Estimating Network Connectedness among S&P 500 Stocks using Machine Learning Techniques

Pushpak Sarkar
University of Washington, Department of Economics

Objective

- Typical financial assets are correlated with each other. But the degree of the correlation varies. This correlation structure acts as an important input for the construction of an optimal portfolio.

- When we are dealing with N financial assets, we need to estimate in total N(N-1)/2 parameters in the correlation/covariance matrix. For large N, this can be a substantial computational requirement. Different machine learning techniques (supervised / unsupervised along with regularization) can be beneficial.

- To estimate the network connectedness and correlation structure, following techniques have been used – 1) LASSO regreesion, 2) Correlated graphical LASSO, 3) Kmeans clustering

Data

- Returns data of stocks in the S&P 500 Index
- Data source: www.kaggle.com
- Frequency: Daily Span: 2013-2018
- Number of stocks: 443 No. of trading days: 1258
- Volatility of each stock is estimated using GARCH (1, 1)

LASSO Regression

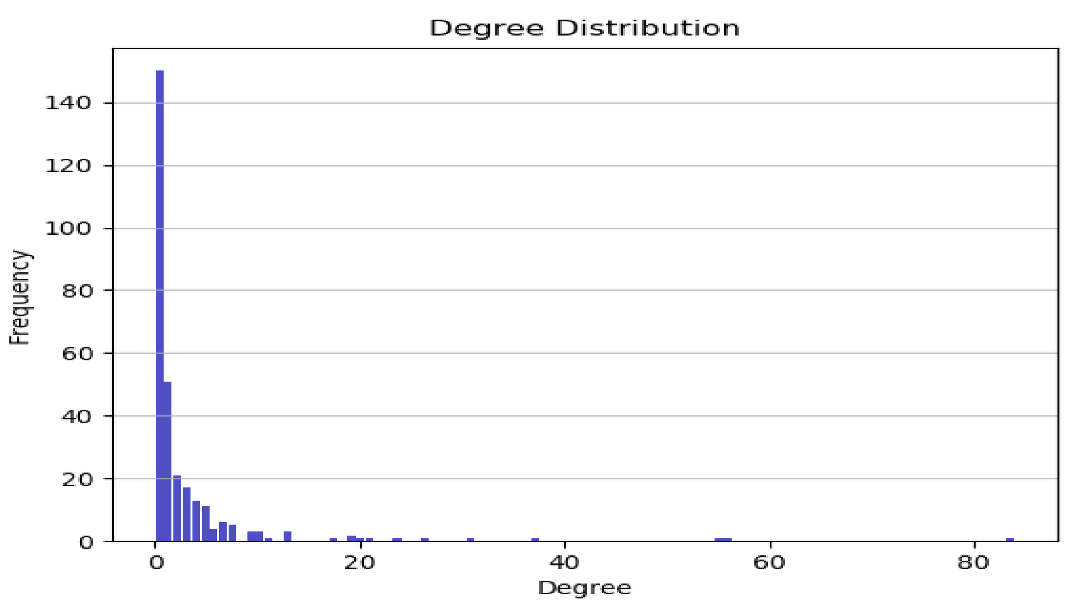
- Model – Firm i's current return is a function of the lagged returns of all firms –

$$r_{i,t} = \beta_{i,1} r_{1,t-1} + \beta_{i,2} r_{2,t-1} + \ldots + \beta_{i,i} r_{i,t-1} + \ldots + \beta_{i,j} r_{j,t-1} + \ldots + \beta_{i,N} r_{N,t-1}$$

- Train and test set (1000 and 258 observations)
- L1 regularization, cross validation k = 5
- Coefficient matrix – 443 X 443 (Sparse)
- Adjacency matrix $A_{i,j}$: $a_{i,j} = 1$ if $\beta_{i,j} \neq 0$ i.e. there is an edge from node j to i
- Out degree of a node – number of edges from that node

Table 1: Summary statistics of degree of nodes

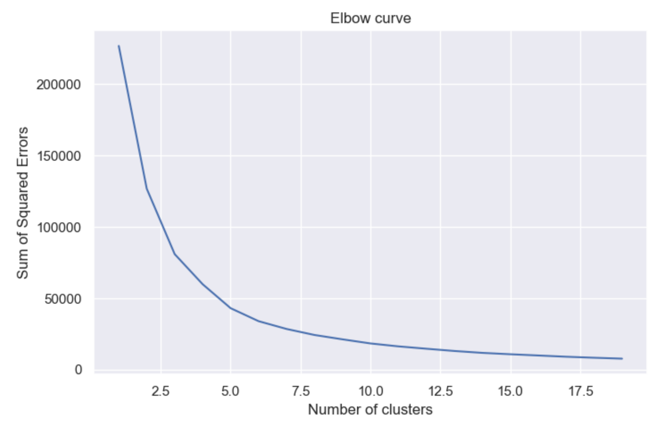
Mean degree of nodes	2.91
Std. dev of degree of nodes	7.92
Minimum of degree of nodes	0
Maximum of degree of nodes	84

