsymbol{I}-\boldsymbol{H}[\boldsymbol{H}^T\boldsymbol{R}^{-1}\boldsymbol{H}]^{-1}\boldsymbol{H}^T\boldsymbol{R}^{-1})(\nabla\boldsymbol{\Phi}+\hat{\boldsymbol{N}}_0+\boldsymbol{\Delta N})\lambda$$

$$=\boldsymbol{S}(\nabla\boldsymbol{\Phi}+\hat{\boldsymbol{N}}_0+\boldsymbol{\Delta N})\lambda \tag{13}$$

with

$$\boldsymbol{S}=\boldsymbol{I}-\boldsymbol{H}[\boldsymbol{H}^T\boldsymbol{R}^{-1}\boldsymbol{H}]^{-1}\boldsymbol{H}^T\boldsymbol{R}^{-1} \tag{14}$$

which was defined in [8]. The estimated phase standard deviation for candidate set $\hat{N}$ is:

$$\sigma_{\Phi|\hat{N}}=\sqrt{\frac{\Delta_\Phi{}^T\Delta_\Phi}{n-k}}$$

with k being the real state number of x (k=4 for single differential GPS and k=3 for double differential GPS).

The target of the ambiguity search is to find the unique and correct candidate set with smallest phase standard deviation. Since $\boldsymbol{\Delta_\Phi}$ is a vector, minimizing the phase standard deviation is equal to minimize the absolute value of each term of $\boldsymbol{\Delta_\Phi}$.

2.2 Property of S Matrix

The S matrix has many nice properties, such as symmetric, zero sum of each row and column ($\sum_{i=1}^{n} s_{ij}=0$, $\sum_{j=1}^{n} s_{ij}=0$), and positive semidefinite.

The following properties are useful for the search space reduction derivation described below:

1) Equal idempotent: $\boldsymbol{S}=\boldsymbol{S}^2=\boldsymbol{S}^3=\cdots$;
2) Rank equal to n-k: *rank*(S)=n-k;
3) Calculation efficiency of Single Value Decomposition (**SVD**) of S matrix: For SVD of $\boldsymbol{S}=\boldsymbol{UXV}^T$, one of the solutions of $\boldsymbol{V}$ is equal to the eigenvector of $\boldsymbol{S}$. The eigenvalue of $\boldsymbol{S}$ is either 1 or 0, and its eigenvectors are real.

It is easy to show the above properties. To make the statement clear without being bothered by mathematical derivation, they are omitted here.

2.3 Derivation Based on Single Value Decomposition for Search Space Reduction

Supposing

$$\boldsymbol{\Delta}_{\Phi_0}=\boldsymbol{S}(\nabla\boldsymbol{\Phi}+\hat{\boldsymbol{N}}_0)\lambda, \tag{16}$$

minimizing the absolute value of $\boldsymbol{\Delta}_{\Phi_0}$ in eqn (3). is equal to estimate $\boldsymbol{\Delta N}$ that

$$\boldsymbol{\Delta}_{\Phi_0}+\boldsymbol{S\Delta N}\lambda=0$$

$$\Rightarrow\quad \boldsymbol{S\Delta N}=-\boldsymbol{\Delta}_{\Phi_0}/\lambda=-\boldsymbol{R}_0 \tag{17}$$

with $\boldsymbol{R}_0$ in unit of cycle.

Since $\boldsymbol{S}$ is not full rank, $\boldsymbol{S}$ can be re-written by using Singular Value Decomposition (**SVD**) as

$$\boldsymbol{S}=\boldsymbol{UXV}^T, \tag{18}$$

where

$$\boldsymbol{U}=[\boldsymbol{u}_1,\boldsymbol{u}_2\cdots\boldsymbol{u}_n],$$

$$\boldsymbol{u}_i{}^T\boldsymbol{u}_i=1,\quad \boldsymbol{u}_i{}^T\boldsymbol{u}_j=0\quad (i\neq j) \tag{19}$$

with $\boldsymbol{u}_i$ being orthonormal vectors and $\boldsymbol{U}$ having full rank n;

