栈与队列

直方图内最大矩形

04-J

就这么着，我有了一所严丝密缝、涂抹灰泥的木板房子，七英尺宽，十五英尺长，立柱有八英尺高...

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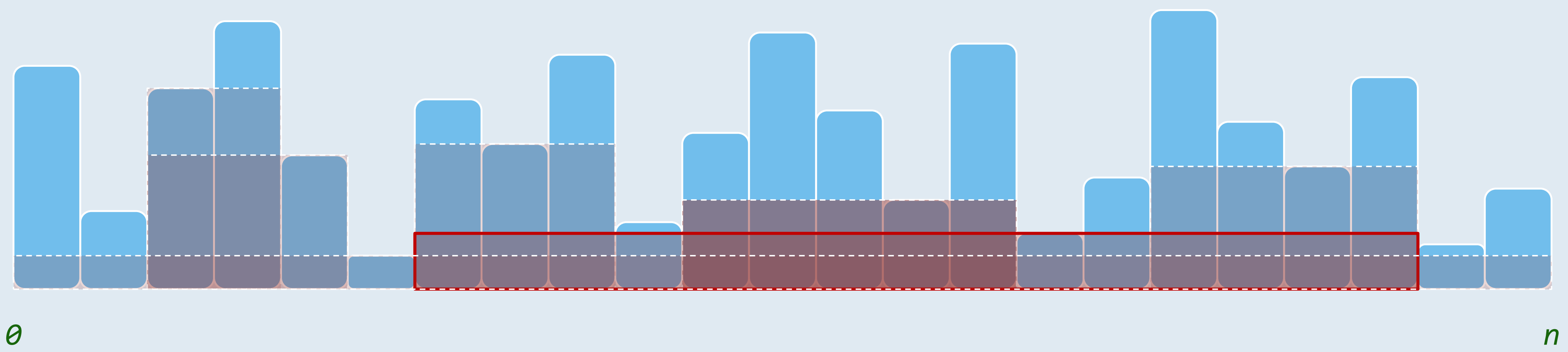
Maximum Rectangle



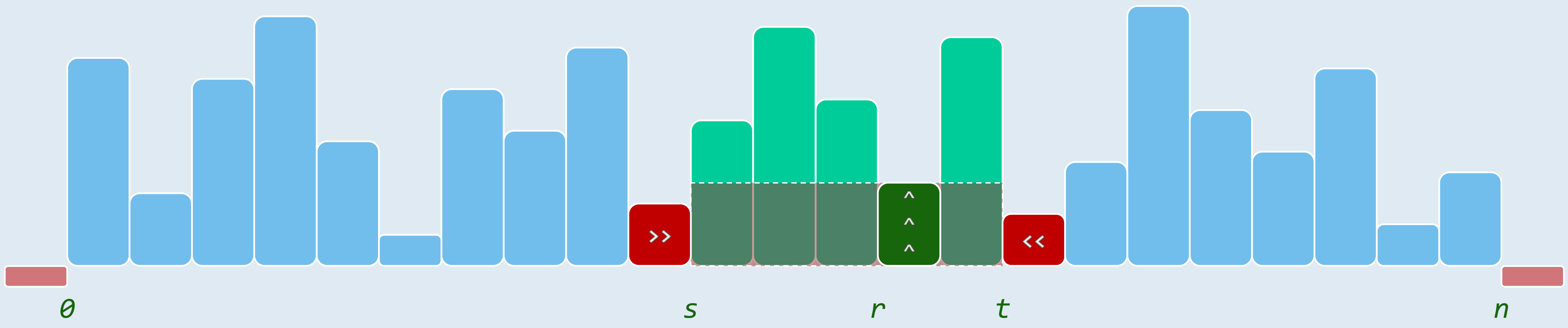
❖ Let H[0,n) be a histogram of non-negative integers

❖ How to find the largest orthogonal rectangle in H[]?

