# Genetic Fuzzy Pareto: Distributed Framework for Multi-Agent Optimization within Uncertain Environment

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***Abstract*—In this paper, a new concept for solving multi-agent optimization problem with genetic fuzzy pareto has been introduced. The solution is generated by genetic fuzzy pareto and Stackelberg game is introduced which uses a leader-follower concept to solve the distributed problem. Further, the changeability valuation is offered to such class of problems. The approach has been extended to multi-agent problems with uncertainties for proving a robust decision making process. The stability of the system is evaluated by introducing perturbations. The process of decision making has been introduced theoretically and the claims have by verified by simulation example of a simple trajectory planning multi-agent problem. During simulation, the scenario of presence of uncertainties in the environment have been considered.**

## I. Introduction

All engineering systems involve some degree of uncertainty in their design, development and operation which is why operational tolerances are always present in the system. Mathematical modeling and simulation is the first step to understand the dynamics and thus design control law for the operation of any dynamic system. But, it is very difficult to have a hundred percent accurate mathematical model of the system because of the presence of uncertainties. Fuzzy systems are termed as universal approximators because of their ability to approximate any real continuous function to a great degree of arbitrary accuracy [7]. They possess a robust behavior and have been widely utilized in several decision making engineering applications where uncertainty is present and a very precise mathematical model can not be obtained [7]. Every engineering system possesses an objective function and a set of constraints, the system must operate in the bounded envelop of those constraints. It is a general optimization problem and the objective of the systems is to minimize the cost and time of operation and attain a final desired state via a smooth transient. In complex machines such as aerospace vehicles, there can be multiple engineering subsystems acting simultaneously. Each system is driven by their own objective function and operational constraints e.g., there would be a set of several control surface actuators in the aircraft. The dynamics of the aircraft are very non-linear as well as coupled. In a closed loop system, the application of one actuator will affect the performance of other control surface actuator to some extent. It scales up the level of the optimization problem and there would be multiple objective function to be optimized and constraints to be satisfied simultaneously which may lead to operational instability of the system.

Such class of problems are categorized as multi-objective optimization problem which is an extended generalization of traditional optimization problems [23]. The solution to multi-objective optimization problems are termed as Pareto optimal solutions as decision maker finds the best possible solutions that satisfy or create a compromise among the multiple objective functions of the system [23] [18]. Pareto optimal solutions are assessed based upon the Pareto efficiency and it is impossible to assign resources to one agent without disturbing and deteriorating the optimal conditions of other agents of the multi-objective optimization problem.

In this paper, the proposed multi-agent optimization problem is solved using genetic fuzzy pareto. The Stackelberg game is introduced in the problem which uses a leader-follower concept to solve the distributed problem. The leader follows its policy to maximize the payoff by tracking the followers response. The roles of the players are assumed to be asymmetric. There have been a number of efforts to solve the Stackelberg game theory problems such as Q-learning [11], SARSA algorithm [2] or backward induction method [1]. Here, the problem is solved by using a novel method of Genetic-Fuzzy Inference System (GFIS). The game theory controller is found by the Mamdani fuzzy inference system. Membership functions and rules are tuned using the Genetic Algorithm. A detailed discussion to these concepts have been provided in later sections of the paper. The problems involving multi-agent systems with uncertainties have been considered for proving a robust decision making. The process of decision making has been introduced theoretically and the claims have by verified by simulation example of a simple trajectory planning multi-agent problem. During simulation, the scenario of presence of uncertainties in the environment have been considered.

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### A. Motivation

Decision making and avoiding the risk by a group of network in multi-agent system plays a huge roll in *robustness* of a system. *Changeability* in this paper refers to the margin of safety and avoiding risk by all the agents such that every agent has the capability to alter its form or behavior. The inclusion of this term, for multi-agent optimization typically comes with the costs over calculation and decision making but the benefits of having this term inside the system can be summarized into the robustness of decision making in presence of unexpected or dynamic environment, which from this point in this paper are referred to uncertainties. Not considering of uncertainties is often costly and leads to instability, chaotic behavior of the system resulting into collapse. Each of the agents reports its decision to the others using the perturbation.