$$\boldsymbol{X}=\begin{bmatrix}\begin{matrix}\sigma_1 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \sigma_{n-4}\end{matrix} & \boldsymbol{0}_{(n-4)\times 4}\\ \boldsymbol{0}_{4\times(n-4)} & \boldsymbol{0}_{4\times 4}\end{bmatrix} \tag{20}$$

with $\sigma_1 = \cdots = \sigma_{n-4} = 1$ for matrix S; and

$$\boldsymbol{V} = [\boldsymbol{v}_1, \boldsymbol{v}_2 \cdots \boldsymbol{v}_n],$$

$$\boldsymbol{v}_i^{T}\boldsymbol{v}_i = 1, \quad \boldsymbol{v}_i^{T}\boldsymbol{v}_j = 0 \quad (i \neq j) \tag{21}$$

with $\boldsymbol{v}_i$ being orthonormal vectors and **V** having full rank n.

To estimate **S** that $\boldsymbol{S\Delta N} = -\boldsymbol{R}_0$, we have

$$\boldsymbol{UXV}^{T}\boldsymbol{\Delta N} = -\boldsymbol{R}_0$$

$$\Leftrightarrow \boldsymbol{XV}^{T}\boldsymbol{\Delta N} = -\boldsymbol{U}^{T}\boldsymbol{R}_0 = \boldsymbol{R}_1$$

$$\Leftrightarrow \begin{bmatrix} \sigma_1 \boldsymbol{v}_1^{T} \\ \vdots \\ \sigma_{n-4}\boldsymbol{v}_{n-4}^{T} \\ \boldsymbol{0}_{4xn} \end{bmatrix} \boldsymbol{\Delta N} = \boldsymbol{R}_1 \tag{22}$$

Eqn. (11) can be re-written as

$$\begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{A}_2 \\ \boldsymbol{0}_{(n-4)x(n-4)} & \boldsymbol{0}_{4x4} \end{bmatrix} \begin{bmatrix} \boldsymbol{N}_{1f} \\ \boldsymbol{N}_{2f} \end{bmatrix} = \begin{bmatrix} \boldsymbol{R}_{11} \\ \boldsymbol{R}_{12} \end{bmatrix} \tag{23}$$

$$\Leftrightarrow \boldsymbol{A}_1\boldsymbol{N}_{1f} + \boldsymbol{A}_2\boldsymbol{N}_{2f} = \boldsymbol{R}_{11}$$

$$\Leftrightarrow \boldsymbol{N}_{1f} = \boldsymbol{A}_1^{-1}(\boldsymbol{R}_{11} - \boldsymbol{A}_2\boldsymbol{N}_{2f}) \tag{24}$$

$$\Rightarrow \boldsymbol{N}_1 = \text{round}(\boldsymbol{N}_{1f}) \tag{25}$$

After calculating the integer $\boldsymbol{N}_1$ and $\boldsymbol{N}_2$, substituting them into eqn. (13) yields

$$\boldsymbol{R} = \boldsymbol{R}_{11} - (\boldsymbol{A}_1\boldsymbol{N}_1 + \boldsymbol{A}_2\boldsymbol{N}_2). \tag{26}$$

which is the residual corresponding to integer set $\boldsymbol{N}_1$ and $\boldsymbol{N}_2$.

It can be shown that:

1) *norm*($\boldsymbol{R}_0$) = *norm*($\boldsymbol{R}_{11}$) and minimizing the norm value of $\boldsymbol{R}$ is equal to minimize the norm value of $\boldsymbol{R}_0$;
2) $\boldsymbol{N}_2$ is formed by four satellites for single differential GPS. However, searching around three satellites is enough in the whole search space.

2.4 Further Calculation Improvement for Implementation

Though the calculation of **SVD** is relatively efficient, a better way for implementation is shown below. Multiplying **S** on both side of eqn. (17) and applying Property 1 of **S** matrix yield

$$\boldsymbol{S\Delta N} = -\boldsymbol{SR}_0$$

$$\Rightarrow \boldsymbol{S}(\boldsymbol{\Delta N} + \boldsymbol{R}_0) = \boldsymbol{0}. \tag{27}$$