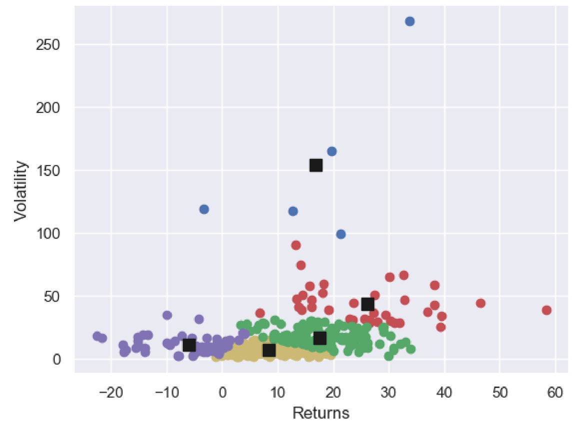


- Most of the firms have degree less than 10
- Histogram skewed to he right
- Firms in the Energy and Gas sector tend to have high degree of connectivity
- Lack of any other meaningful evaluation criteria other than correctly identifying firms belonging to the particular sector

Clustering: kmeans

- Unsupervised learning
- Features – returns and volatility of 443 firms
- Number of clusters – 5





Sectors	Maximum % of firms in a single cluster	Maximum % of firms in two clusters
Communication Services	33%	61%
Consumer Discretionary	48%	72%
Consumer Staples	72%	97%
Energy	67%	93%
Financials	57%	97%
Health Care	38%	73%
Industrials	42%	81%
Information Technology	52%	87%
Materials	43%	71%
Real Estate	41%	72%
Utilities	78%	93%

Graphical LASSO

- Undirected graphical model.
- Estimate correlated graphical structure using regularization technique.

Estimate covariance matrix using Graphical LASSO suggested by Friedman, Hastie, Tibshirani (2007)

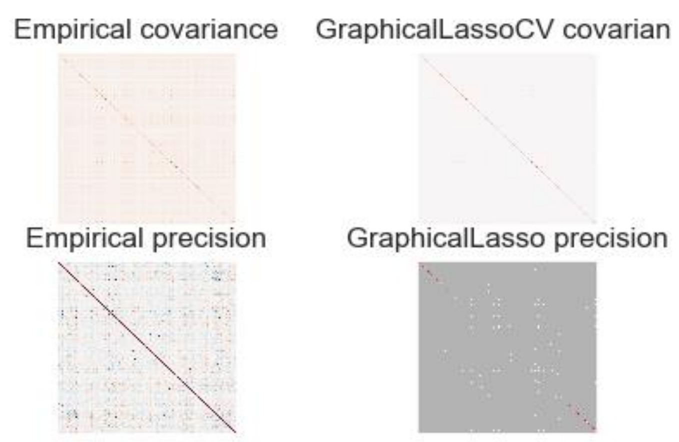
- Inverse covariance matrix or precision matrix contains information about partial covariances between variables
- Partition X = (Z, Y) where $Z = (X_1, X_2, \ldots, X_{p-1})$ and $Y = X_p$

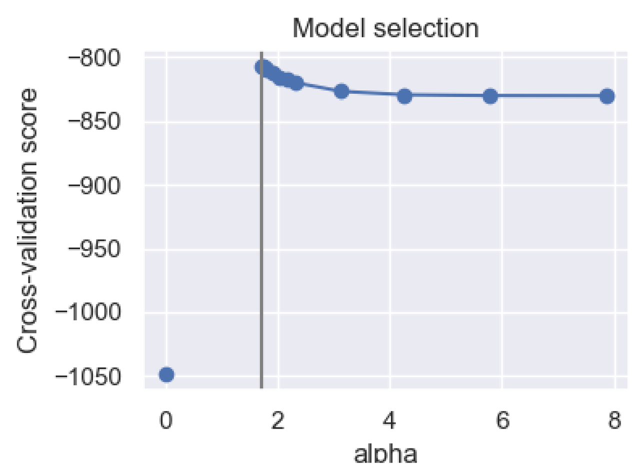
$$Y|Z = z \sim N(\mu_Y + (z - \mu_Z)^T \Sigma_{ZZ}^{-1} \sigma_{ZY}, \sigma_{YY} - \sigma_{ZY}^T \Sigma_{ZZ}^{-1} \sigma_{ZY})$$

$$\Sigma = \begin{bmatrix} \Sigma_{ZZ} & \sigma_{ZY} \\ \sigma_{ZY}^T & \sigma_{YY} \end{bmatrix}$$

Maximize the penalized log-likelihood of the data using a LASSO penalty.