### B. Contribution

This paper extends the previous research by introducing Genetic Fuzzy Pareto optimization method for multi-agents dealing with the uncertainties of the design to aim for the robust decision making of the agents. Furthermore, during this paper, by completely analyzing the perturbation, and implementation of Genetic Fuzzy Pareto, the stability of the system has been studied. At the end, it is tried to solve a simple trajectory planning problem, the applicability of this method has been provided.

This paper is going to proceed with these sections:

## II. Perturbation and Changeability

Changeability in this context refers to the an abstract parameter that gives the flexibility to the agent to change its decision while changing the cost functions upon changing the boundaries of the solution in the environment. The difference between the introduced parameter and the same parameter introduced in other literature is that changeability is absolutely going to increase upon changing in the environment but here this parameter would cause fluctuation in the cost function upon changes in the environment or facing the uncertainties. This parameter has been referred to *flexibility* [22] or a freedom in design parameter in [8].

Here there are basic difference between the flexibility, adaptability and changeability in the systems defined in this paper and the other literature. The changeability framework that has been defined here refers to the allowed changing in the decision based on the constraints applied to the agent. In Ref. [20], this parameter divided by the transition rule and is achieved through *modifiable, robustness and scalability*. In this paper, the implementation of *changeability* can be defined using Figure 1.

Changing in changeability is the result of different behaviors: i) change in the environment causes change in the feasible solution environment ii) uncertainty that the agent deals with and iii) failure happening in the group. Valuation of changeability is critical and are among the hectic part of the solution. On the other hand, perturbation is the decision of the other agents affecting on the active agent. Here, the perturbation is part of the cost function and needs to be

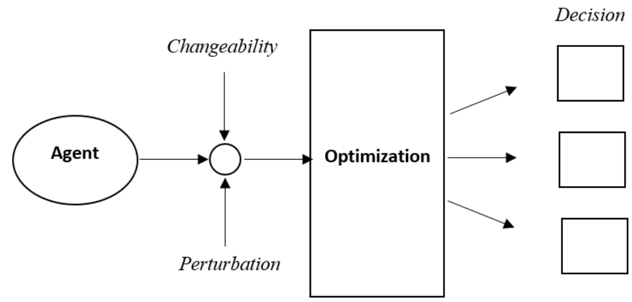


Fig. 1: Implementation of changeability and perturbation

considered as the optimization procedure for each agent. In Figure 1, the cost function of each of the *active* agents are considered along with the changeability into the optimization and then the decision is made.

## III. Applying Changeability in System

Various style of applying changeability in the design systems has been identified. In Ref. [28] real operation analysis has been considered for valuing this parameter and in Ref. [13] Monte Carlo analysis was implemented as the key method of finding this value for the systems. Furthermore in Ref. [16] the basic method of variable expiration was considered and in [24] a time-expanded decision network is created for valuing changeability. But over here, due to unlikely experimentation analysis applications, the aim is to find changeability without using the options theory [8]. In Reference [26], a subset of changeability was design for this purpose and the idea was the maximum amount of variation, using scaling by the expected variations from the solution point, can be applied simultaneously to all the variation parameter without any process failing. This method then continued in Reference [14] with a flexibility matrix. Tackling the problems in this network systems, could only be solved if the network system was robust over the uncertainty over time.

### A. Adjusted Epoch-Era Analysis (A-EEA)

Modeling uncertainty during time $t$ could be done using Epoch-Era Analysis (EEA) as a piece wise constant framework. This approach was first presented by [20] for designing purposes. The base unit of time or epoch, defined by a set of variables relying on the solution space. These variables can be defined using any exogenous factors. Era is defined using a complete set of epochs differentiated by the variables. This framework is capable of letting the agents have the uncertainties as a part of consideration of decision making in solution space. The difference between EEA and A-EEA developed for this system is that in EEA the solution space would not be changing over the time, but what this uncertainty causes is that the agent needs to make decision over the dynamic solution space.

Normalized Pareto Trace for measuring EEA was first introduced in Reference [21] and it is used commonly in the literature. This method considers a passive robustness of a system by calculating how optimal trade-off of cost and decision would be. By implementing the same concept, underneath concepts are defined.