❖ To eliminate possible ambiguity

we can, for example, choose the LEFTMOST one

Maximal Rectangles



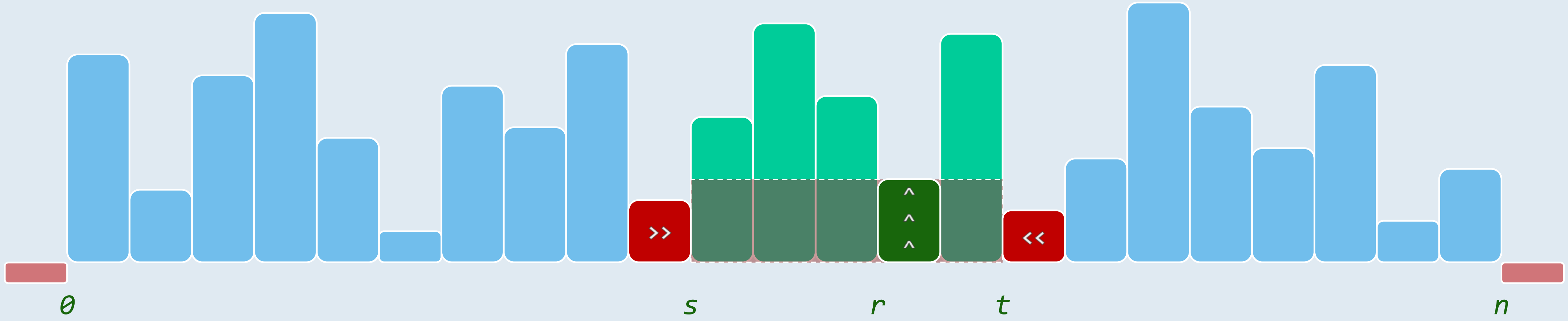
❖ **Maximal rectangle supported by H[r]:** $maxRect(r) = H[r] \cdot (t(r) - s(r))$

$$s(r) = \max\{\ k \mid 0 \le k \le r \text{ and } H[k-1] < H[r]\ \}$$

where

$$t(r) = \min\{\ k \mid r < k \le n \text{ and } H[r] > H[k]\ \}$$

Brute-force



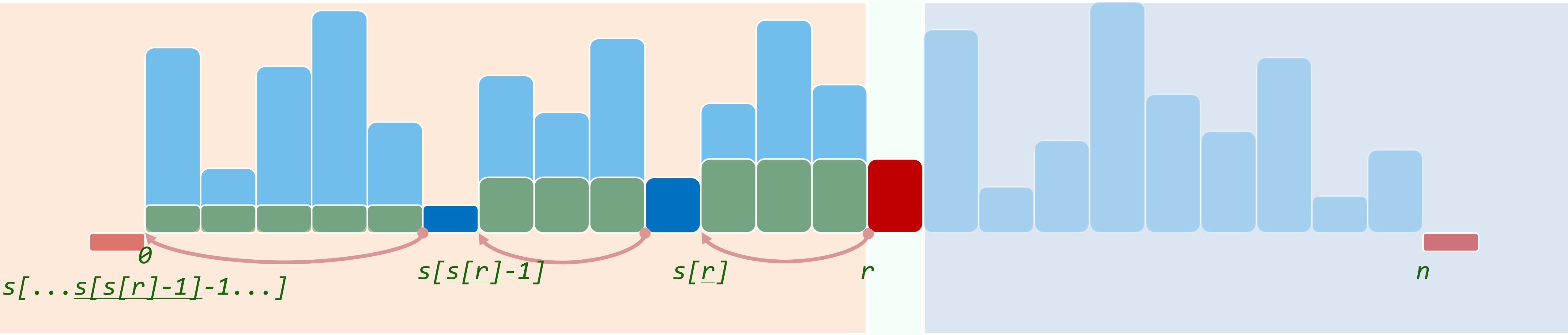
❖ Determining s(r) and t(r) for all r's requires $\mathcal{O}(n^2)$ time

