GEORGIA INSTITUTE OF TECHNOLOGY - GUGGENHEIM SCHOOL OF AEROSPACE ENGINEERING

System Dynamics Notes

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1 MODELING MECHANICAL SYSTEMS

Systems can be described in units of Length, Mass or Time. Measurement systems can be 'absolute' or 'gravitational'. In absolute systems, Mass is chosen as a primary dimension, and force is a derived quantity. In this system, force is defined from mass and acceleration F = ma. In gravitational systems, Force is a primary dimension. In gravitational systems, mass is defined from the ratio of the Force on an object to its acceleration. Mass is defined as $\frac{F}{a}$. SI units are an example of an absolute measurement system (kg, m, s). The British Engineering System (BES) is an example of a gravitational measurement system (lbf, ft, s). All examples will be given in SI units. A chart comparing aboslute and gravitational systems can be found in the appendix.

Mechanical Systems are made up of 3 primary elements.

- Springs: Elements that exert a force/torque proportional to their displacement
- Dampers: Elements that oppose the direction of motion
- Inertial elements: Masses and their moments of inertia.

In our diagrams we use the following notation conventions:

- Capital Letters (X, Y, Z): Positions of elements naturally extended.
- Letter P: Location of an acting force.
- Lower Case Letters (x, y, z): Displacement from an origin or natural length.

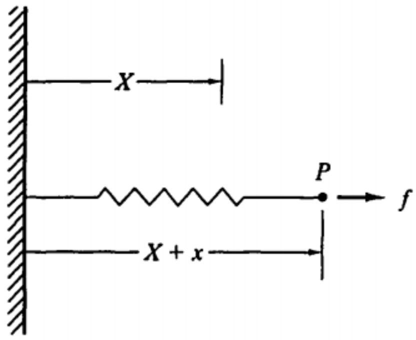

Figure 1.1: Example of a Linear Spring

1.1 Elemental Behavior

Springs

A Linear spring is one whose displacement is proportional to an applied outside force. If a force is acting on a linear spring, the following is true:

$$F \propto x$$

Using empirical values we can determine the spring constant, k, as follows:

$$k = \frac{F}{x}$$

$$F = kx$$

Consider a spring lengthened by an outside force. Due to Newtons 3rd law, the spring will apply and equal and opposite force on its surroundings. If a spring is lengthened, the spring will apply a force on its surroundings towards its center. If the spring is compressed, it will apply a force on its surroundings away from its center. A force is 'positive' if it points towards the center of the spring. A force is 'negative' if it points away from the center of the sping.

A Linear Torsional Spring is one whose displacement is proportional to a torque applied on it by the environment. This is a rotational displacement.

$$\tau \propto \theta$$

Using empirical values, we can determine a spring constant, k, as follows:

$$k = \frac{\tau}{\theta}$$

$$\tau = \frac{k}{\theta}$$

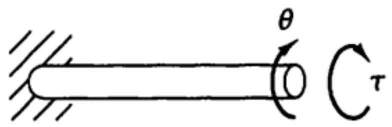

Figure 1.2: Example of a Torsional Spring

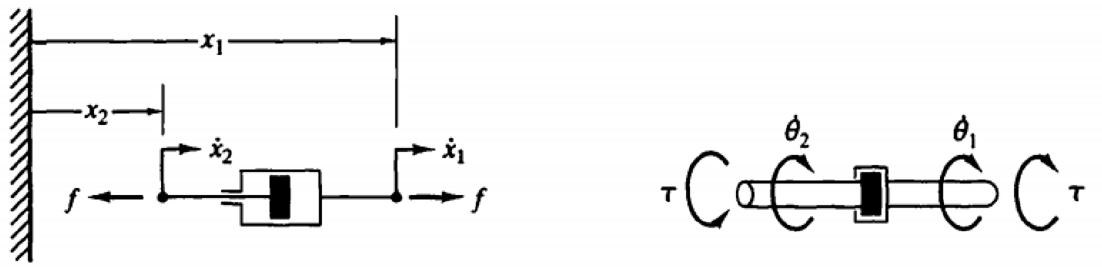

Figure 1.3: Translational and Torisional Dampers

Springs store potential energy. The potential energy of a spring can be found using the following:

$$PE = \frac{1}{2}kx^2$$

DAMPERS

A Damper is a mechanical element that opposes motion. While springs can be used to store potential energy, a damper is used to dissipate kinetic energy as heat. This can be done with a piston and cylinder of oil. As the oil passes the piston, a friction force is exerted. Friction is a non-conservative force. If a friction force is applied over a distance, the total mechanical energy of the system decreases. Mechanical Energy is defined as the sum of Potential Energy and Kinetic Energy.

$$\Sigma MechanicalEnergy = \Sigma PotentialEnergy + \Sigma KineticEnergy$$

There are both translational dampers and torsional dampers. Translational dampers apply forces opposing translational motion. Torsional dampers apply torques opposing rotational motion.

$$F \propto (\dot{x_2} - \dot{x_1})$$
$$F \propto (\dot{\theta}_2 - \dot{\theta}_1)$$