- *Dynamic Fuzzy Pareto Number (D-FPN):* D-FPN of decision $P$ at time $t$ is the smallest percentage $D$ for which the decision is in the fuzzy Pareto set $P_{D_t}$.

$$\forall t \in \{1, ..., T\}$$
$$DFPN(p_t) = min\{D|p_t \subset P_{D_t}\}; \tag{1}$$

  In equation 1, $T$ is the total time till the end of the mission. This parameter is set as a global parameter and can be considered as 0 to 100 percent, regardless of solution space. One of the important matters in this equation is that the D-FPN is not sensitive to small changes in the Pareto frol nts.

- *Dynamic Fuzzy Pareto Score (D-FPS):* For measuring differing amounts of passive robustness versus changeability, D-FPS of each decision must be analyzed.

$$\forall t \in \{1, ..., T\}$$
$$DFPS(p_t) = DFPN(p_t) - DFPN(p_t^*); \tag{2}$$

  Score in equation 2 would simply define the difference in D-FPN score of each decision in pre and post changes analyzing.

- *Overall System Score(OSS):* This parameter is calculated for ranking the overall decisions toward the perturbation $R_{t,i,1}$ in decisions $P_{t,i,1}$ at time $t$ where $i$ is the agents in the network. This parameter depends on the availability of the system for consideration. Equation 3, represents the $OSS$ parameter:

$$OSS(R_{t,i,1}, P_{t,i,1}) = \sum_i J_{P_{t,i,1}} - min(\sum_i J_{P_{t,i,1}+R_{t,i,1}}); \tag{3}$$

  In equation 3, $J = J_{P_{t,i,1}}$ is the cost function of the decisions of the system at time $t$ and $J_R = J_{P_{t,i,1}+R_{t,i,1}}$ is the cost function of the perturbed decisions of the system.

- *fuzzy Pareto Trace (fPT):* This parameter was derived from Reference [25] and allows the system of a fuzzy Pareto front for a buffer from the true Pareto front defined by a perturbation matrix. This parameter is fully explained in Reference [19].

## IV. Dominance

Any fPT of decision $p_t$ for agent $i$ contains the Pareto sets of that decision [20]. Having the decision for each of the agent $i$ would cause the rest of the agents to have other agents coming up with their options and decisions. Here by using fPTs and the definitions of Pareto optimality, the dominance of cost function of $J$ and $J_R$ can be defined as following:

$$\begin{gathered} J_R \textit{ would dominate } J \textit{ if:} \\ J^R + K(J^{max} - J^{min}) \leq J; \textit{ and } J^R \neq J \\ J_i^R + K(J_i^{max} - J_i^{min}) \leq J_i; \textit{ and} \\ J_i^R + K(J_i^{max} - J_i^{min}) < J_i; \exists i \in N \end{gathered} \tag{4}$$

This fuzzy Pareto optimality has been defined in [25] where $K$ represents a user definable value between 0 and 1. $J^{max}$, $J^{min}$, $J_i^{max}$ and $J_i^{min}$ can be found previous to starting the solution. If $K$ is selected as 1, then the fuzzy Pareto frontier will be equal to all the solution space and if it is considered as 0 it is considered in weak Pareto optimal set.

## V. Setting Up the Learning Process

### A. Stackelberg Games

Here, a Stackelberg game which uses a leader-follower concept has been implemented to solve the distributed problem. The roles of the players are assumed to be asymmetric. Over the solution, the leader is set to follow its policy to maximize the payoff by taking the followers response into account. From this point, the set of actions of the leader agent $i$ are in $P_{t,i}$ and the other agents in the network would have a decision sets as:

$$\forall k \in N \quad \& \quad k \neq i; \quad p_{t,k} \in P_{t,k} \tag{5}$$

Each of the agents have cost function due to their decisions with considering a rational action as $\epsilon_{t,k} \in P_{t,k}$ as a reaction to leaders decision $P_{t,i}$. $\epsilon$ is a subset of the actions that can by done by the followers. Then the set of decision which followed by the decision of leader could be determined as equation 6.

$$R^t(P_i) = \{\epsilon \in P_{t,k} : \sum_{k,k\neq i} J_{\epsilon_{t,k}|P_{t,i}} \geq J_{P_{t,k}|P_{t,i}}, \forall P_{t,k}\}; \tag{6}$$

In equation 6, the dependency of the followers decision is shown by the decision of the leader.

