07-A2

BST Application

Range Query: 2D

昔者明王必尽知天下良士之名；既知其名，又知其数；
既知其数，又知其所在。

邓俊辉
deng@tsinghua.edu.cn

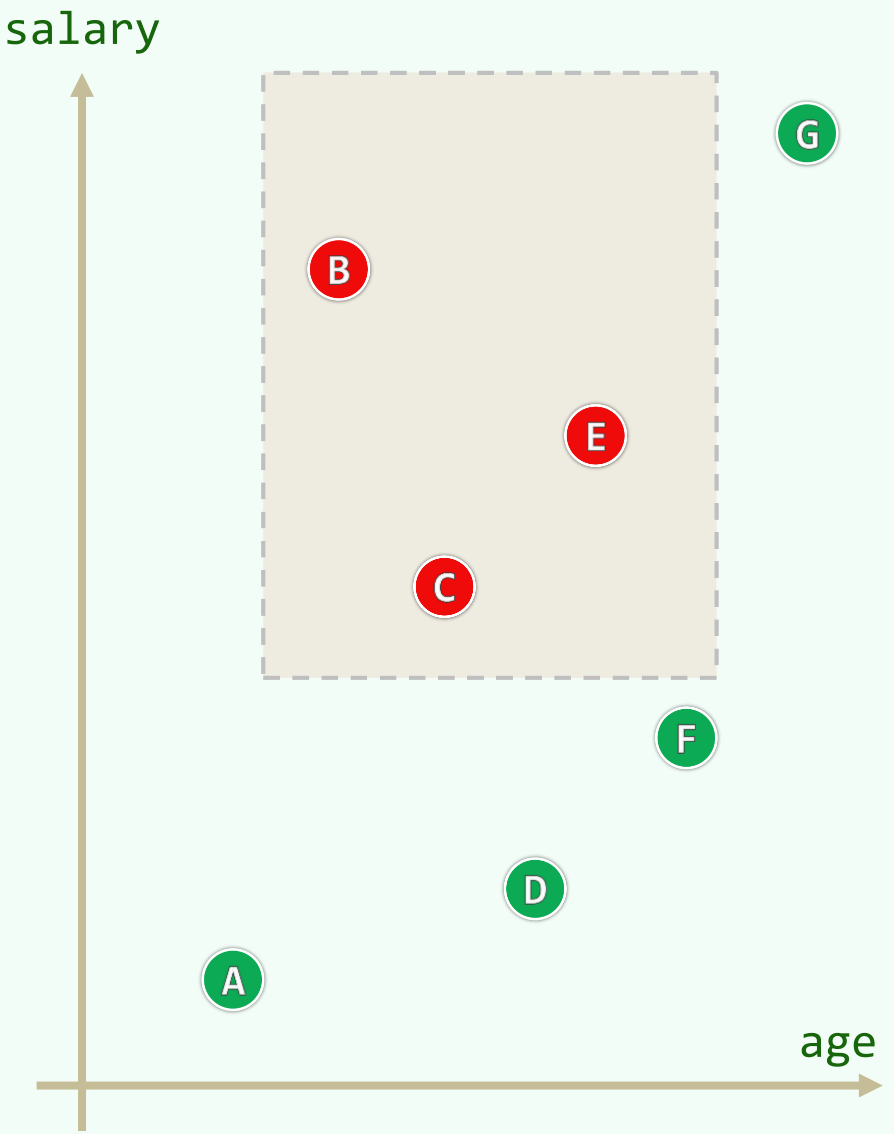
Planar Range Query

- Let $P = \{ p_1, p_2, p_3, \ldots, p_n \}$ be a planar set
- Given $R = (x_1, x_2] \times (y_1, y_2]$
  - COUNTING: $|R \cap P| = ?$
  - REPORTING: $R \cap P = ?$
- Binary search