$$\log(\det) \Sigma^{-1} - \operatorname{trace}(S\, \Sigma^{-1}) - \lambda \, ||\Sigma^{-1}||_1$$





Application – Portfolio Optimization

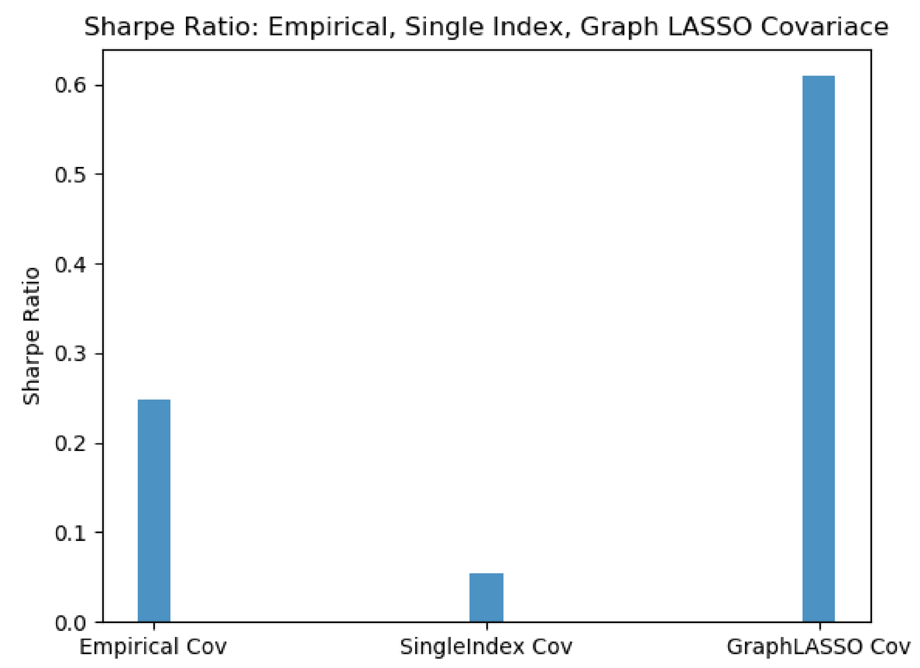
- Once the inverse covariance matrix is estimated, we can directly use it in forming an optimal portfolio following the Markowitz's (1952) mean variance portfolio optimization theory.
- If r_i is the return on asset *i* and wi is the weight of asset *i* in the portfolio, then portfolio return variance is given by $\sigma^2_P = \sum_i w_i^2 \sigma^2_i + \sum_i \sum_{i \neq j} \sigma_i \sigma_j \rho_{ij}$ where ρ_{ij} is the correlation coefficient between returns on asset *i* and *j*
- Portfolio weights are computed by minimizing the variance of the portfolio given the constraint that all weights sum up to 1.
- Minimize portfolio variance ½ $w^T \Sigma w$ subject to $\sum w_i = 1$
- This optimal portfolio is called Global Minimum Variance Portfolio.

Application: Portfolio Optimization

- To evaluate the results, an optimal portfolio based on the empirical covariance matrix S is also computed.
- Use optimal weights estimated from the training set to compute the Sharpe Ratio (SR) = Portfolio return/Portfolio standard deviation or risk adjusted return.
- The Sharpe Ratio of the portfolio constructed from the sparse inverse (using GLASSO) covariance matrix is almost six-times higher than that from the portfolio based on empirical covariance matrix.

Single Index Model

- The return of an asset i is a function of the market return.
- Model: $r_{i,t} = \alpha_i + \beta i \, r_{M,t} + \varepsilon_i$
- Estimate the model and get residuals.
- Estimate the covariance matrix from residuals



- Correlated graphical LASSO perform better
- One can ensure a higher risk adjusted return which is basically the objective of any rational investor.
- Most of the financial variables behave in non-linear fashion. So Neural Network may be worth exploring.

Reference

[1] Diebold, Francis X & Yilmaz, Kamil (2014) Financial and Macroeconomic Connectedness: A Network Approach to Measurement and Monitoring Financial and Macroeconomic Connectedness: A Network Approach to Measurement and Monitoring.
[2] Barigozzi, Matteo & Brownlees, Christian T. (2013) NETS: Network Estimation for Time Series.
[3] Friedman, Jerome, Hastie, Trevor & Tibshirani, Robert (2007) Sparse inverse covariance estimation with the graphical lasso.
[4] Heaton, J.B., Polson, Nick & Witte, Jan (2018) Deep Learning for Finance: Deep Portfolios.
[5] Polson, Nicholas & Sokolov, Vadim (2017) Deep Learning: A Bayesian Perspective.
[6] Markowitz, Harry (1952) Portfolio Selection
[7] https://scikit-learn.org/stable/auto_examples/covariance/plot_sparse_cov.html