The structure of the game, considered in this paper is only based on the agents states and the payoff of this game is considered as the costs associated with all the number of the agents. Then for the definition of the equilibrium for agent $i$ at time $t$:

$$J(P_{t,i}^{\#}, R^t(P_i)^{\#}) \geq J(P_{t,i}, R^t(P_i)); \tag{7}$$

Where the Stackelberg equilibrium is defined as $(P_{t,i}^{\#}, R^t(P_i)^{\#})$. By this means the objective is to satisfy equations 8 and 9.

$$J(P_{t,i}^{\#}), R^t(P_i)^{\#})) = \underset{i\in N}{Stackelberg}(J(P_{t,i}, R^t(P_i))); \tag{8}$$
$$P_{t,i}^{\#} = arg\underset{i\in N}{Stackelberg}(J(P_{t,i}, R^t(P_i))); \tag{9}$$

### B. Genetic Fuzzy Solution for Stackelberg Game

Previous to this challenge, there were number of efforts to solve the Stackelberg game theory problems such as Q-learning [11], SARSA algorithm [2] or backward induction method [1]. Here, the problem is solved by using a novel method of Genetic-Fuzzy Inference System (GFIS). The controller for this game theory is found via the Mamdani fuzzy inference system. Membership functions and rules are tuned using the Genetic Algorithm (GA). This section provides a brief overview of Mamdani FIS and GA. Implementation of these methods are provided in next sections.

*1) Mamdani Fuzzy Inference System (FIS):* Fuzzy logic can be used to infer a set of outputs based on a set of inputs and a set of rules. There are various methods for mapping such as Mamdani and Sugeno to allow fuzzy inference systems to act as universal approximators [21], [27]. This property conveys that FISs can be used to approximate any function to any arbitrary degree of accuracy, by spending more computational effort.

There are three operations, need to be done on input to transform it to desired output, i)fuzzification, ii)inference and iii) defuzzification. The if-then rules which determines the transformation can be shown as:

$$\textbf{If } \mathbb{s}_1 \textit{ is } \mathbb{F}_1 \textbf{ AND ... AND } \mathbb{s}_N \textit{ is } \mathbb{F}_N \textbf{ THEN } \mathbb{A}_N \textit{ is } \mathbb{F}_{\mathbb{k}} \quad (10)$$

Where $\mathbb{F}$ represents the corresponding adjective responding to input $\mathbb{s}$. $\mathbb{A}$ is the objective of the if-then rule and $\mathbb{F}_{\mathbb{k}}$ is the corresponding output.

This single output rule specifies a rule as output and it can be expanded to multiple output by using multiple linguistic variables [27].

$$\textbf{Linguistic } \textit{Variable } n \textbf{ Is } \textit{ adjective } \mathbb{p}$$

Minimum-maximum (min-max) composition is implemented for this system which calculates the minimum (intersection) of the antecedents for each rule. For minmax composition, the MFs of a given output is the union of the effective output MFs for the given output for all rules. The total effective output MF for the output of interest is converted to a single output by a process called defuzzification. For the defuzzification method for the FISs, the centroid method in which the centroid of the total effective output membership function is used as the output.

*2) Genetic Algorithm:* GA as a metaheuristic optimization procedure, it operates based on the theory of evolution via natural selection. For GA a fitness function is assigned to indicate which solutions are most fit, and therefore have a better chance of reproducing [27]. Mutation and crossover of the genes are considered as the next steps. Mutation allows more of the solution space to be explored with random manner. Generally, the process of evolution in generation along with choosing the best elite genes, heuristically improve the solution.

Total game theory and the proposed solution for solving the game theory problem can be followed using algorithm 1.

There are several considerations that might come handy using algorithm 1.

- There are various methods of solving the game problems. Any of the solution can be used as a feed to the solution. During this paper, backward induction with Q-learning process is considered as the solution for comparing the exact results.
- The inputs to the FIS varies due to the number of agents and their states. The important matter is that Genetic-fuzzy systems are used as a offline learning process and after learning is completed, they act robust towards solving the problems. Genetic fuzzy systems over the examples provided at the end of this paper has been proven to provide robust solutions as well as what have been claimed in previous literature [5], [6].

```
while P#_{t,i} ≠ Goal do
    Define the states of the agents;
    Consider cost-to-go;
    Choose a random agent m;
    Define the possible actions and reaction by other
    agents in subsets;
    Solve the game theory considering the cost function;
    Feed the result to Genetic-fuzzy algorithm;
    Find the Gains of the fuzzy controller;
    t ← t + 1;
end
```