From Property 2 of $\boldsymbol{S}$ matrix, there are (n-k) rows of $\boldsymbol{S}$ matrix that are linearly independent. Selecting (n-k) rows and rearranging the $\boldsymbol{S}$ with first (n-4) rows independent, eqn.(27) can be rewritten as

$$\begin{bmatrix} \boldsymbol{S}_{11} & \boldsymbol{S}_{12} \\ \boldsymbol{S}_{21} & \boldsymbol{S}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Delta N}_1 + \boldsymbol{R}_{01} \\ \boldsymbol{\Delta N}_2 + \boldsymbol{R}_{02} \end{bmatrix} = \boldsymbol{0} \tag{28}$$

with $\boldsymbol{S}_{11} \in \Re^{(n-4)x(n-4)}$, $\boldsymbol{S}_{12} \in \Re^{(n-4)x4}$, $\boldsymbol{S}_{21} \in \Re^{4x(n-4)}$, $\boldsymbol{S}_{22} \in \Re^{4x4}$, $\boldsymbol{\Delta N}_1 \in \Re^{(n-4)x1}$, and $\boldsymbol{\Delta N}_2 \in \Re^{4x1}$. From eqn. (28), we have

$$\boldsymbol{S}_{11}(\boldsymbol{\Delta N}_1 + \boldsymbol{R}_{01}) + \boldsymbol{S}_{12}(\boldsymbol{\Delta N}_2 + \boldsymbol{R}_{02}) = \boldsymbol{0} \tag{29}$$

$$\Rightarrow \boldsymbol{\Delta N}_{1f} = -\boldsymbol{T}(\boldsymbol{\Delta N}_2 + \boldsymbol{R}_{02}) - \boldsymbol{R}_{01} \tag{30}$$

with

$$\boldsymbol{T} = \boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{12}. \tag{31}$$

Hence, the ambiguity search space is formed by $\boldsymbol{\Delta N}_2$ subset, while $\boldsymbol{\Delta N}_1$ can be calculated based on eqn. (30) and its integer value is:

$$\boldsymbol{\Delta N}_1 = \text{round}(\boldsymbol{\Delta N}_{1f}). \tag{32}$$

The corresponding residual of each candidate set, formed by searching around $\boldsymbol{\Delta N}_2$ and the calculated $\boldsymbol{\Delta N}_1$ directly, is

$$\boldsymbol{\Delta}_{\Phi 1} = \boldsymbol{\Delta N}_1 - \boldsymbol{\Delta N}_{1f} \tag{33}$$

Hence, the residual vector for all the satellites is

$$\boldsymbol{\Delta}_{\Phi} = \boldsymbol{S}_1\boldsymbol{\Delta}_{\Phi 1} \tag{34}$$

with

$$\boldsymbol{S} = \begin{bmatrix} \boldsymbol{S}_{11} \\ \boldsymbol{S}_{21} \end{bmatrix}. \tag{35}$$

The corresponding phase standard deviation for the candidate set can be calculated from eqn. (15), which is used to validate the correct and unique integer candidate set.

2.5 Apply Transformation on *S* Matrix for Decorrelation

As we have noted that the bigger the diagonal value of the $\boldsymbol{S}$ matrix, the more sensitive to yield the right integer ambiguity. Also, the off-diagonal values of the

$\boldsymbol{S}$ matrix show the correlation among the integer ambiguities for the residual vector.

If there were an normal matrix $\boldsymbol{T}_{\mathrm{r}} \in \Re^{\mathrm{nxn}}$ that

$$\boldsymbol{T}_{\mathrm{r}}\boldsymbol{S} = \boldsymbol{S}_{\mathrm{diag}} \tag{36}$$

with $\boldsymbol{S}_{\mathrm{diag}}$ being diagonal, multiplying $\boldsymbol{T}_{\mathrm{r}}$ on both sides of eqn. (17) yields

$$\boldsymbol{T}_{\mathrm{r}}\boldsymbol{S}\boldsymbol{\Delta}\boldsymbol{\Delta} = -\boldsymbol{T}_{\mathrm{r}}\boldsymbol{R}_0$$

$$\Rightarrow \boldsymbol{S}_{\mathrm{diag}}\boldsymbol{\Delta N} = \boldsymbol{R}_0^{'} . \tag{37}$$

In this case, $\boldsymbol{\Delta N}$ can be solved directly based on one-to-one corresponding. However, due to $\boldsymbol{S}$ being not full rank, $\boldsymbol{T}_{\mathrm{r}}$ does not exist. A sub-optimal solution can be used to facilitate the search.