$$s(r) = \max\{\ k \mid 0 \le k \le r \text{ and } H[k-1] < H[r]\ \}$$

$$t(r) = \min\{\ k \mid r < k \le n \text{ and } H[r] > H[k]\ \}$$

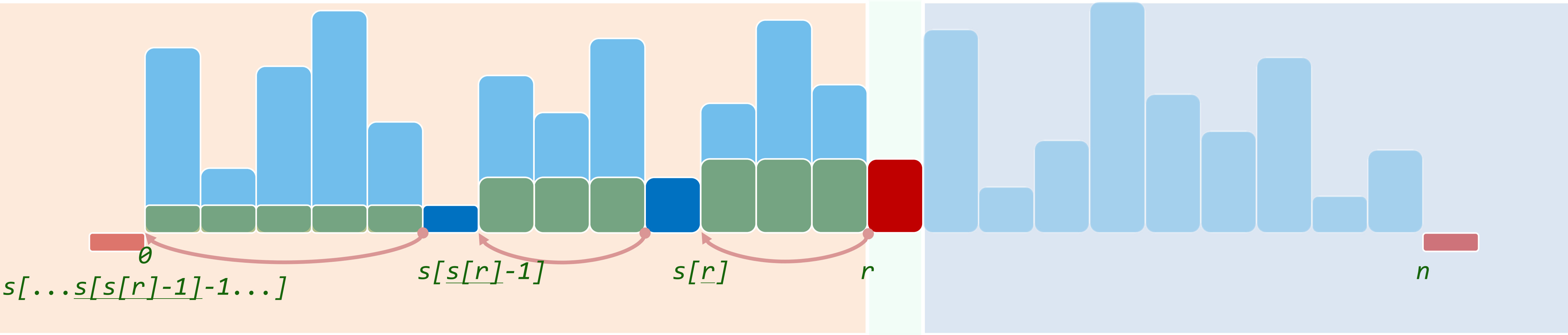
❖ Actually, all s(r)'s can be determined by a **LINEAR** scan of the histogram ...

Using Stack: Algorithm




```
Rank* s = new Rank[n]; Stack<Rank> S;
for ( Rank r = 0; r < n; r++ ) //try using SENTINEL for simplicity by yourself
   while ( !S.empty() && ( H[S.top()] >= H[r] ) ) S.pop(); //until H[top] < H[r]
   s[r] = S.empty() ? 0 : 1 + S.top();   S.push(r); //S is always ASCENDING
while( !S.empty() ) S.pop();
```

Using Stack: Loop Invariant & Correctness



- **All bars are scanned and pushed into S in turn**

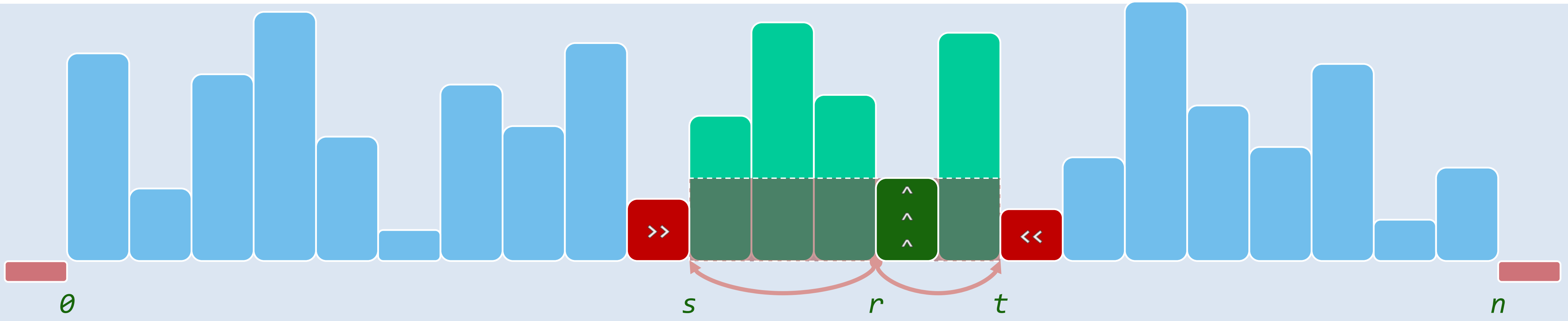
- **After each iteration of the outer loop, S stores a chain in terms of s[]**

  $S[S.size()-1] = S.top() = r$ and $\forall\, 0 \le k < S.size(),\ \ S[k-1]+1 = s[S[k]]$

- **Every bar is popped when it will be of no use thereafter, i.e.**

  $\forall\, 0 \le k < r \ \ but \ \ k \notin S,\ \ \nexists\, r \le t \ \ s.t.\ \ k+1 = s(t)$