**Algorithm 1:** Solution of the Stackelberg game theory using genetic fuzzy solution

## VI. Multi-Agent Application

By use of biologically inspired model of swarms, the multi-agent model has been presented in the absence of the inter-agent distance measurement [3]. The main rule that has been considered in this paper is that the agents collaborating within the swarm, have to keep "equidistant" as possible from closest agents [9]. Hence, a simple pairwise repulsive potential fields for the agents are defined as:

$$\forall i, i-n \in N;$$
$$F_{i,i-n} = L_{i,i-n}(|x_i - x_{i-n}|); \quad (11)$$

In equation 11, $L_{i,i-n}$ is the potential function between agent $i$ and its $n^{th}$ connected agent. In this equation $N$ is the total number of agents. Distance of each of the agents are shown as $x$ matrix is the position of the agent. The overall potential function is considered continuously differentiable and positive definite [4].

*Remark 6.1:* This method has been proved to be efficient previously using different technique of solution [17], [10]. The difference between this method and the previous work is that over the connection model. In [17], all the agents are aware of the position of each other but here each of the agents only aware of the position of the rest of the agents which are located in the indicated vicinity of the agent. This is an indication of connected and disconnected agents.

This problem has been completely explained in Ref. [15]. The scheme of connected and disconnected agents is shown in Figure 2. In this figure the agent is only in touch with the connected agents around itself and hence the total energy for each agent $i$ (repulsive and potential) can be written as:

$$F_{total} = \sum_{\text{Active Agents of } i} L_{i,i-n}(|x_i - x_{i-n}|) + F_{attraction} \quad (12)$$

The aim is to find the control law mentioned as $u_i = -f_i$ for every agent $i$.

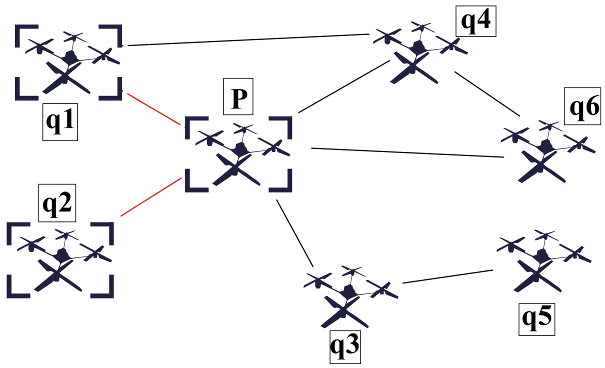


Fig. 2: Connected and disconnected agents in network. Edges with red color represent the connected network

### A. Application to multi-Agents

In this part, multi agents are considered to move inside the boundaries. As an instance, for six agents the artificial potential field function which were considered in this section is as equation 13.

$$F = \sum_{i=1}^{5} \sum_{j=i+1}^{6} (|x_i - x_j|^2 - d_{ij})^2; \tag{13}$$

Where $d_{ij}$ is the parameter introduced for each of the agents and is introduced by [17]. At this point several assumptions were introduced for formulating this problem in distributed manner.

*Assumption 1:* Over the solution, the policy assures that each agent is in active relation within certain radius. If this certain radius is higher then the distributed fashion would change into the centralized control method and all agents are going to decide based on the decisions of others.

*Assumption 2:* Connectivity of the agents to each other are due to being within the specific radius of the other agents.

Then the cost function for the distributed form can as shown in figure 3.

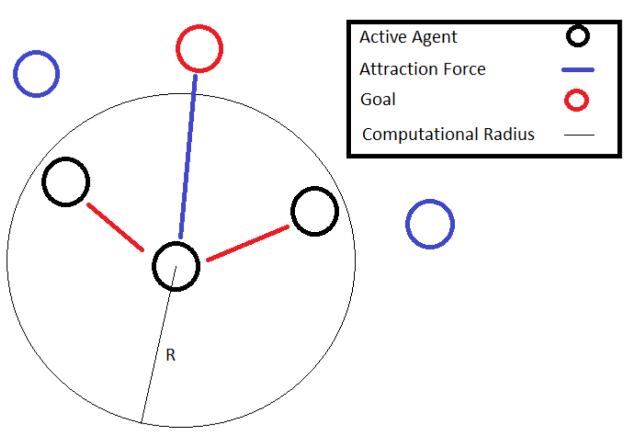


Fig. 3: Connected and disconnected agents in network. Edges with red color represent the connected network

### B. Set Up Adjusted Epoch Era Analysis

In this step, the necessary constructs for Valuing the changeability and perturbation over the uncertainty are created. The variables needs to be considered for the correct decision are:

- Number of the agents connected.
- Controls and states of the agents.
- Forces exerted on each of them.
- Changeability (uncertainty) of the area.