The viscous friction coefficient is found emperically as follows:

$$b = F/(\dot{x}_2 - \dot{x}_1)$$

$$b = F/(\dot{\theta}_2 - \dot{\theta}_1)$$

$$F = b(\dot{x}_2 - \dot{x}_1)$$

$$F = b(\dot{\theta}_2 - \dot{\theta}_1)$$

Inertial Elements

The mass of a system is also critical to its behavior. The translational velocity of an object is constant unless acted on by an outside force. The translational acceleration of an body is dependent on the objects mass and the magnitude of the force acting on the body. In translational motion, every point on the body moves through space.

$$F = ma$$

Analyzing rotational motion is more difficult. The rotational behavior of an object is dependent on the mass of the object AND the object mass distribution relative to a coordinate axis. This 'mass distribution' can be expressed as a scalar known as the 'moment of inertia'. In rotational motion, every point on the body moves except the axis of rotation. If you want the formulas for moment of inertia (J), look them up in a book.

$$\Sigma\tau = J\alpha$$

Forced Response VS Natural Reponse

System behavior is dependent on the systems initial conditions and inputs from the environment. A forced response is caused by a 'forcing function', which is a function describing system inputs from the environment. A natural response is due to initial conditions in the system.

Natural Response Example: A car is going down the road at steady 60 mph. At $t = 0$, the engine cuts out. The car coasts down the road until it comes to a stop. The response after $t = 0$ would be the natural response.

Forced Response Example: A car is going down the road. At $t = 0$, the engine sets to a constant energy input into the car. This is a forced response. There is a user input forcing the behavior of the system.

Example 1

Lets consider a mass mounted to bearings as shown in the image. A person rotates the system until is reaches angular velocity $\omega(0)$ at $t = 0$. At $t = 0$, the person stops cranking the shaft of

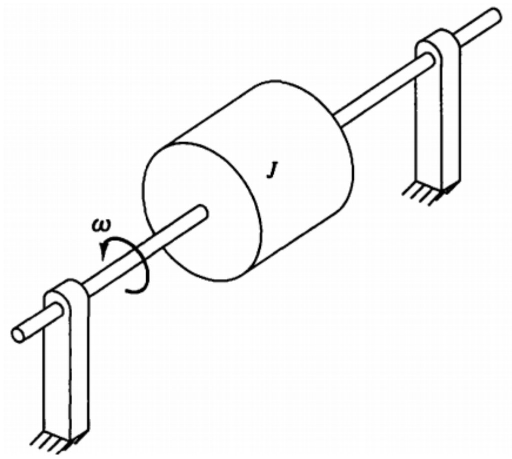

Figure 1.4: Translational and Torisional Dampers

the system. Determine the behavior of the system. Note - This is an example of a natural response.

$$
\begin{aligned}
\omega(0) &= \omega_0 && \text{Initial Conditions}\\
J\dot{\omega} &= -b\omega && \text{Equation of Motion}\\
J\dot{\omega} + b\omega &= 0 && \\
\mathscr{L}(J\dot{\omega} + b\omega) &= \mathscr{L}(0) && \text{Take the Laplace Transform}\\
J(s\mathscr{L}(\omega) - \omega(0)) + b\mathscr{L}(\omega)) &= 0 && \text{Use Laplace Substitution}\\
\mathscr{L}(\omega) &= \omega(0)\frac{1}{(s+\frac{b}{J})} && \text{Solve Laplace Transform}\\
\mathscr{L}^{-1}(\mathscr{L}(\omega)) &= \mathscr{L}^{-1}\left(\omega(0)\frac{1}{(s+\frac{b}{J})}\right) && \text{Take the Inverse Laplce}\\
\omega(t) &= \omega(0)e^{-\left(\frac{b}{J}\right)t} && \text{Solution}
\end{aligned}
$$

1.2 Heading on level 2 (subsection)

$$
A = \begin{bmatrix} A_{11} & A_{21} \\ A_{21} & A_{22} \end{bmatrix} \tag{1.1}
$$

1.2.1 Heading on level 3 (subsubsection)

Heading on level 4 (paragraph)

Systems of units / Quantity	Absolute systems				Gravitational systems	
	Metric			British	Metric engineering	British engineering
	SI	mks	cgs			
Length	m	m	cm	ft	m	ft
Mass	kg	kg	g	lb	$\frac{kg_f\text{-}s^2}{m}$	slug $= \frac{lb_f\text{-}s^2}{ft}$
Time	s	s	s	s	s	s
Force	N $= \frac{kg\text{-}m}{s^2}$	N $= \frac{kg\text{-}m}{s^2}$	dyn $= \frac{g\text{-}cm}{s^2}$	poundal $= \frac{lb\text{-}ft}{s^2}$	kg_f	lb_f
Energy	J = N-m	J = N-m	erg = dyn-cm	ft-poundal	kg_f-m	ft-lb_f or Btu
Power	W $= \frac{N\text{-}m}{s}$	W $= \frac{N\text{-}m}{s}$	$\frac{dyn\text{-}cm}{s}$	$\frac{ft\text{-}poundal}{s}$	$\frac{kg_f\text{-}m}{s}$	$\frac{ft\text{-}lb_f}{s}$ or hp

Figure 2.1: Systems of Units

2 Modeling Systems with Transfer-Functions

2.1 Example of list (3*itemize)

- First item in a list
  - First item in a list
    - First item in a list
    - Second item in a list
  - Second item in a list
- Second item in a list

2.2 Example of list (enumerate)