

# 信号与系统第三次测试 (2022 年)

姓名:

学号:

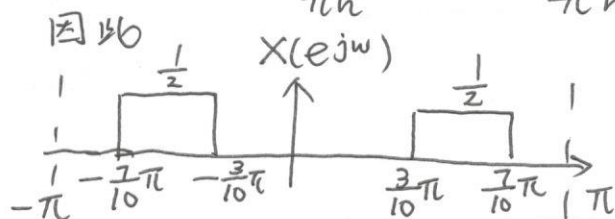
1. 求如下两个信号的离散傅里叶变换

$$(1) x[n] = \frac{\sin(\frac{\pi n}{5}) \cos(\frac{7\pi n}{2})}{\pi n}$$

$$(2) x[n] = (-1)^n$$

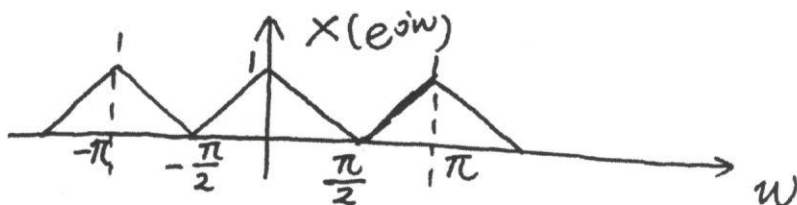
解①  $x[n] = \frac{\sin(\frac{\pi}{5}n) \cos(\frac{1}{2}\pi n)}{\pi n}$

$$= \frac{1}{2} \left[ \frac{\sin(\frac{7}{10}\pi n)}{\pi n} - \frac{\sin(\frac{3}{10}\pi n)}{\pi n} \right] \quad (\text{积化和差公式})$$



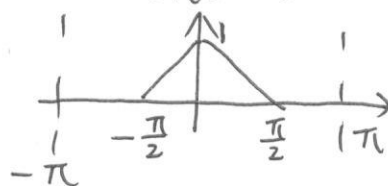
$$(2) X(e^{j\omega}) = 2\pi \sum_{k=-\infty}^{+\infty} \delta(\omega - (2k+1)\pi)$$

2. 求如下信号的离散傅里叶反变换



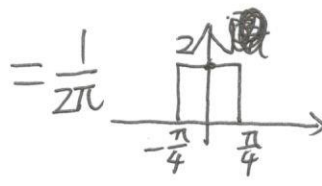
解:

先看



$\downarrow F^{-1}$

$$x[n]$$



$\downarrow F^{-1}$

$$= 2 \frac{\sin(\frac{\pi}{4}n)}{\pi n} \cdot 2 \frac{\sin \frac{\pi}{4}n}{\pi n}$$

$$= 4 \left[ \frac{\sin(\frac{\pi}{4}n)}{\pi n} \right]^2$$

由于原题以  $\pi$  为周期, 因此最终答案

$$x[n] = 4[1 + (-1)^n] \left[ \frac{\sin(\frac{\pi}{4}n)}{\pi n} \right]^2$$

3. 已知某因果系统差分方程为:

$$y[n] - \frac{1}{6}y[n-1] - \frac{1}{6}y[n-2] = x[n] + \frac{1}{3}x[n-1]$$

当  $x[n] = \left(\frac{1}{3}\right)^n u[n]$  时, 求输出  $y[n]$ 。

解:  $(1 - \frac{1}{6}e^{-j\omega} - \frac{1}{6}e^{-j2\omega})Y(e^{j\omega}) = (1 + \frac{1}{3}e^{-j\omega})X(e^{j\omega})$

$$Y(e^{j\omega}) = \frac{(1 + \cancel{\frac{1}{3}e^{-j\omega}})}{(1 - \frac{1}{2}e^{-j\omega})(1 + \cancel{\frac{1}{3}e^{-j\omega}})} \cdot \frac{1}{1 - \frac{1}{3}e^{-j\omega}}$$

$$= \frac{1}{(1 - \frac{1}{2}e^{-j\omega})(1 - \frac{1}{3}e^{-j\omega})}$$

$$= \frac{3}{1 - \frac{1}{2}e^{-j\omega}} - \frac{2}{1 - \frac{1}{3}e^{-j\omega}}$$

$$y[n] = 3\left(\frac{1}{2}\right)^n u[n] - 2\left(\frac{1}{3}\right)^n u[n]$$