### C. Changeability Design

Changeability Design (CD) policy is defined as the inclusion of extra options or margins, which has higher cost for the agent to follow. Policies needs to be defined based on the definition of the problems and in bigger picture, the game policies that has been defined for the game theory.

Here, based on the importance of the uncertainty, three levels are assigned with different costs:

- Level 0 : This level represents the uncertainty over not knowing the environment after a certain radius. This would add a force to the agents in that cells which the agents exists as $F_{\mathbb{1}}$.
- Level 1 : This level represents the uncertainty over the decisions of the communication delays detected inside the network over the perturbation of other agents. This would add a repelling force to agents as $F_{\mathbb{2}}$
- Level 2: Due to map, there are levels of collision which can be combined over the high-levels of 1 and 0. This adds a forces as $F_{\mathbb{3}}$ to the agents.

The policy of the movement for the UAVs is moving through the tessellated part is that each UAV is only capable of occupying the attached cells to itself. Recall that as it mentioned in DFPS parameters, during facing the uncertainty in the system, this parameter, OSS and cost function play a huge role in decision making.

The possible strategy for making the decision is to minimize the cost function.

### D. Dynamic of the Agents

The dynamics of the UAV are considered by using Dubins model for determining the optimal sequence of way-points. Dubins showed that for an approximate model of aircraft dynamics, the optimal motion between a pair of way-points can be chosen among six possible paths[12].

The UAV kinematics can be approximated by the Dubins vehicle, represented by $(x, y, \theta)$. Here, $(x, y)$ represent the position of the UAV in a 2-dimensional plane and $\theta$ represents the heading of the UAV. The UAV kinematics are written as:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} v cos\theta \\ v sin\theta \\ \frac{v}{\rho} u \end{bmatrix} \tag{14}$$

where $v$ is the forward speed of the vehicle, $\rho$ is the radius of curvature of turn, and $u$ is the bounded control input for the UAV.

### E. Results and discussion

*1) Phase1: Reaching to Formation:* There are three agents considered in this paper. The radius of the computation is considered as 10 and agents are align in a straight line to reach to the triangular formation. The initial position of each of the agents has been considered as:

$$\begin{aligned} z_1 &= [1 \quad 5]; \\ z_2 &= [1 \quad 8.5]; \\ z_3 &= [1 \quad 12]; \end{aligned}$$

There is an obstacle in the area placed at [15 26]. These agents need to occupy three points as:

$$\begin{aligned} g_1 &= [10 \quad 7]; \\ g_2 &= [14 \quad 10]; \\ g_3 &= [10 \quad 14]; \end{aligned}$$

*2) Phase2: Moving with Formation:* The structure of potential function explained in this paper suggests to exempt one agent from the control law designed and let it move freely as the leader. Therefore, to run a more general simulation, here, one of the agents is chosen as the leader and moves with a constant speed to a predefined direction. Then, it is anticipated that, after some transient formation, the agent position achieves the triangular formation.

*3) Uncertainty in Environment:* For the whole scenario, the robustness of the proposed method in the presence of measurement noise and agent failure will be tested. For all the agents, the area is considered uncertain and they are not aware of the obstacle or the decisions of the other agents previous to the problem.

The result for this movement has been provided in figure 4.