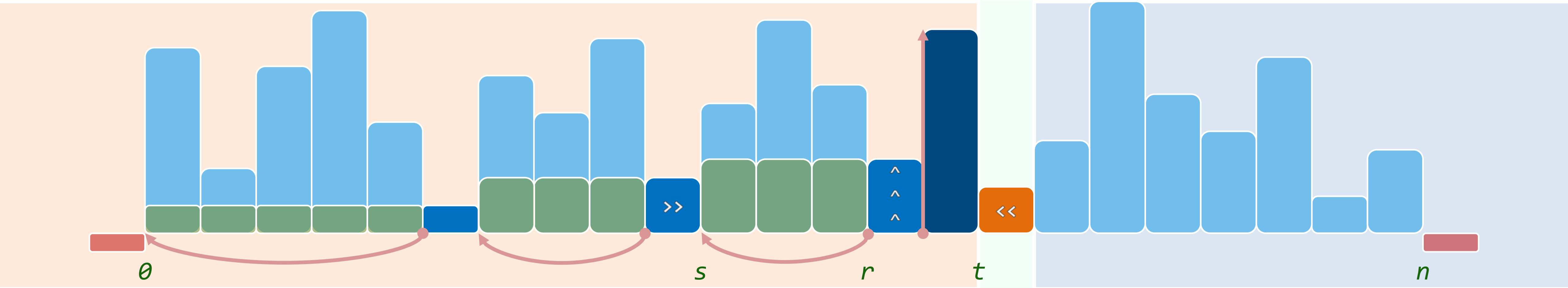
Using Stack: Complexity



- And t(r)'s can be determined by another scan in the **REVERSED** direction
- Hence all maximal rectangles can be computed in $\mathcal{O}(n)$ time and using $\mathcal{O}(n)$ space
- However, what if the histogram is given in an **IN-PLACE** and **ON-LINE** manner?

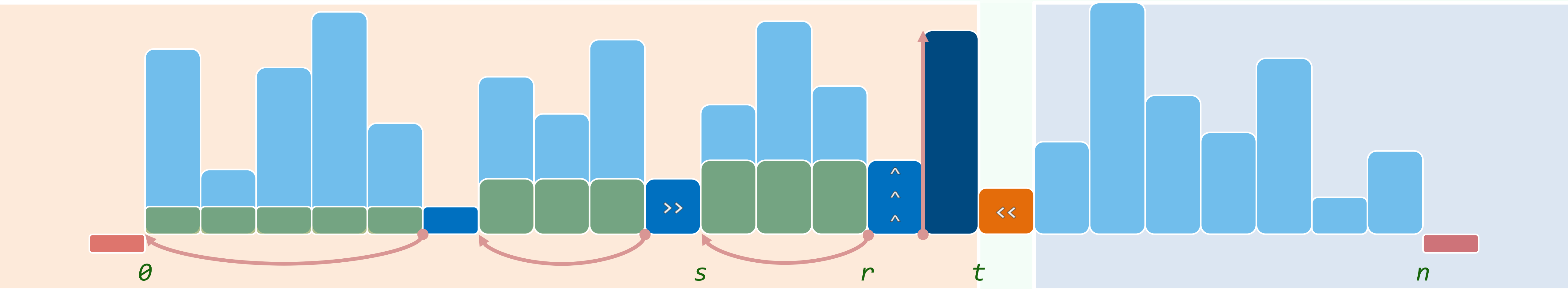
  Note that, the t(r)'s **CAN'T** be determined until the **ENTIRE** input is ready
- Is it possible to compute **BOTH** s(r)'s and t(r)'s by a **SINGLE** scan? //on-fly

One-Pass Scan: Algorithm




```
Stack<Rank> SR;  __int64 maxRect = 0; //SR.2ndTop() == s(r)-1 & SR.top() == r
for ( Rank t = 0; t <= n; t++ ) //amortized-O(n)
   while ( !SR.empty() && ( t == n || H[SR.top()] > H[t] ) )
      Rank r = SR.pop(), s = SR.empty() ? 0 : SR.top() + 1;
      maxRect = max( maxRect, H[r] * ( t - s ) );
   if ( t < n ) SR.push( t );
return maxRect;
```

One-Pass Scan: Loop Invariant & Correctness



- Again, at each iteration of the outer loop, we always have

$$\forall\, 0 \le k < SR.size(),\ \ SR[k-1]+1 = s[SR[k]]$$

- For each bar r popped in the inner loop, we have

$$t(r) = t \quad \text{and} \quad s[r] = SR.top() + 1$$