2.6 Satellite Selection

The criteria of selecting the satellites to search around is of interest. The objective is to minimize the measure error (composed by carrier phase noise and multipath error) that affects the estimate of N_1. The measure error is indicated in T. Therefore, the target is to select the satellites that minimize the element value of matrix $\mathrm{S}_{11}{}^{-1}$ in (n-k)x(n-k) dimensions to decrease the measurement error effect for **N** estimation. This is easy to define, since S_{11} is always positive definite. One of the sub-optical solution is to find the satellite combination that can maximize the norm of S_{11}, which will minimize the element absolute value of T.

2.7 Calculation Efficiency

The approach applies for both L_1, L_2, Lw, and L_{n}. For each search epoch, the approach needs to calculate $\boldsymbol{S}$ matrix, split $\boldsymbol{S}$ matrix, and calculate $\boldsymbol{T}$ matrix once. The total candidate set number of this approach is $\delta\mathrm{N}^3$. For example, if n=7, m=3, and $\delta\mathrm{N} = 4$, the total search number is less than 64.

2.8 Recursive Calculation

For the RTK, solving the integer ambiguity with one epoch is desired. However, due to the $\boldsymbol{S}$ matrix being singular, availability of satellites, and all kinds of noise, there is the possibility of failure for a one-epoch solution. Therefore, recursive calculation is required for both integer ambiguity resolution and ambiguity verification. This is especially important when the geometry of the satellites is bad and the baseline is long.

Eqn. (17) can be re-written as

$$\boldsymbol{S}_{\mathrm{k}}\boldsymbol{\Delta N}_{\mathrm{k}} = \boldsymbol{R}_{\mathrm{k}} \tag{38}$$

for recursive processing. With an initial value for $\boldsymbol{\Delta N_0}$, the recursive calculation is

$$\boldsymbol{\Delta N}_{\mathrm{k}} = \boldsymbol{\Delta N}_{\mathrm{k-1}} + \boldsymbol{K}_{\mathrm{k}}(\boldsymbol{R}_{\mathrm{k}} - \boldsymbol{S}_{\mathrm{k}}\boldsymbol{\Delta N}_{\mathrm{k-1}}) \tag{39}$$

with $\boldsymbol{K}_{\mathrm{k}}$ being the gain of the recursive least square.

Given the assumption that the gain $\boldsymbol{K}_{\mathrm{k}} = \boldsymbol{I}$ is an identity matrix for quick response based on the fact that the carrier phase noise is relatively small and the uncertainty of the ambiguity may be several cycles, eqn. (39) can be re-written as

$$\boldsymbol{\Delta N}_{\mathrm{k}} = \boldsymbol{R}_{\mathrm{k}} + \boldsymbol{B}_{\mathrm{k}}\boldsymbol{\Delta N}_{\mathrm{k-1}} \tag{39}$$

Where

$$\boldsymbol{B}_{\mathrm{k}} = \boldsymbol{H}_{\mathrm{k}}[\boldsymbol{H}_{\mathrm{k}}^{\mathrm{t}}\boldsymbol{R}_{\mathrm{k}}^{-1}\boldsymbol{H}_{\mathrm{k}}]^{-1}\boldsymbol{H}_{\mathrm{k}}^{\mathrm{T}}\boldsymbol{R}_{\mathrm{k}}^{-1} \tag{41}$$

with $\boldsymbol{H}$ and $\boldsymbol{R}$ being defined above.

3 Conclusion

This article presents a methodology of fast ambiguity resolution for RTK by minimizing the search space in the residual domain for each epoch, which provides the search basis for instantaneous ambiguity resolution. The residual sensitive matrix relates the integer ambiguity set to the quadratic measurement residual vector directly for the unique correct integer ambiguity set validation and detection which is computationally efficient. Minimizing the search space based on the residual sensitive matrix not only improves the calculation efficiency and ambiguity resolution time, but also improves the reliability, especially for the RTK embedded with in a GPS receiver with limited capability of calculation.

Related issues, such as calculation improvement, decorrelation, and recursive calculation have also been discussed.

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