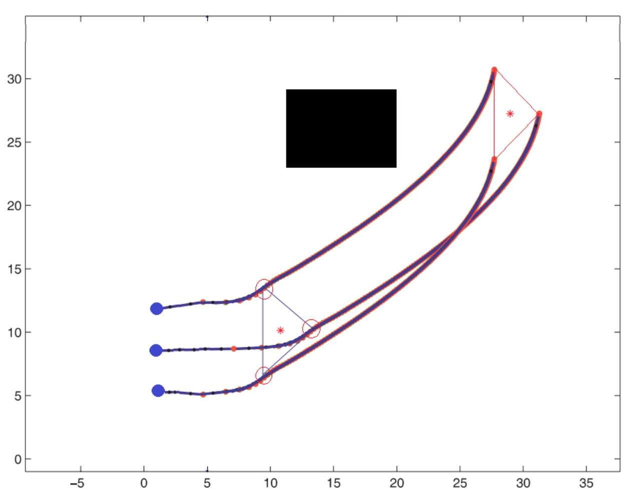

Fig. 4: Result of simulation for three agents

### F. Analyze of the Proposed Method

In case of optimality, the result of this method has been compared with the result of the centralized controller. The key to understand is that since this problem have a stage of solving game theory problem using Genetic Fuzzy, comparing this problem to other methods of solving the same problem is not fair. The result for simulation of centralized formation control matches with the result of proposed method with 96% accuracy of cost function.

The behavior of the cost function for centralized and distributed form of the controller is shown in figure 5.

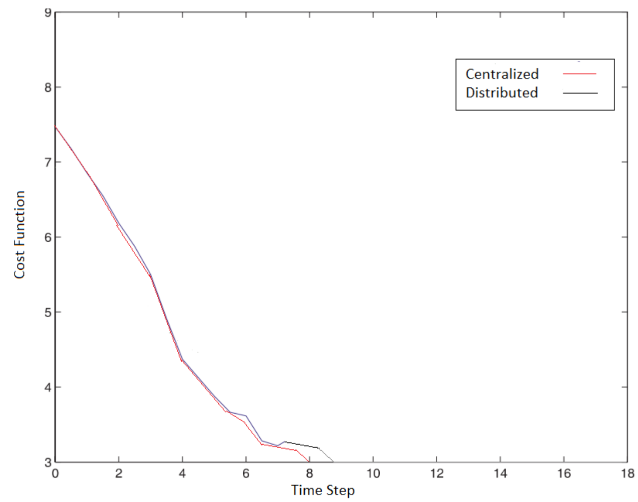


Fig. 5: Cost of centralized and distributed method of simulation for three agents

## VII. Conclusion

In this paper, the formation control problem of a class of multi-agent systems within uncertain environment has been investigated. During this paper, first by introducing a new novel concepts and parameters as Genetic Fuzzy Pareto solution the basic of this method has been introduced. After that by implementing the game theory inside the problem, a solution generated and the changeability valuation is offered to the problem. The process of decision making has been introduced and at the end, all the theoretical results were verified by simulation examples, and good performance of the proposed controller was shown even in the case of presence in uncertain environment.

## References


[1] Robert J Aumann. Backward induction and common knowledge of rationality. *Games and Economic Behavior*, 8(1):6–19, 1995.

[2] Bikramjit Banerjee, Sandip Sen, and Jing Peng. On-policy concurrent reinforcement learning. *Journal of Experimental & Theoretical Artificial Intelligence*, 16(4):245–260, 2004.

[3] William Bialek, Andrea Cavagna, Irene Giardina, Thierry Mora, Edmondo Silvestri, Massimiliano Viale, and Aleksandra M Walczak. Statistical mechanics for natural flocks of birds. *Proceedings of the National Academy of Sciences*, 109(13):4786–4791, 2012.

[4] Chien Chern Cheah, Saing Paul Hou, and Jean Jacques E Slotine. Region-based shape control for a swarm of robots. *Automatica*, 45(10):2406–2411, 2009.

[5] N Ernest, D Carroll, C Schumacher, M Clark, K Cohen, et al. Genetic fuzzy based artificial intelligence for unmanned combat aerial vehicle control in simulated air combat missions. *J Def Manag*, 6(144):2167–0374, 2016.

[6] N Ernest and K Cohen. Fuzzy logic based intelligent agents for unmanned combat aerial vehicle control. *J Def Manag*, 6(139):2167–0374, 2015.

[7] Michela Fazzolari, Rafael Alcala, Yusuke Nojima, Hisao Ishibuchi, and Francisco Herrera. A review of the application of multiobjective evolutionary fuzzy systems: Current status and further directions. *IEEE Transactions on Fuzzy systems*, 21(1):45–65, 2013.