  doesn't help this kind of query
- You might consider to

  expand the counting method using

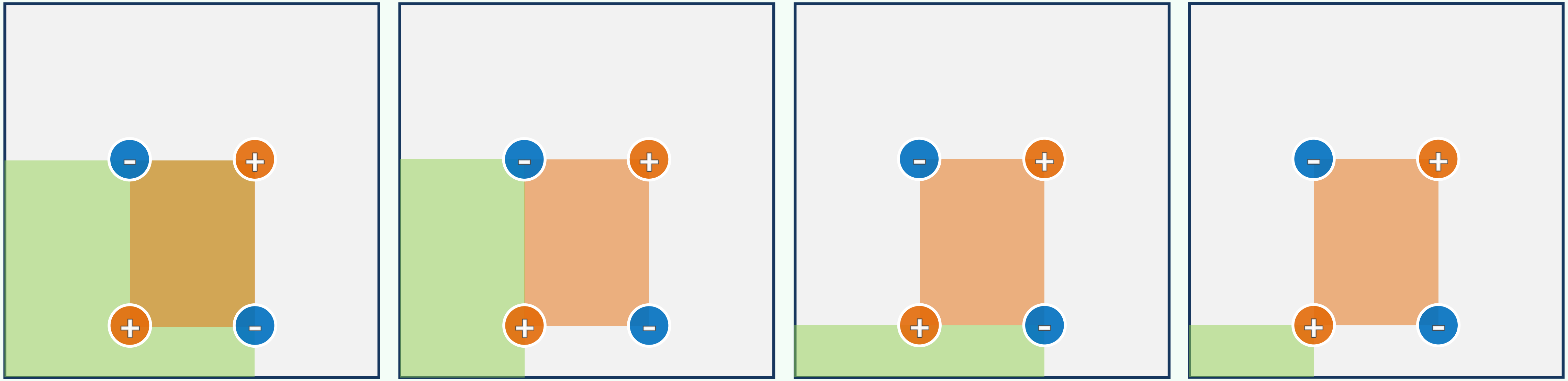
  the **Inclusion-Exclusion Principle**



Preprocessing

- $\forall$ point (x, y), let $n(x, y) = \left|((0, x] \times (0, y]) \cap P\right|$

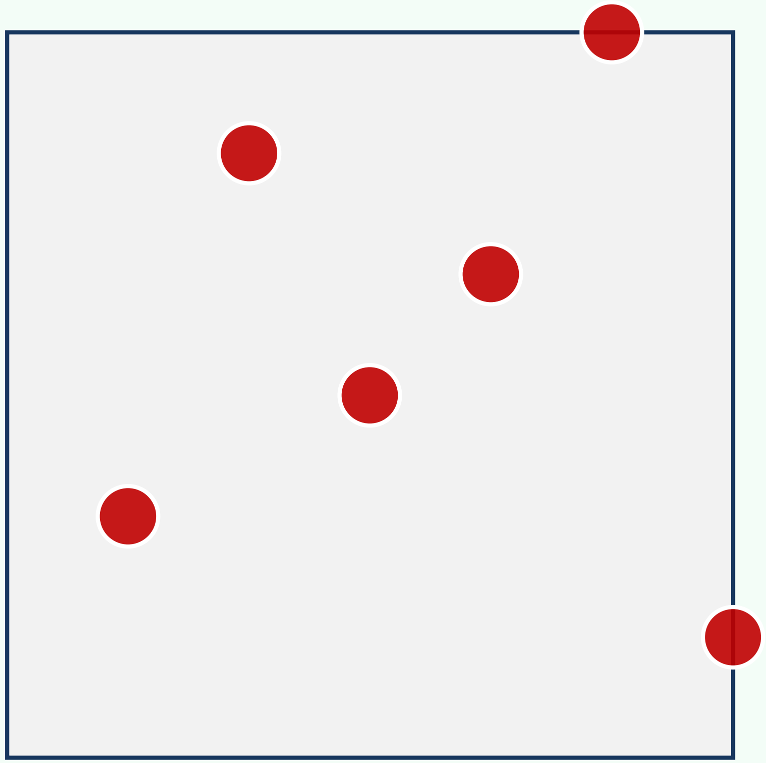
- **This requires $\mathcal{O}(n^2)$ time/space**