[8] Matthew E Fitzgerald and Adam M Ross. Mitigating contextual uncertainties with valuable changeability analysis in the multi-epoch domain. In *Systems Conference (SysCon), 2012 IEEE International*, pages 1–8. IEEE, 2012.

[9] Gonzalo A Garcia and Shawn S Keshmiri. Biologically inspired trajectory generation for swarming uavs using topological distances. *Aerospace Science and Technology*, 54:312–319, 2016.

[10] Veysel Gazi. Swarm aggregations using artificial potentials and sliding-mode control. *IEEE Transactions on Robotics*, 21(6):1208–1214, 2005.

[11] Dongbing Gu and Huosheng Hu. Fuzzy multi-agent cooperative Q-learning. In *Proc. of the 2005 IEEE International Conference on Information Acquisition*, pages 193–197, 2005.

[12] Jason T Isaacs and Joao P Hespanha. Dubins traveling salesman problem with neighborhoods: a graph-based approach. *Algorithms*, 6(1):84–99, 2013.

[13] Johnathan Mun. *A primer on applying Monte Carlo simulation, real options analysis, knowledge value added, forecasting, and portfolio optimization*. PhD thesis, Naval Postgraduate School, 2010.

[14] Andrew Olewnik and Kemper Lewis. A decision support framework for flexible system design. *Journal of Engineering Design*, 17(1):75–97, 2006.

[15] Reza Olfati-Saber and Richard M Murray. Distributed cooperative control of multiple vehicle formations using structural potential functions. In *IFAC World Congress*, volume 15, pages 242–248. Citeseer, 2002.

[16] Jeff G Pierce. *Designing flexible engineering systems utilizing embedded architecture options*. PhD thesis, Vanderbilt University, 2010.

[17] Bijan Ranjbar-Sahraei, Faridoon Shabaninia, Alireza Nemati, and Sergiu-Dan Stan. A novel robust decentralized adaptive fuzzy control for swarm formation of multiagent systems. *IEEE Transactions on Industrial Electronics*, 59(8):3124–3134, 2012.

[18] Jafar Razmi, Ehsan Jafarian, and Saman Hassanzadeh Amin. An intuitionistic fuzzy goal programming approach for finding pareto-optimal solutions to multi-objective programming problems. *Expert Systems with Applications*, 65:181–193, 2016.

[19] Adam M Ross, Hugh L McManus, Donna H Rhodes, Daniel E Hastings, and AM Long. Responsive systems comparison method: Dynamic insights into designing a satellite radar system. *AIAA Space 2009*, pages 1–25, 2009.

[20] Adam M Ross, Donna H Rhodes, and Daniel E Hastings. Defining changeability: Reconciling flexibility, adaptability, scalability, modifiability, and robustness for maintaining system lifecycle value. *Systems Engineering*, 11(3):246–262, 2008.

[21] Timothy J Ross. *Fuzzy logic with engineering applications*. John Wiley & Sons, 2009.

[22] Joseph H Saleh, Gregory Mark, and Nicole C Jordan. Flexibility: a multi-disciplinary literature review and a research agenda for designing flexible engineering systems. *Journal of Engineering Design*, 20(3):307–323, 2009.

[23] Ricardo C Silva and Akebo Yamakami. Definition of fuzzy pareto-optimality by using possibility theory. In *IFSA/EUSFLAT Conf.*, pages 1234–1239. Citeseer, 2009.

[24] Matthew R Silver and Olivier L De Weck. Time-expanded decision networks: A framework for designing evolvable complex systems. *Systems Engineering*, 10(2):167–188, 2007.

[25] Rudolf Marcel Smaling. *System architecture analysis and selection under uncertainty*. PhD thesis, Massachusetts Institute of Technology, 2005.

[26] Ross Edward Swaney and Ignacio E Grossmann. An index for operational flexibility in chemical process design. part i: Formulation and theory. *AIChE Journal*, 31(4):621–630, 1985.

[27] Alex R Walker, Philip T Putman, and Kelly Cohen. Solely magnetic genetic/fuzzy-attitude-control algorithm for a cubesat. *Journal of Spacecraft and Rockets*, 52(6):1627–1639, 2015.

[28] Jennifer Wilds, Jason Bartolomei, Richard de Neufville, and Daniel Hastings. Real options 'In' a micro air vehicle. In *Conference on Systems Engineering Research*, 2007.