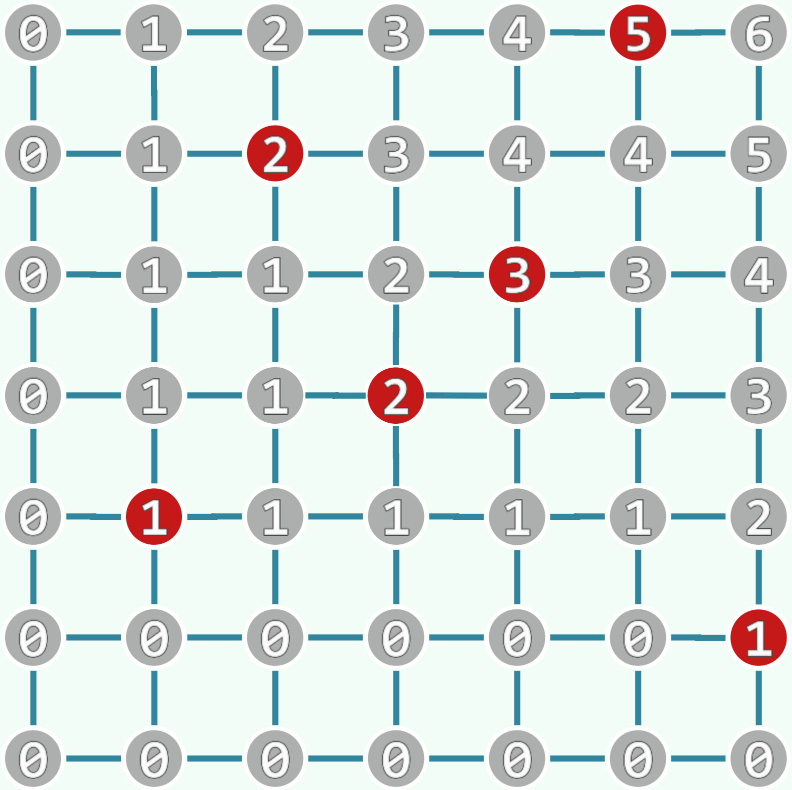


Domination

❖ A point (u, v) is called to be **DOMINATED** by point (x, y) if

$u \leq x$ and $v \leq y$

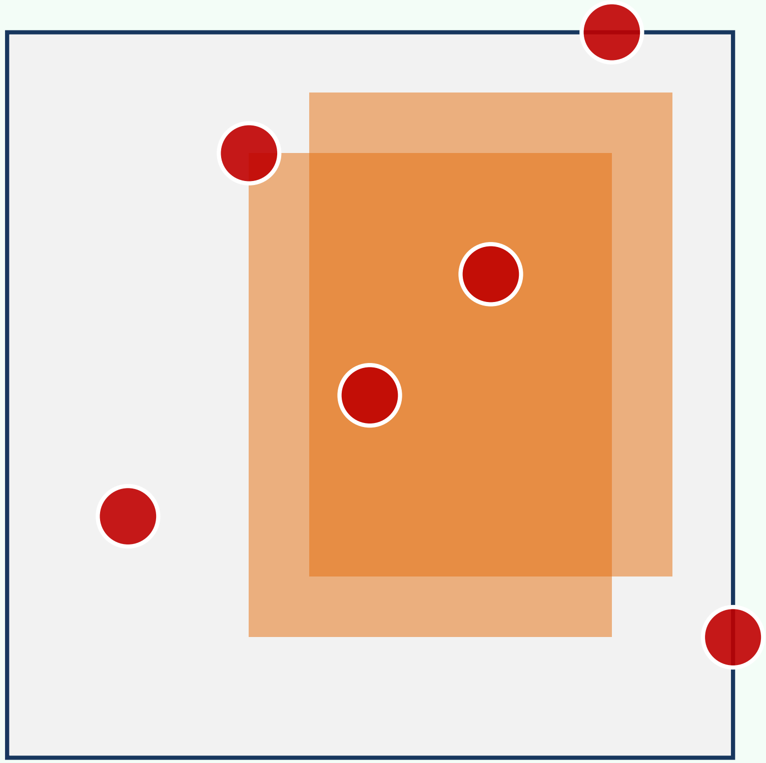


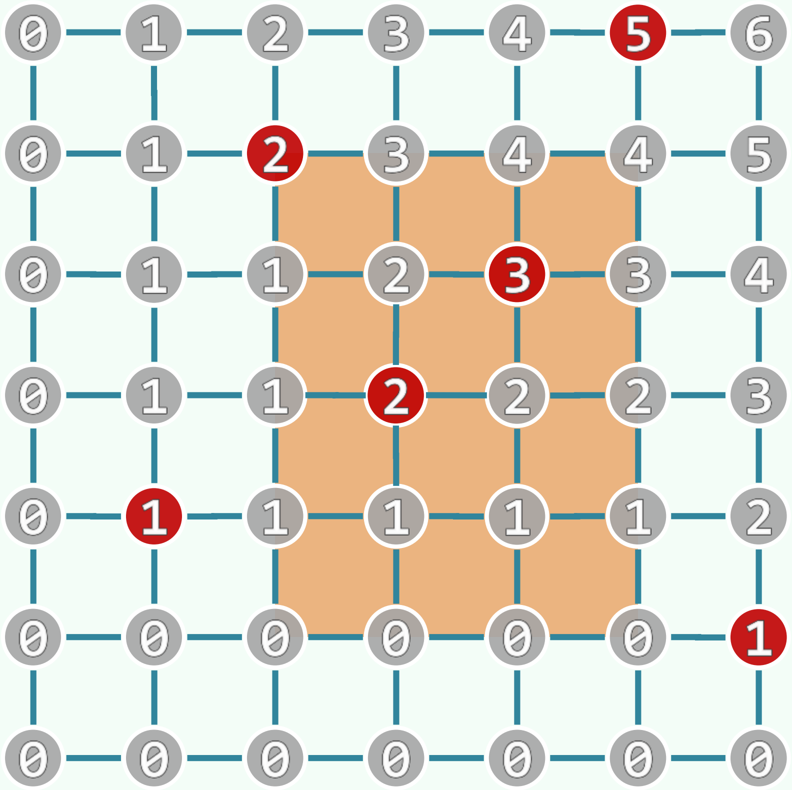


Inclusion-Exclusion Principle

❖ **Then for any rectangular range $\mathcal{R} = (x_1, x_2] \times (y_1, y_2]$, we have**

$$\left|\mathcal{R} \cap \mathcal{P}\right| \;=\; n(x_1, y_1) + n(x_2, y_2) - n(x_1, y_2) - n(x_2, y_1)$$





Performance

- Each query needs only $\mathcal{O}(\log n)$ time
- Uses $\Theta(n^2)$ storage and even more for higher dimensions
- To find a better solution, let's go back to the **1D** case ...